Rome, 8 September '08

Progress on QCD evolution equations

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In honour of Giorgio Parisi for his 60th birthday

In the years 1976-78 I have done a few papers on QCD with Giorgio

Charmed Quarks and Asymptotic Freedom in Neutrino Scattering. Guido Altarelli (Rome U. & INFN, Rome) , G. Parisi (Frascati) , R. Petronzio (Rome U. & INFN, Rome).
Phys.Lett.B63:183,1976. Cited 75 times

Asymptotic Freedom in Parton Language. Guido Altarelli (Ecole Normale Superieure) , G. Parisi (IHES, Bures-sur-Yvette) .
Nucl.Phys.B126:298,1977. Cited 3719 times

Transverse Momentum in Drell-Yan Processes. Guido Altarelli (Rome U. & INFN, Rome) , G. Parisi (Ecole Normale Superieure) , R. Petronzio (CERN) . **Phys.Lett.B76:351,1978**.
Cited 169 times

Transverse Momentum of Muon Pairs Produced in Hadronic Collisions. Guido Altarelli (Rome U. & INFN, Rome) , G. Parisi (Ecole Normale Superieure) , R. Petronzio (CERN) .
Phys.Lett.B76:356,1978. Cited 143 times

In particular the “French” paper was on the QCD evolution equations for parton densities.

- Still a subject of great actual interest. I review recent progress

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Paper **1** to **4** of **4**

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1) **Review of particle physics. Particle Data Group.**
By Particle Data Group (S. Eidelman *et al.*). 2004.
Published in **Phys.Lett.B592:1,2004**.

2) **Review of particle physics. Particle Data Group.**
By Particle Data Group (K. Hagiwara *et al.*). 2002.
Published in **Phys.Rev.D66:010001,2002**.

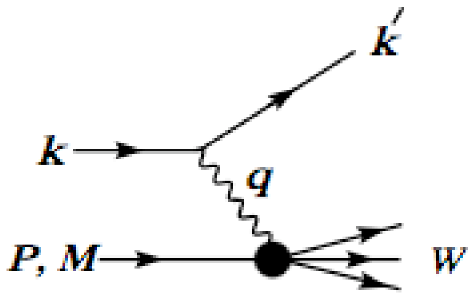
3) **Measurements of Omega and Lambda from 42 high redshift supernovae.**
By Supernova Cosmology Project (S. Perlmutter *et al.*). LBNL-41801, LBL-41801, Dec 1998. 33pp.
The Supernova Cosmology Project.
Published in **Astrophys.J.517:565-586,1999**.
e-Print: **astro-ph/9812133**

4) **Asymptotic Freedom in Parton Language.**
Guido Altarelli (Ecole Normale Superieure) , G. Parisi (IHES, Bures-sur-Yvette) . LPTENS-77-6, Mar 1977. 33pp.
Published in **Nucl.Phys.B126:298,1977**.

Deep Inelastic Scattering

$\ell + N \rightarrow \ell' + X,$

$\ell = e, \mu, \nu$



- Many structure functions
- $F_i(x,Q^2)$: two variables
- Neutral currents, charged currents
- Different beams and targets
- Different polarization

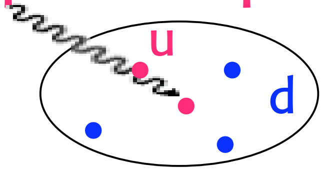
A fundamental role in the development of QCD:

from the beginning: Establishing quarks and gluons as partons
Constructing a field theory of strong int.ns
along the years: Quantitative testing of QCD
Totally inclusive
QCD theory of scaling violations crystal clear
(based on ren. group and operator exp.)
Q^2 dependence tested at each x value)
Measuring q and g densities in the nucleon
Instrumental to compute all hard processes
Measuring α_s
Always presenting new challenges:
Structure functions at small x
Polarized parton densities

In the '70's a great role in establishing QCD

- Approximate Scaling
- Success of Naive Parton Model Bjorken, Feynman

From constituent quarks (real? fictitious?) to parton quarks (real!)



- $R = \sigma_L/\sigma_T$ ---> 0 Spin 1/2 quarks
- ~50% of momentum carried by neutrals Gluons
- Quark charges:

$\sigma_L \sim 0$

$F = 2F_1 \sim F_2/x$

...... = small sea

$F\gamma p = 4/9\ u(x) + 1/9\ d(x) + \ldots\ldots$

$F\gamma n = 4/9\ d(x) + 1/9\ u(x) + \ldots\ldots$

$F\nu p \sim F\bar{\nu}n = 2\ d(x) + \ldots\ldots$

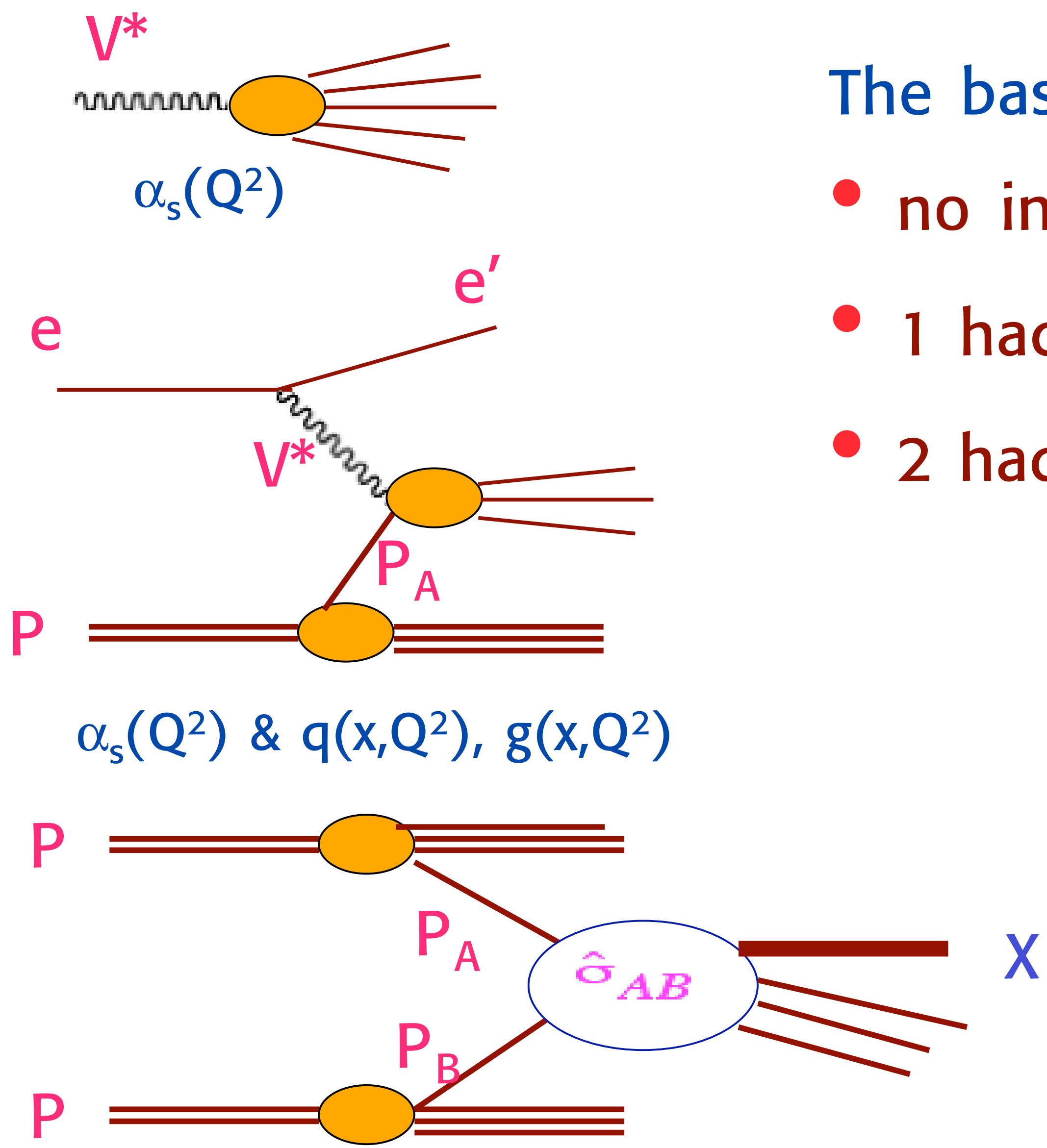
$F\nu n \sim F\bar{\nu}p = 2\ u(x) + \ldots\ldots$

$$\int (u - \bar{u})dx = 2$$

$$\int (d - \bar{d})dx = 1$$

$$\int (s - \bar{s})dx = 0$$

F= F(x), u=u(x), d= d(x):
naive parton model (scaling)



The basic experimental set ups:

- no initial hadron (....LEP, ILC, CLIC)
- 1 hadron (....HERA, LHeC)
- 2 hadrons (....SppS, Tevatron, LHC)

Progress in particle physics needs their continuous interplay to take full advantage of their complementarity

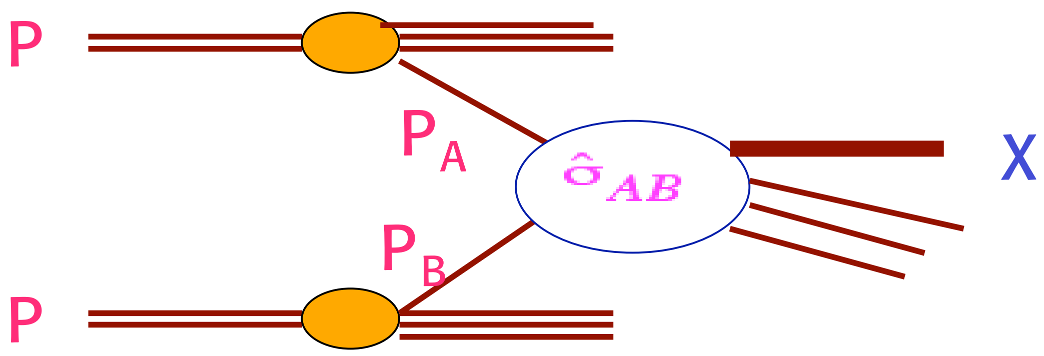
Parton densities extracted from DIS are used to compute hard processes, via the Factorisation Theorem:

x times density of parton A

$$\sigma(s) = \sum_{A,B} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} \, p_A(x_1, Q^2) \, p_B(x_2, Q^2) \, \hat{\sigma}_{AB}(x_1 x_2 s, Q^2)$$

reduced X-section

For example, at hadron colliders



$V = \gamma^*, W, Z$

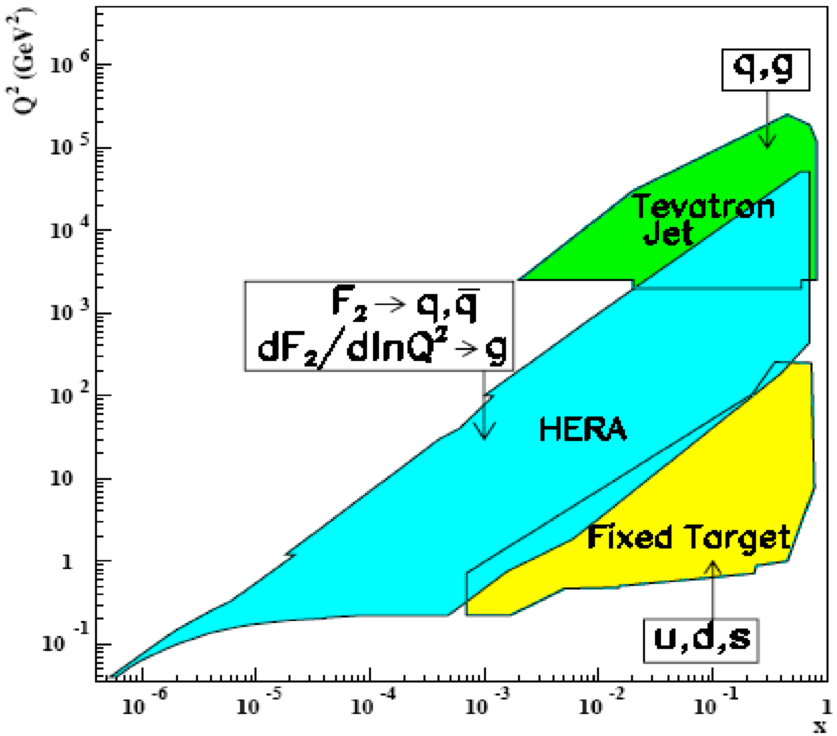
X=V, jets, QQ, H.....

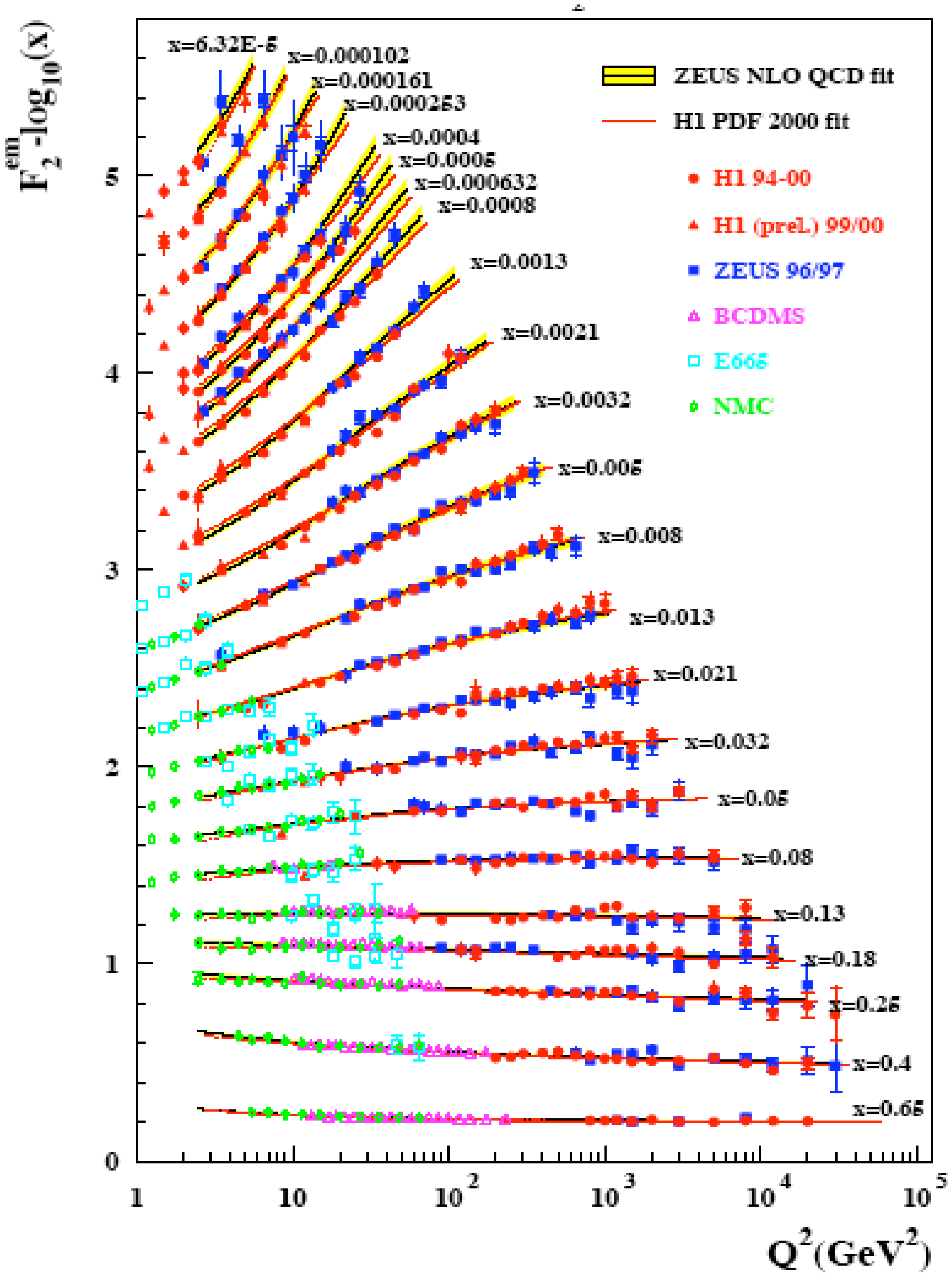
Q=b,c,t

•Very stringent tests of QCD
•Feedback on constraining parton densities

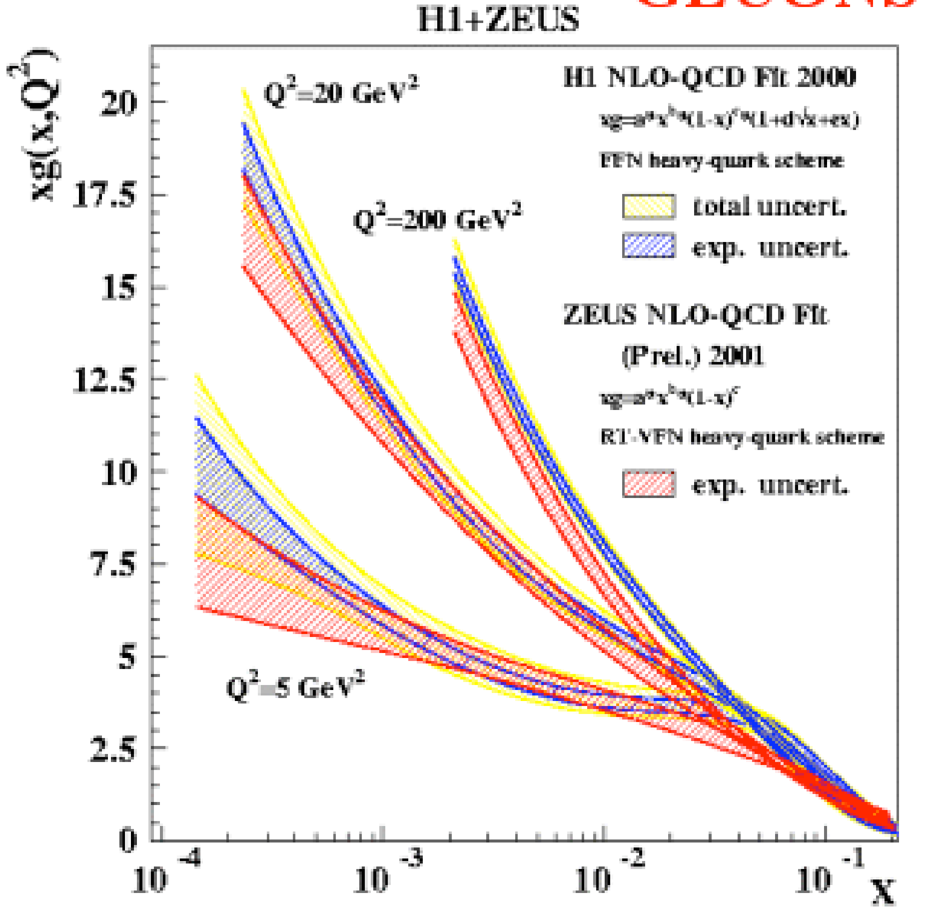
Great progress in the DIS data culminated at HERA

Proton Structure Function $F_2(x,Q^2)$





GLUONS



H1

$$\alpha_s(M_Z^2) = 0.1150 \pm 0.0017(exp)^{+0.0009}_{-0.0005}\,(model)$$
$$\pm 0.005(\text{theory})$$

ZEUS (prel.)

$$\alpha_s(M_Z^2) = 0.117 \pm 0.001(stat + uncorr) \pm 0.005(corr)$$

theory error to be evaluated

Progress in experiment has been matched by impressive achievements in theory

For example in the theory of scaling violations

The scaling violations are clearly observed
and the (N)NLO QCD fits are remarkably good.

These fits to $F_i(x,Q^2)$ provide

- an impressive set of QCD tests
- measurements of $q(x,Q^2)$, $g(x,Q^2)$
- measurements of $\alpha_s(Q^2)$

$$\frac{\partial q_i(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\left\{P_{q_iq_j}(y,\alpha_S)q_j(\frac{x}{y},Q^2)\ +\ P_{q_ig}(y,\alpha_S)g(\frac{x}{y},Q^2)\right\}$$

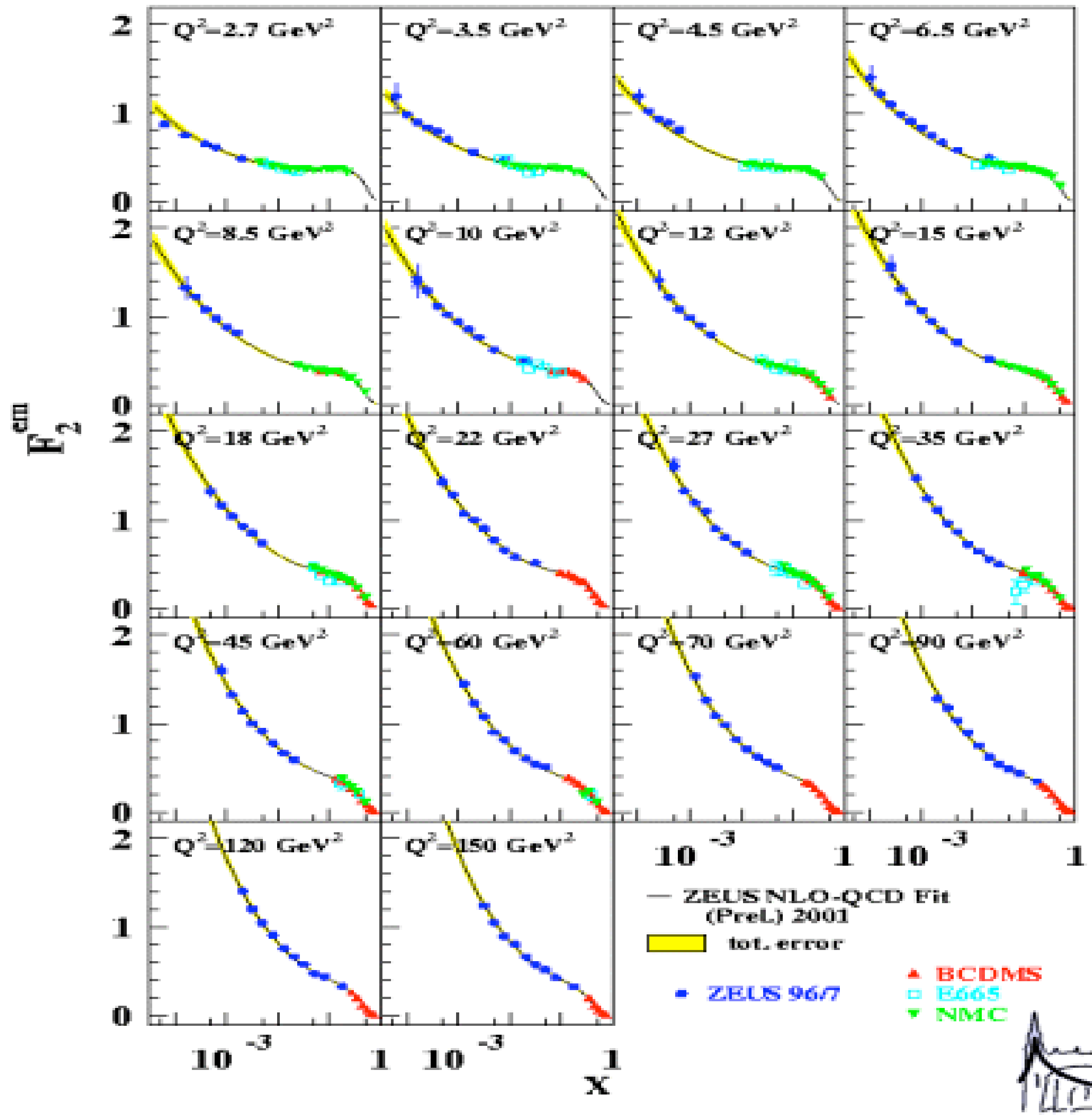
$$\frac{\partial g(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\left\{P_{gq_j}(y,\alpha_S)q_j(\frac{x}{y},Q^2)\ +\ P_{gg}(y,\alpha_S)g(\frac{x}{y},Q^2)\right\}$$

Gribov, Lipatov; Altarelli, Parisi; Dokshitser

Example of NLO QCD evolution fit to HERA data

ZEUS

NLO fits to HERA data are not perfect but amazingly good!!



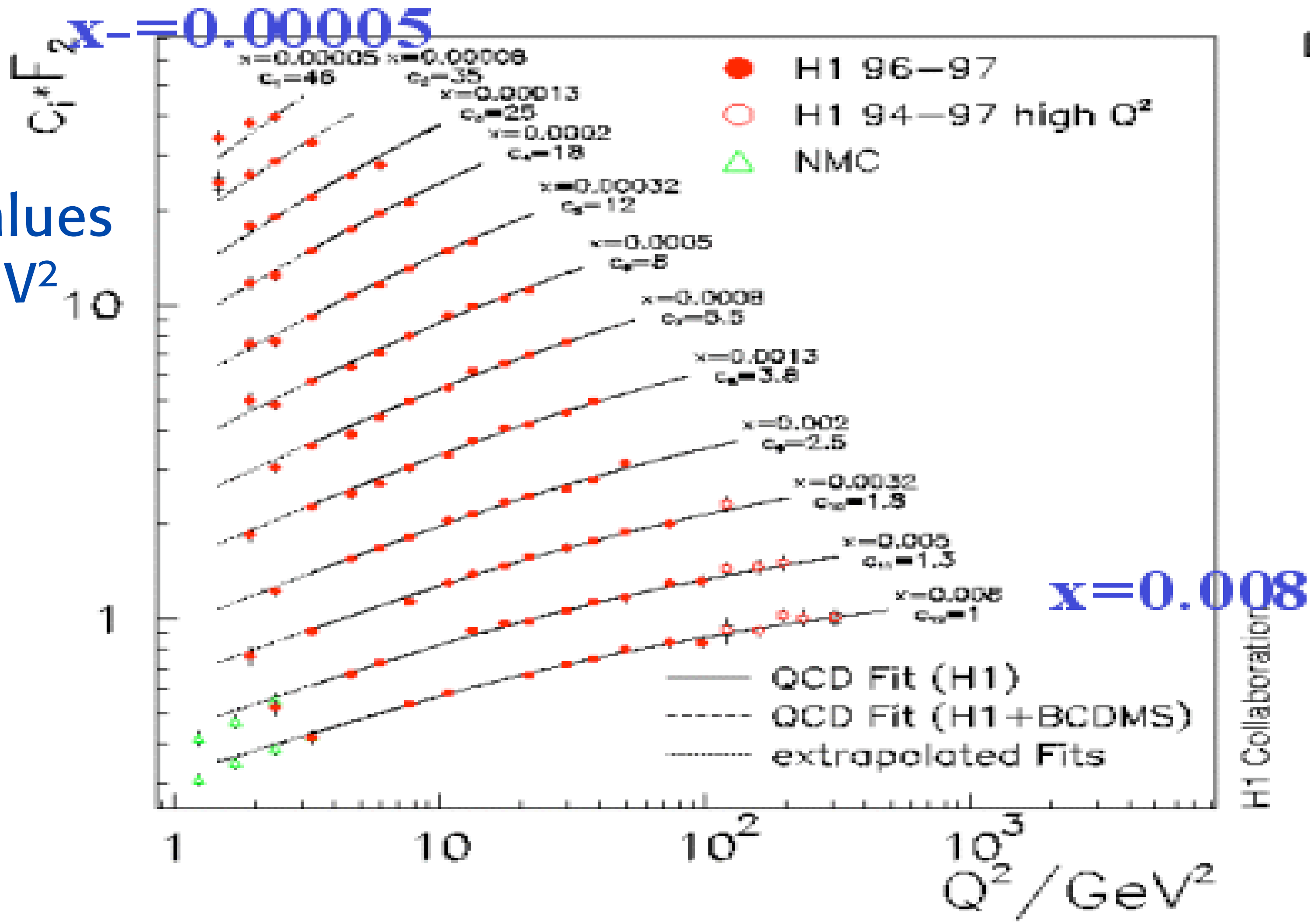
Incr. Q^2

EPS HEP 2001

Even at small x the NLO fit is rather good!

But terms in $(\alpha_s \log 1/x)^n$ should be important!!

At HERA for Q^2 values 3, 10, 10^2, 10^3 GeV^2 $\alpha_s \log 1/x$ can be as large as 4.3, 3.0, 1.2, 0.6



Splitting functions stimulated the development of the most advanced computational techniques over the years

For over a decade all splitting funct.s P have been known to only NLO accuracy: $\alpha_s P \sim \alpha_s P_1 + \alpha_s^2 P_2 + \ldots\ldots$

Floratos et al; Gonzales-Arroyo et al; Curci et al; Furmanski et al

Then the complete, analytic NNLO results have been derived for the first few moments (N<13,14).

Larin, van Ritbergen, Vermaseren+Nogueira

Finally, in 2004, the calculation of the NNLO splitting functions has been totally completed $\alpha_s P \sim \alpha_s P_1 + \alpha_s^2 P_2 + \alpha_s^3 P_3 + \ldots\ldots$

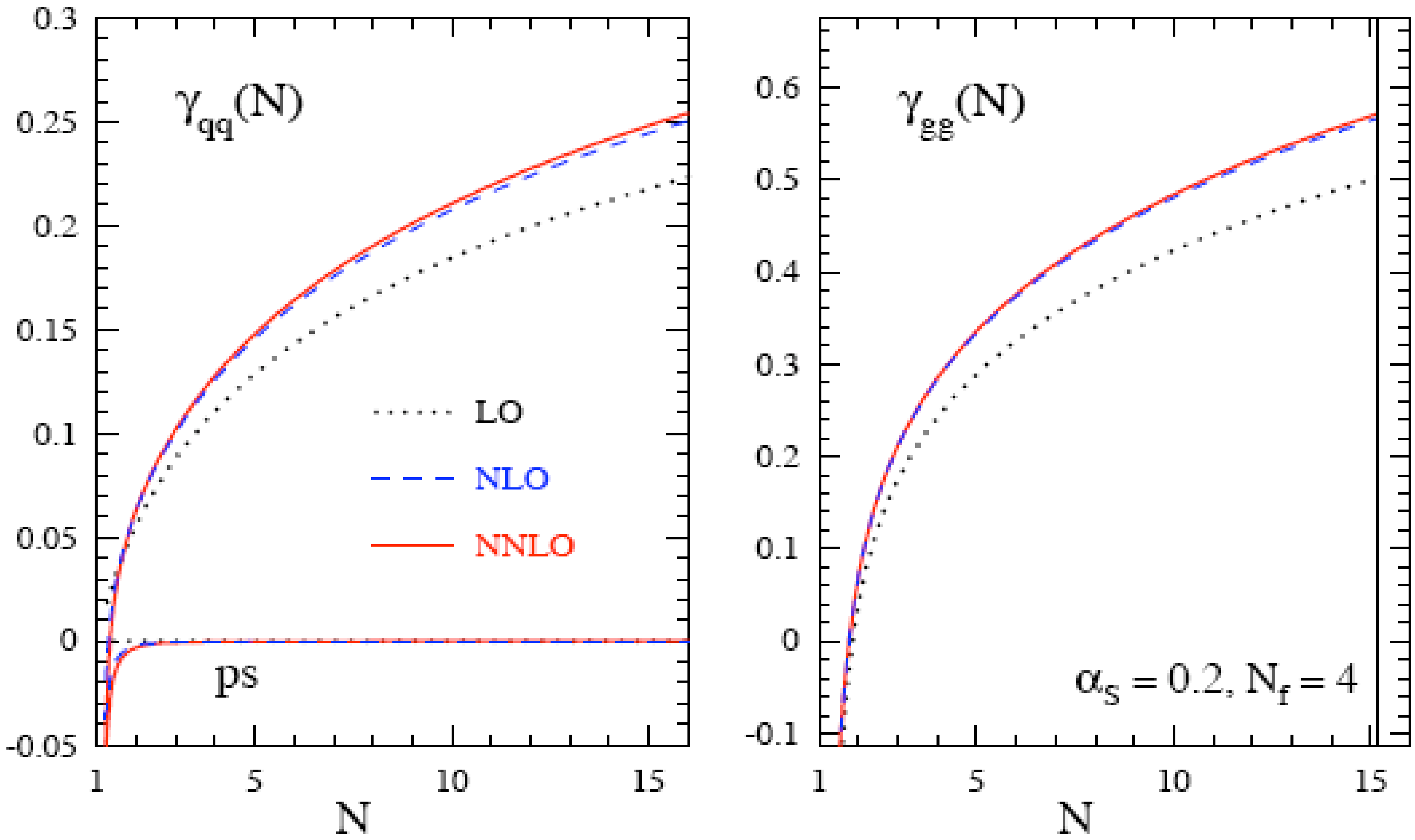
Moch, Vermaseren, Vogt '04

A really monumental, fully analytic, computation

NNLO singlet splitting functions A completely analytical result

Moch, Vermaseren, Vogt '04

Anomalous dimensions vs N, the Mellin index



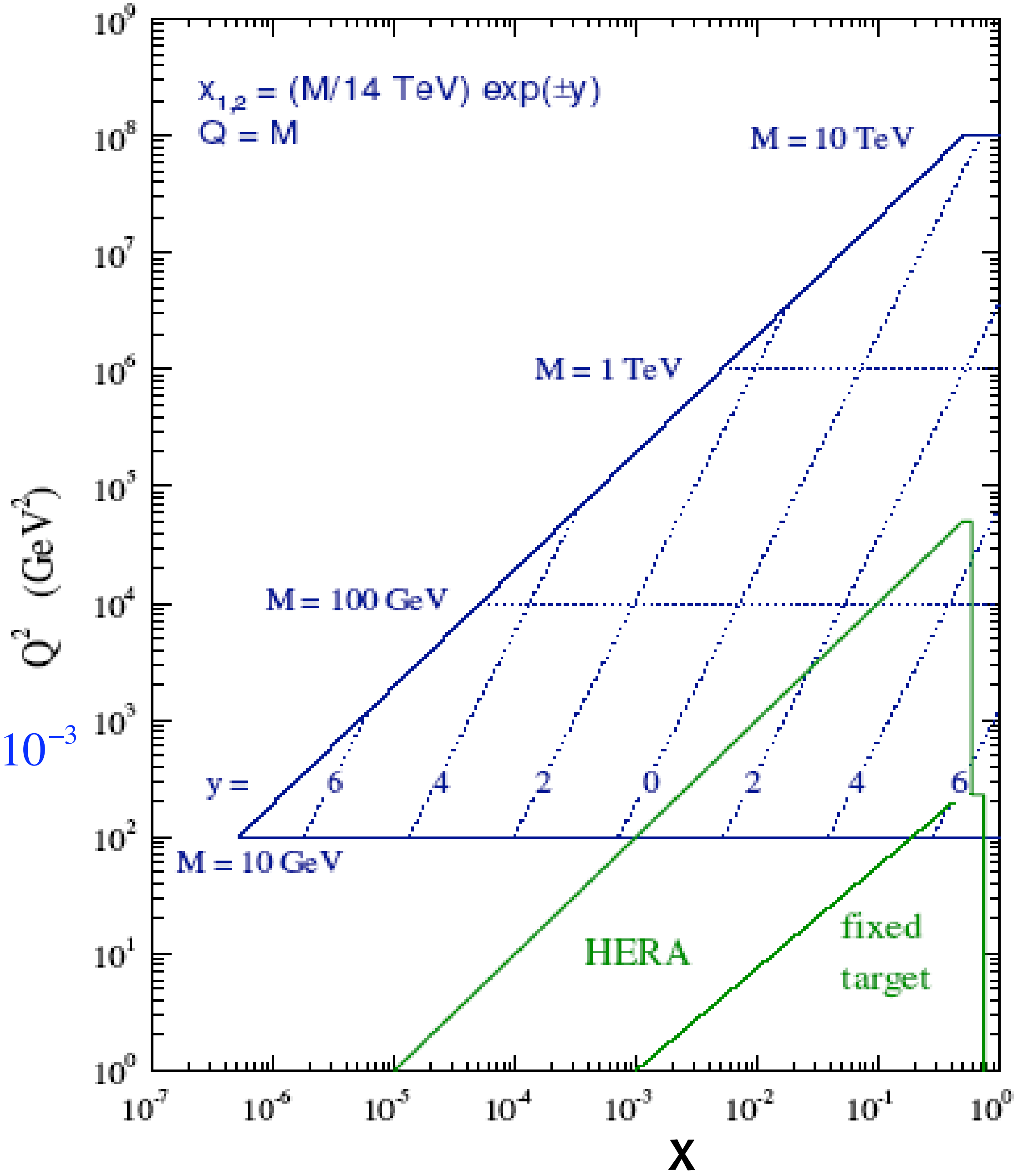
Good convergence is apparent

At HERA for $Q^2 > 1$ GeV^2
$1/x < 10^5$
bulk of data at
$1/x \sim 10^3$ --10^4

At the LHC for producing
a M = 10 GeV mass
(a bb pair) $1/x <$ few 10^6
depending on the
rapidity y

$$x_1 x_2 s = (2m_b)^2 \Rightarrow \bar{x} = \sqrt{x_1 x_2} \sim \frac{2m_b}{\sqrt{s}} \sim 0.7 \cdot 10^{-3}$$

At HERA&LHC
at small x and realistic Q^2
$(\alpha_s \log 1/x)^n \sim o(1)$
must be controlled!



The problem is clear:

- At HERA & LHC at small x the terms in $(\alpha_s \log 1/x)^n$ cannot be neglected in the singlet splitting function
- BFKL have computed all coeff.s of $(\alpha_s \log 1/x)^n$ (LO BFKL)
- Just adding the sequel of $(\alpha_s \log 1/x)^n$ terms leads to a dramatic increase of scaling violations which is not observed (a too strong peaking of F_2 and of gluons is predicted)
- The inclusion of running coupling effects in BFKL is an issue
- Later, also all coeff.s of $\alpha_s(\alpha_s \log 1/x)^n$ (NLO BFKL) have been calculated
- (Fortunately) they completely destroy the LO BFKL prediction

⊕ The problem is to find the correct description at small x

Our goal is to construct a relatively simple, closed form, improved anomalous dimension $\gamma_I(\alpha,N)$ or splitting function $P_I(\alpha,x)$

$P_I(\alpha,x)$ should

- reproduces the perturbative results at large x
- based on physical insight resum BFKL corrections at small x
- properly include running coupling effects
- be sufficiently simple to be included in fitting codes

The comparison of the result with the data provides a new quantitative test of the theory

Moments

$$\xi = \log\frac{1}{x}; \qquad t = \log\frac{Q^2}{\mu^2}$$

$$G(x, Q^2) \equiv G(\xi, t) = x[g(x, Q^2) + k\Sigma(x, Q^2)]$$

Singlet quark

For each moment: singlet eigenvector with largest anomalous dimension eigenvalue

$$G(N, t) = \int_0^1 x^{N-1} G(x, Q^2)\,dx = \int_0^\infty e^{-N\xi} G(\xi, t)\,d\xi$$

Mellin transf. (MT)

$$G(\xi, t) = \int_{-i\infty}^{+i\infty} e^{N\xi} G(N, t)\frac{dN}{2\pi i}$$

Inverse MT ($\xi>0$)

t-evolution eq.n

$$\boxed{\frac{d}{dt}G(N, t) = \gamma(N, \alpha(t))G(N, t)}$$

γ: anom. dim

$$\gamma(N, \alpha) = \alpha\gamma_{1l}(N) + \alpha^2\gamma_{2l}(N) + \alpha^3\gamma_{3l}(N) + \ldots$$

Pert. Th.: $\underbrace{\text{LO} \quad \text{NLO} \quad \text{NNLO}}_{\text{known}}$

Moch, Vermaseren, Vogt '04

Recall:

$$\gamma(N) = \int_0^1 x^N P(x) dx$$

$$P(x) = \frac{\alpha}{x}\left(\alpha \log \frac{1}{x}\right)^n \Leftrightarrow \gamma(N) = n!\left(\frac{\alpha}{N}\right)^{n+1}$$

splitting function anomalous dimension

At 1-loop:

$$\alpha \cdot \gamma_{1l}(N) = \alpha \cdot \left[\frac{1}{N} - A(N)\right]$$

This corresponds to the "double scaling" behavior at small x:

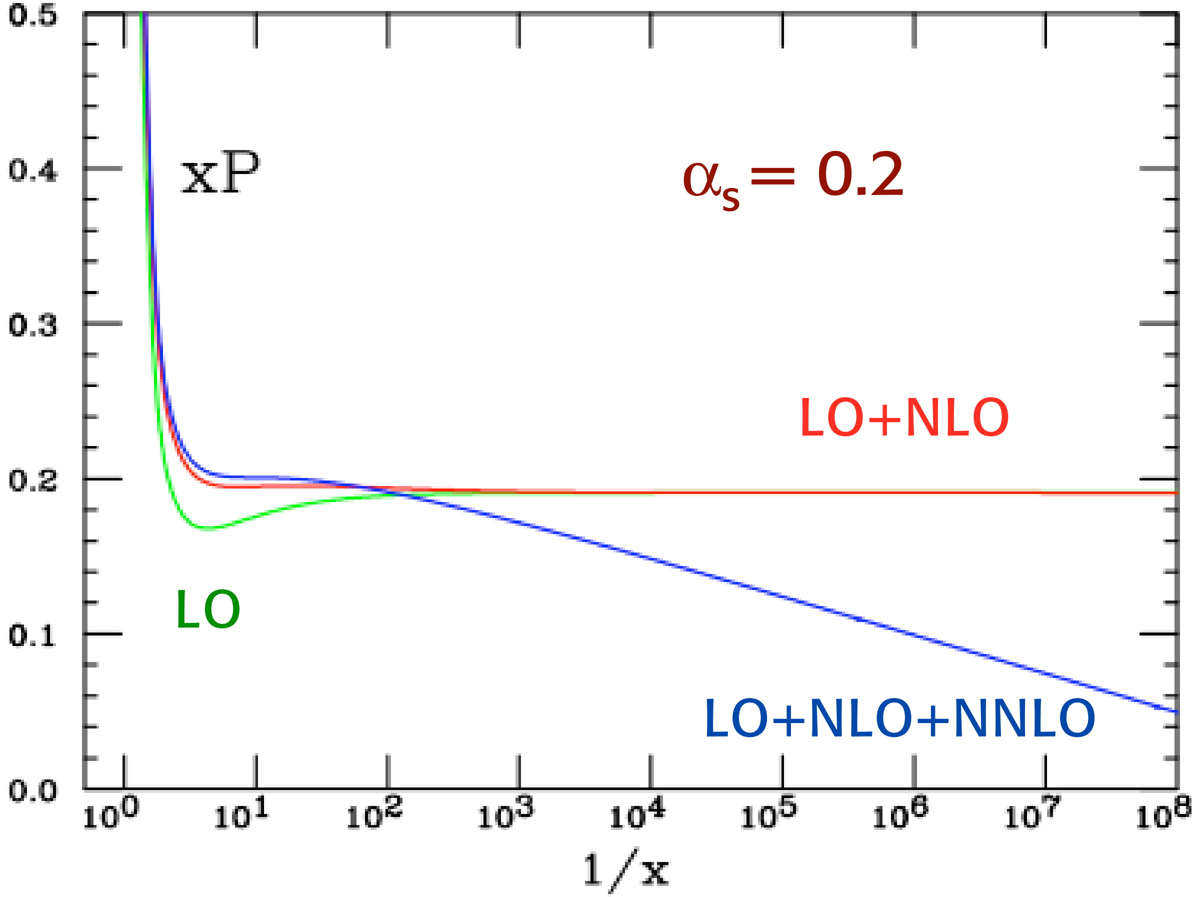
$$G(\xi, t) \sim \exp\left[\sqrt{\frac{4n_C}{\pi\beta_0} \cdot \xi \cdot \frac{\log Q^2/\Lambda^2}{\log \mu^2/\Lambda^2}}\right]$$

$\beta(\alpha) = -\beta_0 \alpha^2 + \ldots$

A. De Rujula et al '74/Ball, Forte '94

Amazingly supported by the data

The singlet splitting function in perturbation theory

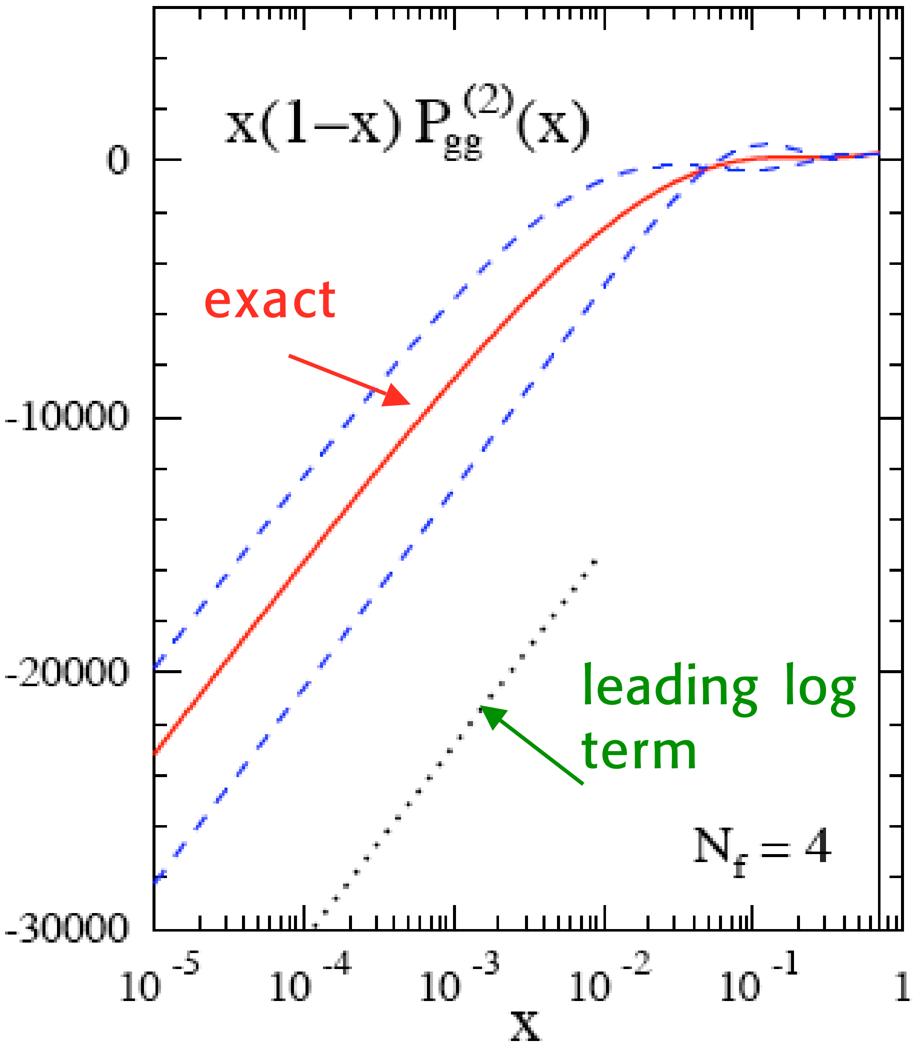


due to $\alpha_s{}^3 \log 1/x$ term

$$\alpha x P_{1l} + \alpha^2 x P_{2l} + \alpha^3 x P_{3l} + \ldots\ldots \sim \alpha + \cancel{\alpha^2 \log 1/x} + \alpha^2 + \cancel{\alpha^3 (\log 1/x)^2} + $$
$$+ \alpha^3 (\log 1/x) + \ldots.$$

accidentally missing

Moch et al found that the approximation to the 2-loop singlet splitting function in terms of leading logs is not good



In principle the BFKL approach provides a tool to control $(\alpha/N)^n$ corrections to $\gamma(N, \alpha)$, that is $(\alpha \log 1/x)^n$ to $xP(x,Q^2)/\alpha$

Define t- Mellin transf.:

$$G(\xi, M) = \int_{-\infty}^{+\infty} e^{-Mt} G(\xi, t) dt$$

with inverse:

$$G(\xi, t) = \int_{-i\infty}^{+i\infty} e^{Mt} G(\xi, M) \frac{dM}{2\pi i}$$

ξ-evolution eq.n (BFKL) [at fixed α]:

$$\boxed{\frac{d}{d\xi} G(\xi, M) = \chi(M, \alpha) G(\xi, M)}$$

with $\chi(M, \alpha) = \alpha \cdot \chi_0(M) + \alpha^2 \cdot \chi_1(M) + \ldots$

known

χ_0, χ_1 contain all info on $(\alpha \log 1/x)^n$ and $\alpha(\alpha \log 1/x)^n$

Bad behaviour, bad convergence

At 1-loop:

$$\psi(M) = \Gamma'(M)/\Gamma(M)$$

$$\alpha\chi_0(M) = \frac{\alpha n_C}{\pi}\int_0^1 [z^{M-1} + z^{-M} - 2]\frac{dz}{1-z} = \frac{\alpha n_C}{\pi}\cdot[2\psi(1) - \psi(M) - \psi(1-M)]$$

Near M=0:

$$\alpha\chi_0(M) \sim \frac{\alpha n_C}{\pi}[\frac{1}{M} + 2\zeta(3)M^2 + 2\zeta(5)M^4 + ...]$$

Note the 1/M behaviour and that the constant and linear terms in M are missing

At M=1/2

$$\lambda_0 = \alpha\chi_0\left(\frac{1}{2}\right) = \frac{\alpha n_C}{\pi}4\ln 2 = \alpha c_0 \sim 2.65\alpha \sim 0.5$$

The minimum value of $\alpha\chi_0$ at M=1/2 is the Lipatov intercept:

$$\lambda_0 = \alpha\chi_0\left(\frac{1}{2}\right) = \frac{\alpha n_C}{\pi} 4\ln 2 = \alpha c_0 \sim 2.65\alpha \sim 0.5$$

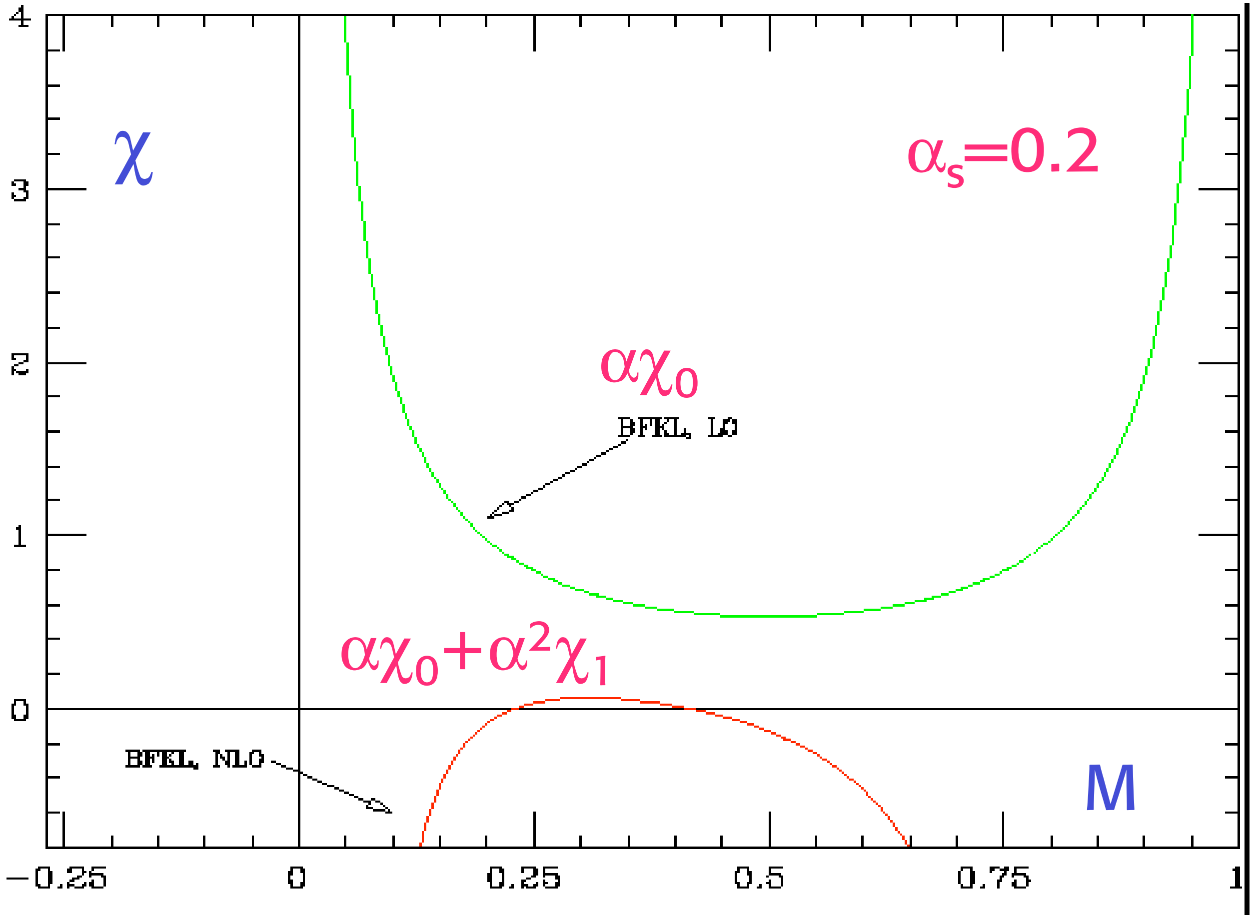
It corresponds to (for x->0, Q^2 fixed):

$xP(x) \sim \alpha x^{-\lambda 0}$

Too hard, not supported by data

But the NLO terms are very large

χ_1 totally overwhelms χ_0!!



Basic ingredients of our resummation procedure

- Duality relation $\chi(\gamma(\alpha, N), \alpha) = N$

  from consistency of ξ and t evolution

- Momentum conservation $\chi(0, \alpha) = 1$

  as $\gamma(\alpha,1) = 0$

- Symmetry properties of the BFKL kernel

- Running coupling effects

Based on G.A., R. Ball, S.Forte:

NPB 575,313,'00
hep-ph/0001157 (lectures)
NPB 599,383,'01, hep-ph/0104246

More specifically on
NPB 621,359,'02, NPB 674,459,'03
hep-ph/0310016

and finally, on our most recent works:
hep-ph/0606323, NPB 742,1,'06,
NPB 799,199,'08

Related work (same physics, same conclusion, different techniques): Ciafaloni, Colferai, Salam, Stasto; Thorne&White

In the region of t and x where both

$$\frac{d}{dt}G(N,t) = \gamma(N,\alpha)G(N,t)$$

$$\frac{d}{d\xi}G(\xi,M) = \chi(M,\alpha)G(\xi,M)$$

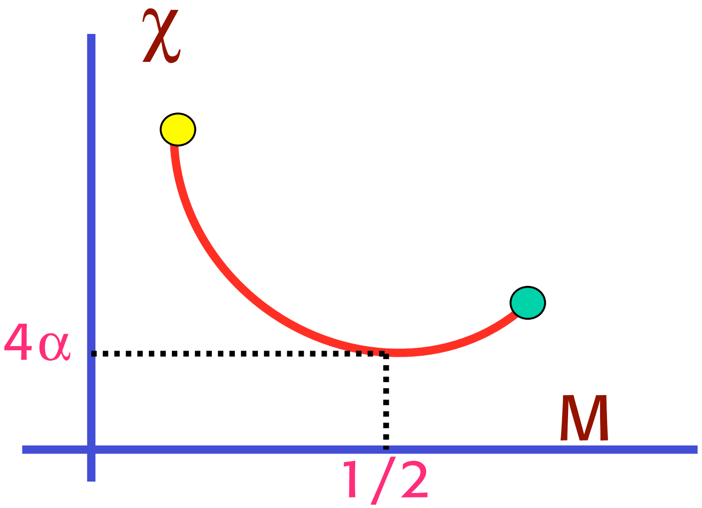
are approximately valid, the "duality" relation holds:

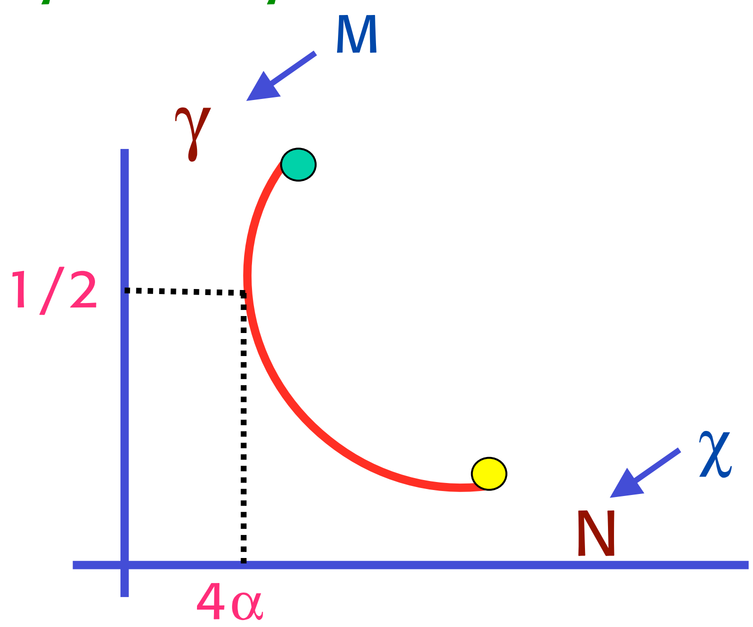
$$\boxed{\chi(\gamma(\alpha,N),\alpha) = N}$$

Note: γ is leading twist while χ is all twist.
Still the two perturbative exp.ns are related and improve each other.
Non perturbative terms in χ correspond to power or exp. suppressed terms in γ.

$\chi(\gamma(N)) = N$

Graphically duality is a reflection





Example: if $\quad \chi(M,\alpha) = \alpha\left[\frac{1}{M} + \frac{1}{1-M}\right] \quad \Longrightarrow$

$$\Longrightarrow \quad \alpha\left[\frac{1}{\gamma} + \frac{1}{1-\gamma}\right] = N \quad \Longrightarrow \quad \gamma = \frac{1}{2}\left[1 \pm \sqrt{1 - \frac{4\alpha}{N}}\right]$$

Note: γ contains $(\alpha/N)^n$ terms

For example at 1-loop: $\chi_0(\gamma_s(\alpha,N))=N/\alpha$

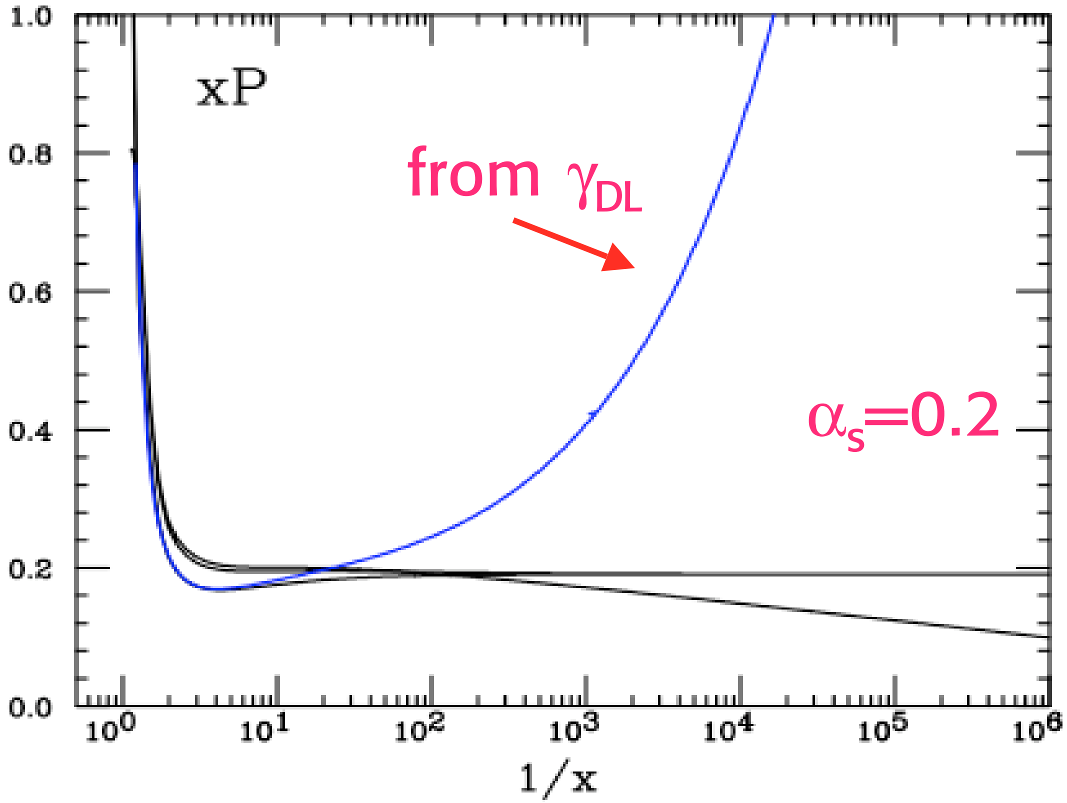
χ_0 improves γ by adding a series of terms in $(\alpha/N)^n$:

$\chi_0 \to \gamma_s\left(\frac{\alpha}{N}\right)$ $$\gamma_s\left(\frac{\alpha}{N}\right) = \sum_k c_k\left(\frac{\alpha}{N}\right)^k$$

$$\boxed{\gamma_{DL}(\alpha, N) = \alpha \cdot \gamma_{1l}(N) + \gamma_s\left(\frac{\alpha}{N}\right) + \ldots \text{ -double count.}}$$

γ_{DL} is the naive result from GLAP+(LO)BFKL

The data discard such a large rise at small x



Similarly it is very important to improve χ by using γ_{1l}

Near M=0, $\chi_0 \sim 1/M$, $\chi_1 \sim -1/M^2$.......

Duality + momentum cons. ($\gamma(\alpha, N=1)=0$)

$$\chi(\gamma(\alpha, N), \alpha) = N \longrightarrow \chi(0, \alpha) = 1$$

$$\lim_{M \to 0} \chi(M, \alpha) \approx \frac{\alpha}{M+\alpha}$$

$$\begin{cases} \gamma(\chi(M)) = M & \rightarrow \gamma_{1l} \Rightarrow \chi_s\left(\frac{\alpha}{M}\right) \\ \chi_s\left(\frac{\alpha}{M}\right) = \sum_k d_k \left(\frac{\alpha}{M}\right)^k & \end{cases}$$

$$\boxed{\chi_{DL}(M, \alpha) = \alpha \cdot \chi_0(M) + \chi_s\left(\frac{\alpha}{M}\right) + \ldots \text{ -double count.}}$$

Double Leading Expansion

$$\gamma(N,\alpha) = \alpha\cdot\gamma_{1l}(N) + \ldots \sim \alpha\cdot\left[\frac{1}{N} - A(N)\right]$$

Momentum conservation: $\gamma(1,\alpha)=0$ $\longrightarrow$ $A(1)=1$

Duality: $\gamma(\chi(M)) = M$ $\longrightarrow$ $\alpha\cdot\left[\frac{1}{\chi} - A(\chi)\right] = M$ $\longrightarrow$

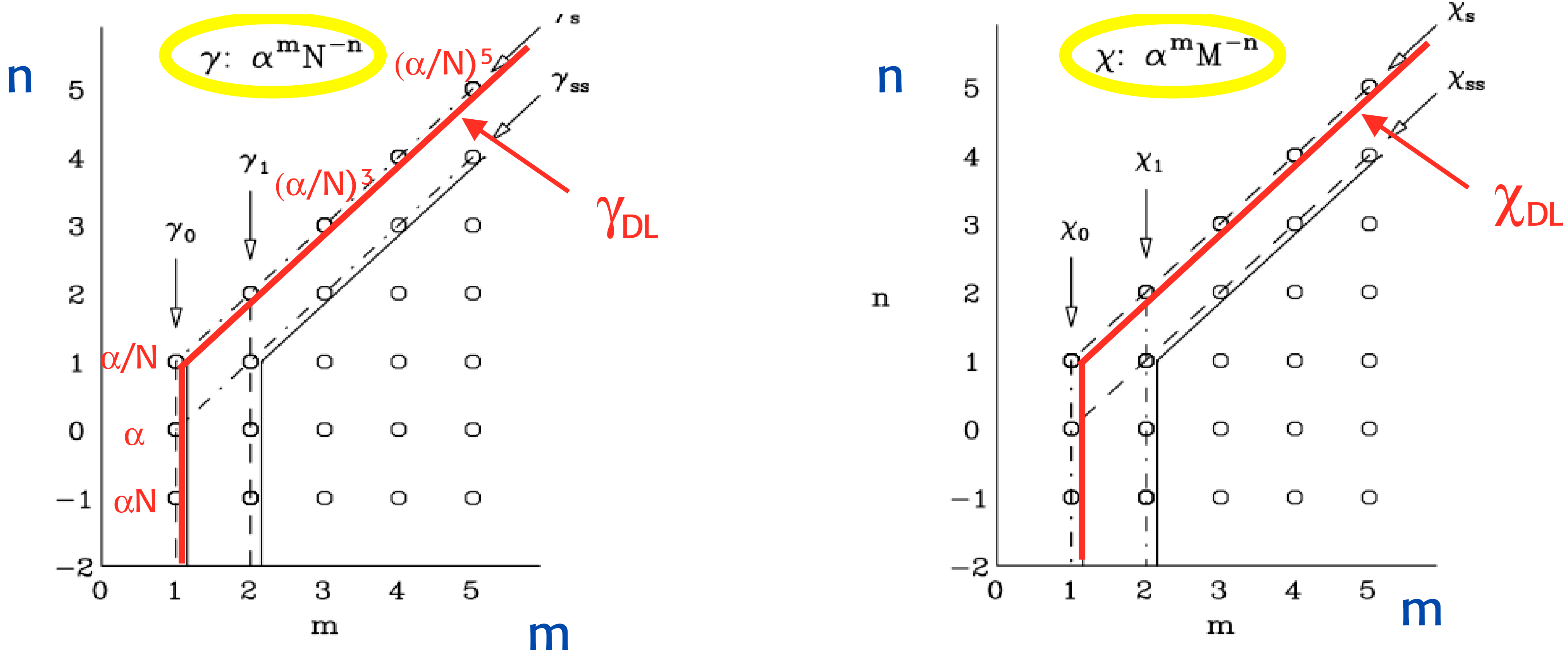
$\longrightarrow$ $\chi = \dfrac{\alpha}{M+\alpha A(\chi)}$ $\longrightarrow$ $\chi(M\sim 0) \sim \dfrac{\alpha}{M+\alpha A(1)} \sim \dfrac{\alpha}{M+\alpha}$

$$\chi_{DL}(M,\alpha) = \alpha\cdot\chi_0(M) + \chi_s\left(\frac{\alpha}{M}\right) + \ldots \text{ -double count.}$$

$$\chi_0(M) = \alpha\cdot\left[\frac{1}{M} + 0(M^2)\right]$$

$$\gamma_{DL}(\alpha, N) = \alpha \cdot \gamma_{1l}(N) + \gamma_s\left(\frac{\alpha}{N}\right) + \ldots \text{ -double count.}$$

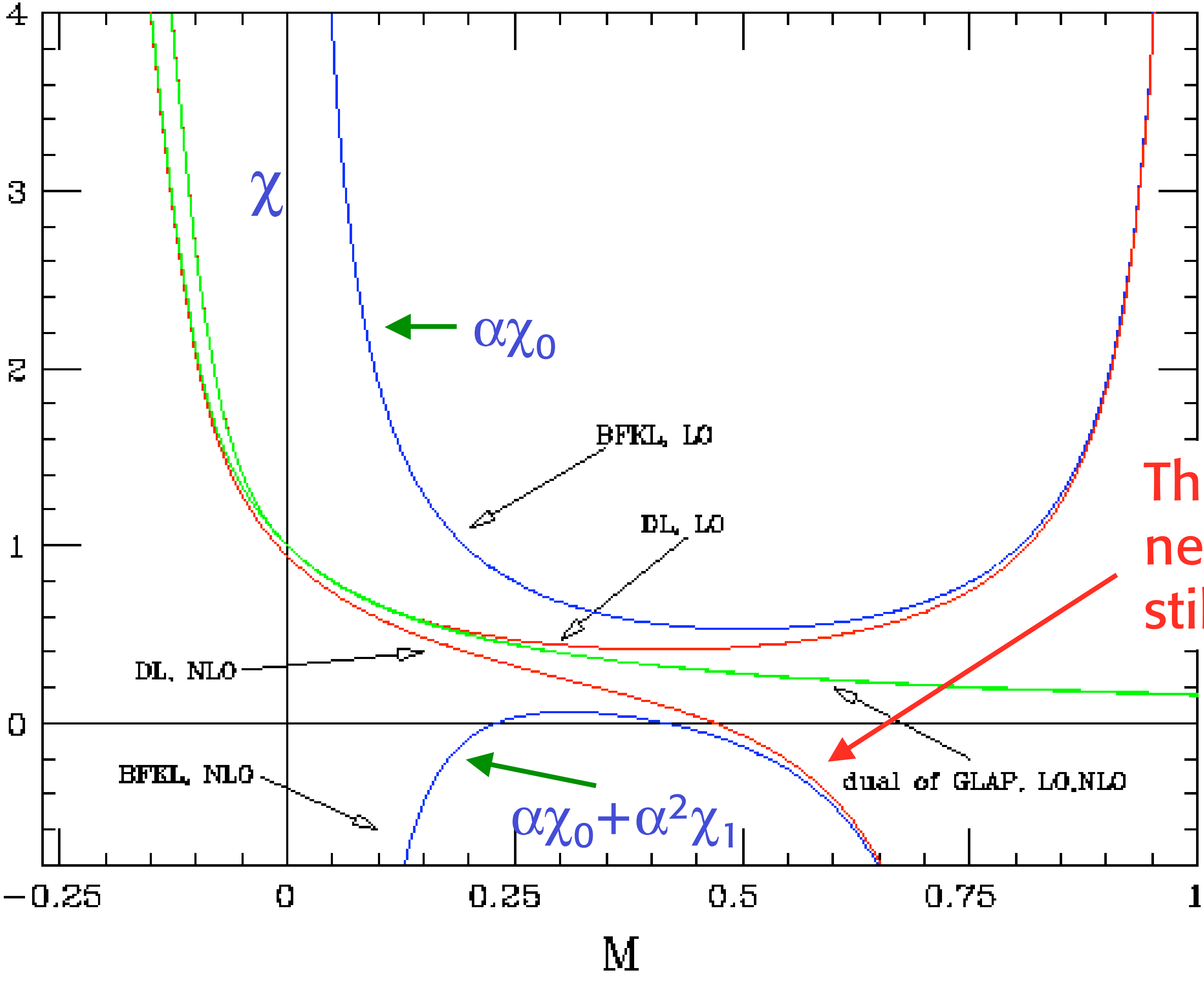
$$\chi_{DL}(M, \alpha) = \alpha \cdot \chi_0(M) + \chi_s\left(\frac{\alpha}{M}\right) + \ldots \text{ -double count.}$$



In the DL expansion one sums over "frames" rather than over vertical lines like in ordinary perturb. theory

DL, LO: $\chi_{DL}(M, \alpha) = \alpha \cdot \chi_0(M) + \chi_s\left(\frac{\alpha}{M}\right) + \ldots$ -double count.

BFKL, LO



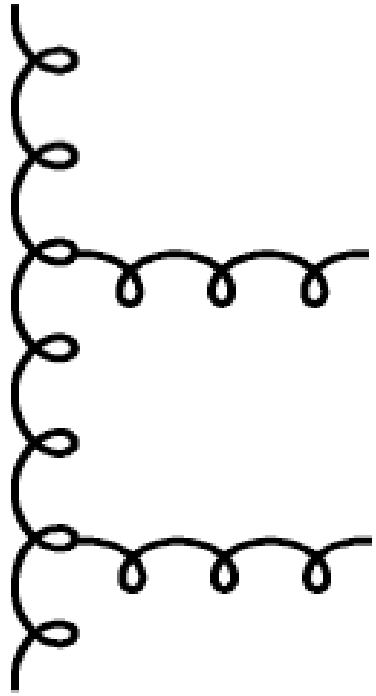
The NLO-DL is good near M=0, but it is still bad near M=1

Can be fixed by symmetrization

Symmetrization

G. Salam '98

$k^2 \Leftrightarrow Q^2$



$Q^2 \Leftrightarrow k^2$

The BFKL kernel is symmetric under exchange of the external gluons

This implies a symmetry under M <--> 1- M for $\chi(\alpha,M)$ broken by two effects:

- Running coupling effects ($\alpha(Q^2)$ breaks the symmetry)
- The change of scale from the BFKL symm. scale $\xi=\ln(s/Qk)$ to the DIS scale $\xi=\ln(s/Q^2)$

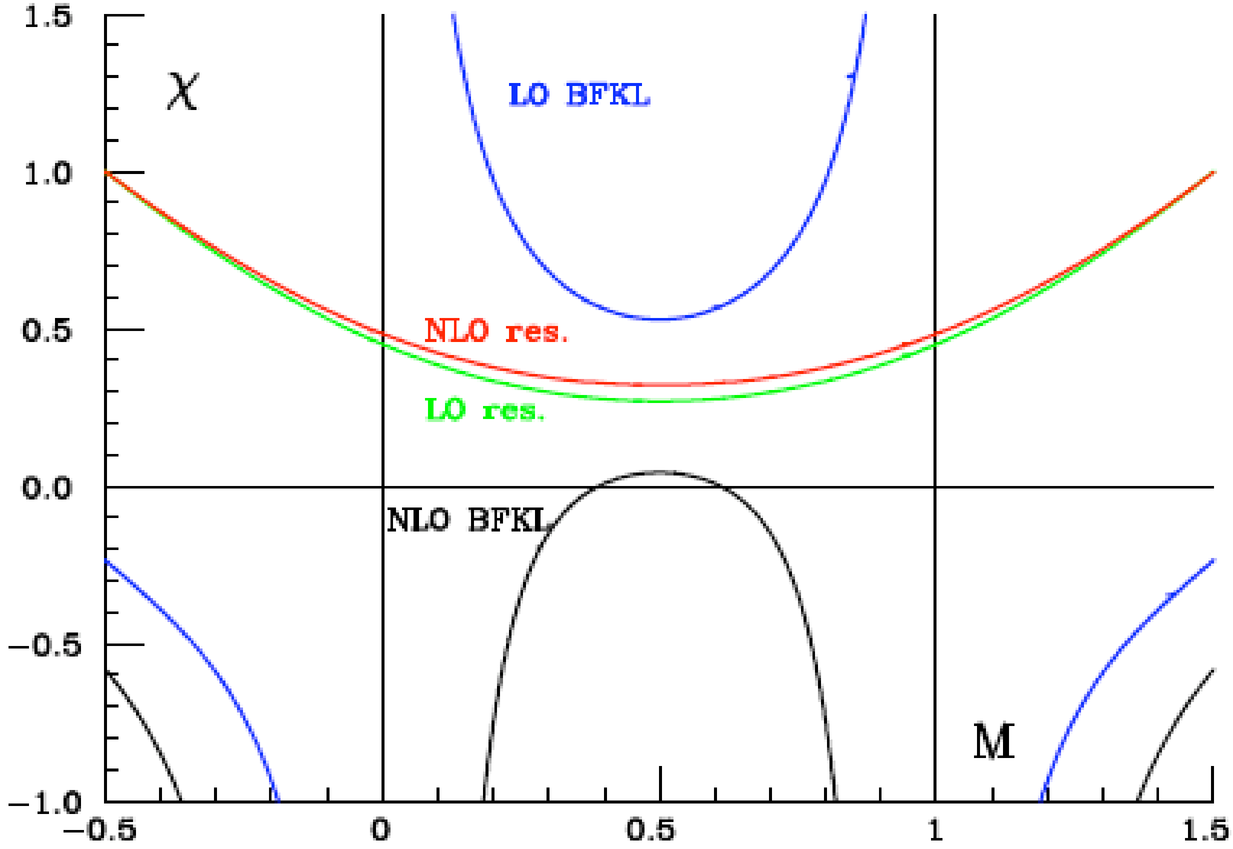
$$\chi_{DIS}\left(M+\frac{\chi_{SYMM}(M)}{2}\right) = \chi_{SYMM}(M)$$

$$\chi = \chi_{DIS}$$

Symmetrization makes χ regular at M=0 AND M=1

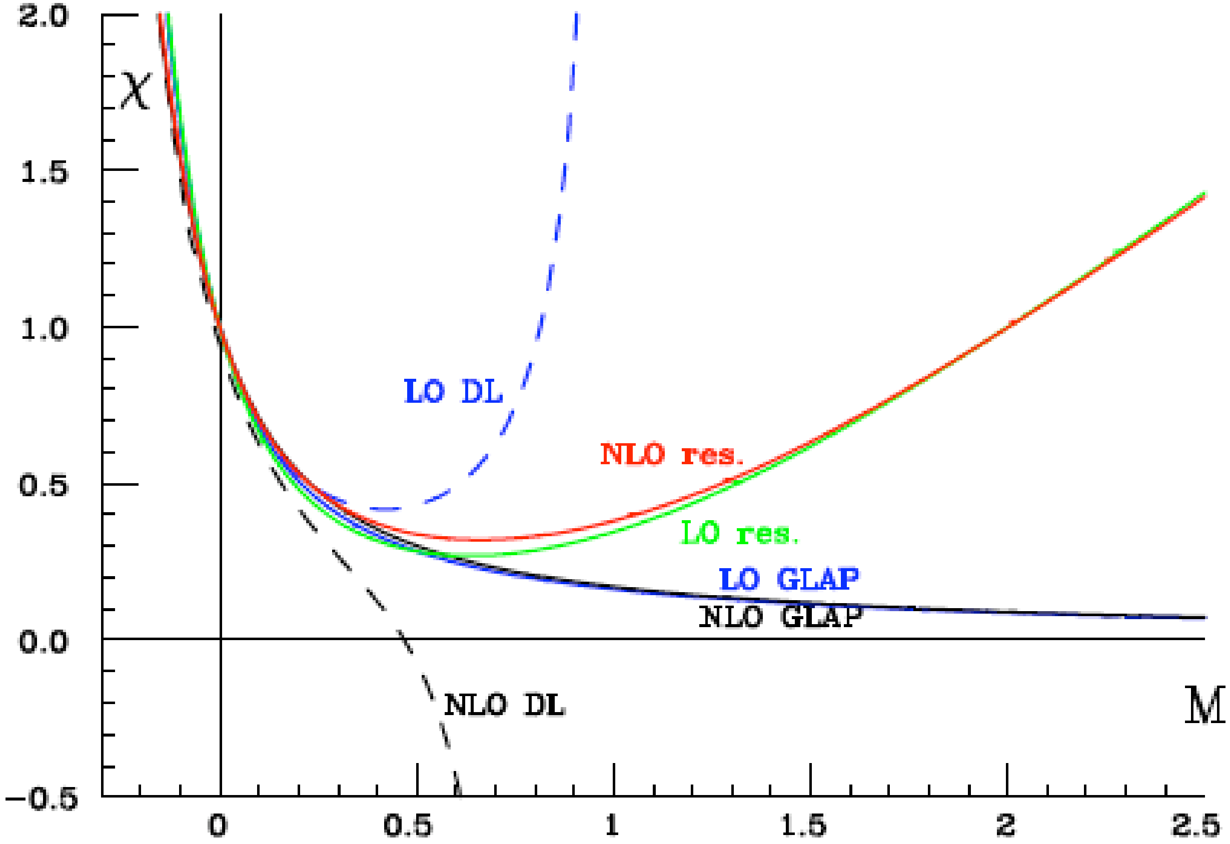
In symmetric variables:

fixed coupling: α=0.2



Note how the symmetrized LO DL and NLO DL are very close!

The same now in DIS variables



All χ curves have a minimum and follow GLAP closer.

The remaining ingredient is the running of the coupling.

A considerable further improvement is obtained by including running coupling effects

Recall that the x-evolution equation was at fixed α

$$\frac{d}{d\xi}G(\xi, M) = \chi(M, \alpha)G(\xi, M)$$

The implementation of running coupling in BFKL is not simple. In fact in M-space α becomes an operator

$$\alpha(t) = \frac{\alpha}{1 + \beta_0 \alpha t} \Rightarrow \frac{\alpha}{1 - \beta_0 \alpha \frac{d}{dM}}$$

In leading approximation:

$$\frac{d}{d\xi}G(\xi, M) = \chi(M, \alpha)G(\xi, M)$$

$$\frac{d}{d\xi}G(\xi, M) = \frac{\alpha}{1 - \beta_0 \alpha \frac{d}{dM}}\chi_0(M)G(\xi, M)$$

By taking a second MT the equation can be written as [F(M) is a boundary condition]

$$\left(1 - \beta_0 \alpha \frac{d}{dM}\right) NG(N, M) + F(M) = \alpha\chi_0(M) G(N, M)$$

It can be solved iteratively

$$G(N, M) = \frac{F(M)}{N - \alpha\chi_0(M)} + \frac{\alpha\beta_0}{N - \alpha\chi_0(M)} \frac{d}{dM} \frac{F(M)}{N - \alpha\chi_0(M)} + \ldots$$

or in closed form:

$$G(N, M) = H(N, M) +$$

$$+ \int_{M_0}^{M} dM' \exp\left[\frac{M - M'}{\beta_0 \alpha} - \frac{1}{\beta_0 N} \int_{M'}^{M} \chi_0(M'')\, dM''\right] \frac{F(M')}{\beta_0 \alpha N}$$

H(N,M) is a homogeneous eq. sol. that vanishes faster than all pert. terms and can be dropped.

The small x behaviour is controlled by the minimum of $\chi(M)$

We make a quadratic expansion of $\chi(M)$ near the minimum.

$$\chi_q(\hat{\alpha}_s, M) = [c(\hat{\alpha}_s) + \tfrac{1}{2}\kappa(\hat{\alpha}_s)(M - M_s)^2]$$

We can solve the equation exactly:

For c, k proportional to α : the solution is an Airy function

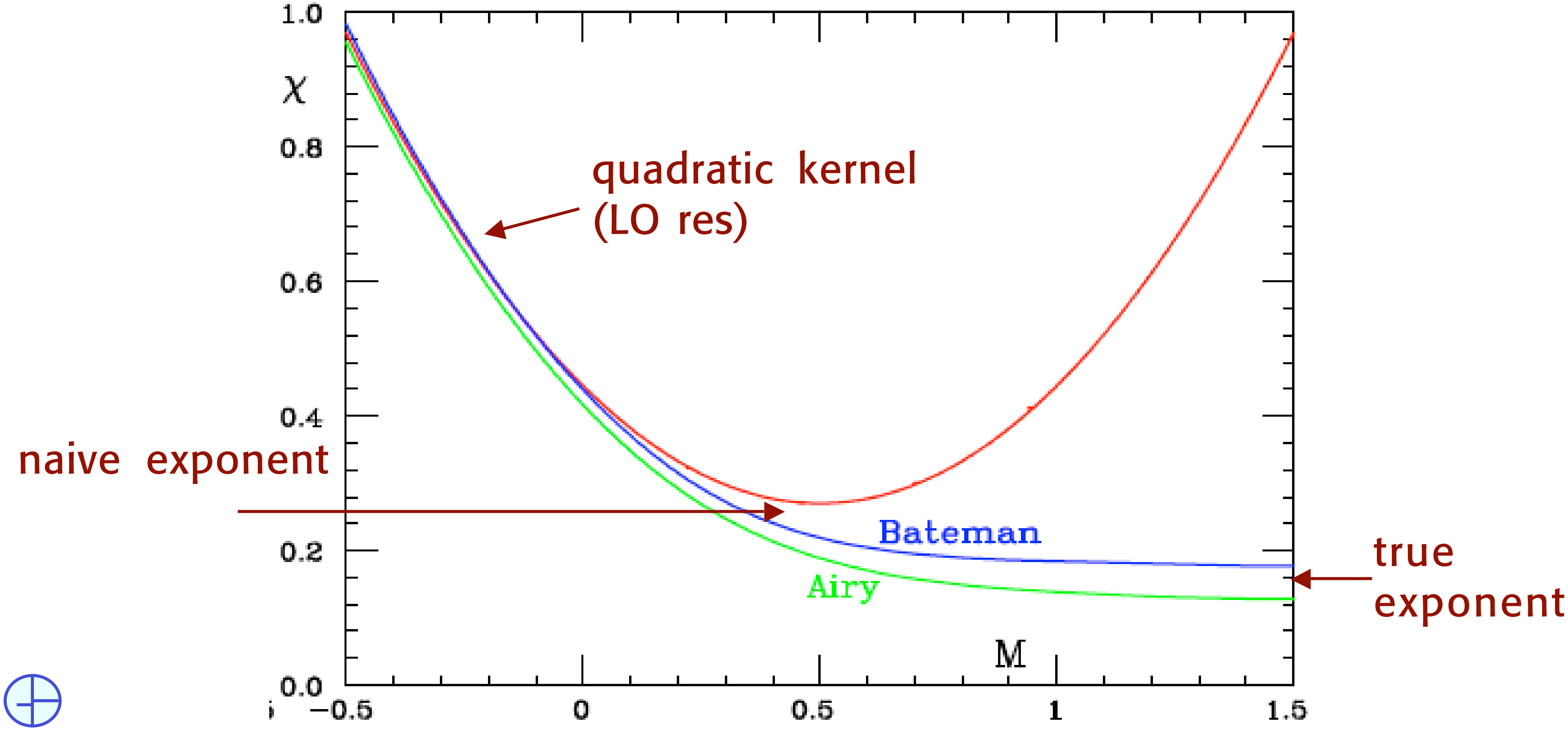
For example, if we take $\chi(\alpha,M) \sim \alpha\, \chi_0(M)$

For general $c(\alpha), \kappa(\alpha)$, to the required accuracy, it is sufficient to make a linear expansion in $\hat{\alpha} - \alpha$

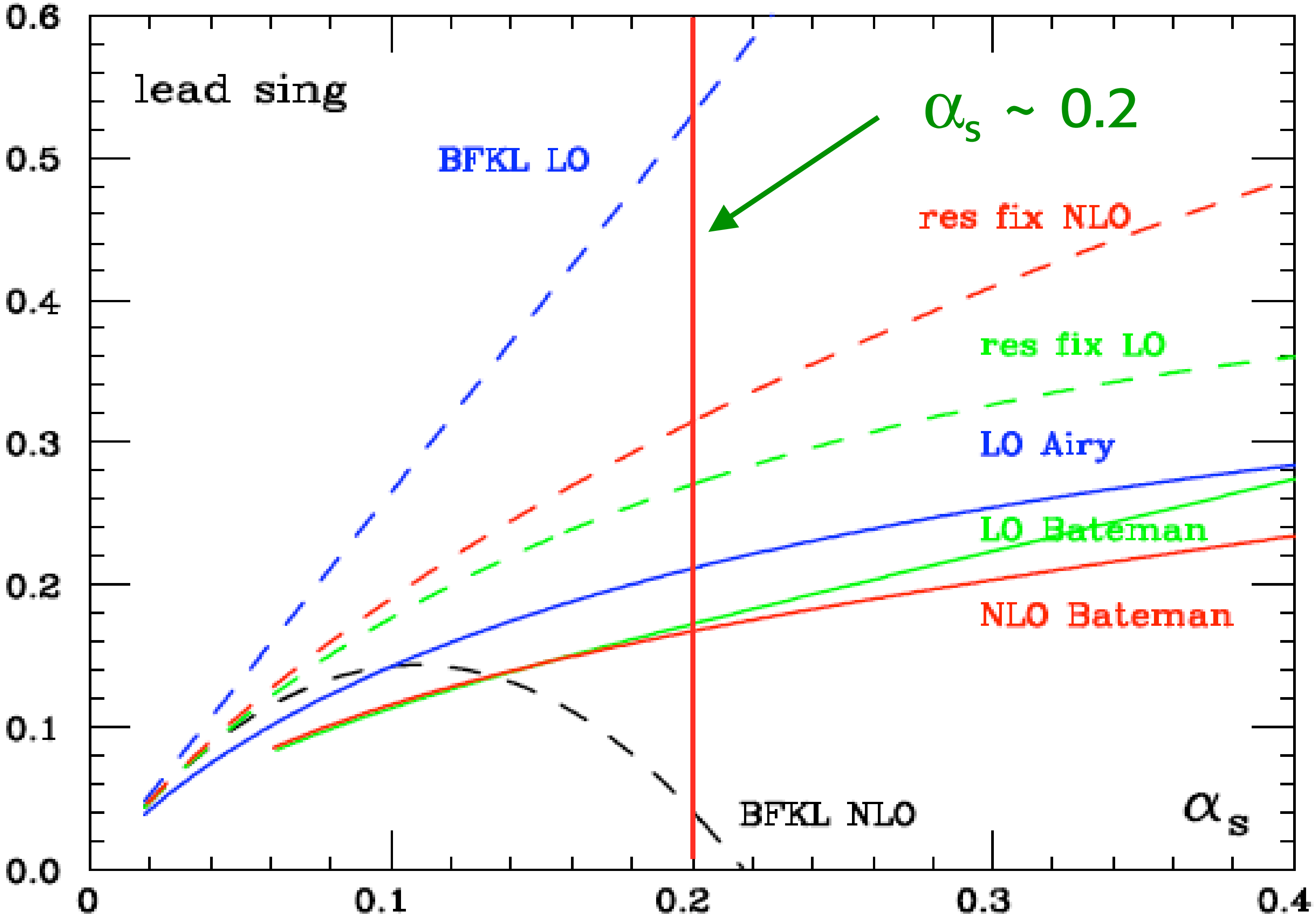
the solution is a Bateman function.

The asymptotic small x behaviour is considerably softened by the running!

Note that the running effect is not replacing $\alpha \to \alpha(Q^2)$ in the naive exponent



DL resummation with symmetrization and running coupling effects progressively soften the small x behaviour



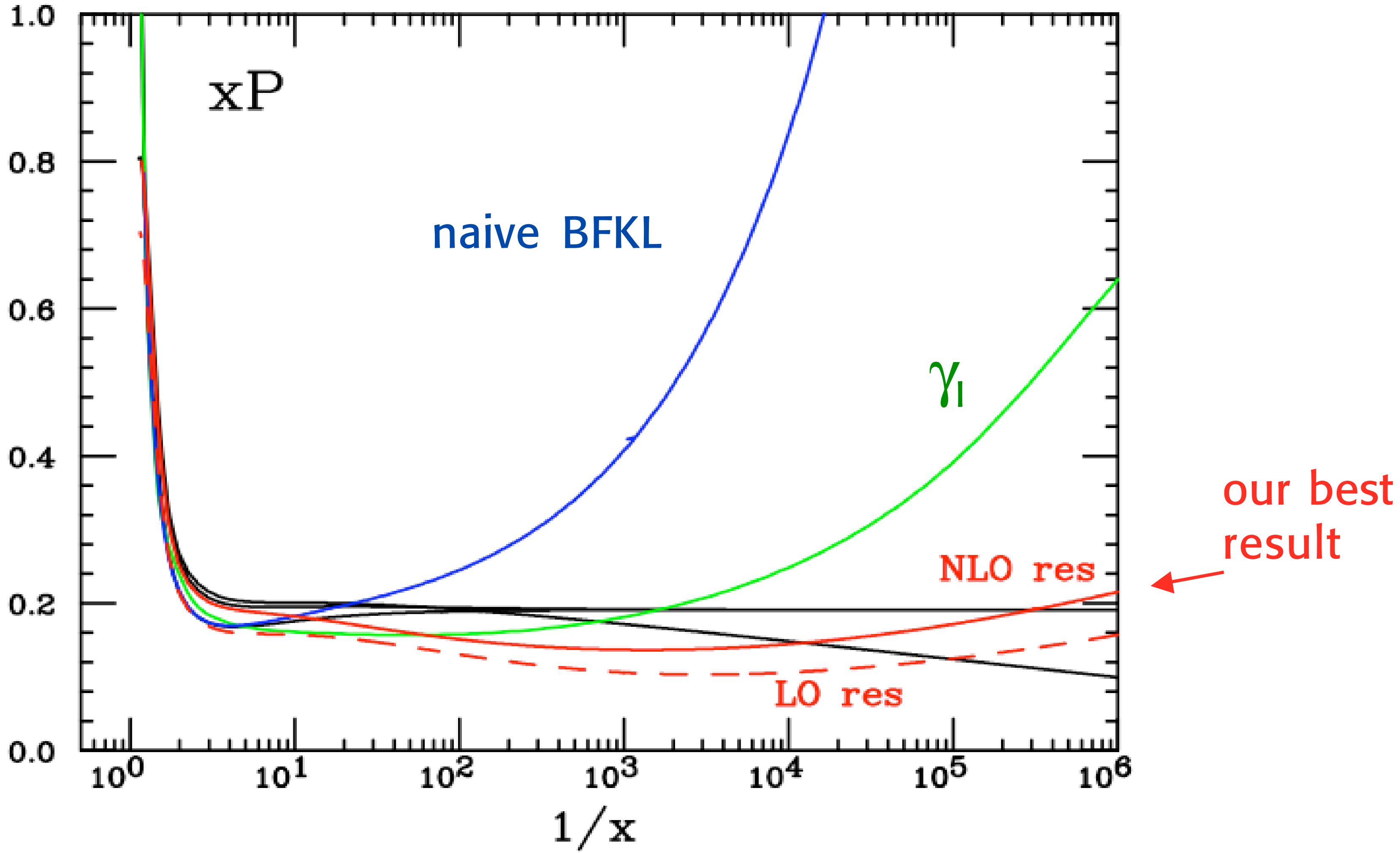
The goal of our recent work is to use these results to construct a relatively simple, closed form, improved anom. dim. $\gamma_I(\alpha,N)$ or splitting funct.n $P_I(\alpha,x)$

$P_I(\alpha,x)$ should

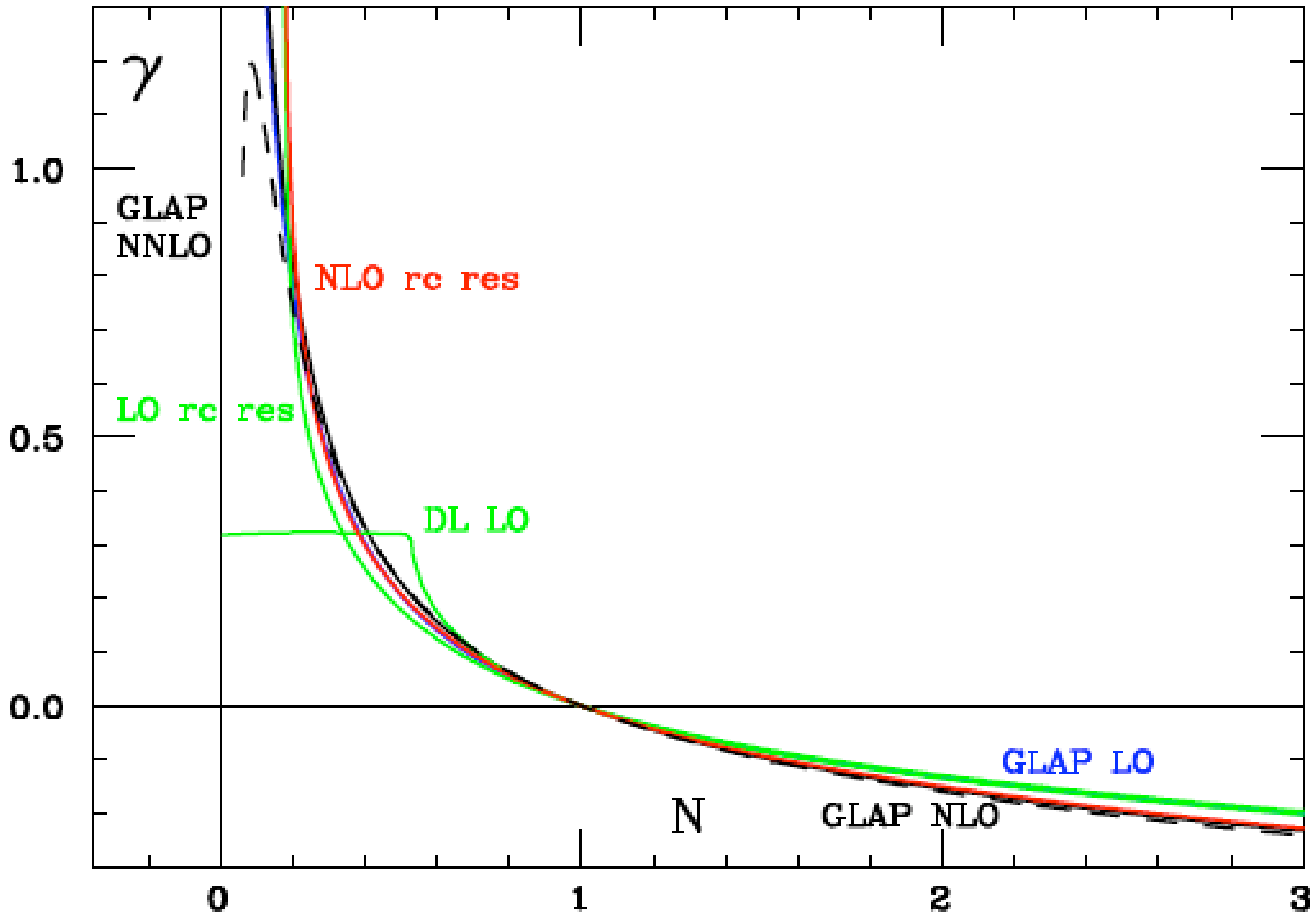
- reproduces the perturbative results at large x
- based on physical insight resum BFKL corrections at small x
- include running coupling effects
- be sufficiently simple to be included in fitting codes

The comparison of the result with the data provides a qualitatitevely new test of the theory

Here are the complete results using the DL resummation, symmetry and running coupling effects at LO and NLO

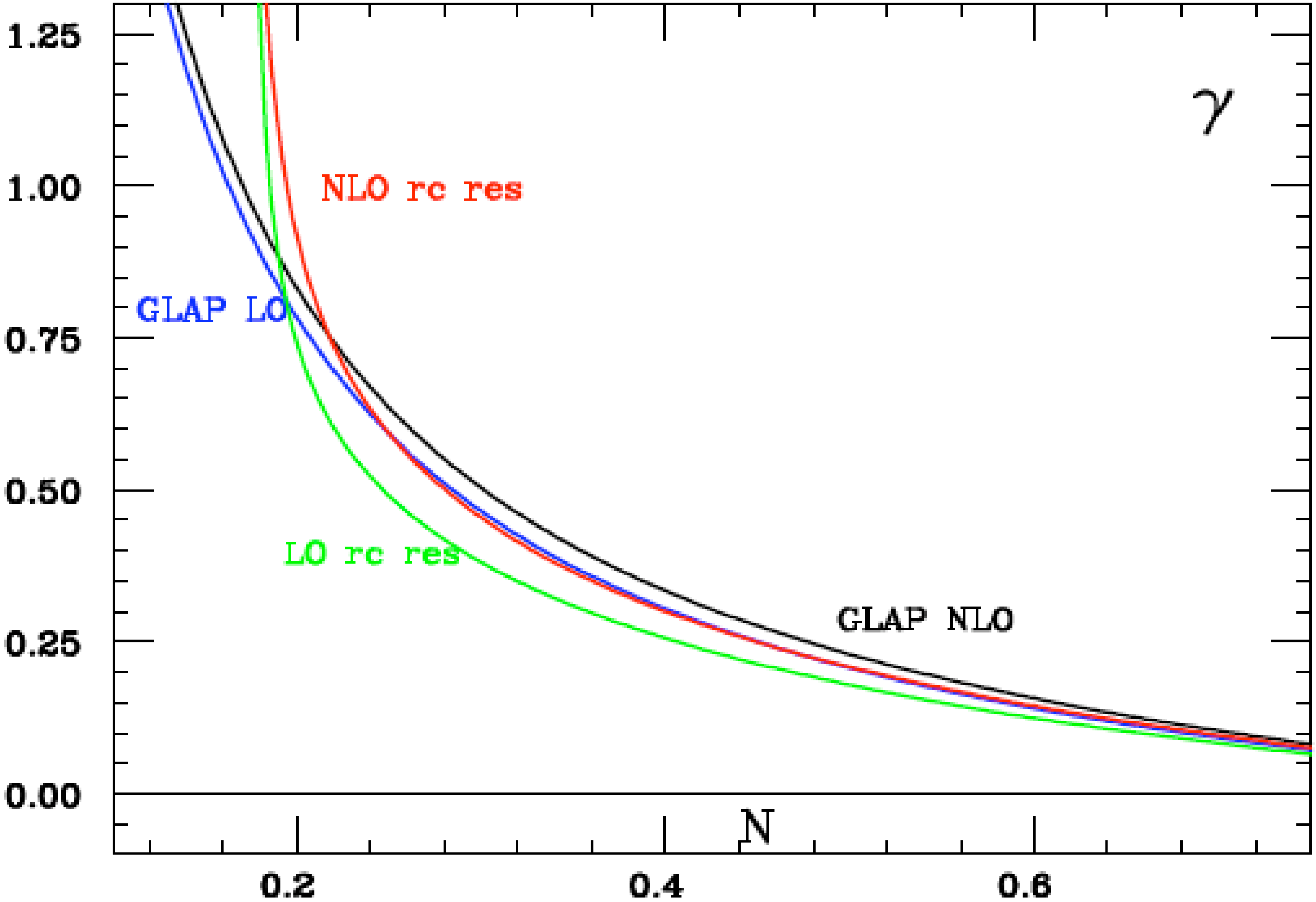


The anomalous dimension



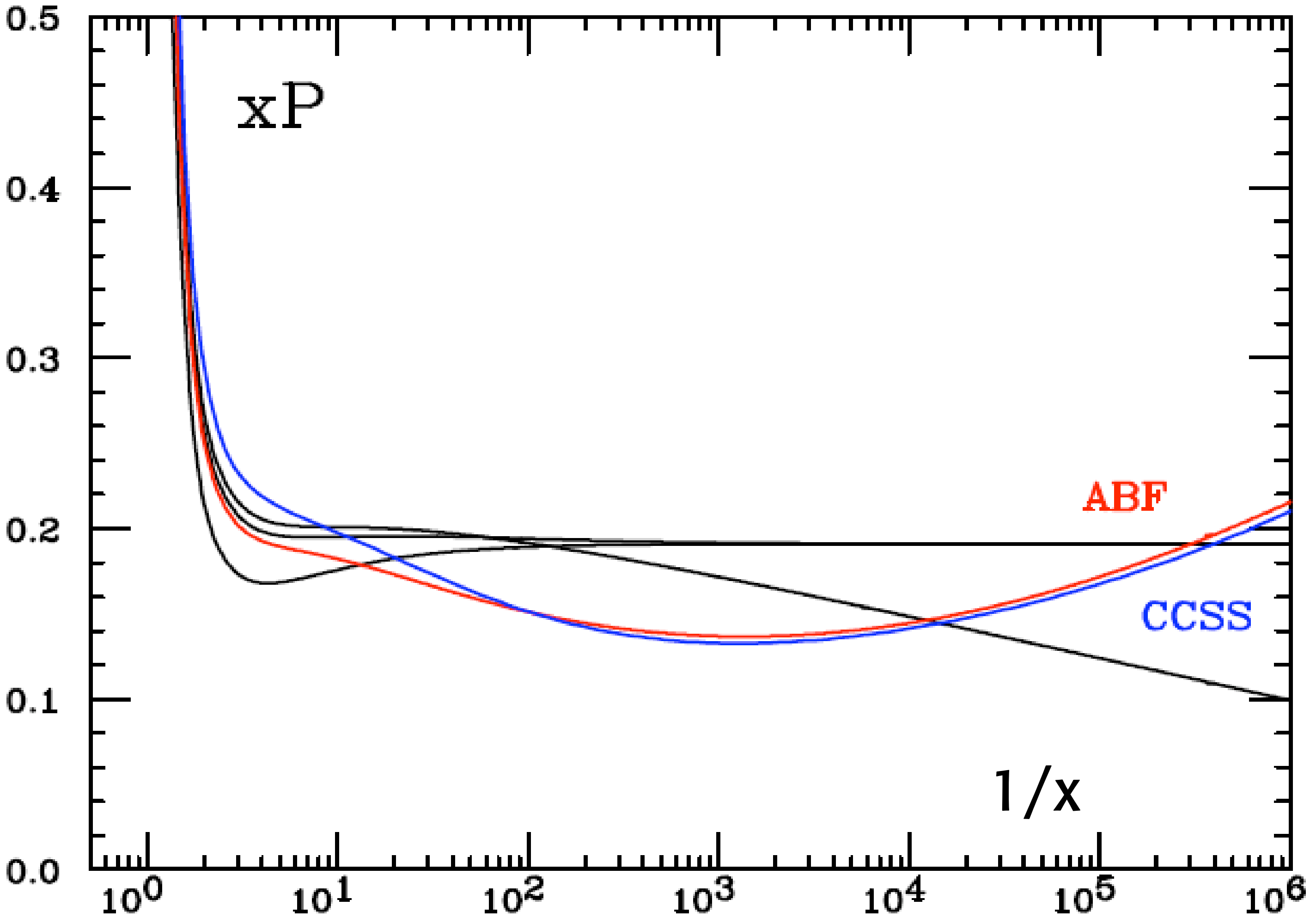
An expanded view at small N

The perturbative γ has poles at N=0. The resummed at $N_B>0$



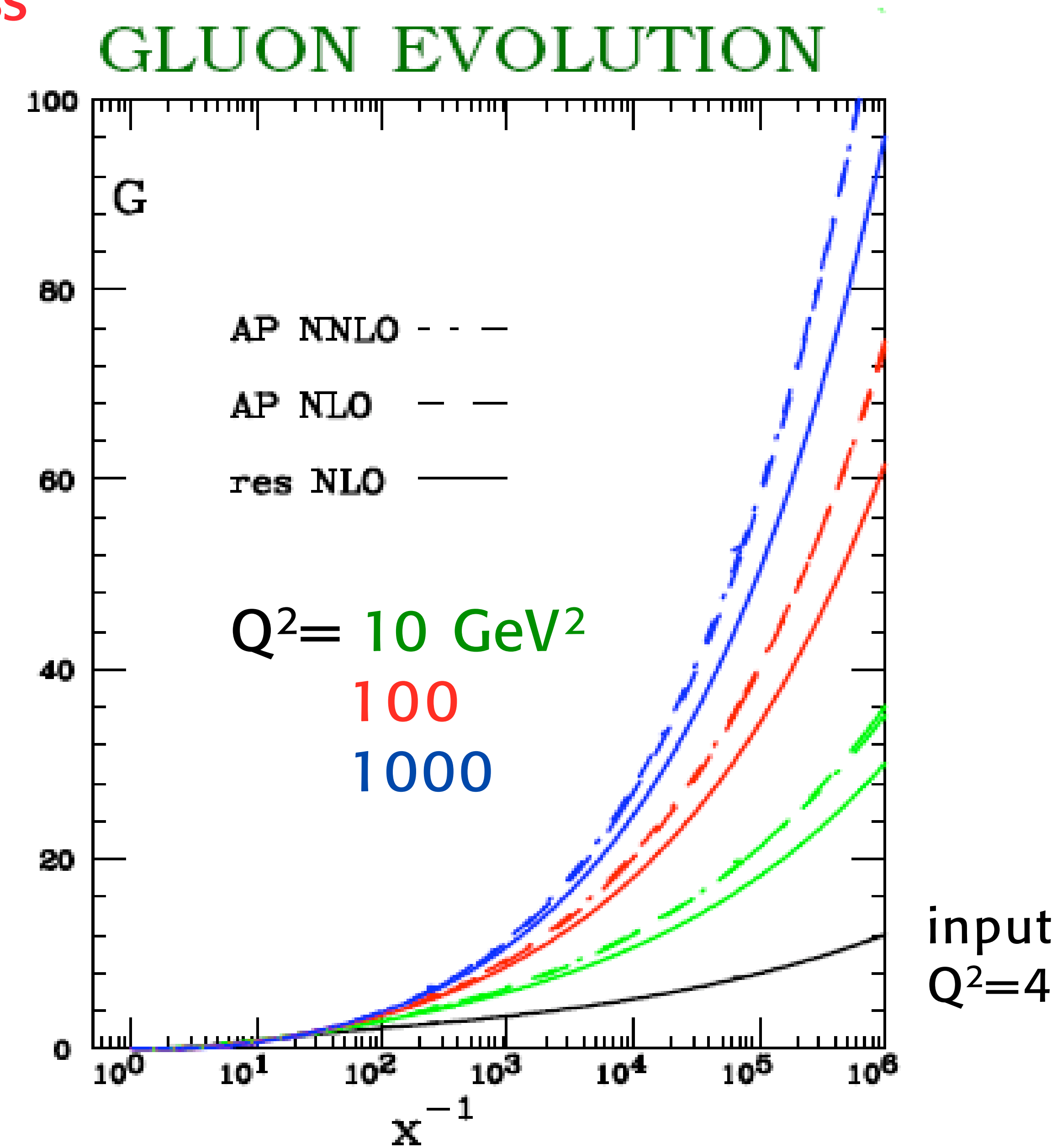
The comparison with Ciafaloni et al (CCSS) is simply too good not to be in part accidental (given the th ambiguities in each method)

The main diff. with CCSS is that they solve numerically the running coupling eqn. (no quadratic expansion near minimum). They do not include NLO GLAP



Due to the dip thereare less scaling violations at HERA than from NLO

Example: the resummed gluon at not too small x is less enhanced



As an effect of the dip there is less evolution for F_2 than at NLO (while for NNLO the opposite is true)

Initial pdfs at Q_0 = 2GeV adjusted so that $F_2^{Res} = F_2^{NLO}$ etc.

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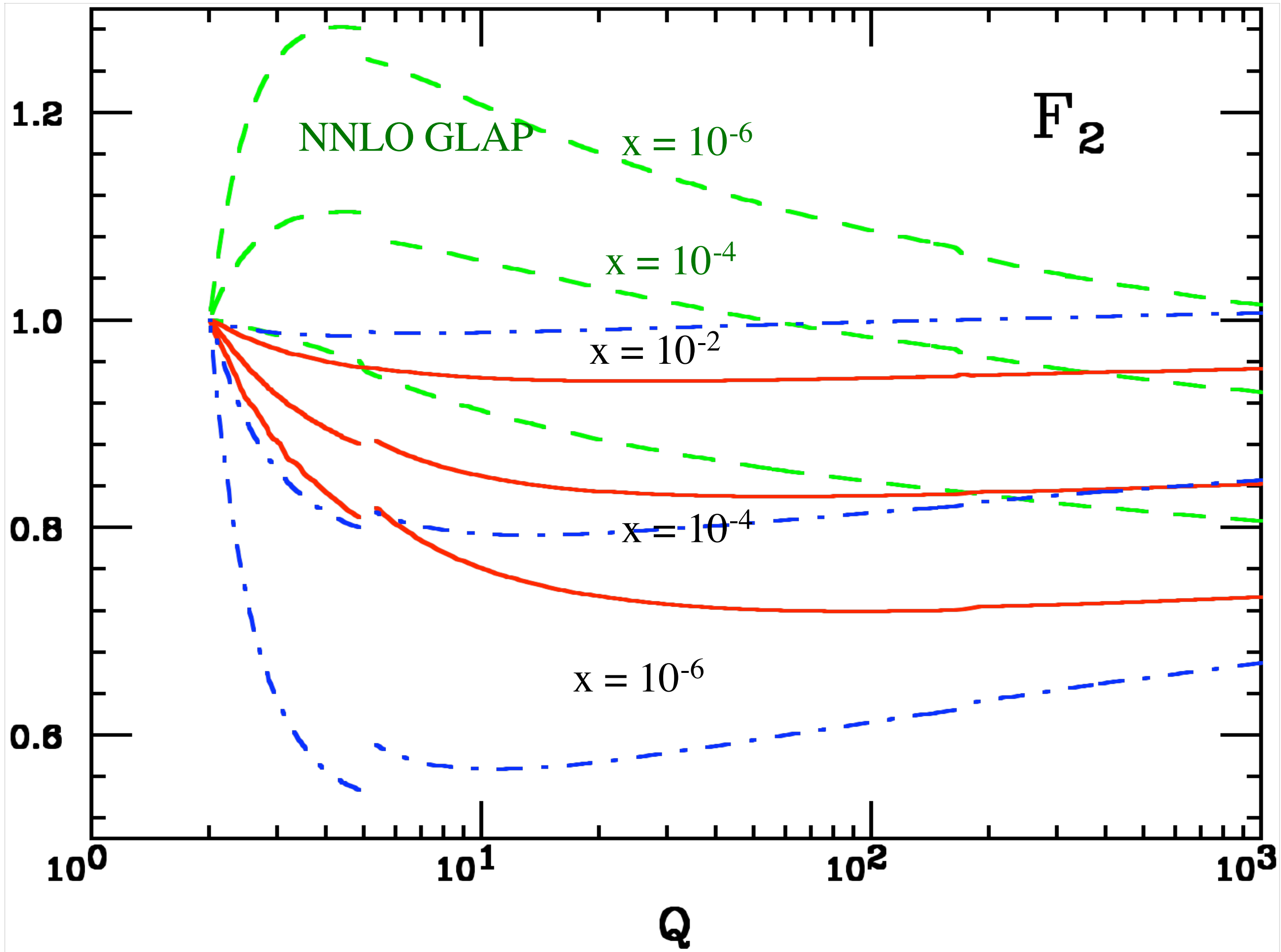
$$K \equiv \frac{F_2^{Res}}{F_2^{NLO}}$$

Effect of resummation opposite to NNLO



NLO Res: Q_0

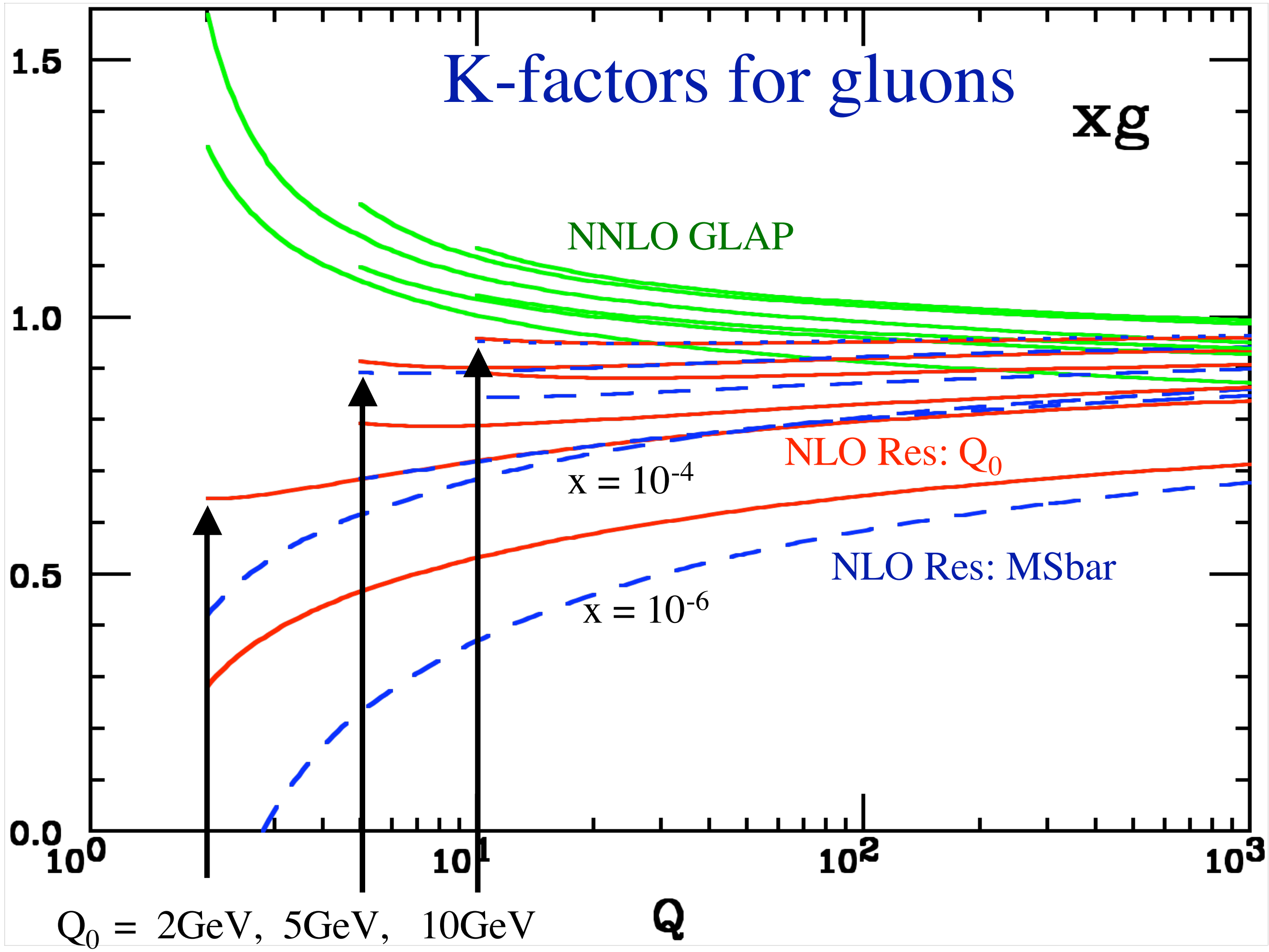
NLO Res: MSbar

Neglecting resummation makes a 10-20% error on pdf's in going from HERA to the LHC

Initial pdfs at Q_0 = 2, 5 and 10GeV adjusted so that $F_2^{Res} = F_2^{NLO}$ etc.

$$K \equiv \frac{g^{Res}}{g^{NLO}}$$

Resummation: fewer gluons at LHC



Summary and Conclusion

- The matching of perturbative QCD evolution at large x and of BFKL at small x is now understood.
- Duality, momentum conservation, symmetry under gluon exchange of the BFKL kernel and running coupling effects are essential
- The resulting asymptotic small x behaviour is much softened with respect to the naive BFKL, in agreement with the data.
- We used these results to construct an improved splitting function that reduces to the pert. result at large x and incorporates BFKL with running coupling effects at small x.
- The impact of the small x corrections is small-to-moderate at the LHC, but would be large at a future hadron supercollider

Extra slides

Improved anomalous dimension

1st iteration: optimal use of $\gamma_{1l}(N)$ and $\chi_0(M)$

$$\gamma_I(\alpha, N) = \alpha\gamma_{1l}(N) + \gamma_s\left(\frac{\alpha}{N}\right) - \frac{\alpha n_c}{\pi N} +$$
$$+\gamma_A(\alpha, N) - \frac{1}{2} + \sqrt{\frac{2}{k_0}\left(\frac{N}{\alpha} - c_0\right)} + \frac{1}{4}\beta_0\alpha - \text{mom sub}$$

Properties:

- Pert. Limit α->o , N fixed

$$\gamma_I(\alpha, N) \longrightarrow \alpha\gamma_{1l}(N) + o(\alpha^2)$$

- Limit α->o , α /N fixed

$$\gamma_I(\alpha, N) \longrightarrow \alpha\gamma_{1l}(N) + \gamma_s\left(\frac{\alpha}{N}\right) - \frac{\alpha n_c}{\pi N} + o(\alpha\,\alpha/N)$$

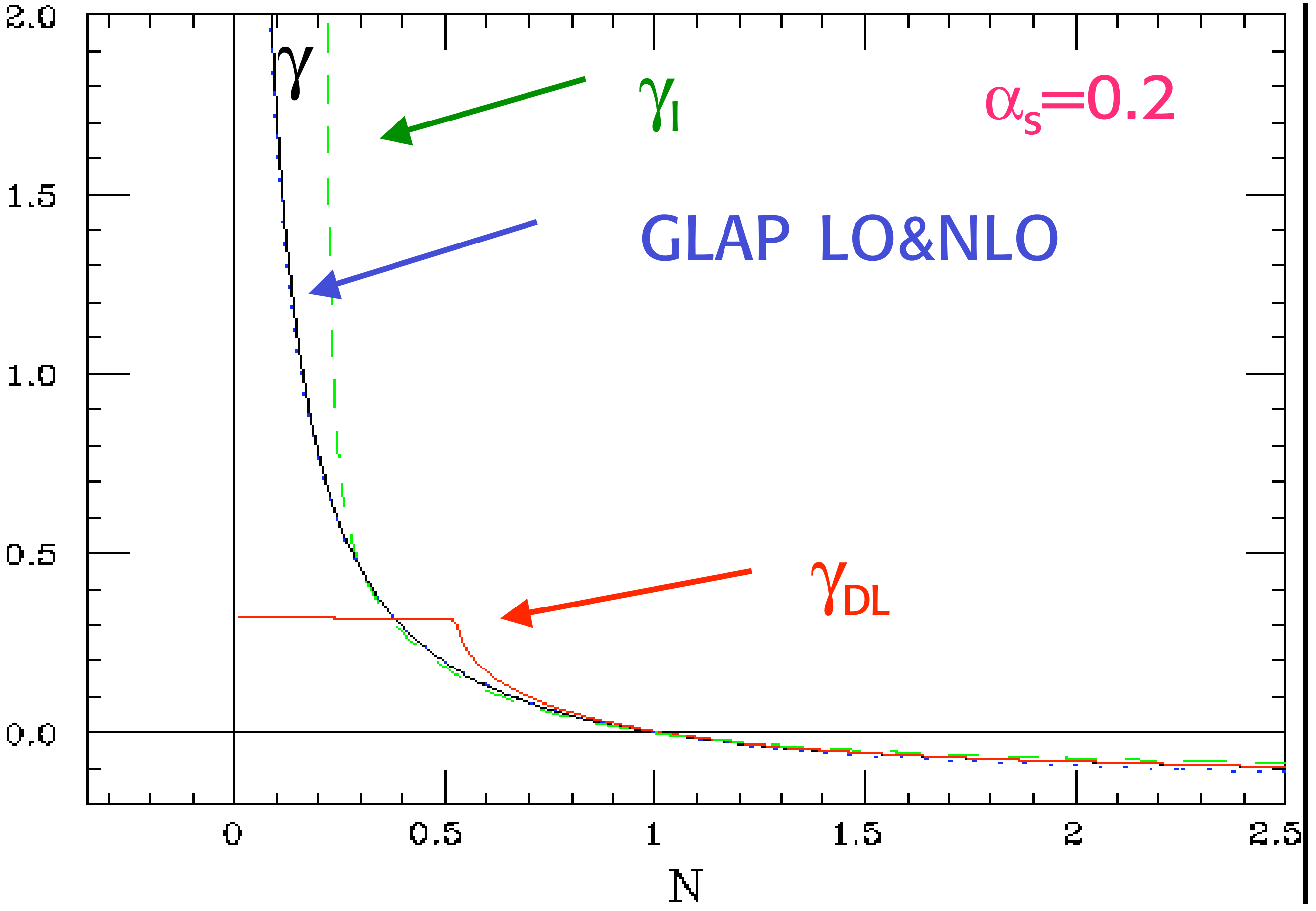
$\alpha\gamma_{1l}(N) \longrightarrow$ Pole in 1/N

$\gamma_s\left(\frac{\alpha}{N}\right) \longrightarrow$ Cut with branch in $\alpha\, c_0$

the Airy term cancels the cut and introduces a pole at $N=N_0$

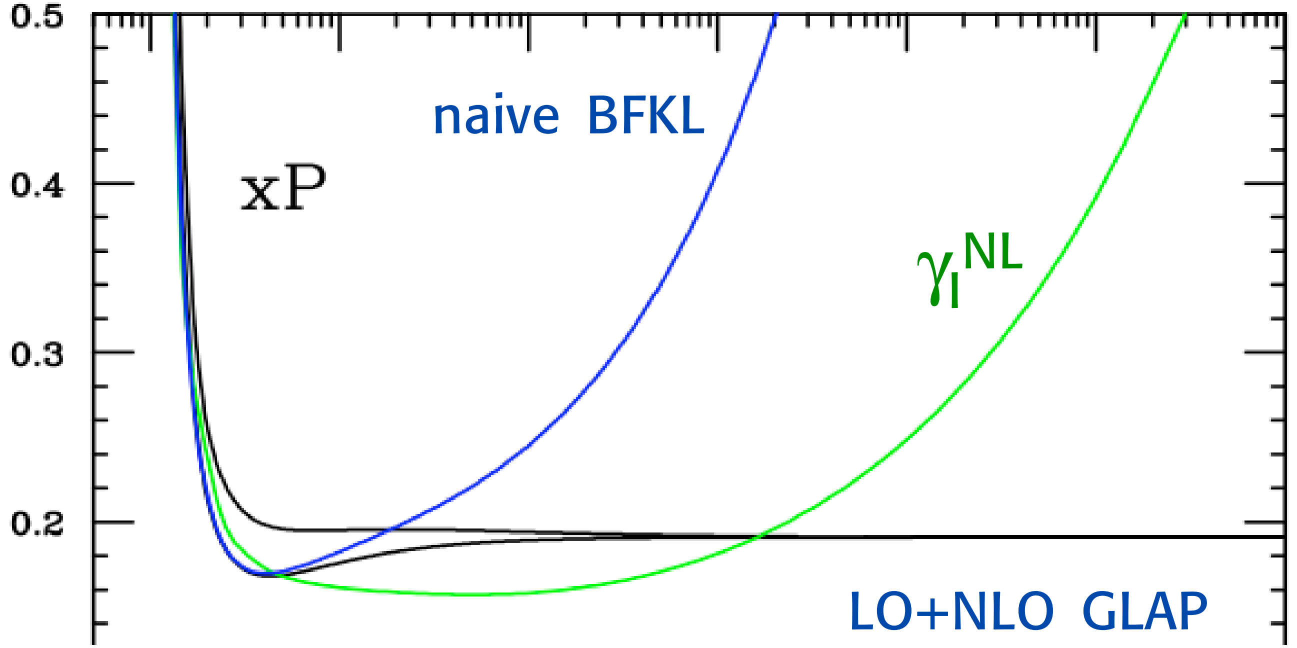
- $$\gamma_I(\alpha, N) = \alpha\gamma_{1l}(N) + \gamma_s\left(\frac{\alpha}{N}\right) - \frac{\alpha n_c}{\pi N} +$$
$$+\gamma_A(\alpha, N) - \frac{1}{2} + \sqrt{\frac{2}{k_0}\left(\frac{N}{\alpha} - c_0\right)} + \frac{1}{4}\beta_0\alpha - \text{mom sub}$$

- $$\gamma_{DL}(\alpha,N) = \alpha\gamma_{1l}(N) + \gamma_s\left(\frac{\alpha}{N}\right) - \frac{\alpha n_c}{\pi N} - \text{mom sub}$$



Also adding the NLO pert. anom. dim., this is the best one can get from $\chi_0(M)$.

$$\gamma_I^{NL}(\alpha_s, N) = [\alpha_s\gamma_0(N) + \alpha_s^2\gamma_1(N) + \gamma_s(\tfrac{\alpha_s}{N}) - \tfrac{n_c\alpha_s}{\pi N}] +$$
$$+ \gamma_A(c_0, \alpha_s, N) - \tfrac{1}{2} + \sqrt{\tfrac{2}{\kappa_0\alpha_s}[N - \alpha_s c_0]} + \tfrac{1}{4}\beta_0\alpha_s(1 + \frac{\alpha_s}{N}c_0) - \text{mom. sub.}$$



Here, since we started from χ_0, symmetrization was not used and we only have naive BFKL + running coupling effect