# Effective Shell-Model Hamiltonians from Realistic NN Potentials

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Elba XII Workshop - Electron-Nucleus Scattering XII

# A brief introduction for non-experts

What is a realistic effective shell-model hamiltonian ?

# An example: $^{19}F$

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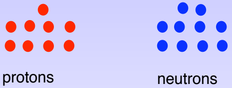


- 9 protons & 10 neutrons interacting
- spherically symmetric mean field (e.g. harmonic oscillator)
- 1 valence proton & 2 valence neutrons interacting in a truncated model space

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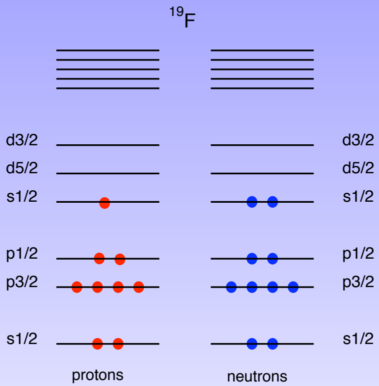


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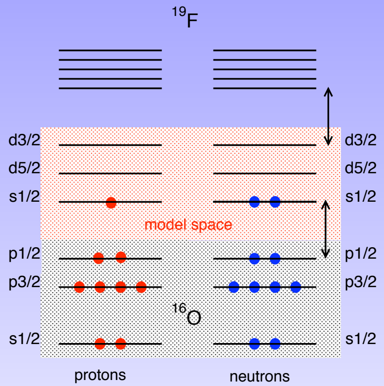


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# Effective shell-model hamiltonian

The shell-model hamiltonian has to take into account in an effective way all the degrees of freedom not explicitly considered

**Two alternative approaches**

- phenomenological
- microscopic

$$V_{NN} \ (+V_{NNN}) \Rightarrow \text{many-body theory} \Rightarrow H_{\text{eff}}$$

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The eigenvalues of $H_{\text{eff}}$ belong to the set of eigenvalues of the full nuclear hamiltonian

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# Workflow for a realistic shell-model calculation

1. Choose a realistic *NN* potential (*NNN*)
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# Realistic nucleon-nucleon potential: $V_{NN}$

At present several realistic $V_{NN}$s are available: CD-Bonn, Argonne V18, Nijmegen, ...

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**How to handle the short-range repulsion ?**

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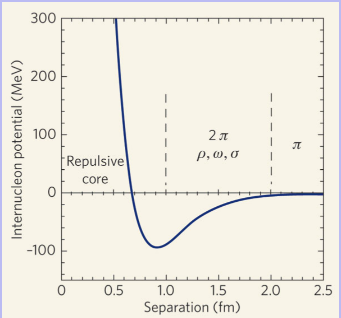


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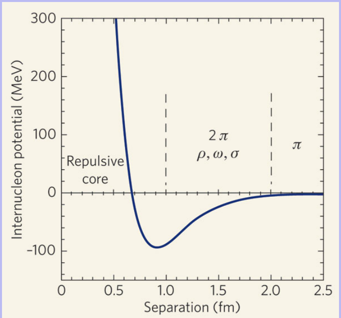


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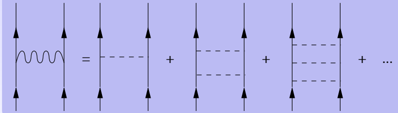

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New approach $\Rightarrow$ low-momentum *NN* potentials:

- $V_{\rm low-k}$
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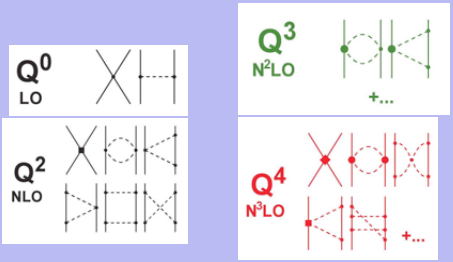

# The shell-model effective hamiltonian

## A-nucleon system Schrödinger equation

$$H|\Psi_\nu\rangle = E_\nu|\Psi_\nu\rangle$$

with

$$H = H_0 + H_1 = \sum_{i=1}^{A}(T_i + U_i) + \sum_{i<j}(V_{ij}^{NN} - U_i)$$

## Model space

$$|\Phi_i\rangle = [a_1^\dagger a_2^\dagger \ldots a_n^\dagger]_i|c\rangle \Rightarrow P = \sum_{i=1}^{d}|\Phi_i\rangle\langle\Phi_i|$$

## Model-space eigenvalue problem

$$H_{\rm eff} P|\Psi_\alpha\rangle = E_\alpha P|\Psi_\alpha\rangle$$

# The shell-model effective hamiltonian

$$\left(\begin{array}{c|c} PHP & PHQ \\ \hline QHP & QHQ \end{array}\right) \quad \begin{array}{c} \mathcal{H} = X^{-1}HX \\ \Longrightarrow \\ Q\mathcal{H}P = 0 \end{array} \quad \left(\begin{array}{c|c} P\mathcal{H}P & P\mathcal{H}Q \\ \hline 0 & Q\mathcal{H}Q \end{array}\right)$$

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## Folded-diagram expansion

$\hat{Q}$-box vertex function

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

$\Rightarrow$ Recursive equation for $H_{\rm eff}$ $\Rightarrow$ iterative techniques (Krenciglowa-Kuo, Lee-Suzuki, ...)

$$H_{\rm eff} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} \cdots ,$$

generalized folding

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

The $\hat{Q}$-box can be calculated perturbatively

$$\frac{1}{\epsilon - QHQ} = \sum_{n=0}^{\infty} \frac{(QH_1Q)^n}{(\epsilon - QH_0Q)^{n+1}}$$

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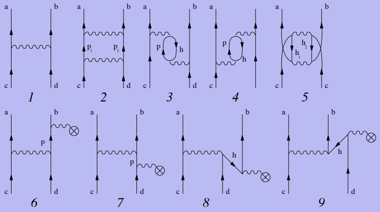

- $H^{\rm eff}$ for systems with one and two valence nucleons
- $\hat{Q}$-box $\Rightarrow$ Goldstone diagrams up to third order in $V_{NN}$ (up to 2p-2h core excitations )
- Padè approximant [2|1] of the $\hat{Q}$-box

$$[2|1] = V^0_{Qbox} + V^1_{Qbox} + V^2_{Qbox}(1 - (V^2_{Qbox})^{-1} V^3_{Qbox})^{-1}$$

# Test case: *p*-shell nuclei

- $V_{NN} \Rightarrow$ chiral N$^3$LO potential by Entem & Machleidt
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- Single-particle energies and residual two-body interaction are derived from the theory. No empirical input

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# Convergence checks

## The intermediate-state space $Q$

$Q$-space is truncated: intermediate states whose unperturbed excitation energy is greater than a fixed value $E_{max}$ are disregarded

$$|\epsilon_0 - QH_0Q| \leq E_{\rm max} = N_{\rm max}\hbar\omega$$

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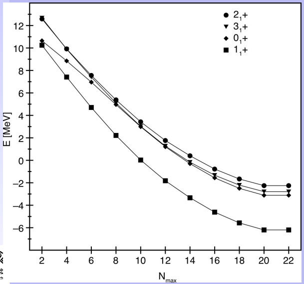


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# Convergence checks

## Order-by-order convergence

Compare results from $H^{\rm eff}_{\rm 1st}$, $H^{\rm eff}_{\rm 2nd}$, $H^{\rm eff}_{\rm 3rd}$ and $H^{\rm eff}_{\textrm{Padè}}$

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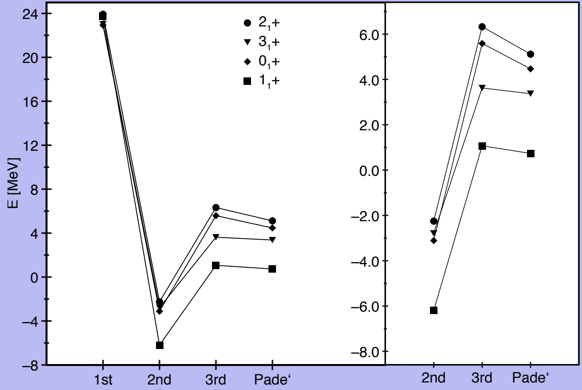

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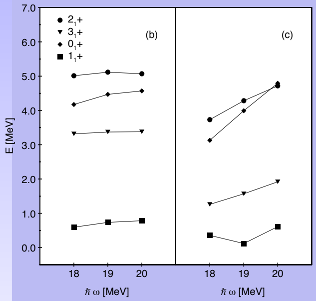

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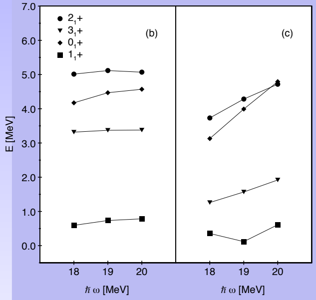
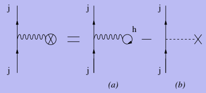



HF-insertions




- zero in a self-consistent basis
- neglected in most applications
- neglect introduces relevant dependence on $\hbar\omega$

# Benchmark calculation

Approximations are under control ... and what about the accuracy of the results ?

Compare the results with the “exact” ones

*ab initio* no-core shell model (NCSM)

P. Navrátil, E. Caurier, Phys. Rev. C **69**, 014311 (2004)

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$$H_{int} = \left(1 - \frac{1}{A}\right) \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j=1}^{A} \left( V_{ij}^{NN} - \frac{\mathbf{p_i} \cdot \mathbf{p_j}}{mA} \right) =$$

$$= \left[ \sum_{i=1}^{A} \left( \frac{p_i^2}{2m} + U_i \right) \right] + \left[ \sum_{i<j=1}^{A} \left( V_{ij}^{NN} - U_i - \frac{p_i^2}{2mA} - \frac{\mathbf{p_i} \cdot \mathbf{p_j}}{mA} \right) \right]$$

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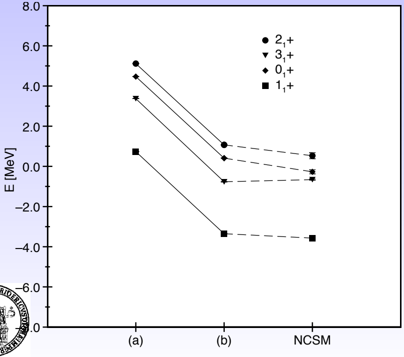


(a) not translationally invariant Hamiltonian
(b) purely intrinsic hamiltonian

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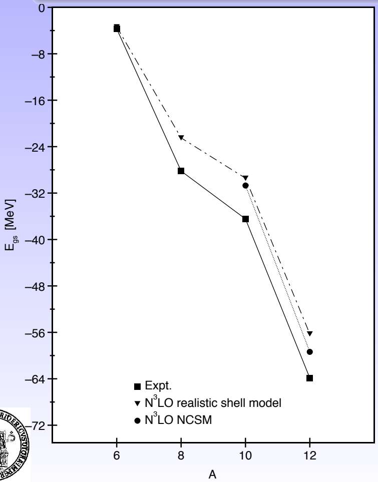
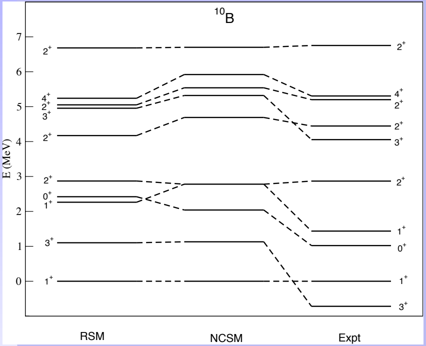



- ground-state energies for $N = Z$ nuclei
- discrepancy grows with the number of valence nucleons

## $^{10}$B relative spectrum

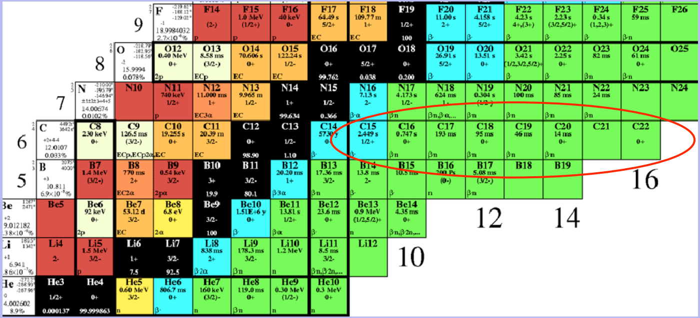
# Benchmark calculation

## $^{10}$B relative spectrum




- discrepancy $\leq$ 1 MeV
- minor role of many-body correlations

## Neutron-rich Carbon isotopes



Selected for a Viewpoint in *Physics*
PRL 104, 062701 (2010) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2010

## Observation of a Large Reaction Cross Section in the Drip-Line Nucleus $^{22}$C

K. Tanaka,[1] T. Yamaguchi,[2] T. Suzuki,[2] T. Ohtsubo,[3] M. Fukuda,[4] D. Nishimura,[4] M. Takechi,[4,1] K. Ogata,[5] A. Ozawa,[6] T. Izumikawa,[7] T. Aiba,[3] N. Aoi,[1] H. Baba,[1] Y. Hashizume,[6] K. Inafuku,[8] N. Iwasa,[8] K. Kobayashi,[2] M. Komuro,[2] Y. Kondo,[9] T. Kubo,[1] M. Kurokawa,[1] T. Matsuyama,[3] S. Michimasa,[1,*] T. Motobayashi,[1] T. Nakabayashi,[9] S. Nakajima,[2] T. Nakamura,[9] H. Sakurai,[1] R. Shinoda,[2] M. Shinohara,[9] H. Suzuki,[10,6] E. Takeshita,[1,†] S. Takeuchi,[1] Y. Togano,[11] K. Yamada,[1] T. Yasuno,[6] and M. Yoshitake[2]

$^{19,20,22}\mathrm{C} + p$

10 $^{22}$C counts per hour

TABLE I. Reaction cross sections ($\sigma_R$) in millibarns.

| $A$ | $\sigma_R$ |
|---|---|
| 19 | 754(22) |
| 20 | 791(34) |
| 22 | 1338(274) |

RIKEN Radioactive Isotope Beam Factory (RIBF)

# $^{22}$C structure

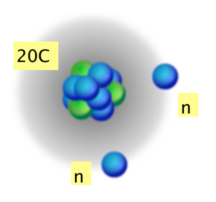
**$^{22}$C is the heaviest Borromean nucleus ever observed**

Borromean nucleus ⇒ weakly bound nucleus that has, considered as a three-body system, no bound states in the binary subsystems.

$^{22}$C is weakly bound $S_{2n}^{\mathrm{est}} = 420\mathrm{keV}$

$^{21}$C is unstable for one-neutron emission

M. Langevin *et al.*, Phys. Lett. B **150**, 71 (1985)



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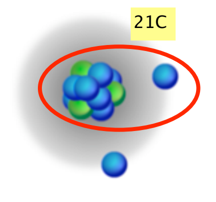

# Shell-model calculation

PHYSICAL REVIEW C **81**, 064303 (2010)

**Shell-model calculations for neutron-rich carbon isotopes with a chiral nucleon-nucleon potential**

L. Coraggio,[1] A. Covello,[1,2] A. Gargano,[1] and N. Itaco[1,2]
[1]*Istituto Nazionale di Fisica Nucleare, Complesso Universitario di Monte S. Angelo, Via Cintia, I-80126 Napoli, Italy*
[2]*Dipartimento di Scienze Fisiche, Università di Napoli Federico II, Complesso Universitario di Monte S. Angelo, Via Cintia, I-80126 Napoli, Italy*

- Realistic potential $V_{NN} \Rightarrow$ N$^3$LOW
- $\Rightarrow H_{\mathrm{eff}}$
- Single-particle energies & effective interaction from the theory. No empirical input

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PHYSICAL REVIEW C **81**, 064303 (2010)

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[1]*Istituto Nazionale di Fisica Nucleare, Complesso Universitario di Monte S. Angelo, Via Cintia, I-80126 Napoli, Italy*
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- Realistic potential $V_{NN} \Rightarrow$ N$^3$LOW
- $\Rightarrow H_{\mathrm{eff}}$
- Single-particle energies & effective interaction from the theory. No empirical input

# Shell-model calculation

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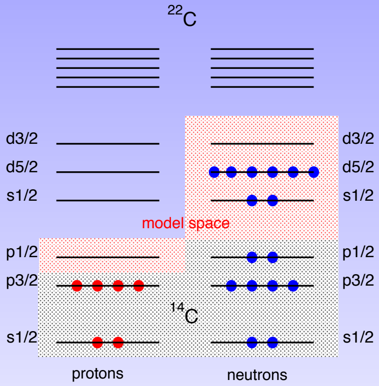
# Model space




$^{14}$C inert core

**Neutrons:**

shell *sd* $\Rightarrow$ 1s1/2, 0d5/2, 0d3/2

**Protons:**

0p1/2

# Results

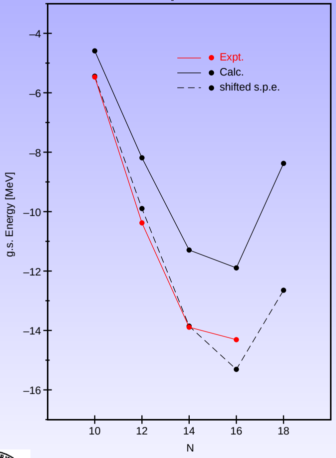
- $^{22}$C & drip line
- N=14 shell closure

Single-particle energies

| nlj | $\epsilon$ MeV |
|---|---|
| 1s1/2 | 0.0 (-0.793) |
| 0d5/2 | 5.914 |
| 0d3/2 | 1.394 |

# Results: $^{22}$C & drip line

g.s energies of even mass Carbon isotopes




$S_n(^{21}C) = -1.6$ MeV

$S_{2n}(^{22}C) = 456$ keV

# Results: $^{22}C$ & drip line

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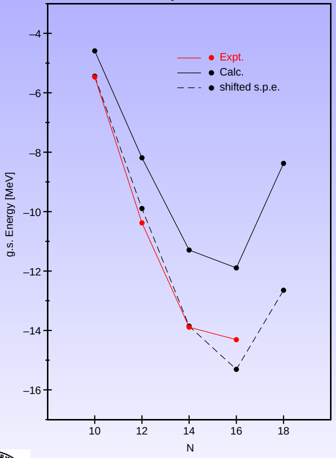


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Experimentally the N=14 shell closure in oxygen isotopes disappears in carbon isotopes

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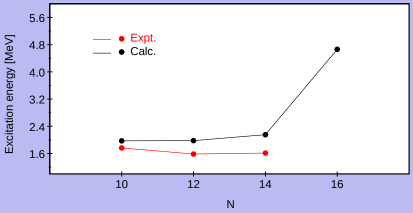
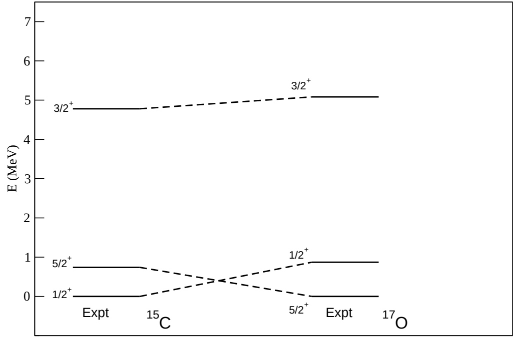
## Excitation energies of $2_1^+$ states in heavy carbon isotopes



# Results: N=14 shell closure

The pn monopole component of $V_{\rm eff}$ reproduces correctly the g.s. inversion in $^{15}$C and $^{17}$O

$$\epsilon_j(^{17}O) = \epsilon_j(^{15}C) + V^{\rm mon}_{p\frac{1}{2};j} \times 2$$



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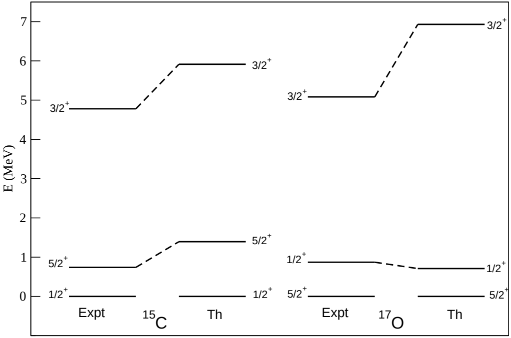

# Concluding remarks

- Approximations involved may be kept under control
- Effective shell-model hamiltonians derived from realistic *NN* potentials are reliable
- Realistic shell-model calculations $\Rightarrow$ fully microscopic description of nuclear structure
  - no parameters modified *ad hoc*
  - enhanced predictive power
- Effective realistic shell-model hamiltonians may be employed successfully to describe the properties of nuclei close to the drip lines
  - drip-lines position
  - properties of loosely bound nuclei
  - evolution of nuclear properties when varying $N/Z$
  - shell-quenching
  - new magic numbers

# Effective Shell-Model Hamiltonians from Realistic NN Potentials

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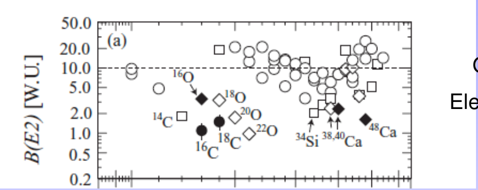
Elba XII Workshop - Electron-Nucleus Scattering XII

# Results: $B(E2; 2_1^+ \rightarrow 0_1^+)$ transition probabilities

Reduced Experimental $B(E2; 2_1^+ \rightarrow 0_1^+) \Rightarrow$ effective charges quenching




Ong *et al* Phys. Rev. C **78**, 014308 (2008)

Elekes *et al* Phys. Rev. C **79**, 011302 (2009)

**$B(E2; 2_1^+ \rightarrow 0_1^+)$ in $e^2 fm^4$**

| | Calc. | Expt. |
|---|---|---|
| $^{16}$C | 1.8 | $2.6 \pm 0.2 \pm 0.7$<br>$4.15 \pm 0.73$ |
| $^{18}$C | 3.0 | $4.3 \pm 0.2 \pm 1.0$ |
| $^{20}$C | 3.7 | $< 3.7$ |

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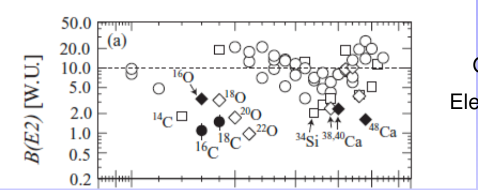


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