A Regge Model for Nucleon-Nucleon Scattering Amplitudes

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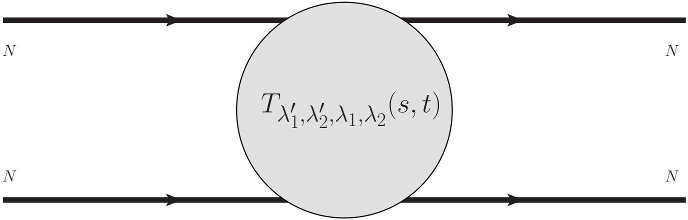
Outline

Objective

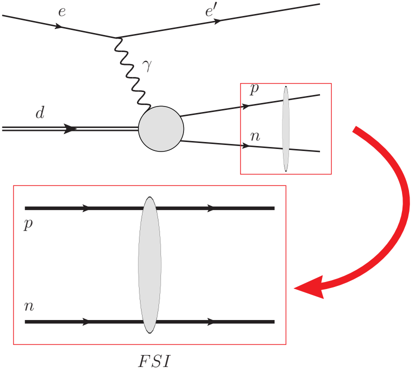


- Our goal is to calculate the nucleon-nucleon scattering amplitudes.
  - Utilize isospin (I) to describe proton-proton (pp) and proton-neutron (pn)
- Full spin dependence, in order to describe all observables.
  - Helicity basis, $\lambda_i = \vec{S} \cdot \hat{p}$
  - $(++;++), (++;+-), (+-;+-), (++;--), (+-;-+)$
- Ability to extrapolate to high energies where pn data is sparse ($s > 6\ GeV^2$).

We need a relativistic, fully spin dependent model.

Motivation

- Describe final state interactions in electrodisintegration of the deuteron.
- A calculation by Jeschoneck and Van Orden, utilize the scattering amplitudes as input.
  - Data is available with high energy nucleons.



Why Regge Theory?

- We need to model this process at mid to high energies ($s > 6$ GeV^2).
- Many methods are ineffective at these energies.
- Regge phenomenology has had great historical success and scales to high energies.
- We need a parameterization method.
  - Fit to available data (mostly low energy).
  - Confidence to extrapolate to higher energies.

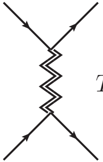
Regge theory allows us to construct a relativistic, fully spin dependent model, over a large energy range.

Concepts of Regge Analysis

Analyze $T(s,t)$ with continuous, complex angular momentum.

$s-channel$	$t-channel$
$N+N \rightarrow N+N$	$N+\overline{N} \rightarrow \overline{N}+N$

$T(s,t)$



$T(s,t)$

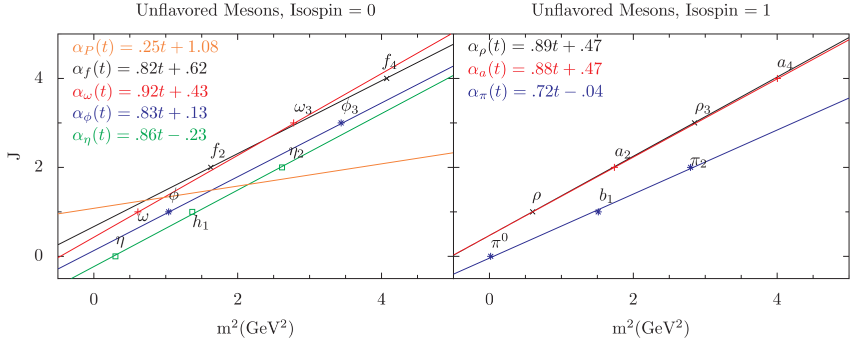
$s = 4E_{cm}^2$	$t = 4E_t^2$
$t = -2\vec{p}^2(1-cos(\theta))$	$s = -2\vec{p_t}^2(1+cos(\theta_t))$
$\cos(\theta) = 1 + \frac{2t}{s-4m^2}$	$\cos(\theta_t) = -1 - \frac{2s}{t-4m^2}$

t-channel analysis gives approximation to s-channel process

$$T_{Regge}^{N\bar{N}\rightarrow N\bar{N}}(s,t) \rightarrow \lim_{s\rightarrow\infty} T^{NN\rightarrow NN}(s,t) \propto \left(-1 - \frac{2s}{t-4m^2}\right)^{J\rightarrow\alpha(t)}$$

Regge Parameters

- The Regge trajectories, $J \to \alpha(t) = \alpha_0 + \alpha' t$, interpolate between physical mesons.
- Regge exchanges should have good quantum numbers, PGI.



- For full angular dependence, we use additional trajectories.

Regge Poles

- Regge exchanges characterized by residue $\beta(t)$ and trajectory $\alpha(t)$.

$$R(s,t) = \xi(t)\beta(t)\left(-1 + \frac{2s}{4m^2 - t}\right)^{\alpha(t)}$$

$$\xi(t) = \begin{cases} e^{-i\pi(\alpha(t)+\delta)/2} & \zeta_{PG} = +1 \\ -ie^{-i\pi(\alpha(t)+\delta)/2} & \zeta_{PG} = -1 \end{cases}$$

$$\alpha(t) = \alpha_0 + \alpha' t$$

$$\beta(t) = (B_1 + B_2 t)e^{ct}$$

Applying Regge Theory to Nucleon-Nucleon Scattering

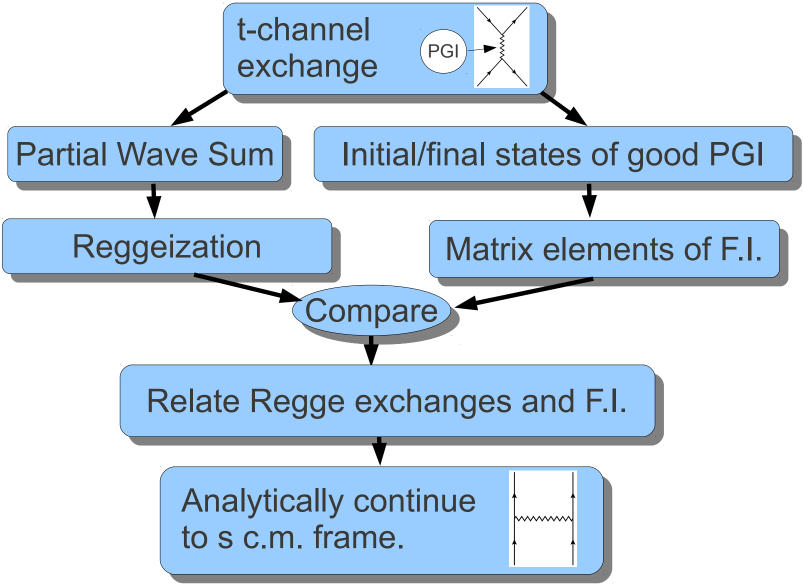
- Regge exchanges are in the crossed channel, and the helicity crossing relations are complicated.

Utilizing the Fermi invariants makes crossing trivial.

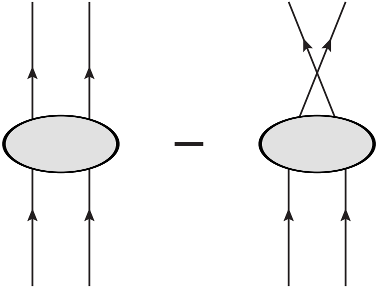
$$\begin{aligned}\hat{T} &= F_S^I(s,t) I^{(1)} \cdot I^{(2)} - F_P^I(s,t)(i\gamma_5)^{(1)} \cdot (i\gamma_5)^{(2)} \\ &+ F_V^I(s,t)\gamma^{\mu(1)}\gamma_\mu^{(2)} + F_A^I(s,t)(\gamma_5\gamma^\mu)^{(1)}(\gamma_5\gamma_\mu)^{(2)} \\ &+ F_T^I(s,t)\sigma^{\mu\nu(1)}\sigma_{\mu\nu}^{(2)}\end{aligned}$$

- Spin dependence explicitly dealt with.
- Analytically continuing scalar functions trivial.

Overview of the Calculation



Helicity Amplitudes



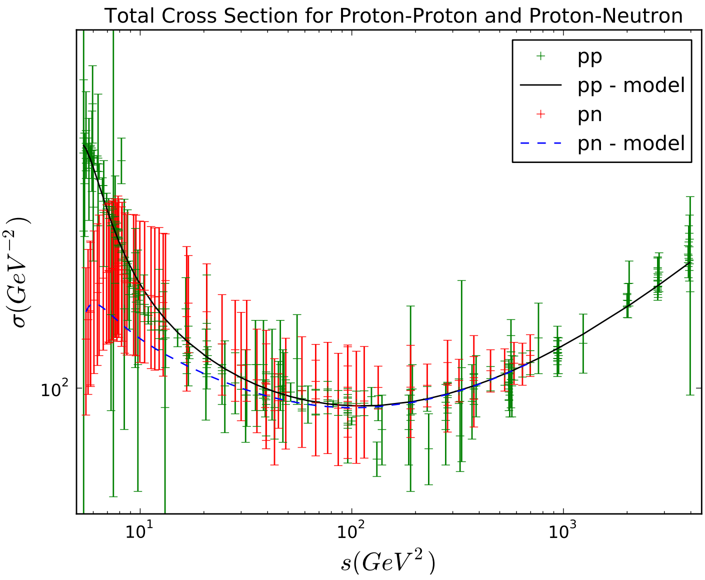
- s-channel helicity amplitudes in terms of the invariants.

$$T_i^{pp\to pp} = \sum_j \left\{ C_{ij}^t \left[F_j^0(s,t) + F_j^1(s,t)\right] - C_{ij}^u \left[F_j^0(s,u) + F_j^1(s,u)\right]\right\}$$

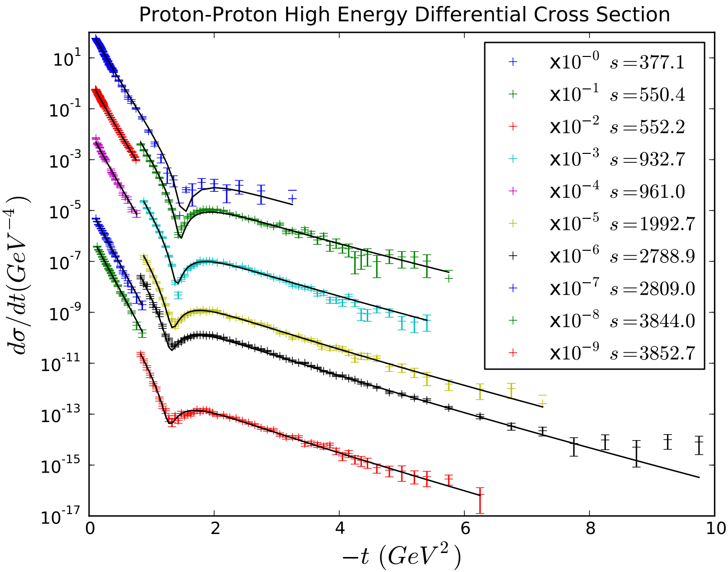
$$T_i^{pn\to pn} = \sum_j \left\{ C_{ij}^t \left[F_j^0(s,t) - F_j^1(s,t)\right] - 2C_{ij}^u F_j^1(s,u)\right\}$$

$$i \to (++;++), (++;+-), (+-;+-), (++;--), (+-;-+)$$
$$j \to S, V, T, P, A$$

Total Cross Section



Proton-Proton High Energy Differential Cross Section



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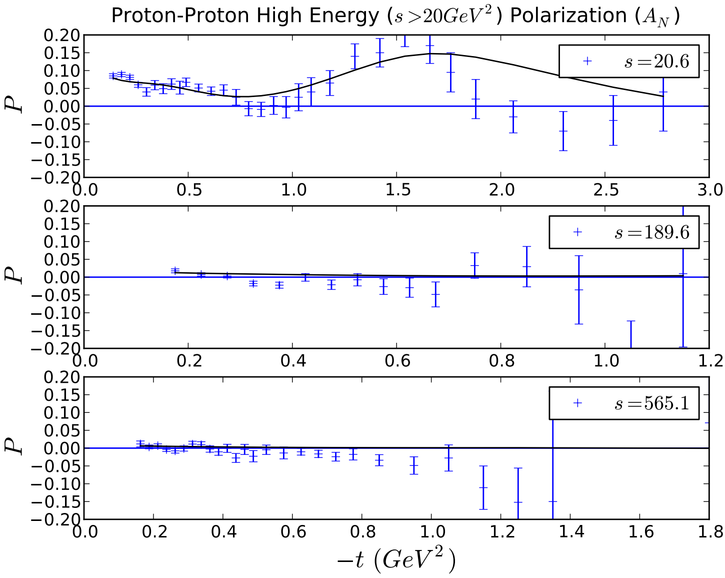
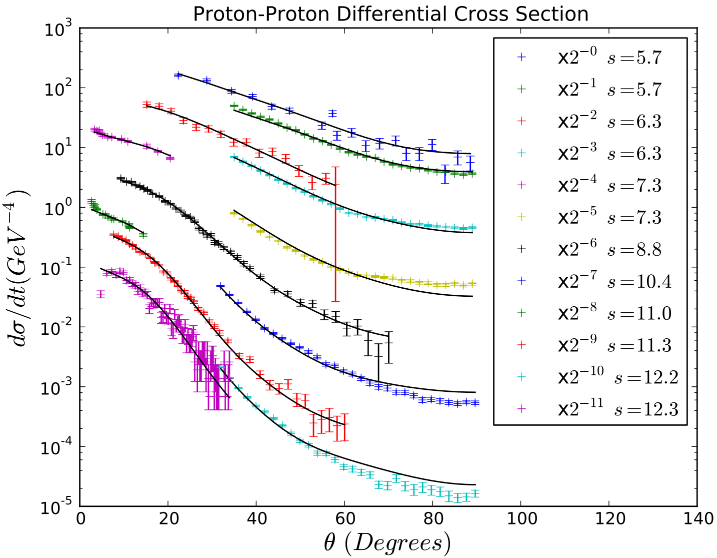
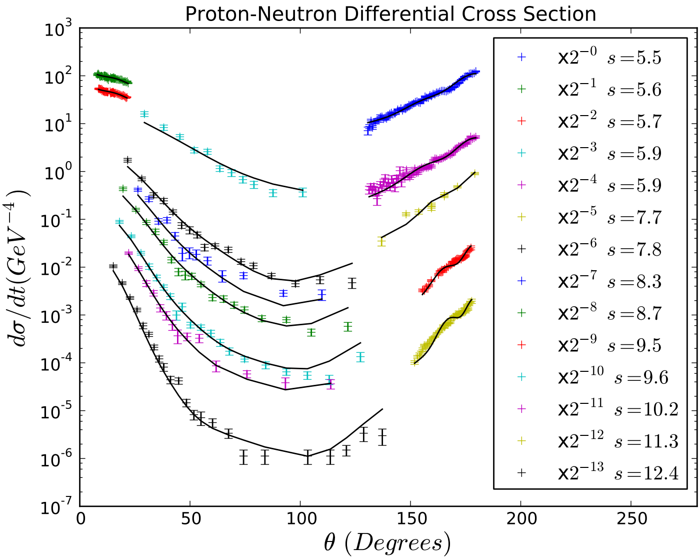


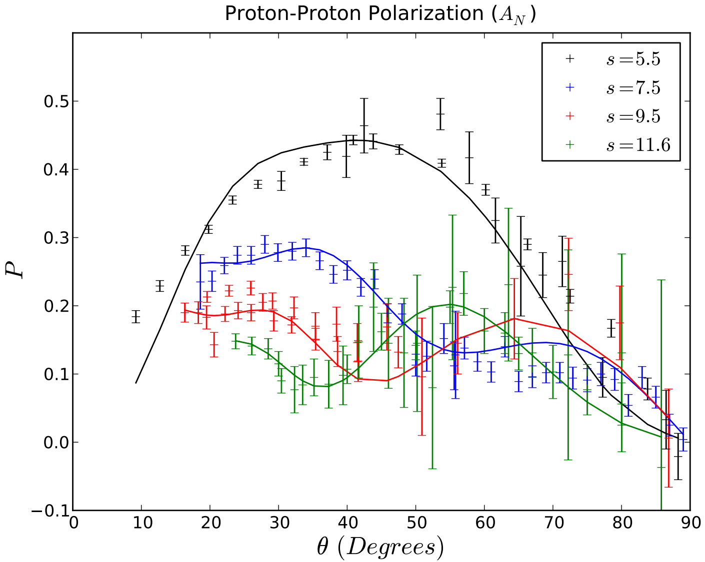
Proton-Proton Differential Cross Section



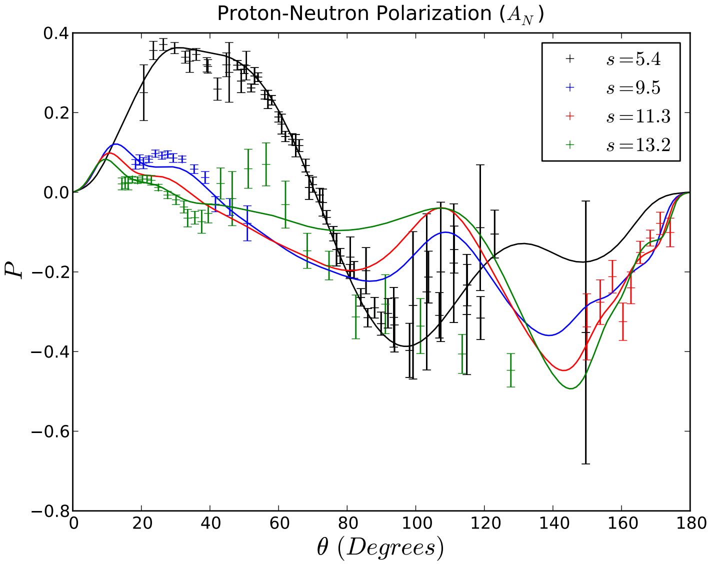
Proton-Neutron Differential Cross Section



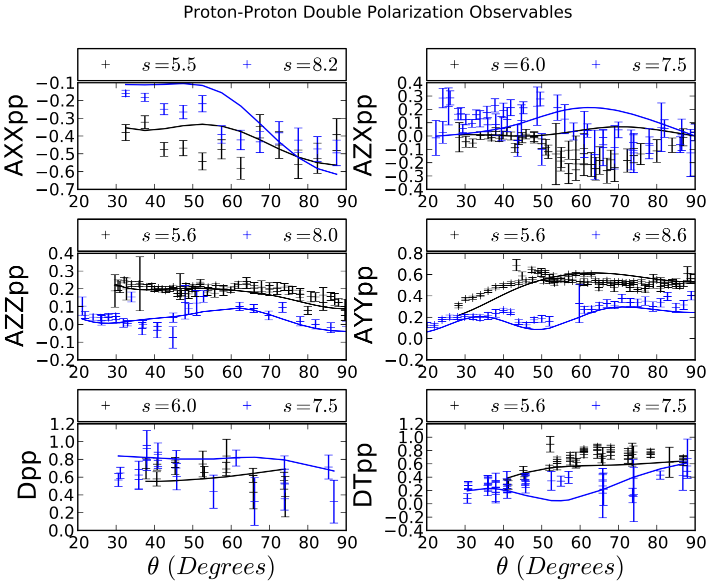
Proton-Proton Polarization



Proton-Neutron Polarization



Proton-Proton Double Spin Observables



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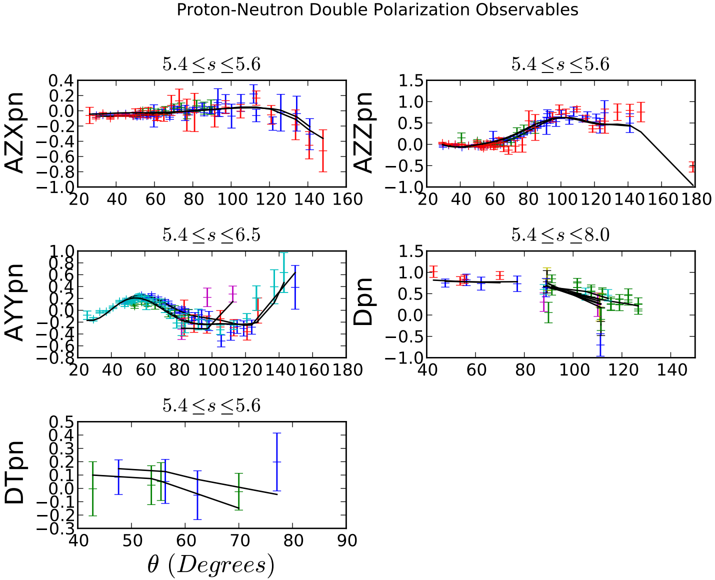
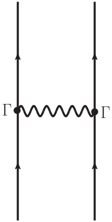


Summary and Outlook

- We have constructed a relativistic, fully spin dependent model, which describes the high energy nucleon-nucleon scattering amplitudes.
- While we describe the data well, we believe we can improve upon the fit.
- Utilize the amplitudes to describe final state interactions in the $D(e, e'p)n$ calculation.
- When complete, we will provide the amplitudes via a program or code for the community.

Electromagnetic Effects

- In order to describe proton-proton scattering we also need to take into account electromagnetic interactions.



$$\Gamma = F_1 \gamma^\mu - \frac{F_2}{2m} i\sigma^{\mu\nu} q_\nu$$

$$F_1 = \frac{G_E - G_M t/4m^2}{1 - t/4m^2} \quad F_2 = \frac{G_M - G_E}{1 - t/4m^2}$$

$$G_E = G_M/2.79 = (1 - t/.71)^{-2}$$

- To approximate higher order effects, we also allow for a helicity dependent phase for the dominant amplitudes.

$$T^{EM}_{++,++} \approx T^{EM}_{+-,+-} \approx e^{i\phi_1(t)} T^{EM}_{++,++}|_{1photon}$$

$$T^{EM}_{++,+-} \approx A e^{i\phi_2(t)} T^{EM}_{++,+-}|_{1photon}$$

- $X2 = 4.9$
- $(X_{SIGpp}, 189.85449675079789, 192, 0.98882550391040569)$
- $(X_{SIGpn}, 25.075946302709312, 72, 0.34827703198207377)$
- $(X_{DSGppH}, 1648.5956834235128, 758, 2.1749283422473784)$
- $(X_{DSGppL}, 30534.265011369982, 3447, 8.8582143926225658)$
- $(X_{DSGpnA}, 3107.4561391554071, 745, 4.1710820659804124)$
- $(X_{PppH}, 704.17520994637914, 250, 2.8167008397855167)$
- $(X_{Ppp}, 11369.075182146697, 3136, 3.6253428514498394)$
- $(X_{Ppn}, 1115.4155961287843, 493, 2.2625062801800899)$
- $(X_{AZXpp}, 4394.0279028217074, 568, 7.7359646176438508)$
- $(X_{AZXpn}, 100.8000053578187, 81, 1.2444445105903543)$
- $(X_{Dpp}, 387.06625135748175, 188, 2.0588630391355411)$
- $(X_{Dpn}, 52.317494816433403, 37, 1.4139863463900919)$
- $(X_{AYYpp}, 6880.520955580645, 1587, 4.3355519568876151)$
- $(X_{AYYpn}, 215.03225731892957, 117, 1.8378825411874322)$
- $(X_{AZZpp}, 1073.3736097413077, 608, 1.7654171212850456)$
- $(X_{AZZpn}, 199.71357425625374, 89, 2.243972744452289)$
- $(X_{DTpp}, 1068.2075172285674, 281, 3.8014502392475711)$
- $(X_{DTpn}, 3.2812763867581172, 8, 0.41015954834476465)$
- $(X_{AXXpp}, 1060.4130983805419, 276, 3.8420764434077603)$