Griffiths singularity in the two dimensional random bond disordered Ising ferromagnet

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Abstract

For the two dimensional random bond disordered Ising ferromagnet, we measured bulk data of the magnetic susceptibility (χ) and correlation length (ξ) up to $\xi \simeq 536$, with the use of a novel finite size scaling Monte Carlo technique. Our data are exclusively consistent with normal power-law critical behaviors with only one singular point at criticality, disproving the existence of Griffiths singularity even in an extremely deep scaling region. The critical exponents of χ and ξ increase continuously with the strength of disorder.

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The critical behavior of the two-dimensional (2D) randomly disordered Ising ferromagnet is an outstanding problem in both condensed matter and statistical physics. By the random disorder is meant either a random site dilution or random- valued positive coupling. The effect of the fluctuations of the quenched random disorder on the critical behavior is of main interest. McCoy and Wu [1] made the first successful attempt for such a model, and proved that the specific heat (C_v) is non-divergent in the 2D Ising system with one-directional and correlated random bond disorder. According to McCoy and Wu, the divergence of C_v is caused by the coherence of all the bonds acting together, so destroying the coherence by introducing one or two directional bond disorders makes C_v finite.

Many authors, however, regarded the non-diverging behavior of C_v in the McCoy -Wu model as a characteristic of the one-dimensional correlated disorder. Especially, some authors, based on their observation that the continuum limit of the 2D random bond disordered Ising model is a certain type of Gross-Neveu model, predicted $\eta = 1/4$ [2] along with double-logarithmically diverging critical behavior of C_v [2] [3]. Namely, for weakly disordered 2D random bond disordered Ising (RBDI) ferromagnet it was predicted:

$$\xi \sim t^{-\nu}[1 + C\ln(1/t)]^{\tilde{\nu}}, \quad \nu = 1,\ \tilde{\nu} = 1/2 \tag{1}$$

$$\chi \sim \xi^{2-\eta}, \quad \eta = 1/4 \tag{2}$$

$$C_v \sim t^{-\alpha}\ln[1 + C\ln(1/t)] + C', \quad \alpha = 0 \tag{3}$$

with t representing the reduced (inverse) temperature. A heuristic criterion initially given by Harris [4] is reflected in these expressions; that is, a random quenched disorder is relevant only if α of the pure system is positive, and only in deep scaling region close to criticality.

Now the theoretical prediction of $\eta = 1/4$ has been confirmed by a different theoretical approach [5] along with by numerous numerical studies [6] [7] [8]. The predicted behavior of C_v, however, contradicts some other analytical results [9] [10] and recent extensive numerical work on the 2D randomly site diluted Ising system (RSDI) [8]. Various numerical methods [8] [11] [12] [13] report that the presence of strong random site dilutions actually changes the universality class; nevertheless, a conclusive numerical evidence lacks in the 2D random bond disordered Ising (RBDI) case where the effect of disorder is much weaker than in RSDI, albeit some numerical results supporting the scenario of the logarithmic correction [6] [14] [15].

Another important issue on the 2D RSDI was first addressed by Griffiths [16], who argued that magnetization as a function of external field (H) becomes non-analytic at $H = 0$ below a temperature nowadays referred to a Griffiths temperature (T_G). Being identified as the critical temperature of the fictitious pure system with the largest value of the coupling allowed in the random system, T_G is independent of the concentration of random dilution, and lies *above* the the critical temperature (T_c). Griffiths phase which refers to a range of temperature where Griffiths singularity takes place, i.e., $T_c \leq T \leq T_G$, however, prolongs with the degree of dilution.

Griffiths singularity has been elaborated and extended by many authors [17] [18]; especially, it is claimed that in Griffiths phase free energy is singular as a function of temperature as well at $H = 0$ [19]. Ziegler's [10] argument contrasts among others, however, in that only two singular points (not a Griffiths *phase*) exist in the 2D randomly disordered Ising ferromagnet at which both χ and ξ diverge but C_v does not.

Griffiths singularity is believed to be generic in any disordered system, including spin glasses [20] and random fields [21]. However, few numerical attempts to prove or to disprove the presence of Griffiths singularity were made, mainly because it is not clear how to treat Griffiths singularity in the context of either the standard finite size scaling (FSS) or Monte Carlo renormalization group method. In fact, the non-existence of Griffiths singularity is prerequisite for a reliable application of the Monte Carlo renormalization group method to a randomly disordered system [13].

In this Letter, we attempt to clarify the unresolved issues of the 2D RBDI ferromagnet by the MC measurements of the bulk data (thermodynamic data) up to an extraordinarily deep scaling region [22]. In general, with a knowledge of bulk data (thermodynamic data) in a deep scaling region all the necessary informations regarding a critical behavior can be gained in the most straightforward way. If Griffiths phase indeed exists, one expects to observe either a divergence or a discontinuity in the behaviors of $\chi(t)$ and $\xi(t)$ at temperatures other than criticality. The broad range of the bulk data is also necessary to test against the claims of some authors [15] [23] [24] that only some *effective* γ and ν increase with the strength of disorder.

Practically, a physical quantity measured on a finite system with a linear size of L at a t, say $P_L(t)$, converges to its thermodynamic value (bulk value), $P_\infty(t)$, under the *thermodynamic* condition $\xi_L/L \geq r$. The value of r which is independent of temperature is approximately 6 for the 2D pure Ising ferromagnet with periodic boundary condition [25], but increases with the strength of random disorder [8]. Consequently, with the use of traditional Monte Carlo methods it is prohibitively difficult to measure proper thermodynamic values at temperatures sufficiently close to a critical point.

Our evaluations of bulk values are based on a novel technique recently developed by Kim [26]. Using this technique, the bulk data can be extrapolated with MC data obtained on much smaller lattice. The technique is based on the fundamental formula of finite size scaling [26]. Namely, for a multiplicative renormalizable quantity P

$$P_L(t) = P_\infty(t) q_P(x), \quad x = \xi_L/L, \tag{4}$$

where P_L denotes P defined on a finite lattice of linear size L. Notice that here we use a scaling variable different from the traditional one, ξ_∞/L, and that $q_P(x)$ represents a universality class. Because of a correction to FSS mainly due to certain irrelevant operators, Eq.(4) is valid only for $L \geq L_{min}$, where $L_{min} \simeq 20$ for most models without crossover in the critical behavior.

The outline of the technique is as follows: The functional value of $q_P(x)$ is easily determined numerically for some discrete values of x, at a temperature where P_∞ including ξ_∞ is already known. Since $q_P(x)$ has no explicit temperature dependence, for a given x', the value of $q_P(x')$ at any other temperature can be interpolated based on these known values of $q_P(x)$. Once $q_P(x')$ is determined, P_∞ is easily computed from Eq.(4) by the measurement of P_L with the value of L corresponding to the x'. An interpolation at larger x' makes this technique more efficient. At the price of using modestly small values of L this technique requires very precise measurements; nevertheless, the cost of computer resources for the required precision is much less than simulations of huge lattice system.

Our Hamiltonian is defined as

$$H = -\sum_{<ij>} J_{ij} S_i S_j, \quad S_i = \pm 1, \tag{5}$$

where the sum is over all the nearest neigbors of lattice. J_{ij} is defined to be positive, e.g., taking either a positive valued J or J' randomly with probability p and $1-p$ respectively. For $p = 1/2$, the system is self-dual [27] with the self-dual point given by

$$\tanh(J) = \exp(-2J') \tag{6}$$

A self-dual point equals the critical point of a system, provided the system has only one critical point. We fix $J = 1$ and $p = 1/2$ without any loss of generality, and consider three different values of J', i.e., $J' = 0.9$, 0.25, and 0.1. Accordingly, the inverse Griffiths temperature, β_G, is equivalent to the critical point of pure Ising system, i.e. $\beta_G = \ln(\sqrt{2}+1)/2$, with the corresponding inverse self-dual points (critical points) given by $\beta_c = 0.4642819\ldots$, $0.80705185\ldots$, and $1.10389523\ldots$ for $J' = 0.9$, 0.25, and 0.1 respectively.

Our raw-data for each J' are obtained by choosing a realization of random J', then running Monte Carlo simulations in Wolff's one cluster algorithm with periodic boundary conditions; for each realization, measurements were taken over 10 000 configurations each of which was separated by 2-8 one cluster updatings according to auto-correlation times. The procedure is then repeated for different realizations of J'. The average over all the different realizations converges as the numbers of the random realization increase; basically this mean value of a physical quantity is something physically interesting. To achieve the necessary precision for our technique, the numbers of the different realizations we used are approximately 20 - 40, 150 - 250, and 300 - 600 for $J' = 0.9$, 0.25, and 0.1 respectively; yet, in general, the fluctuation among different realizations of the random disorder is more significant than the statistical error for a given realization. Consequently, we ignore here the latter in our calculation of the statistical error of a mean value. The largest value of L we used to extract our bulk values is just 240. The total computing time spent for this work amounts to more than 600 CPU hours in unit of CRAY YMP832 Supercomputer.

For $J' = 0.9$, 0.25, and 0.1 respectively, the ranges of our correlation lengths thus evaluated are over $5.7(1) \leq \xi \leq 536.0(8.1)$ ([5.7(1), 536.0(8.1)]), [5.8(1), 429.2(15.5)], and [5.0(2), 403.4(22.3)], corresponding to the range of β (inverse temperature) over $0.42 \leq \beta \leq 0.4638$ ([0.42, 0.4638]), [0.70, 0.805], and [0.87, 1.097]. We advertently choose the range of β such that $\xi(\beta) \geq 5$, so that the effect of certain nonconfluent correction to scaling can be unimportant in the fits. (See below.) Our bulk data are summerized in Figs.(1) and (2), which depict $\ln \xi(t)$ and $\ln \chi(t)$ as a function of $|\ln t|$ respectively. Note that the slopes in Figs.(1) and (2) respectively represent ν and γ.

The data show no sign of singular behavior such as a divergence or a discontinuity at the temperatures in the Griffiths phase, up to $t \simeq 1.1 \times 10^{-3}$. The almost perfect straight lines in such broad ranges of the bulk data, showing that both γ and ν do not change with t, clearly prove that they do not represent the effective values of the critical exponents but the asymptotic ones [28]. Thus, the data for each J' are exclusively consistent with normal power-law singularities having one and only one singular point at criticality. With the conspicuous differences in the slope, the values of ν and γ obviously increase with decreasing J'.

Fixing the critical points at the self-dual points in the χ^2 fits and assuming a pure power-law type critical behavior, we obtain $\nu = 1.00(0)$ and $\gamma = 1.75(1)$ ([1.00(1), 1.75(1)]), [1.09(1), 1.90(2)], and [1.23(2), 2.13(3)] for $J' = 0.9$, 0.25, and 0.1 respectively. Notice that $\eta = 2 - \gamma/\nu$ remains a constant i.e., $\eta = 1/4$ within the statistical errors, irrespective of the values of J'. Assuming a scaling function with a nonconfluent correction term, e.g., $\xi(t) \sim t^{-\nu}(1 + at)$, yields no significant change in the estimate of the critical exponent, e.g., $\nu = 1.075(7)$ for $J' = 0.25$.

For $J' = 0.9$ the estimated values of ν and γ are virtually the same as those for the pure system. Nevertheless, we do not take this as an evidence that the critical exponents are strictly the same in the two systems, due to the general tendency of increasing ν and γ with decreasing J'. Rather, it appears that the values of ν and γ increase so mildly for weakly disordered system ($J' \simeq 1$) that they are extremely hard to be distangled from those of the pure system.

Distinguishing a tiny increase of ν from a multiplicative logarithmic correction by comparing χ^2/N_{DF} values of the two fits is practically very difficult; accordingly, for $J' = 0.9$ and 0.25 the data fit to the Eqs.(1) and (2) as well as to the pure power-laws. For $J' = 0.1$, however, the ξ (χ) data fit to a pure power-law much better than to Eq.(1) (Eq.(2)): The values of χ^2/N_{DF} for the two fits of the ξ data are 4.9 and 0.7, respectively to Eq.(1) and to the pure power-law type. Thus, for a strong RBDI case at least, the scenario of the logarithmic correction is clearly unfavored by the data. Although for $J' = 0.9$ and 0.25 we are unable to determine the correct type of the critical behavior through the χ^2 fit, the universal scaling functions, $q_P(x)$ in Eq.(4), for the two values of J' are definitely different from each other, as shown in Fig.(3), indicating that they indeed belong to different universality classes.

According to the standard theory of FSS, the value of Binder's cumulant ratio [29] at criticality ($U_L(t=0)$) for a fixed geometry is another indicator of a universality class [30]. Our data of $U_L(t=0)$ in the range $20 \leq L \leq 100$ are tabulated in Tab.(1), showing that $U_L(t=0)$ for each J' is indeed independent of L within the statistical errors, and that $U_L(t=0)$ increases with decreasing J'. Note that $U_L(t=0)$ for $J' = 0.9$ are virtually the same as for the pure system, i.e., $U_L(t=0) = 1.832(1)$ [25], as ν and γ are for the same J'. Indeed, the behavior of $U_L(t=0)$ as a function of J' confirms our claim that ν and γ increase continuously with the strength of the disorder.

It is unlikely that the increase of $U_L(t=0)$ is owing to a logarithmic correction [14] for the following two reasons: (i) Notice that the value of $\tilde{\nu}$ in Eq.(1) is fixed regardless of the strength of the random disorder, i.e., only the non-universal constant C varies with it. Therefore, the continuous variance of $U_L(t=0)$ with respect to J' is improbable in the context of the scenario of the logarithmic correction. (ii) More crucially, it is observed [31] that $U_L(t=0)$ changes very mildly along the critical line of the Ashkin-Teller model along which ν varies continuously.

Some remarks are in order: (i) Because of the mild variation of the critical exponents, along with the hyperscaling relation, $\alpha = 2 - D\nu$, we expect that it would be very difficult to observe a finite peak of C_v in weakly disordered case. Notice, however, that this is the case even in the 3D (pure) XY model [32], where $\alpha < 0$. Moreover, in this model the specific heat at criticality increase with L monotonically [32], so that it can be fitted to a double logarithmic function. This is a clear demonstration that a mild increase of the specific heat at criticality, for some finite values of L, cannot be an evidence for its divergence at criticality [6]. (ii) According to Ziegler [33], his singular point manifests itself at the length scale $L \simeq 900$ for $J' = 0.25$. With our largest $\xi \simeq 429$ and with our observation that $r \simeq 8$ for $J' = 0.25$, our largest length scale amounts to $L \simeq 429 \times 8 \simeq 3430$, which is far beyond his threshold length scale. (iii) In the context of the renormalization group approach, the continuous variation of a critical exponent implies the existence of a line of the fixed points, as in the 2D Ashkin- Teller model. We conjecture that even in the 3D randomly disordered Ising ferromagnet the critical exponents would vary continuously, instead of the presence of an additional fixed point traditionally advocated [34].

To conclude: We have studied the critical behavior of the 2D RBDI ferromagnet with unprecedented extensiveness and precision, so that some conclusive results of its critical behavior are obtained. Our data unambiguously show that both ν and γ increase continuously with the strength of bond disorder. Any scenarios of Griffiths singularity are not respected by the data.

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TABLES

TABLE I. Binder's cumulant ratio at the self-dual points, for the three values of J'. Note that $U_L(t=0)$ for each J' does not vary with L within the statistical errors, thus showing that each self-dual point is indeed the critical point. Also, it is clear that $U_L(t=0)$ increases with decreasing J', although for $J'=0.9$ it is virtually the same as in the pure system.

	$J'=0.90$	$J'=0.25$	$J'=0.10$
L=20	1.834(1)	1.850(3)	1.862(3)
L=40	1.833(3)	1.847(3)	1.858(3)
L=60	1.832(1)	1.851(4)	1.854(4)
L=80	1.833(1)	1.849(3)	1.862(4)
L=100	1.832(2)	1.855(6)	1.858(4)

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Figure Captions

Fig.(1): $\ln\xi$ versus $|\ln t|$. The dotted lines represent the results of the best χ^2 fits assuming pure power-law type singularity. The values of the slope , which correspond to the values of ν, are 1.00, 1.09, and 1.23 respectively for $J' = 0.90$, 0.25, and 0.10. The data for the pure Ising model ($J' = 1$) are taken from the well- known exact formula, over the range $4.75 \leq \xi \leq 425.9$. Notice that they are precisely on the line of $\nu = 1$, showing that the correct scaling region already sets in at a temperature where $\xi \simeq 5$.

Fig.(2): $\ln\chi$ versus $|\ln t|$. Here the slope of each line corresponds to the value of γ, for each J'.

Fig.(3): q_P versus x. The upper two *curves* are for $P = \xi$, while the lower two are for $P = \chi$. The data on each curve actually represent data-collapse from different temperatures. (The detailed pictures will be presented elsewhere.) For each P, it is obvious that each J' characterizes a different curve, i.e., a different universality class.