High-momentum components

how to (and how *not* to) measure them

Ingo Sick

A short look to other fields

- atoms
- liquids
- nucleons (?)

Main topic: nuclei

Reason for interest:

- high-k = signature of physics beyond mean-field
  - short-range correlations
- does not involve phenomenology of MF
- directly related to underlying V_{NN}

Emphasis of talk

- not so much: how to measure high-k components
- rather
  - what have learned from past attempts
  - how *not* to try

Atoms

High-quality wave functions calculable

strongly dominated by mean-field aspects

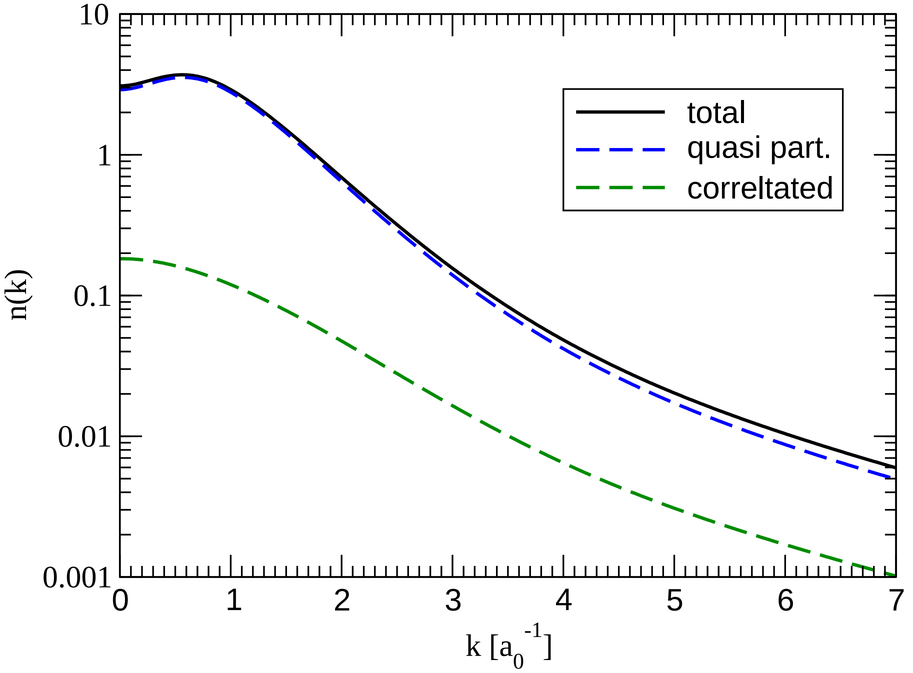
effect of e-e correlations small

Typical measurements

Compton profile (γ,e), positron annihilation (e^+,2γ), seldom (e,2e)

For atom like *e.g.* Neon

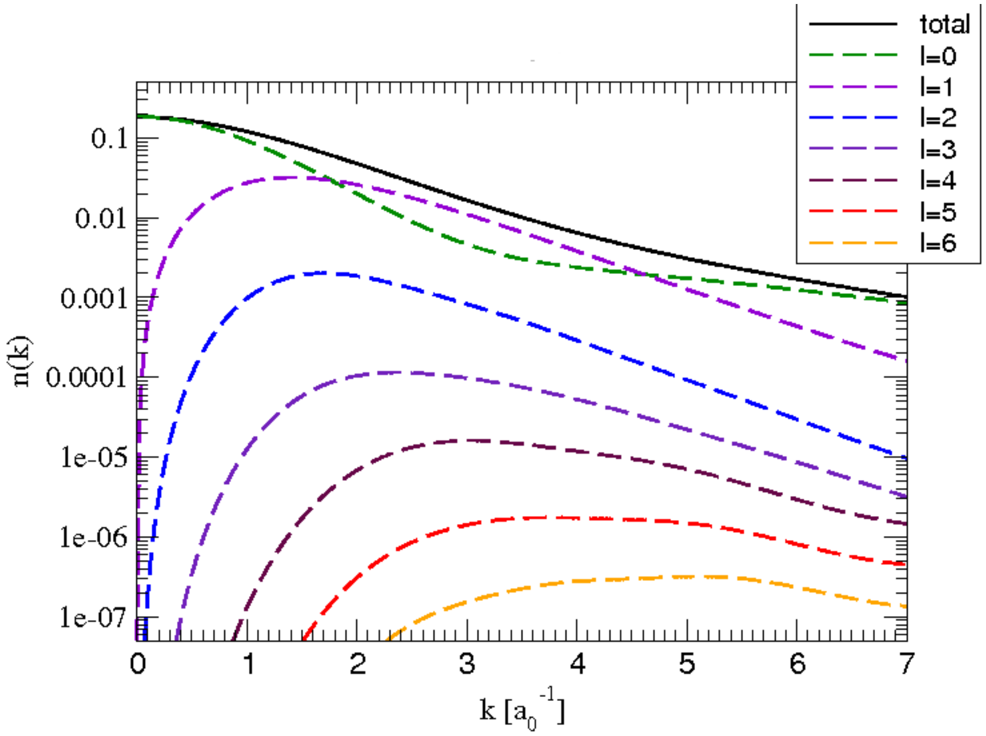
calculation Barbieri *et al.*



Find:

small effect of correlations

Contributions of individual shells



high-*k* tail from 1s-state only

high-*k* from confinement to nuclear neighborhood

***not* due to e-e correlations**

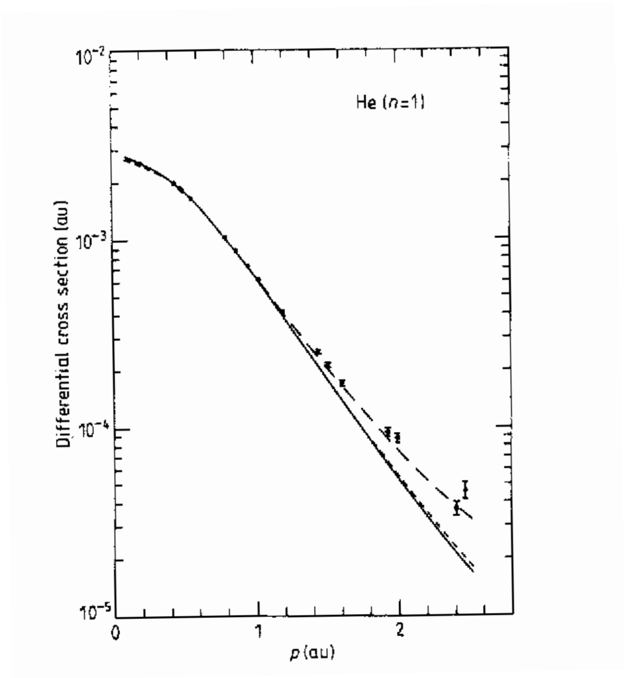
High-k in atomic helium

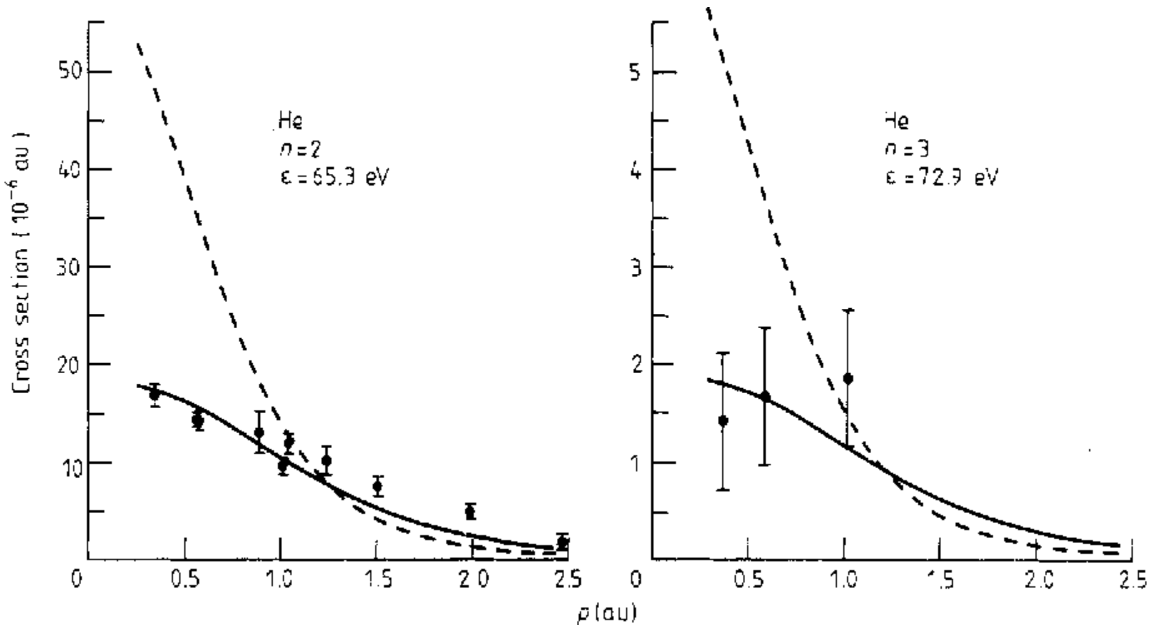
studied via (e,2e), Cook *et al.*

small effect of correlations (—— → - - -)

$\ll$ than Coulomb distortion (- - - → — — —)

(e,2e) to excited states





solid : correlated wave function

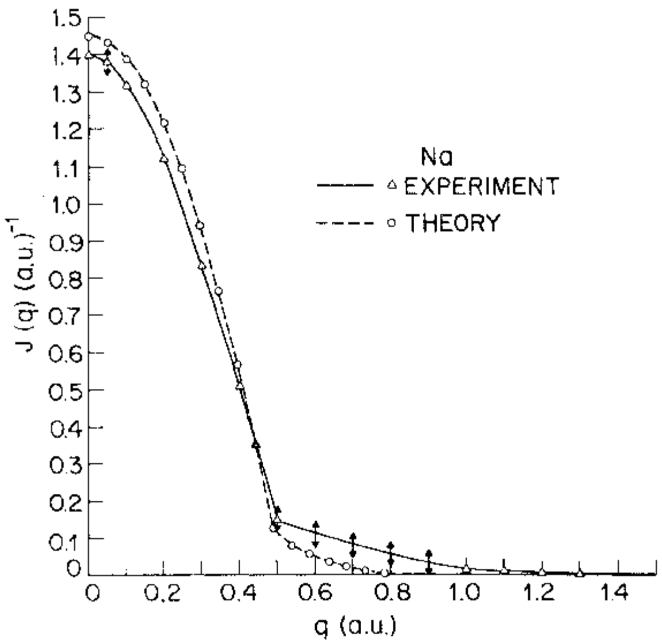
dashed: HF

correlations visible in $He^+(2s)$ and $He^+(3s)$

(see discussion of $S(\boldsymbol{k}, E)$ below)

Main interest in atomic high-k

not so much: e-e correlations
rather: molecular structure of solid
(lattice leading to high-k tail)
correlated conduction electrons
see Compton profile of Na (core-e removed)

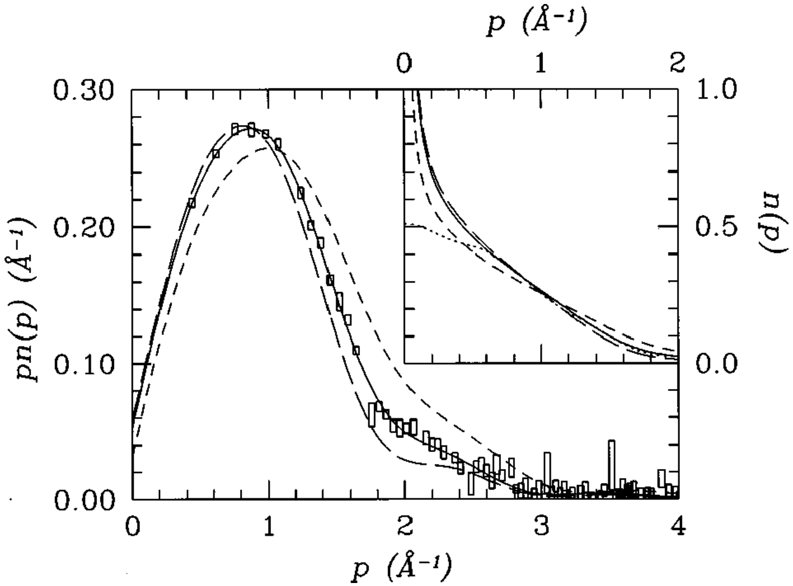


Liquids

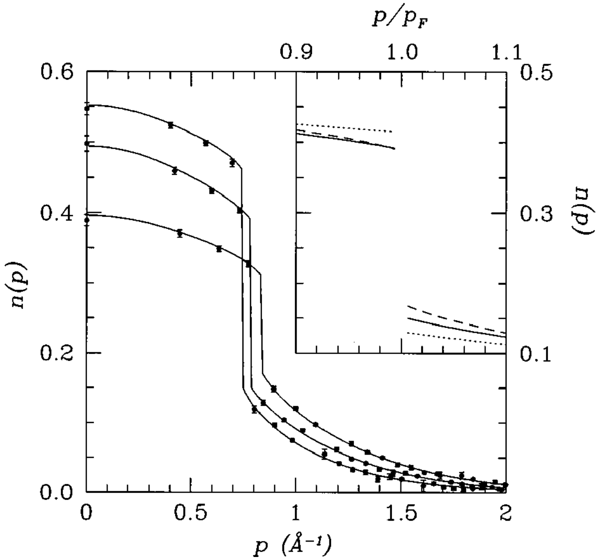
prototypes: L^4He, L^3He, mixtures
strong correlations due to repulsive core of He-He interaction
Lennard-Jones type potential $r^{-12} - r^{-6}$
sophisticated calculations, *e.g.* Diffusion Monte-Carlo

Examples: Moroni *et al.*

^{4}He

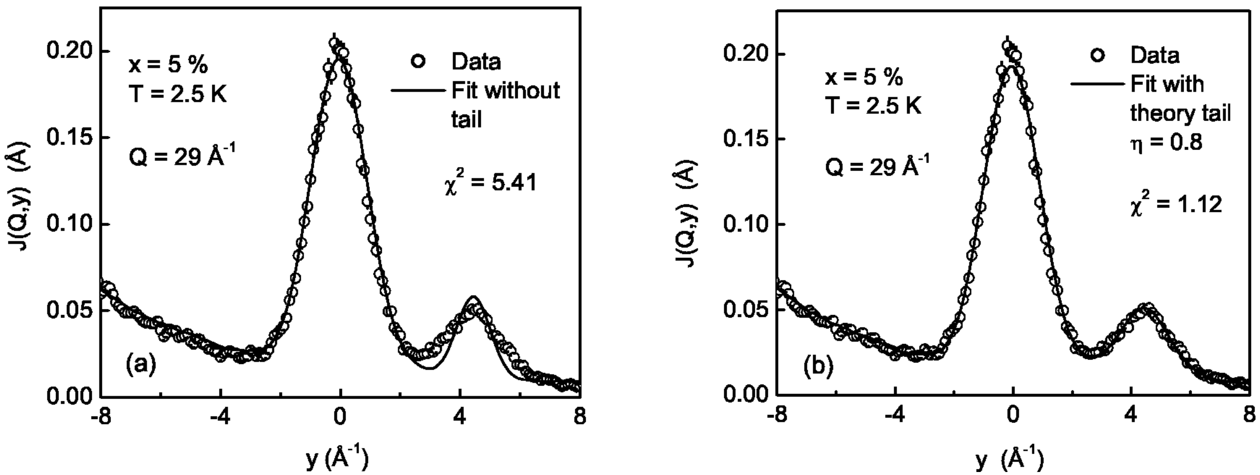


^{3}He

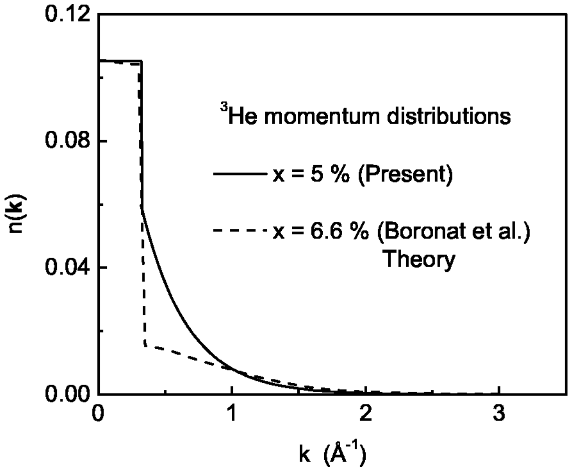


Data: from (n,n'), ∼1eV, Diallo *et al.*

quasi-elastic scattering: high-k in tails of q.e. peak



Better agreement in dip *with* tail of $n(k)$



Main interest to condensed matter physics:

not high-k

rather % Bose condensate $\rightarrow \delta(k=0)$ peak

should occur for superfluid L^4He

$\delta(y=0)$ hardly visible on q.e.-peak. Reason: FSI

Detailed studies of FSI-effects in (n,n'): of interest to nuclear physics!

main effect: folding of IA response

width of folding function proportional to σ_{tot} of He-He interaction

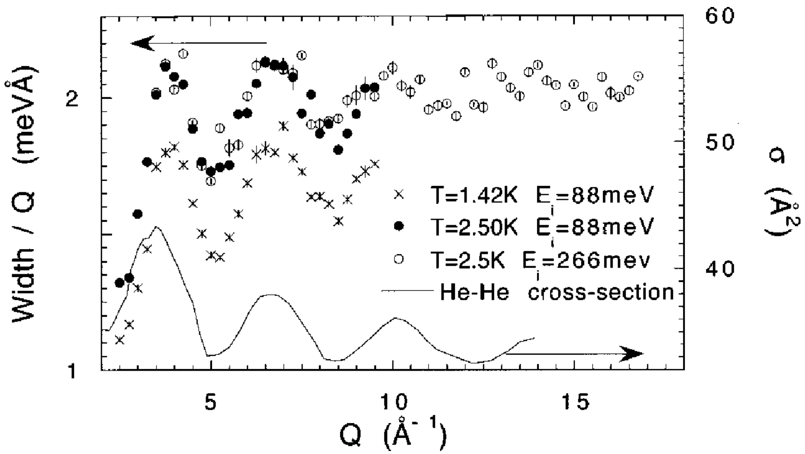
smears out δ-function peak

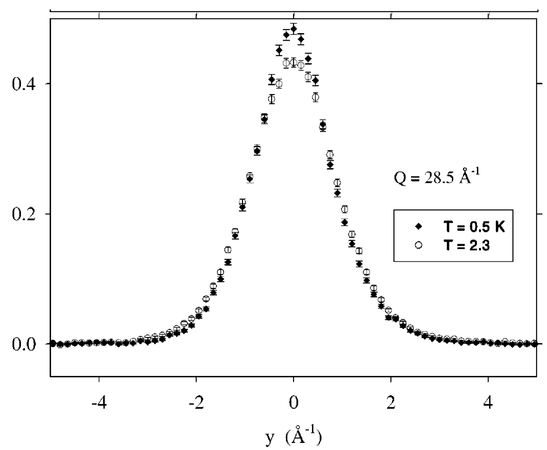
FSI-theory can be verified

σ_{tot} is oscillating function of recoil-He energy

$\rightarrow$ folding width oscillating function of q

nicely observed in data $\rightarrow$ can see effect of BC





High-k tail in nucleon structure functions?

know virtually nothing
DIS data at large x obscured by resonances
interpretation based on constituents with mass $x \cdot m_N$ murky anyway

theoretical predictions??

none I am aware of
finite lattice spacing not helpful

asymptotic freedom $\rightarrow$ minimal high-k??

would be interesting!

Nuclei

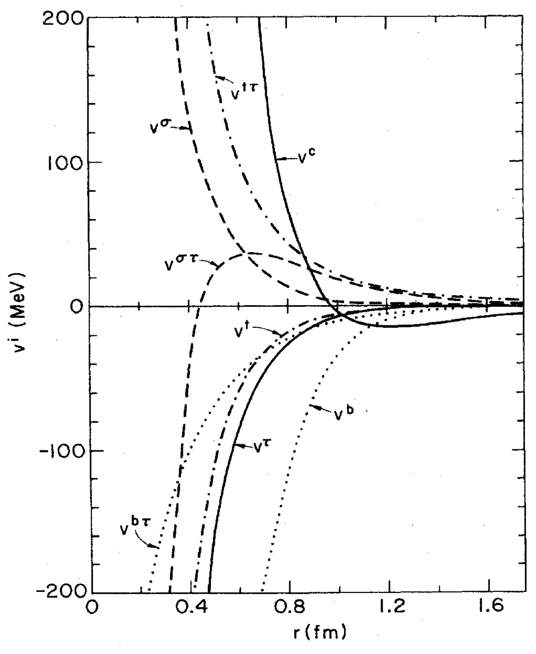
Important high-k components

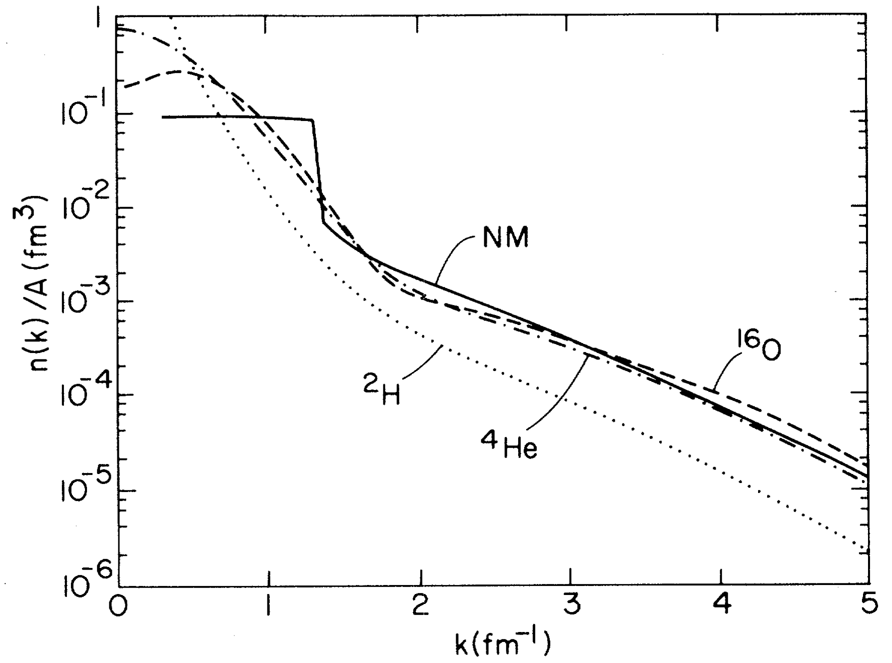
V_{NN} in some channels strongly repulsive at small r

channel dependence complicates exact solution of Schrödinger equation

core leads to high-k tail of $n(k)$

rather universal for nuclei A=2...∞



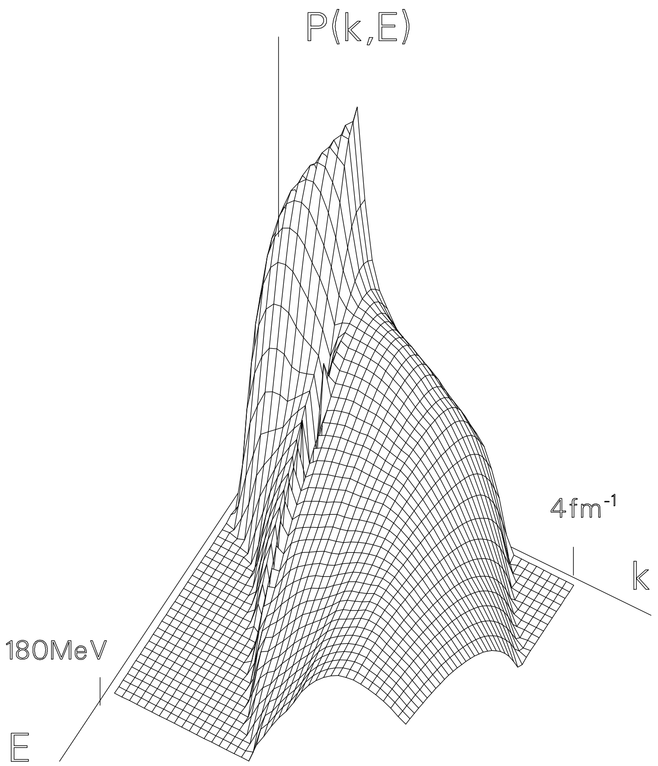


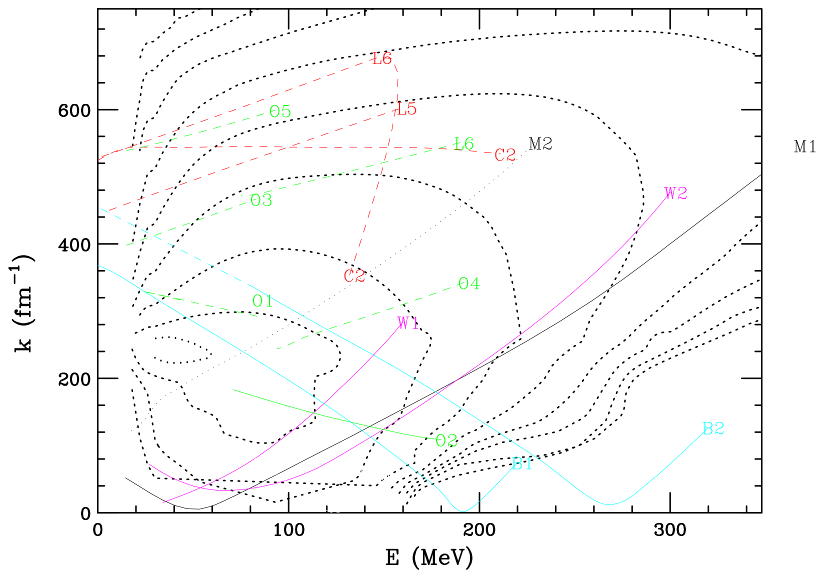
→ search for high-k popular theme... leading mostly to failures!

Most important insight

high-k occur only at large removal energy E

when hit high-k nucleon correlated partner with $-k$ also "freed"
costs energy $E \sim (-k)^2/2m_N$



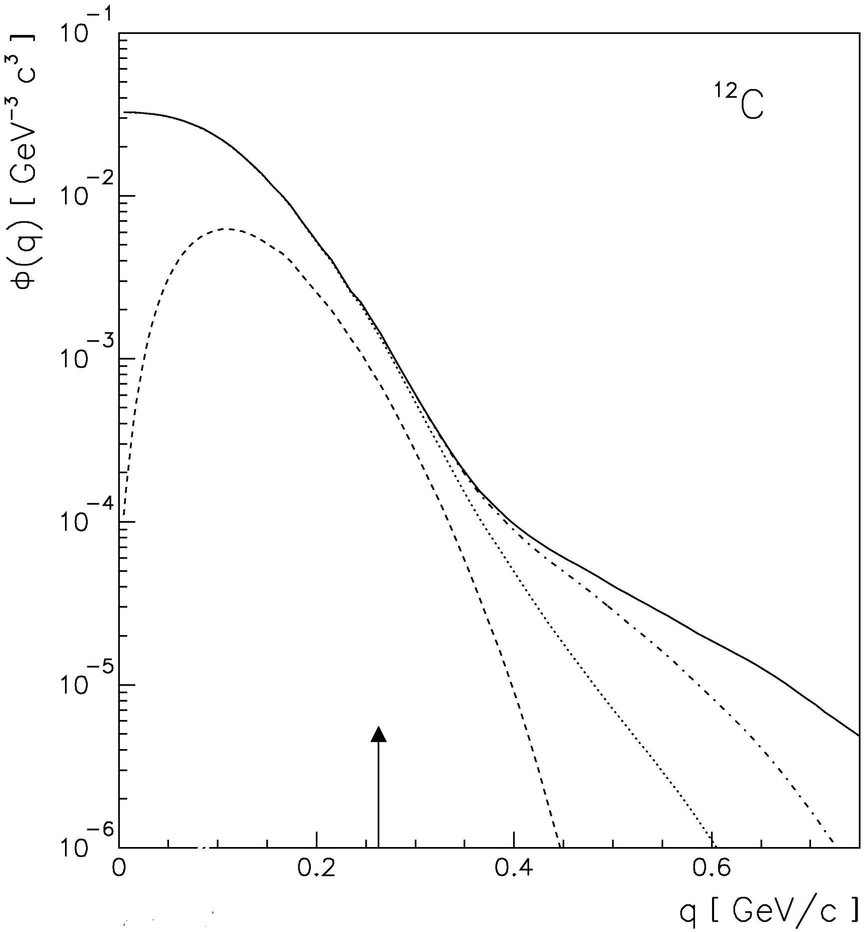


in all processes have to conserve momentum ***and*** energy

$\Rightarrow$ must discuss data in terms of $S(k, E)$, *not* $n(k)$

Example for correlation large k .... large E

$n(k)$ with cut-off in E



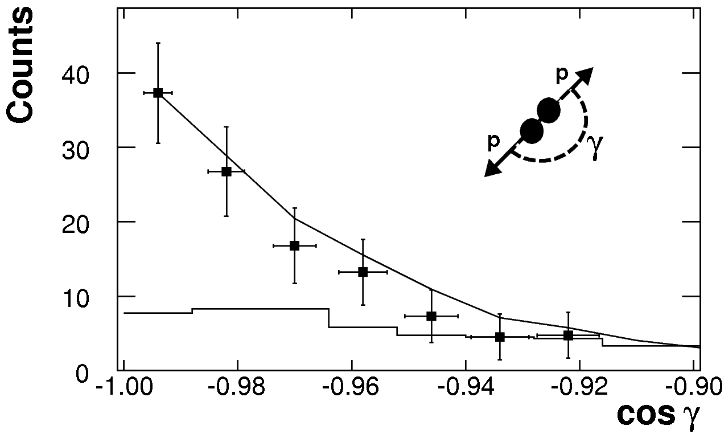
$E < 30 MeV$ - - -
$E < 100 MeV$ .....
$E < 200 MeV$ -.-.-.
$E < 500 MeV$ ———-

at low E find only mean-field strength

to get the full high-k strength must include *really large* E!

Theoretical picture confirmed by experiment

$^{12}C(e,e'2p)$ Shneor *et al.*



correlated nucleon is back-to-back with high-k nucleon

accounts, together with not-observed (e,e'pn), for all high-k strength (np/pp from Wiringa *et al.*)

Consequences of large E

high-k strength moved to large energy-loss

there most often covered up by low-k strength + inelastic processes

for examples see below

What do we know even *without* measuring high-k?

1. $n(k)$ from exact calculations for A=3÷11,16,∞

can today solve Schrödinger equation for best NN-potentials
Faddeev, CBF, AFMC, GFMC, ..
calculations are phenomenally successful
explain many observables

in particular explain binding energy

Koltun sumrule $\quad \mathrm{BE/A} = \langle E \rangle - \langle T \rangle$
$\pm$ 1MeV $\quad \sim$50MeV

$\langle T \rangle$ quite accurate $\rightarrow$ can trust $n(k)$ at large k

2. $S(k, E)$ for A=3,4 and ∞

calculated using exact methods
situation similar to the one for $n(k)$

3. $S(k, E)$ for other nuclei

$S(k, E)$ from NM calculations has been split into MF + correlated parts

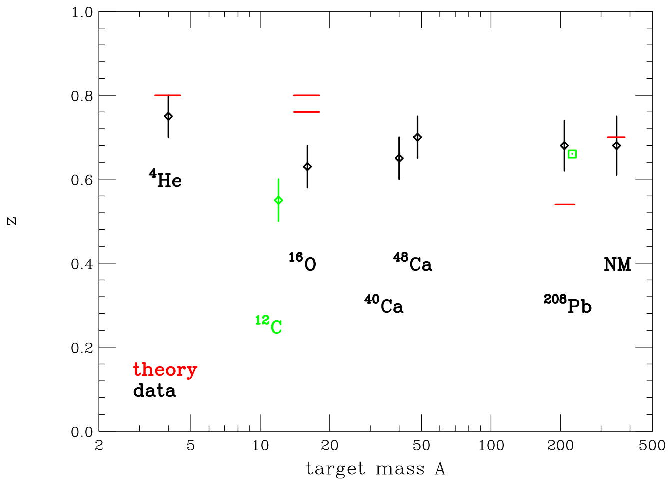
calculate $S(k, E)$ in LDA: S_{MF} from MF calculations fit to *e.g.* $\rho(r)$
add S_{corr} from NM for different NM-densities

4. Integrated correlated (high-k) strength known

occupation s_{MF} of mean-field orbits measured

$1-s_{MF}$ yields integrated correlated strength

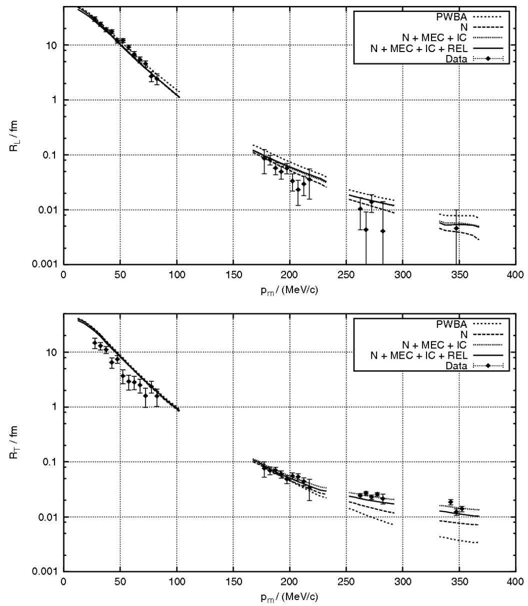
agrees well with theoretical predictions



5. Large-k fall-off same as for deuteron

same short-range V_{NN} $\rightarrow$ same fall-off

know quite well from experiment



Minimum requirement: calculate observable with $S(k, E)$ in PWIA (easy!)

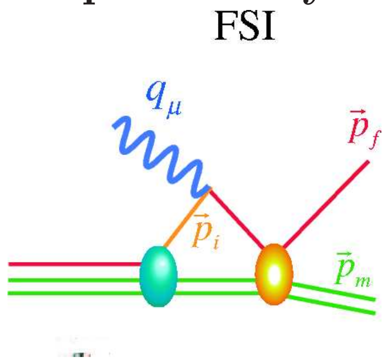
if σ_{PWIA} deviates by more than 30% from σ_{exp} then non-IA processes dominate

no point in trying to determine $S(k, E)$ or $n(k)$

Common pitfalls

1. PWIA no good, in most cases multi-step processes dominant

probability of high-k small, very spread out in E
multi-step processes, even if not dominant, have similar probability
move strength around
cover up large-k/large-E strength



Treatment of FSI using DWBA inadequate

Im(V) supposed to account for absorption
works only for (essentially) elastic channels
nucleon is not "swallowed up" but reappears
interacting nucleon moved to larger energy loss/different momentum
there can simulate high-k/high-E strength

FSI must be treated with approach like Glauber
need to follow fate of interacting nucleon(s)

FSI = *additive* contribution, *not* multiplicative
fraction of dominant low-k/E strength moved to region relevant for large-k/E

$\Longrightarrow$ popular idea of "removal" of FSI via cross-section ratios is an illusion

2. Low momentum transfer to nucleon maximizes FSI!

initial and final-state of high-k N must be orthogonal

in limit of momentum transfer = 0:

FSI (which orthogonalizes) *cancels entirely* high-k contribution
(Amado+Woloshyn, 1977)

$\longrightarrow$ cannot use low-q reactions for *quantitative* study

unless have total control of FSI
.... which is more difficult than calculating $S(k, E)$

Example: (x,p) reaction at backward angles

***the* process used in the 80^{ies}**

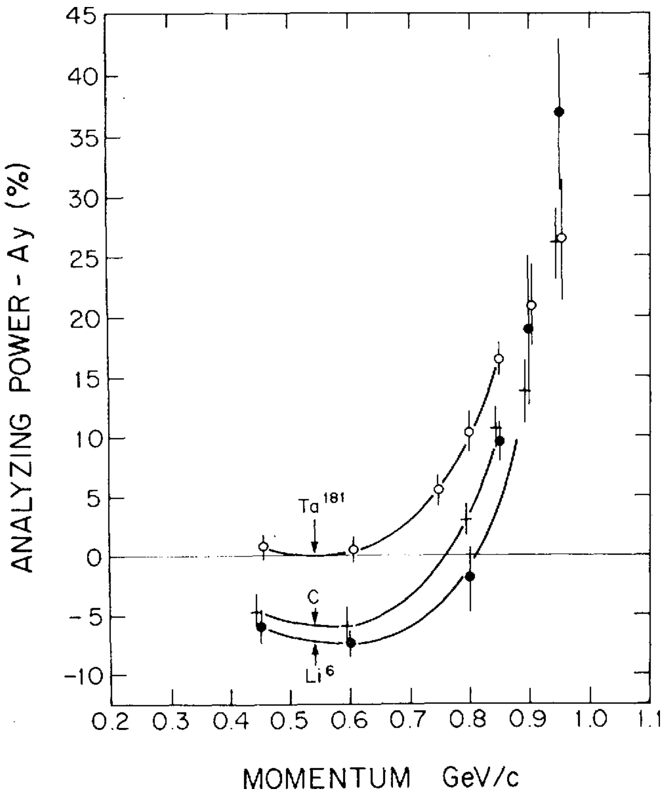
Idea: dump energy, but little momentum in nucleus

observe high-momentum, backward angle proton

reasoning: proton must have had high-k in initial state

Later insight: backward high-momentum protons not from high-k

analyzing power measured (Frankel *et al.*)
should be large and negative for 2-body mechanism



but is positive for high momenta

remember Amado+Woloshyn!

3. Sub-threshold data often dominated by FSI

Idea: bombard nucleus with probe

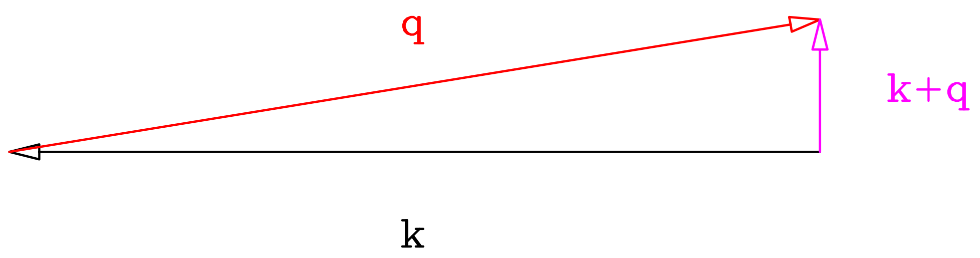
observe strength of process at energy subthreshold on nucleon, allowed on nucleus

reasoning: works if nucleon had large $\vec{k}$ opposite to $\vec{q}$

Typical processes: (x,K), (x,$\bar{p}$), (x,π), (e,e') at x >1

Example: inclusive electron scattering at large q, low ω

Idea: to get low $\omega \sim (\vec{k}+\vec{q})^2/2M$ for large q: need $\vec{k} \sim -\vec{q}$, $\rightarrow$ large k



Problem: low $\vec{k}+\vec{q} \rightarrow$ large FSI

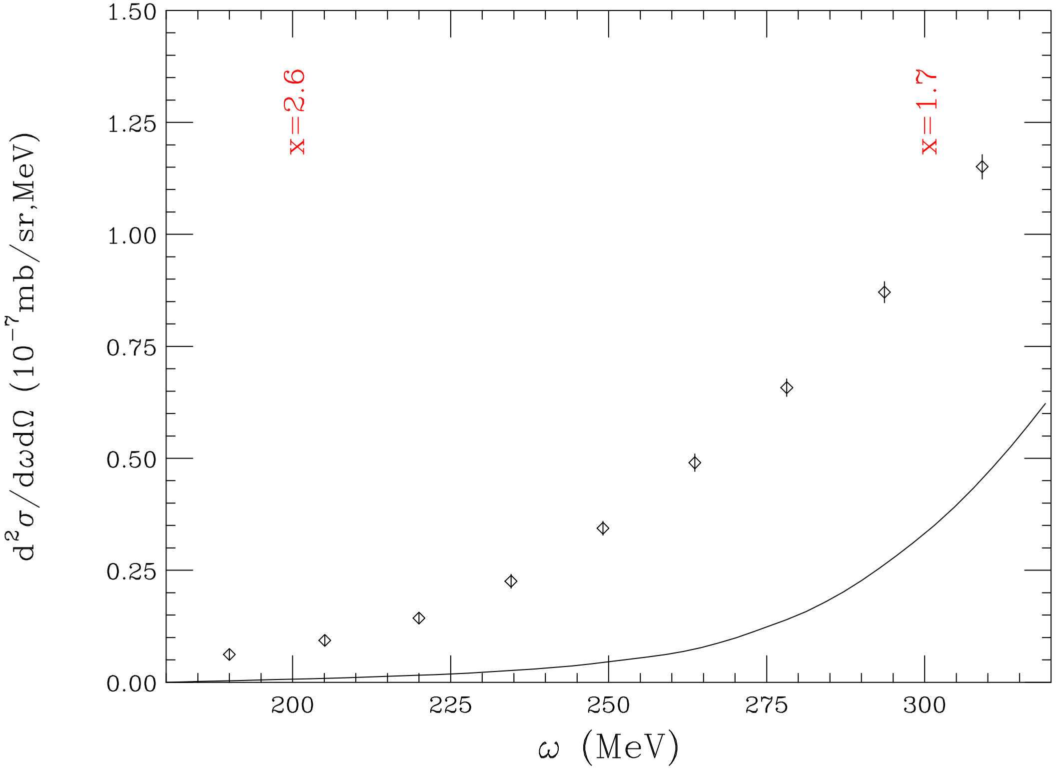
is important in tail of quasi-elastic peak

is not easy to calculate

Two types of proof for large FSI: see A), B)

A) ^{3}He(e,e') in threshold region, $x \sim 1.5 \div 3$

For ^3He have *exact* $S(k,E)$ from Faddeev calculation
is as good as deuteron $n(k)$

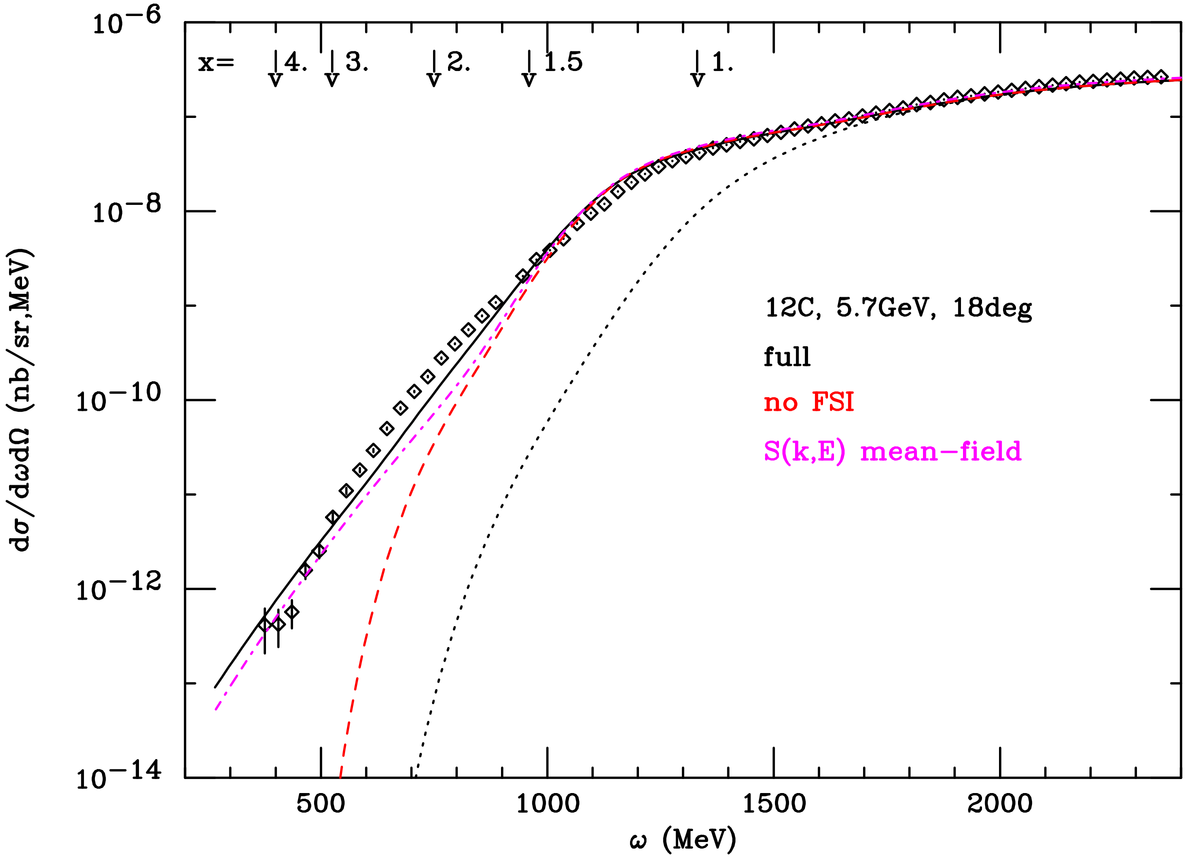


find σ_{PWIA} at large x factor 3÷10 too small
need FSI to get close to data

understanding of small contribution of large k:
large E moves most higk-k strength to large ω, under Q.E. peak

B) Recent (e,e') at $x \sim 2 \div 3$

5.7GeV, 18°, ^{12}C



——— full $S(k,E)$ and FSI
- - - - - full $S(k,E)$, no FSI
-.-.-.-.-. no high-k, with FSI

effect of FSI: folding of IA result
remember FSI in L^{4}He

folding function from particle-$S(k,E)$
Benhar *et al.*

σ_{PWIA} at large x much too small

effect of large-k minimal, FSI dominates (Benhar *et al.* 1989)

cross section ratios $\frac{\sigma_A}{\sigma_{A'}}$ *a la JLab* $\Longrightarrow$ ratios of FSI, not ratios of $n(k)$!

4. Naive associations are misleading

In DIS: x .... scattering from parton with mass xM_N

large x .... large mass???

but: partons are *not* particles with physical properties!

to discuss away unphysical xM_N-assumption must go to IMF

In QES: above idea taken over naively

identify x with "structures" having mass $\sim xM_N$

"structure" somehow connected to "correlations" *i.e.* high-k

Problem of association x with mass xM_N-"structure"

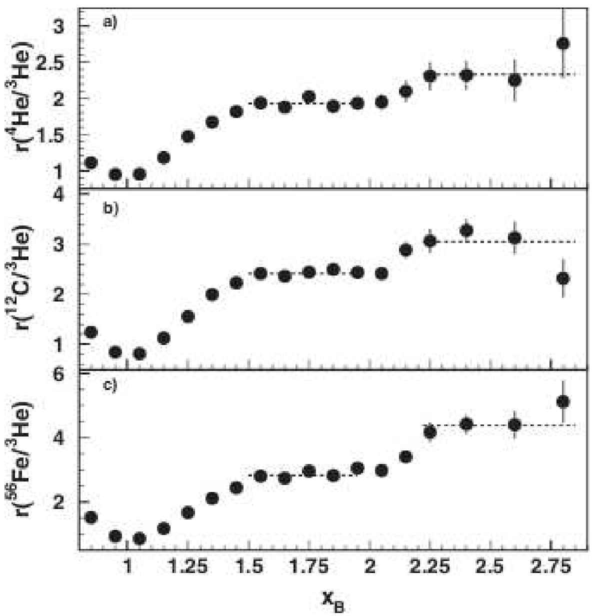
"structure" must recoil as a whole

then "structure" with *e.g.* $x \sim 3$ would have form factor similar to 3N-system

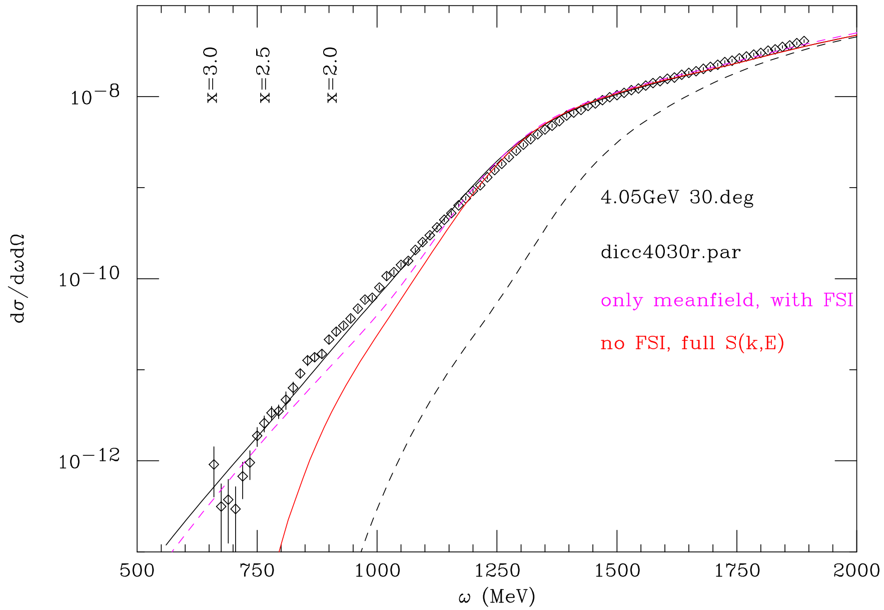
$\rightarrow$ cross section $\sim$5 orders of magnitude too small!

How then to understand large-x data?

Ratio of "correlations"?



Ratio of FSI!



Physics from drawing horizontal lines
through not-so-constant points?

Do real calculation!
S(k,E) contains *all* correlations!

To understand process: must specify reaction mechanism
"scattering off structure" ... fuzzy concept

scattering off nucleon + FSI well defined, explains data

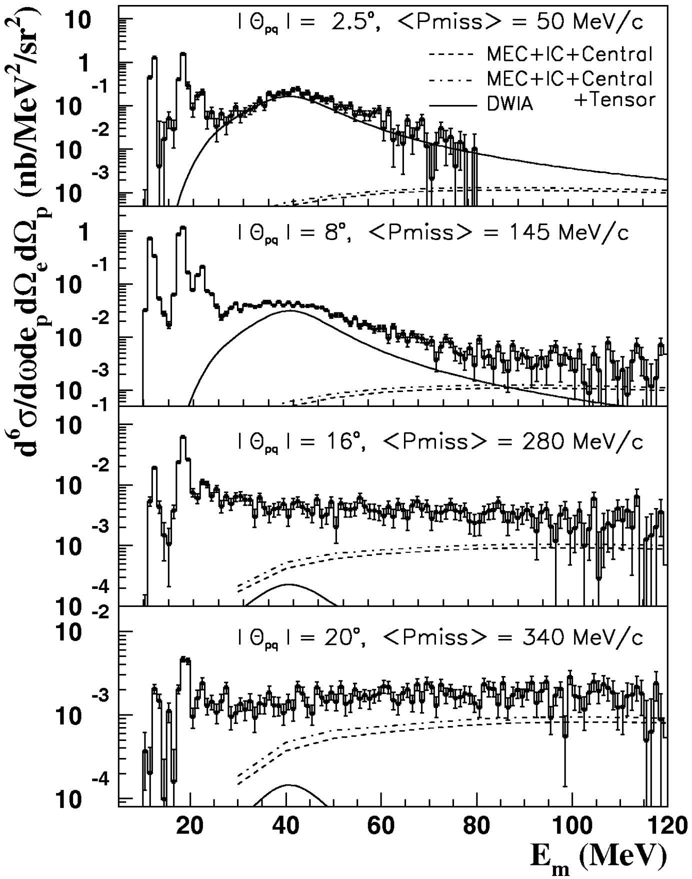
Which approach to measure high k/E *does* work?

Best tool known: (e,e'p) at large q

but only if

- FSI minimized by choice of kinematics
- kinematics such as to minimize Δ-excitation

example for *maximal* FSI+MEC: $^{16}O(e,e'p)$ at $Q^2 \sim 0.8$ (Liyanage *et al.*)



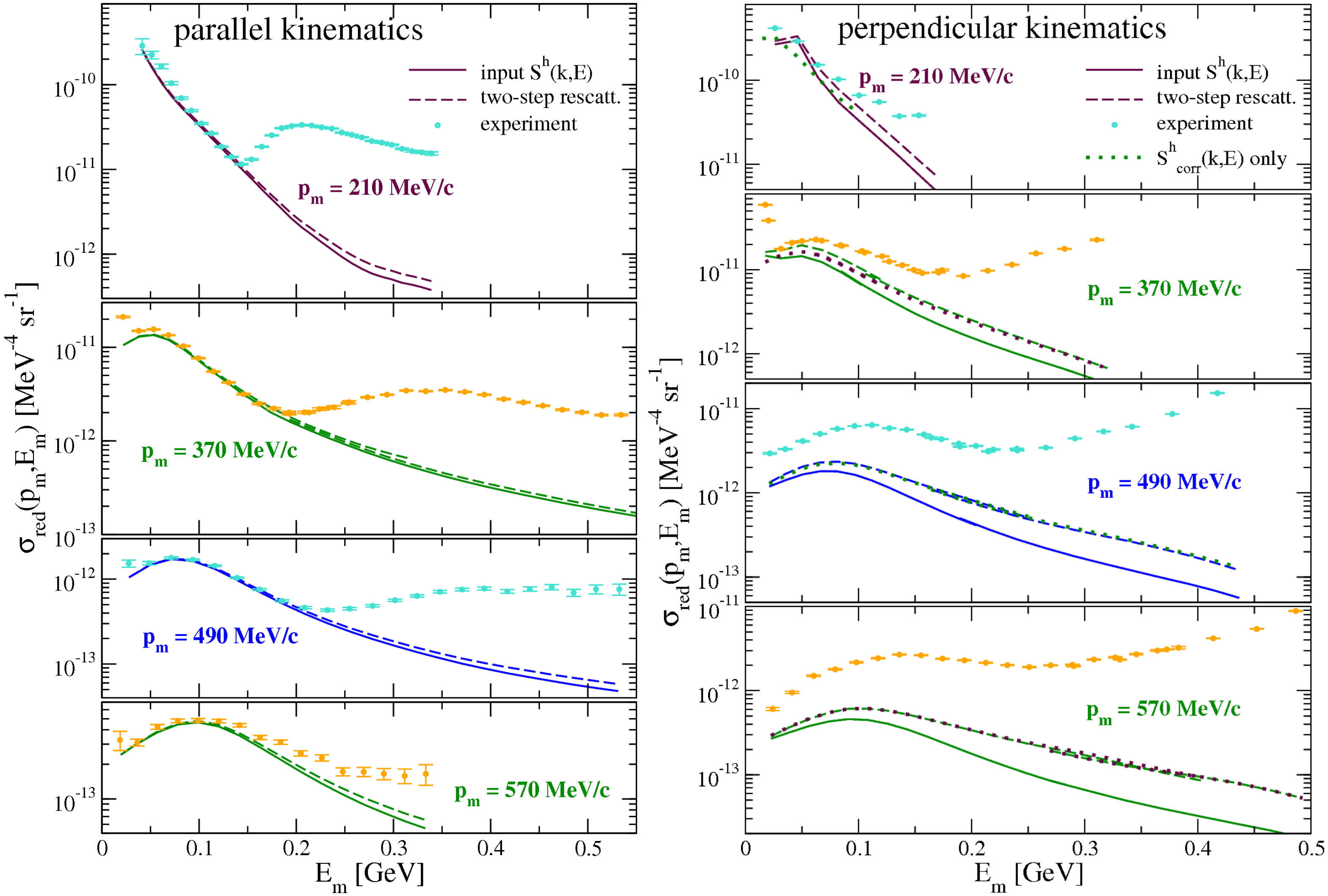
perpendicular kinematics maximizes
multi-step processes + MEC

multistep $\rightarrow$ strength at large E
cannot even see 1s-peak at $k > 250 MeV/c$
let alone the high-k/E-strength
(weaker, more spread out)

(e,e'p) with minimized FSI:

parallel kinematics, $\vec{q}$ parallel to $\vec{k}$

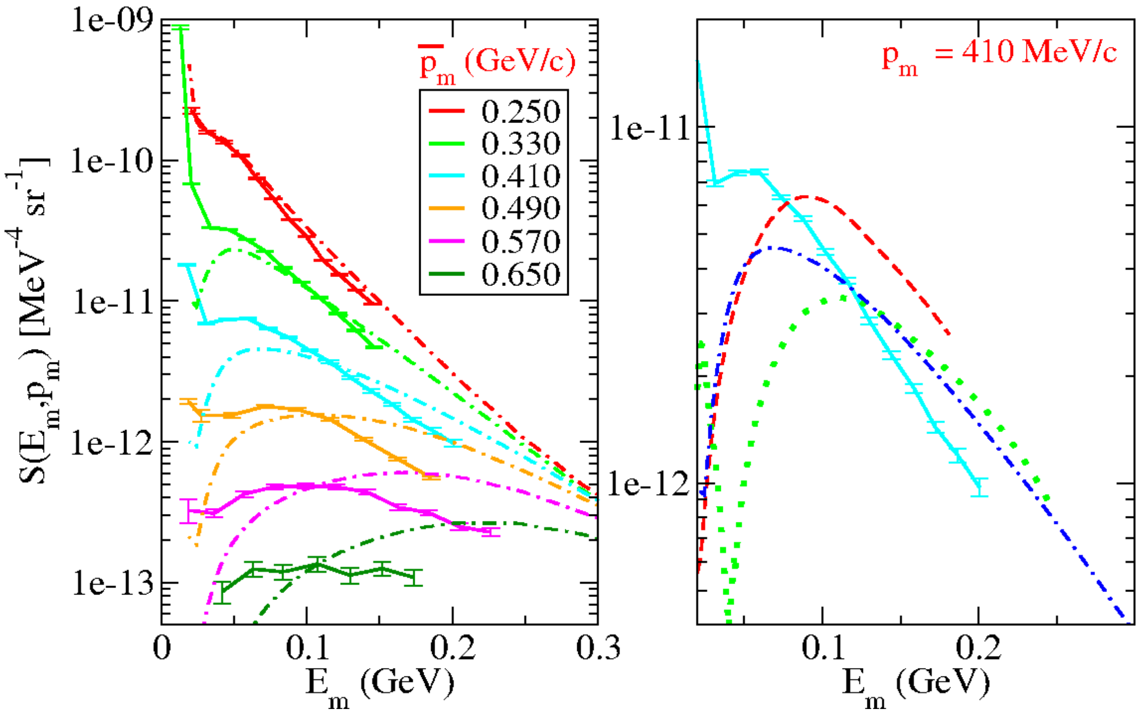
effect multistep reactions in Glauber (Barbieri *et al.*)



JLab experiment Rohe *et al.*, 2004

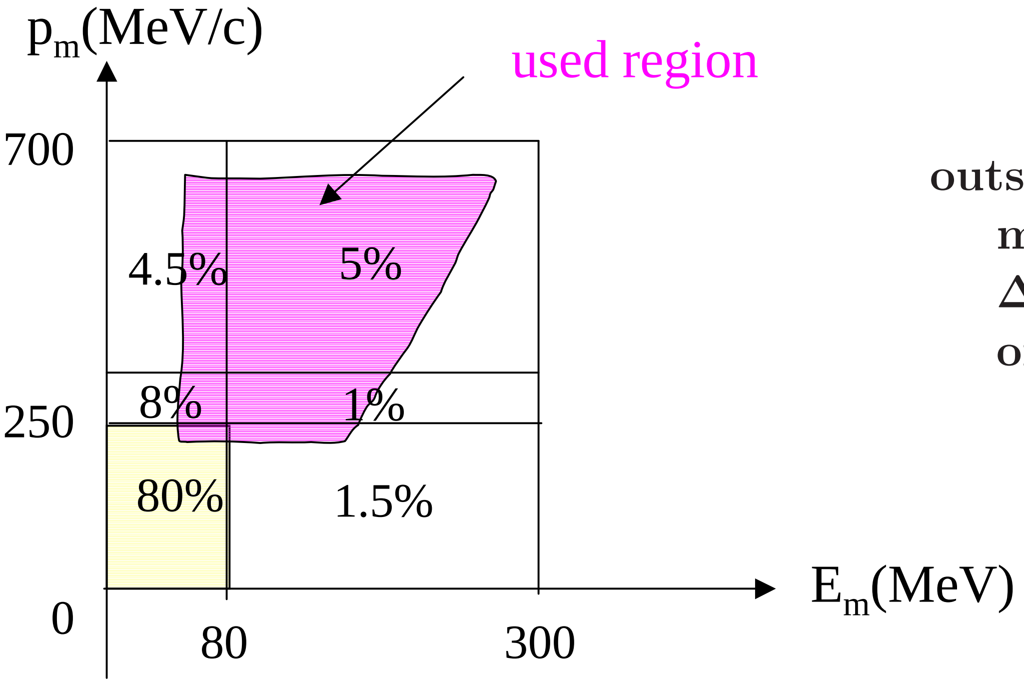
as close to parallel kinematics as was practical

Results: Spectral function



Find ± satisfactory correspondence with theory
in detail: find shift of $S(k,E)$ to smaller E
at present not understood

Comparison of integrated strength: possible for restricted region

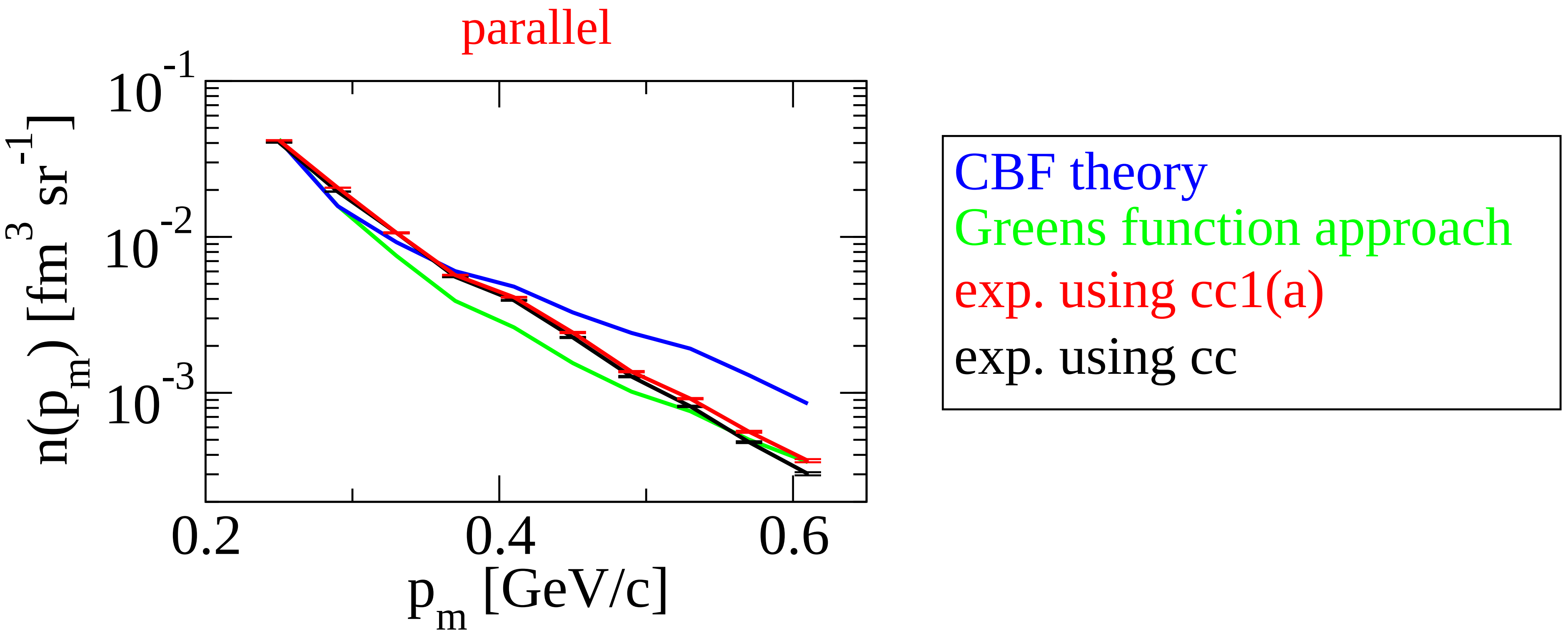


outside used region:
- mean-field dominates
- Δ too important
- or no data

# of correlated protons in ^{12}C	used region	total
integral over S from experiment	0.59	
integral over S from CBF	0.64	1.32
integral over S from SGGF	0.61	1.27

→ good agreement
→ can believe total from theory
→ 21%, integrated over $\boldsymbol{k}, \boldsymbol{E}$
→ agrees with $1 - s_{MF}$

Momentum distribution in "used region"



measure believable high-*k*-tail for first time
find rather good agreement with theory

..... but both data and theory could stand some improvement

Final insight

Don't even think about measuring $n(k)$ at large k!

every measuring process must conserve momentum *and* energy

large k always involve large E
large k and large E are inseparable

can only measure *together* !

Think only about measuring $S(k, E)$

if one measures $S(k, E)$ over large enough a region in E

then one can obtain $n(k)$ from an integral over $S(k, E)$

or compare data and theory integrated over the same E-region

Some references

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