Theory of quarkonium radiative transitions

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Outline

1. Motivation
2. Energy scales and EFTs
3. Magnetic transitions
4. Electric transitions
5. Conclusions and outlook

based on

Nora Brambilla, Yu Jia and Antonio Vairo

Model-independent study of magnetic dipole transitions in quarkonium

PRD 73 (2006) 054005 [arXiv:hep-ph/0512369]

and

Nora Brambilla, Piotr Pietrulewicz and Antonio Vairo

Model-independent study of electric dipole transitions in quarkonium

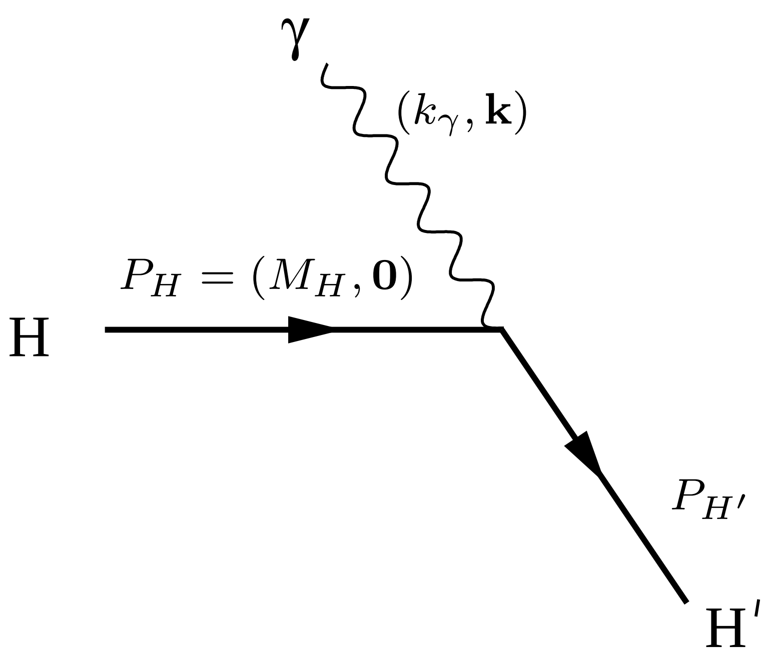
PRD 85 (2012) 094005 [arXiv:1203.3020]

Radiative transitions: basics

Two dominant single-photon-transition processes:

E1 electric dipole transitions

M1 magnetic dipole transitions



$$k_\gamma = \frac{{M_H}^2 - {M_{H'}}^2}{2M_H}$$

$$P_{H'} = \left(\sqrt{k_\gamma^2 + M_{H'}^2}, -\mathbf{k}\right)$$

Radiative transitions: basics

Two dominant single-photon-transition processes:
E1 electric dipole transitions
M1 magnetic dipole transitions

M1 transitions in the non-relativistic limit

$$\Gamma^{(0)}_{n^3S_1 \to n'^1S_0\,\gamma} = \frac{4}{3}\,\alpha\,e_Q^2\,\frac{k_\gamma^3}{m^2}\left|\int_0^\infty dr\,r^2\,R_{n'0}(r)\,R_{n0}(r)\,j_0\left(\frac{k_\gamma r}{2}\right)\right|^2$$

If $k_\gamma \langle r \rangle \ll 1$ $\quad j_0(k_\gamma r/2) = 1 - (k_\gamma r)^2/24 + \ldots$

- $n = n'$ allowed transitions
- $n \neq n'$ hindered transitions

Radiative transitions: basics

Two dominant single-photon-transition processes:

E1 electric dipole transitions

M1 magnetic dipole transitions

E1 transitions in the non-relativistic limit

$$\Gamma^{(0)}_{n^3P_J \to n'^3S_1\,\gamma} = \frac{4}{9}\,\alpha\,e_Q^2\,k_\gamma^3\,\left|\int_0^\infty dr\,r^3\,R_{n'0}(r)\,R_{n0}(r)\right|^2$$

Note that, for equal energies and masses, M1 transitions are suppressed by a factor $1/(m\langle r\rangle)^2 \sim v^2$ with respect to E1 transitions, which are much more common.

For relativistic corrections, the literature is based essentially on one work that uses

- a relativistic equation with scalar and vector potentials;
- a non-relativistic reduction;
- and relativistic invariance to calculate recoil corrections.

- `Grotch Owen Sebastian PR D30 (1984) 1924`

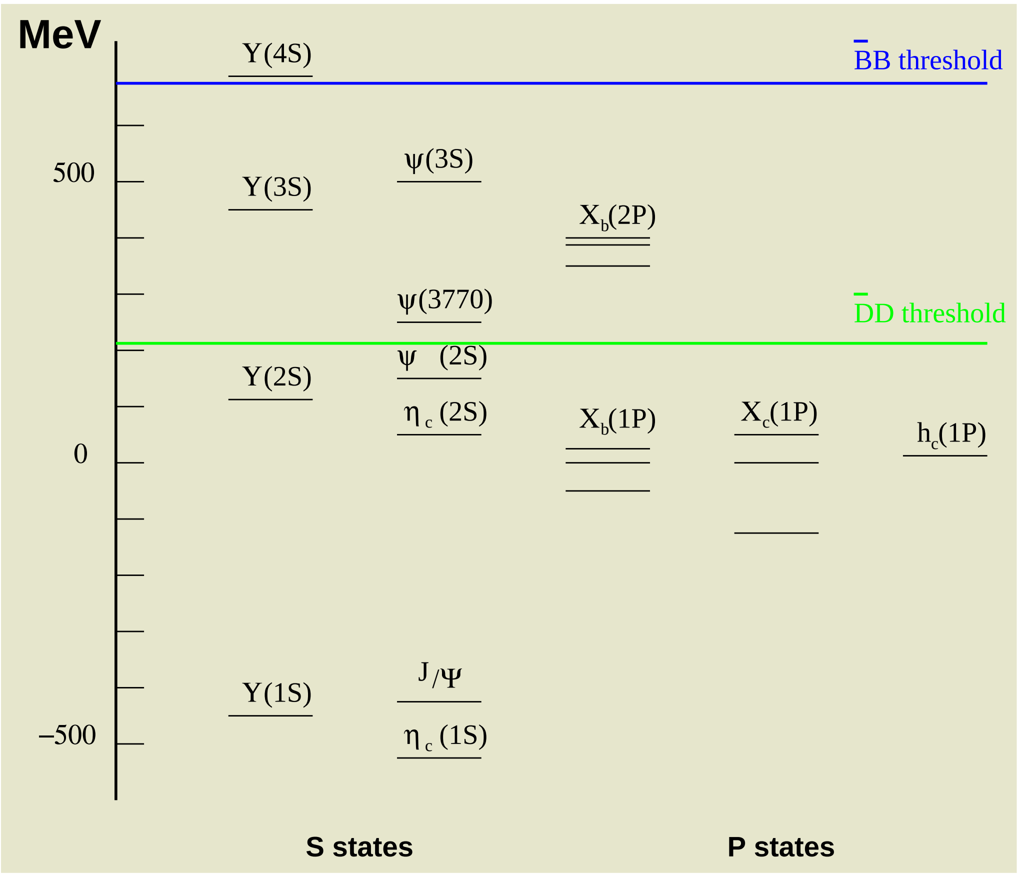
Differently a theory of radiative transitions should

- follow from QCD;
- clearly define when a relativistic/non-relativistic, weak/strong-coupling, multipole/non-multipole expansions are feasible;
- be improvable in a systematic way;
- provide reliable error bars for the determination of physical observables.

An approach that fulfills these requirement and takes advantage of the hierarchy of energy scales of the problem is based on the construction of a suitable hierarchy of effective field theories.

◦ Brambilla Pineda Soto Vairo RMP 77 (2005) 1423

Energy scales



Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

The non-relativistic hierarchy

$$m \gg mv \sim p \sim 1/r \gg mv^2 \sim E$$

exists regardless of the fact that

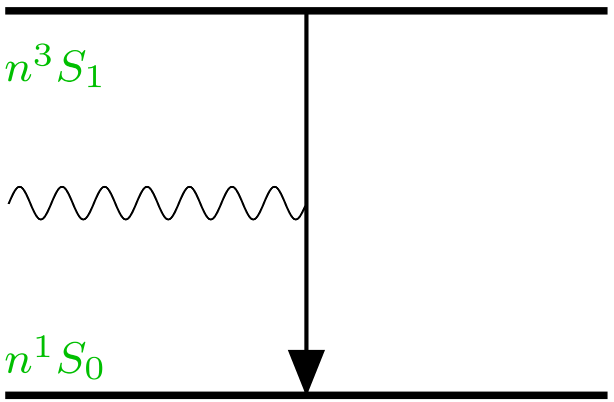
$$mv \gtrsim \Lambda_{\rm QCD}$$

Energy scales

The photon energy k_γ:

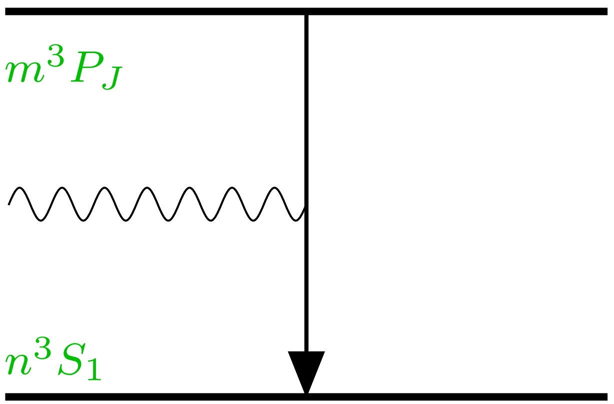
$k_\gamma \sim mv^4$

- for M1 allowed transitions

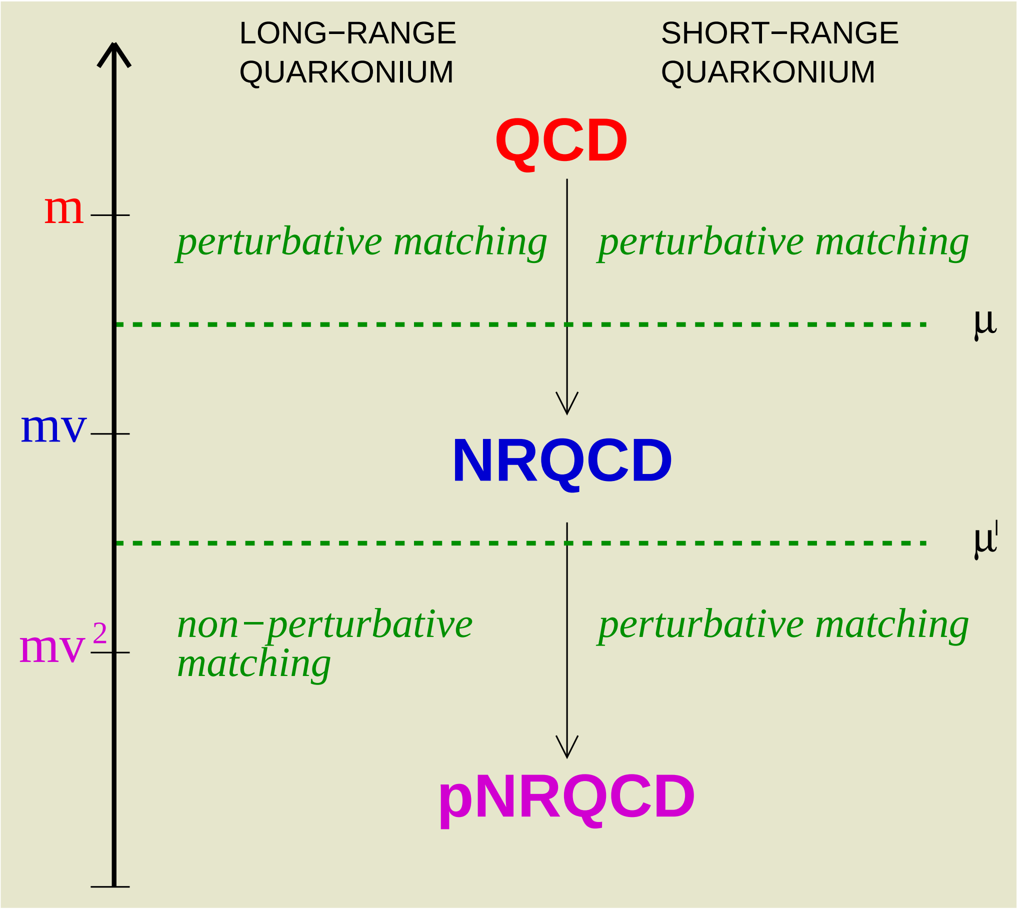


$k_\gamma \sim mv^2$

- for M1 hindered transitions
- for E1 transitions



Non-relativistic Effective Field Theories

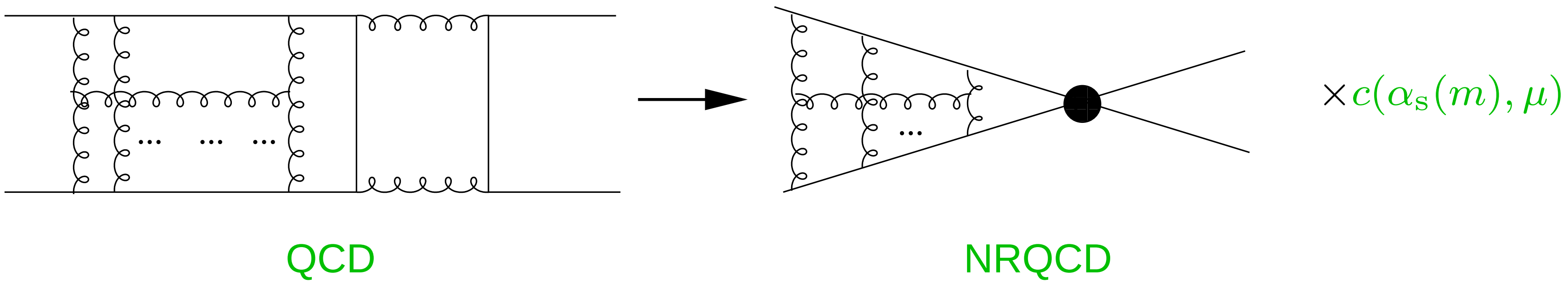


- Caswell Lepage PLB 167(86)437
- Lepage Thacker NP PS 4(88)199
- Bodwin et al PRD 51(95)1125, ...

- Pineda Soto NP PS 64(98)428
- Brambilla et al PRD 60(99)091502
- Brambilla et al NPB 566(00)275
- Kniehl et al NPB 563(99)200
- Luke Manohar PRD 55(97)4129
- Luke Savage PRD 57(98)413
- Grinstein Rothstein PRD 57(98)78
- Griesshammer NPB 579(00)313
- Luke et al PRD 61(00)074025
- Hoang Stewart PRD 67(03)114020, ...

- They exploit the expansion in v/factorization of low and high energy contributions.
- They are renormalizable order by order in v.
- In perturbation theory, RG techniques provide resummation of large logs.

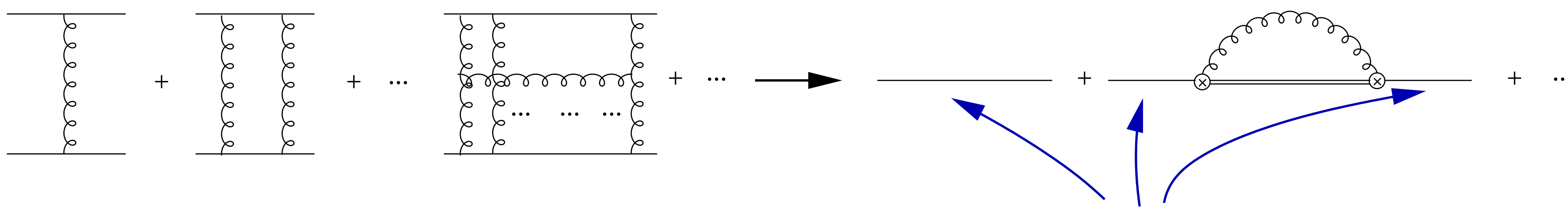
NRQCD



The effective Lagrangian is organized as an expansion in $1/m$ and $\alpha_s(m)$:

$$\mathcal{L}_{\rm NRQCD} = \sum_n \frac{c_n(\alpha_s(m), \mu)}{m^n} \times O_n(\mu, mv, mv^2, ...)$$

pNRQCD



$$\frac{1}{E - p^2/m - V(r, \mu', \mu)}$$

NRQCD pNRQCD

The effective Lagrangian is organized as an expansion in $1/m$, $\alpha_s(m)$ and r:

$$\mathcal{L}_{\text{pNRQCD}} = \int d^3r \sum_n \sum_k \frac{c_n(\alpha_s(m), \mu)}{m^n} \times V_{n,k}(r, \mu', \mu)\, r^k \times O_k(\mu', mv^2, \ldots)$$

- $V_{n,0}$ are the potentials in the Schrödinger equation.
- $V_{n,k\neq 0}$ are the couplings with the low-energy degrees of freedom, which provide corrections to the potential picture.

pNRQCD: degrees of freedom

- Degrees of freedom at scales lower than mv:

  Q-$\bar{Q}$ states, with energy $\sim \Lambda_{\rm QCD}$, mv^2 and momentum $\lesssim mv$
  $\Rightarrow$ (i) singlet $\rm S$ (ii) octet $\rm O$ [if $mv \gg \Lambda_{\rm QCD}$]

  Gluons with energy and momentum $\sim \Lambda_{\rm QCD}$, mv^2 [if $mv \gg \Lambda_{\rm QCD}$]

  Photons of energy and momentum lower than mv.

- Power counting:

  $p \sim \dfrac{1}{r} \sim mv$;

  photons (and weakly coupled gluons) are multipole expanded:

$$A^{\rm em}(R, r, t) = A^{\rm em}(R, t) + \mathbf{r} \cdot \boldsymbol{\nabla} A^{\rm em}(R, t) + \ldots$$

  and scale like $(\Lambda_{\rm QCD}$ or $mv^2)^{\rm dimension}$.

pNRQCD: Lagrangian

$$\mathcal{L}_{\rm pNRQCD} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} - \frac{1}{4} F^{\rm em}_{\mu\nu} F^{\mu\nu\, {\rm em}}$$

$$+ \int d^3 r \, {\rm Tr} \left\{ {\rm S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) {\rm S} \right.$$

$$\left. + \, {\rm O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) {\rm O} \right\}$$

LO in r

$$+ {\rm Tr} \left\{ {\rm O}^\dagger \mathbf{r} \cdot g\mathbf{E} \, {\rm S} + {\rm S}^\dagger \mathbf{r} \cdot g\mathbf{E} \, {\rm O} \right\}$$

$$+ \frac{1}{2} {\rm Tr} \left\{ {\rm O}^\dagger \mathbf{r} \cdot g\mathbf{E} \, {\rm O} + {\rm O}^\dagger {\rm O} \mathbf{r} \cdot g\mathbf{E} \right\}$$

[if $mv \gg \Lambda_{\rm QCD}$]

$$+ \cdots$$

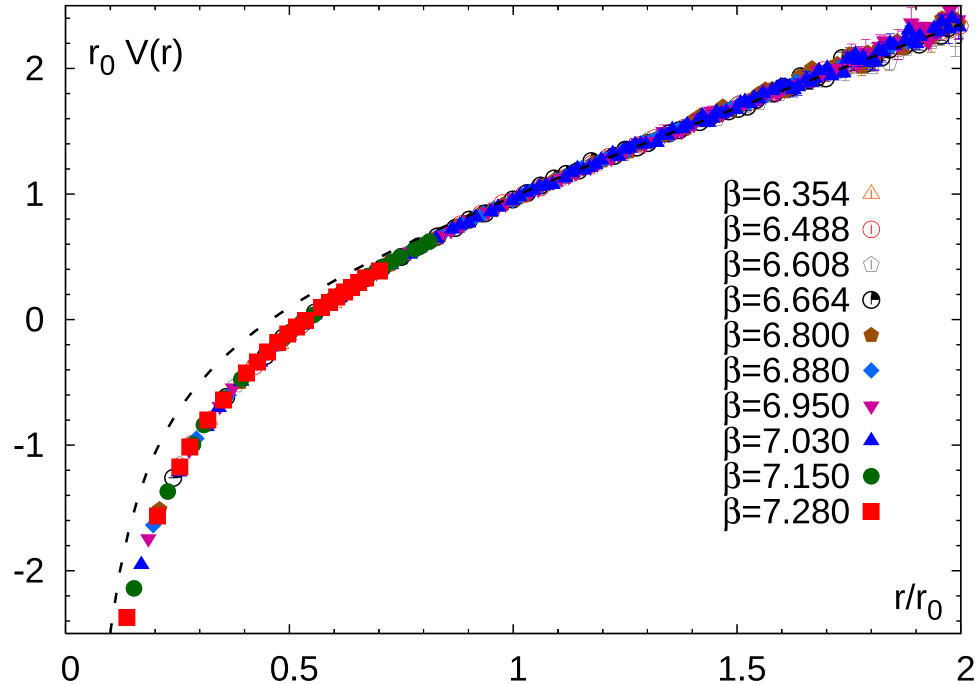
NLO in r

$$+ \mathcal{L}_\gamma^{\rm M1} + \mathcal{L}_\gamma^{\rm E1}$$

The static potential

$$V_s = V_s{}^{(0)} + \frac{V_s{}^{(1)}}{m} + \frac{V_s{}^{(2)}}{m^2} + \ldots$$

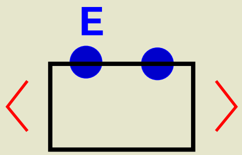
$$V_s^{(0)} = \lim_{T\to\infty} \frac{i}{T} \ln\langle W(r\times T)\rangle = \lim_{T\to\infty} \frac{i}{T} \ln \left\langle \boxed{} \right\rangle$$

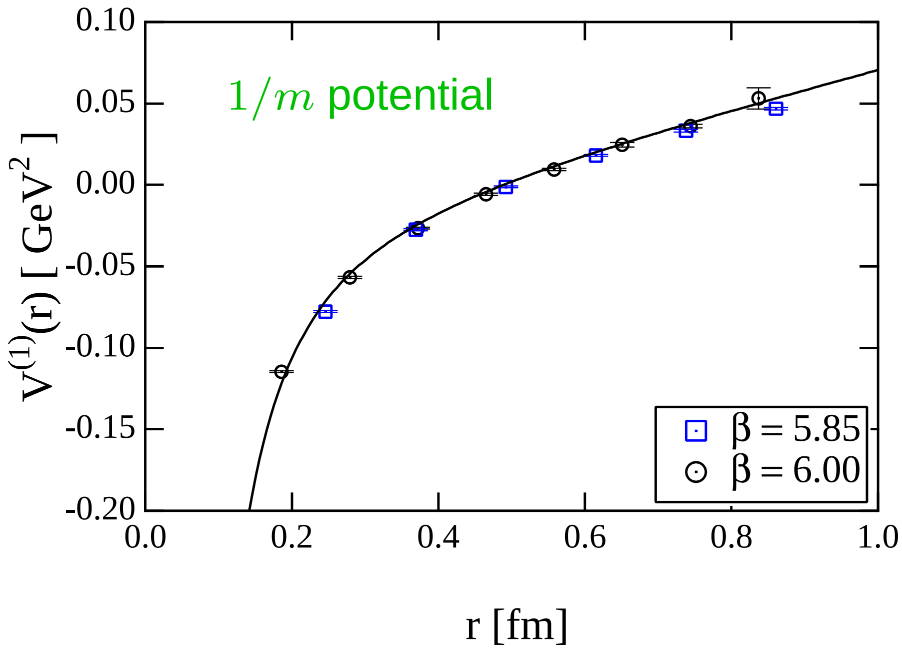


- Bazakov et al PR D85 (2012) 054503

The $1/m$ potential

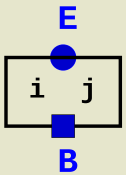
$$V_s^{(1)} = -\frac{1}{2}\int_0^\infty dt\, t$$



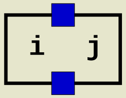


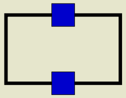
- Koma Koma Wittig PoS LAT2007 (2007) 111

Spin-dependent $1/m^2$ potentials

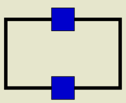
$V_{\mathrm{SD}}^{(2)} = \frac{1}{r}\left(c_F \epsilon^{kij} \frac{2r^k}{r} i \int_0^\infty dt\, t \left\langle\right.$ 

$\left.\right\rangle - \frac{1}{2} V_s^{(0)\prime} \right) (\mathbf{S}_1 + \mathbf{S}_2) \cdot \mathbf{L}$

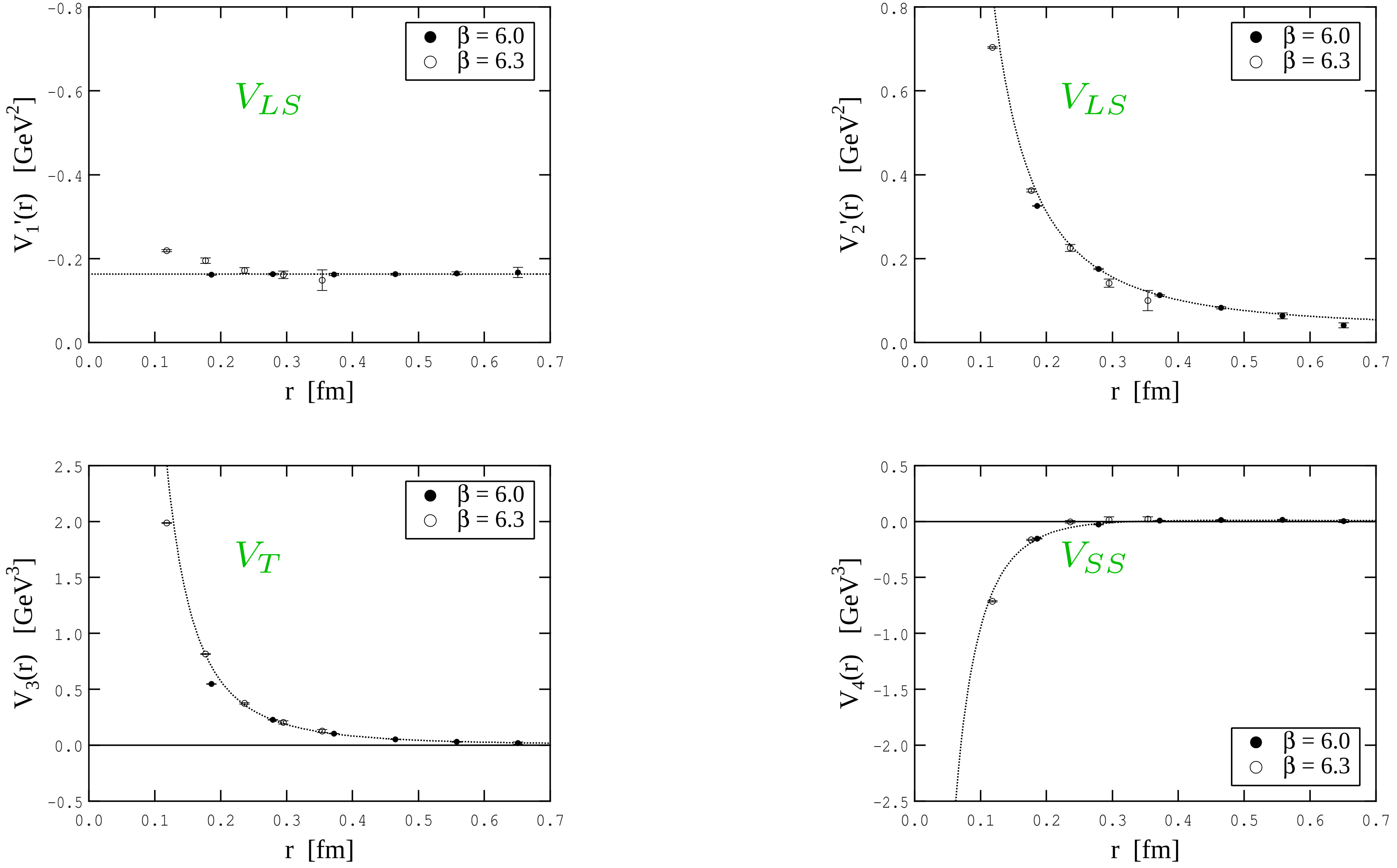
$- {c_F}^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left(\left\langle\right.\right.$ 

$\left.\right\rangle - \frac{\delta_{ij}}{3} \left\langle\right.$  $\left.\left.\right\rangle\right)$

$\times \left(\mathbf{S}_1 \cdot \mathbf{S}_2 - 3(\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S}_2 \cdot \hat{\mathbf{r}}) \right)$

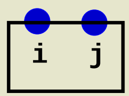
$+ \left(\frac{2}{3} {c_F}^2 i \int_0^\infty dt \left\langle\right.\right.$  $\left.\left.\right\rangle - 4\left(d_2 + \frac{4}{3} d_4 \right) \delta^{(3)}(\mathbf{r}) \right) \mathbf{S}_1 \cdot \mathbf{S}_2$

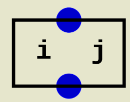
Spin-dependent $1/m^2$ potentials



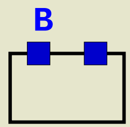
- Koma Koma NPB 769 (2007) 79

Spin-independent $1/m^2$ potentials

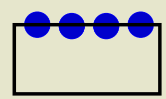
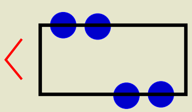
$V_{\mathrm{SI}}^{(2)} = p^i \Bigg(i \int_0^\infty dt\, t^2 \, \Big\langle$ 

$\Big\rangle + \Big\langle$ 

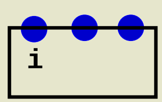
$\Big\rangle \Bigg) p^j$

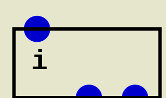
$-\frac{c_F^2}{2} i \int_0^\infty dt \, \Big\langle$ 

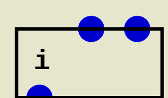
$\Big\rangle + \left(d_1 + \frac{4}{3} d_3 + \frac{4}{3} \pi \alpha_{\mathrm{s}} c_D \right) \delta^{(3)}(\mathbf{r})$

$-i \int_0^\infty dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 (t_2 - t_3)^2 \Bigg(\Big\langle$  $\Big\rangle + \Big\langle$  $\Big\rangle \Bigg)$

$+ \int_0^\infty dt_1 \int_0^{t_1} dt_2 (t_1 - t_2)^2 \nabla^i$

$\times \Bigg(\Big\langle$ 

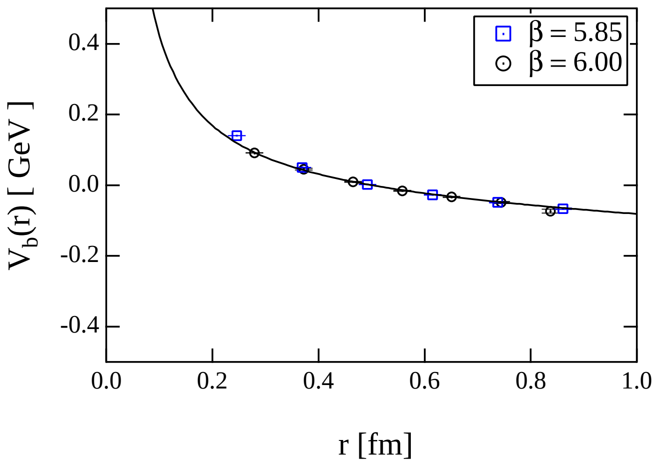
$\Big\rangle + \frac{1}{2} \Big\langle$ 

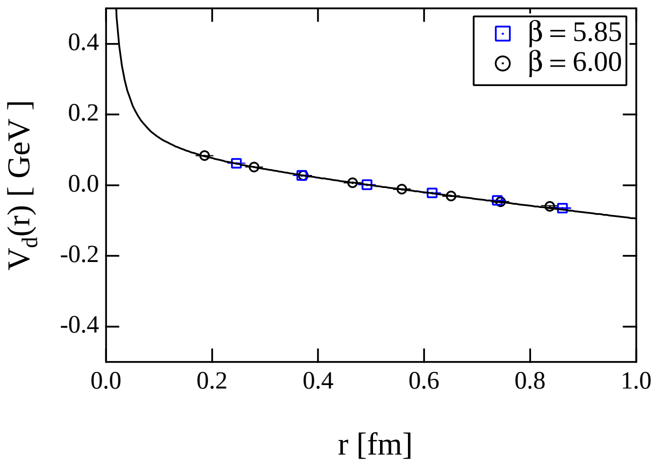
$\Big\rangle + \frac{1}{2} \Big\langle$ 

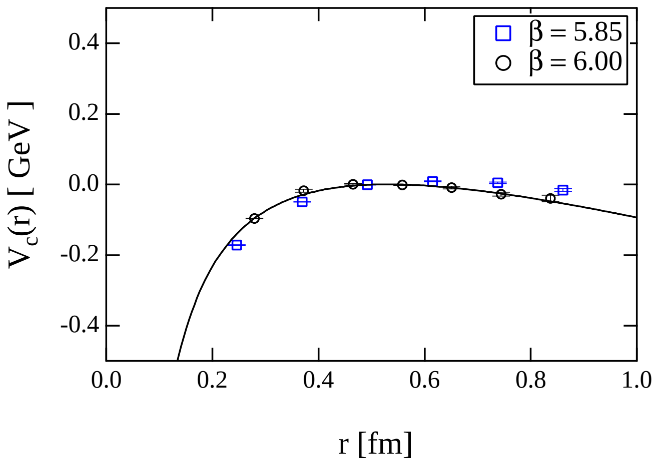
$\Big\rangle \Bigg)$

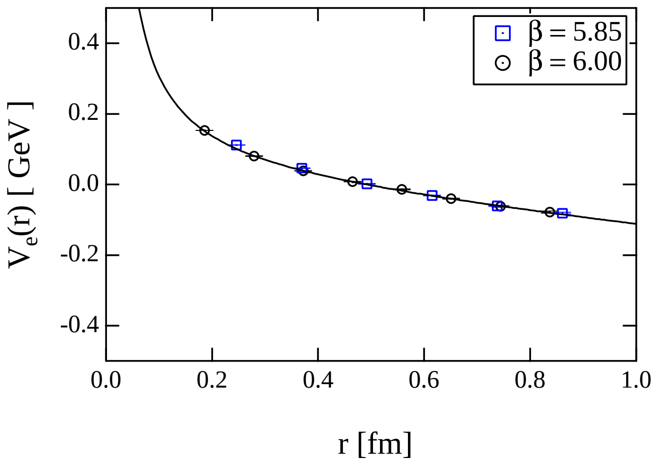
$$-2 b_3 f_{abc} \int d^3\mathbf{x}\, g \langle\!\langle G^a_{\mu\nu}(\mathbf{x}) G^b_{\mu\alpha}(\mathbf{x}) G^c_{\nu\alpha}(\mathbf{x}) \rangle\!\rangle^c_\Box$$

Spin-independent p^2/m^2 potentials









- Koma Koma Wittig PoS LAT2007 (2007) 111

$$\mathcal{L}_\gamma^{\mathrm{M1}}$$

$$\begin{aligned}
\mathcal{L}_\gamma^{\mathrm{M1}} \quad = \quad & \mathrm{Tr}\Bigg\{ \frac{1}{2m}\, V_1 \,\left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot ee_Q \mathbf{B}^{\mathrm{em}}\right\} \mathrm{S} \\
& + \frac{1}{2m}\, V_1 \,\left\{\mathrm{O}^\dagger, \boldsymbol{\sigma}\cdot ee_Q \mathbf{B}^{\mathrm{em}}\right\} \mathrm{O} \qquad\qquad [\text{if } mv \gg \Lambda_{\mathrm{QCD}}] \\
& + \frac{1}{4m^2}\, \frac{V_2}{r} \,\left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot \left[\hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times ee_Q \mathbf{B}^{\mathrm{em}}\right)\right]\right\} \mathrm{S} \\
& + \frac{1}{4m^2}\, \frac{V_3}{r} \,\left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot ee_Q \mathbf{B}^{\mathrm{em}}\right\} \mathrm{S} \\
& + \frac{1}{4m^3}\, V_4 \,\left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot ee_Q \mathbf{B}^{\mathrm{em}}\right\} \boldsymbol{\nabla}_r^2 \mathrm{S} \\
& + \frac{1}{4m^3}\, V_5 \,\left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}^i ee_Q \mathbf{B}^{\mathrm{em}\, j}\right\} \boldsymbol{\nabla}_r^i \boldsymbol{\nabla}_r^j \mathrm{S} \\
& + \frac{1}{4m^3}\, \frac{V_6}{r^2} \,\left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot ee_Q \mathbf{B}^{\mathrm{em}}\right\} \mathrm{S} + \cdots \Bigg\}
\end{aligned}$$

$$\mathcal{L}_\gamma^{\mathrm{E1}}$$

$$\begin{aligned}
\mathcal{L}_\gamma^{\mathrm{E1}} \quad = \quad & \mathrm{Tr}\Big\{ V_A^{\mathrm{em}}\ \mathrm{S}^\dagger \mathbf{r}\cdot ee_Q \mathbf{E}^{\mathrm{em}} \mathrm{S} \\
& + V_B^{\mathrm{em}}\ \mathrm{O}^\dagger \mathbf{r}\cdot ee_Q \mathbf{E}^{\mathrm{em}} \mathrm{O} \qquad\qquad [\text{if } mv \gg \Lambda_{\mathrm{QCD}}] \\
& + \frac{1}{24}\ V_7\ \mathrm{S}^\dagger \mathbf{r}\cdot [(\mathbf{r}\cdot\boldsymbol{\nabla})^2 ee_Q \mathbf{E}^{\mathrm{em}}]\mathrm{S} \\
& + \frac{i}{4m}\ V_8\ \mathrm{S}^\dagger \{\boldsymbol{\nabla}\cdot, \mathbf{r}\times ee_Q \mathbf{B}^{\mathrm{em}}\}\mathrm{S} \\
& + \frac{i}{12m}\ V_9\ \mathrm{S}^\dagger \{\boldsymbol{\nabla}_r\cdot, \mathbf{r}\times [(\mathbf{r}\cdot\boldsymbol{\nabla}) ee_Q \mathbf{B}^{\mathrm{em}}]\}\mathrm{S} \\
& + \frac{1}{4m}\ V_{10}\ [\mathrm{S}^\dagger, \boldsymbol{\sigma}]\cdot [(\mathbf{r}\cdot\boldsymbol{\nabla}) ee_Q \mathbf{B}^{\mathrm{em}}]\mathrm{S} \\
& - \frac{i}{4m^2}\ V_{11}\ [\mathrm{S}^\dagger, \boldsymbol{\sigma}]\cdot (ee_Q \mathbf{E}^{\mathrm{em}} \times \boldsymbol{\nabla}_r)\mathrm{S} \\
& - \frac{1}{16m^2}\ V_{12}\ \left[\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot \left[-i\boldsymbol{\nabla}_r\times, \mathbf{r}^i(\boldsymbol{\nabla}^i ee_Q \mathbf{E}^{\mathrm{em}})\right]\right]\mathrm{S} + \cdots \Big\}
\end{aligned}$$

Matching

The coefficients V are calculated like the potentials by matching Green's functions in QCD with Green's functions in the EFT. They get contributions from

- hard modes ($\sim m$):

$$\bar{\psi}(i\not{D} - m)\psi \to \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m} + \frac{c_F^{\rm em}}{2m}\boldsymbol{\sigma}\cdot e e_Q \mathbf{B}^{\rm em} + \cdots \right)\psi$$

where

$$c_F^{\rm em} \equiv 1 + \kappa = 1 + 2\frac{\alpha_{\rm s}}{3\pi} + \ldots$$

is the quark magnetic moment;

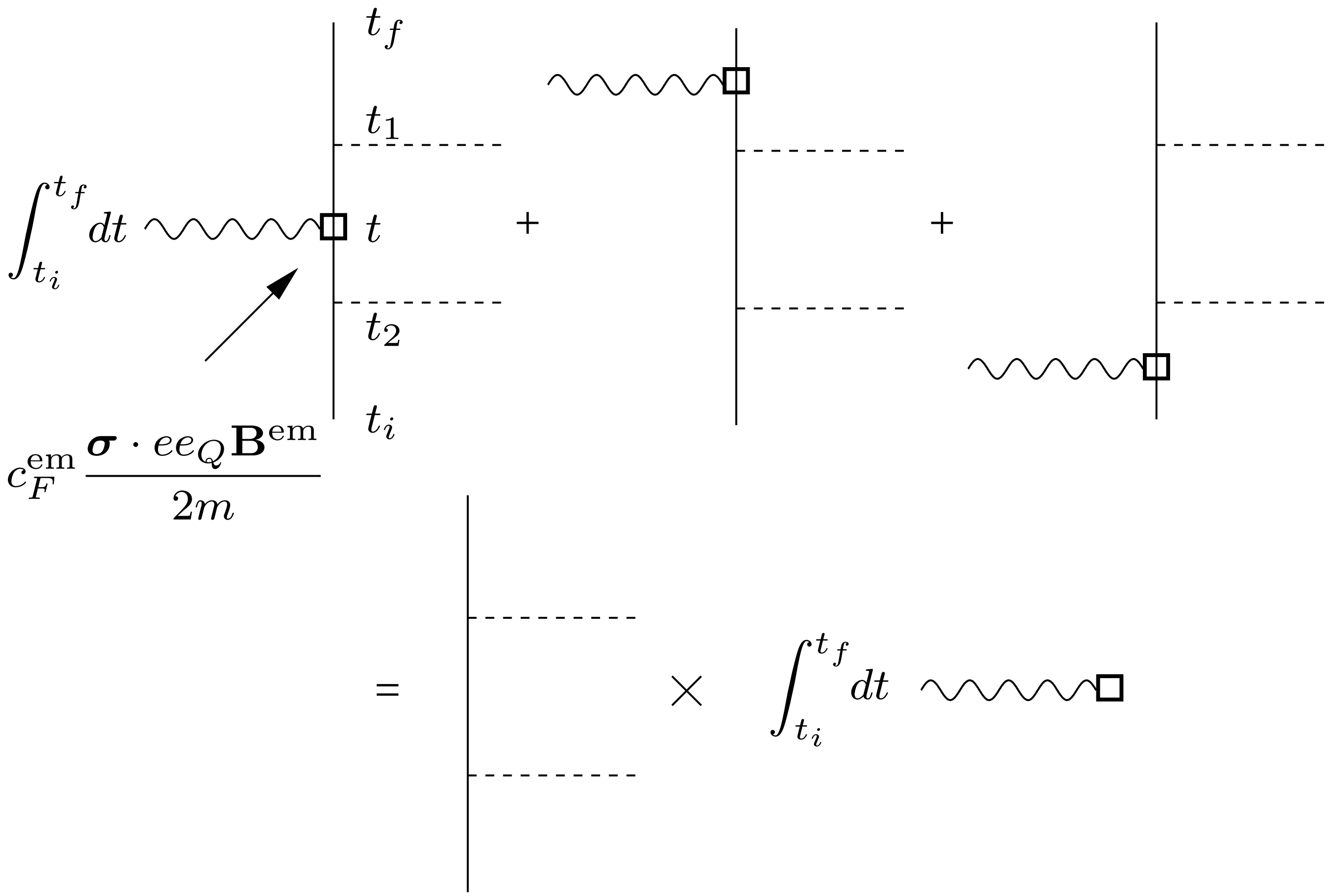
- soft modes ($\sim mv$).

M1 operator at $\mathcal{O}(1)$

$$V_1 \left\{ S^\dagger, \frac{\boldsymbol{\sigma} \cdot e\mathbf{B}^{\mathrm{em}}}{2m} \right\} S$$

$$V_1 = \Bigg(\text{hard}\Bigg) \times \Bigg(\text{soft}\Bigg)$$

- $\Bigg(\text{hard}\Bigg) = c_F^{\mathrm{em}} = 1 + \dfrac{2\alpha_{\mathrm{s}}(m_c)}{3\pi} + \cdots$
- Since $\boldsymbol{\sigma} \cdot e\mathbf{B}^{\mathrm{em}}(\mathbf{R})$ behaves like the identity operator to all orders V_1 does not get soft contributions.



Diagrammatic factorization of the magnetic dipole coupling in the SU(3)$_f$ limit.

- *The argument is similar to the factorization of the QCD corrections in* $b \to u\, e^- \bar{\nu}_e$*, which leads to* $\mathcal{L}_{\rm eff} = -4G_F/\sqrt{2}\; V_{ub}\; \bar{e}_L \gamma_\mu \nu_L\; \bar{u}_L \gamma^\mu b_L$ *to all orders in* $\alpha_{\rm s}$*.*

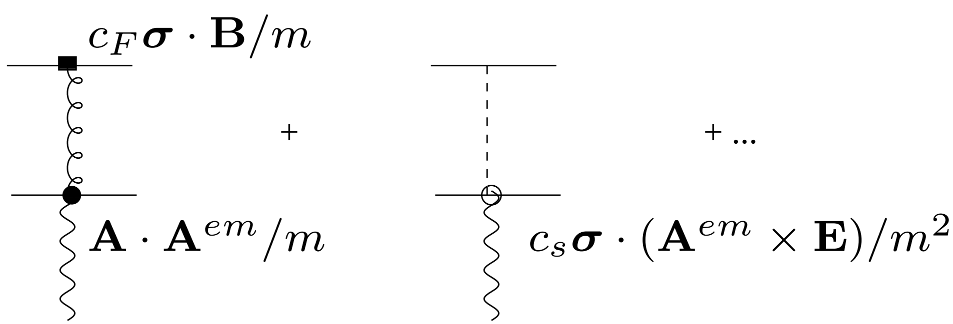
M1 operator at $\mathcal{O}(1)$

$$V_1 \left\{ S^\dagger, \frac{\boldsymbol{\sigma} \cdot e\mathbf{B}^{\rm em}}{2m} \right\} S$$

- $$\boxed{V_1 = 1 + \frac{2\alpha_{\rm s}(m_c)}{3\pi} + \cdots}$$
- No large quarkonium anomalous magnetic moment!
  - Dudek Edwards Richards PR D73 (2006) 074507

M1 operators at $\mathcal{O}(v^2)$

$$\frac{1}{4m^2}\,\frac{V_2}{r}\,\left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\left[\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times ee_Q\mathbf{B}^{\mathrm{em}}\right)\right]\right\}\mathrm{S} \quad \text{and} \quad \frac{1}{4m^2}\,\frac{V_3}{r}\,\left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot ee_Q\mathbf{B}^{\mathrm{em}}\right\}\mathrm{S}$$



$= \left(\text{hard}\right)\times\left(\text{soft}\right)$

- to all orders $\left(\text{hard}\right) = 2c_F - c_s = 1$; $\left(\text{soft}\right) = r^2V_s'/2$

  because of reparameterization/Poincaré invariance

  ◦ `Brambilla Gromes Vairo PL B576 (2003) 314`

- Therefore $\boxed{V_2 = r^2V_s'/2}$ and $\boxed{V_3 = 0}$

- No scalar interaction!

M1 operators at $\mathcal{O}(v^2)$

$$V_4 \left\{ S^\dagger, \frac{\boldsymbol{\sigma}\cdot e\mathbf{B}^{em}}{4m^3} \right\} \boldsymbol{\nabla}_r^2 S \text{ and } V_5 \left\{ S^\dagger, \frac{\boldsymbol{\sigma}^i e\mathbf{B}^{em\,j}}{4m^3} \right\} \boldsymbol{\nabla}_r^i \boldsymbol{\nabla}_r^j S$$

$$V_4 = \left(\text{hard}\right) \times \left(\text{soft}\right) \qquad\qquad V_5 = \left(\text{hard}\right)' \times \left(\text{soft}\right)'$$

- $\left(\text{hard}\right) = 1 \qquad \left(\text{hard}\right)' = \mathcal{O}(\alpha_s)$
  because of reparameterization invariance
  - `Manohar PR D56 (1997) 230`
- $\left(\text{soft}\right) = \left(\text{soft}\right)' = 1$
- $\boxed{V_4 = 1}$ and $\boxed{V_5 = \mathcal{O}(\alpha_s \text{ hard scale})}$

M1 operators at $\mathcal{O}(v^2)$

$$\frac{1}{4m^3}\,\frac{V_6}{r^2}\,\left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot ee_Q\mathbf{B}^{\mathrm{em}}\right\}\mathrm{S}$$

$$V_6 = \left(\text{hard}\right)\times\left(\text{soft}\right)$$

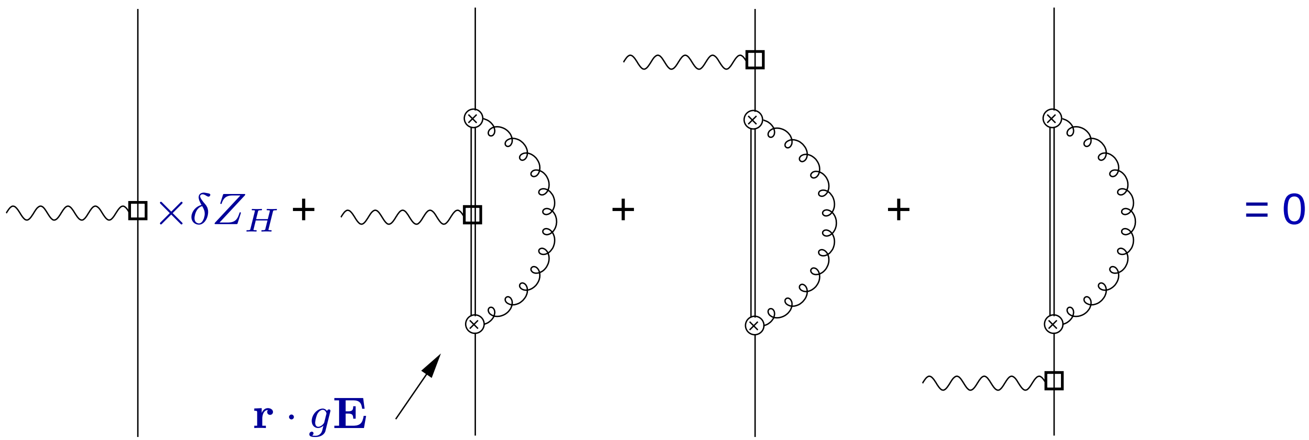
The soft part of V_6 is not protected by any symmetry, in the weak coupling it is suppressed by at least one power of α_{s}.

- $V_6 = \mathcal{O}(\alpha_{\mathrm{s}}$ soft scale$)$
- For strongly coupled quarkonia this is the only non-perturbative contribution at $\mathcal{O}(v^2)$!

$\mathcal{O}(v^2)$ corrections to the quarkonium states

For weakly coupled quarkonia, photons may couple to octets:

$$V_1 \left\{ O^\dagger, \frac{\boldsymbol{\sigma}\cdot e\mathbf{B}^{em}}{2m} \right\} O \qquad \text{[if } mv \gg \Lambda_{\rm QCD}\text{]}$$



- If $mv^2 \sim \Lambda_{\rm QCD}$ the above graphs are potentially of order $\Lambda^2_{\rm QCD}/(mv)^2 \sim v^2$.
- The contribution vanishes because $\boldsymbol{\sigma}\cdot e\mathbf{B}^{em}(\mathbf{R})$ behaves like the identity operator.
- For weakly coupled quarkonia there are no non-perturbative contributions at $\mathcal{O}(v^2)$!

$J/\psi \to \eta_c(1S)\,\gamma$

$\Gamma_{J/\psi\to\eta_c(1S)\,\gamma}$ enters into many charmonium BR.
Its uncertainty sets typically their experimental errors.

$$\Gamma_{J/\psi\to\eta_c(1S)\,\gamma} = (1.85 \pm 0.08 \pm 0.28)\,\text{keV}$$

- `CLEO coll PRL 102 (2009) 011801`

This should be compared with $\Gamma_{J/\psi\to\eta_c(1S)\,\gamma}$ in the non-relativistic limit, which gives about 2.83 keV (for $m_c = M_{J/\psi}/2 = 1548$ MeV). This calls for large relativistic corrections or a large quarkonium anomalous magnetic moment (excluded).

$J/\psi \to \eta_c(1S)\,\gamma$

Up to relative order v^2 the transition $J/\psi \to \eta_c(1S)\,\gamma$ is completely accessible by perturbation theory.

$$\Gamma_{J/\psi\to\eta_c(1S)\,\gamma} = \int \frac{d^3k}{(2\pi)^3}\,(2\pi)\delta(E_p^{J/\psi} - k - E_k^{\eta_c})|\langle\gamma(k)\eta_c|\mathcal{L}_\gamma|J/\psi\rangle|^2$$

$$= \frac{16}{3}\alpha e_c^2\,\frac{k_\gamma^3}{M_{J/\psi}^2}\left[1 + C_F\,\frac{\alpha_{\rm s}(M_{J/\psi}/2)}{\pi} - \frac{2}{3}(C_F\,\alpha_{\rm s}(p_{J/\psi}))^2\right]$$

The normalization scale for the $\alpha_{\rm s}$ inherited from κ is the charm mass ($\alpha_{\rm s}(M_{J/\psi}/2) \approx 0.35 \sim v^2$), and for the $\alpha_{\rm s}$, which comes from the Coulomb potential, is the typical momentum transfer $p_{J/\psi} \approx mC_F\alpha_{\rm s}(p_{J/\psi})/2 \approx 0.8$ GeV $\sim mv$.

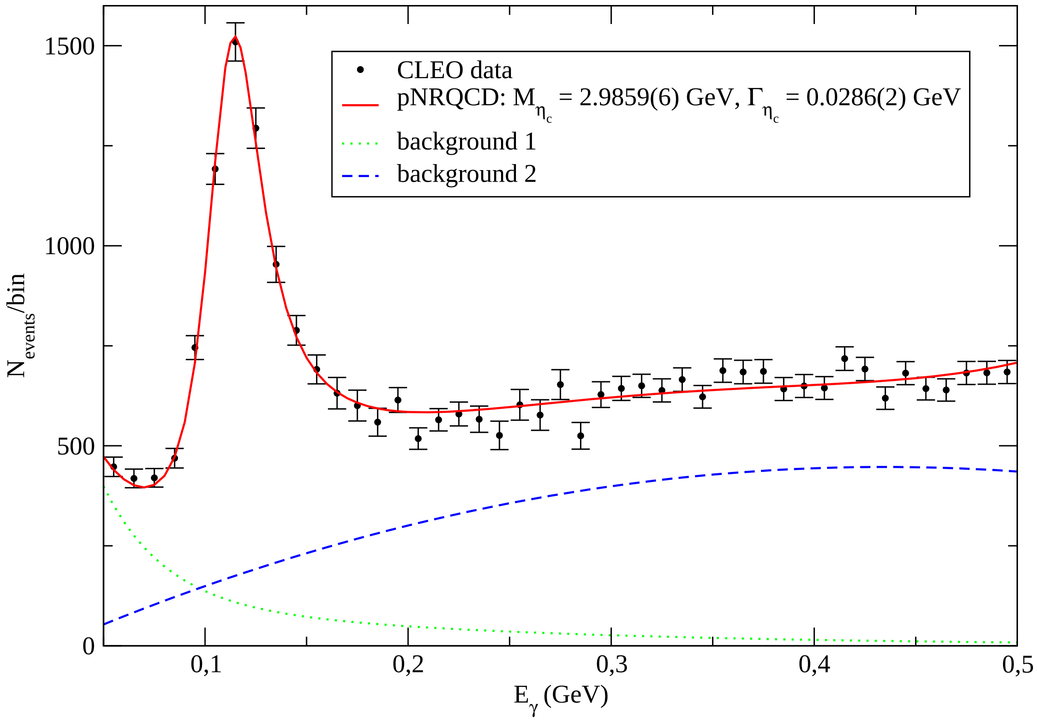
$$\Gamma_{J/\psi\to\eta_c(1S)\,\gamma} = (1.5 \pm 1.0)\ \text{keV}$$

$$\Gamma_{J/\psi\to\eta_c(1S)\,\gamma} = \frac{16}{3}\alpha e_c^2 \frac{k_\gamma^3}{M_{J/\Psi}^2}\left(1+\frac{4}{3}\frac{\alpha_s(M_{J/\Psi}/2)}{\pi} - \frac{2}{3}\frac{\langle 1|rV_s'|1\rangle}{M_{J/\Psi}} + 2\frac{\langle 1|V_s|1\rangle}{M_{J/\Psi}}\right)$$

- If $V_s = -\frac{4}{3}\frac{\alpha_s(\mu)}{r}$: $-\frac{2}{3}\frac{\langle 1|rV_s'|1\rangle}{M_{J/\Psi}} + 2\frac{\langle 1|V_s|1\rangle}{M_{J/\Psi}} = -\frac{32}{27}\alpha_s(\mu)^2 < 0$
- *If* $V_s = \sigma r$: $-\frac{2}{3}\frac{\langle 1|rV_s'|1\rangle}{M_{J/\Psi}} + 2\frac{\langle 1|V_s|1\rangle}{M_{J/\Psi}} = \frac{4}{3}\frac{\sigma}{M_{J/\Psi}}\langle 1|r|1\rangle > 0$

  A scalar interaction would add a negative contribution $-2\langle 1|V^{\rm scalar}|1\rangle/M_{J/\Psi}$.

Photon line shape in $J/\psi \to \eta_c(1S)\,\gamma \to X\,\gamma$



$$\Gamma_{J/\psi \to \eta_c(1S)\,\gamma} = (0.94 \pm 0.09\,\text{(stat)})\ \text{keV}$$

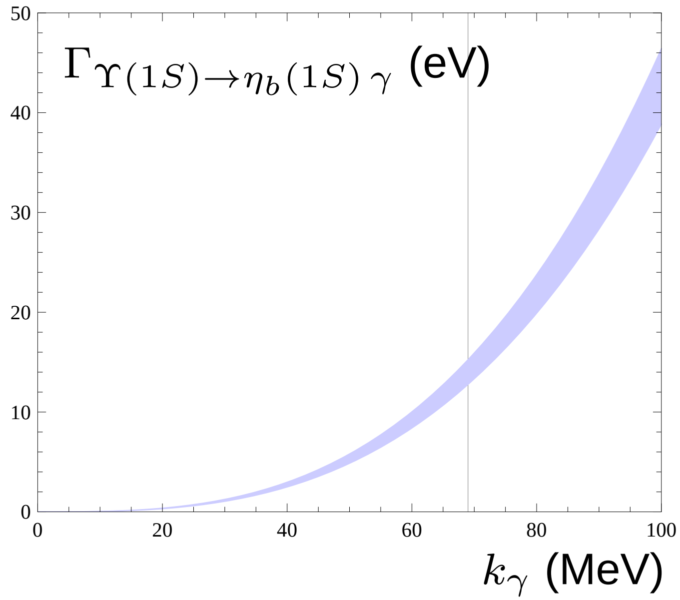
- Brambilla Roig Vairo AIP CP 1343 (2011) 418, in preparation

$\Gamma_{\Upsilon(1S)\to\eta_b(1S)\,\gamma}$

$\eta_b(1S)$ has been discovered in $\Upsilon(3S) \to \eta_b(1S)\,\gamma$ and $\Upsilon(2S) \to \eta_b(1S)\,\gamma$ by BABAR in 2008 and 2009, and in $\Upsilon(3S) \to \eta_b(1S)\,\gamma$ by CLEO in 2010, leading to the PDG average

$$M_{\eta_b(1S)} = (9391.0 \pm 2.8) \text{ MeV}$$

from which it follows



$$\Gamma_{\Upsilon(1S)\to\eta_b(1S)\,\gamma} = (k_\gamma/69 \text{ MeV})^3\,(14.0 \pm 1.2) \text{ eV}$$

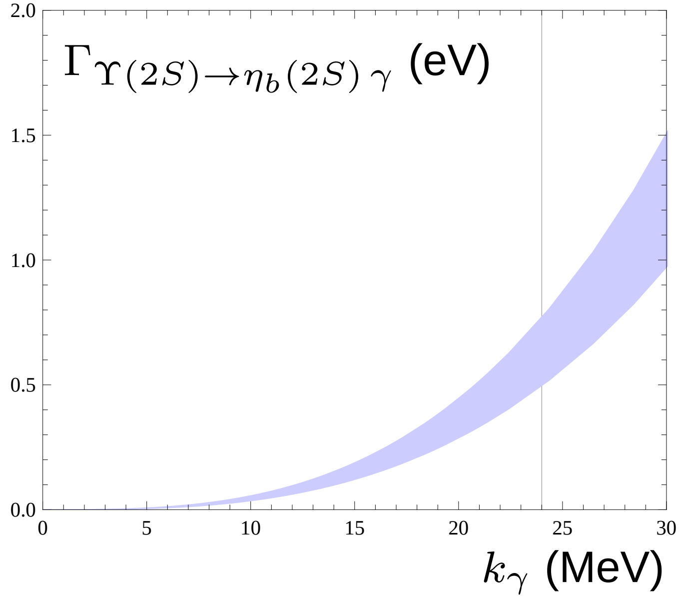
$\Gamma_{\Upsilon(2S)\to\eta_b(2S)\,\gamma}$

From the first evidence for $\eta_b(2S)$ (in $h_b(2P) \to \eta_b(2S)\,\gamma$) by the BELLE collaboration,

$$M_{\eta_b(2S)} = (9999.0 \pm 3.5^{+2.8}_{-1.9}) \text{ MeV}$$

○ `BELLE coll arXiv:1205.6351`

it follows



$$\Gamma_{\Upsilon(1S)\to\eta_b(2S)\,\gamma} = (k_\gamma/24 \text{ MeV})^3\,(0.63 \pm 0.14) \text{ eV}$$

M1 hindered transitions

- One new operator contributes:

$$-\frac{1}{16m^2}\,V_{12}\,\left[\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\left[-i\boldsymbol{\nabla}_r\times, \mathbf{r}^i(\boldsymbol{\nabla}^i ee_Q\mathbf{E}^{\rm em})\right]\right]\mathrm{S}$$

  that gets only hard contributions: $\boxed{V_{11} = V_{12} = 2c_F^{\rm em} - 1}$.

- Two new wave function corrections contribute:

  (1) induced by the spin-spin potential;

  (2) recoil correction induced by the spin-orbit potential;

  Due to the recoil, the final state develops a nonzero P-wave component suppressed by a factor $v\,k_\gamma/m$ (through the spin-orbit operator $-\frac{1}{4\,m^2}\frac{V_S^{(0)\,\prime}}{2}\,\mathrm{Tr}\left\{\{\mathrm{S}^\dagger,\boldsymbol{\sigma}\}\cdot[\hat{\mathbf{r}}\times(-i\boldsymbol{\nabla})]\,\mathrm{S}\right\}$), which, in a $n^3S_1 \to n'\,{}^1S_0\,\gamma$ transition, can be reached from the initial ${}^3S_1$ state through a $1/v$ enhanced E1 transition.

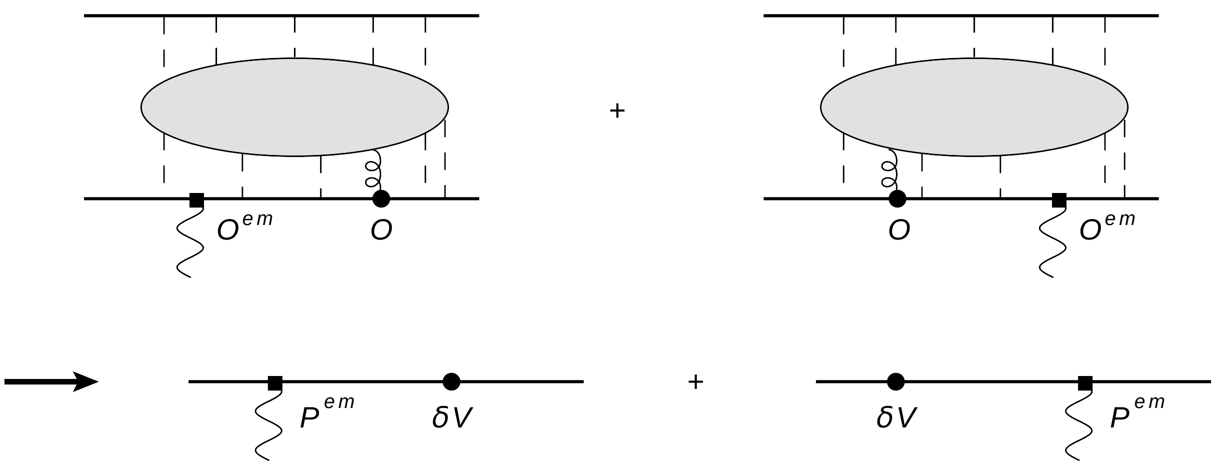
E1 transitions: matching

The matching proceeds like in the M1 case.

- No soft (non-perturbative) contributions to any of the matching coefficients.

$$V_A^{\rm em} = V_B^{\rm em} = 1$$
$$V_7 = V_8 = V_9 = 1$$
$$V_{10} = c_F^{\rm em}$$

Ex.



E1 transitions

$$\Gamma_{n^3P_J \to n'^3S_1\,\gamma} = \Gamma^{(0)}\left[1 + R_{nn'}^{S=1}(J) - \frac{k_\gamma^2}{60}\frac{I_5(n1\to n'0)}{I_3(n1\to n'0)} - \frac{k_\gamma}{6m} + \kappa\frac{k_\gamma}{2m}\left(\frac{J(J+1)}{2} - 2\right)\right]$$

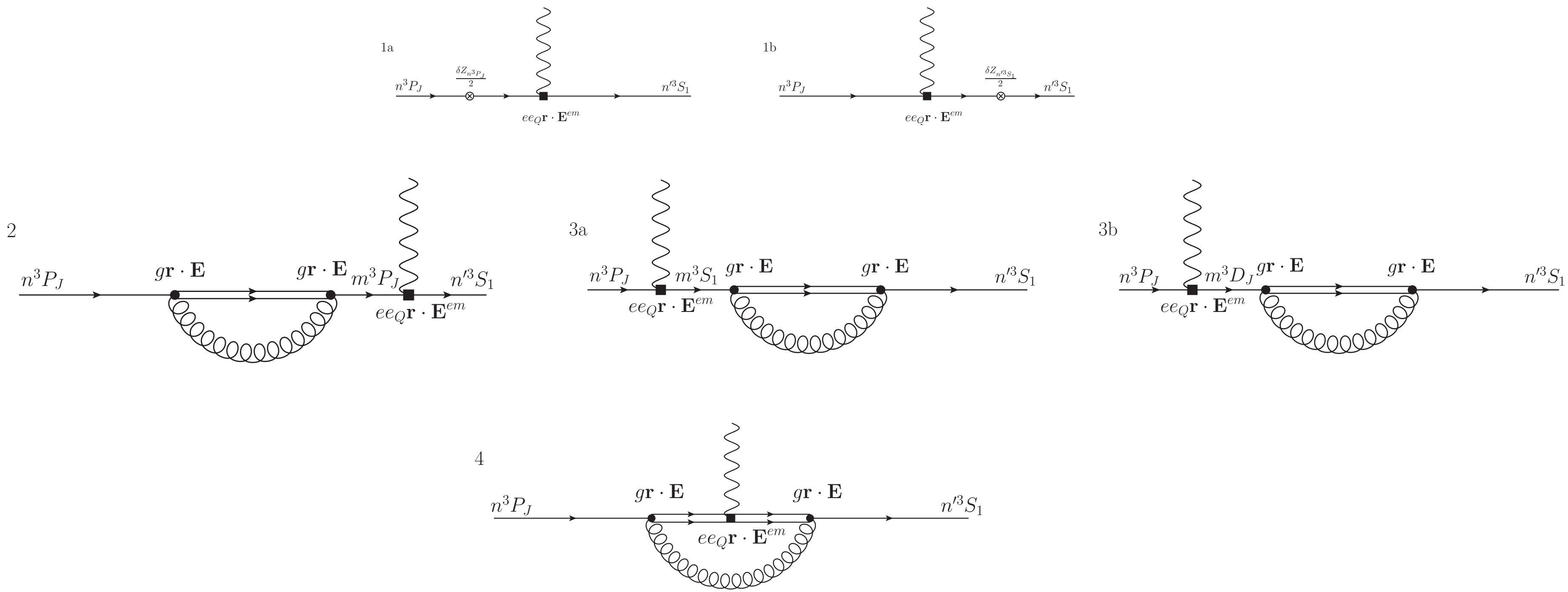
$$\Gamma_{n^1P_1 \to n'^1S_0\,\gamma} = \Gamma^{(0)}\left[1 + R_{nn'}^{S=0} - \frac{k_\gamma}{6m} - \frac{k_\gamma^2}{60}\frac{I_5(n1\to n'0)}{I_3(n1\to n'0)}\right].$$

where $I_N(nL \to n'L') = \int_0^\infty dr\, r^N\, R_{n'L'}(r)\, R_{nL}(r)$

and $R_{nn'}^{S=1}(J)$, $R_{nn'}^{S=0}$ are wave-function corrections.

E1 transitions

Octets contribute to E1 transitions in the weak coupling (unlike M1 transitions).



- This contribution would be encoded, in the weak coupling, in $R_{nn'}^{S=1}(J)$ and $R_{nn'}^{S=0}$.
- Gluon fields coupled to octets scale like mv^2 and are, in general, non perturbative.

Summary: M1 vs E1 transitions

At relative order v^2,

- weakly coupled M1 transitions (like $J/\psi \to \eta_c\,\gamma$) are completely accessible by perturbation theory;
- weakly coupled E1 transitions (perhaps of relevance only for bottomonium 1P$\to$ 1S transitions) are affected by non-perturbative corrections induced by the electromagnetic coupling to unbound quark-antiquark color octet states;
- strongly coupled M1 transitions get non-perturbative contributions through the wave functions and one non-perturbative $1/m^3$ operator;
- strongly coupled E1 transitions (which are most if not all E1 transitions) get non-perturbative contributions only through the wave functions.

Conclusions

EFTs provide a systematic approach to quarkonium radiative transitions with the potential to allow for precision phenomenology (line-shape parameters, branching fractions) and to address some more fundamental issues (nature of the bound state, nature of the confining interaction, quarkonium magnetic moment).

For what concerns the traditional Grotch Owen Sebastian formulas, they hold under the following conditions:

- there is no scalar interaction;
- the quarkonium anomalous magnetic moment is small and positive: $2\alpha_{\rm s}/(3\pi) + ...$;
- in the strong-coupling regime, the contribution from a $1/m^3$ operator is added to M1 transitions;
- they are valid up to relative order v^2.

Outlook

It remains to be done (among others)

- the completion of the non-perturbative matching of the last operator relevant for M1 transitions at relative order v^2;
- a consistent phenomenological analysis where the wave functions are obtained by solving the bound state equation (ideally) with the lattice potentials;
- the inclusion of relativistic corrections to the expression of the photon line shape close to the transition energies.