Nuclear Symmetry Energy & the R-mode Instability of Neutron Stars

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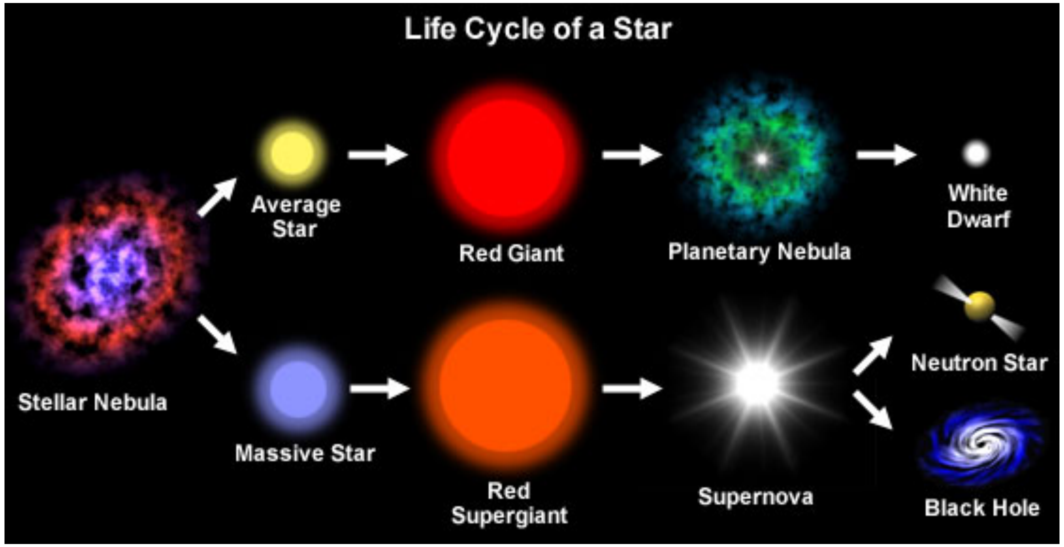
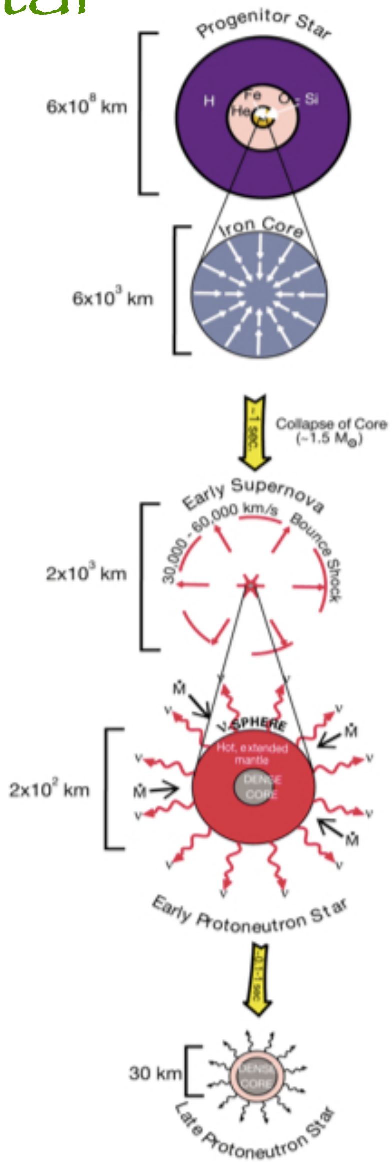
The birth of a Neutron Star

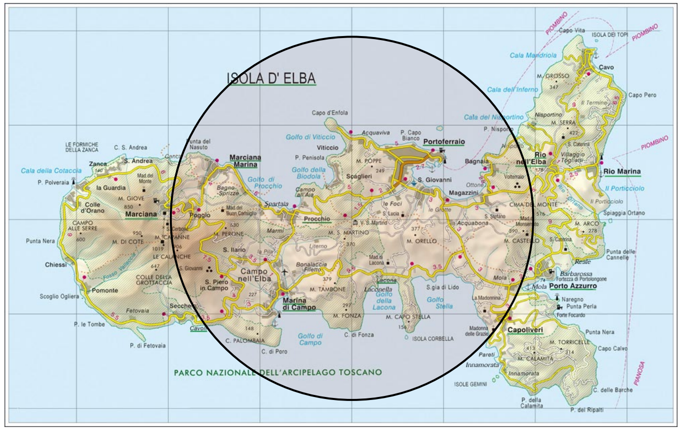
Neutron stars are a type of stellar compact remnant that can result from the gravitational collapse of a massive star ($8\ M_{\odot} < M < 25\ M_{\odot}$) during a Type II, Ib or Ic supernova event.





Some known facts about neutron stars

- Mass: $M \sim 1 - 2\, M_{\odot}$
- Radius: $R \sim 10 - 12$ km
- Density: $\rho \sim 10^{14} - 10^{15}\ g/cm^3$



$\rho_{universe} \sim 10^{-30}\ g/cm^3$

$\rho_{sun} \sim 1.4\ g/cm^3$

$\rho_{earth} \sim 5.5\ g/cm^3$

- Baryonic number: $N_b \sim 10^{57}$ ("giant nuclei")
- Magnetic field: $B \sim 10^{8\ldots16}$ G ($10^{4\ldots12}$ T)

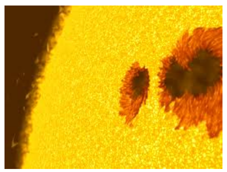
$0.3 - 0.5 G$

Earth

$10^3 - 10^4 G$

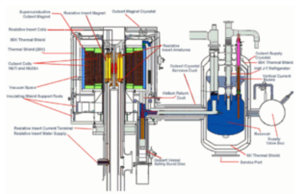
Magnet

$10^5 G$



Sunspots

$4.5x10^5 G$

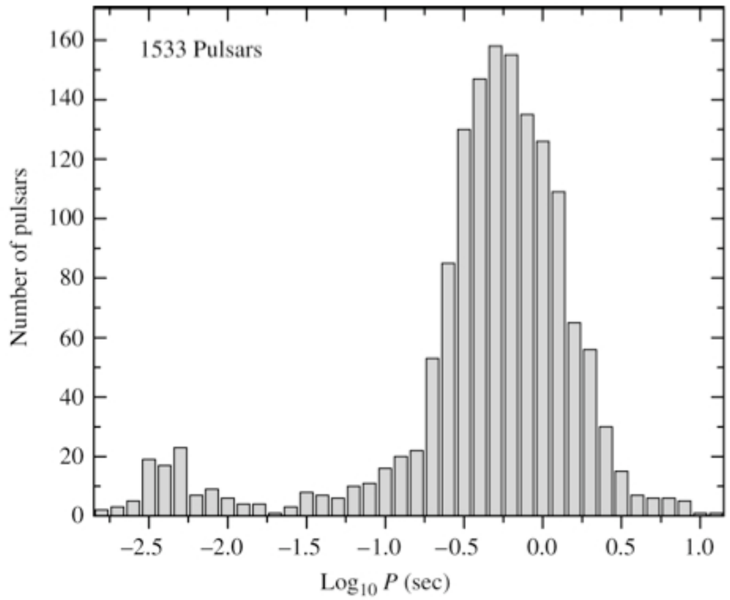


Largest continuous field in lab. (FSU, USA)

$2.8x10^7 G$

Largest magnetic pulse in lab. (Russia)

- Electric field: $E \sim 10^{18} V/cm$
- Temperature: $T \sim 10^{6...11}$ K
- Rotational period distribution
  ➔ two types of pulsars:
  - pulsars with P ~ s
  - pulsars with P ~ ms



Shortest rotational period: $P_{B1937+2} \approx 1.58$ ms until the last discovery: PSR in Terzan 5: $P_{J1748-2446ad} \approx 1.39$ ms

- Accretion rates: 10^{-10} to 10^{-8} $M_{\odot}$/year

Observation of neutron stars

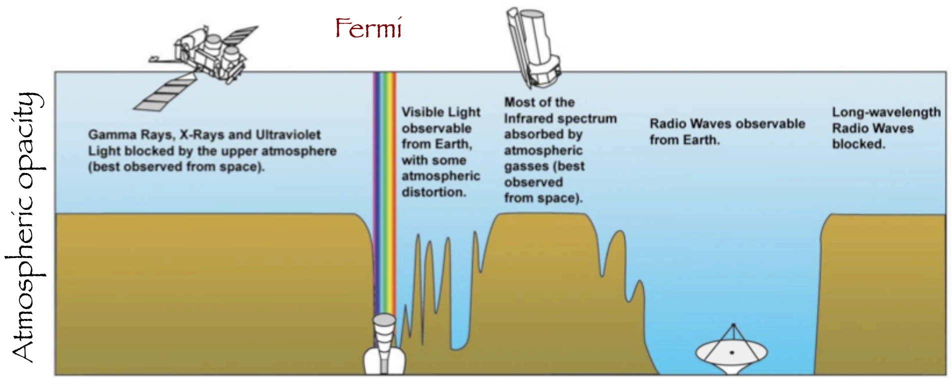
X- and γ-ray telescopes

Chandra

Fermi

Space telescopes

HST (Hubble)



Optical telescopes

VLT (Atacama, Chile)

Radio telescopes

Arecibo (Puerto Rico): 305 m

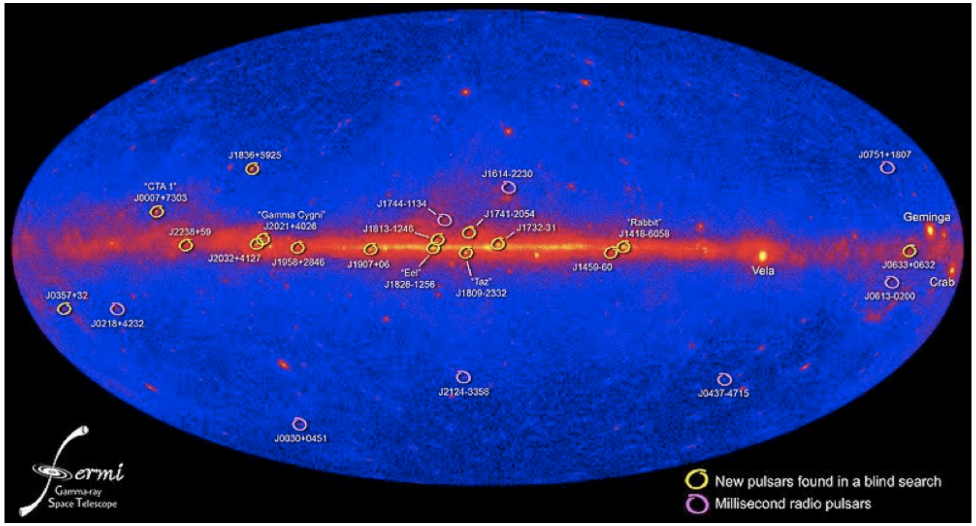
Green Banks (USA): 100 m

Nançay (France): 94 m

Nowadays more than 2000 pulsars are known

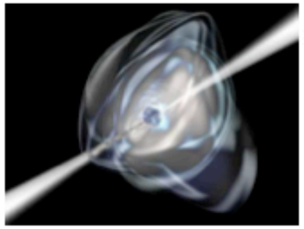
Observables

- Period (P, dP/dt)
- Masses
- Luminosity
- Temperature
- Magnetic Field
- Gravitational Waves (future)

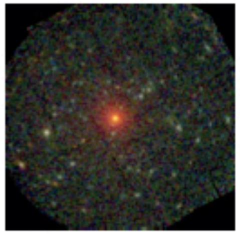


http://www.phys.ncku.edu.tw/~astrolab/mirrors/apod_e/ap090709.html

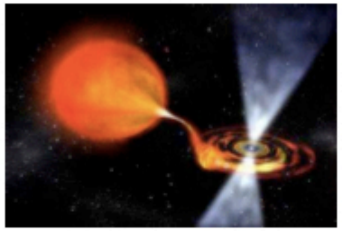
The 1001 astrophysical faces of neutron stars



Anomalous X-ray Pulsars

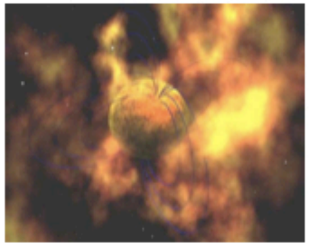


dim isolated neutron stars



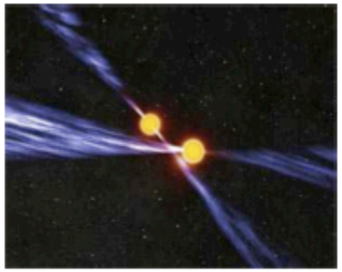
X-ray binaries

bursting pulsars

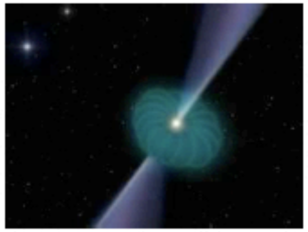


Soft Gamma Repeaters

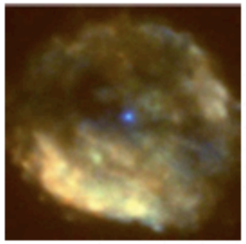
pulsars



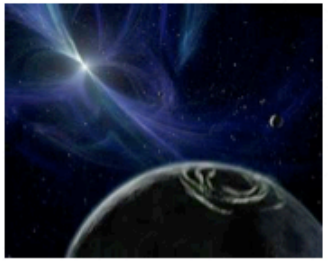
binary pulsars



Rotating Radio Transients



Compact Central Objects



planets around pulsar

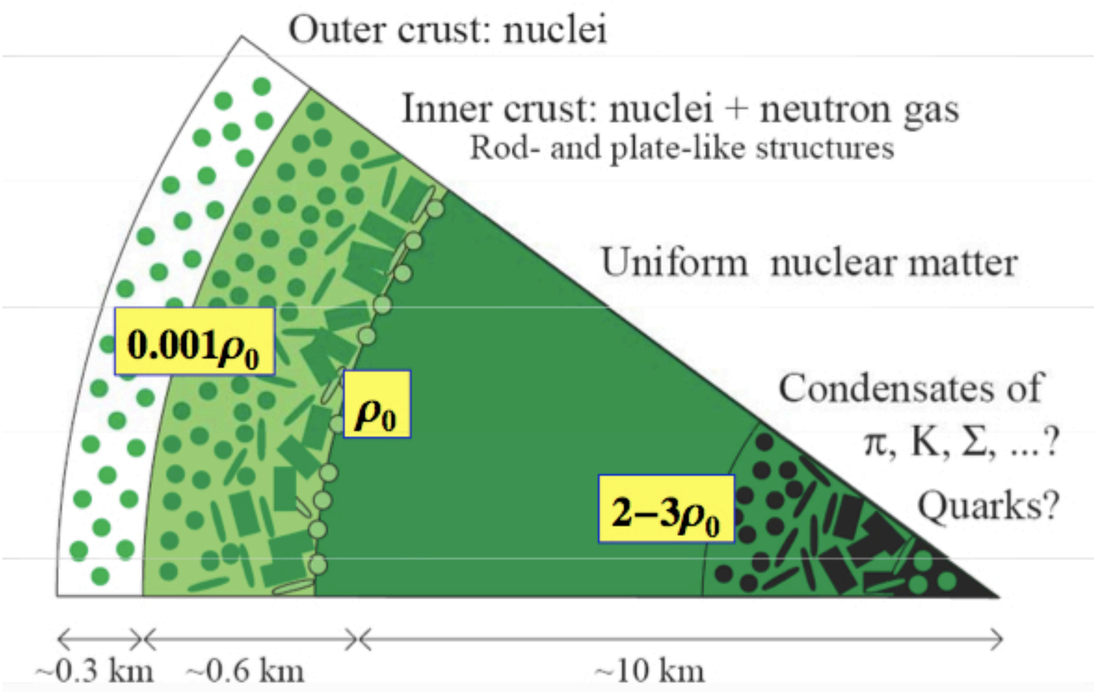
Anatomy of a Neutron Star

Equilibrium composition determined by

- ✓ Charge neutrality

$$\sum_i q_i \rho_i = 0$$

- ✓ Equilibrium with respect to weak interacting processes



$$b_1 \rightarrow b_2 + l + \bar{\nu}_l$$
$$b_2 + l \rightarrow b_1 + \nu_l$$

$$\longrightarrow \quad \mu_i = b_i \mu_n - q_i \left(\mu_e - \mu_{\nu_e} \right), \quad \mu_i = \frac{\partial \varepsilon}{\partial \rho_i}$$

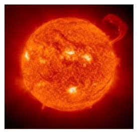
Structure equations for neutron stars

Large gravitational potential
on neutron star's surface

$$\rightarrow \quad \frac{2GM}{c^2 R} \sim 0.2 - 0.4$$

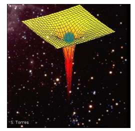
General Relativity is needed
to describe their structure

$\sim 10^{-10}$



$\sim 10^{-5}$

$\sim 10^{-4} - 10^{-3}$



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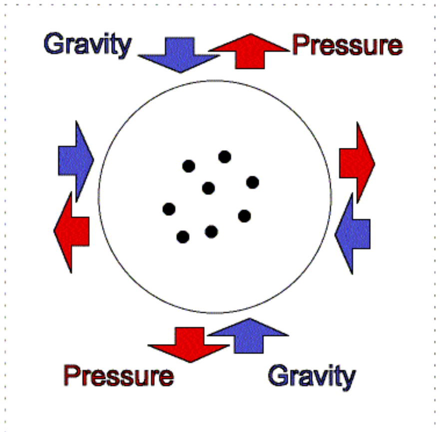
Tolman-Oppenheimer-Volkoff (TOV) equations describe the hydrostatic equilibrium
of a non-rotating neutron star in GR



$$\frac{dP}{dr} = -G\frac{m(r)\varepsilon(r)}{r^2}\left(1+\frac{P(r)}{c^2\varepsilon(r)}\right)\left(1+\frac{4\pi r^3 P(r)m(r)}{c^2}\right)\left(1-\frac{Gm(r)}{c^2 r}\right)^{-1}$$

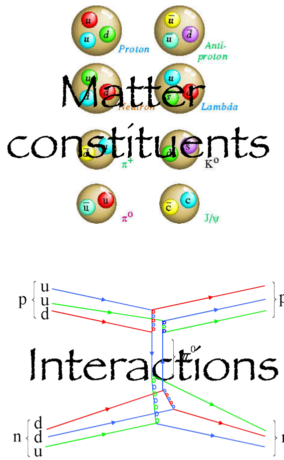
$$\frac{dm}{dr} = 4\pi r^2\varepsilon(r)$$

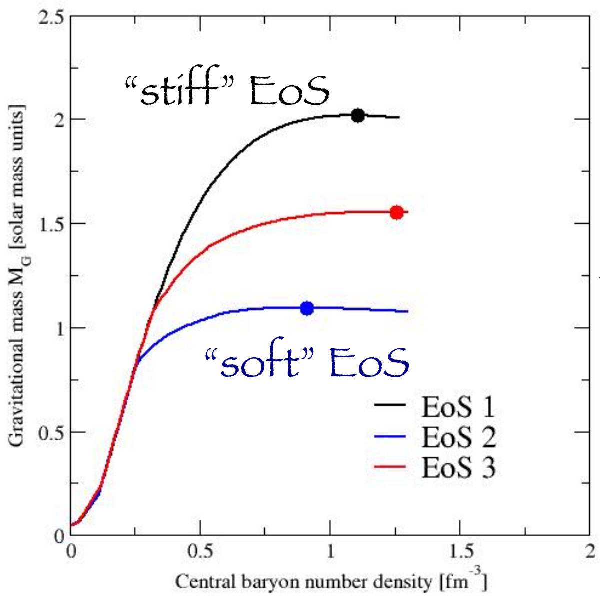
boundary conditions

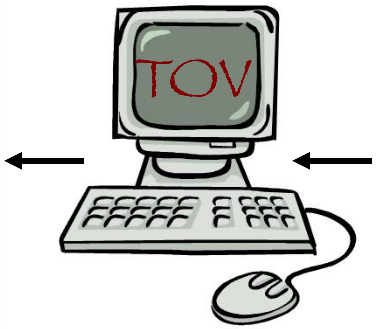
$$P(0) = P_o\,, \quad m(0) = 0$$
$$P(R) = 0\,, \quad m(R) = M$$

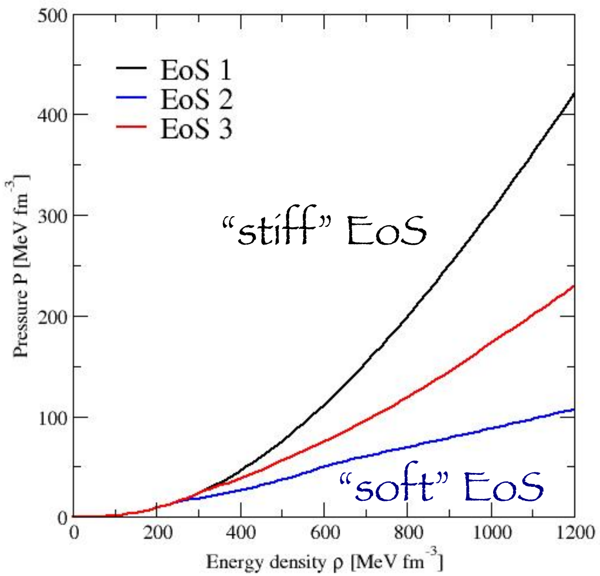
The role of the Equation of State

The only ingredient needed to solve the structure equations of neutron stars is the (poorly known) EoS (i.e., p(ε)) of dense matter







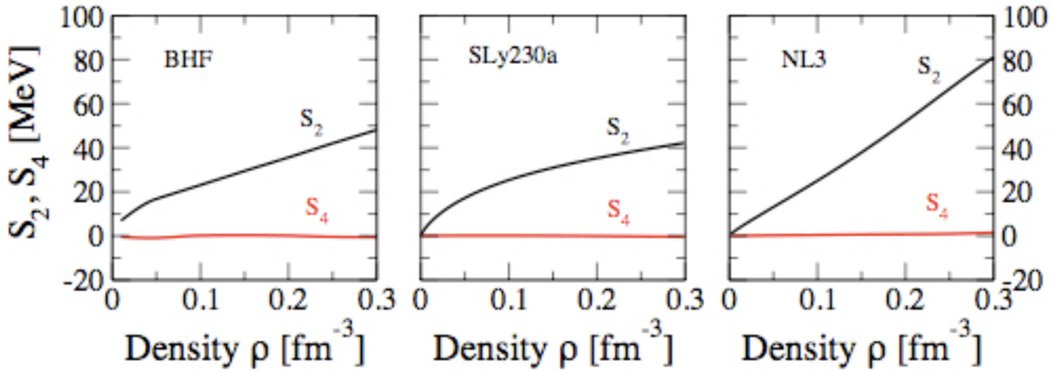


Equation of State of Asymmetric Nuclear Matter

Charge symmetry ➔ expansion of $(E/A)_{ANM}$ on even powers of isospin asymmetry $\beta=(\rho_n-\rho_p)/(\rho_n+\rho_p)$

$$\frac{E}{A}(\rho,\beta) = E_{SNM}(\rho) + S_2(\rho)\beta^2 + S_4(\rho)\beta^4 + O(6)$$

$$E_{SNM}(\rho) = \frac{E}{A}(\rho,\beta=0), \quad S_2(\rho) = \frac{1}{2}\frac{\partial^2 E/A}{\partial\beta^2}\bigg|_{\beta=0}, \quad S_4(\rho) = \frac{1}{24}\frac{\partial^4 E/A}{\partial\beta^4}\bigg|_{\beta=0}$$



In good approximation:

$$S_2(\rho) \sim \frac{E}{A}(\rho,\beta=1) - \frac{E}{A}(\rho,\beta=0)$$

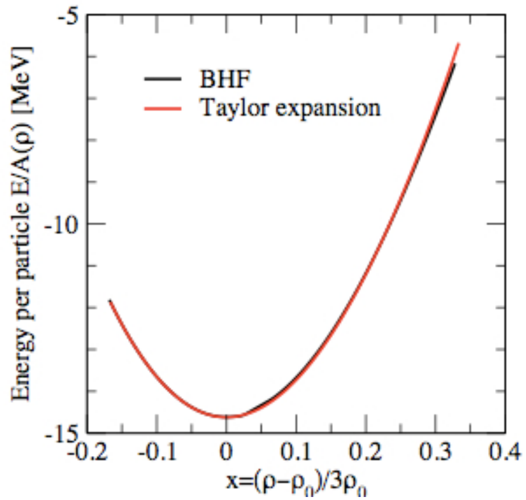
$E_{SNM}(\rho)$ commonly expanded around saturation density ρ_0

$$E_{SNM}(\rho) = E_0 + \frac{K_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{Q_0}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3 + O(4)$$

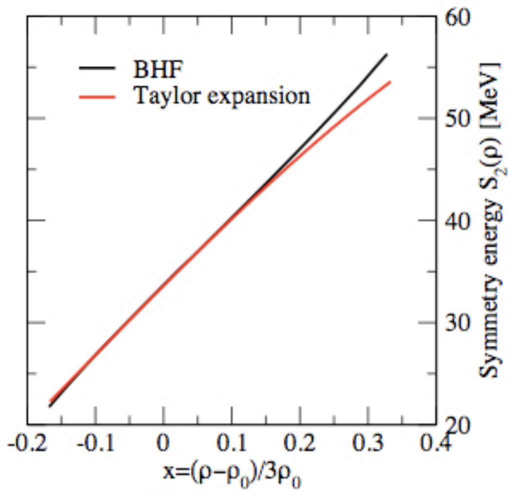
$$E_0 = E_{SNM}(\rho=\rho_0) \approx -16\,MeV$$

$$K_0 = 9\rho_0^2 \frac{\partial^2 E_{SNM}(\rho)}{\partial\rho^2}\bigg|_{\rho=\rho_0} \approx 240 \pm 20\,MeV$$

$$Q_0 = 27\rho_0^3 \frac{\partial^3 E_{SNM}(\rho)}{\partial\rho^3}\bigg|_{\rho=\rho_0} \approx -500 \div 300\,MeV$$



Similarly $S_2(\rho)$ can be also characterized with few bulk parameters around ρ_0

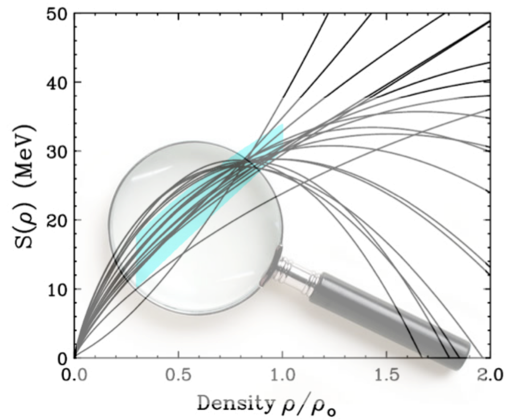


$$S_2(\rho) = E_{sym} + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{K_{sym}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \frac{Q_{sym}}{6}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^3 + O(4)$$

$$L = 3\rho_0 \frac{\partial S_2(\rho)}{\partial\rho}\bigg|_{\rho=\rho_0} \quad K_{sym} = 9\rho_0^2 \frac{\partial^2 S_2(\rho)}{\partial\rho^2}\bigg|_{\rho=\rho_0} \quad Q_{sym} = 27\rho_0^3 \frac{\partial^3 S_2(\rho)}{\partial\rho^3}\bigg|_{\rho=\rho_0}$$

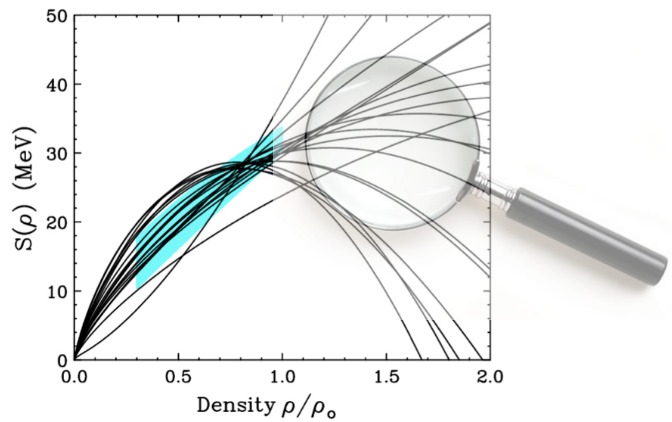
Less certain & predictions of different models vary largely

Symmetry Energy Sensitive Observables



- Sub-saturation densities

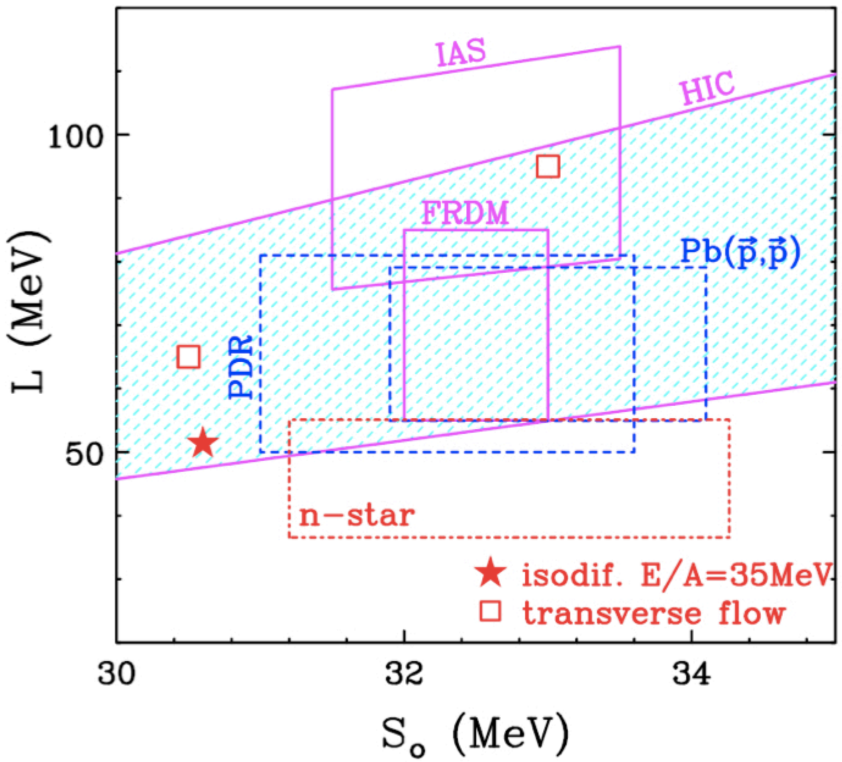
✓ Neutron skin thickness in heavy nuclei
✓ Giant & pygmy resonances in neutron-rich nuclei
✓ n/p & t/^{3}He ratios in nuclear reactions
✓ Isospin fragmentation & isospin scaling in nuclear multi-fragmentation
✓ Neutron-proton correlation functions at low relative momenta
✓ Isosin diffusion/transport in heavy ion collisions
✓ Neutron-proton diffential flow



- Supra-saturation densities

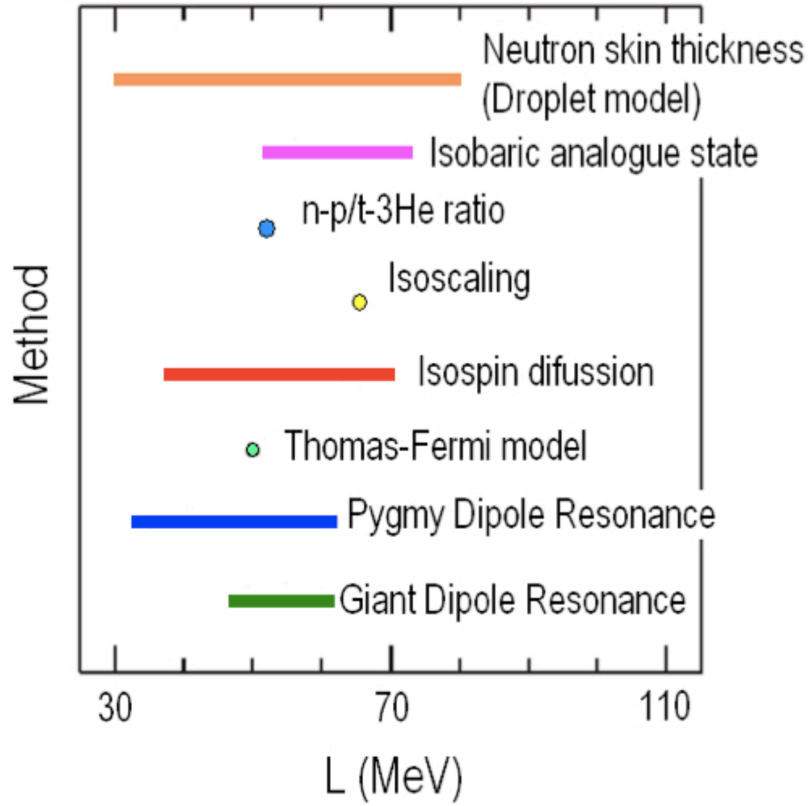
✓ π^-/π^+ & K^-/K^+ ratios in heavy ion collisions
✓ Neutron-proton differential transverse flow
✓ n/p ratio of squeezed out nucleons perpendicular to the reaction plane
✓ Nucleon elliptic flow at high transverse momenta

Symmetry Energy versus L



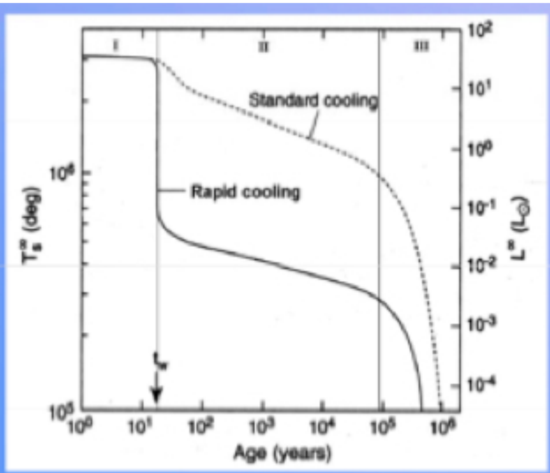
M. B. Tsang et al, PRC in press
arXiv 1204.0466 (2012)

Recent extracted values of L



(Adapted from D. V. Shetty & S. J. Yennello, Pramana 75, 259 (2010))

Neutron Star Cooling & Symmetry Energy



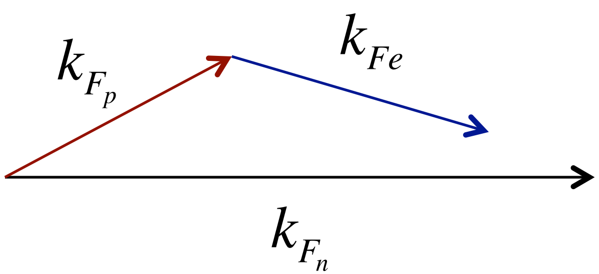
I. Core relaxation epoch
II. Neutrino cooling epoch
III. Photon cooling epoch

- Fast: e.g., Direct URCA

$$n \rightarrow p + l + \bar{\upsilon}_l$$
$$l + p \rightarrow n + \upsilon_l$$

- Slow: e.g., Modified URCA

$$N + n \rightarrow N + p + l + \bar{\upsilon}_l$$
$$N + l + p \rightarrow N + n + \upsilon_l$$

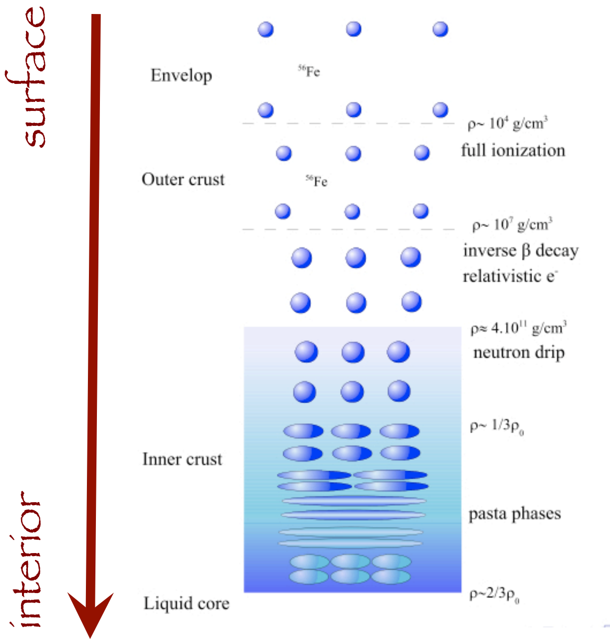


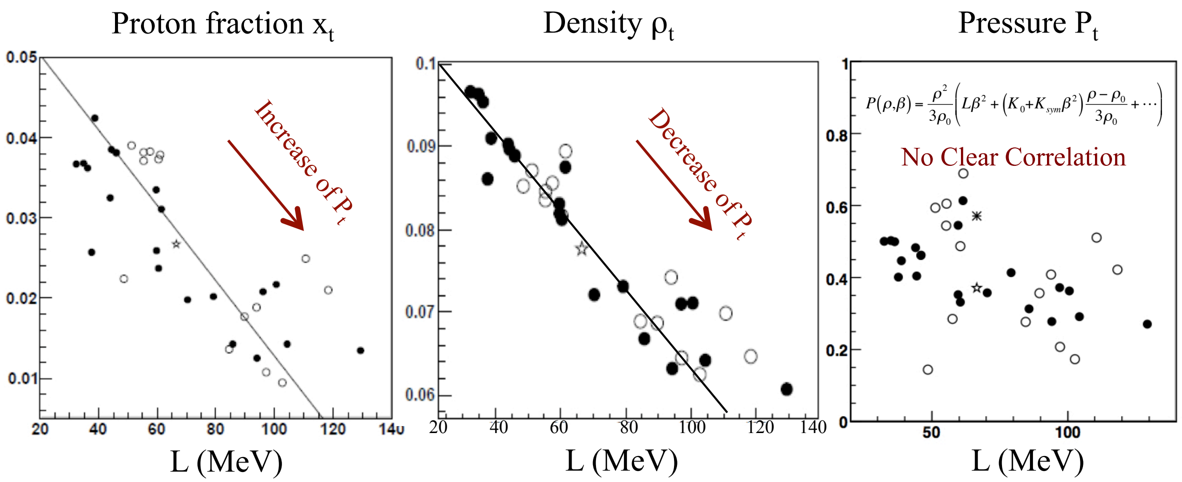
Direct URCA cannot occur unless x_p> 11%-15%

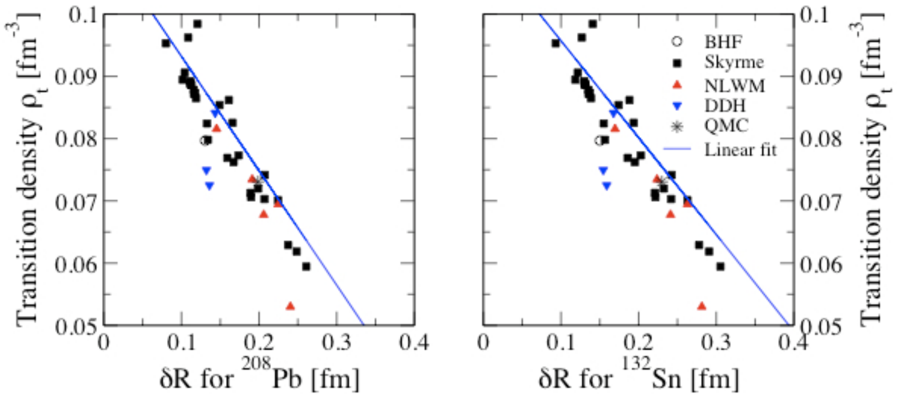
Larger Symmetry Energy ➔ Larger x_p ➔ Earlier onset of Direct URCA

$$\mu_n - \mu_p = 4(1-2x_p)S_2(\rho) = \mu_l - \mu_{\upsilon_l} \Rightarrow \frac{x_p}{1-2x_p} = \frac{4S_2(\rho)}{\hbar c(3\pi^2\rho)^{1/3}}$$

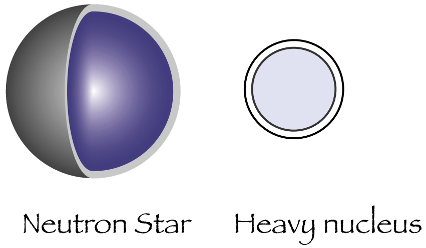
Crust-core Transition & Symmetry Energy







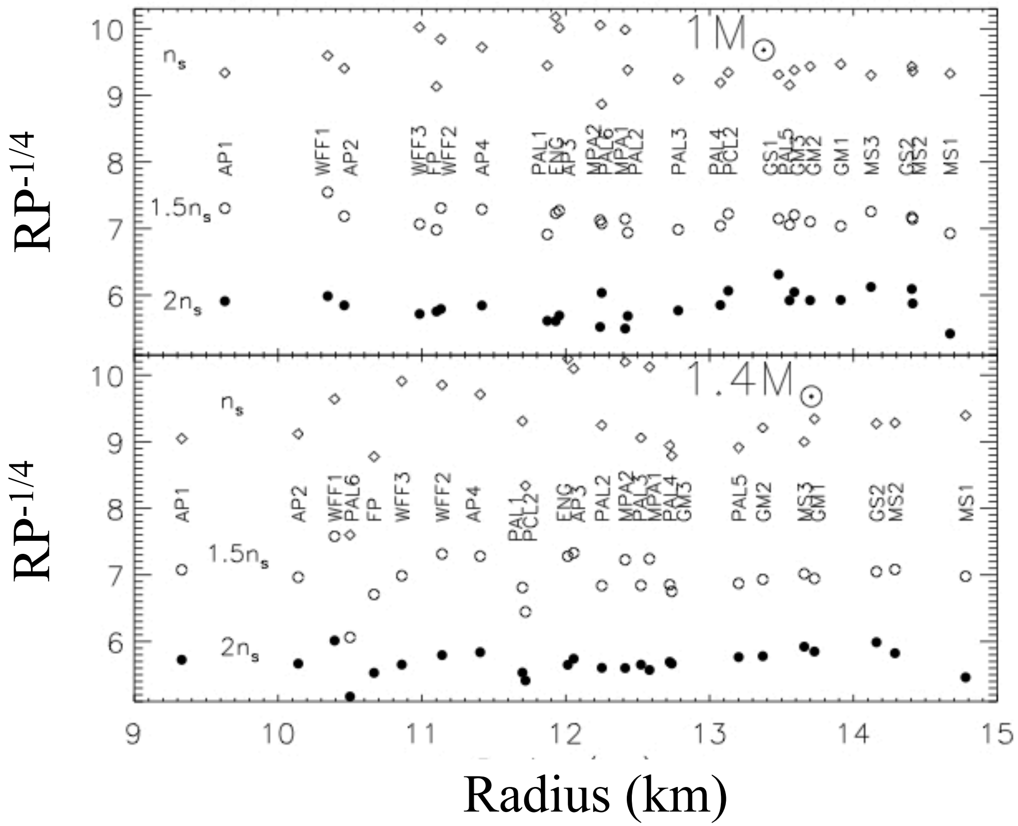
Inverse correlation between δR and ρ_t
(Horowitz & Piekarewicz)



Crust & Neutron Skin made out of neutron rich matter at similar densities ➔ Both governed by EoS at $\rho < \rho_0$ (particularly by $S_2(\rho)$ & its derivatives)

Pressure – Radius Correlation

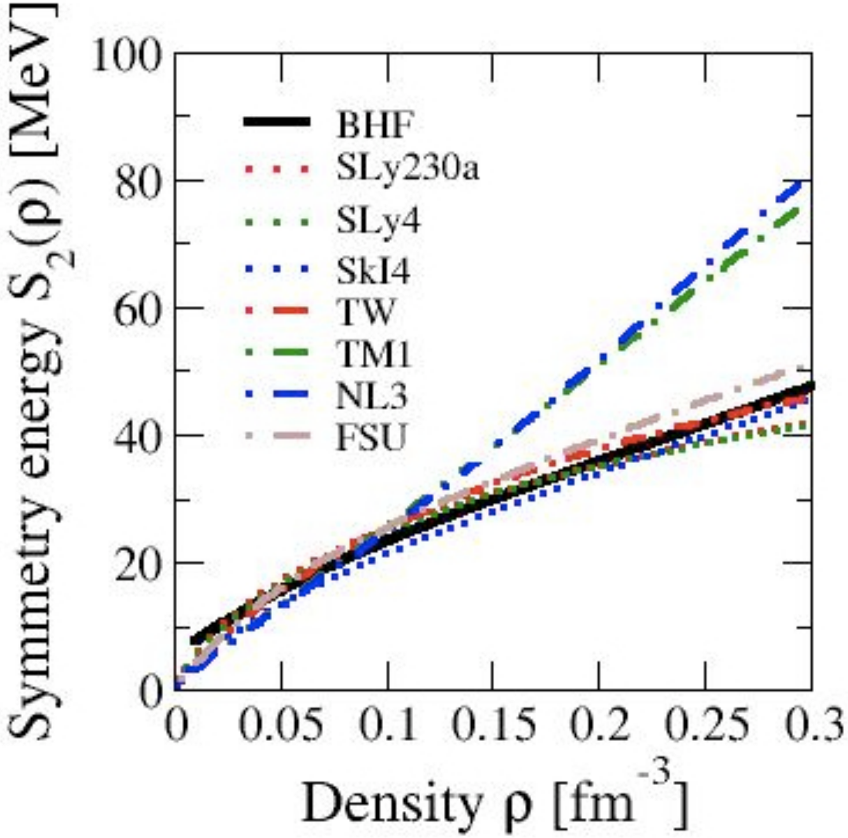
The analysis of this correlation can put stringent constraints in $S_2(\rho)$

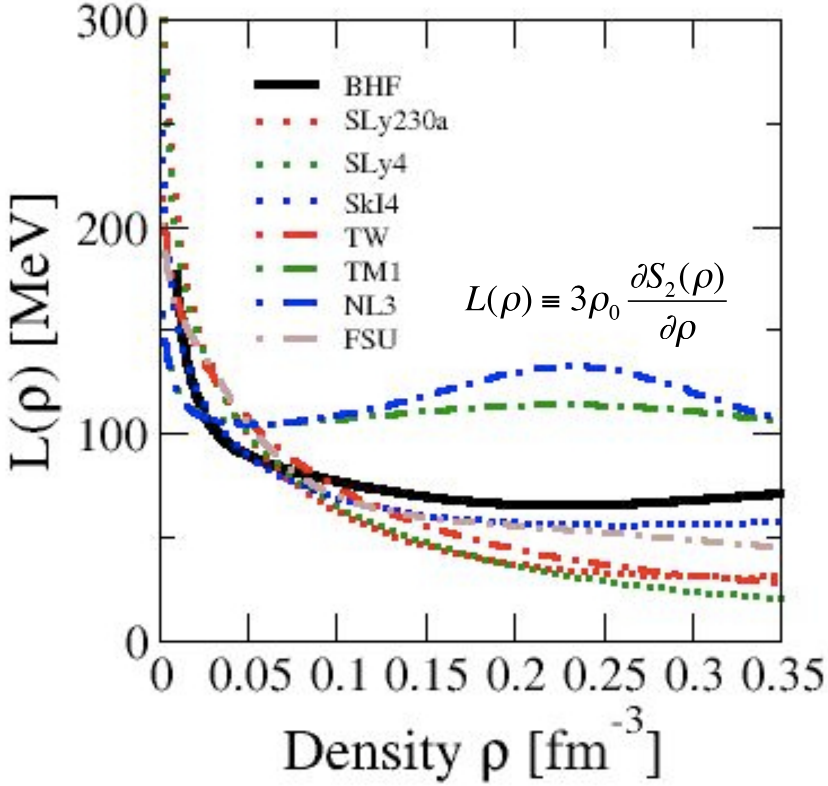


$$P(\rho,\beta) = \frac{\rho^2}{3\rho_0}\left(L\beta^2 + \left(K_0 + K_{sym}\beta^2\right)\frac{\rho-\rho_0}{3\rho_0} + \cdots\right)$$

Lattimer & Prakash, ApJ 550, 426 (2001)

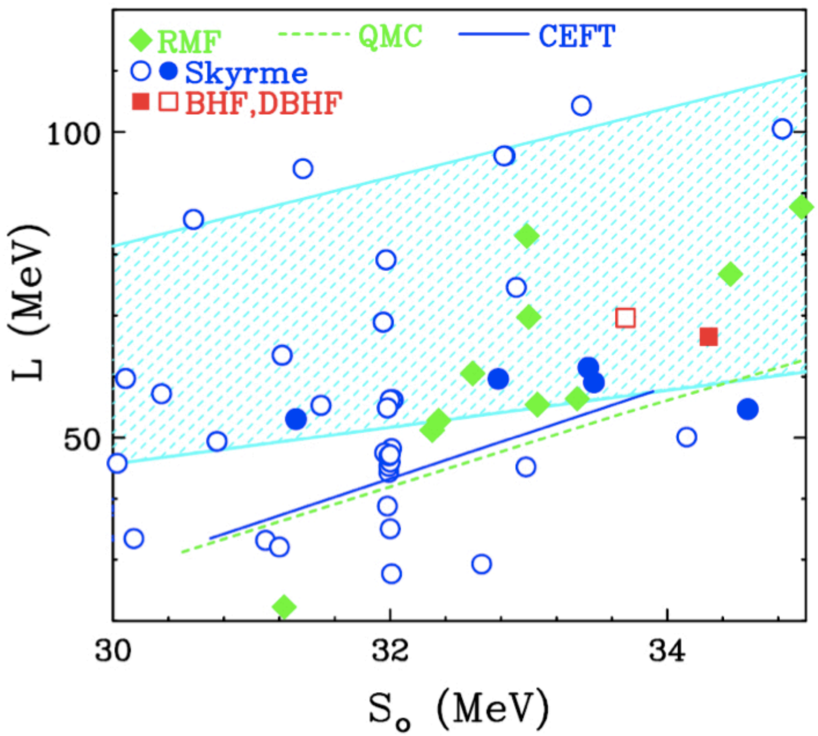
Theoretically ...





I.V., C. Providência, A. Polls & A. Rios, PRC 80, 045806 (2009)

Nevertheless, in spite of the experimental & theoretical efforts carried out $S_2(\rho)$ is still uncertain, specially at high densities



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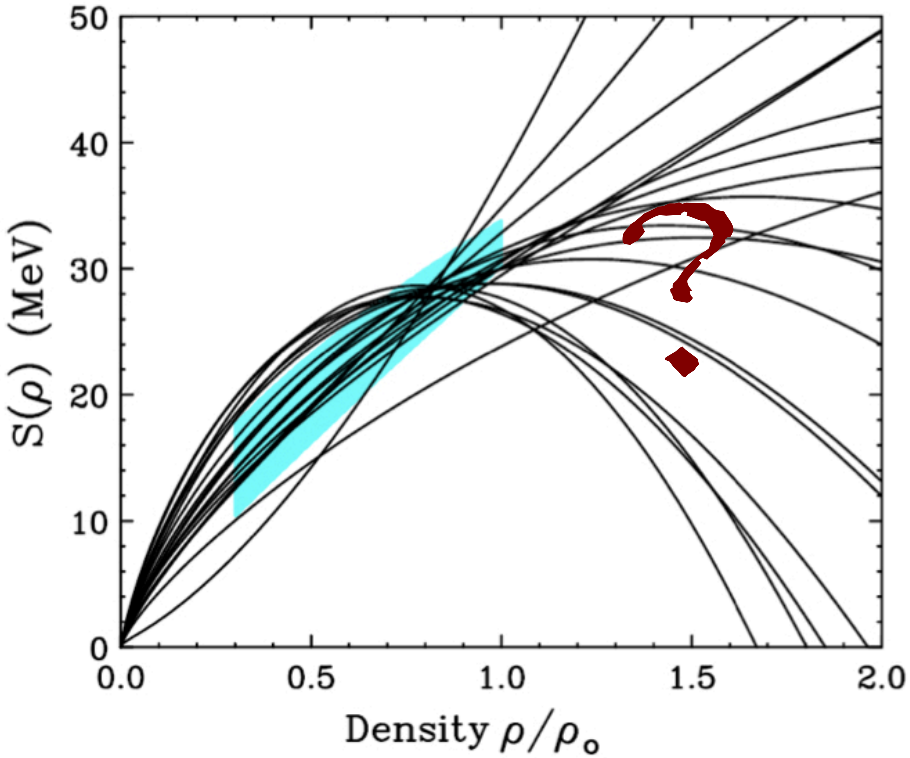

Brown, PRL 85, 5296 (2001)

In this talk …

Study the role of the symmetry energy slope parameter L on the r-mode instability of neutron stars by using both microscopic (BHF, APR & AFDMC) and phenomenological (Skyrme & RMF) approaches of the nuclear matter EoS

based on: Phys. Rev. C 85, 045808 (2012)

Ω_{Kepler} : absolute upper limit on the rotational frequency of neutron stars

(matter ejected from equator for $\Omega > \Omega_{Kepler}$)

$$\Omega_{Kepler} \approx 7800 \sqrt{\left(\frac{M}{M_{sun}}\right)\left(\frac{10km}{R}\right)^3} s^{-1}$$

But instabilities can prevent neutron stars from reaching Ω_{Kepler} ➔ more stringent limit in rotation

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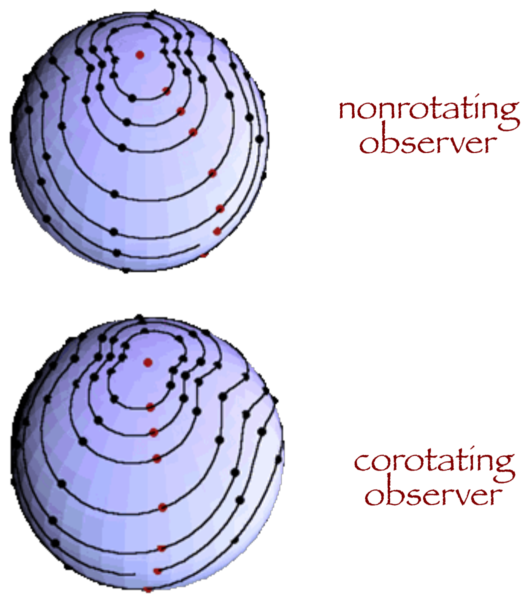
R-mode instability : toroidal mode of oscillation

✓ restoring force: Coriolis

✓ emission of GW in hot & rapidly rotating NS (CFS mechanism)

- GW (viscosity) grow (stabilizes) the mode
- GW potentially detectable ➔ information on information on internal structure of NS ➔ constraints on EoS (particularly $E_{sym}(\rho)$)



Picture by Prof. B. J. Owen

Microscopic approaches

BHF with Av18 + UIX

- $$\frac{E}{A}(\rho,\beta) = \frac{1}{A}\sum_{\tau}\sum_{k\le k_{F_\tau}}\left(\frac{\hbar^2k^2}{2m_\tau} + \frac{1}{2}\mathrm{Re}\left[U_\tau(\vec{k})\right]\right)$$

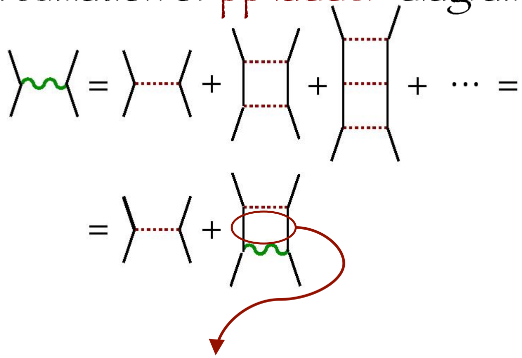
Infinite sumation of two-hole line diagrams

✓ $$G(\omega) = V + V\frac{Q}{\omega - E - E' + i\eta}G(\omega)$$

✓ $$E_\tau(k) = \frac{\hbar^2k^2}{2m_\tau} + \mathrm{Re}\left[U_\tau(k)\right]$$

✓ $$U_\tau(k) = \sum_{\tau'}\sum_{k'\le k_{F_{\tau'}}}\left\langle \vec{k}\vec{k}'\middle|G(\omega = E_\tau(k) + E_{\tau'}(k'))\middle|\vec{k}\vec{k}'\right\rangle_{\mathcal{A}}$$

Partial sumation of pp ladder diagrams



✓ Pauli blocking

✓ Nucleon dressing

APR & AFDMC parametrized

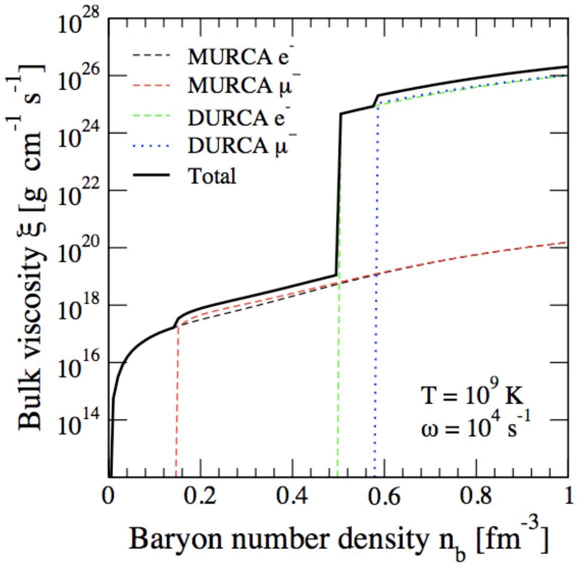
- $$\frac{E}{A}(\rho,\beta) = E_0u\frac{u-2-s}{1+us} + S_0u^\gamma\beta^2, \qquad u = \rho/\rho_0$$ Heiselberg & Hjorth-Jensen Phys. Rep. 328, 237 (2000)

- $$\frac{E}{A}(\rho,\beta) = E_0 + a(\rho-\rho_0)^2 + b(\rho-\rho_0)^3e^{\gamma(\rho-\rho_0)} + C_s\left(\frac{\rho}{\rho_0}\right)^{\gamma_s}\beta^2$$ Gandolfi et al., MNRAS 404, L35 (2010)

Phenomenological approaches

- Skyrme
  - Lyon group SLy: SLy0-SLy10, Sly230a
  - SkI family: SkI1-SkI6
  - Rs, Gs, SGI, SkMP, SkO, SkO', SkT4-5, SV
- Relativistic mean field models
  - Non-linear Walecka models (NLWM) with constant coupling constants: GM1, GM3, TM1, NL3, NL3-II, NL-SH
  - Density dependent hadronic models (DDH) with density dependent coupling constants: DDME1, DDME2, TW99, PK1, PK1R, PKDD

Bulk & shear viscosities



$$\xi = \xi_{MURCA} + \xi_{DURCA} = \sum_{Nl} \frac{|\lambda_{Nl}|}{\omega^2} \left|\frac{\partial P}{\partial X_l}\right| \frac{\partial \varsigma_l}{\partial n_b} + \sum_{l} \frac{|\lambda_l|}{\omega^2} \left|\frac{\partial P}{\partial X_l}\right| \frac{\partial \varsigma_l}{\partial n_b}$$

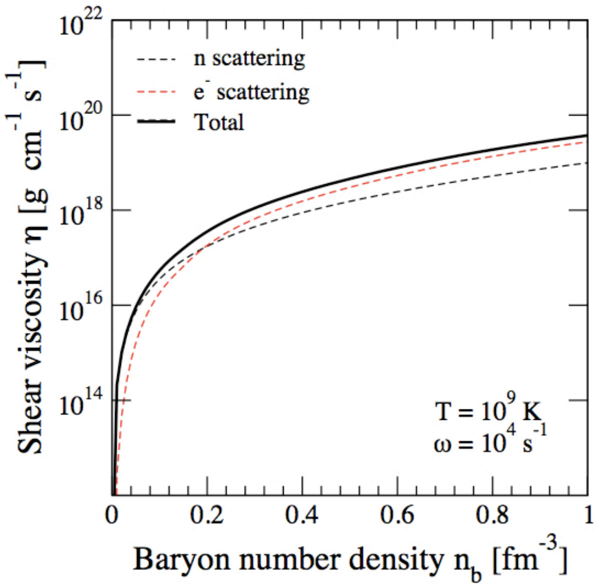
✓ Modified URCA

$$N + n \rightarrow N + p + l + \bar{\nu}_l, \quad N + p + l \rightarrow N + n + \nu_l$$

✓ Direct URCA

$$n \rightarrow p + l + \bar{\nu}_l, \quad p + l \rightarrow n + \nu_l$$

Haensel et al., AA 357, 1157 (2000); AA 372, 130 (2001)



$$\eta = \eta_n + \eta_e$$

✓ n scattering [1]

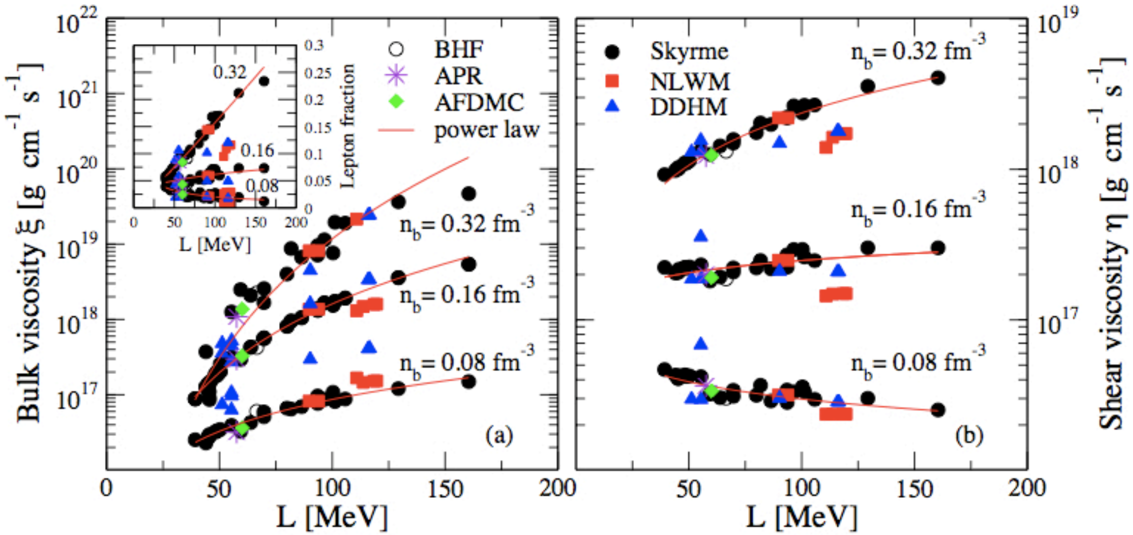
$$\eta_n = 2 \times 10^{18} \left(\frac{\rho}{10^{15}\, g cm^{-3}} \right)^{9/4} \left(\frac{T}{10^9 K} \right)^{-2}$$

✓ e^- scattering [2]

$$\eta_e = 4.26 \times 10^{-26} \left(x_p n_b \right)^{14/9} T^{-5/3}$$

[1] Cutler & Lindblom., ApJ 314, 234 (1987)
[2] Shternin & Yakovlev, PRD 78, 063006 (2008)

L dependence of ξ and η

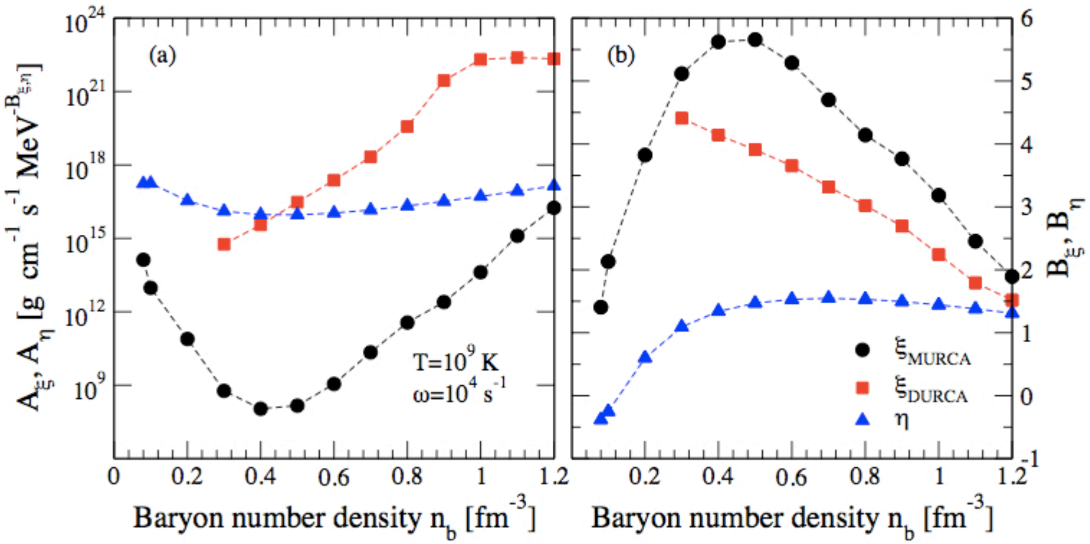


- ✓ ξ increases with L for all densities
- ✓ η increases with L for $n_b > n_0$ & decreases with L for $n_b < n_0$

consequence of lepton fraction dependence

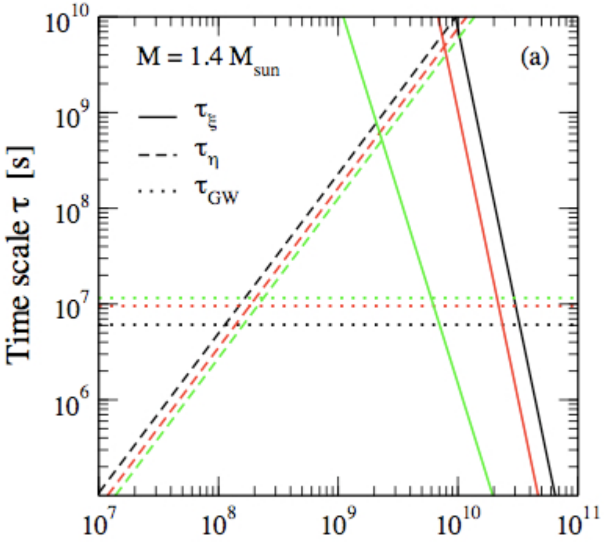
L dependence described by simple power laws

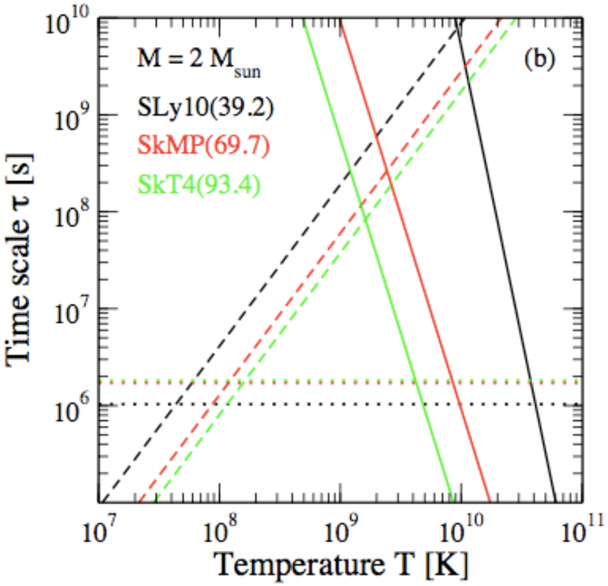
$$\xi = A_\xi L^{B_\xi}, \eta = A_\eta L^{B_\eta}$$



Dissipative time scales of r-modes

$$\frac{1}{\tau_i} = -\frac{1}{2E}\left(\frac{dE}{dt}\right)_i$$





- $$\frac{1}{\tau_\xi} = \frac{4\pi}{690}\left(\frac{\Omega^2}{\pi G\bar{\rho}}\right)^2 R^{2l-2}\left[\int_0^R \rho r^{2l+2}dr\right]^{-1}\int_0^R \xi\left(\frac{r}{R}\right)^2\left[1+0.86\left(\frac{r}{R}\right)^2\right]r^2dr$$
- $$\frac{1}{\tau_\eta} = (l-1)(2l+1)\left[\int_0^R \rho r^{2l+2}dr\right]^{-1}\int_0^R \eta r^{2l}dr$$
- $$\frac{1}{\tau_{GW}} = \frac{32\pi G\Omega^{2l+2}}{c^{2l+3}}\frac{(l-1)^{2l}}{[(2l+1)!!]^2}\left(\frac{l+2}{l+1}\right)^{2l+1}\int_0^R \rho r^{2l+2}dr$$

✓ τ_{GW} larger for models with larger L

Larger L ➔ stiffer EoS ➔ less compact star ➔ τ_{GW} larger

✓ τ_ξ & τ_η smaller for models with larger L

τ_ξ (τ_η) decrease with ξ (η) but ξ(η) increase with L

✓ τ_{GW}, τ_ξ & τ_η decrease when increasing M

Given an EoS: the more massive the star the denser it is

➔ τ_{GW}, τ_ξ ~$(\rho/\xi)R^2$ & τ_η ~$(\rho/\eta)R^2$ decrease

R-mode instability region

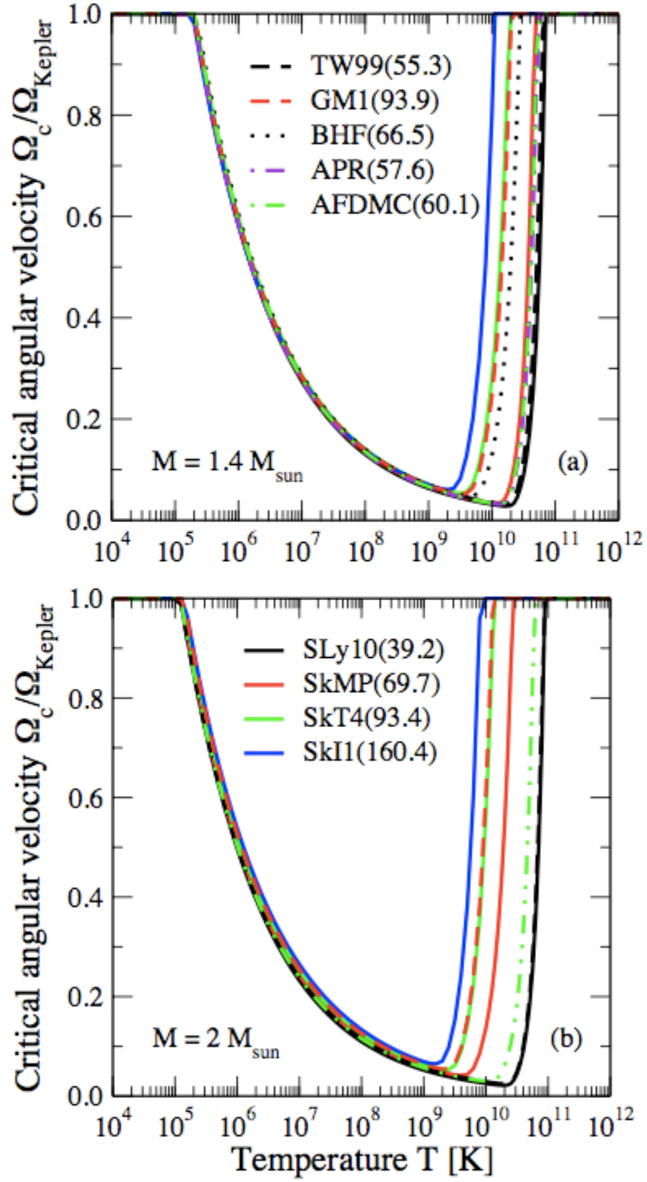
- time dependence of an r-mode $\sim e^{i\omega t - t/\tau}$

$$\frac{1}{\tau(\Omega,T)} = -\frac{1}{\tau_{GW}(\Omega)} + \frac{1}{\tau_{\xi}(\Omega,T)} + \frac{1}{\tau_{\eta}(T)}$$

➔ $\frac{1}{\tau(\Omega_c,T)} = 0$ r-mode instability region

$\Omega < \Omega_c$ stable

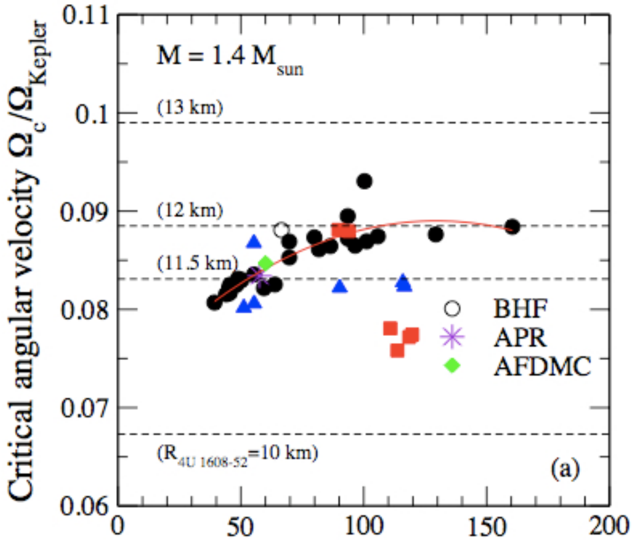
$\Omega > \Omega_c$ unstable

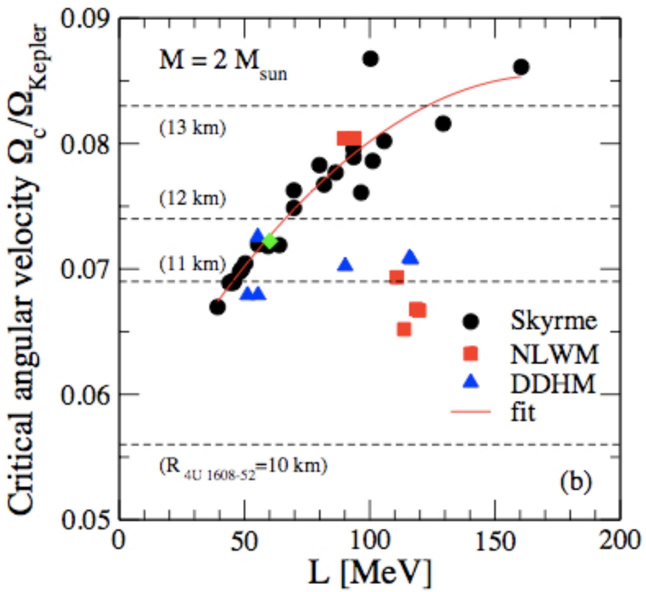


✓ instability region smaller for models with larger L (increase of ξ & η with L)

✓ instability region larger for more massive star (time scales decrease when M increases)

Constraining L from LMXBs





- observational constraints from pulsar in LMXB 4U 1608-52

✓ estimated core temperature $T \sim 4.55 \times 10^8$ K

✓ measured spin frequency 620 Hz

radius: 10, 11.5, 12 or 13 km

mass: 1.4 or 2 $M_\odot$

$$\Omega_{Kepler} \approx 7800\sqrt{\left(\frac{M}{M_{sun}}\right)\left(\frac{10km}{R}\right)^3}\, s^{-1}$$

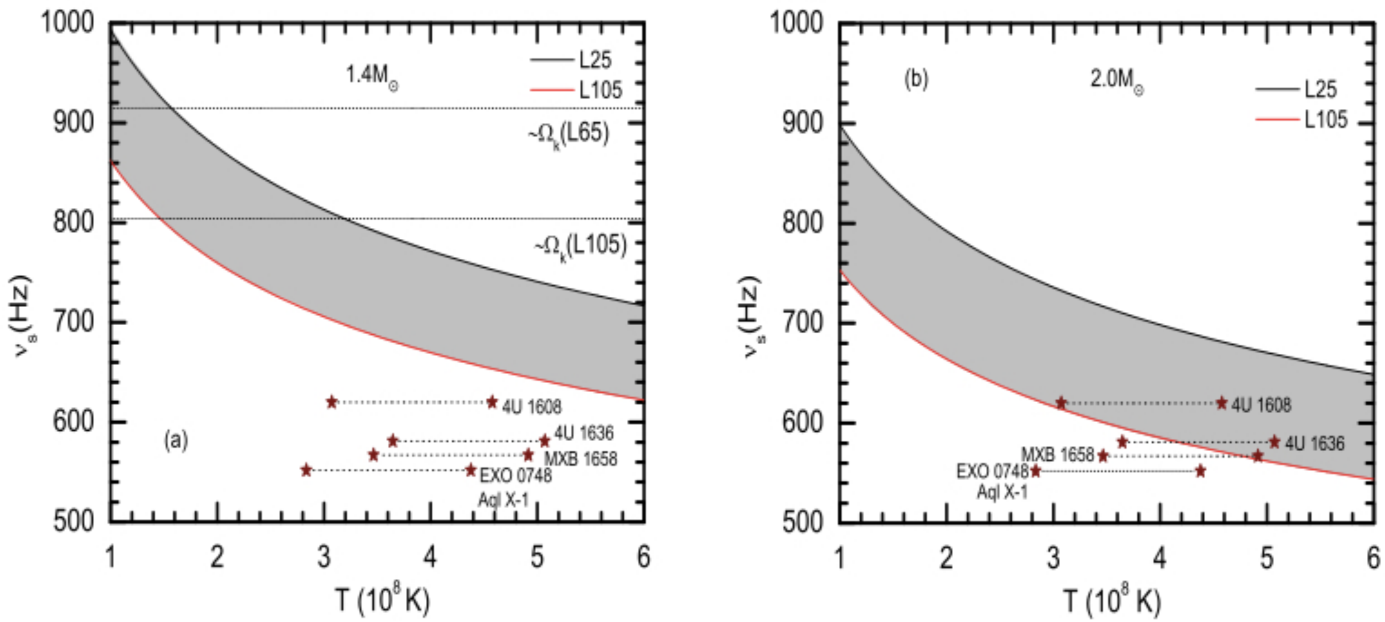
✓ No constraint on L if

- $R_{4U\,1608\text{-}52} < 11$ km ($\Omega_c > \Omega$)
- $R_{4U\,1608\text{-}52} > 12(13)$ km & M=1.4(2)$M_\odot$ ($\Omega_c < \Omega$)

✓ L > 50 MeV if (assuming 4U 1608-52 stable)

- $R_{4U\,1608\text{-}52}$ is 11.5-12(11.5-13) km & M=1.4(2)$M_\odot$

This is in contrast with the recent work of Wen, Newton & Li where they obtain L< 60 MeV (PRC 85, 025801 (2012))



However, they consider electron-electron scattering at the crust-core ($\rho<\rho_0$) boundary as the main dissipation mechanism. Therefore, their calculation of is done in a region wher η decreases with L

Summary & Conclusions

Study the role of the symmetry energy slope parameter L on the r-mode instability of neutron stars by using both microscopic and phenomenological approaches of the nuclear matter EoS

- r-mode instability region smaller for models with larger L (increase of ξ & η with L).

- L dependence of ξ & η can be described by simple power laws $\xi=A_\xi L^{B_\xi}$ & $\eta=A_\eta L^{B_\eta}$.

- Constraints on L from LMXB 4U 1608-52: L > 50 MeV if 4U 1608-52 assumed stable with 11.5-12 (11.5-13) km and M=1.4(2) $M_\odot$. Otherwise no constraints.


MERCI! THANK YOU!
FRAPAR.