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Absence of ageing in the remanent magnetization in Migdal–Kadanoff spin glasses

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Abstract. We study the non-equilibrium behaviour of three-dimensional spin glasses in the Migdal–Kadanoff approximation. This approximation is exact for disordered hierarchical lattices which have a unique ground state and equilibrium properties correctly described by the droplet model. Extensive numerical simulations show that this model lacks ageing in the remanent magnetization as well as a maximum in the magnetic viscosity in disagreement with experiments as well as with numerical studies of the Edwards–Anderson model. This result strongly limits the validity of the droplet model (at least in its simplest form) as a good model for real spin glasses.

Spin glasses are disordered magnets, which for low impurity concentrations above the Kondo regime display interesting non-equilibrium phenomena. In particular, a freezing of the dynamics appears at a temperature T_c, below which slow relaxation phenomena manifest through non-stationary effects in the zero-field-cooled magnetization. In this regime different non-equilibrium phenomena have been observed such as ageing, remanence and several memory as well as chaotic effects [1]. Despite the great activity devoted to understanding the nature of the low-temperature phase in three-dimensional spin glasses (for a review of numerical simulations, experiments and theory, see [2]) many questions regarding the ground state (e.g., its shape and its uniqueness) and the type of excitations still remain unanswered.

The mean-field picture for spin glasses [3] (i.e. the results obtained for the Sherrington–Kirkpatrick model), despite its great theoretical interest, it is not able to furnish a real-space picture of the type of excitations present in spin glasses. To fill this gap, and based on domain wall scaling arguments (initially proposed by McMillan and Bray and Moore), Fisher and Huse proposed what has been termed as the *droplet model* for spin glasses [4]. In the droplet model there are two unique ground states related by spin inversion symmetry. Thermal fluctuations activate droplets which are supposed to be compact domains of typical size L and fractal surface of dimension $d_s \geqslant d-1$. These excitations cost a free energy which grows as $\Upsilon(T)L^{\theta}$, where θ is a zero-temperature exponent and $\Upsilon(T)$ is a temperature-dependent stiffness constant. The idea that excitations in spin glasses are compact droplets is the simplest description that finds its most successful application in the study of phase transitions in ordered systems. Despite its inherent simplicity, the droplet model has a severe limitation, i.e. its main assumptions remain to be proven with a correct microscopic theory. If one of its key assumptions were wrong then the whole set of predictions emerging from the model would need to be revisited.

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From the dynamical point of view the main difference between mean-field-like and droplet pictures can be found in the behaviour of the ageing component of the remanent magnetization. In fact, in the ageing regime, even if both predict a dependence of the response function $R_{ag}(t, t_w)$ on the waiting time t_w, once the integrated response $M_{ag}(t, t_w) = \int_{t_w}^{t} \mathrm{d}s R_{ag}(t, s)$ is considered, different behaviours can be found in the limit $t, t_w \to \infty$, with t/t_w constant. In the droplet model, as in usual coarsening models, this limit is zero, while in spin-glass mean-field models it is finite. The presence of this *anomalous response* is one of the original features of the spin-glass mean-field models.

Now, which is the appropriate microscopic model that correctly describes the spin-glass transition? The simplest proposal was put forward by Edwards and Anderson almost 20 years ago [5], who introduced a random bond nearest-neighbour interaction model, the so-called Edwards–Anderson (EA) model. It is widely believed that the EA model is a real spin glass, i.e. it reproduces the majority of results found experimentally in the laboratory. So the question is whether the droplet model [4] is the appropriate theory to describe the phenomenology already contained in the EA model. Despite the large number of numerical works devoted to this question (see the reviews [6, 7]) there is still no universal agreement on this.

Our work has been motivated by recent results by Moore *et al* [8] who found that finite-size effects in the Migdal–Kadanoff (MK) approximation of the three-dimensional EA model are mean-field-like. In the thermodynamical limit the MK approximation is known to be described by the droplet model with $d_s = d - 1$ and $\theta \simeq 0.26$ [10, 11]. Consequently, they suggested that the droplet model could also explain the vast majority of numerical simulation results for the EA model obtained during the last decade (which, on the other hand, have been taken by the Rome group as evidence against the droplet picture). This is an interesting observation whose physical meaning and consequences need to be better understood—one should note that this observation was anticipated some time ago in a theoretical study of the one-dimensional spin-glass chain [9]. This controversy has centred around the study of the spin-glass equilibrium properties. Now it is time to check whether non-equilibrium behaviour is well reproduced by the droplet model. This is of the upmost importance because experimental measurements in spin glasses in the low-temperature regime are always taken in the out-of-equilibrium regime.

In this paper we wish to show that the MK model lacks one of the key features of real spin glasses found in the laboratory, i.e. ageing in the zero-field-cooled magnetization. Consequently, the physics contained in the MK approximation corresponds to the coarsening of a disordered ferromagnet being far from what is observed in real spin glasses.

The EA model in the presence of a field is defined by

$$\mathcal{H} = -\sum_{(i,j)} J_{ij}\sigma_i\sigma_j - h\sum_i \sigma_i \tag{1}$$

where the site indices run on the nodes of a cubic lattice, (i, j) stands for nearest-neighbour pairs, the spins take values $\sigma_i = \pm 1$ and the couplings are extracted from a Gaussian distribution of zero average and unit variance. Following [8] we will consider the three-dimensional EA model in the MK approximation which amounts to considering a hierarchical lattice that is constructed iteratively by replacing each bond by eight bonds as indicated in figure 1. Denoting by g the number of generations then the total number of bonds is 8^g, which corresponds to the number of sites for a cubic lattice with lattice size $L = 2^g$.

The order parameter can be defined through the equilibrium autocorrelation function,

$$q_{\mathrm{EA}} = \lim_{t\to\infty}\lim_{V\to\infty} \frac{\sum_{i=1}^{V} x_i \langle\sigma_i(t)\sigma_i(0)\rangle}{\sum_{i=1}^{V} x_i} \tag{2}$$

where $V = 8^g = L^3$ is the volume, and the averages $\langle\cdots\rangle$ are taken over dynamical histories starting from different equilibrium initial conditions at time zero. The x_i parameters are

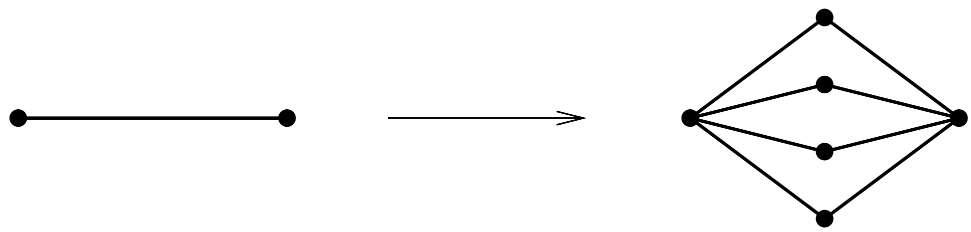

Figure 1. Elementary step in the construction of the hierarchical lattice, where the MK approximation is exact.

weights which take into account the fact that a given site is connected with a different number of bonds depending on its generation level (i.e. depending on which iteration in the recursive construction of the lattice that site was generated). Our results concentrate on the choice $x_i = 1$, i.e. all sites are identically weighted. However, in addition, the results obtained with $x_i = c_i$, where c_i is the connectivity of site i (so all bonds are identically weighted), corroborate our conclusions†.

We have concentrated our attention on the study of the relaxational dynamics in the low-temperature phase $T < T_c \simeq 0.88$ [10], below which $q_{\rm EA}$ defined in equation (2) differs from zero. We have used Monte Carlo dynamics with the Metropolis algorithm and random updating‡. In our runs we follow typical ageing-experiment scheduling: that is, at $t = 0$ we quench the system from infinite temperature to a finite one $T < T_c$ without the magnetic field, letting the system evolve for a time t_w. At time t_w we switch the magnetic field on. For subsequent times ($t > t_w$) the system continues to relax in a magnetic field h and we then measure the following two quantities: (a) the autocorrelation function

$$C(t, t_w) = \frac{\sum_{i=1}^{V} x_i \sigma_i(t)\sigma_i(t_w)}{\sum_{i=1}^{V} x_i} \tag{3}$$

and (b) the zero-field-cooled susceptibility defined by

$$\chi_{\rm ZFC}(t, t_w) = \lim_{h\to 0} \frac{M_{\rm ZFC}(t, t_w)}{h} \tag{4}$$

where

$$M_{\rm ZFC}(t, t_w) = \frac{\sum_{i=1}^{V} x_i[\sigma_i(t) - \sigma_i(t_w)]}{\sum_{i=1}^{V} x_i}. \tag{5}$$

The limit in equation (4) is usually ignored, because we always work in the linear response regime. All the data we present have been obtained with a magnetic field of intensity $h = 0.1$ and we have checked that the same susceptibility is obtained by doubling the perturbing field.

We have performed extensive numerical simulations for $g = 5$ ($L = 32$) and $g = 6$ ($L = 64$) at two different temperatures ($T = 0.7$, 0.5) and many values of t_w. We obtain the same results for both temperatures. Here we present only those for $T = 0.7$, while the complete set of data will be reported elsewhere. Note that the ratio, $T/T_c \simeq 0.8$, used here is very similar to that used in many experiments [12].

† We think that this way of giving the same weight to all the links is quite unnatural. A better way to achieve it is to consider observables that only depend on the links, such as the link–link correlation function $C_{\rm link}(t) = N_{\rm link}^{-1} \sum_{(i,j)} \langle \sigma_i(t)\sigma_j(t)\sigma_i(0)\sigma_j(0)\rangle$, which tends to the link overlap $q_{\rm link}$ in the limit of equation (2). In this case the natural perturbing field would be a little change in the interaction strength: $J_{ij} \to J_{ij} + h\epsilon_{ij}$. In addition, in this case the results are in agreement with our conclusions.

‡ In each time step every spin is updated c_i times.

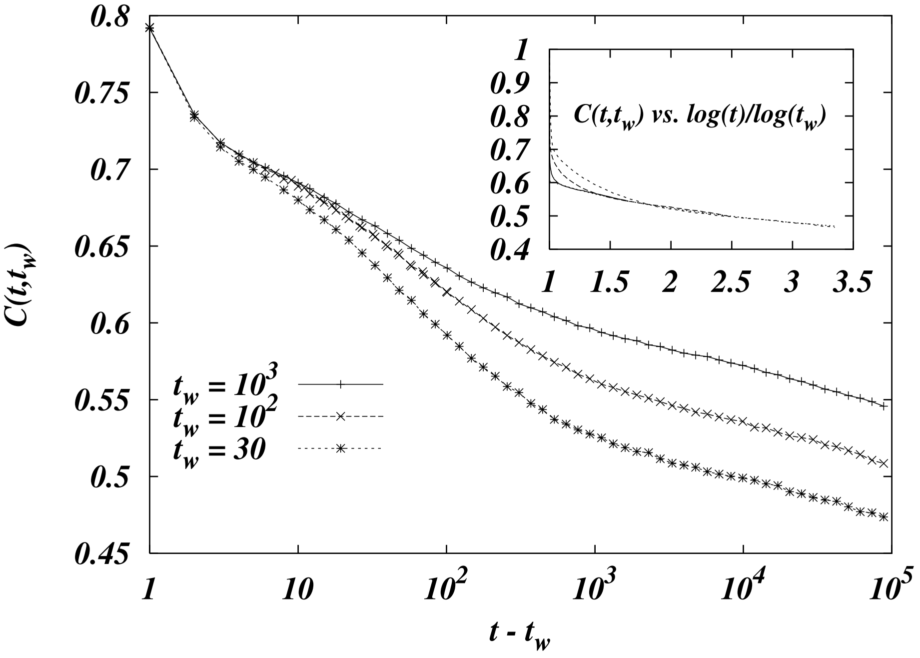

Figure 2. The correlation functions for $g = 6$, $T = 0.7$ and different t_w clearly show ageing. The inset shows the same curves as a function of the scaling variable $\log(t)/\log(t_w)$.

In figure 2 we show the autocorrelation function for different values of t_w. One can observe the presence of ageing characteristics of many glassy systems.

Following general assumptions, in the asymptotic regime $t_w \to \infty$, the correlation function decomposes into two terms, each one governing a different time regime. In the quasi-equilibrium regime, $t - t_w \ll t_w$, the system is in some sort of local equilibrium and the correlation functions are time-translationally invariant. In the ageing regime $t - t_w \gg t_w$, the system ages and the correlation functions depend on both times through non-trivial scaling relations. So in general, one can write [13]

$$C(t, t_w) = C_{st}(t - t_w) + C_{\mathrm{ageing}}(t, t_w) \tag{6}$$

with $\lim_{\tau\to\infty} C_{st}(\tau) = q_{\mathrm{EA}}$. In equilibrium the ageing part vanishes and one recovers the previous result of equation (2).

The difference between the experimental data and the EA model on one hand, and the MK approximation and the droplet model on the other, is the large-times scaling of the dynamical functions. As can be seen in the inset of figure 2, in the MK approximation we find that the ageing part of the autocorrelation function is well described, in the large-times limit, by a function of the ratio $\log(t)/\log(t_w)$. On the other hand, in experiments and in the EA model,the scaling is far from the $\log(t)/\log(t_w)$ and similar to t/t_w.

Because of the use of the scaling variable, in the inset of figure 2 the data corresponding to the quasi-equilibrium regime collapse on the line $\log(t)/\log(t_w) = 1$. Thus we can estimate the value of q_{EA} as the limit of the scaling function for $\log(t)/\log(t_w) \to 1^+$, i.e. $q_{\mathrm{EA}} \simeq 0.6$ (a value compatible with [8]).

Our most striking result is found for the zero-field-cooled susceptibility $\chi_{\mathrm{ZFC}}(t, t_w)$ shown in figure 3. In the MK approximation there is no dependence of the susceptibility

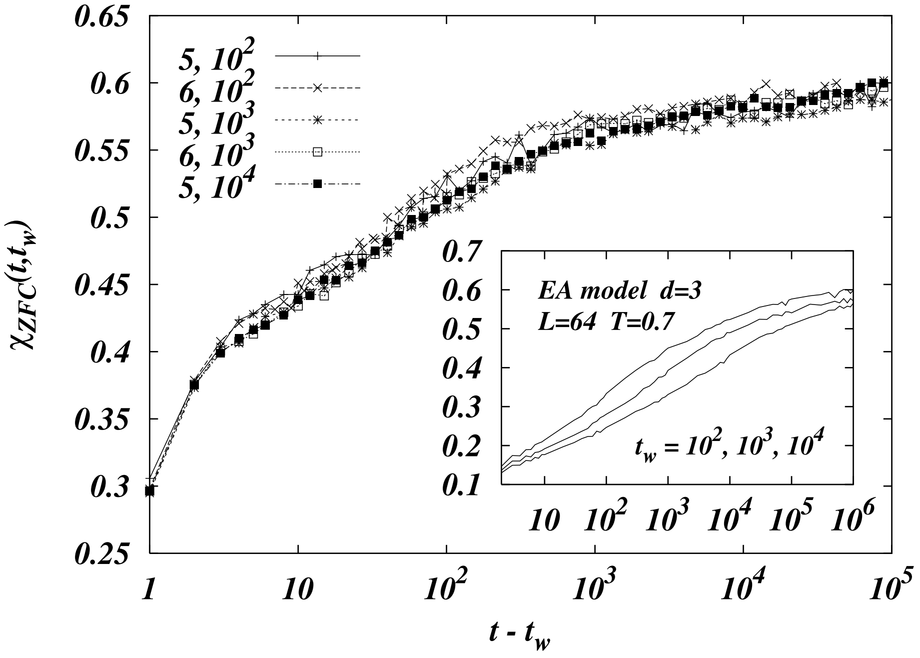

Figure 3. The zero-field-cooled susceptibility data do not show any ageing. In the legend we report the values of g and t_w. In the inset we show the same quantity measured in the EA model, which closely resembles the experimental data.

on t_w. We believe that such a result, which is characteristic of droplet models and kinetic growth [14], makes the droplet model, at least in its simplest form, inadequate for the description of the EA one. Ageing in both zero-field-cooled and field-cooled magnetization is so commonly found in experiments on spin glasses that it is not clear to us how this result can be explained by the standard droplet theory. Note also that the peak in the magnetic viscosity $S(t, t_w) = \partial \chi_{\mathrm{ZFC}}(t, t_w)/\partial \log(t - t_w)$ (experimentally very well observed [15]) is completely absent in the MK approximation. We recall that ageing in $\chi_{\mathrm{ZFC}}(t, t_w)$, with a peak in the $S(t, t_w)$, is naturally found in the EA model (see the inset of figure 3) as well as in mean-field models. It then remains to be explained why these ageing effects are naturally and easily observed in the EA model and not in the MK approximation.

Finally, we consider the analysis of the fluctuation–dissipation ratio which is useful to compare the results obtained in the MK approximation with those obtained in the EA and coarsening models [16, 17]. In the quasi-equilibrium regime ($t - t_w \ll t_w$) the system is in local equilibrium. Consequently, both correlation and susceptibility are time-translationally invariant and the fluctuation–dissipation theorem (FDT) is satisfied,

$$T\chi_{\mathrm{ZFC}}(t - t_w) = 1 - C(t - t_w). \tag{7}$$

In the ageing regime ($t - t_w \gg t_w$) the system ages and FDT is violated. Then it is useful to define the so-called fluctuation–dissipation ratio [16]

$$X(t, t_w) = \frac{T R(t, t_w)}{\frac{\partial C(t,t_w)}{\partial t_w}} \tag{8}$$

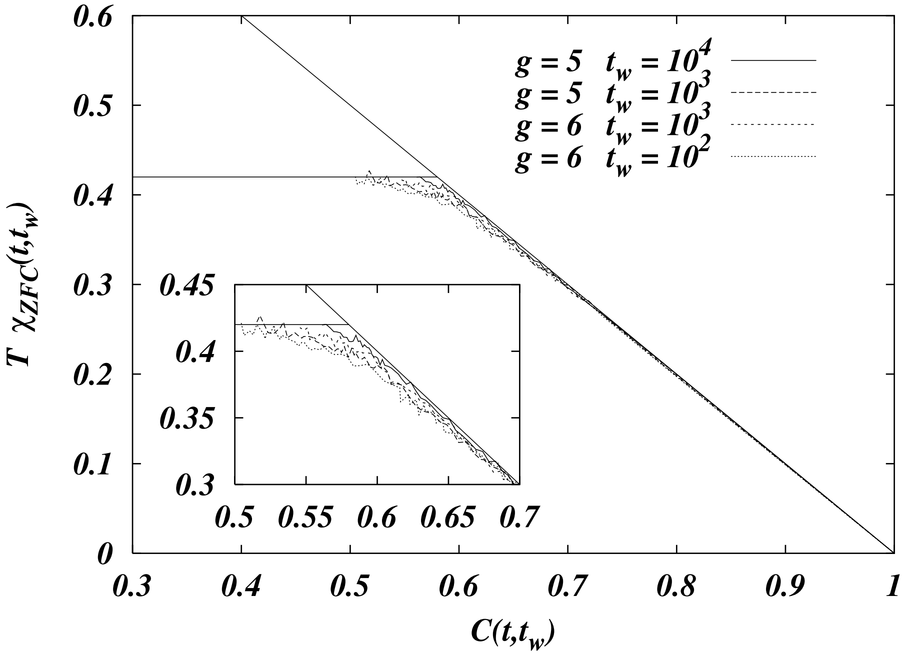

Figure 4. The absence of response in the ageing regime is suggested by the rapid convergence of the χ_{ZFC} versus C curves to the plotted lines. Note that the horizontal line denotes an upper bound for the data.

which in the asymptotic long-time limit $t, t_w \to \infty$ may be uniquely expressed as function of the correlation $C(t, t_w)$ yielding

$$T\chi_{\mathrm{ZFC}}(t, t_w) = \int_{C(t,t_w)}^{1} X(C)\,\mathrm{d}C. \tag{9}$$

Moreover, the $X(C)$ can be related to equilibrium quantities [18]. The previous expression reduces to equation (7) in the quasi-equilibrium regime where $X = 1$. A plot of $T\chi_{\mathrm{ZFC}}(t, t_w)$ as a function of $C(t, t_w)$ is expected to show two different behaviours. For $q_{\mathrm{EA}} < C < 1$ we have $X = 1$ and so the curve $T\chi_{\mathrm{ZFC}}$ versus C has slope -1. For $C < q_{\mathrm{EA}}$ the X may be a non-vanishing function of C and we have $T\chi_{\mathrm{ZFC}}(t, t_w) = (1 - q_{\mathrm{EA}}) + \int_{C(t,t_w)}^{q_{\mathrm{EA}}} X(C)\,\mathrm{d}C$. In coarsening models, $X = 0$ for $C < q_{\mathrm{EA}}$ and so the function $\chi_{\mathrm{ZFC}}(C)$ is flat for $C < q_{\mathrm{EA}}$. In figure 4 we show χ_{ZFC} as a function of C for different values of g and t_w, which show that the behaviour rapidly converges to that of coarsening models and strongly differs from that observed in finite-dimensional EA spin glasses [17]. The horizontal line in figure 4 shows the infinite time limit of the susceptibility, extrapolated from the data of figure 3 and from those for the field-cooled magnetization (not shown). It is an upper bound for the plotted curves, thanks to the positiveness of the X ratio. From figure 4 we can also get an estimate for the q_{EA} order parameter, defined as the abscissa value where the curves leave the FDT line ($T\chi_{\mathrm{ZFC}} = 1 - C$). This point is converging very reasonably in the large-times limit, near to the intersection of the two lines, giving $q_{\mathrm{EA}} \simeq 0.6$ (as already found from the data of figure 2). Figure 4 adds more evidence to support the fact that spin glasses in the MK approximation do not capture all the key features of finite-dimensional spin glasses as we understand them from the three-dimensional EA model.

To summarize, we have shown that in the MK approximation spin glasses do not show ageing in the integrated response function. This ageing is experimentally observed in real spin glasses through zero-field-cooled and field-cooled measurements and constitutes one of the key features which distinguishes spin glasses from other disordered systems. In addition, the study of the fluctuation–dissipation ratio suggests that relaxation in this model is driven by coarsening similar to conventional ferromagnets [14, 17].

In our opinion, the trivial dynamics observed in this model is mainly due to the lack of strong frustration in the hierarchical lattice. Even if locally there is some frustration, the spin-glass ground state on the hierarchical lattice can be calculated in polynomial time and this would suggest the existence, on large scales, of only one phase dominating the dynamics. Then, the relaxational dynamics should not greatly differ from the dynamics taking place in a slightly frustrated coarsening model. In contrast, in spin-glass dynamics the number of competing phases is very high and this competition generates a very rich dynamical behaviour, e.g. ageing in the zero-field-cooled magnetization. In the limit of zero temperature, the existence of many phases is reflected by the presence of a large number of almost degenerate configurations, thus making the ground state search a very difficult problem.

One could argue that these results for the MK approximation are not extrapolable to the droplet model because, in the general case, the inequality $d_s \geqslant d - 1$ could restore ageing. Despite this possibility our results unambiguously show that the MK model is not a good model for realistic spin glasses. A new class of excitations or droplets must be present in spin glasses. The droplet model in its simplest version does not capture the physics behind real spin glasses. One possible generalization of the droplet model (which would no longer be simply droplet) is to consider two kinds of basic excitations in a spin glass: on small length scale the usual droplets and, in addition, system-size scale collective rearrangements [19]. The second kind of excitations are, at present, ignored in the droplet model (they are exponentially rare), but they could be responsible for the many mean-field-like features observed in finite-dimensional spin glasses. In terms of a very simplified energy landscape, the two excitations would correspond, respectively, to the local movements of the system in a single ‘valley’ and to the jumps from one valley to an other. In our opinion, a new theory comprehensive of the small-scale droplets and the system-size scale excitations (with a clear real-space picture) would be welcome and could hopefully terminate the longstanding discussion on finite-dimensional spin glasses.

Acknowledgments

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