

Ab initio calculations of light-ion reactions

Workshop on “Electron-Nucleus Scattering XII”
 June 25-29, 2012
 Elba, Italy

Petr Navratil | TRIUMF

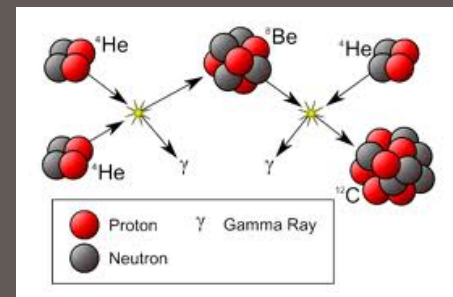
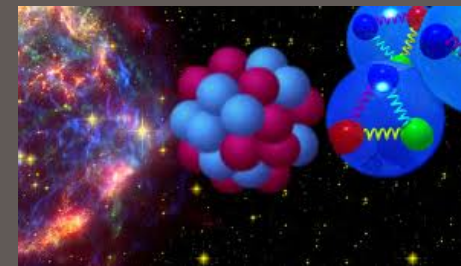
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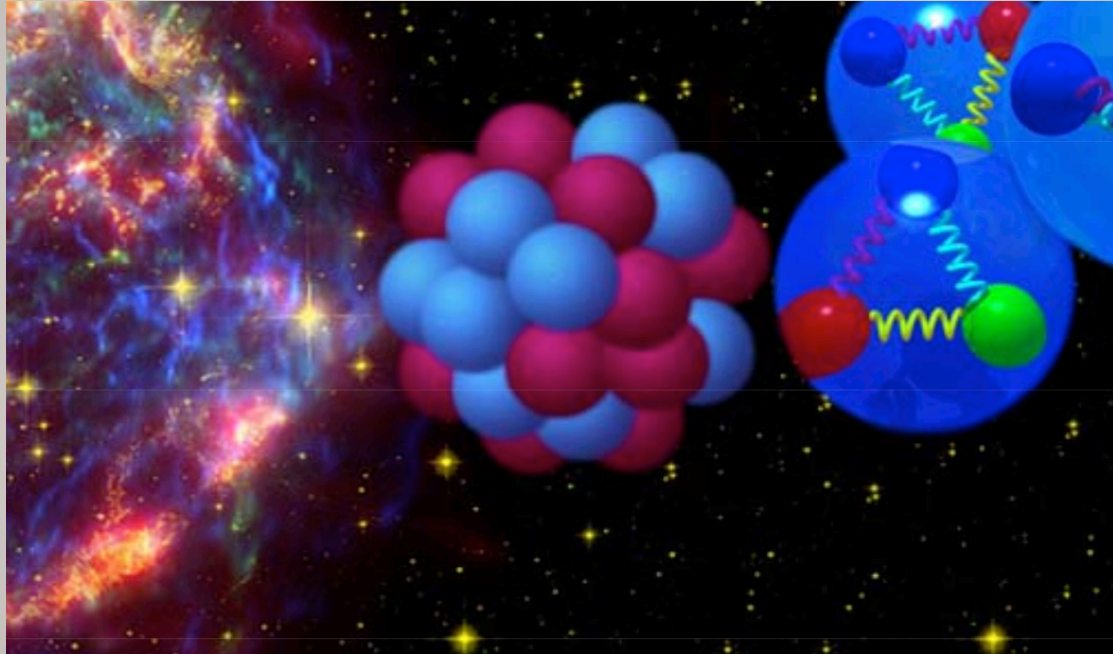
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Marciana Marina, Isola d'Elba, Italy.



Connection
to
Astrophysics



Connection
to
QCD

- Nuclear forces – from chiral EFT
- Many-body techniques – NCSM, NCSM/RGM
- Results for bound states, resonances, reactions

Nuclei from the first principles

- **First principles for Nuclear Physics:**

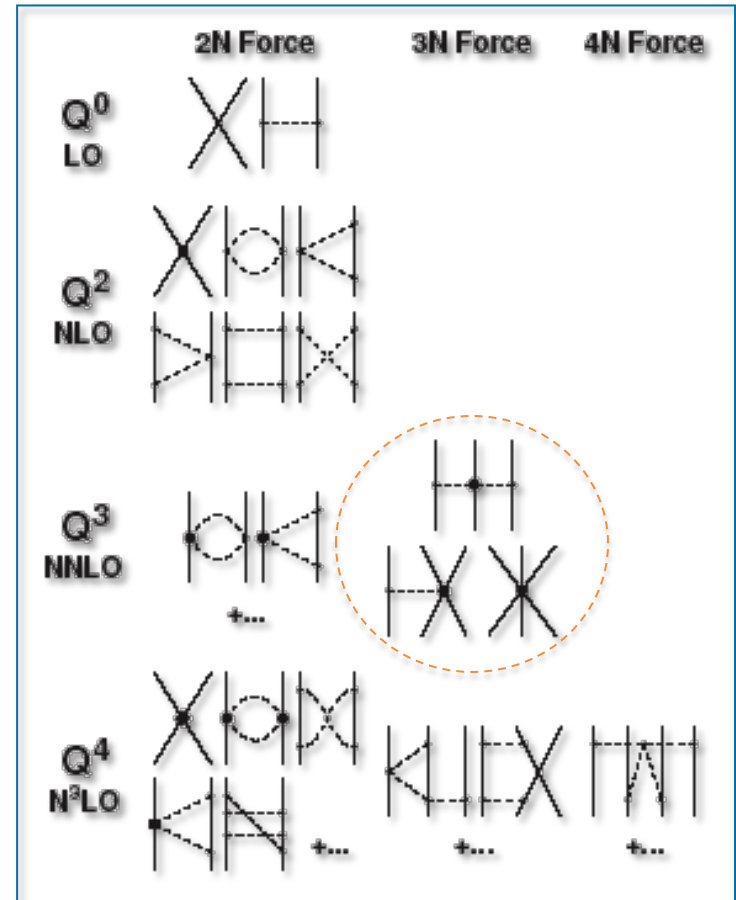
- **QCD**

- Non-perturbative at low energies
 - Lattice QCD in the future

- ***For now a good place to start:***

- **Inter-nucleon forces from chiral effective field theory**

- Based on the symmetries of QCD
 - Degrees of freedom: nucleons + pions
 - Systematic low-momentum expansion to a given order
 - Hierarchy
 - Consistency
 - Low energy constants (LEC)
 - Fitted to data
 - Can be calculated by lattice QCD

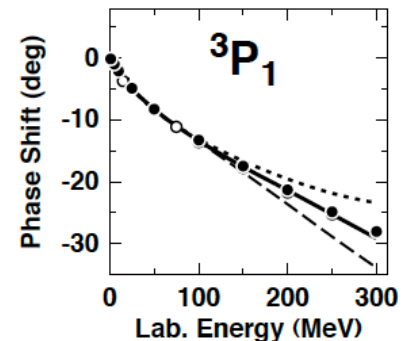
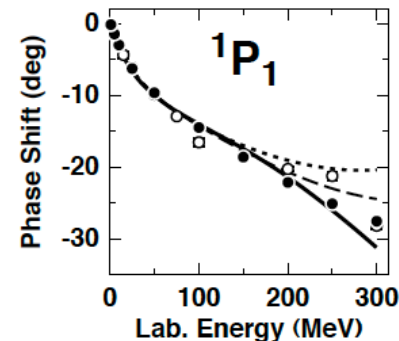
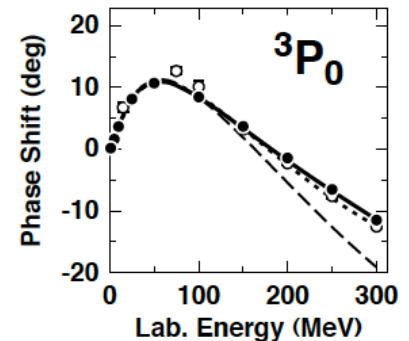
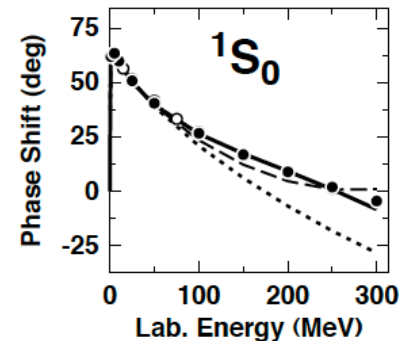
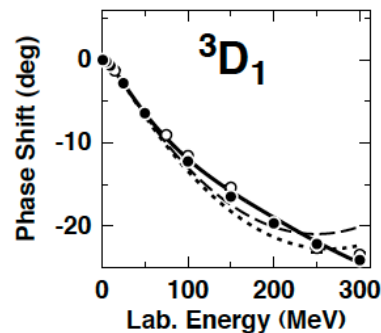
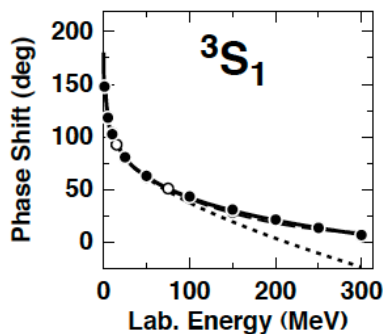


The NN interaction from chiral EFT

PHYSICAL REVIEW C **68**, 041001(R) (2003)

Accurate charge-dependent nucleon-nucleon potential at fourth order of chiral perturbation theory

D. R. Entem^{1,2,*} and R. Machleidt^{1,†}



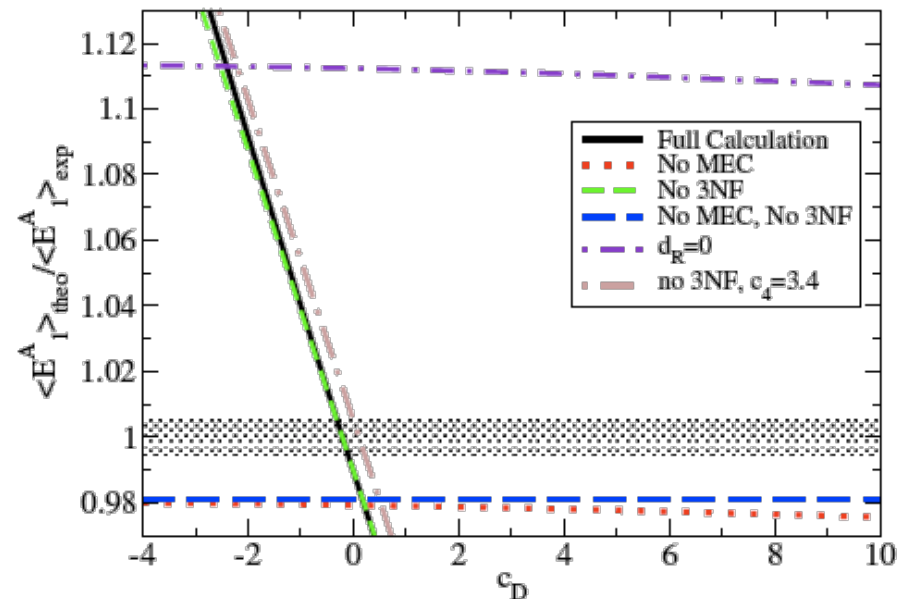
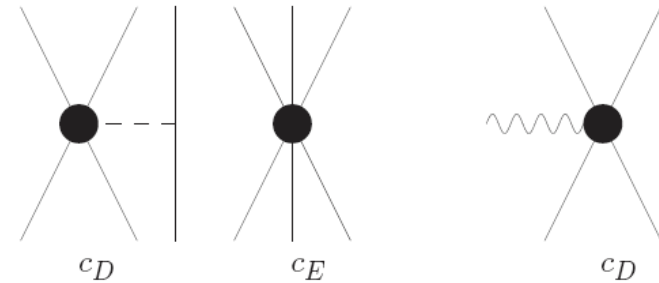
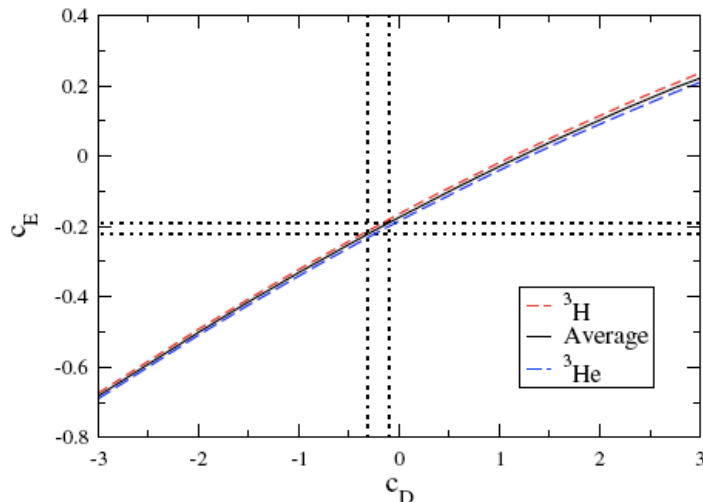
- 24 LECs fitted to the np scattering data and the deuteron properties
 - Including c_i LECs ($i=1-4$) from pion-nucleon Lagrangian

Determination of NNN LECs c_D and c_E from the triton binding energy and the half life

- **Chiral EFT:** c_D also in the two-nucleon contact vertex with an external probe
- Calculate $\langle E_1^A \rangle = |\langle {}^3\text{He} || E_1^A || {}^3\text{H} \rangle|$
 - Leading order GT
 - N²LO: one-pion exchange plus contact

- **A=3 binding energy constraint:**

$$c_D = -0.2 \pm 0.1 \quad c_E = -0.205 \pm 0.015$$

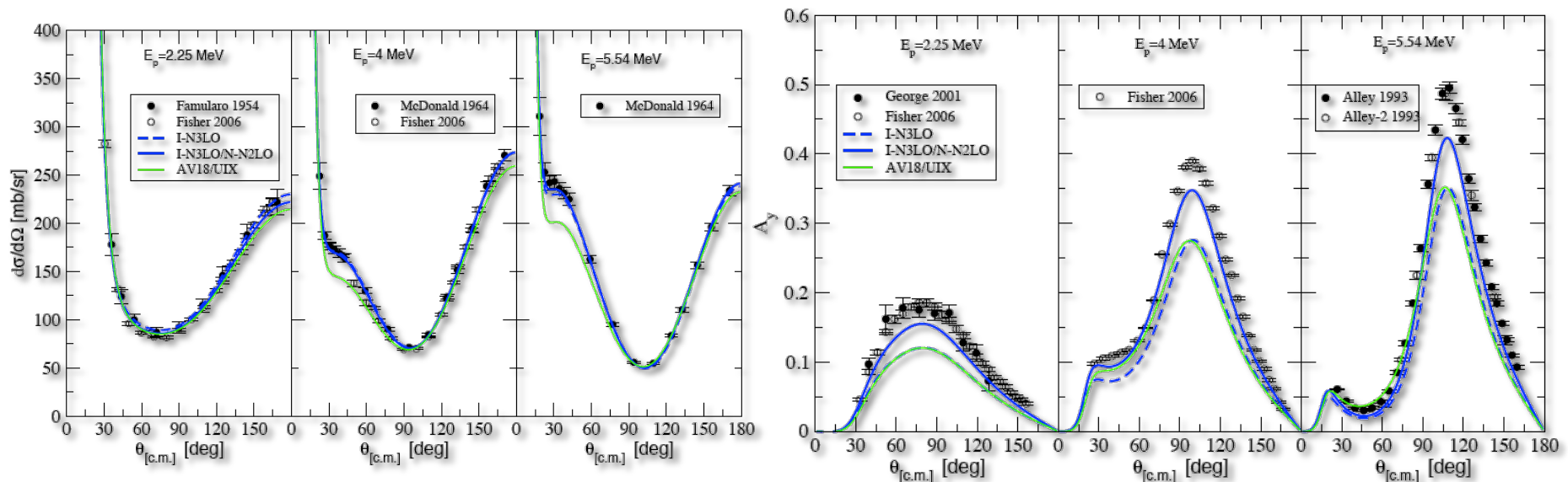


A=3,4 bound states

	³ H		³ He		⁴ He	
	$E_{\text{g.s.}}$	$\langle r_p^2 \rangle^{1/2}$	$E_{\text{g.s.}}$	$\langle r_p^2 \rangle^{1/2}$	$E_{\text{g.s.}}$	$\langle r_p^2 \rangle^{1/2}$
<i>NN</i>	-7.852(4)	1.651(5)	-7.124(4)	1.847(5)	-25.39(1)	1.515(2)
<i>NN + NNN</i>	-8.473(4)	1.605(5)	-7.727(4)	1.786(5)	-28.50(2)	1.461(2)
Expt.	-8.482	1.60	-7.718	1.77	-28.296	1.467(13) [31]

Proton-³He elastic scattering with χ EFT NN+NNN

- Variational calculations in hyperspherical-harmonics basis
 - M. Viviani, L. Girlanda, A. Kievski, L. E. Marcucci, and S. Rosati, arXiv:1004.1306
- A_y puzzle resolved with the chiral N³LO NN plus local chiral N²LO NNN**



Chiral NN+NNN Hamiltonian provides the best agreement with the cross section and analyzing power data and with the new TUNL PSA analysis

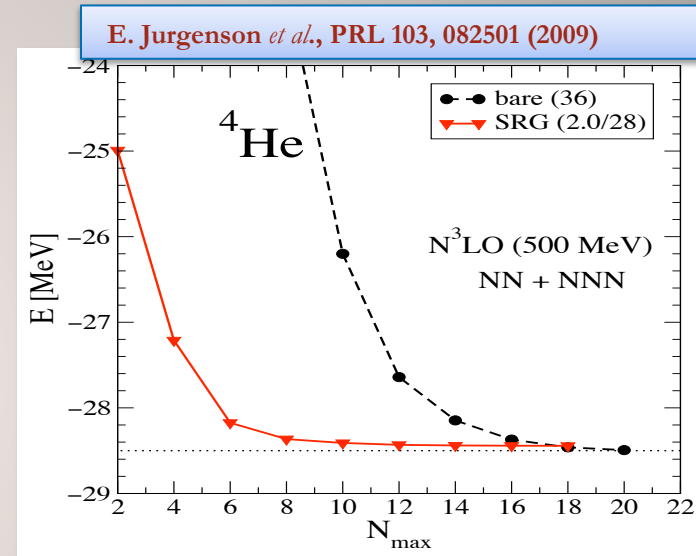
No-core shell model combined with the resonating group method (NCSM/RGM)

- **The NCSM:** An approach to the solution of the A -nucleon bound-state problem

- Accurate nuclear Hamiltonian
- Finite harmonic oscillator (HO) basis
 - Complete $N_{max} \hbar\Omega$ model space
- Effective interaction due to the model space truncation
 - Similarity-Renormalization-Group evolved NN(+NNN) potential
- Short & medium range correlations
- No continuum



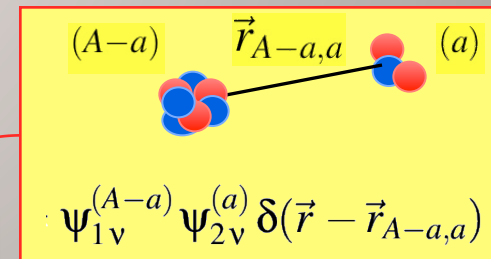
$$\Psi^A = \sum_{N=0}^{N_{max}} \sum_i c_{Ni} \Phi_{Ni}^A$$



- **The RGM:** A microscopic approach to the A -nucleon scattering of clusters

- Long range correlations, relative motion of clusters

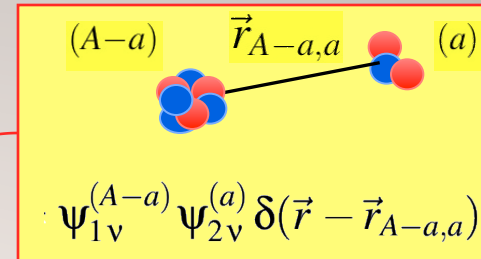
$$\Psi^{(A)} = \sum_v \int d\vec{r} \phi_v(\vec{r}) \hat{\mathcal{A}} \Phi_{v\vec{r}}^{(A-a,a)}$$



Ab initio NCSM/RGM: Combines the best of both approaches
 Accurate nuclear Hamiltonian, consistent cluster wave functions
 Correct asymptotic expansion, Pauli principle and translational invariance

The *ab initio* NCSM/RGM in a snapshot

- Ansatz: $\Psi^{(A)} = \sum_{\mathbf{v}} \int d\vec{r} \phi_{\mathbf{v}}(\vec{r}) \hat{\mathcal{A}} \Phi_{\mathbf{v}\vec{r}}^{(A-a,a)}$



eigenstates of $H_{(A-a)}$ and $H_{(a)}$ in the *ab initio* NCSM basis

- Many-body Schrödinger equation:

$$H\Psi^{(A)} = E\Psi^{(A)}$$

$$T_{\text{rel}}(r) + \mathcal{V}_{\text{rel}} + \bar{V}_{\text{Coul}}(r) + H_{(A-a)} + H_{(a)}$$

$$\sum_{\mathbf{v}} \int d\vec{r} \left[\mathcal{H}_{\mu\mathbf{v}}^{(A-a,a)}(\vec{r}', \vec{r}) - E \mathcal{N}_{\mu\mathbf{v}}^{(A-a,a)}(\vec{r}', \vec{r}) \right] \phi_{\mathbf{v}}(\vec{r}) = 0$$

$$\langle \Phi_{\mu\vec{r}'}^{(A-a,a)} | \hat{\mathcal{A}} H \hat{\mathcal{A}} | \Phi_{\mathbf{v}\vec{r}}^{(A-a,a)} \rangle$$

Hamiltonian kernel

$$\langle \Phi_{\mu\vec{r}'}^{(A-a,a)} | \hat{\mathcal{A}}^2 | \Phi_{\mathbf{v}\vec{r}}^{(A-a,a)} \rangle$$

Norm kernel

realistic nuclear Hamiltonian

Norm kernel (Pauli principle)

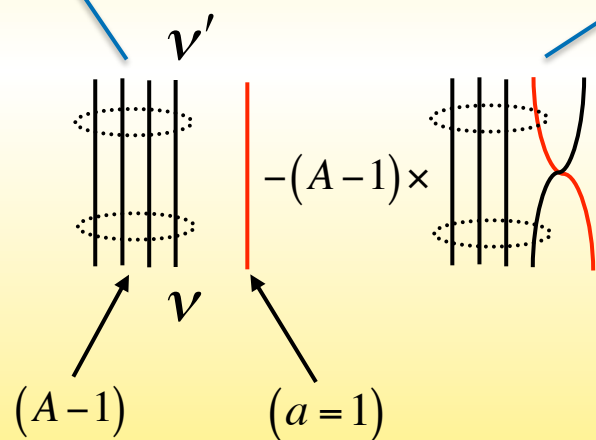
Single-nucleon projectile

$$\langle \Phi_{v'r'}^{J\pi T} | \hat{A}_{v'} \hat{A}_v | \Phi_{vr}^{J\pi T} \rangle = \left\langle \begin{array}{c} (A-1) \\ \text{red, blue, blue} \\ \text{red} \\ r' \quad (a'=1) \end{array} \middle| 1 - \sum_{i=1}^{A-1} \hat{P}_{iA} \middle| \begin{array}{c} (A-1) \\ \text{red, blue, blue} \\ \text{red} \\ r \quad (a=1) \end{array} \right\rangle$$

$$N_{v'v}^{J\pi T}(r', r) = \underbrace{\delta_{v'v} \frac{\delta(r' - r)}{r'r}}_{\text{Direct term}} - (A-1) \sum_{n'n} R_{n'\ell'}(r') R_{n\ell}(r) \underbrace{\langle \Phi_{v'n'}^{J\pi T} | \hat{P}_{A-1,A} | \Phi_{vn}^{J\pi T} \rangle}_{\text{Exchange term}}$$

$$\text{SD} \langle \psi_{\mu_1}^{(A-1)} | a^+ a | \psi_{\nu_1}^{(A-1)} \rangle_{\text{SD}}$$

Direct term:
Treated exactly!
(in the full space)



Exchange term:
Obtained in the model space!
(Many-body correction due to
the exchange part of the inter-
cluster antisymmetrizer)

$$\delta(r - r_{A-a,a}) = \sum_n R_{n\ell}(r) R_{n\ell}(r_{A-a,a})$$

Hamiltonian kernel (projectile-target potentials)

Single-nucleon projectile

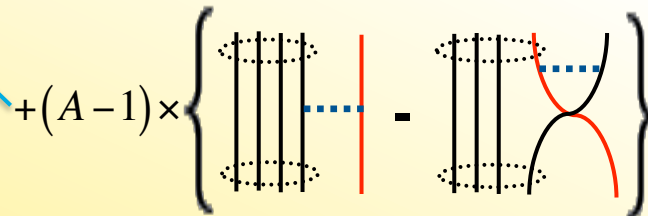
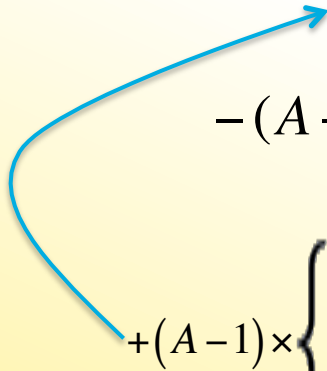
$$\langle \Phi_{v'r'}^{J\pi T} | \hat{A}_{v'} H \hat{A}_v | \Phi_{vr}^{J\pi T} \rangle = \left\langle \begin{array}{c} (A-1) \\ \text{red, blue} \\ \text{red} \\ r' \quad (a'=1) \end{array} \middle| H \left(1 - \sum_{i=1}^{A-1} \hat{P}_{iA} \right) \middle| \begin{array}{c} (A-1) \\ \text{red, blue} \\ \text{red} \\ r \quad (a=1) \end{array} \right\rangle$$

$$H_{v'v}^{J\pi T}(r', r) = \left[T_{rel}(r) + \bar{V}_{Coul}(r) + \varepsilon_{\alpha_1}^{I_1^{\pi_1} T_1'} \right] N_{v'v}^{J\pi T}(r', r)$$

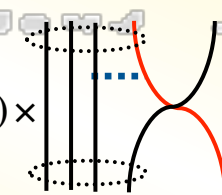


$$+ (A-1) \sum_{n'n} R_{n'\ell'}(r') R_{n\ell}(r) \langle \Phi_{v'n'}^{J\pi T} | V_{A-1,A} (1 - \hat{P}_{A-1,A}) | \Phi_{vn}^{J\pi T} \rangle$$

$$- (A-1)(A-2) \sum_{n'n} R_{n'\ell'}(r') R_{n\ell}(r) \langle \Phi_{v'n'}^{J\pi T} | \hat{P}_{A-1,A} V_{A-2,A-1} | \Phi_{vn}^{J\pi T} \rangle$$



Direct potential: in the model space
(interaction is localized!)



Exchange potential:
in the model space

Solving the RGM equations

- Input: Realistic nuclear Hamiltonian, eigenfunctions of nucleon clusters
 - Macroscopic degrees of freedom: nucleon clusters
 - Unknowns: relative wave function between the two clusters
- Non-local integral-differential coupled-channel equations:

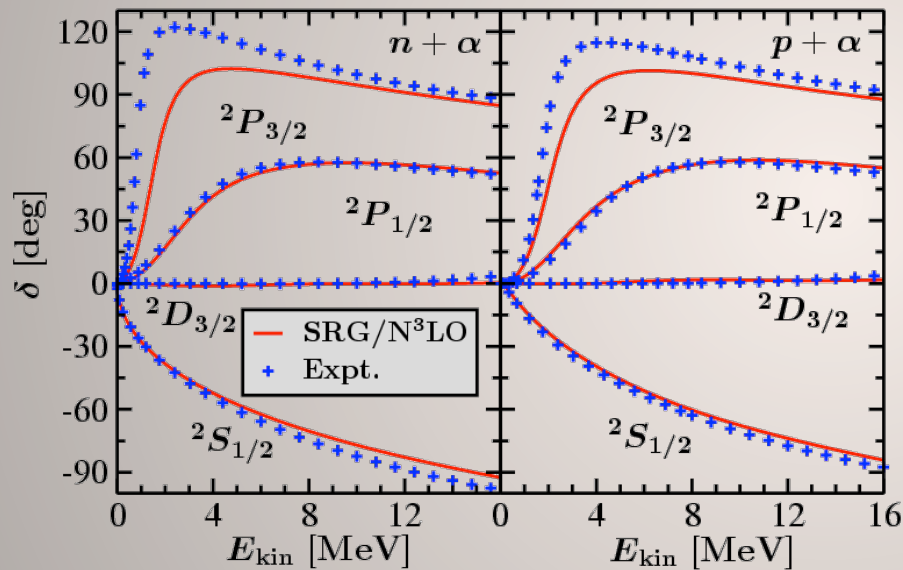
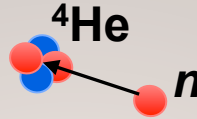
$$\left[T_{rel}(r) + V_C(r) + E_{\alpha_1}^{(A-a)} + E_{\alpha_2}^{(a)} \right] u_{\nu}^{(A-a,a)}(r) + \sum_{a'v'} \int dr' r' W_{av,a'v'}(r,r') u_{\nu'}^{(A-a',a')}(r') = 0$$

- Solve with R-matrix theory on Lagrange mesh imposing
 - Bound state boundary conditions → eigenenergy + eigenfunction
 - Scattering state boundary conditions → Scattering matrix
 - Phase shifts
 - Cross sections
 - ...

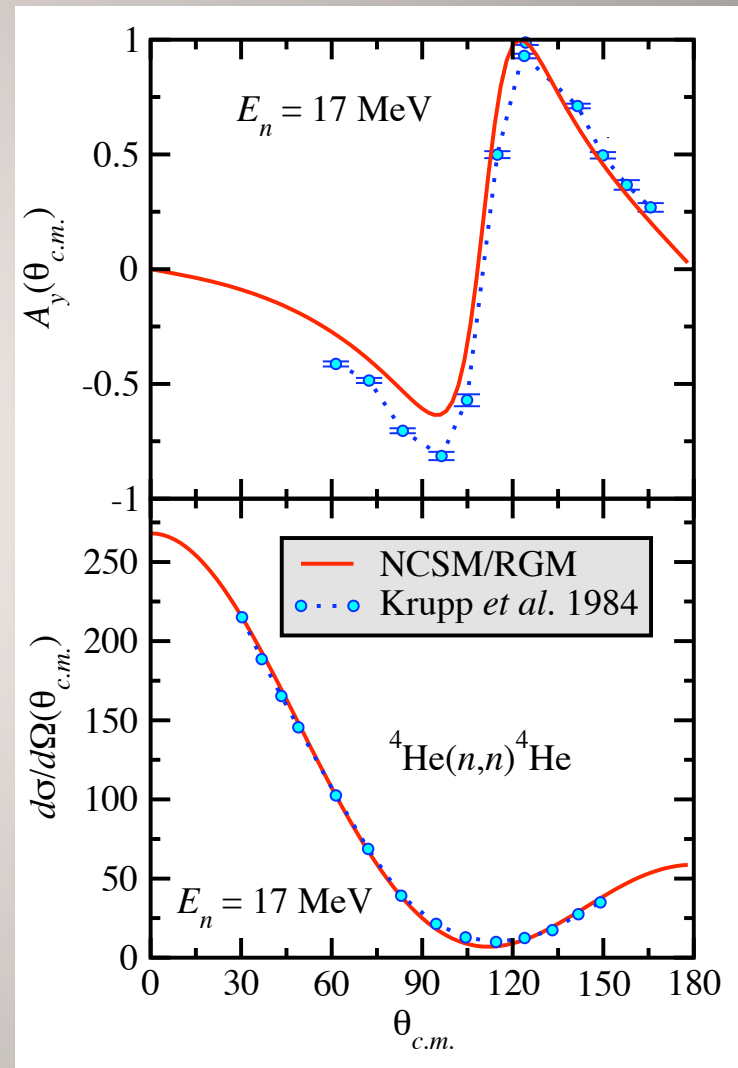
The R-matrix theory on Lagrange mesh is an elegant and powerful technique, particularly for calculations with non-local potentials

The best system to start with: $n+{}^4\text{He}$, $p+{}^4\text{He}$

- NCSM/RGM calculations with
 - $N + {}^4\text{He}(\text{g.s.}, 0^+0)$
 - SRG- $N^3\text{LO}$ NN potential with $\Lambda=2.02 \text{ fm}^{-1}$

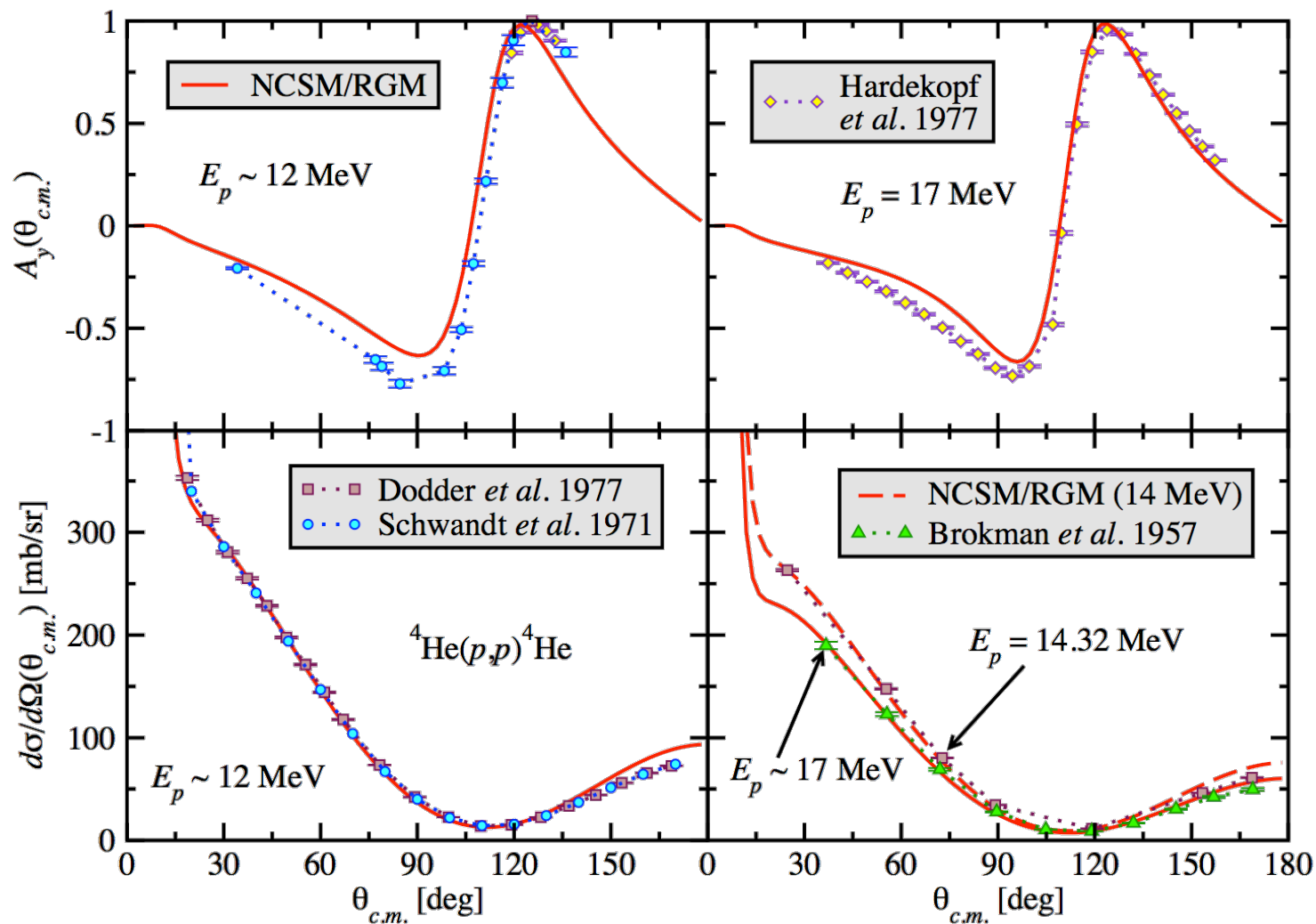


- Differential cross section and analyzing power @17 MeV neutron energy
 - Polarized neutron experiment at Karlsruhe



NNN missing: Good agreement only for energies beyond low-lying $3/2^-$ resonance

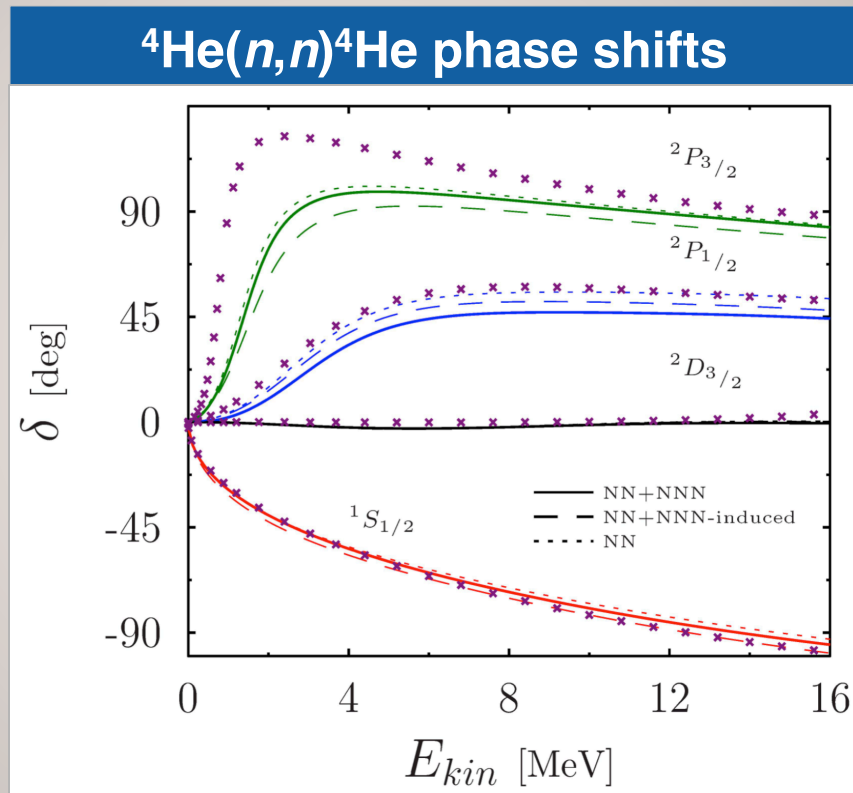
$p+^4\text{He}$ differential cross section and analyzing power



n - ^4He scattering with NN+NNN interactions

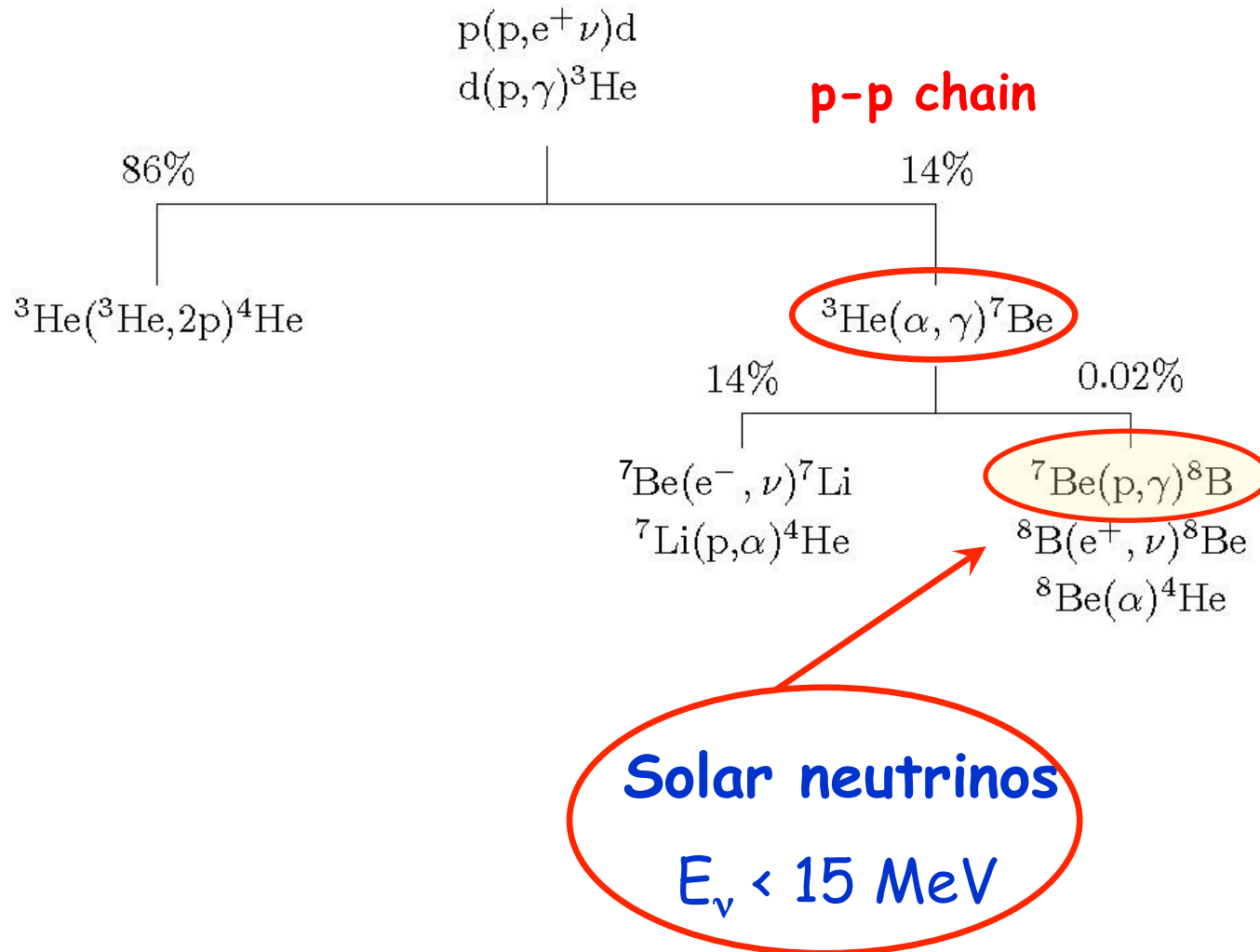
G. Hupin, J. Langhammer, S. Quaglioni, P. Navratil, R. Roth, work in progress

$n + ^4\text{He}(\text{g.s.})$, SRG-($N^3\text{LO NN} + N^2\text{LO NNN}$ potential with $(\lambda=2 \text{ fm}^{-1})$).



Largest splitting between P waves obtained with NN+NNN. Need ^4He excited states and study with respect to SRG λ

Solar p - p chain



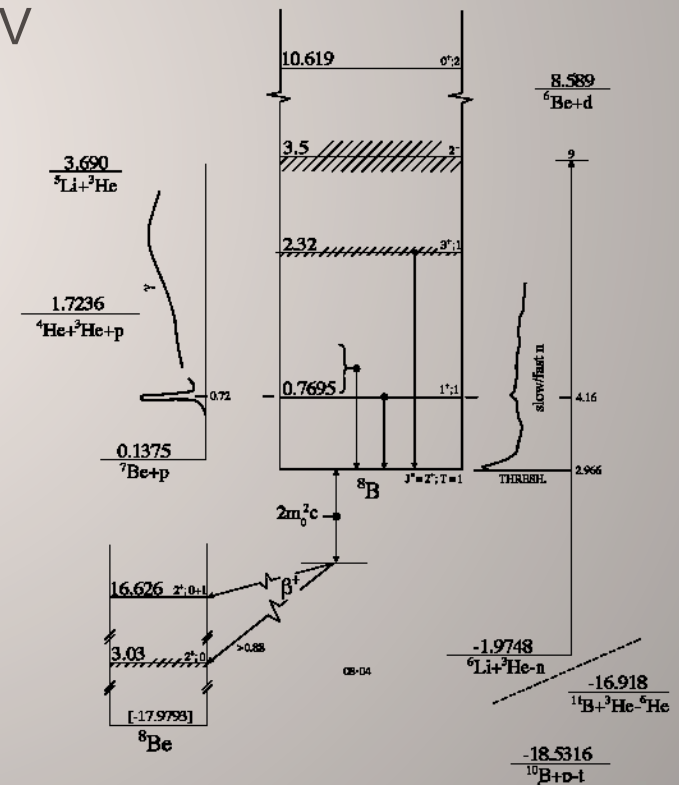
${}^7\text{Be}(p,\gamma){}^8\text{B}$ S-factor

- S_{17} one of the main inputs for understanding the solar neutrino flux
 - Needs to be known with high precision
- Current evaluation has uncertainty $\sim 10\%$
 - Theory needed for extrapolation to ~ 10 keV

$$S(E) = E\sigma(E) \exp[2\pi\eta(E)]$$

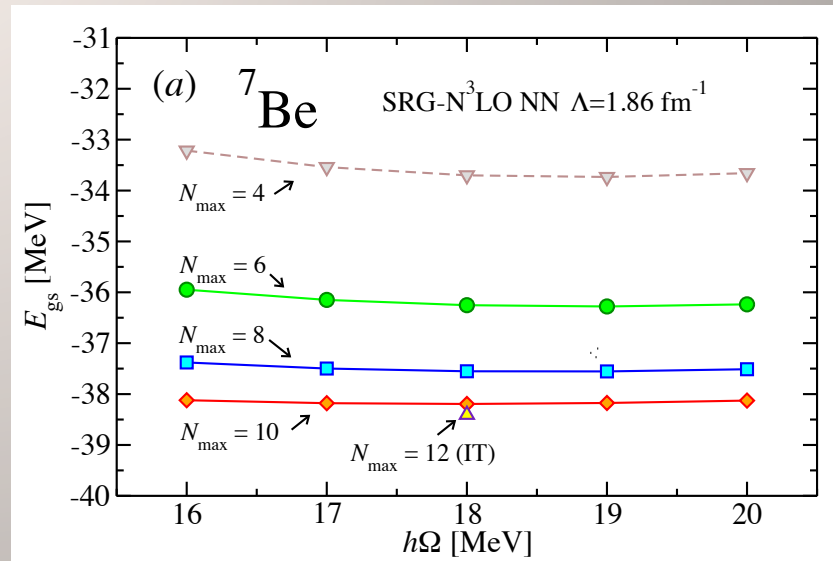
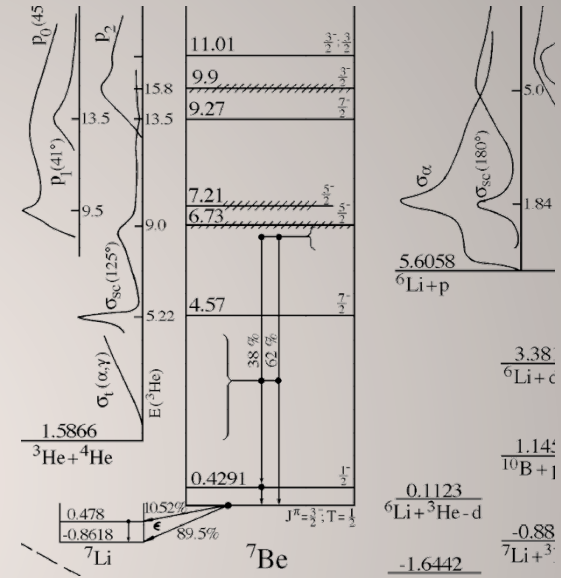
$$\eta(E) = Z_{A-a}Z_a e^2 / \hbar v_{A-a,a}$$

$$\langle {}^8\text{B}_{\text{g.s.}} | E1 | {}^7\text{Be}_{\text{g.s.}} + p \rangle$$



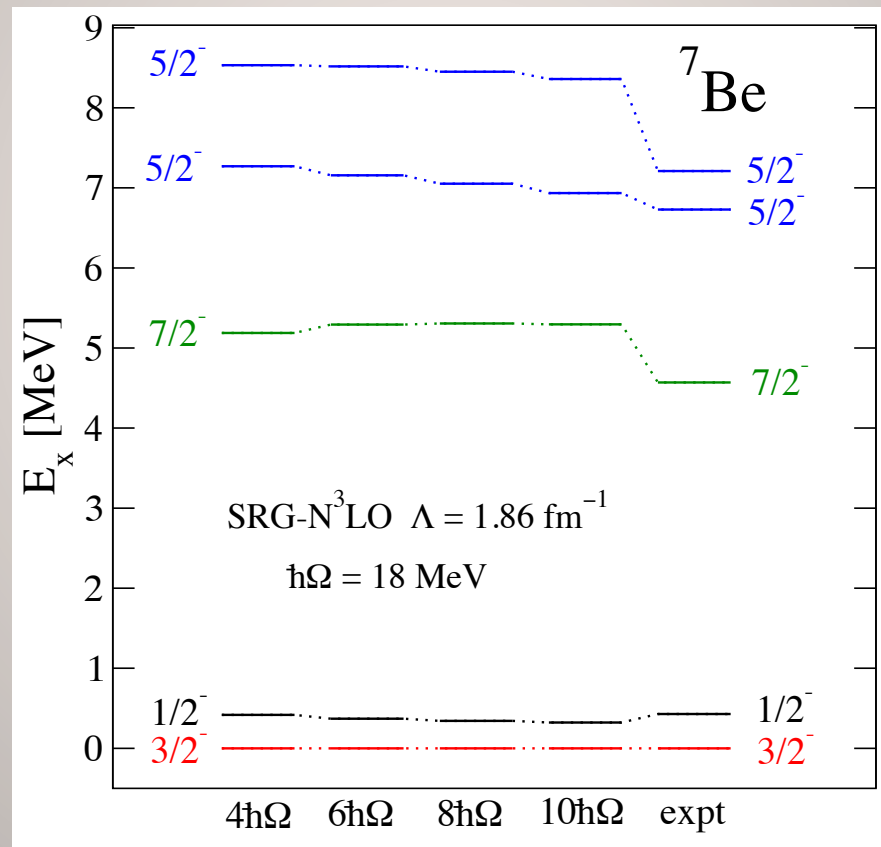
${}^7\text{Be}(p,\gamma){}^8\text{B}$ radiative capture: Input - NN interaction, ${}^7\text{Be}$ eigenstates

- Similarity-Renormalization-Group (SRG) evolved chiral $N^3\text{LO}$ NN interaction
 - Accurate
 - Soft: Evolution parameter Λ
 - Study dependence on Λ
- ${}^7\text{Be}$
 - NCSM up to $N_{\text{max}}=10$, Importance Truncated NCSM up to $N_{\text{max}}=14$
 - Variational calculation
 - optimal HO frequency from the ground-state minimum
 - For the selected NN potential with $\Lambda=1.86\text{ fm}^{-1}$: $\hbar\Omega=18\text{ MeV}$



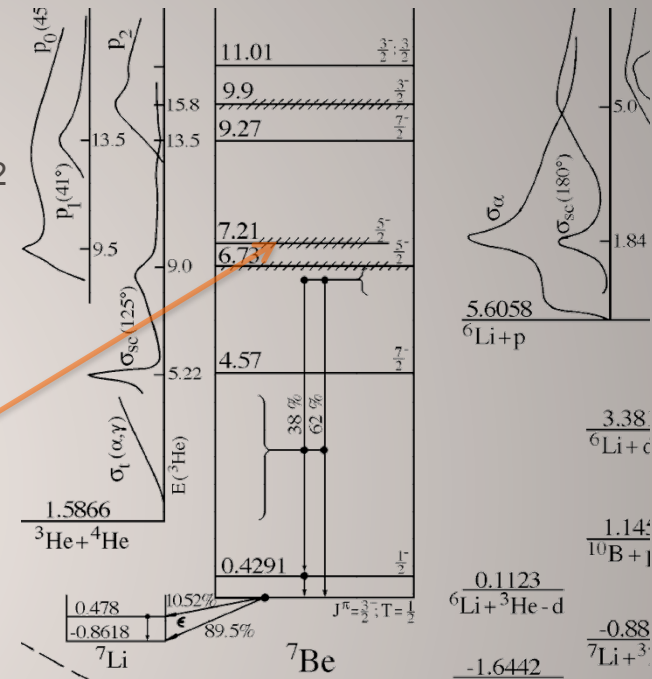
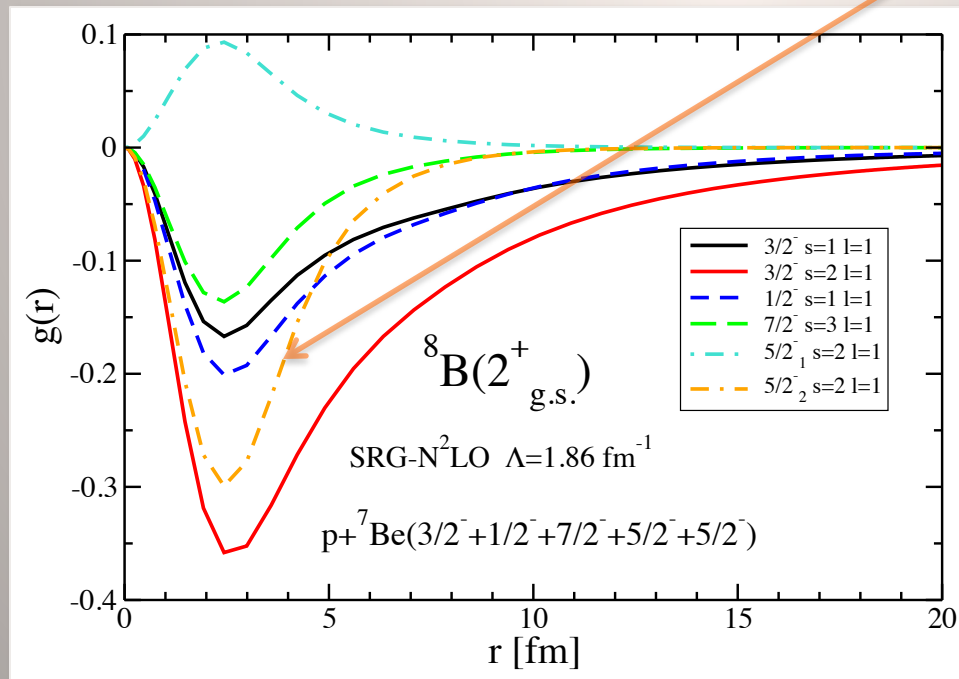
Input: ${}^7\text{Be}$ eigenstates

- Excited states at the optimal HO frequency, $\hbar\Omega = 18 \text{ MeV}$

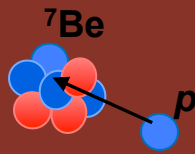


Structure of the ${}^8\text{B}$ ground state

- NCSM/RGM p - ${}^7\text{Be}$ calculation
 - five lowest ${}^7\text{Be}$ states: $3/2^-$, $1/2^-$, $7/2^-$, $5/2^-_1$, $5/2^-_2$
 - Soft NN SRG- N^3LO with $\Lambda = 1.86 \text{ fm}^{-1}$
- ${}^8\text{B}$ 2^+ g.s. bound by 136 keV (Expt 137 keV)
 - Large P -wave $5/2^-_2$ component



5/2₂⁻ state of ${}^7\text{Be}$ should be included in ${}^7\text{Be}(p, \gamma){}^8\text{B}$ calculations



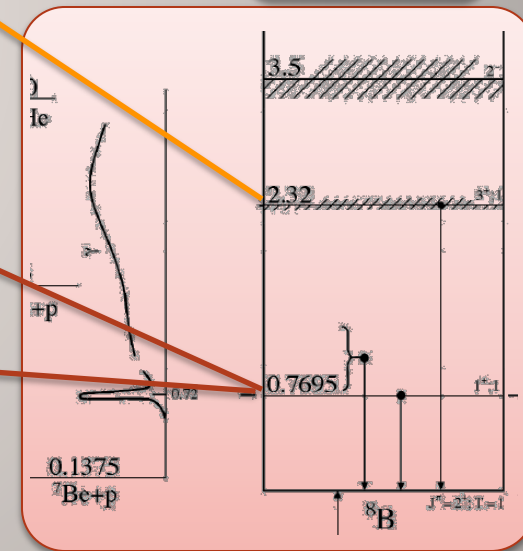
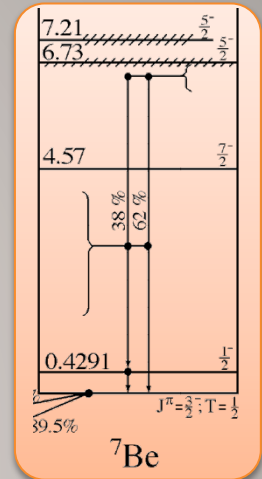
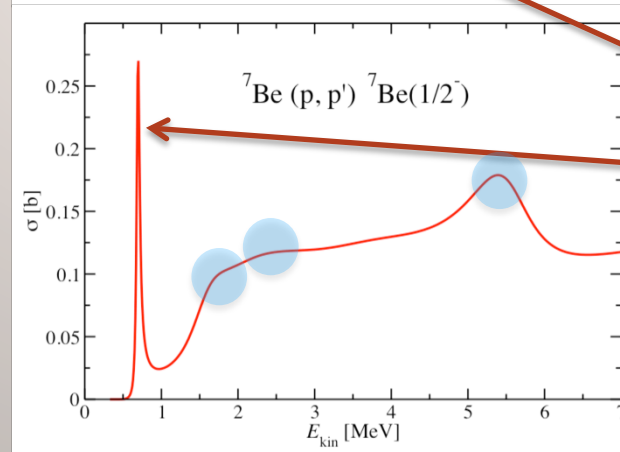
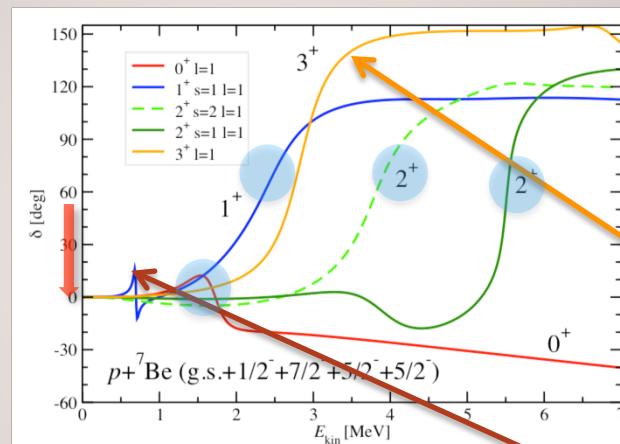
p - ${}^7\text{Be}$ scattering

- NCSM/RGM calculation of p - ${}^7\text{Be}$ scattering
 - ${}^7\text{Be}$ states $3/2^-$, $1/2^-$, $7/2^-$, $5/2^-_1$, $5/2^-_2$
 - Soft NN potential (SRG- N^3LO with $\Lambda = 1.86 \text{ fm}^{-1}$)

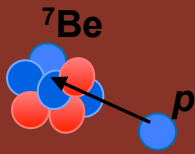
${}^8\text{B}$ 2^+ g.s. bound by 136 keV
(expt. bound by 137 keV)

New 0^+ , 1^+ , and two 2^+ resonances predicted

$s=1$ $l=1$ 2^+ clearly visible in (p,p') cross sections

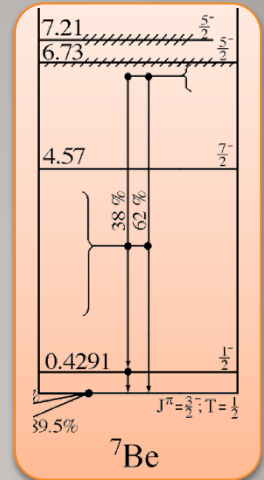


PRC **82**, 034609 (2010)

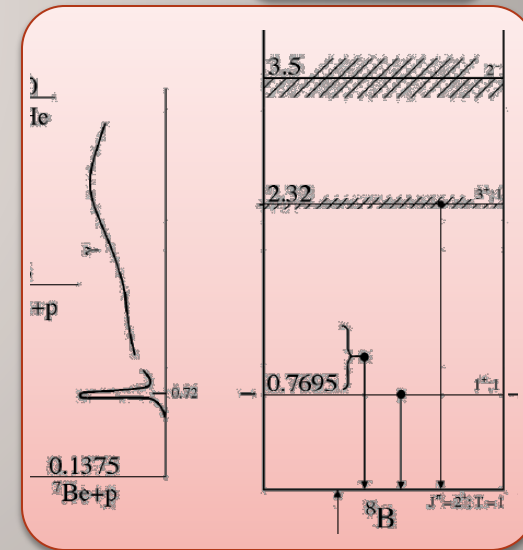
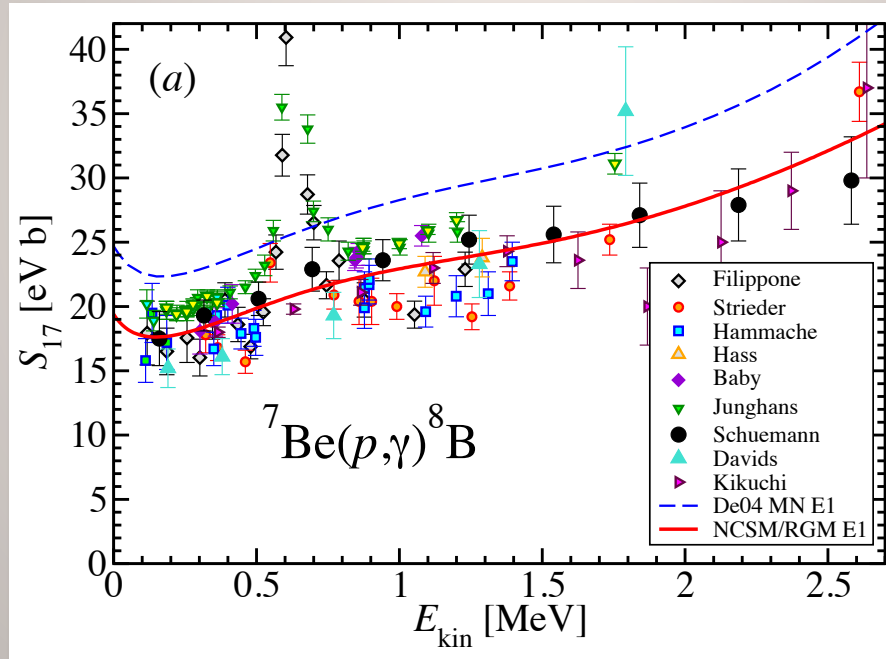


${}^7\text{Be}(p,\gamma){}^8\text{B}$ radiative capture

- NCSM/RGM calculation of ${}^7\text{Be}(p,\gamma){}^8\text{B}$ radiative capture
 - ${}^7\text{Be}$ states $3/2^-$, $1/2^-$, $7/2^-$, $5/2^-_1$, $5/2^-_2$
 - Soft NN potential (SRG- N^3LO with $\Lambda = 1.86 \text{ fm}^{-1}$)

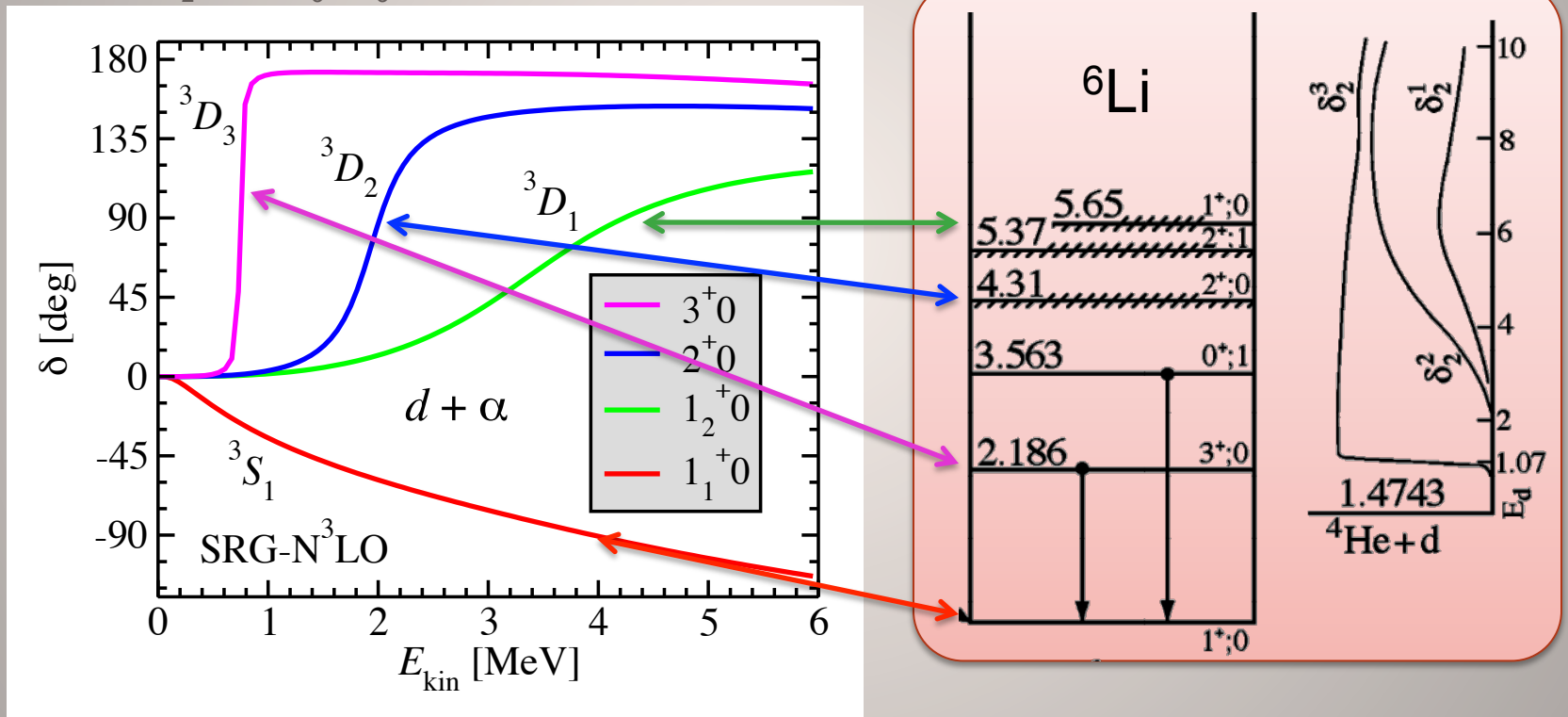


${}^8\text{B}$ 2^+ g.s. bound by 136 keV
 (expt. 137 keV)
 $S(0) \sim 19.4(0.7) \text{ eV b}$
 Data evaluation:
 $S(0) = 20.8(2.1) \text{ eV b}$



NCSM/RGM *ab initio* calculation of *d*-⁴He scattering

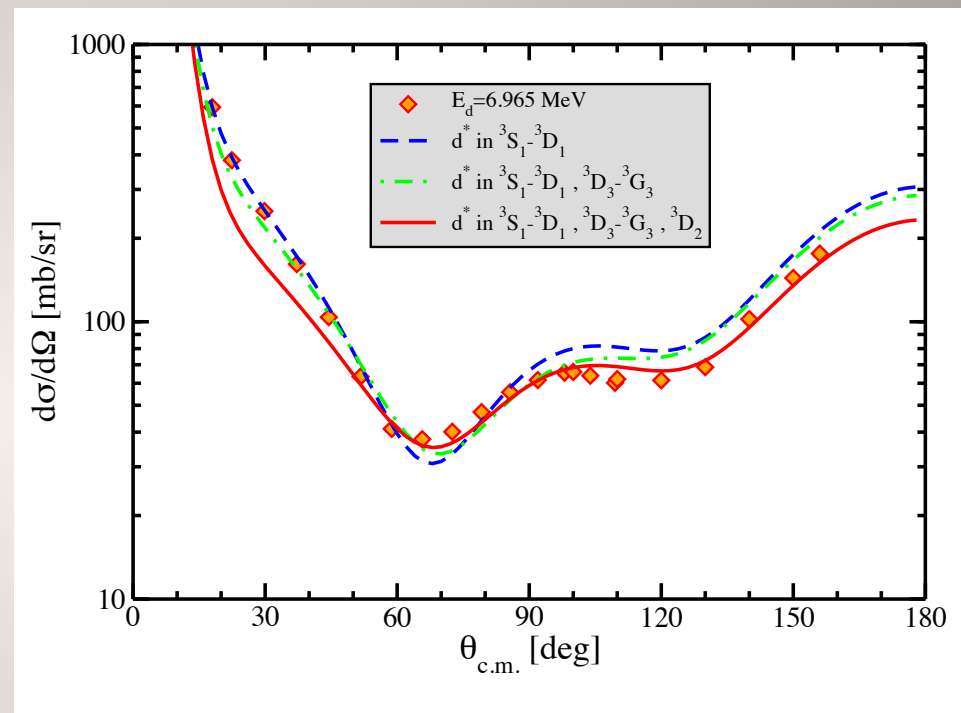
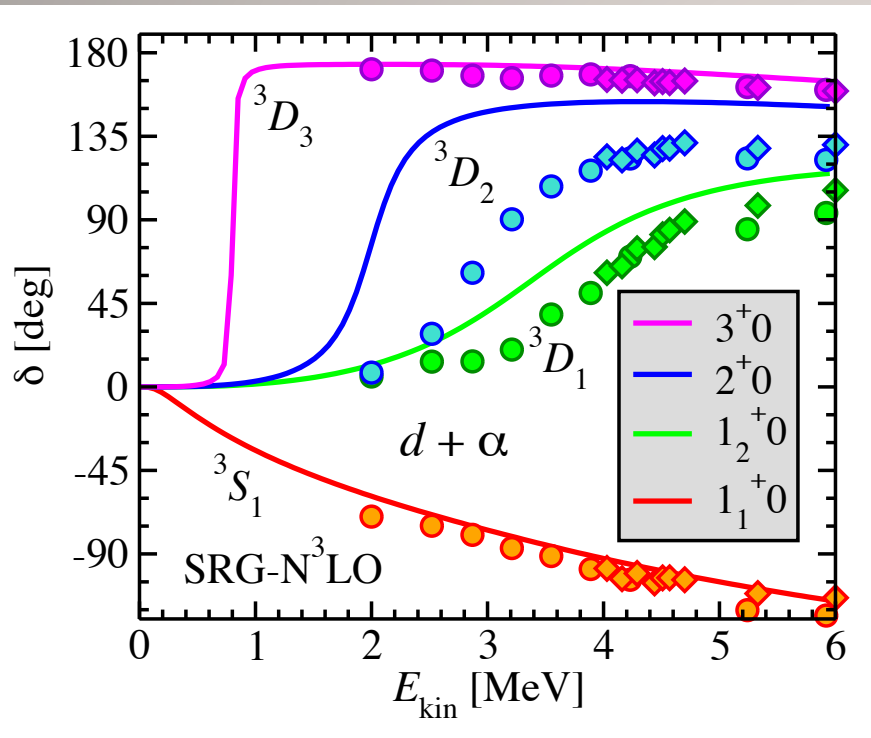
- NCSM/RGM calculation with *d* + ⁴He(g.s.) up to $N_{\max} = 12$
 - SRG-N³LO potential with $\Lambda = 1.5 \text{ fm}^{-1}$
 - Deuteron breakup effects included by continuum discretized by pseudo states in ³S₁-³D₁, ³D₂ and ³D₃-³G₃ channels



- The 1⁺0 ground state bound by 1.9 MeV (expt. 1.47 MeV)
- Calculated T=0 resonances: 3⁺, 2⁺ and 1⁺ in correct order close to expt. energies

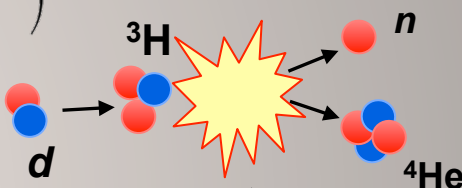
NCSM/RGM *ab initio* calculation of d - ^4He scattering

PHYS. REV. C **83**, 044609 (2011)



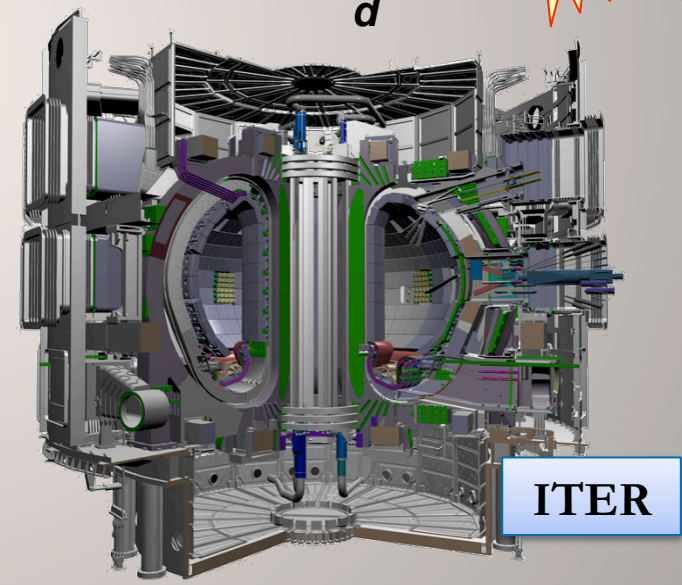
Scattering provides a strict test of NN and NNN forces
 Important to include 6-nucleon correlations
 – deuteron (virtual) breakup ...

Ab initio calculation of the ${}^3\text{H}(d,n){}^4\text{He}$ fusion

$$\int dr r^2 \begin{pmatrix} \left\langle \begin{matrix} \mathbf{r}' \\ n \end{matrix} \left| \hat{A}_1(H-E)\hat{A}_1 \right| \begin{matrix} \mathbf{r} \\ \alpha \end{matrix} \right\rangle & \left\langle \begin{matrix} \mathbf{r}' \\ n \end{matrix} \left| \hat{A}_1(H-E)\hat{A}_2 \right| \begin{matrix} \mathbf{r} \\ {}^3\text{H} \end{matrix} \right\rangle \\ \left\langle \begin{matrix} \mathbf{r}' \\ d \end{matrix} \left| \hat{A}_2(H-E)\hat{A}_1 \right| \begin{matrix} \mathbf{r} \\ \alpha \end{matrix} \right\rangle & \left\langle \begin{matrix} \mathbf{r}' \\ d \end{matrix} \left| \hat{A}_2(H-E)\hat{A}_2 \right| \begin{matrix} \mathbf{r} \\ {}^3\text{H} \end{matrix} \right\rangle \end{pmatrix} \begin{pmatrix} \frac{g_1(r)}{r} \\ \frac{g_2(r)}{r} \end{pmatrix} = 0$$




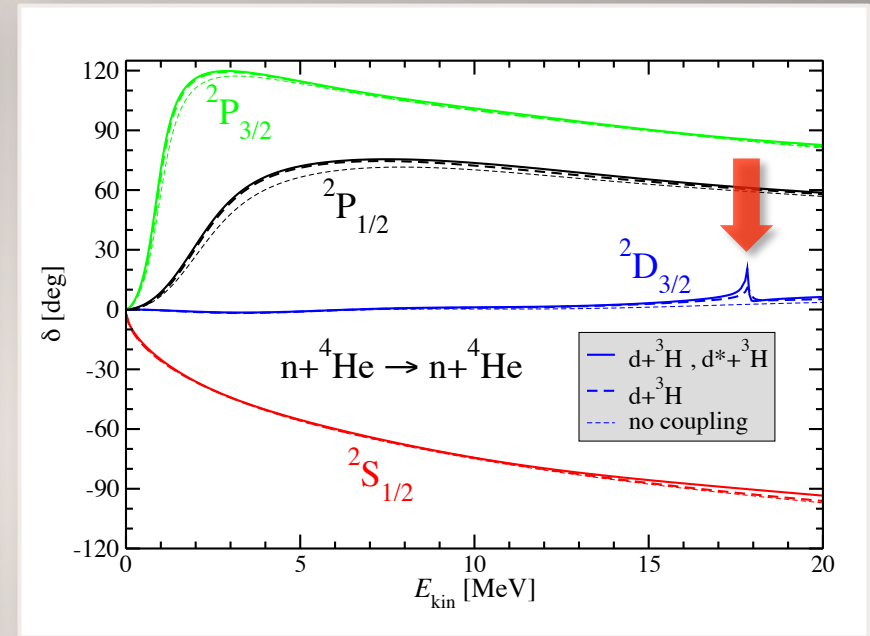
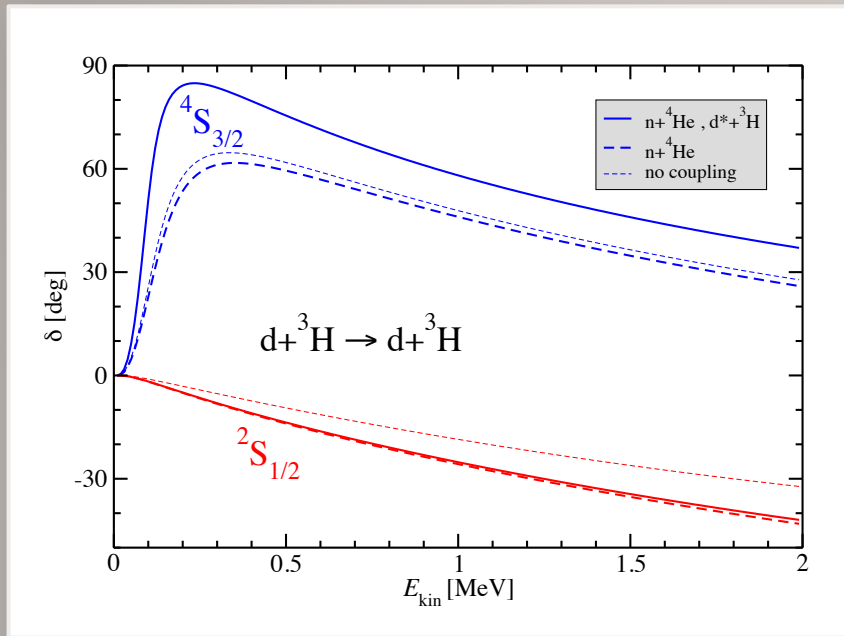
NIF



ITER

energy generation

$d+{}^3\text{H}$ and $n+{}^4\text{He}$ elastic scattering: phase shifts



- $d+{}^3\text{H}$ elastic phase shifts:

- Resonance in the ${}^4\text{S}_{3/2}$ channel
- Repulsive behavior in the ${}^2\text{S}_{1/2}$ channel → Pauli principle

d^* deuteron pseudo state in ${}^3\text{S}_1$ - ${}^3\text{D}_1$ channel:
deuteron polarization, virtual breakup

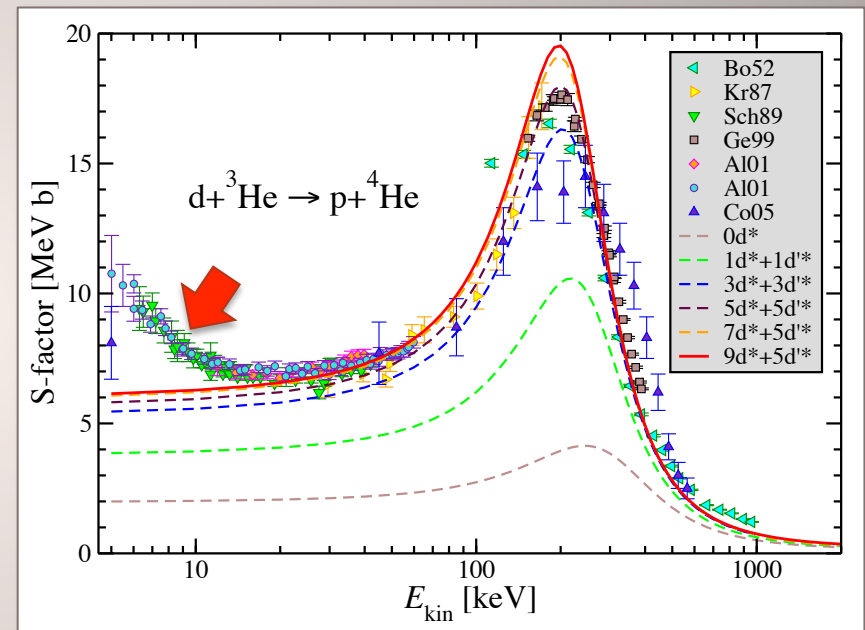
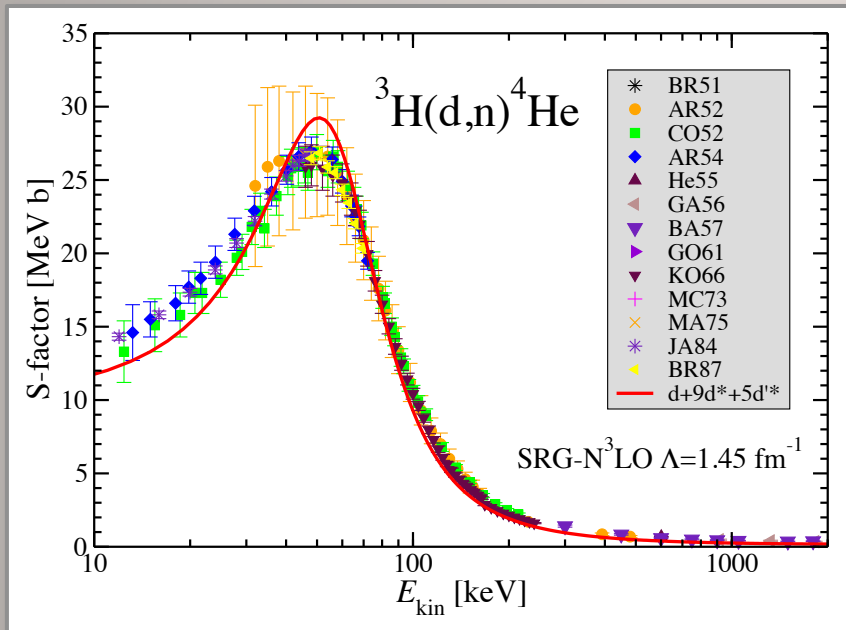
- $n+{}^4\text{He}$ elastic phase shifts:

- $d+{}^3\text{H}$ channels produces slight increase of the P phase shifts
- Appearance of resonance in the $3/2^+$ D -wave, just above $d-{}^3\text{H}$ threshold

The $d-{}^3\text{H}$ fusion takes place through a transition of $d+{}^3\text{H}$ is S -wave to $n+{}^4\text{He}$ in D -wave:
Importance of the **tensor force**

${}^3\text{H}(d,n){}^4\text{He}$ & ${}^3\text{He}(d,p){}^4\text{He}$ fusion

- NCSM/RGM with $SRG-N^3LO$ NN potentials



Potential to address unresolved fusion research related questions:

${}^3\text{H}(d,n){}^4\text{He}$ fusion with polarized deuterium and/or tritium,

${}^3\text{H}(d,n\gamma){}^4\text{He}$ bremsstrahlung,

Electron screening at very low energies ...

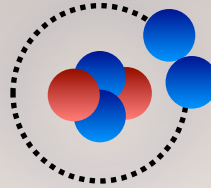
P.N., S. Quaglioni,
PRL **108**, 042503 (2012)

Borromean halo nuclei: He isotopes

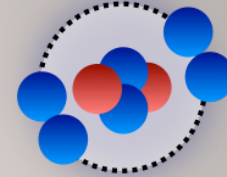
^4He



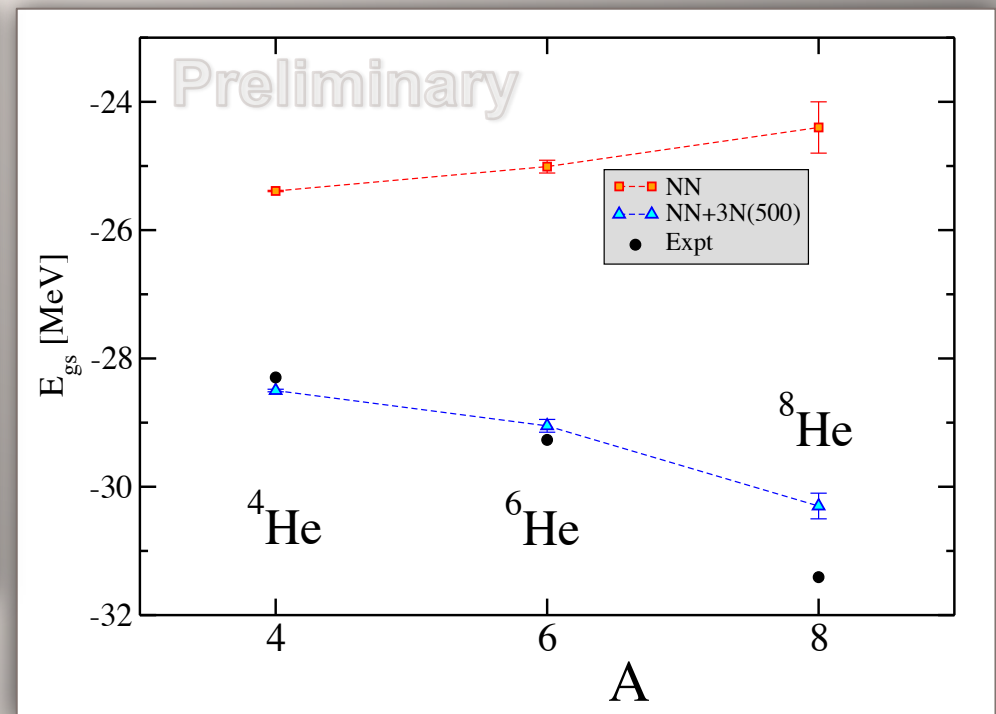
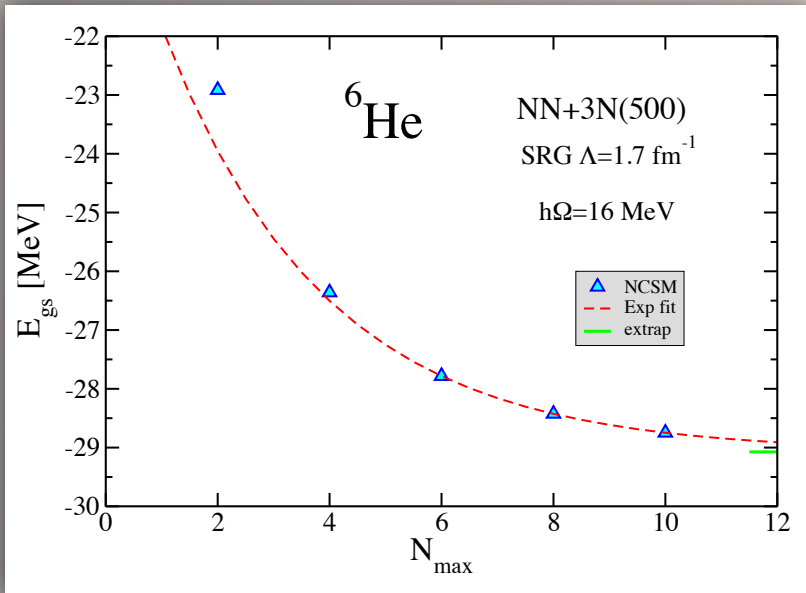
^6He



^8He



- ^6He and ^8He with chiral $N^3\text{LO NN} + N^2\text{LO 3N}$
 - chiral $N^3\text{LO NN}$: ^4He underbound, ^6He and ^8He unbound
 - chiral $N^3\text{LO NN} + N^2\text{LO 3N(500)}$: ^4He OK, both ^6He and ^8He bound



**NNN interaction important
to bind neutron rich nuclei**

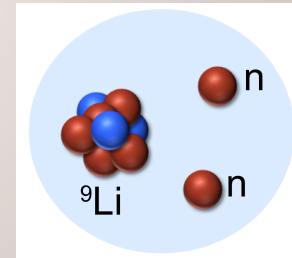
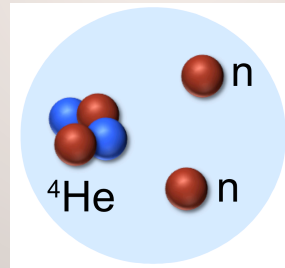
Three-body clusters in *ab initio* NCSM/RGM

- Starts from:

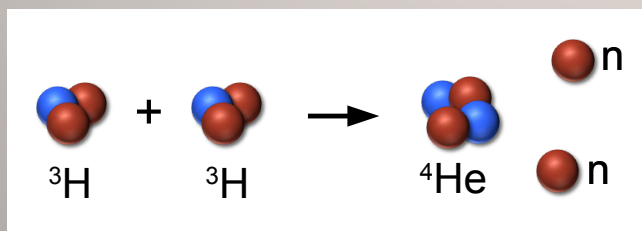
$$\Psi_{RGM}^{(A)} = \sum_{v_2} \int g_{v_2}(\vec{r}) \hat{A}_{v_2} \underbrace{|\phi_{v_2\vec{r}}\rangle}_{\text{2-body channels}} d\vec{r} + \sum_{v_3} \iint G_{v_3}(\vec{x}, \vec{y}) \hat{A}_{v_3} \underbrace{|\Phi_{v_3\vec{x}\vec{y}}\rangle}_{\text{3-body channels}} d\vec{x} d\vec{y}$$



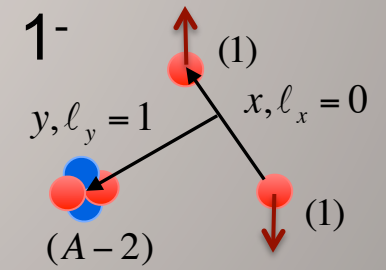
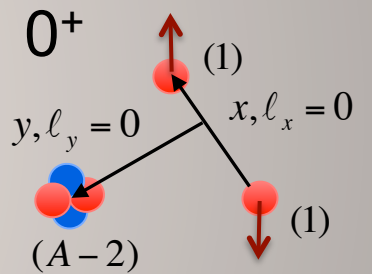
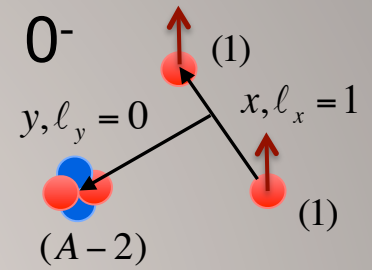
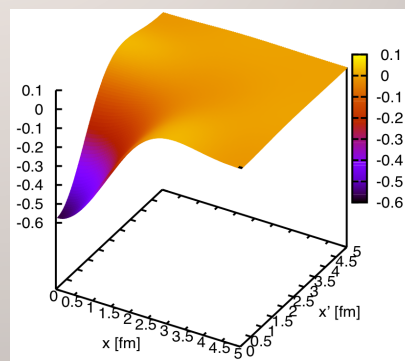
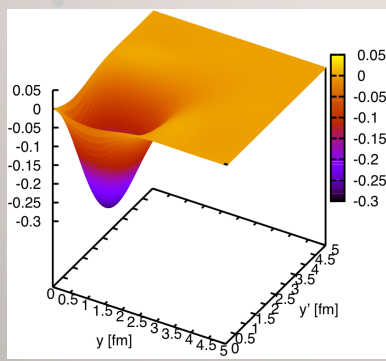
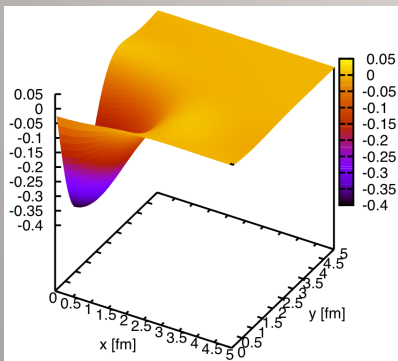
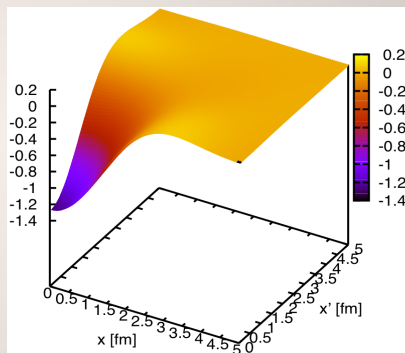
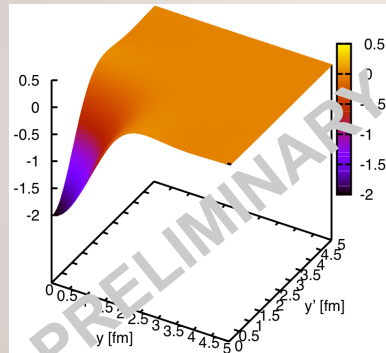
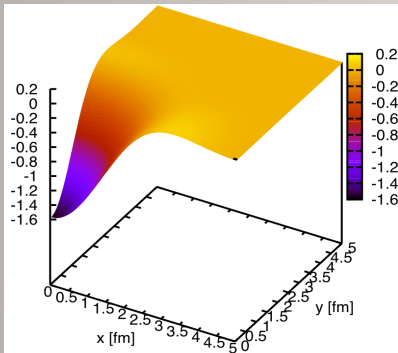
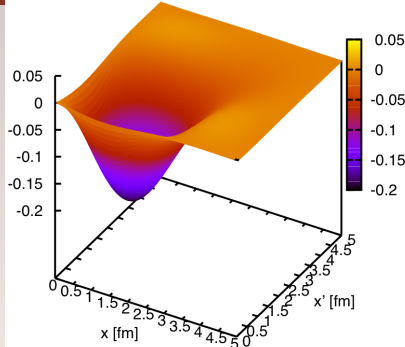
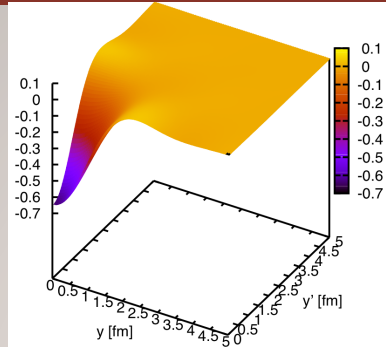
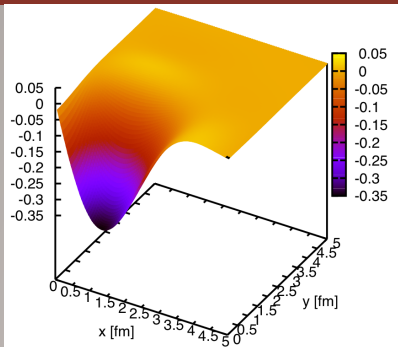
- Two-neutron halo nuclei



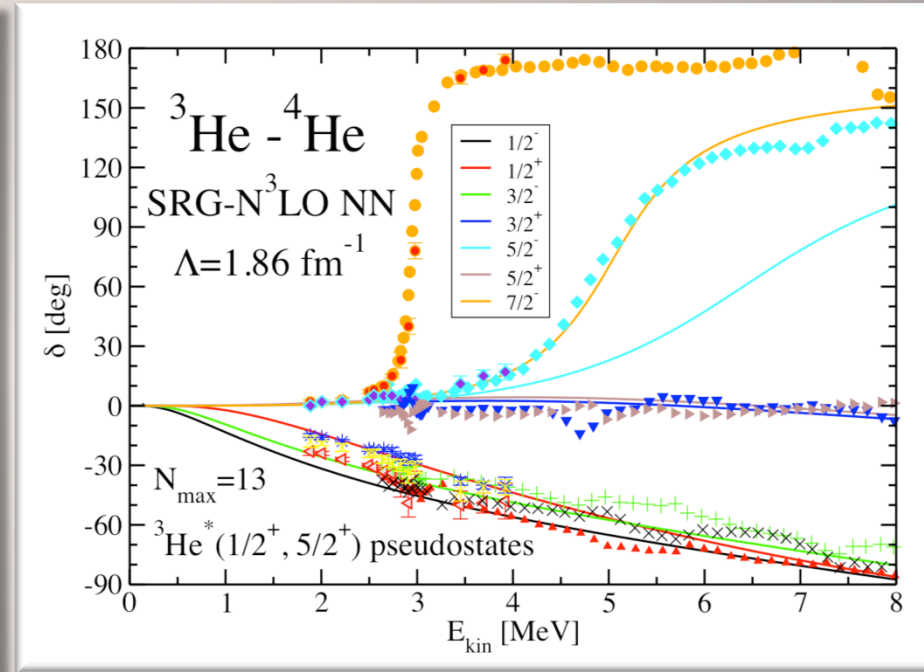
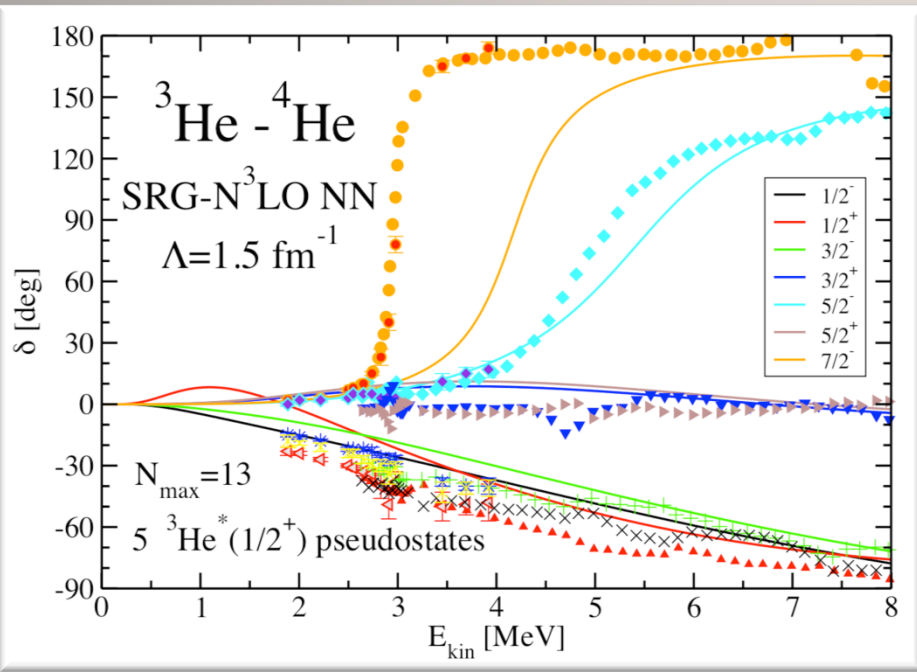
- Transfer reactions with three-body continuum final states



Norm kernel for $n+n+{}^4\text{He}$



Ab initio calculations of ${}^3\text{He}+\alpha$ scattering: First results (preliminary, incomplete)



Calculations for $a=3$ projectile under way:
 Soft SRG interactions ($\Lambda=1.5 \text{ fm}^{-1}$, $\Lambda=1.86 \text{ fm}^{-1}$)
 Virtual breakup of ${}^3\text{He}$ included by pseudostates (in $1/2^+$, $5/2^+$ channels so far)

New developments: NCSM with continuum

NCSM



$$|\Psi_A^{J^{\pi T}}\rangle = \sum_{Ni} c_{Ni} |ANiJ^{\pi T}\rangle$$

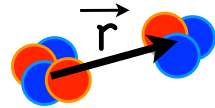
New developments: NCSM with continuum

NCSM



$$|\Psi_A^{J^\pi T}\rangle = \sum_{Ni} c_{Ni} |ANiJ^\pi T\rangle$$

NCSM/RGM



$$|\Psi_A^{J^\pi T}\rangle = \sum_{\nu} \int d\vec{r} \chi_{\nu}(\vec{r}) \hat{A} \Phi_{\nu\vec{r}}^{J^\pi T(A-a,a)}$$

$$\mathcal{H}\chi = E\mathcal{N}\chi$$

$$\bar{\chi} = \mathcal{N}^{+\frac{1}{2}} \chi$$

$$(\mathcal{N}^{-\frac{1}{2}} \mathcal{H} \mathcal{N}^{-\frac{1}{2}}) \bar{\chi} = E \bar{\chi}$$

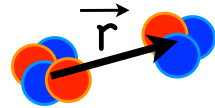
New developments: NCSM with continuum

NCSM



$$|\Psi_A^{J^\pi T}\rangle = \sum_{Ni} c_{Ni} |ANiJ^\pi T\rangle$$

NCSM/RGM



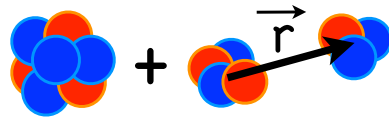
$$|\Psi_A^{J^\pi T}\rangle = \sum_{\nu} \int d\vec{r} \chi_{\nu}(\vec{r}) \hat{A} \Phi_{\nu\vec{r}}^{J^\pi T(A-a,a)}$$

$$\mathcal{H}\chi = E\mathcal{N}\chi$$

$$\bar{\chi} = \mathcal{N}^{+\frac{1}{2}} \chi$$

$$(\mathcal{N}^{-\frac{1}{2}} \mathcal{H} \mathcal{N}^{-\frac{1}{2}}) \bar{\chi} = E \bar{\chi}$$

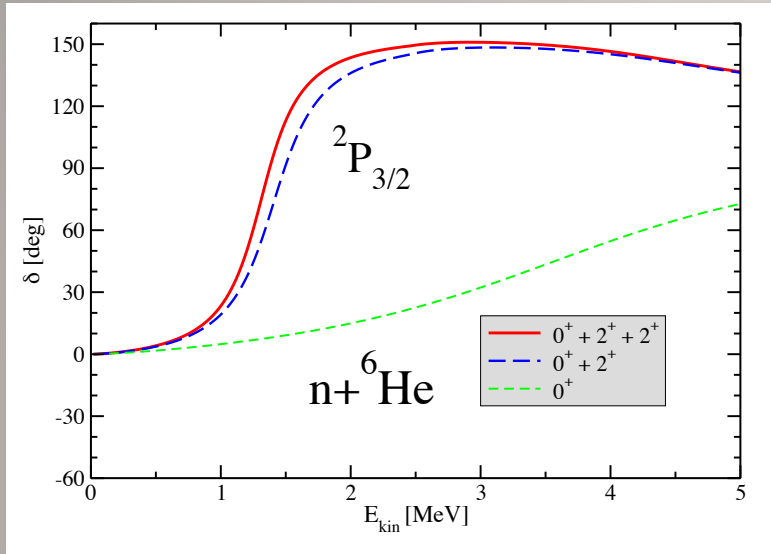
NCSMC



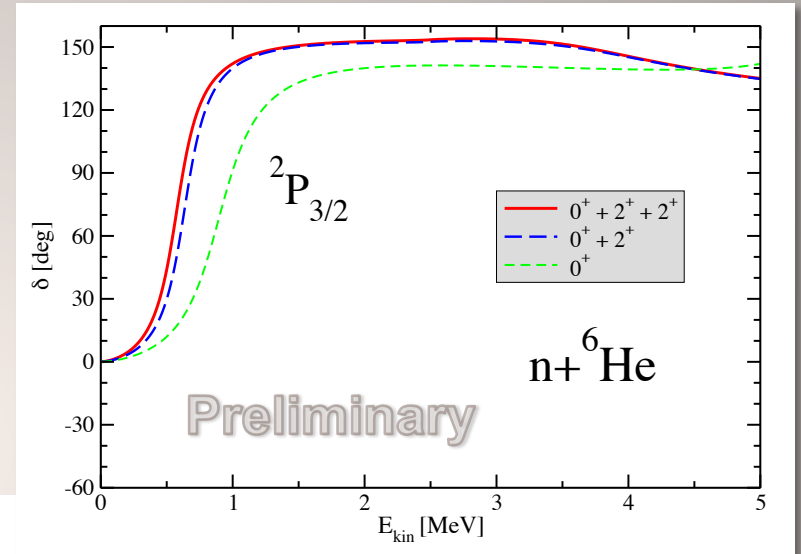
$$|\Psi_A^{J^\pi T}\rangle = \sum_{\lambda} c_{\lambda} |A\lambda J^\pi T\rangle + \sum_{\nu} \int d\vec{r} \left(\sum_{\nu'} \int d\vec{r}' \mathcal{N}_{\nu\nu'}^{-\frac{1}{2}}(\vec{r}, \vec{r}') \bar{\chi}_{\nu'}(\vec{r}') \right) \hat{A} \Phi_{\nu\vec{r}}^{J^\pi T(A-a,a)}$$

$$\begin{pmatrix} H_{NCSM} & \bar{h} \\ \bar{h} & \mathcal{N}^{-\frac{1}{2}} \mathcal{H} \mathcal{N}^{-\frac{1}{2}} \end{pmatrix} \begin{pmatrix} c \\ \bar{\chi} \end{pmatrix} = E \begin{pmatrix} 1 & \bar{g} \\ \bar{g} & 1 \end{pmatrix} \begin{pmatrix} c \\ \bar{\chi} \end{pmatrix}$$

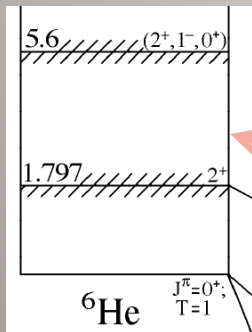
NCSM with continuum: ${}^7\text{He} \leftrightarrow {}^6\text{He}+n$



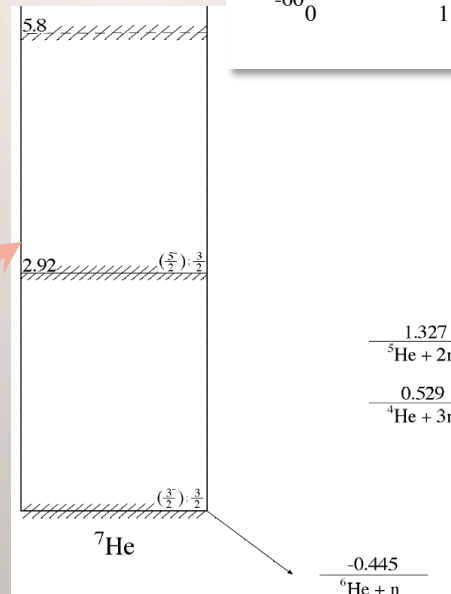
NCSM/RGM
with up to three ${}^6\text{He}$ states



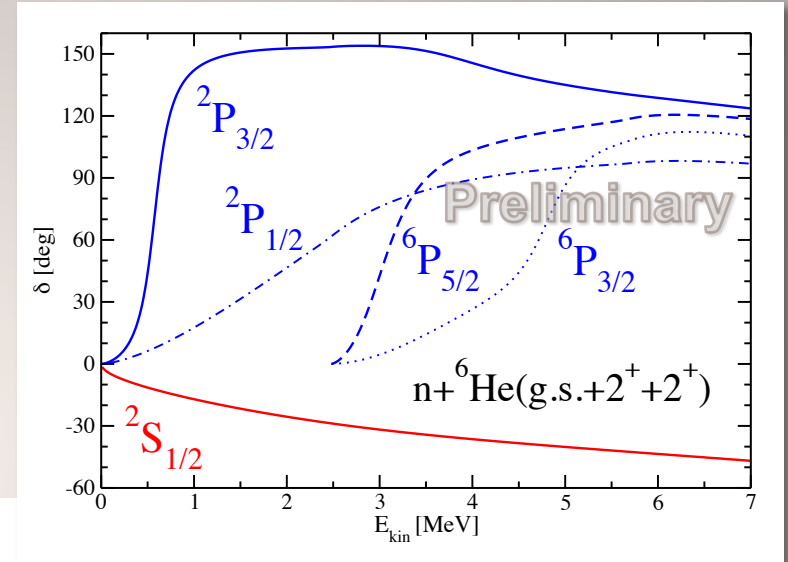
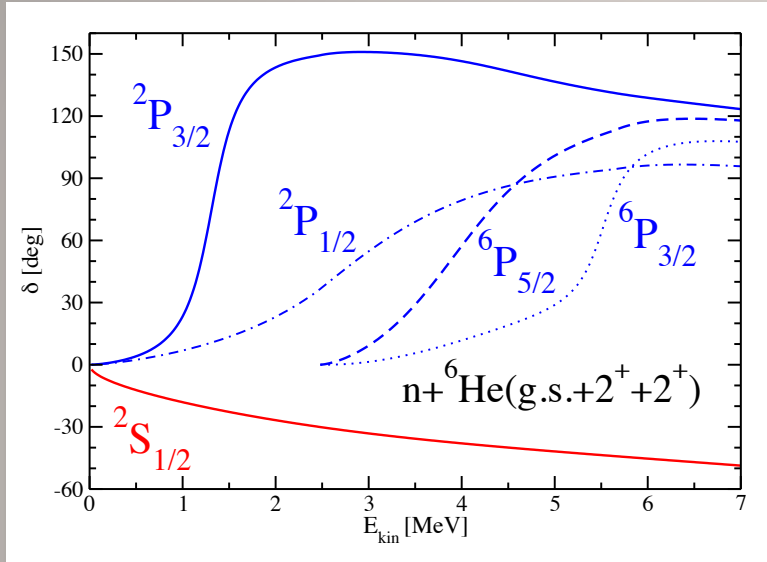
NCSMC
with up to three ${}^6\text{He}$ states
and three ${}^7\text{He}$ eigenstates
More 7-nucleon correlations
Fewer target states needed



Expt.

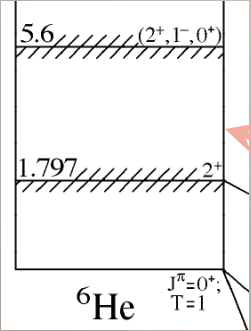


NCSM with continuum: ${}^7\text{He} \leftrightarrow {}^6\text{He}+n$

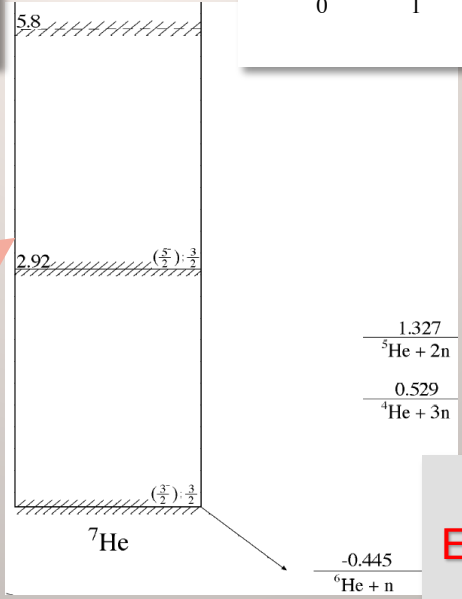


NCSM/RGM
with three ${}^6\text{He}$ states

NCSMC
with three ${}^6\text{He}$ states
and three ${}^7\text{He}$ eigenstates
More 7-nucleon correlations
Fewer target states needed



Expt.



Experimental controversy:
Existence of low-lying $1/2^-$ state
... not seen in this calculations

Conclusions and Outlook

- With the NCSM/RGM approach we are extending the *ab initio* effort to describe low-energy reactions and weakly-bound systems
- The first ${}^7\text{Be}(p,\gamma){}^8\text{B}$ *ab initio* S-factor calculation PLB 704 (2011) 379
- Deuteron-projectile results with SRG- N^3LO *NN* potentials:
 - d - ${}^4\text{He}$ scattering PRC 83, 044609 (2011)
 - First *ab initio* study of ${}^3\text{H}(d,n){}^4\text{He}$ & ${}^3\text{He}(d,p){}^4\text{He}$ fusion PRL 108, 042503 (2012)
- Under way:
 - n - ${}^8\text{He}$ scattering and ${}^9\text{He}$ structure
 - ${}^3\text{He}$ - ${}^4\text{He}$ and ${}^3\text{He}$ - ${}^3\text{He}$ scattering calculations
 - *Ab initio* NCSM with continuum (NCSMC)
 - Three-cluster NCSM/RGM and treatment of three-body continuum
 - Inclusion of **NNN** force
- To do:
 - Alpha clustering: ${}^4\text{He}$ projectile