From nuclei to infinite matter using Quantum Monte Carlo methods

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Microscopic Theories of Nuclear Structure, Dynamics and Electroweak Currents
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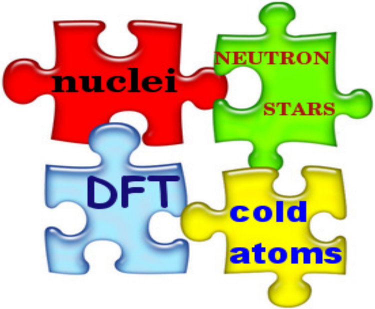
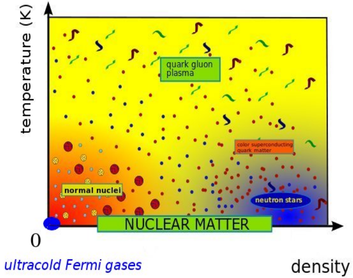
Overview





- Structure of light nuclei
- Electro-weak response functions of nuclei
- Cold atoms and low density-neutron matter
- High density neutron matter, symmetry energy and neutron stars
- Inhomogeneous neutron matter, ab-initio vs DFT

Motivation

At "nuclear" energies, understanding the structure of nuclei and electroweak interactions very challenging and important!

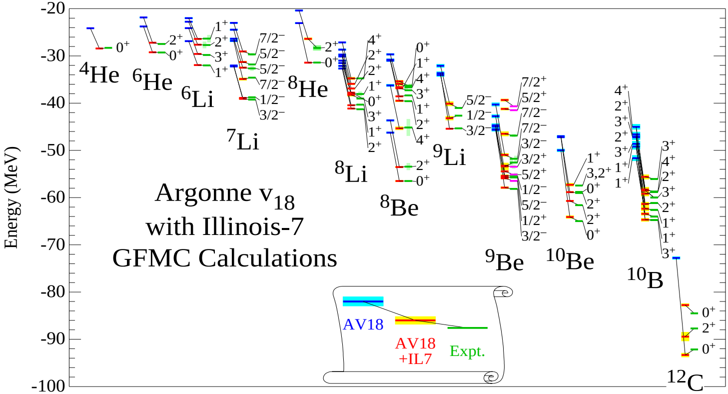
Understanding Nuclei:

- Nuclear interactions and structure
- Exotic nuclei - neutron rich
- Electroweak processes

Relevance: (just few examples...)

- Neutrino scattering in nuclei (neutrino oscillation experiments)
- Neutrinoless Double Beta Decay
- Neutrino interactions in supernovae and neutron stars, nucleosynthesis

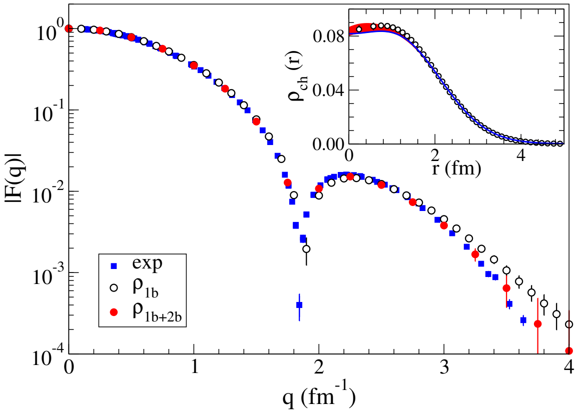
Light nuclei spectrum computed with GFMC



Carlson, Gandolfi, Pederiva, Pieper, Schiavilla, Schmidt, Wiringa, RMP (2015)

Also radii, densities, matrix elements, ...

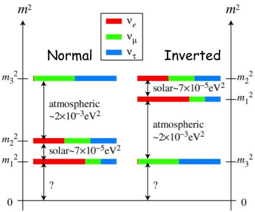
$$|F(q)| = \langle\psi|\rho_q|\psi\rangle \qquad \rho_q = \sum_i \rho_q(i) + \sum_{i<j} \rho_q(ij)$$



Lovato, Gandolfi, Butler, Carlson, Lusk, Pieper, Schiavilla, PRL (2013)

Neutrino oscillations

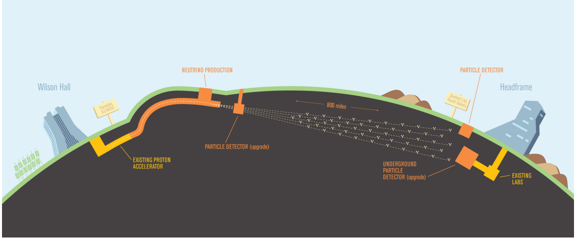
DUNE - Deep Underground Neutrino Experiment - to measure neutrino oscillations and CP violation



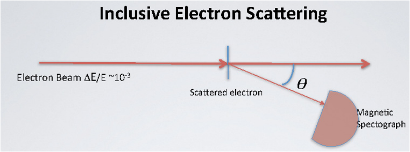
Simplified 2 flavors evolution (CP violation non included):

$$P_{\alpha\to\beta} = \sin^2(2\theta_{\alpha\beta})\sin^2\left(1.267\frac{\Delta m^2_{\alpha\beta}L}{E}\frac{GeV}{eV^2 km}\right)$$

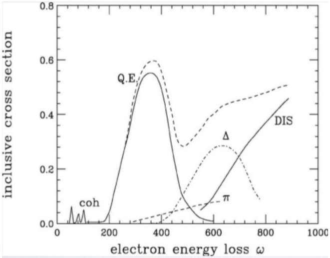
Need to know E!



Electron energy easy to know:

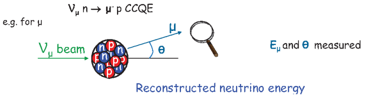


Electron scattering in nuclei:



Benhar, Day, Sick, RMP (2008)

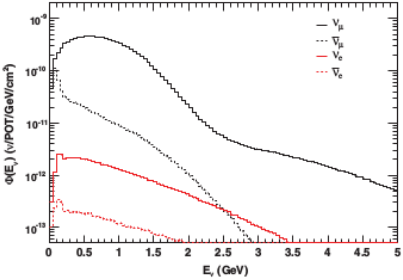
E_ν difficult to reconstruct. Example: CCQE process



$$\overline{E_\nu} = \frac{E_\mu - m_\mu^2/(2M)}{1-(E_\mu - P_\mu\cos\theta)/M}$$

Neutral current process even more difficult.

Simulation of neutrino energy distribution:



MiniBooNE Coll., PRD (2009)

Knowledge of cross-section
+ near detector
= determination of E_ν

Inclusive scattering

Electron scattering:

$$\left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}\epsilon'\mathrm{d}\Omega}\right)_{\nu/\overline{\nu}} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_M \left[\frac{Q^4}{q^4} R_L(q,\omega) + \left(\frac{Q^2}{2q^2} + \tan^2\frac{\theta}{2}\right) R_T(q,\omega)\right]$$

R_T and R_L transverse and longitudinal response functions.

Neutrino scattering:

$$\left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}\epsilon'\mathrm{d}\Omega}\right)_{\nu/\overline{\nu}} = \frac{G^2}{2\pi^2} k'\epsilon' \cos^2\frac{\theta}{2}\left[R_{00}(q,\omega) + \frac{\omega^2}{q^2} R_{zz}(q,\omega) - \frac{\omega}{q} R_{0z}(q,\omega) + \left(\tan^2\frac{\theta}{2} + \frac{Q^2}{2\,q^2}\right) R_{xx+yy}(q,\omega) \mp \tan\frac{\theta}{2}\sqrt{\tan^2\frac{\theta}{2} + \frac{Q^2}{q^2}}\, R_{xy}(q,\omega)\right]$$

R_{00}, R_{zz}, R_{0z}, R_{xx+yy}, and R_{xy} neutrino response functions.

R_{xy} is important for ν vs $\bar{\nu}$ processes.

Response functions

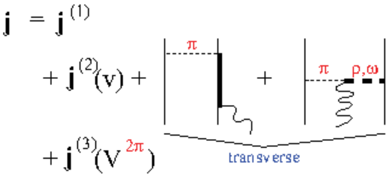
Response to an operator $\hat{J}$:

$S(q,\omega) = \sum_n \left|\langle\psi_n|\hat{J}(q)|\psi_0\rangle\right|^2 \delta(E_n - E_0 - \omega)$

Euclidean (or imaginary-time) response:

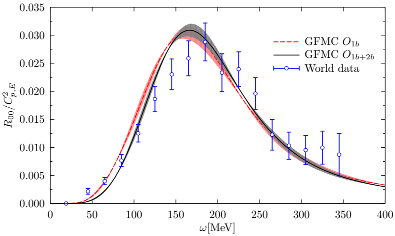
$$S(q,\tau) = \int d\omega\, e^{-\tau\omega} S(q,\omega) = \langle\psi_0|\hat{J}^\dagger(q) e^{-i\tau H} \hat{J}(q)|\psi_0\rangle$$

Using QMC we can calculate **exactly** $S(q,\tau)$



Electromagnetic response functions of ^{12}C

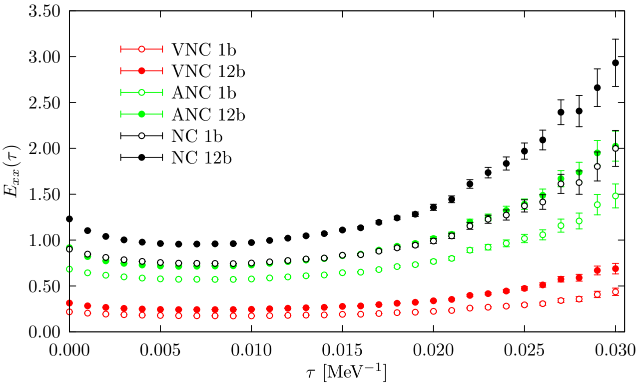
Electromagnetic longitudinal and transverse response functions of ^{12}C (q=570 MeV)



Lovato et al., PRL (2016).

Role of two-body currents crucial to reproduce data.

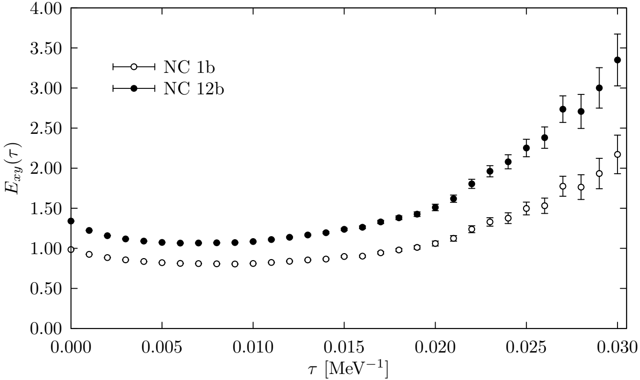
Transverse vector, axial, and neutral current euclidean response functions, with one- and two-body operators. ^{12}C, q=570 MeV



Lovato, Gandolfi, Carlson, Pieper, Schiavilla, PRC (2015)

Euclidean electroweak response functions of ^{12}C

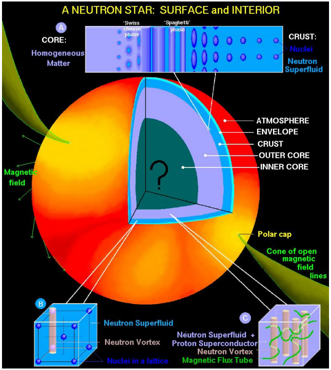
R_{xy} term responsible for ν vs $\bar{\nu}$ response. ^{12}C, q=570 MeV



Lovato, Gandolfi, Carlson, Pieper, Schiavilla, PRC (2015)

Two-body currents enhances the ν vs $\bar{\nu}$ response.

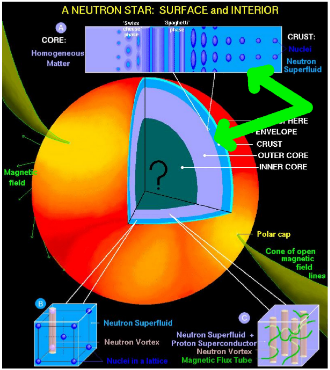
Neutron star is a wonderful natural laboratory



D. Page

- Atmosphere: atomic and plasma physics
- Crust: physics of superfluids (neutrons, vortex), solid state physics (nuclei)
- Inner crust: deformed nuclei, pasta phase
- Outer core: nuclear matter
- Inner core: hyperons? quark matter? π or K condensates?

Low-density neutron matter and unitary Fermi gas



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Very Low Density Neutron Matter: cold atoms

Low density neutron matter $\rightarrow$ unitary limit:

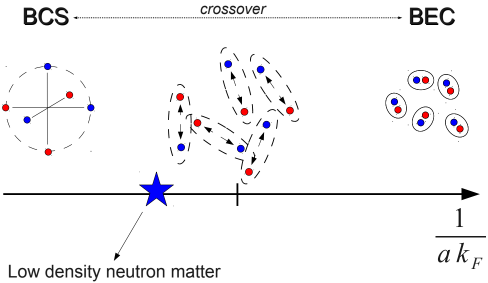
$$r_{eff} \ll r_0 \ll |a| \,, \quad r_{eff} = 0 \,, \; |a| = \infty$$

If these limits are satisfied the result is model independent

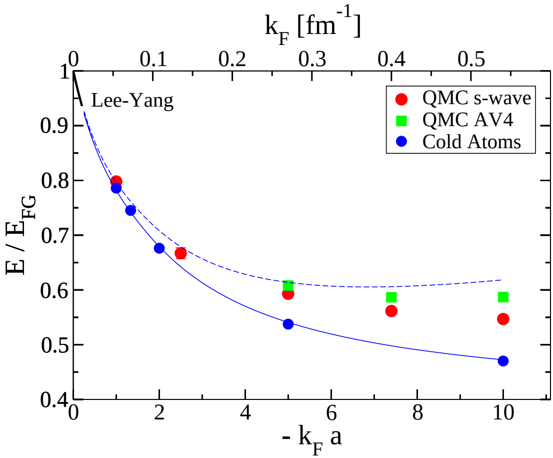
Only one scale: $\rightarrow E = \xi E_{FG}$

Why neutron matter and cold atoms are so similar?

- NN scattering length is large and negative, $a = -18.5$ fm
- NN effective range is small, $r_{eff} = 2.7$ fm



Fermi gas and low-density neutron matter

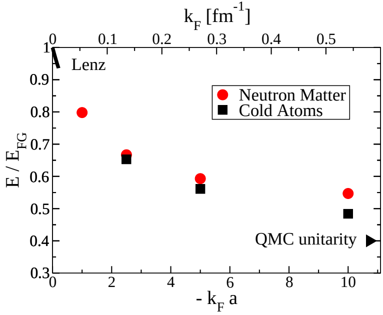


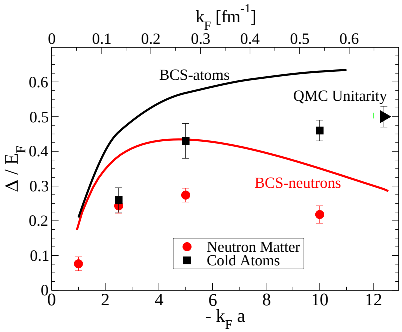
Carlson, Gandolfi, Gezerlis, PTEP 01A209 (2012).

EOS and pairing gap important to understand the neutron star crust, but also neutron-rich nuclei.

Dilute neutron matter

EOS and pairing gap of low–density neutron matter:





Gezerlis, Carlson (2008)

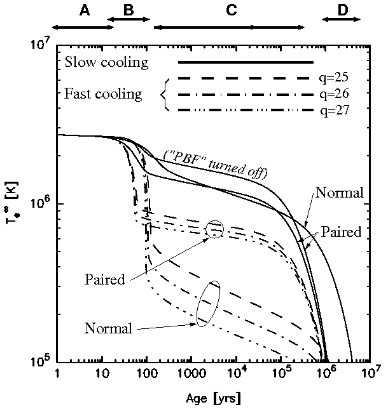
Pairing gap and neutron stars

The pairing gap is fundamental for the cooling of neutron stars.

Neutron star crust made of nuclei arranged on a lattice surrounded by a gas of neutrons.

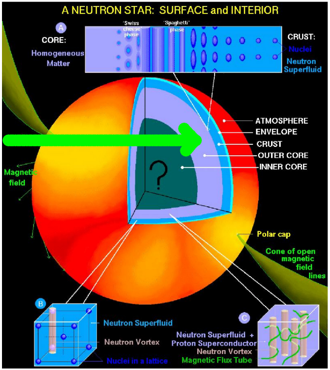
Specific heat suppressed by superfluidity (similarly to the superconducting mechanism).

Cooling dependent to the pairing gap!



D. Page (2012)

Neutron matter at saturation densities



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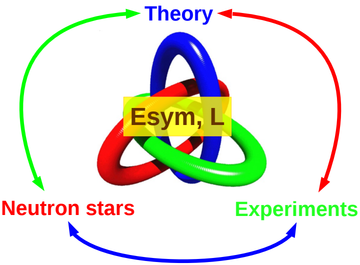
Neutron matter equation of state

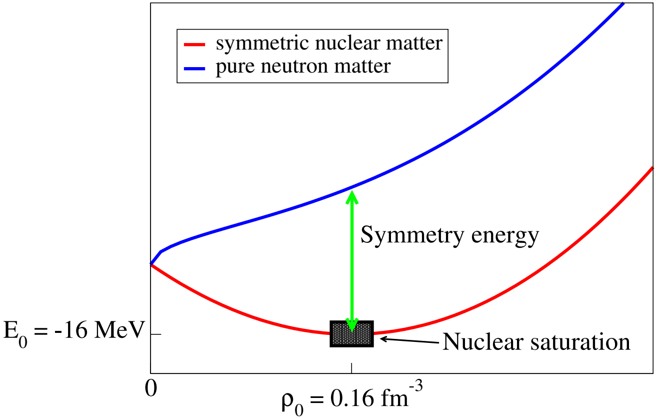
Why to study neutron matter at nuclear densities?

- EOS of neutron matter gives the symmetry energy and its slope.
- The three-neutron force ($T = 3/2$) very weak in light nuclei, while $T = 1/2$ is the dominant part. No direct $T = 3/2$ experiments available.

Why to study symmetry energy?



What is the Symmetry energy?

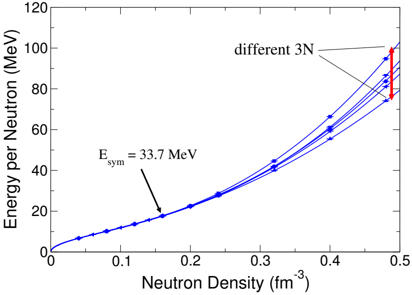


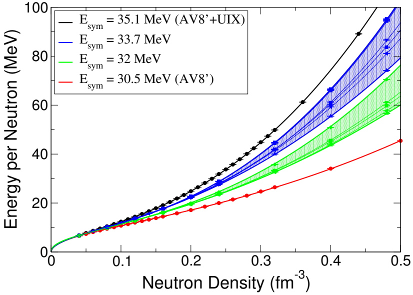
Assumption from experiments:

$$E_{SNM}(\rho_0) = -16MeV\,, \quad \rho_0 = 0.16fm^{-3}\,, \quad E_{sym} = E_{PNM}(\rho_0) + 16$$

Neutron matter and symmetry energy

Uncertainty of 3N vs uncertainty of E_{sym}



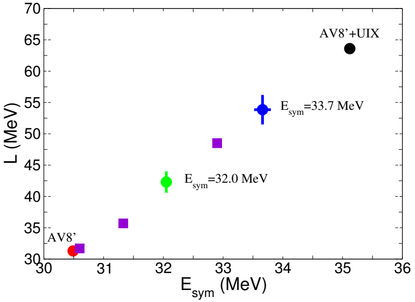


Gandolfi, Carlson, Reddy PRC (2012).

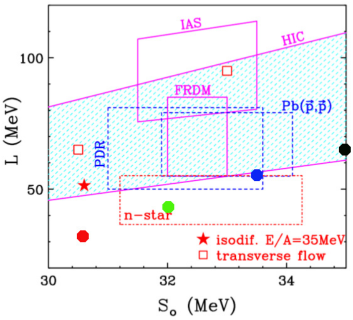
Neutron matter and symmetry energy

From the EOS, we can fit the symmetry energy around ρ_0 using

$$E_{sym}(\rho) = E_{sym} + \frac{L}{3}\frac{\rho - 0.16}{0.16} + \cdots$$



Gandolfi *et al.*. EPJA (2013)



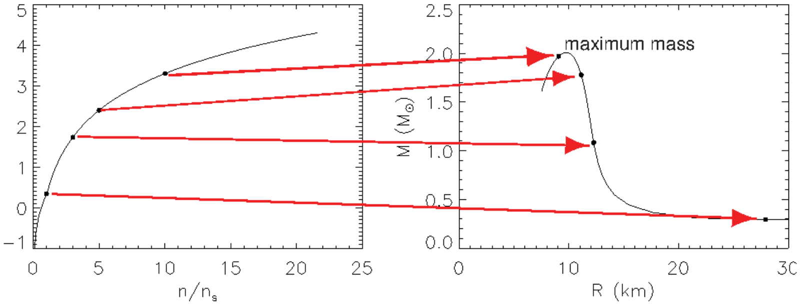
Tsang *et al.*, PRC (2012)

Neutron matter and neutron star structure

TOV equations:

$$\frac{dP}{dr} = -\frac{G[m(r) + 4\pi r^3 P/c^2][\epsilon + P/c^2]}{r[r - 2Gm(r)/c^2]},$$

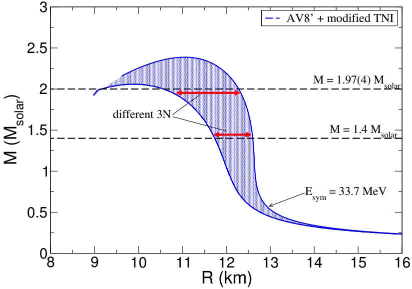
$$\frac{dm(r)}{dr} = 4\pi\epsilon r^2,$$

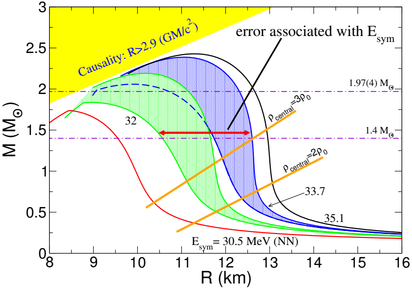


J. Lattimer

Neutron star structure

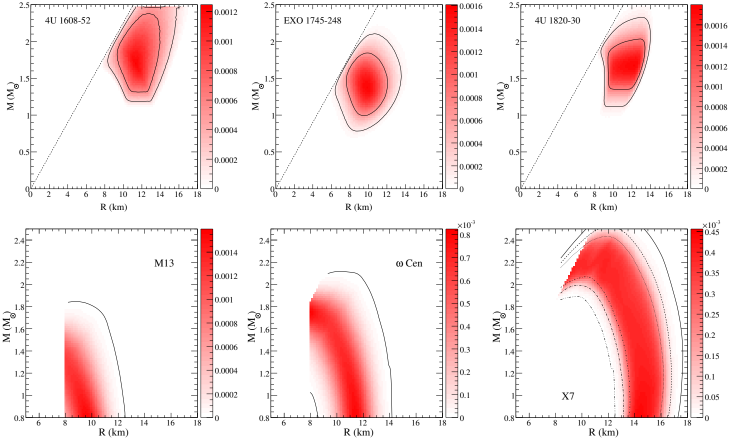
EOS used to solve the TOV equations to get the Mass-Radius relationship.





Accurate measurement of E_{sym} would put a constraint to the radius of neutron stars, **OR** observation of M and R would constrain E_{sym}!

Observations of the mass-radius relation are becoming available:

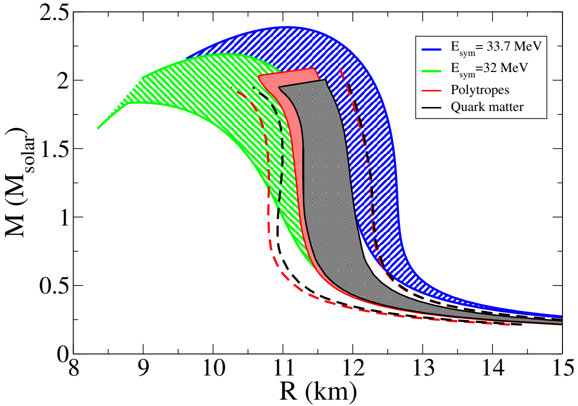


Steiner, Lattimer, Brown, ApJ (2010)

Neutron star observations can be used to 'measure' the EOS and constrain E_{sym} and L.

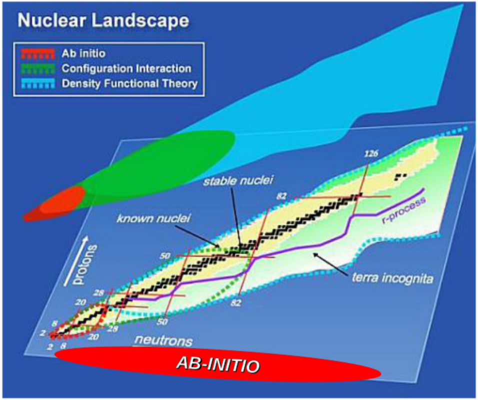
Neutron star matter really matters!

Here an 'astrophysical measurement'



Steiner, Gandolfi PRL (2012).

Nuclei and DFT



Large nuclei and inhomogeneous matter have to be studied using density functional theory.

Note: if we know the nuclear DF, then we know the ground-state of any nuclear system.

Density Functional theory

For any system, if we have the functional $E[\rho]$, we can exactly solve the ground-state. In the real life it's almost impossible to have the exact one.

Reasonable guess for cold atoms and nuclear systems:

$$E[\rho] = \xi E_{FG} + F[\nabla\rho] + \ldots$$

$$E[\rho] = E_{FG} + F[\rho_p, \rho_n] + G[\nabla\rho_p, \nabla\rho_n] + \ldots$$

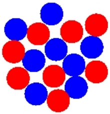
Various parameters, $\sim 14-18$, fitted to

- Nuclear matter EOS, mostly $F[\rho_p, \rho_n]$
- Neutron matter EOS, mostly $F[\rho_n]$
- Selected nuclei, mostly $G[\nabla\rho_p, \nabla\rho_n]$

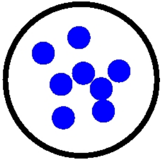
What about non-homogeneous neutron matter, $G[\nabla\rho_n]$?

Neutron drops

What are, and why to study neutron drops?



NP self–bound



N confined

They model **inhomogeneous neutron matter**. Ab-initio→DFT

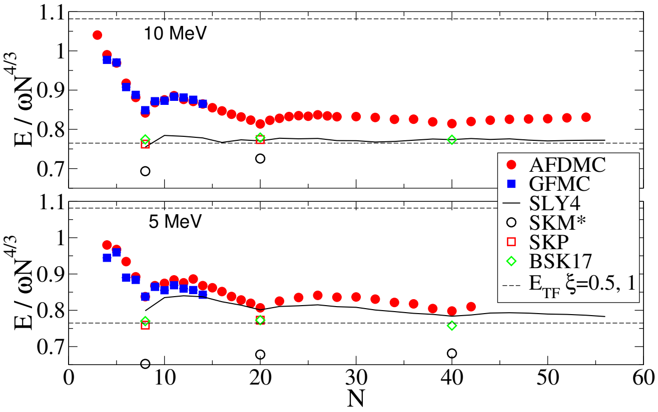
Neutrons are confined by an external potential:

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \sum_i V_{ext}(r_i)$$

V_{ext} tuned to change boundary conditions and densities.

Neutron drops, harmonic oscillator well

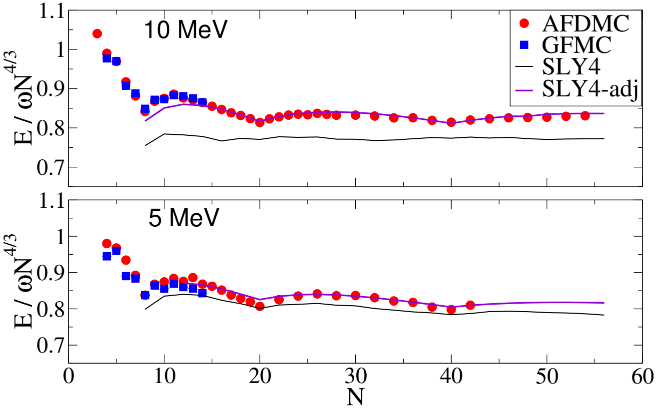
External well: harmonic oscillator with $\hbar\omega$=5, 10 MeV.



Skyrme systematically overbind neutron drops.

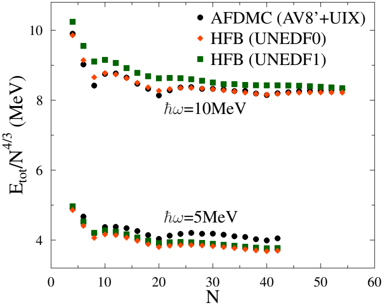
Neutron drops, adjusted Skyrme force

The **missing repulsion** obtained by (mainly) adjusting the gradient term $G_d[\nabla\rho_n]^2$.

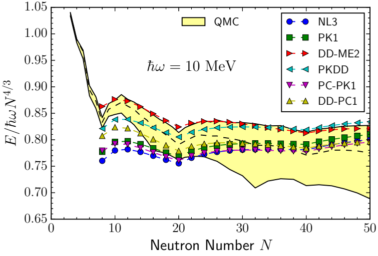


Gandolfi, Carlson, Pieper PRL (2011).

Ab-initio calculations meant as "experimental data" to benchmark DFT:



M. Kortelainen, et al. Phys. Rev. C 85, 024304 (2012)



Zhao, Gandolfi, Phys. Rev. C 94, 041302(R) (2016)

Gradient term

Impact:

- Medium large neutron-rich nuclei
- Phases in the crust of neutron stars
- Experimental measurement of symmetry energy of nuclear matter
- Benchmark other theoretical methods,
  Maris, Vary, Gandolfi, Carlson, Pieper, PRC (2013)

All the above fields need predictions because experiments are (almost) not available!

Conclusions

- QMC powerful tool to study strongly interacting systems
- Energy and other properties of nuclei well described
- Electron scattering in ^{12}C calculated using GFMC. Good agreement with experiments. One- and two-body vector currents tested.
- Two-body axial currents show a similar enhancement in Euclidean response functions and sum rules.
- Properties of low-density neutron matter similar with cold atoms (experiments can be done)
- Strong relations between E_{sym} and the structure of neutron stars
- Neutron star observations becoming relevant in nuclear physics
- Ab-initio calculations important to constrain nuclear density functionals

Last lesson...

The last but very important lesson.

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Always acknowledge the funding agencies!!!

Los Alamos NATIONAL LABORATORY EST.1943

NUCLEI Nuclear Computational Low-Energy Initiative

www.computingnuclei.org

U.S. DEPARTMENT OF ENERGY