A comparative analysis of three-nucleon potentials in nuclear matter

Alessandro Lovato

In collaboration with Omar Benhar, Stefano Fantoni and K. E. Schmidt

Outline

- Ab initio many body method
- Nuclear Hamiltonian: 2- and 3- body potentials
- Chiral NNN potential and its local form
- Local Chiral NNN contact term cutoff dependence
- Comparative study of local three-body potential in nuclear matter
- Conclusions
Our aim is to perform *ab initio* calculations in nuclear matter. The interaction must not be affected by uncertainties involved in many body techniques.

*Ab initio* many body calculations

- Fully predictive
- Their approximations can be estimated
- Provide a test for the interaction itself
A “Realistic” nuclear hamiltonian

The non relativistic Hamiltonian describing nuclear matter is

\[ H = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{j \geq i=1}^{A} v_{ij} + \ldots \]

(Some) Realistic nucleon-nucleon (NN) potentials

- Argonne \( v_{18} \), \( v_8' \)
  - Mainly phenomenological
  - Local in coordinate space

- CD-BONN
  - Meson-exchange based
  - Nonlocal

- Chiral N\(^3\)LO
  - From Chiral Lagrangians
  - Nonlocal

Parameters of these potentials obtained

Fitting the \( \sim 4300 \) nn and np Nijmegen scattering data below 350 MeV with \( ^2 \approx 1 \).

Fitting the binding energy of the deuteron

No many-body method involved in the fit

Potentials with high predictive power, suitable for “ab initio” calculations.
When two body potential only is considered:

- Bad description of three and more nuclei bound and scattering states.

Pieper et al. 2001
NN potential is not enough

When two body potential only is considered:

- Equilibrium density of symmetric nuclear matter overestimated.

\[ \rho_0 = 0.16 \text{ fm}^{-3} \]

\[ \text{AL et al 2011} \]
NNN potential

Originates from the fact that nucleons are not elementary particles.

Three-body force is **NOT** an iteration of the two-body force.
**NNN potential**

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Three-body force is **NOT** an iteration of the two-body force

**Two-body force**

*Bogner's idea*
NNN potential

Originates from the fact that nucleons are not elementary particles.

Three-body force is **NOT** an iteration of the two-body force

---

**Two-body force**

**Three-body force**

*Bogner's idea*
Urbana IX three body potential

UIX potential consists of two contributions

\[ V^{2\pi} \text{ – Fujita Myiazawa} \]

\[ V^R \text{ – scalar repulsive term} \]

Cyclic sum of three permutations

\[
V^{2\pi} = A^{2\pi} (O_{123}^{2\pi} + O_{231}^{2\pi} + O_{312}^{2\pi})
\]

\[
O_{123}^{2\pi} = \left( \{\hat{X}_{12}, \hat{X}_{23}\}\{\tau_{12}, \tau_{13}\} + \frac{1}{4} [\hat{X}_{12}, \hat{X}_{23}] [\tau_{12}, \tau_{23}] \right)
\]

\[ \hat{X}_{ij} = Y(m_\pi r)\sigma_{ij} + T(m_\pi r)S_{ij} \]

Cyclic sum of three permutations

\[
V^R = U_0 \sum_{cycl} T^2(m_\pi r_{12}) T^2(m_\pi r_{23})
\]

Cutoff functions of OPE
Urbana IX three body potential

UIX potential has two parameters

- $A^{2\pi}$ adjusted to reproduce the observed binding energies of $^3\text{H}$.
- $U_0$ tuned for FHNC/SOC calculations to reproduce the empirical equilibrium density of SNM

Lagaris and Pandharipande argued that, because of correlations, the relative weight of the contribution depends upon the density of the system:

- **High density**
  - $V^R$
  - $\hat{V}^{2\pi}$

- **Low density**
  - $V^R$
  - $\hat{V}^{2\pi}$
Urbana IX three body potential

Improved description of energy levels of light nuclei

Pieper et al 2001
Urbana IX three body potential

Improved description of energy levels of light nuclei

Pieper et al. 2001

Still some discrepancies with experimental data!
Urbana IX three body potential

SNM saturation density is well reproduced

FHNC/SOC: $v_8' + UIX$

FHNC/SOC: $v_8'$

$\rho_0 = 0.16 \text{ fm}^{-3}$

AL et al 2011
Urbana IX three body potential

SNM saturation density is well reproduced

\[ E(\rho_0) = -16 \text{ MeV} \]

AL et al 2011

FHNC/SOC: \( v_8' + \text{UIX} \)

FHNC/SOC: \( v_8' \)

SNM is underbound
Urbana IX three body potential

SNM saturation density is well reproduced

\[ E(\rho_0) = -16 \text{ MeV} \]

FHNC: \( v_6' + v_{12}(\rho) \)

FHNC: \( v_6' + V_{123} \)

FHNC: \( v_6' \)

AFDMC: \( v_6' + v_{12}(\rho) \)

AFDMC: \( v_6' \)

AL et al 2011

SNM is underbound
Urbana IX three body potential

The value of the n-d scattering length obtained with Argonne $v_{18} + UIX$ is not compatible with experimental data

<table>
<thead>
<tr>
<th>$2a_{nd} (\text{fm})$</th>
<th>$v_{18} + UIX$</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.578</td>
<td>0.645 ± 0.003 ± 0.007</td>
<td></td>
</tr>
</tbody>
</table>

Kievsky et al 2010

Theoretical problems

$V^{2\pi}$: no a priori reasons to stop at the first order in the perturbative expansion in the coupling constant $g_0 \sim 10$.

Adjusting $U_0$ to reproduce the correct value of $\rho_0$, calculated within the FHNC/SOC framework, makes the potential affected by the uncertainties of the many-body technique. “ab initio”?
We need to go beyond the UIX potential!
How to go beyond Urbana IX?

Same strategy used for NN potential?

Decomposition of NN potential in spin-isospin structures respecting the symmetry of the interaction

![Equation](https://example-equation-image)

Argonne $v_{18}$, or $v_8'$

Fitting the **huge amount** of nn and pn data

Shape of the radial functions $v^p(r)$

Following the same strategy adopted for the NN potential seems not to be feasible without an additional theoretical guidance.

- Large variety of different possible structures in the three-nucleon force
- Difficulties in extracting information of three nucleon force from NNN data
Effective theories describing phenomena which occur at a chosen length (or energy) range.

**THE SYSTEM**

- Pions with external momentum of the order of $m_\pi$
- Non-relativistic nucleons with spatial-momentum of the order of $m_\pi$.

QCD is NOT perturbative at this scale.

Lattice QCD

Effectively theories describing phenomena which occur at a chosen length (or energy) range.

ChPT

Aoki 2010
Chiral Perturbation Theory

If u and d quarks are massless, QCD is invariant under the **chiral symmetry group**

\[ G = SU(2)_R \times SU(2)_L \]

For the hadronic spectrum to be represented, **G is spontaneously broken**

\[ G = SU(2)_R \times SU(2)_L \rightarrow H = SU(2)_I \]

**Goldstone theorem**

- Three massless pseudoscalar bosons appear.
- Goldstone bosons decouple in small momentum limit.

**Pions!**

Expansion in powers of \( q/\Lambda \)
Chiral Perturbation Theory (ChPT)

- Quarks u and d do have mass, pions are pseudogolstone bosons and have small masses, vanishing in the chiral limit.

- The interactions of Goldstone bosons vanish at zero momentum transfer.

Chiral Perturbation Theory

Effective Lagrangian and physical amplitudes are expanded in powers of

\[ \frac{Q}{\Lambda} \]

Soft scale \( m_\pi \)

Hard scale \( \Lambda_\chi \approx m_\rho \)

The order of the chiral nuclear potential is related with the power of \( Q/\Lambda \)

- LO
- NLO
- NNLO
- NNNLO

\[ 1 \quad (Q/\Lambda_\chi)^2 \quad (Q/\Lambda)^3 \quad (Q/\Lambda)^4 \]

Higher order in ChPT involve more Feynman diagrams. **Weinberg scheme!**
Chiral 3-body potential at NNLO

In a theory without explicit \( \Delta \) degrees of freedom, the first contribution to the chiral 3NF appears at \( N^2\text{LO} \) in the Weinberg counting scheme.

The interaction is described by three different physical mechanisms:

- **Two-pion exchange (TPE)**
- **One-pion exchange (OPE)**
- **Contact term**

Cyclic sum:

\[
V^x(1, 2, 3) = V^x(1 : 2, 3) + V^x(2 : 1, 3) + V^x(3 : 1, 2)
\]
Fourier transforming the Chiral NNLO 3-body potential, originally derived in momentum space, yields a local expression in coordinate space:

**NNLOL potential**

\[
V^\chi(3:12) = \int \frac{d^3q_1}{(2\pi)^3} \frac{d^3q_2}{(2\pi)^3} \tilde{V}^\chi(3:12) F_\Lambda(q_1^2) F_\Lambda(q_2^2) e^{iq_1 \cdot r_{13}} e^{iq_2 \cdot r_{23}}
\]

\[
F_\Lambda(q_i^2) = \exp\left(-\frac{q_i^4}{\Lambda^4}\right)
\]

- Depends on transferred momenta
- Generates powers of \(q/\Lambda\) beyond NNLO

Radial functions appearing in the coordinate space version of the chiral NNLO potential:

\[
z_n(r) = \frac{4\pi}{m_\pi^3} \int \frac{d^3q}{(2\pi)^3} \frac{F_\Lambda(q^2)}{(q^2 + m_\pi^2)^n} e^{iq \cdot r}
\]

\[
= \frac{2}{\pi m_\pi^3} \int dq q^2 \frac{F_\Lambda(q^2)}{(q^2 + m_\pi^2)^n} j_0(qr)
\]

Navratil 2007
**TPE term**

\[
= c_1 V_1(3 : 12) + c_3 V_3(3 : 12) + c_4 V_4(3 : 12)
\]

**Tucson Melbourne** potential has **the same spin-isospin structure** of the chiral NNLOL potential TPE term.

\[
c_1 = \frac{a}{m_\pi^2} , \quad c_3 = 2b , \quad c_4 = -4d , \quad F_\Lambda(q^2) = \left(\frac{\Lambda^2 - m_\pi^2}{\Lambda^2 + q^2}\right)^2 .
\]

**UIX** potential: the anticommutator and the commutator terms in \(V^{2\pi}\) correspond to \(V_3\) and \(V_4\)

\[
c_3 W_0 = 4A_{2\pi} , \quad c_4 W_0 = 4C_{2\pi}
\]

\(V_1\) term **is not present** in UIX.
When applied to an antisymmetric wavefunction

\[ V^{\text{OPE}}(3:12)\mathcal{A}_{12}|\Psi\rangle \]

All structures lead to the same expression, it is convenient to consider only one

\[ V^{\text{OPE}}(3:12) = -c_D V_0^D \frac{(\sigma_2 \cdot q_2)}{(q_2^2 + m_\pi^2)} \left[ \alpha_1(\sigma_1 \cdot q_2)\tau_{12} + \alpha_2(\sigma_1 \cdot q_2)\tau_{13} + \alpha_3 q_2(\sigma_1 \times \sigma_3)\vec{\tau}_2 \cdot (\vec{\tau}_1 \times \vec{\tau}_3) + 1 \leftrightarrow 2 \right] \]

Regulator dependence on the transferred momentum \( q_2 \)

The antisymmetrization operator also exchanges the positions of nucleons

In principle all the different terms need to be considered.
In principle all the different terms need to be considered. Because of $A_{123}$, all terms coming from the Lagrangian give the same contribution. Regulator dependence on the transferred momentum $q_2$.

- **TM′** potential has a repulsive three-nucleon contact term without isospin dependence:

$$V_E(3:12) = c_E W_0^E$$

- **UIX**: $V^R$ although is not a contact term, has the same structure of the contact term of NNLOL:

$$U_0 \quad \longleftrightarrow \quad c_E W_0^E$$
Contact term

Repulsive term radial functions are different

**NOT** smeared delta function
NNLO contact term issue in nuclear matter

Full momentum space expression of NNLO contact term

\[ V^{\text{cont}}(3:12) = c_E (\beta_1 + \beta_2 \sigma_{12} + \beta_3 \tau_{12} + \beta_4 \sigma_{12} \tau_{12} + \beta_5 \sigma_{12} \tau_{23} + \beta_6 (\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \vec{\sigma}_3 + (\vec{\tau}_1 \times \vec{\tau}_2) \cdot \vec{\tau}_3 + 1 \leftrightarrow 2) \]
NNLO contact term issue in nuclear matter

Full momentum space expression of NNLO contact term

\[ V^{\text{cont}}(3 : 12) = c_E (\beta_1 + \beta_2 \sigma_{12} + \beta_3 \tau_{12} + \beta_4 \sigma_{12} \tau_{12} + \beta_5 \sigma_{12} \tau_{23} \\
+ \beta_6 (\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \vec{\sigma}_3 + (\vec{\tau}_1 \times \vec{\tau}_2) \cdot \vec{\tau}_3 + 1 \leftrightarrow 2) \]

Fourier transform of the isospin and scalar term yields

\[ V_E^\tau(3 : 12) = c_E^\tau W^E_0 \tau_{12} z_0(r_{13}) z_0(r_{23}) \quad \text{and} \quad V_E^I(3 : 12) = c_E^I W^E_0 z_0(r_{13}) z_0(r_{23}) \]
NNLOL contact term issue in nuclear matter

Full momentum space expression of NNLO contact term

$$V_{\text{cont}}^{3 : 12} = c_E (\beta_1 + \beta_2 \sigma_{12} + \beta_3 \tau_{12} + \beta_4 \sigma_{12} \tau_{12} + \beta_5 \sigma_{12} \tau_{23} + \beta_6 (\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \vec{\sigma}_3 + (\vec{r}_1 \times \vec{r}_2) \cdot \vec{r}_3 + 1 \leftrightarrow 2)$$

Fourier transform of the isospin and scalar term yields

$$V_E^T(3 : 12) = c_E^T W_0^E \tau_{12} z_0(r_{13}) z_0(r_{23})$$

$$V_E^I(3 : 12) = c_E^I W_0^E z_0(r_{13}) z_0(r_{23})$$

Convenient normalization for the radial function

$$Z_0(r) = \frac{(4\pi)}{m^3_{\pi}} z_0(r)$$

$$\lim_{\Lambda \to \infty} Z_0(r) = \delta(r)$$
NNLOL contact term issue in nuclear matter

In the limit of infinite cutoff

$$\sum_{\text{cycl}} V_0^E \delta(r_{13}) \delta(r_{23}) \tau_{12} A_{123} = - \sum_{\text{cycl}} V_0^E \delta(r_{13}) \delta(r_{23}) A_{123}$$

The terms are equivalent!

Crucial for the equivalence

The exchange of particles lying in the same position is the identity!

$$e^{ik_{ij} \cdot r_{ij}} \delta(r_{ij}) = \delta(r_{ij})$$

At finite values of the cutoff this is not true anymore.

Not considering all the terms lead to ambiguities in both PNM and SNM
For Pauli principle, in PNM the expectation value of a three-body contact term is zero.

\[
\langle V_{E}^{I,\tau} \rangle_{PNM} = O\left(\frac{q^4}{\Lambda^4}\right)
\]

For reproducing the binding energies of light nuclei and \(2a_{nd}\) and requires a repulsive \(V_E\)

\[
\begin{align*}
\Lambda (\text{MeV}) & \quad \langle V_{E}^{I,\tau_{12}} \rangle_{PNM}^{FG} / A (\text{MeV}) \\
300 & \quad 9.15 \\
400 & \quad 5.95 \\
500 & \quad 3.60 \\
600 & \quad 2.15 \\
700 & \quad 1.30 \\
800 & \quad 0.81 \\
\infty & \quad 0 
\end{align*}
\]

Furthermore

\[
\langle \tau_{12} \rangle_{PNM} = 1 \quad \Rightarrow \quad \begin{cases} 
\langle V_{E}^{I} \rangle_{PNM} > 0 \\
\langle V_{E}^{\tau_{12}} \rangle_{PNM} < 0
\end{cases}
\]

Fixing only one of the contact terms of NNLOL on low energy observables leads to ambiguity in PNM.
NNLOL contact term issue in nuclear matter

Assume that reproducing the binding energies of light nuclei and $^2\alpha_{nd}$ and requires a repulsive $V_E$

\[ c_E^+ < 0 \]
\[ c_E^- > 0 \]

**Symmetric Nuclear Matter (SNM)**

### Infinite cutoff

\[
\frac{\langle V_{E12}^{T} \rangle_{SNM}^{FG}}{A} = -\frac{3}{16} \rho^2 V_0^E
\]

### Finite cutoff

<table>
<thead>
<tr>
<th>$\Lambda$ (MeV)</th>
<th>$\langle V_{E12}^T \rangle_{SNM}^{FG}/A$ (MeV)</th>
<th>$\langle V_{E1}^I \rangle_{SNM}^{FG}/A$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>-2.61</td>
<td>10.21</td>
</tr>
<tr>
<td>400</td>
<td>-3.61</td>
<td>8.15</td>
</tr>
<tr>
<td>500</td>
<td>-4.37</td>
<td>6.93</td>
</tr>
<tr>
<td>600</td>
<td>-4.87</td>
<td>6.30</td>
</tr>
<tr>
<td>700</td>
<td>-5.15</td>
<td>5.98</td>
</tr>
<tr>
<td>800</td>
<td>-5.30</td>
<td>5.81</td>
</tr>
<tr>
<td>$\infty$</td>
<td>-5.55</td>
<td>5.55</td>
</tr>
</tbody>
</table>

The two contact term are equivalent in the limit of infinite cutoff only.
Kievsky et al. in 2010 have found the best-fit values for the TM' and NNOL 3-body potentials plus Argonne $v_{18}$ NN potential to simultaneously reproduce

\[
\begin{align*}
B(^3H) &= -8.482 \text{ MeV} \\
B(^4He) &= -28.30 \text{ MeV} \\
^2a_{nd} &= 0.645 \pm 0.003 \pm 0.007 \text{ fm}
\end{align*}
\]
3-body potential analysis

Kievsky et al. in 2010 have found the best-fit values for the TM' and NNOL 3-body potentials plus Argonne $v_{18}$ NN potential to simultaneously reproduce

\[
\begin{align*}
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\end{align*}
\]

### Chiral NNLOL potential

<table>
<thead>
<tr>
<th>Potential</th>
<th>$c_3$ (MeV$^{-1}$)</th>
<th>$c_4$ (MeV$^{-1}$)</th>
<th>$c_D$</th>
<th>$c_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NNLOL$_1$</td>
<td>-0.00448</td>
<td>-0.001963</td>
<td>-0.5</td>
<td>0.100</td>
</tr>
<tr>
<td>NNLOL$_2$</td>
<td>-0.00448</td>
<td>-0.002044</td>
<td>-1.0</td>
<td>0.000</td>
</tr>
<tr>
<td>NNLOL$_3$</td>
<td>-0.00480</td>
<td>-0.002017</td>
<td>-1.0</td>
<td>-0.030</td>
</tr>
<tr>
<td>NNLOL$_4$</td>
<td>-0.00544</td>
<td>-0.004860</td>
<td>-2.0</td>
<td>-0.500</td>
</tr>
</tbody>
</table>

### MIXED APPROACH

Fit of all $c_i$ required

\[
\begin{align*}
c_1 &= 0.00081 \text{ MeV}^{-1} \\
\Lambda_x &= 700 \text{ MeV} \\
\Lambda &= 500 \text{ MeV}
\end{align*}
\]
Kievsky et al. in 2010 have found the best-fit values for the TM' and NNOL 3-body potentials plus Argonne v18 NN potential to simultaneously reproduce

\[
\begin{align*}
B(\text{}^3\text{H}) &= -8.482 \text{ MeV} \\
B(\text{}^4\text{He}) &= -28.30 \text{ MeV} \\
^2a_{nd} &= 0.645 \pm 0.003 \pm 0.007 \text{ fm}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Potential</th>
<th>(b(m_\pi^{-3}))</th>
<th>(d(m_\pi^{-3}))</th>
<th>(c_E)</th>
<th>(\Lambda(m_\pi))</th>
</tr>
</thead>
<tbody>
<tr>
<td>TM_1'</td>
<td>-8.256</td>
<td>-4.690</td>
<td>1.0</td>
<td>4.0</td>
</tr>
<tr>
<td>TM_2'</td>
<td>-3.870</td>
<td>-3.375</td>
<td>1.6</td>
<td>4.8</td>
</tr>
<tr>
<td>TM_3'</td>
<td>-2.064</td>
<td>-2.279</td>
<td>2.0</td>
<td>5.6</td>
</tr>
</tbody>
</table>

\(a = -0.87 m_\pi^{-1}\)
The correlated basis ground state is defined by

\[ |\Psi_0\rangle = \frac{\hat{F}|\Phi_0\rangle}{\langle \Phi_0|\hat{F}^\dagger \hat{F}|\Phi_0\rangle^{1/2}} \]

The Fermi gas wave function is a Slater determinant of plane waves

\[ \Phi_0 = \mathcal{A}[\phi_1(x_1) \ldots \phi_A(x_A)] \]

The correlation operator reflects the structure of the NN potential

\[ F = S\left( \prod_{j>i=1}^A F_{ij} \right) = S\left( \prod_{j>i=1}^A \sum_{p=1}^6 f^p(r_{ij}) O^p_{ij} \right) \]

Variational approach!

Variational principle holds

\[ E_V = \langle \Psi_0 | H | \Psi_0 \rangle = \frac{\langle \Phi_0 | F^\dagger H F | \Phi_0 \rangle}{\langle \Phi_0 | F^\dagger F | \Phi_0 \rangle} \geq E_0 \]

The correlation functions are determined variationally by minimizing \( E_V \)
Correlated basis function

Hamiltonian expectation value

$$E_V = \frac{\int d x_1 \ldots d x_A \Phi_0^*(x_1 \ldots x_A) F^\dagger H F \Phi_0(x_1 \ldots x_A)}{\int d x_1 \ldots d x_A \Phi_0^*(x_1 \ldots x_A) F^\dagger F \Phi_0(x_1 \ldots x_A)}$$

Integration over the coordinates of a huge number of particles.

Key point of the cluster decomposition: expansion of $F^\dagger F$ in powers of

$$h_{ij} \equiv f_{ij}^2 - 1, \quad 2 f_{ij} f_{ij}^p, \quad f_{ij}^{p>1} f_{ij}^{q>1}$$

The expansion can be represented by generalized Mayer diagrams

- **Massive summation of relevant diagrams**
  - Scalar diagrams: FHNC
  - Operatorial diagrams: SOC

No elementary diagrams

At most two operators arrive at a given point of the diagram.

Violation of variational principle?
3-body potentials in CBF

Expectation value of the three body potential

\[ \langle V \rangle = \frac{\int d x_1 \ldots d x_A \Phi_0^*(x_1 \ldots x_A) F^\dagger V_{123} F \Phi_0(x_1 \ldots x_A)}{\int d x_1 \ldots d x_A \Phi_0^*(x_1 \ldots x_A) F^\dagger F \Phi_0(x_1 \ldots x_A)} \]

Diagrams involved for the contact term \( V_E \)

Diagrams for TPE and OPE
3-body potentials in CBF: simulated annealing

Variational energy depends on a set of parameters

\[ E_V = E_V(d_c, d_t, \beta_p, \alpha_p) \]

For the minimization of \( E_V \)  

Simulated annealing procedure
3-body potentials in CBF: simulated annealing

Variational energy depends on a set of parameters

\[ E_V = E_V(d_c, d_t, \beta_p, \alpha_p) \]

For the minimization of \( E_V \)  

**Simulated annealing procedure**
3-body potentials in CBF: simulated annealing

Variational energy depends on a set of parameters

\[ E_V = E_V(d_c, d_t, \beta_p, \alpha_p) \]

For the minimization of \( E_V \) \rightarrow \text{Simulated annealing procedure}

Metropolis algorithm

\[ s = \{d_c, d_t, \beta_p, \alpha_p\} \quad s' = \{d'_c, d'_t, \beta'_p, \alpha'_p\} \quad \text{with} \quad P_{s,s'} = \exp \left[ -\frac{E(s') - E(s)}{T} \right] \]

As \( T \) is lowered, the parameters stay closer to the minimum of \( E_V \)

To keep the violations under control \rightarrow \text{Constrained optimization}

\[
\begin{align*}
|E_{PB} - E_{JF}| < 10\% T_F \\
|\rho \int d\vec{r}_{12} (g^c(r_{12}) - 1) + 1| < 0.03 \\
|\frac{\rho}{3} \int d\vec{r}_{12} g^\tau (r_{12}) + 1| < 0.03
\end{align*}
\]
AFDMC simulations for PNM with 66 neutrons in periodic box system.

For 66 fermions difference around 2% of the asymptotic value of TABC.

Gandolfi et al. 2009

<table>
<thead>
<tr>
<th>A</th>
<th>(E/A ) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>56.51</td>
</tr>
<tr>
<td>38</td>
<td>53.50</td>
</tr>
<tr>
<td>66</td>
<td>55.43</td>
</tr>
<tr>
<td>114</td>
<td>56.58</td>
</tr>
<tr>
<td>(\infty)</td>
<td>55.71</td>
</tr>
</tbody>
</table>
TM' results for PNM

Energy per nucleon (MeV) vs. density (fm$^{-3}$) for FHNC and AFDMC calculations with $v_8$ + TM1, TM2, and TM3.
Equations of State very close to each other

Is this happening because three body force are designed to reproduce $^2a_{nd}$ also?
TM' results for SNM

![Graph showing energy per nucleon (MeV) vs. density (fm⁻³) for different models of TM'.]

<table>
<thead>
<tr>
<th></th>
<th>TM'₁</th>
<th>TM'₂</th>
<th>TM'₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho₀ ) (fm⁻³)</td>
<td>0.12</td>
<td>0.13</td>
<td>0.14</td>
</tr>
<tr>
<td>( E₀ ) (MeV)</td>
<td>-9.0</td>
<td>-8.8</td>
<td>-9.4</td>
</tr>
<tr>
<td>K (MeV)</td>
<td>266</td>
<td>243</td>
<td>249</td>
</tr>
</tbody>
</table>

Experimental values

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho₀ ) (fm⁻³)</td>
<td>0.16</td>
</tr>
<tr>
<td>( E₀ ) (MeV)</td>
<td>-16.0</td>
</tr>
<tr>
<td>K (MeV)</td>
<td>240</td>
</tr>
</tbody>
</table>
Chiral NNLOL results for PNM

- FHNC: $v_8' + NNLO_1$
- AFDMC: $v_8' + NNLO_1$

- FHNC: $v_8' + NNLO_2$
- AFDMC: $v_8' + NNLO_2$

- FHNC: $v_8' + NNLO_3$
- AFDMC: $v_8' + NNLO_3$

- FHNC: $v_8' + NNLO_4$
- AFDMC: $v_8' + NNLO_4$
Chiral NNLOL results for PNM

**Very Soft**
Chiral NNLOL results for SNM

NNLOL\textsubscript{4} has a large negative $c_E$

- stiffest SNM EoS
- softest PNM EoS
Chiral NNLOL results for SNM

![Graph showing energy per nucleon vs. density for different NNLOL calculations.]

<table>
<thead>
<tr>
<th>Density</th>
<th>NNLOL(_1)</th>
<th>NNLOL(_2)</th>
<th>NNLOL(_3)</th>
<th>NNLOL(_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho_0) (fm(^{-3}))</td>
<td>0.21</td>
<td>0.20</td>
<td>0.19</td>
<td>0.17</td>
</tr>
<tr>
<td>(E_0) (MeV)</td>
<td>-15.2</td>
<td>-14.6</td>
<td>-14.6</td>
<td>-12.9</td>
</tr>
<tr>
<td>(K) (MeV)</td>
<td>198</td>
<td>252</td>
<td>220</td>
<td>310</td>
</tr>
</tbody>
</table>

**Experimental values**

<table>
<thead>
<tr>
<th>Density</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho_0) (fm(^{-3}))</td>
<td>0.16</td>
</tr>
<tr>
<td>(E_0) (MeV)</td>
<td>-16.0</td>
</tr>
<tr>
<td>(K) (MeV)</td>
<td>240</td>
</tr>
</tbody>
</table>
Conclusions

- Contact term of NNLOL potential suffers of cutoff dependence. Its contribution in nuclear matter cannot be evaluated fitting low energy observables.

- No one of the potential considered simultaneously explains the binding energy and the saturation density of SNM.

- NNLOL$_4$ and TM$_3'$ potentials provide reasonable value for the saturation density of SNM although not involving any parameters adjusted to reproduce it.

- Deriving potential from chiral perturbation theory is still a promising approach.

NNNLO potential is now available as well as NNNNNLO contact terms!
More low energy observables are needed