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Aging, memory and rejuvenation: some lessons from simple models

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Abstract. Many recent experiments probed the off equilibrium dynamics of spin glasses and other glassy systems through temperature cycling protocols, and observed memory and rejuvenation phenomena. Here we show through numerical simulations, using powerful algorithms, that such features can already be observed to some extent in simple models such as two dimensional ferromagnets. We critically discuss these results and review some aspects of the literature in the light of our findings.

1. Introduction

One of the main field of research in ill-condensed matter over the last few years was certainly the off equilibrium dynamics of glassy systems. These studies led to the emergence of satisfying pictures and useful concepts that now allow a good qualitative understanding of many experimental facts, such as aging [1, 2]. For instance, if one quenches a glassy system from its high temperature phase to its low temperature phase, this system will *age* and the longer the experimentalist will wait, the slower the system will be (which is indeed what aging is in real life). To be more precise, after a quench at $t = 0$, an observable like the magnetic susceptibility under a field applied at time t_w —the so-called waiting time— will typically decay following a scaling function $f(t/t_w)$. This generic picture of aging is now well documented and quite ubiquitous, being observed in many experimental and theoretical situations [1, 2]. However, one of the most striking feature in the dynamics of these systems, which is not well taken into account so far, is certainly their dependence to the complete history, so that more complex procedures than a simple quench are of great interest. Indeed, following the early seminal work of Struick and Kovacs [3], a number of more elaborate experiments have been performed in a wide class of glassy materials such as polymers [4], colloidal suspensions under a shear [5], disordered or frustrated magnets [6, 7] or, for what will matter here, spin glasses [8, 9, 10]. Interesting and impressive hysteresis effects have been observed; they are commonly referred to as *memory* and *rejuvenation*.

Let us briefly discuss these effects in the context of spin glasses (and refer for instance to [1, 8, 9, 10, 11] for a more exhaustive description). In standard experiment, a temperature cycle is performed: a system (with a glass transition at T_g) is at first quenched from its high temperature phase to $T_1 < T_g$ and then kept at this temperature for a while, before being cooled again to $T_2 < T_1$. After another time interval, it is brought back to T_1. Two striking effects are observed. 1) As the system is brought to T_2 its dynamics witnesses a large restart, although it looked almost equilibrated at T_1; in particular its susceptibility is initially much larger than what would be after the same time in a direct quench at T_2. In the aging phenomenology a system that responds more is *younger* thus the name *rejuvenation* for this effect that has been observed in many materials [4, 5, 8]. 2) After the stage at T_2, when brought back to T_1, the system may behave (depending on the material and/or the parameters of the experiment) as if the temperature cycle has not been done at all and its susceptibility seems just to follow the T_1 curve from where it was left in the first stage at T_1. In AgMn spin glass for instance [11], there are no differences (apart from a short transient) between the susceptibility obtained for a long quench at T_1, and the one obtained in a temperature cycle if one just removes by hand all the data corresponding to the time spent at T_2. This is called the *memory* effect as the system, despite its rejuvenation, remembers how it was when it left T_1.

It is fair to say that we are still far from a complete theoretical understanding of these two effects and their co-existence, apart from simple phenomenological descriptions [9, 11, 10, 12, 13, 14]. Numerical simulations would be of great help in the understanding of this problem, but at the moment they have produced a number of contradictory claims: while some authors [11] advocate also for off-equilibrium typical configurations the presence of a property called temperature chaos [15] (equilibrium configurations at different temperatures are completely reshuffled for sufficiently large systems), others claim to observe these effects [13, 16] in the absence of any chaos, a conclusion that has also been challenged [14]. It was even provocatively asked if the Edwards-Anderson spin glass model was able to reproduce experimental findings [12], or if other models would be more appropriate [17]. Many questions were raised by the interpretation of numerical simulations, and in this situation it is natural for a physicist to come back to what he knows best: the ferromagnetic models we simulated in our early courses. Doing such quenches and T-cycling simulations in the $2d$ Ising and XY models will indeed provide some interesting lessons [18] as we shall now discuss.

2. Models and methods

We consider T-cycle experiments ($T = \infty \to T_1 < T_g \to T_2 < T_1 \to T_1$) in Monte Carlo (MC) simulations. We use large system sizes (typically $L \approx 10^3$) to avoid finite size effects and equilibration. We consider two models defined on a $2d$ square lattice: the first has Ising spins and Hamiltonian $H = -\sum S_i S_j$, and the second has 2-component vector spins of unit length and Hamiltonian $H = -\sum \vec{S}_i \vec{S}_j$, where the sums act on neighboring spins. While the Ising model undergoes a standard second order ferromagnetic transition at T_c , the XY model possesses a remarkable quasi-ordering characterized by a line of critical points going from $T = 0$ up to a transition temperature T_{KT} [19]. Finally, we briefly discuss finite dimensional spin glasses with Gaussian random couplings (where we use, for $4d$, $L \approx 20$). We express all temperatures in units of T_c or T_{KT} and consider Glauber as well as Kawasaki dynamics.

A few words on the numerical methods used in this work. So far simulations computed magnetic susceptibilities from correlation functions, assuming the validity of the Fluctuation-Dissipation Theorem (FDT); we will see that this can be sometime quite dangerous at short times, when the system is still strongly out of equilibrium. Instead, we used a generalization of a recently proposed algorithm to compute directly the linear response to a (DC or AC) magnetic field without *physically* putting the field [20, 21]. We will unfortunately skip here these quite technical, but important, points (addressing the reader to a more detailed paper [18]) and instead will focus on the results.

3. Effective temperatures in coolings and heatings

Let us start by few remarks on coolings from a initial high temperature T_i to a final lower one T_f and on heatings from a low temperature T_i to a higher one T_f. We concentrate on the $2d$ XY model (that will be useful later on) where some analytical results can be obtained in both cases, when the system is initially equilibrated at T_i (under the so-called *spin waves* *approximation* [22]). One can show that the correlation between a configuration at times t_w and $t > t_w$ scales [22] as

$$C(t, t_w) \propto (1/t_w)^{\frac{\eta(T_f)}{2}} \left(1 + \frac{1}{4\frac{t}{t_w}(\frac{t}{t_w}+1)}\right)^{\frac{\eta(T_f)-\eta(T_i)}{4}} , \qquad (1)$$

where $\eta(T)$ is a critical exponent, roughly proportional to the temperature T [22, 23]. The use of the FDT allow us to estimate the magnetic susceptibility at time t to a field applied at time t_w or, more conveniently, the susceptibility under an oscillatory field of frequency $\omega \approx 1/t_w$: roughly $\chi(\omega, t) \approx (1 - C(t + 1/\omega, t))/T$. Using expression (1) we see that starting from a high temperature T_i and cooling down to $T_f < T_i$, then $\eta(T_f) < \eta(T_i)$ and therefore $C(t + 1/\omega, t)$ increases with t. As a consequence, when the system ages one observes that its susceptibility decreases towards its equilibrium value, as it is well known. However, when heating from a low temperature T_i to $T_f > T_i$ one has now $\eta(T_f) > \eta(T_i)$ and $C(t+1/\omega, t)$ then decreases with t and thus the susceptibility increases with t; in this case one observes a kind of *inverse aging* where the system is initially too correlated for the new temperature T_f, so that it has to uncorrelate with time.

At short times after a change of temperature from T_i to T_f the system is strongly out of equilibrium and the FDT is violated, so that an effective temperature T_{eff} [24] can be defined. Computing it in the Langevin formalism we found [18]

$$T_{eff} = T_f \left(1 + \frac{1}{\omega t}\frac{T_i - T_f}{T_f}\right) = T_f \left(1 - \frac{1}{\omega t}\right) + T_i \frac{1}{\omega t} . \qquad (2)$$

This shows that, although $T_{eff} = T_f$ at large time (where FDT is valid), at shorter time, when $t = \mathcal{O}(1/\omega)$, T_{eff} is a weighted average of T_i and T_f. All of that is in fact completely general and we will see from our data that this scenario holds equally well for ferromagnets and for spin glasses: the moral of this story is that FDT overestimates the real susceptibility in coolings, and underestimates it in heatings.

4. Temperature Cycle experiment in Ising model

We now turn our attention to the numerical data in Fig.1(a), obtained from a T-cycling simulation in the $2d$ Ising model with Glauber dynamics. A clear restart of the aging dynamics (a rejuvenation) is observed when cooling from $T_1 = 0.8$ to $T_2 = 0.4$, while no memory effect is seen (going back to T_1, the susceptibility is lower than the last point in the first stage at T_1). Apart from this lack of memory (that we will discuss in the next paragraph), this looks amazingly, and perhaps surprisingly, similar to the curves obtained in cycling simulations of spin glasses [13, 16]; this rises the following questions.

First, should we be really surprised? After all, the temperature is changed so that *something* has to happens as the system tries to equilibrate in the new environment. Yet, it is easy to show in a simulation that this equilibration will be very fast, and hardly observable, if we would start from a state with spatially homogeneous magnetization. However, we know that after a quench the system is far from equilibrium. It is composed by domains of positive and negative magnetizations separated by interfaces —or domain walls— that grow with time [23]. While the bulk part of domains is indeed equilibrating almost instantaneously to the new temperature, the interface itself needs more time to re-equilibrate and this is the origin of the signal observed in Fig.1(a). The reader then may find that this is a bit a trivial effect. How could this looks so impressive in Fig.1(a)? We even observe that the susceptibility estimated by the FDT is initially *larger* at T_2 than at T_1, and we would thus be tempted to think that this is the sign of a very strong rejuvenation effect (as it is sometimes claimed in the literature [16, 25]). This is not

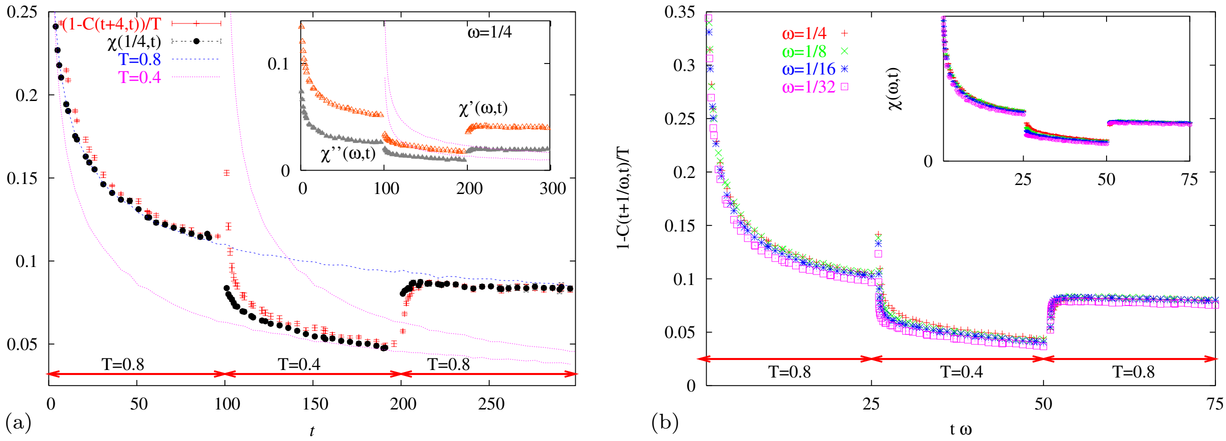

Figure 1. T-cycling experiments in the $2d$ Ising model with Glauber dynamics.
(a) Susceptibility at time $t + 1/\omega$ under a DC field applied at time t, by assuming FDT, i.e. $(1 - C(t + 1/\omega, t))/T$, and using the exact algorithm. The inset shows χ' and χ'' obtained with the exact algorithm for AC field of frequency ω. A transient rejuvenation effect is observed upon cooling but no memory (full lines are data for direct quenches to T_1 and T_2). (b) Same data for larger time scales, i.e. smaller ω, as a function of the rescaled time $t\,\omega$, using FDT (main plot) and the exact algorithm (inset). As all times are rescaled rejuvenation vanishes fastly in the exact susceptibility while a transient signal is still present assuming FDT.

completely true. Indeed, it is possible to show [18] that at large $\beta = 1/T$, while the asymptotic equilibrium susceptibility χ_{bulk} behaves as $\beta e^{-\beta}$, the aging part χ_{dw} due to the domains walls scales as β, essentially because there are spins in zero local field on these walls (this is in fact nothing else than the division by T in the dynamic FDT formula). χ_{dw} is therefore very sensitive to T-changes so that $\chi(T_2)$ can be made arbitrary high by lowering T_2, while nothing really changes in the physics of the system. The relative high of the susceptibility at different temperatures is therefore not a very good measure for a restart of the dynamics. Finally, we can check that, as predicted in the last section, we strongly overestimate (*resp.* underestimate) the early time regime of the susceptibility upon cooling (*resp.* heating) using the FDT, which therefore enhances artificially the rejuvenation effect (a comment also made in [16]). The reason for that can be easily understood: when a given spin is strongly out of equilibrium (for instance when the temperature is changed), it will be forced to flip and this will affect the correlation function. However, the susceptibility is only sensitive to flips due to thermal fluctuations, not to those driven by off-equilibrium relaxation: this is the origin of the large discrepancy between the FDT approximate and the exact susceptibility [18].

Still, a signal is observed in the exact susceptibility, so that the puzzled reader may rightfully ask why then is there no rejuvenation in *real* ferromagnets? First of all, this is not completely true as rejuvenation is observed in some particular class of frustrated magnets [6], but the answer to this question is that before claiming any experimental relevance one has to do numerical simulations on the same time scales than experiments, i.e. in the $t \to \infty$ limit. As can be seen in Fig.1(b), the rejuvenation signal tends to vanish if one rescales all time scales by the period P of the oscillating field and send $P \to \infty$ (because the time needed to re-equilibrate interfaces is finite). Again, notice that while this is clear on the correct susceptibility, the approximate one still reports a misleading remaining signal in the $P \to \infty$ limit. Observing a plot such as the one in Fig.1(a) is thus meaningless without a systematic large time study. These points obviously weaken many conclusions that have been obtained from simulations so far.

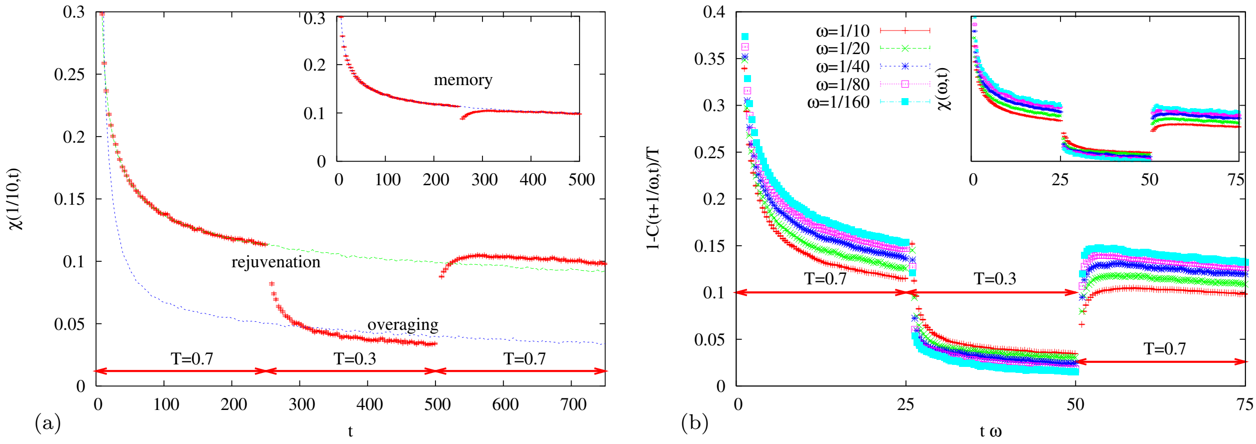

Figure 2. Similar plots to Fig.1 but for Kawasaki dynamics. (a) Exact DC susceptibility in a T-cycle; memory and rejuvenation are both observed (in the inset we remove the part at T_2 to show better memory). (b) As for Glauber, the rejuvenation signal vanishes at larger time scales, although in a much slower way.

5. The magic of 2d Kawasaki dynamics

While we observed a (transient) rejuvenation in Fig.1(a), memory is lacking. This is easily understood: the coarsening dynamics is almost T-independent, domains grows as $\sqrt{t}$ at any low temperatures, and thus the susceptibility measured coming back at T_1 is lower than the one the system had when it first left T_1: essentially there are much less domain walls! If, however, the coarsening at $T_2 < T_1$ is *much slower*, so that the density of interface would not decreases too fast, we may expect a good memory effect.

This can be achieved easily just by switching to Kawasaki dynamics. While it is well known that domains then coarsen as $t^{1/3}$ at large times [23], it has been shown recently that, due to initial moves that requires thermal activation, the Kawasaki dynamics of the $2d$ ferromagnet get stuck for time scales shorter than $\tau = \exp(8\beta)$ [26] so that the grow is only logarithmic in this regime (where the system is actually hardly distinguishable from a spin glass; even its T_{eff} resembles those of mean field disordered systems [26]). This strong temperature dependence of domains growth is sufficient to add memory to our rejuvenation effect. In Fig.2(a), we now observe a quasi perfect memory effect due to the freezing of the coarsening dynamics at T_2, so that back to T_1 the dynamics continues where it has left (apart from a short, fast transient). We also see overaging, another interesting effect observed in some experiments (that we will not discuss here). The $2d$ Kawasaki model is probably the simplest model in finite dimension that display these phenomena. This demonstrates that memory and rejuvenation can be observed numerically even in simple models without disorder. All that are good news, given the experimental ubiquity of these effects but it again demonstrates that caution has to be taken when interpreting such data. Indeed, the rejuvenation signal tends to decrease, although slowly, at larger time scales (Fig.2(b)).

An alternative simple way to introduce such a T-dependence in the dynamics is to add small disorder and/or frustration in the couplings (or in the magnetic field), in which case the dynamics at T_2 could again be slow enough to allow the observation of memory. The recipe how to cook a model with memory and rejuvenation is thus quite simple. This explains the results of [16], where they observed similar effects in site-diluted ferromagnet. All these results actually resemble what is experimentally observed in disordered [6] and frustrated magnets [7], probably because the underlying mechanism of interfaces pining is similar.

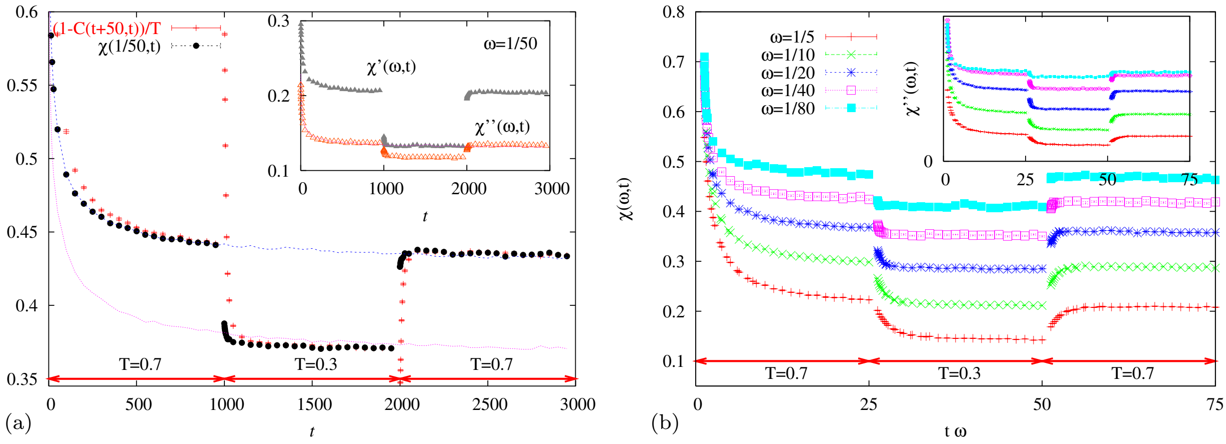

Figure 3. T-cycling in the $2d$ XY model (same presentation of data as in Fig.1).
(a) The FDT-based susceptibility enhanced artificially the rejuvenation effect.
(b) Rejuvenation fastly vanishes at large time scales.

6. Temperature Cycle experiment in 2d XY model

We turn briefly to the $2d$ XY model where the situation is quite different: here we expect a rejuvenation signal from equilibrium physics since the equilibrium correlation function essentially behaves as $C(r) \propto r^{-T}$ so that all length scales have to be re-equilibrated upon T-changes, as is evidenced by Eq.(1). This model thus seems to be a good illustration of the "many length scales" ideas advocated in [9]. It was suggested in [17] that the $2d$ XY model may capture most of the experimental spin glass phenomenology, but our numerical studies for this model are in disagreement with this picture [18]. Firstly, as can be checked in the data of Fig.3(a), the FDT violations we reported in the Ising model are even stronger in the XY model, so that the impressive rejuvenation effect previously seen in the correlation is actually very tiny for the susceptibility. Secondly, due to the form of the correlation in Eq.(1), which is typical of aging at criticality, the large time limit makes all the rejuvenation effect to concentrate in a vanishing small time window, see Fig. 3(b).

All this makes the result of [17] a bit artificial. While the mechanism of re-adaptation at all scales is certainly relevant to glassy dynamics, the use of the XY model is not really justified, mainly because it is indeed a very special critical system, and critical dynamics is quite different from the one observed in usual aging. Unfortunately surfing on a critical line does not seem to be sufficient to interpret spin glass experiments.

7. Conclusion and discussion

Once again, studying simple models provided important lessons. Firstly, contrary to what was believed, it is not so hard to observe either rejuvenation and memory in simulations at finite times, in fact even a simple ferromagnetic model can do that. We also showed that assuming FDT enhanced artificially the rejuvenation effect, and that one can have a larger susceptibility at a lower temperature without any restart of the dynamics. Therefore careful interpretations of simulation have to be made, and the long time limit has to be studied before doing any comparison with experimental data. All these points are valid for spin glasses, as can be checked in Fig.4(a).

In recent years, many authors concluded that since memory and rejuvenation can be observed without temperature chaos, this concept is irrelevant (we saw indeed that these effects can be obtained almost in any models if one tunes properly the parameters). Nevertheless the

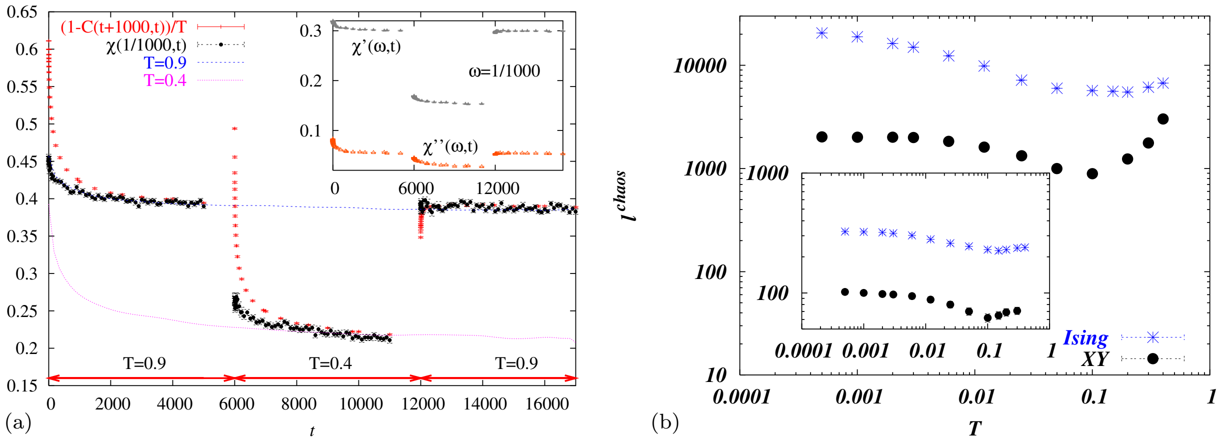

Figure 4. (a) T-cycling in $4d$ spin glasses; data look very similar to what is obtained in the ferromagnets (in the inset data in oscillatory field). (b) Chaotic length, beyond which temperature chaos is observable, as a function of T in the $3d$ XY and Ising spin glasses from real space renormalization (after [29], $\Delta T = 0.001$ in the main plot, $\Delta T = 0.01$ in the inset). Chaos is much stronger for continuous spins.

phenomenology we observed remains quite far from what is observed in spin glass experiments when looking more closely. Firstly, the large time limit is different. Secondly, in spin glasses like AgMn the rejuvenation can be complete, so that the susceptibility at T_2 after the stage at T_1 is the same as in a direct quench at T_2 (these are the only real rejuvenations according to [11]); it is hardly the case for all the models considered here. It has been argued that temperature changes are not instantaneous in experiments so that numerical quenches have to be also progressive in order to obtain (maybe) a complete rejuvenation [13, 16]. We expect that this will not affect too much the exact susceptibility, but rather its approximation based on FDT and its "unphysical" part. It is finally important to mention the *second rejuvenation* effect that is sometime observed in Heisenberg spin glass when heating back to T_1 [8, 27, 11], so that the dynamics looks like quenched from a higher temperature in this reheating step. This is not observed in the models considered here, nor in simulations of spin glass, and can neither be understood within the XY model, even qualitatively. This suggests something new is at work, and this might well be *temperature chaos*. This is an old issue in spin glass community that for a while shared many common points with the Loch Ness monster: many people talk about it, yet no one really saw it. It seems now that it exists in mean field as well as finite dimensional systems [28] and it can also be shown that the lengths beyond which chaotic effects are observable should be shorter for continuous spins than for Ising spins [29], see also Fig.4(b), which would explain nicely why rejuvenation effects are stronger for Heisenberg spin glasses, and why Ising samples do not seem to display a second rejuvenation.

As usual, it would be certainly funny to look to these statements in a few years, when most of these questions will have hopefully found their answers.

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