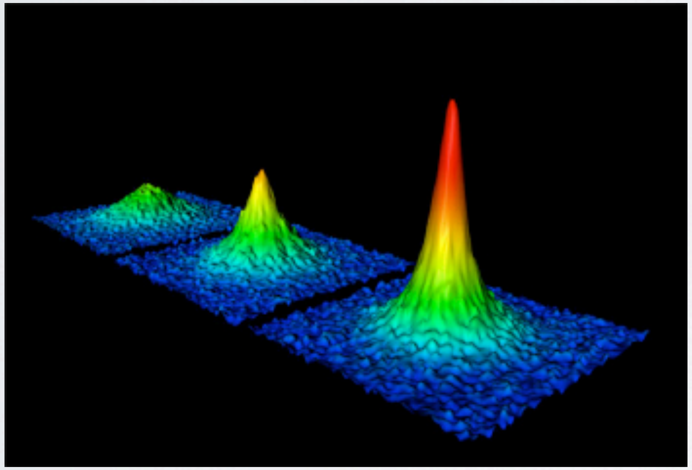
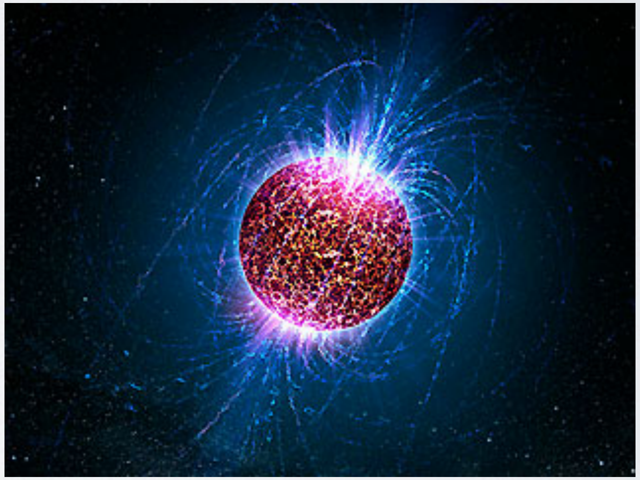
# Comparison and Contrast:
# Cold Atoms and Dilute Neutron Matter

J. Carlson - LANL

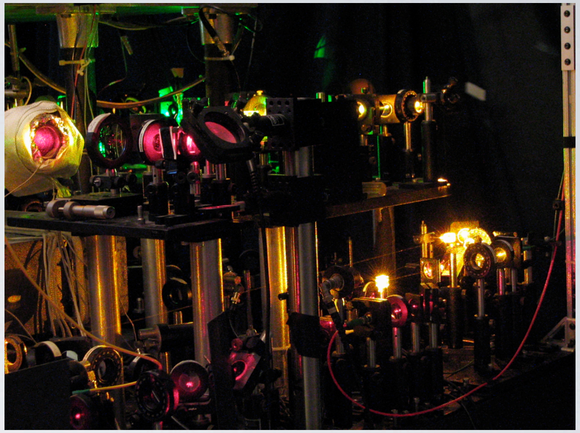
Fermi Condensates



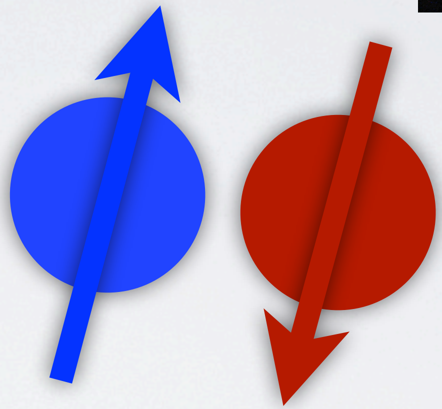
C. Regal et al. PRL 2004

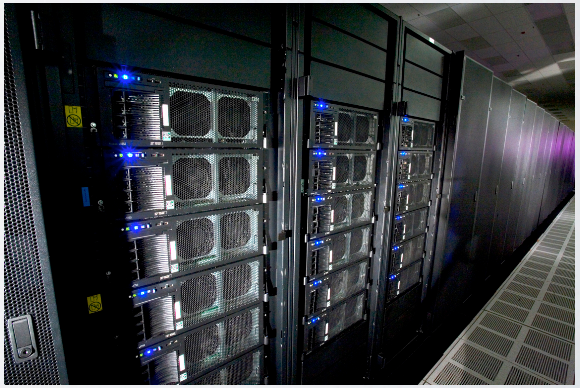


- Introduction
- Homogeneous Matter:
    - Equation of State (T=0)
    - Contact
    - Gap
- Linear Response:
    - RF Response
    - Spin Response
    - Density Response
- Inhomogeneous (trapped) Matter
    - Local Density approximation and Density functional Theory
- Future Outlook



MIT Optical Trap





roadrunner

# Interactions:

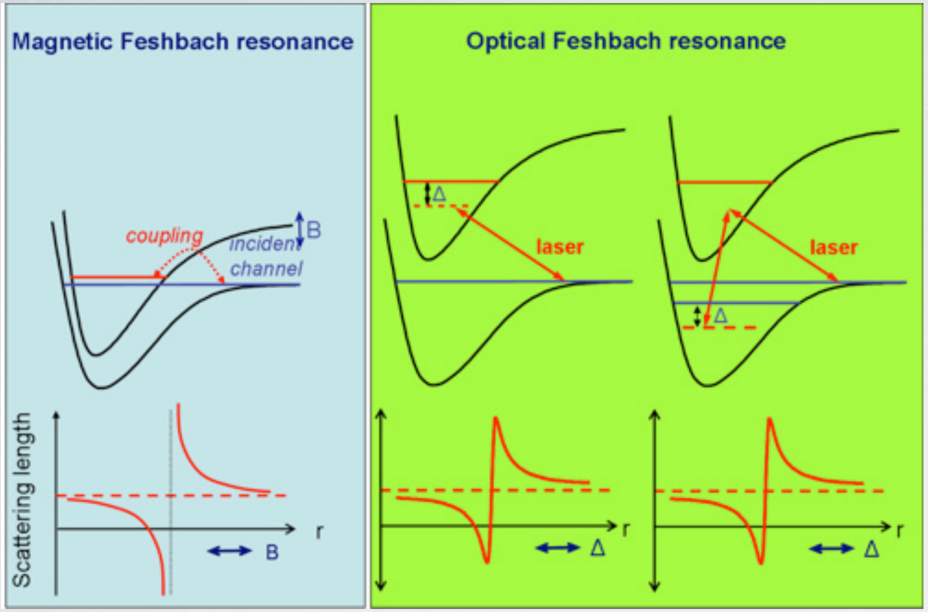


Diagram from Innsbruck

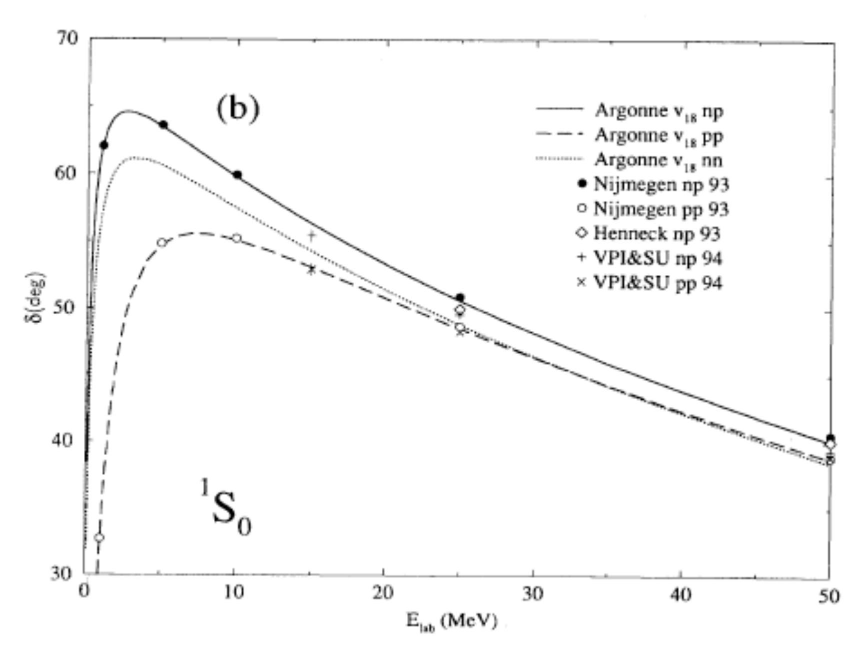
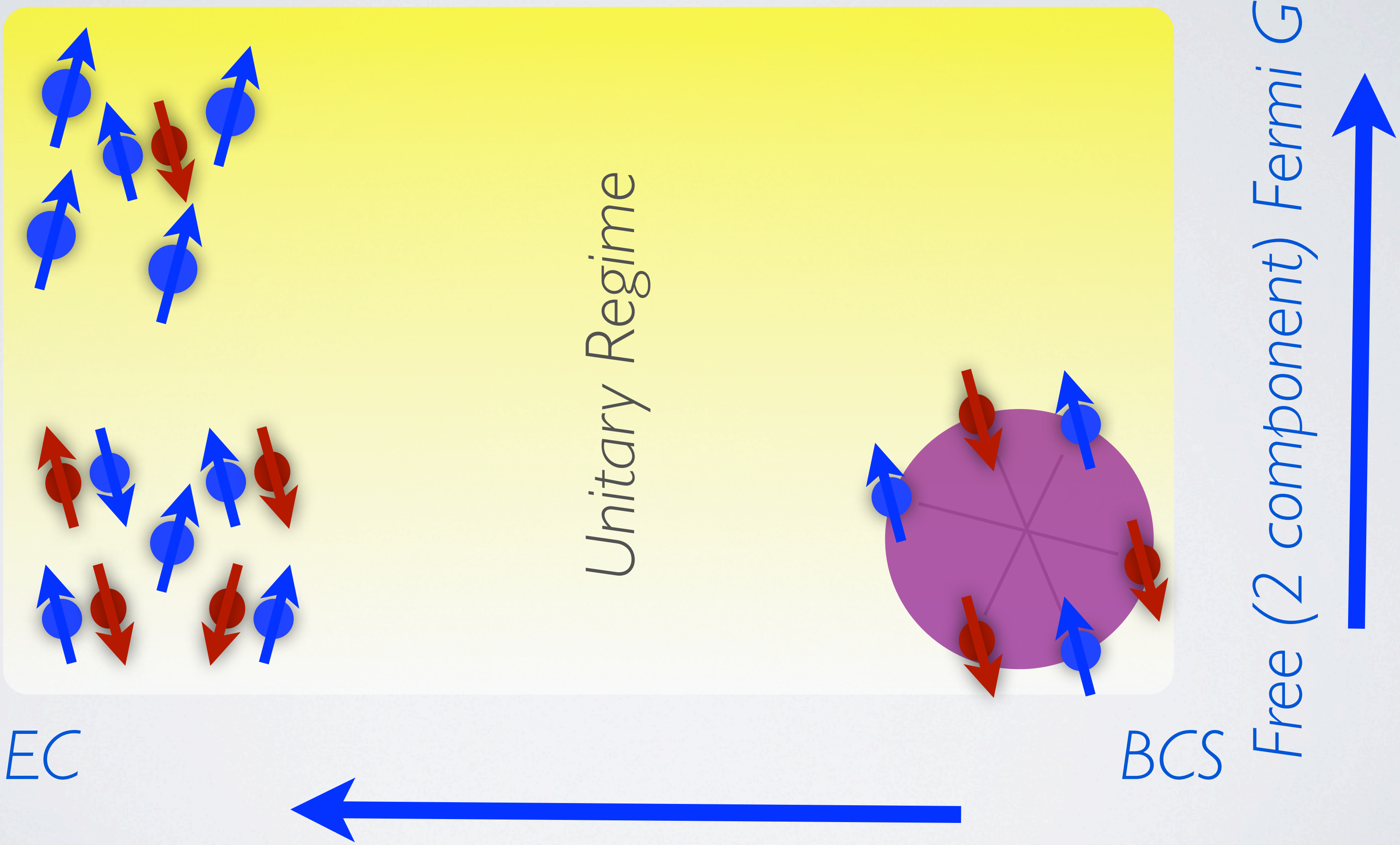


FIG. 2. Phase shifts in the $^1S_0$ channel for *np*, *nn*, and *pp* scattering, compared to various partial-wave phase-shift analyses.

Fermions: $^6$Li, $^{40}$K
Density ~ 1/ μm$^3$
Temperature ~ 200 nK ~ 0.1 Ef

NN phase shifts ($^1S_0$)
Density ~ 1 / (10 fm)$^3$
T ~ 0

# BCS-BEC Transition (T=0):



*Free (1 component) Fermi Gas*

*Bose-Fermi Mixture*

*Unitary Regime*

*Free (2 component) Fermi Gas*

Polarization

*BEC*

*BCS*

Interaction Strength

# *Cold Fermi Atoms: Physics Interests*

- (nearly) Free Fermions
- (nearly) Free Bosons
- ‘Universality’ and the BCS-BEC transition
- Polarons
- Efimov States
- Superfluid Fermions (s-, p-, d-wave,... pairing)
- Exotic Polarized Superfluids (FFLO, breached pair,...)
- PseudoGap States
- Itinerant Ferromagnetism
- `Perfect’ Fluids
- Reduced Dimensionality
- More than pairing (3-,4-body condensates, ...)
- Bose, Fermi Hubbard Models,
- .....

# *Unitarity*

Unitarity = limit of 0 pair binding

a  = ∞

All quantities multiples of Fermi Gas at same ρ
At zero polarization, expect strong pairing

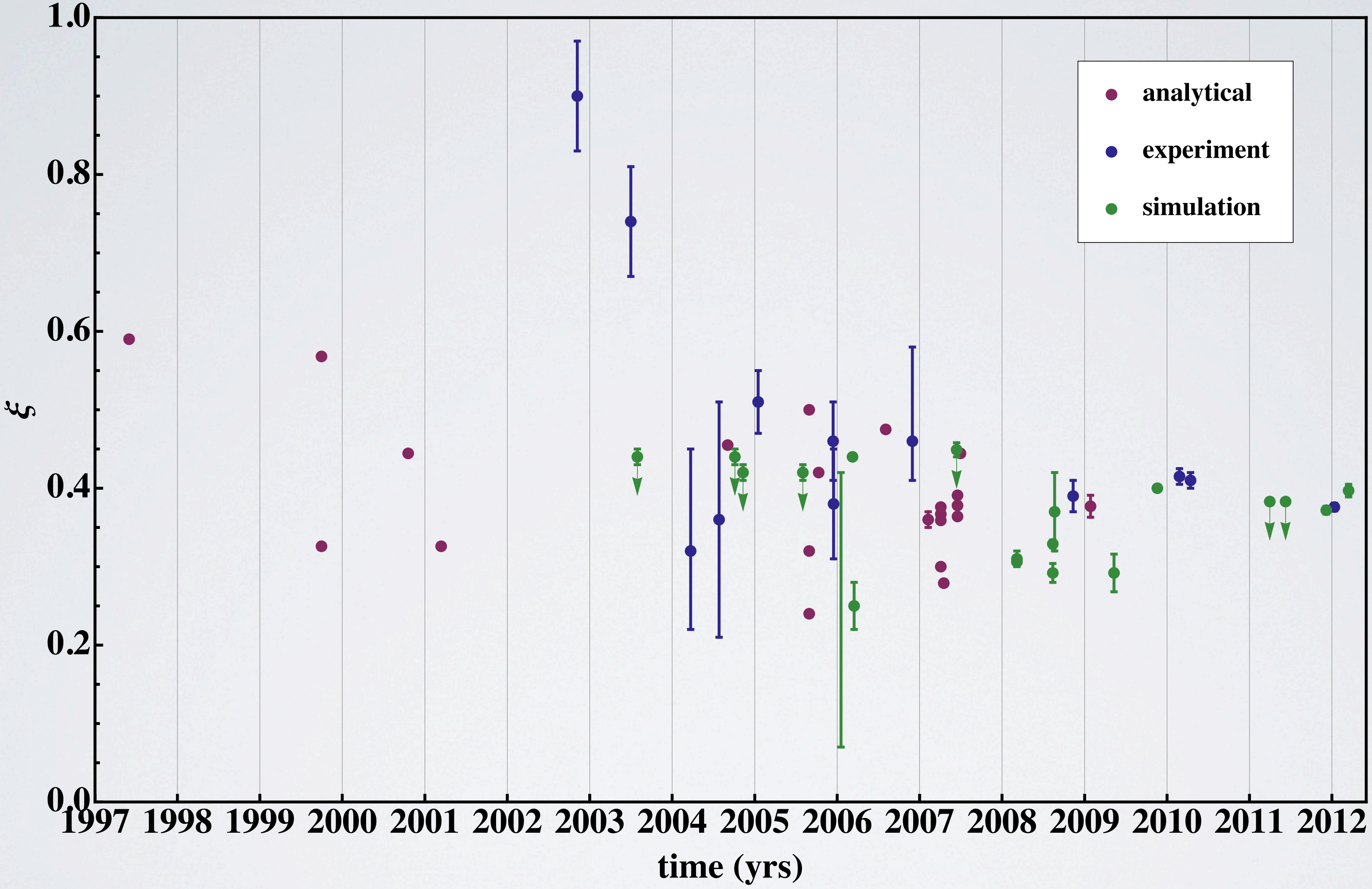
$$E = \xi \; E_{FG} = \xi \; \frac{3}{5} \; \frac{\hbar^2 k_F^2}{2m}$$

$$\Delta \; = \; \delta \; \frac{\hbar^2 k_F^2}{2m}$$

$$T_c \; = \; t \; \frac{\hbar^2 k_F^2}{2m}$$

Values of ξ, δ, t are independent of ρ

# ξ: Experiments, Analytic, and Computational



Endres, Kaplan, Lee and Nicholson, arXiv:1203.3169 (2012)

# Zero Temperature Simulations

Quantum Monte Carlo:

$$\begin{aligned}\Psi &= \exp[-H\tau]\ \Psi_0 \\ \Psi_0 &= \Psi_{BCS} = \prod_k \ [v_k/u_k]\ a_\uparrow^\dagger(k) a_\downarrow^\dagger(-k)\ |0\rangle\end{aligned}$$

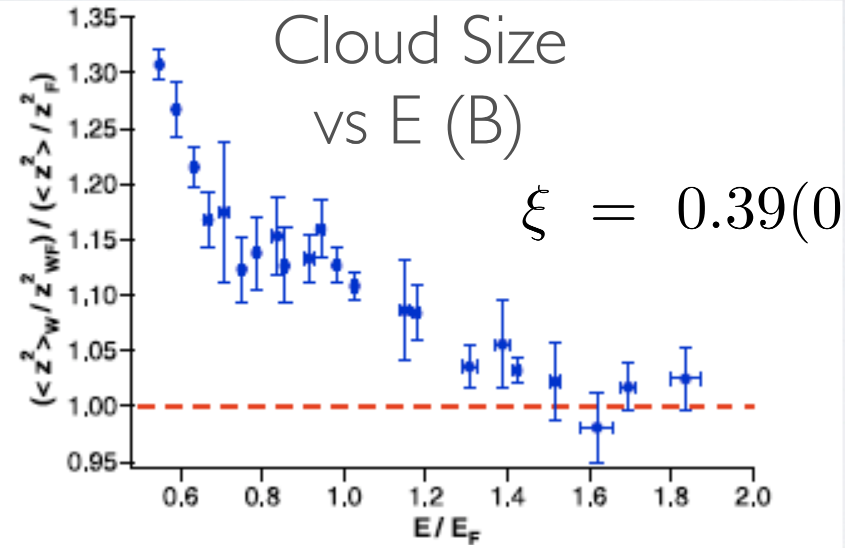
Branching Random Walk: DMC coordinate space
AFMC orbitals

Computational requirements from workstation (Total Energy)
to largest supercomputers
(exotic superfluids)

AFMC exact for unpolarized systems (finite lattices)
good for finite temperature

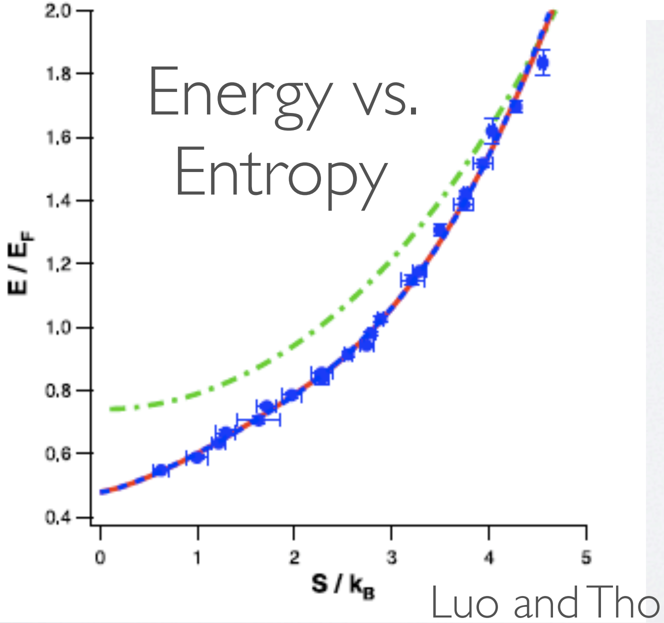
DMC in continuum, but fixed-node approximation
good for polarized systems

# *Experiments at Unitarity: # ↑ = # ↓*
# *Cloud Size and Sound Velocity*



Cloud Size
vs E (B)

$$\xi = 0.39(02)$$

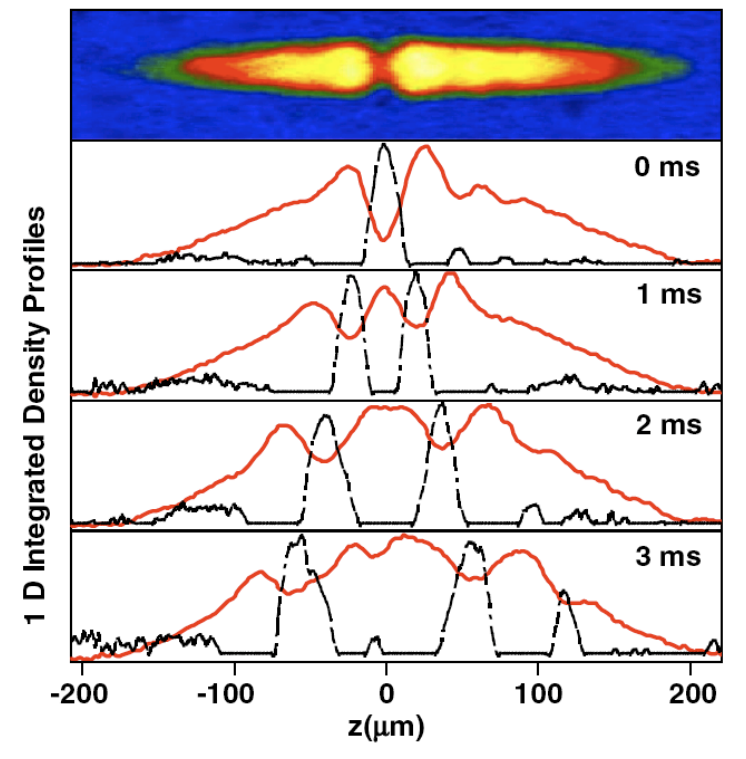


Energy vs.
Entropy

Luo and Thomas, JLTP, 2009

Sound Propagation

Joseph, et al., PRL 2007

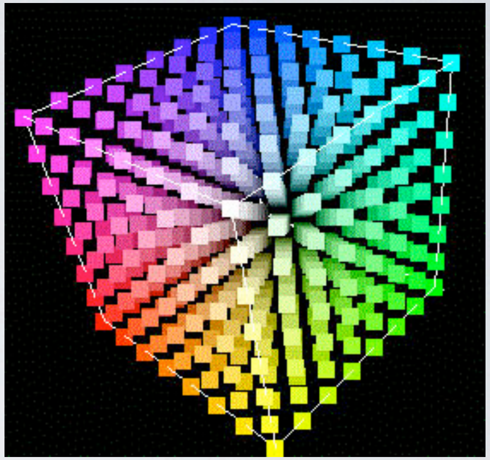


$$\frac{c_0}{v_f} = \frac{\xi^{1/4}}{\sqrt{(5)}}$$

scaling verified as ρ
varied by 30

# Improved Lattice (AFMC) Methods for Unitary Gas

BCS importance function
no sign problem
control of lattice size, N,
effective range



At finite (small) effective range:

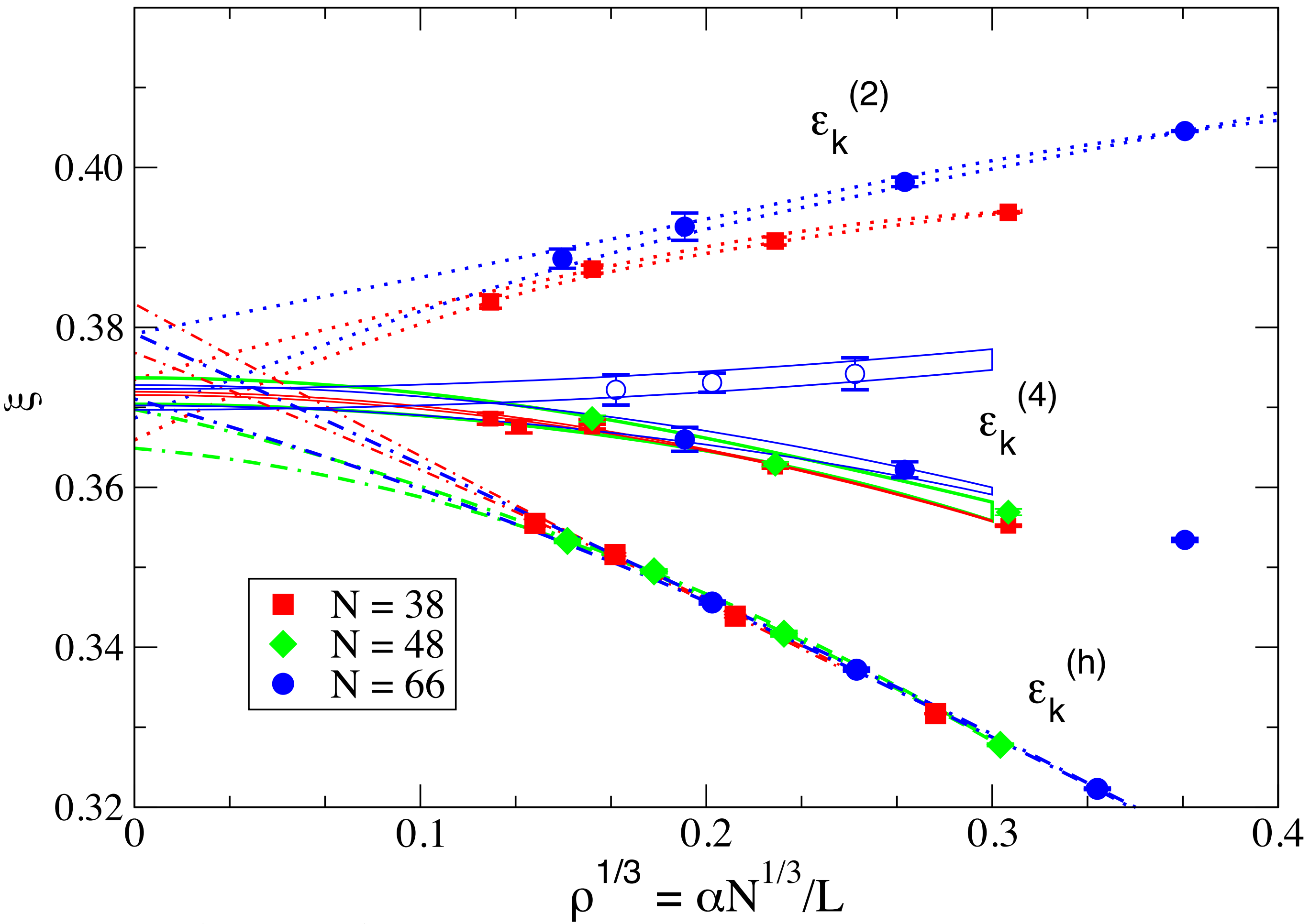
$$E \,/\, E_{FG} \; = \; \xi \; + \; \mathcal{S}\, k_F \, r_e$$

$\xi$ and $\mathcal{S}$ are universal parameters

Can measure neutron matter EOS (including effective range corrections) in cold atoms

# Unitary Fermi Gas (lattice)

$E/E_{FG}$



$$\xi = 0.372(0.005)$$

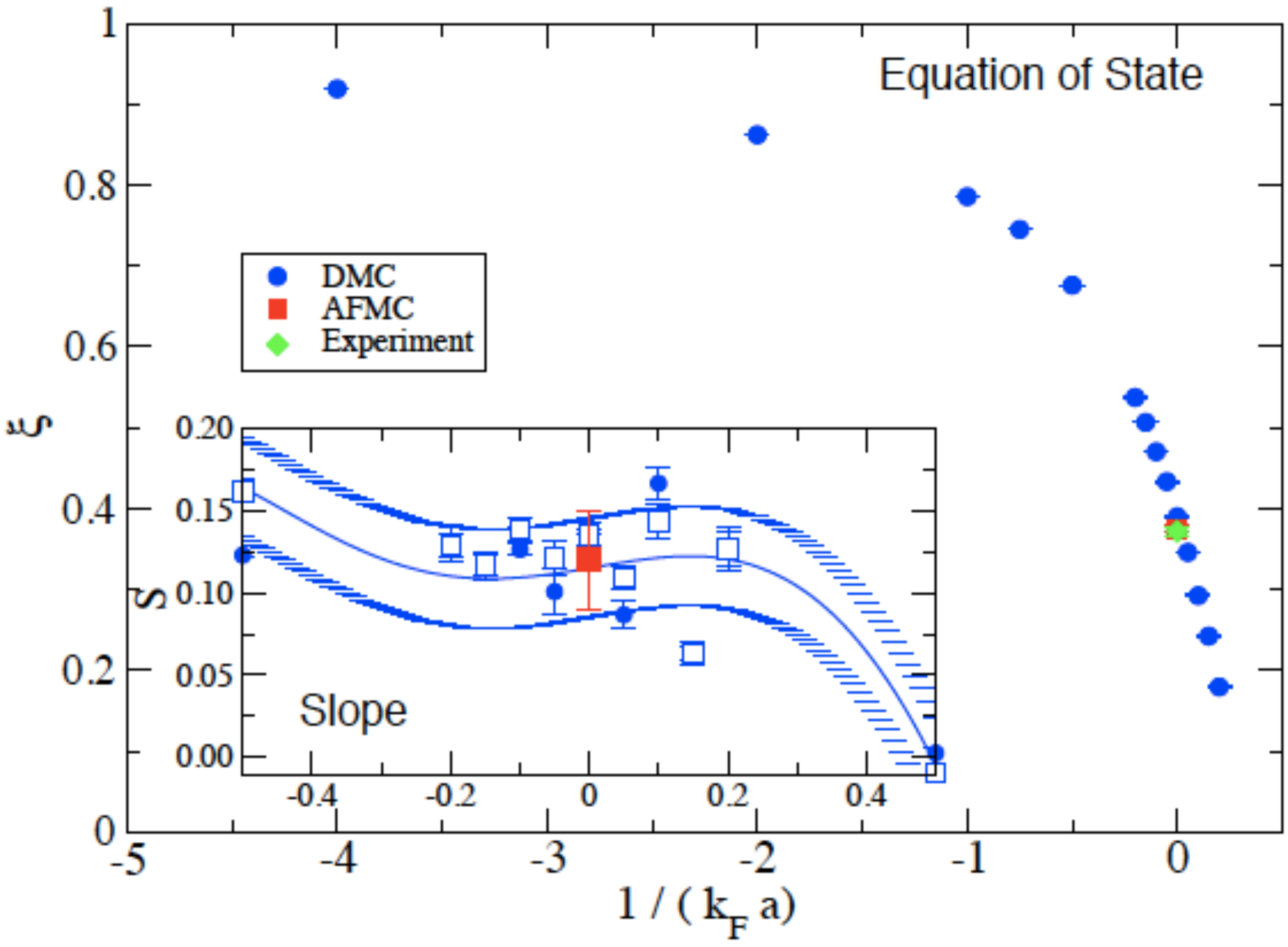
$$\xi = 0.376(0.005)$$ from experiment (MIT)

arXiv:1110.3309 (Hu, et al)

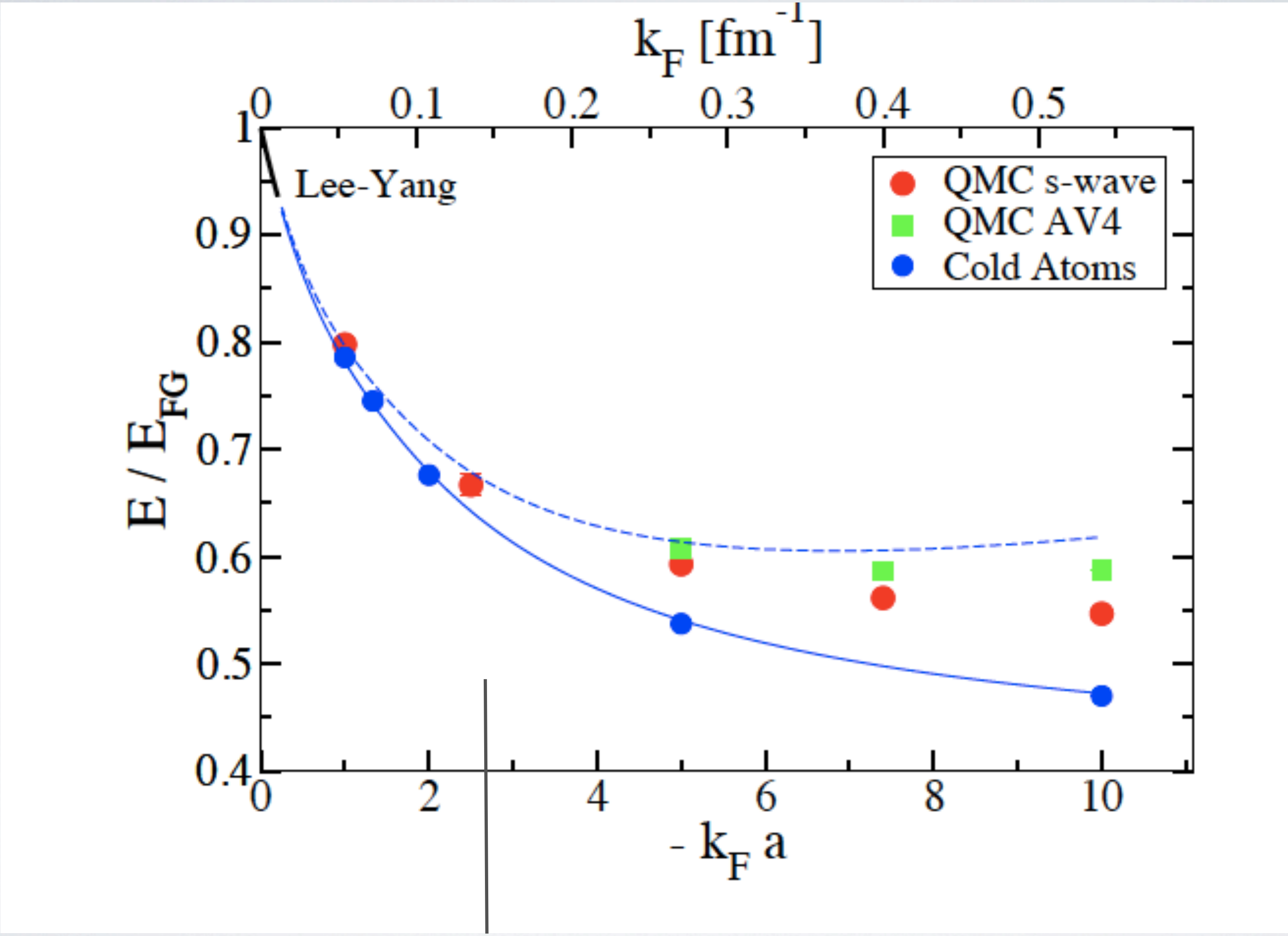
up to $27^3$ lattice, 66 particles

K.E. Schmidt, S. Zhang, S. Gandolfi, JC, PRA 2011

# Cold Atom Equation of State vs $k_F$ a



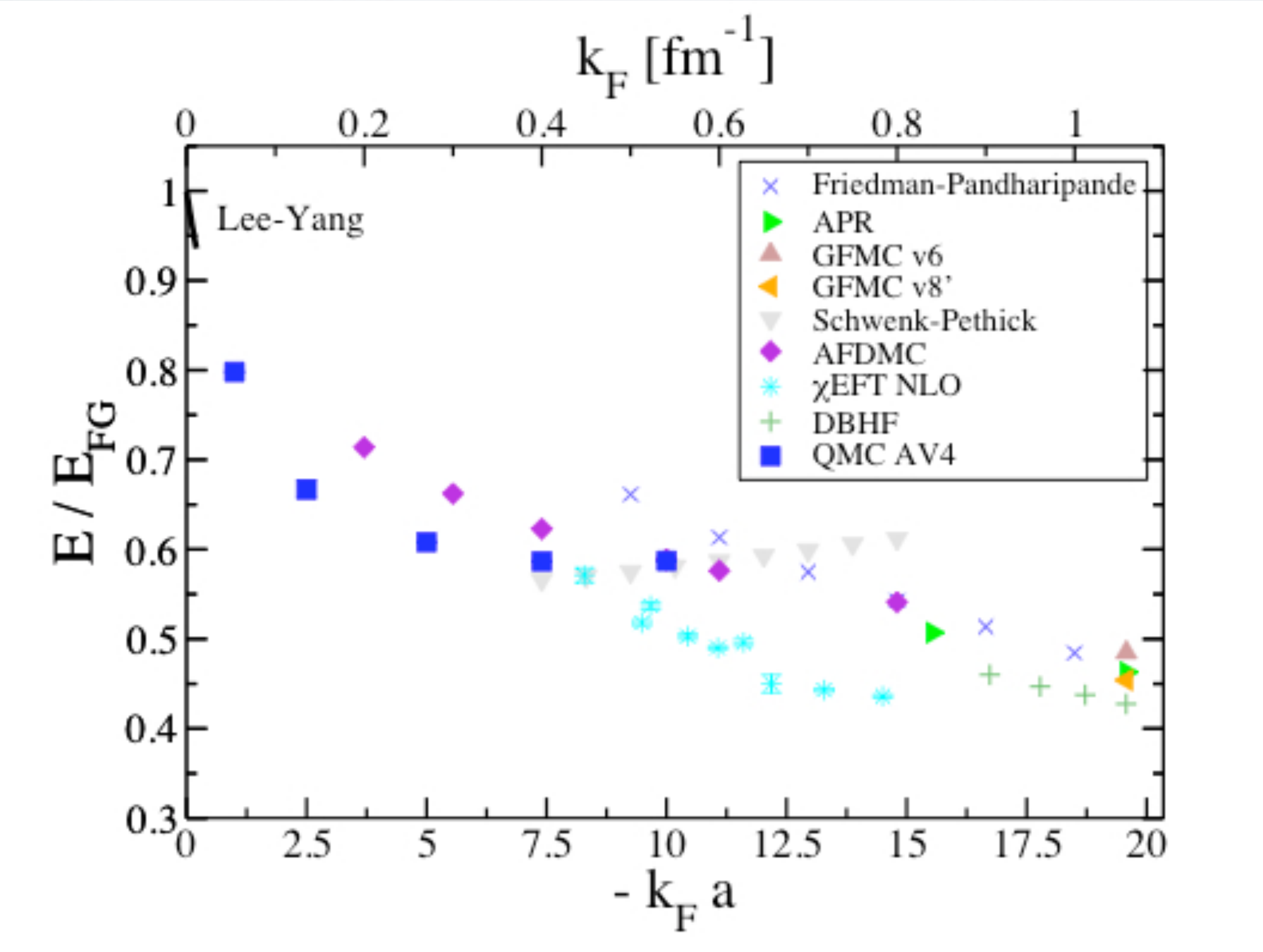
# Equation of State: Cold Atoms vs. Neutron Matter



$k_F r_e \sim 0.3$

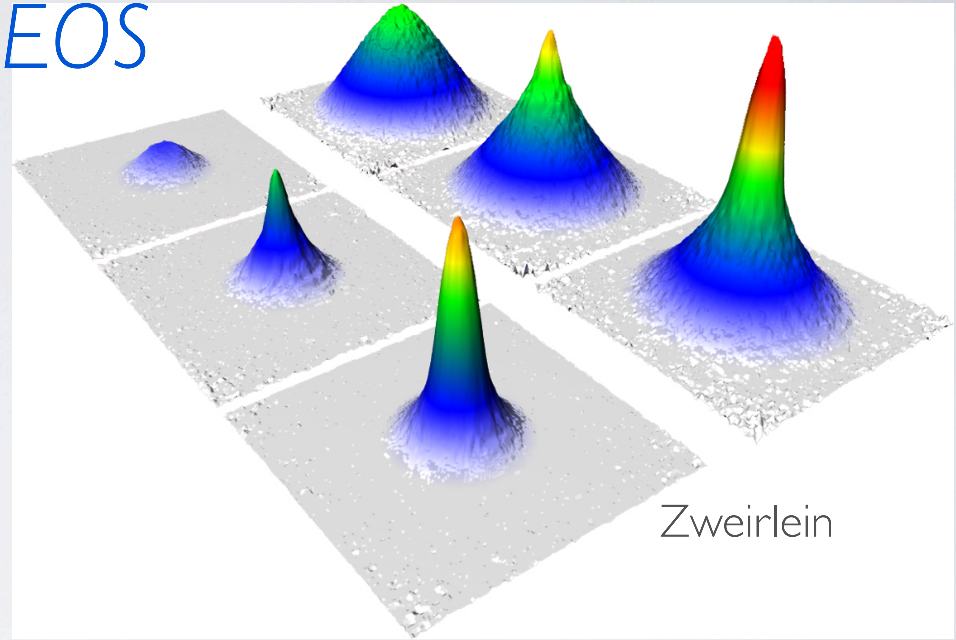
A. Gezerlis, J. C., 2008,2010

# Neutron Matter EOS

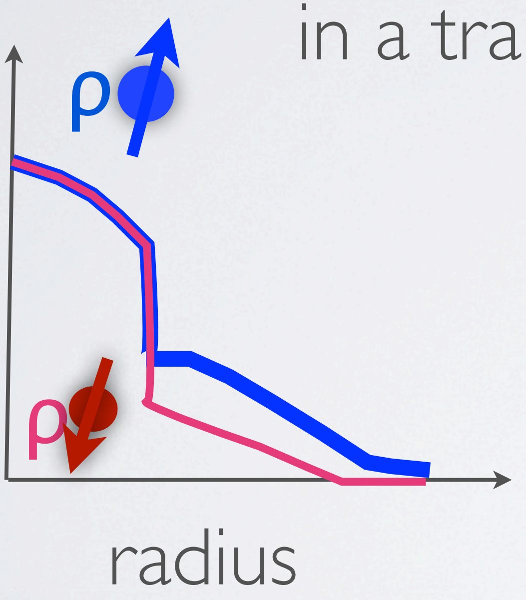


Neutron Matter EOS strongly constrained
at low-moderate densities

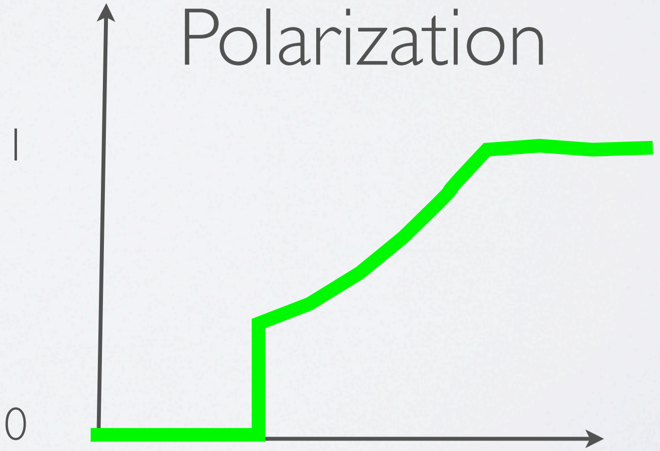
# *Pairing Gap at Unitarity from the polarized EOS*


Zweirlein

Spin up, down densities in a trap



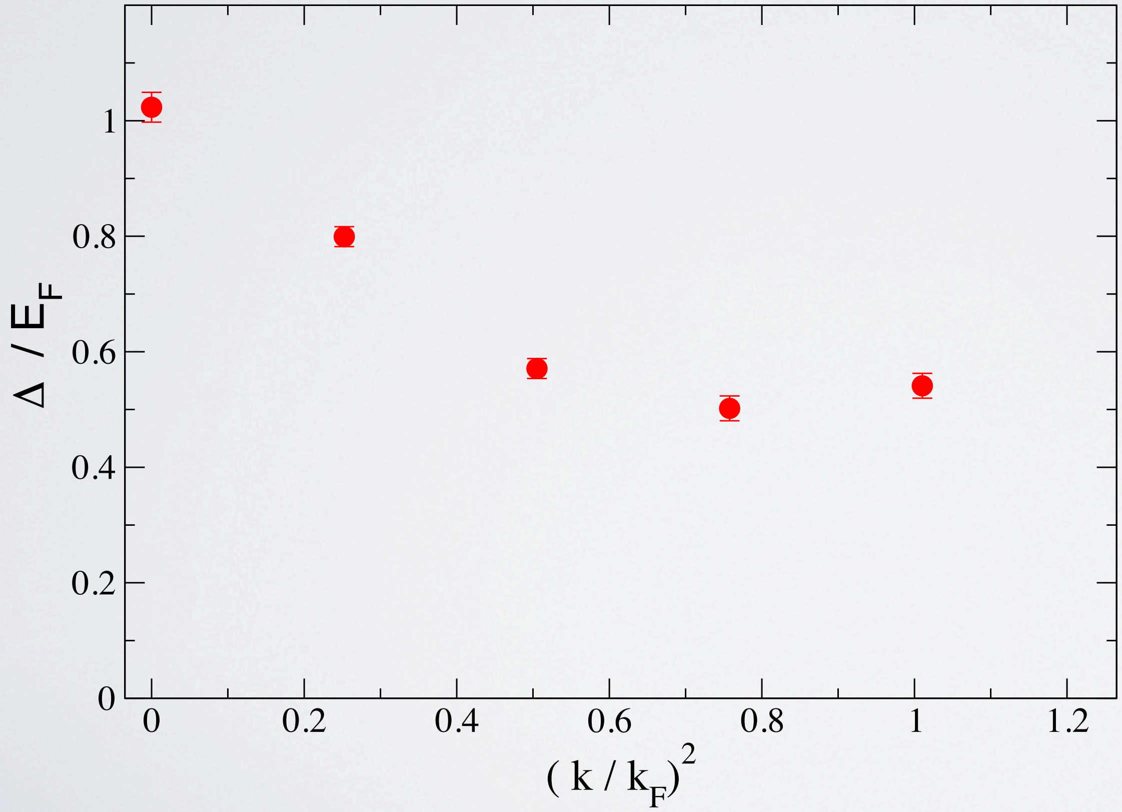
Polarization



# *Quasiparticle Dispersion in cold Atoms*

Add one  to fully-paired system

Energy cost for an unpaired particle: **μ + Δ**



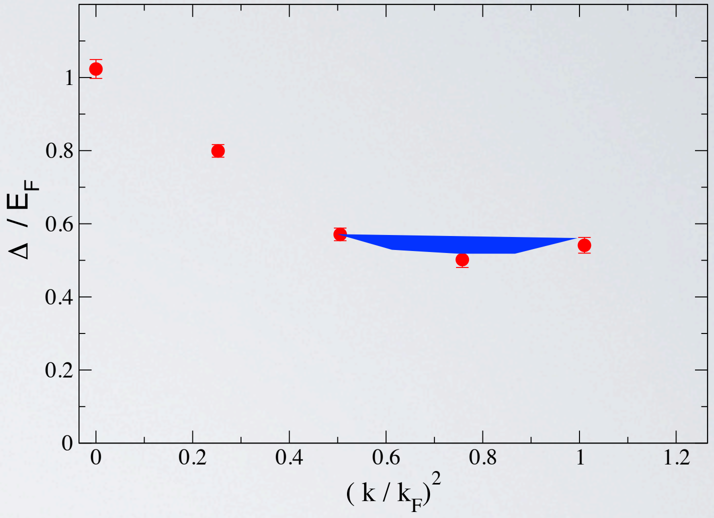
$$\Delta \;=\; \delta \; \frac{\hbar^2 k_F^2}{2m}$$

$$\delta \;=0.50\;(03)$$

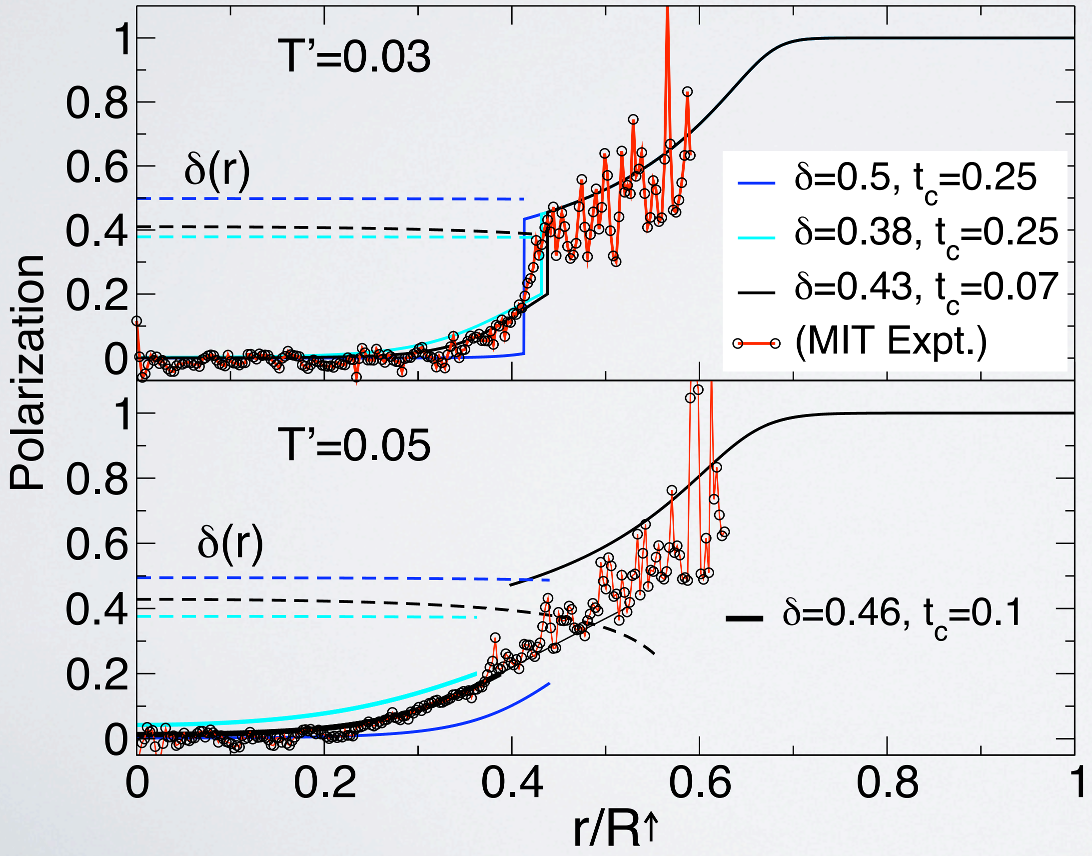
$$(k_{min}/k_f)^2 \;=0.80(10)$$

JC and Reddy, PRL 2005

# *Pairing Gap at Unitarity - Experiment*



## Polarization in a trap



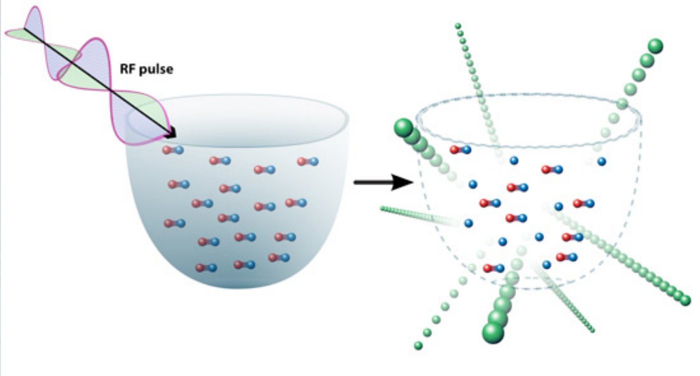
Largest **Δ**/Ef !

$$\Delta = \delta \frac{\hbar^2 k_F^2}{2m}$$

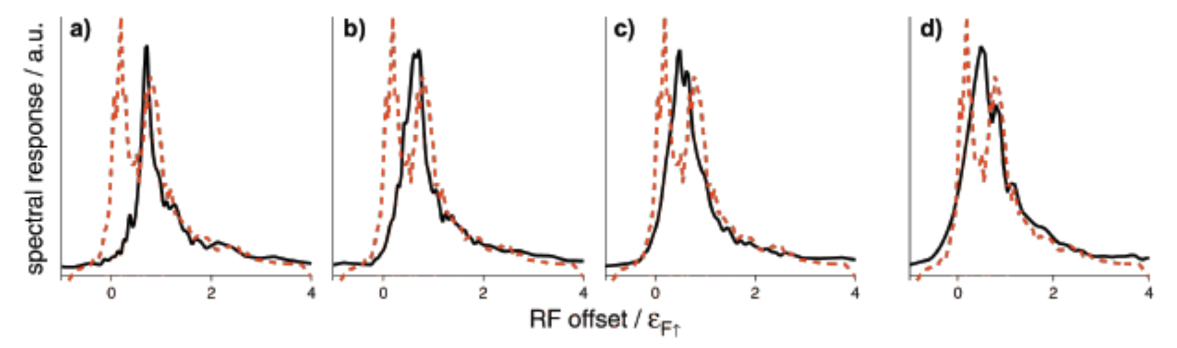
$$\delta = 0.45(05)$$

JC and Reddy, PRL 2007
analyzing MIT data

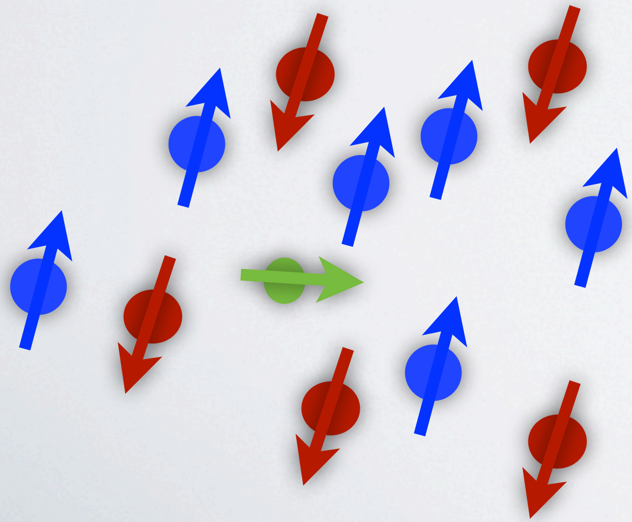
# *Pairing Gap at Unitarity - Experiment RF response*

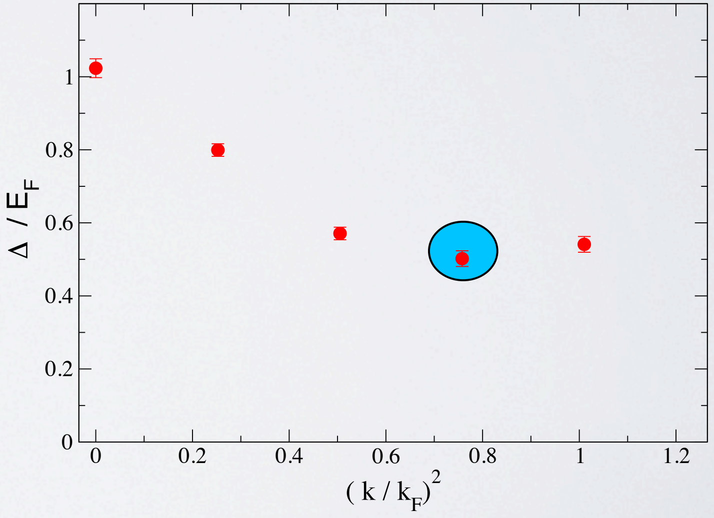


Credit: Greg Kuebler, JILA



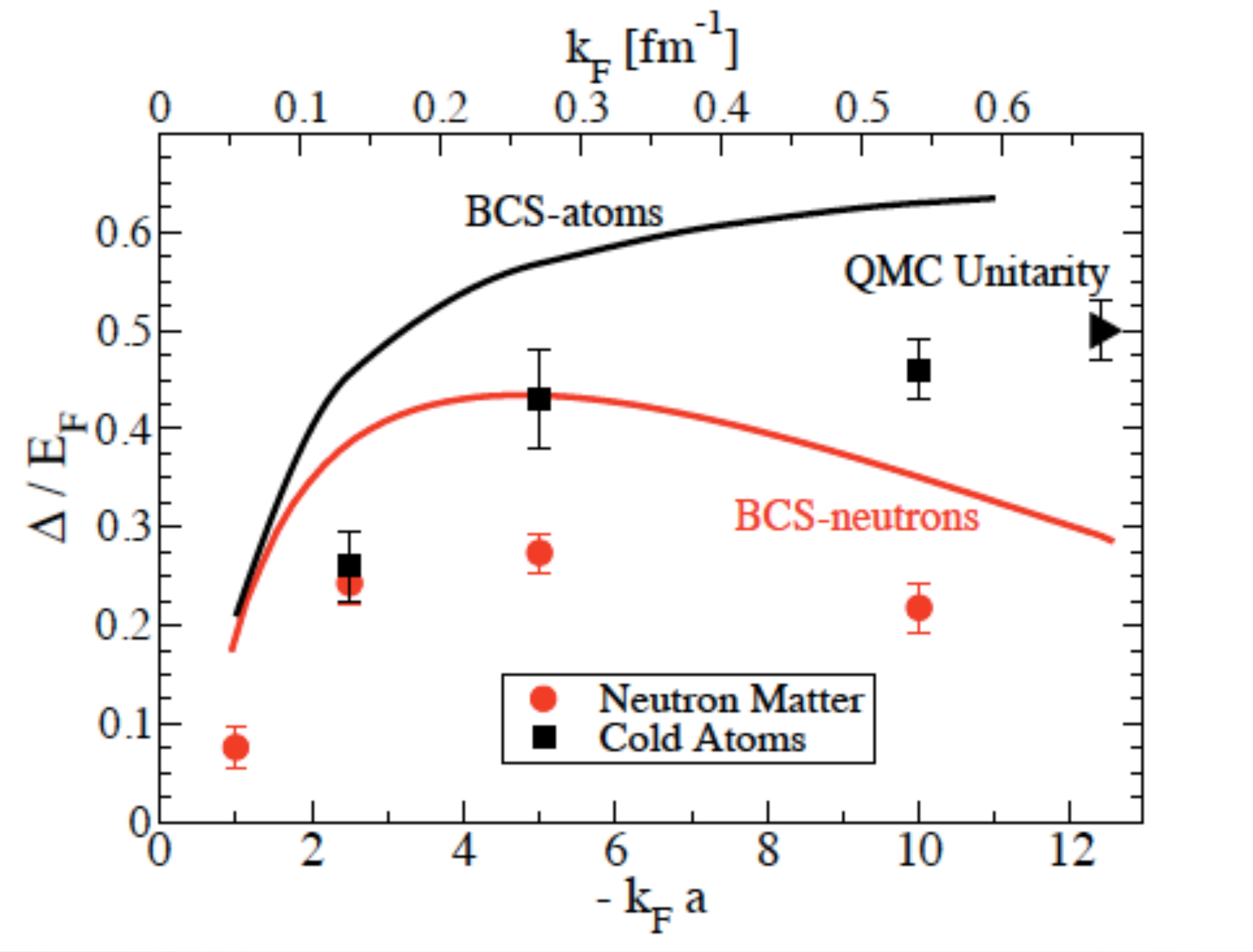
Shift of response of paired vs. unpaired atoms





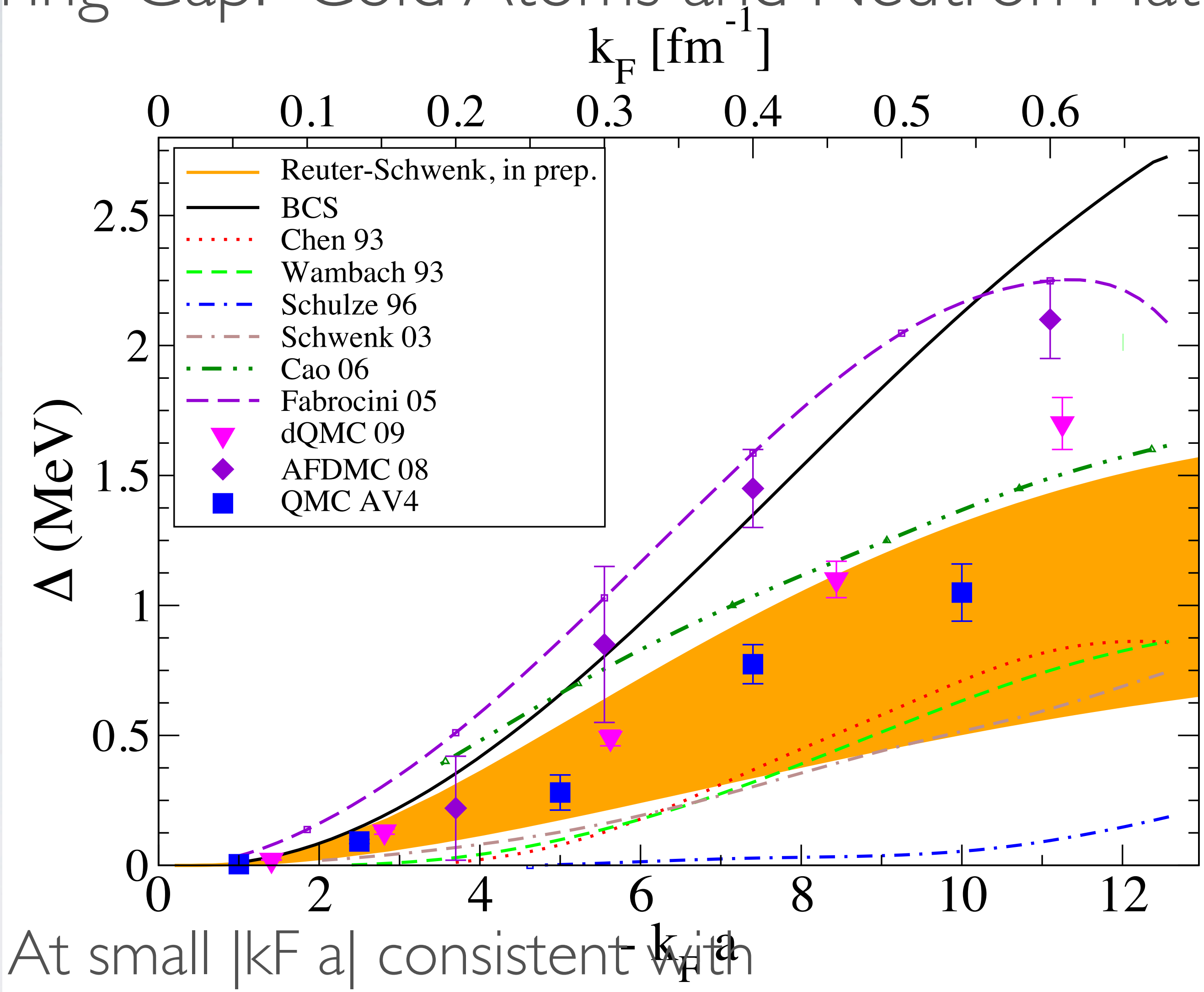
Shin, Ketterle, ... 2008

# Neutron Matter and Cold Atom Pairing Gap



Transition from weak pairing to near unitarity

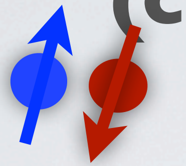
# Pairing Gap: Cold Atoms and Neutron Matter



At small |kF a| consistent with
Gorkov polarization suppression of BCS
At large densities consistent with
cold atom results

# Tying short-range to long-range physics (contact)

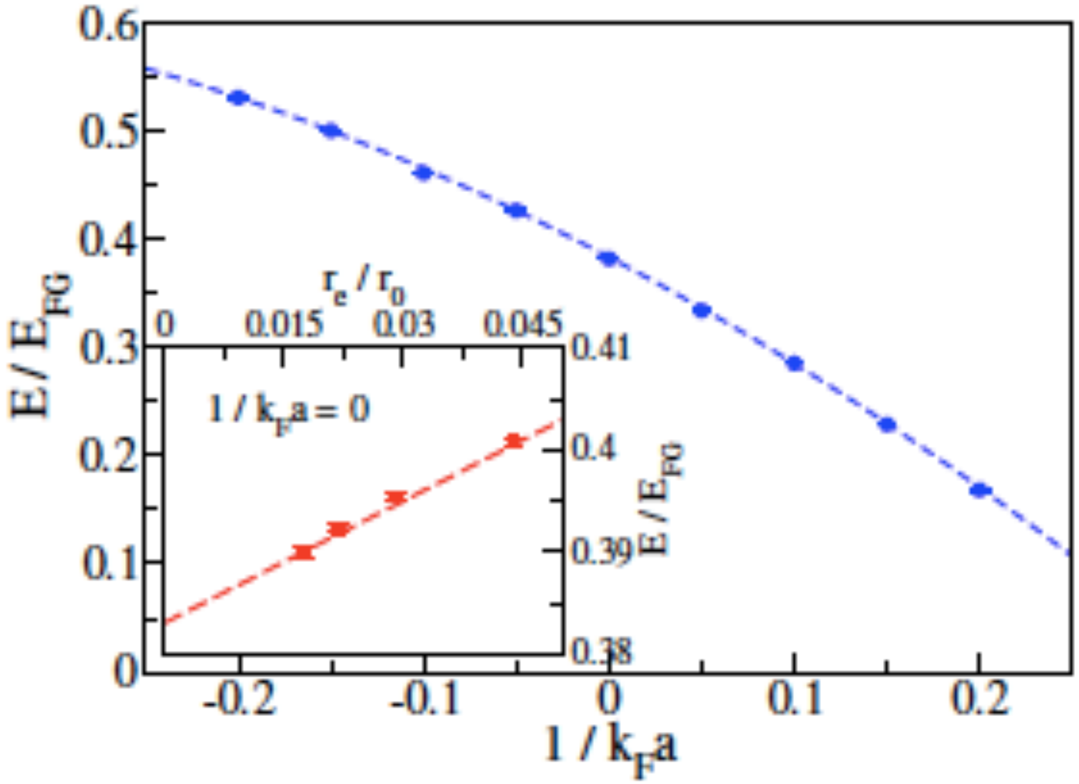
Probability of



at same point

Tan, Annals of Phys. 2008

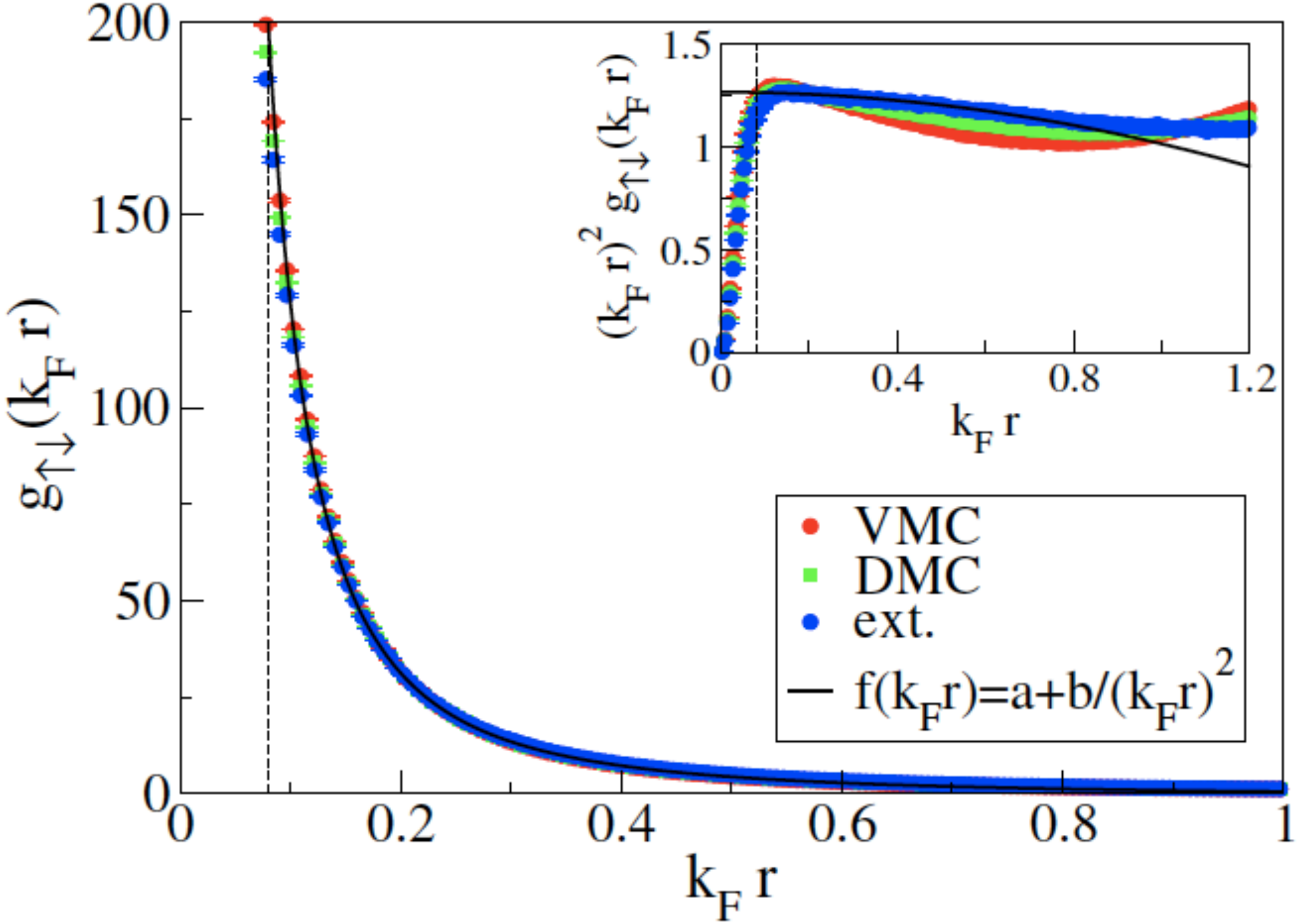
$$\frac{dE}{da^{-1}} = -\frac{\hbar^2 2\pi n A^2}{m} \rightarrow C = 8\pi^2 n^2 A^2.$$



$$\frac{C}{k_F^4} = \frac{2\zeta}{5\pi} = 0.1147(3)$$

Gandolfi, et al,
PRA 2011

# Pair Distribution Function



# Momentum Distribution

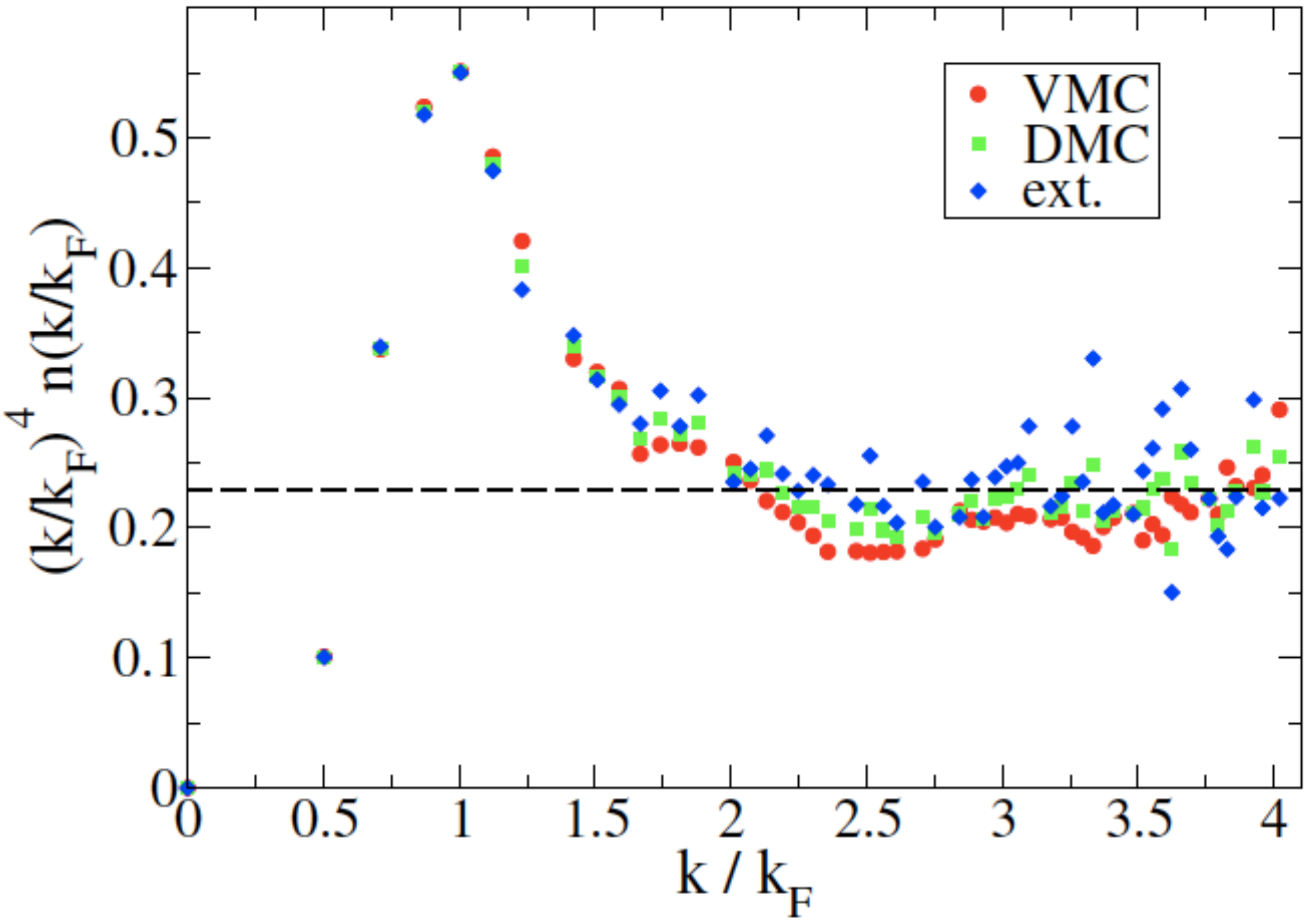
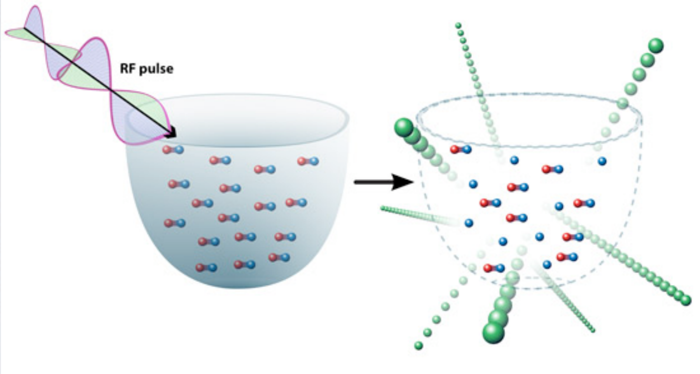


Fig. 8. Momentum distribution scaled by $k^4$ for cold atoms at unitarity.

# Beyond the Equation of State: Structure and Dynamics

Credit: Greg Kuebler, JILA



Linear Response:

$$S(k,\omega) = \langle 0|O^\dagger(k)|f\rangle\langle f|O(k)|0\rangle\delta(\omega - (E_f - E_0))$$

RF response (q=0): Flip atom to a new HF state

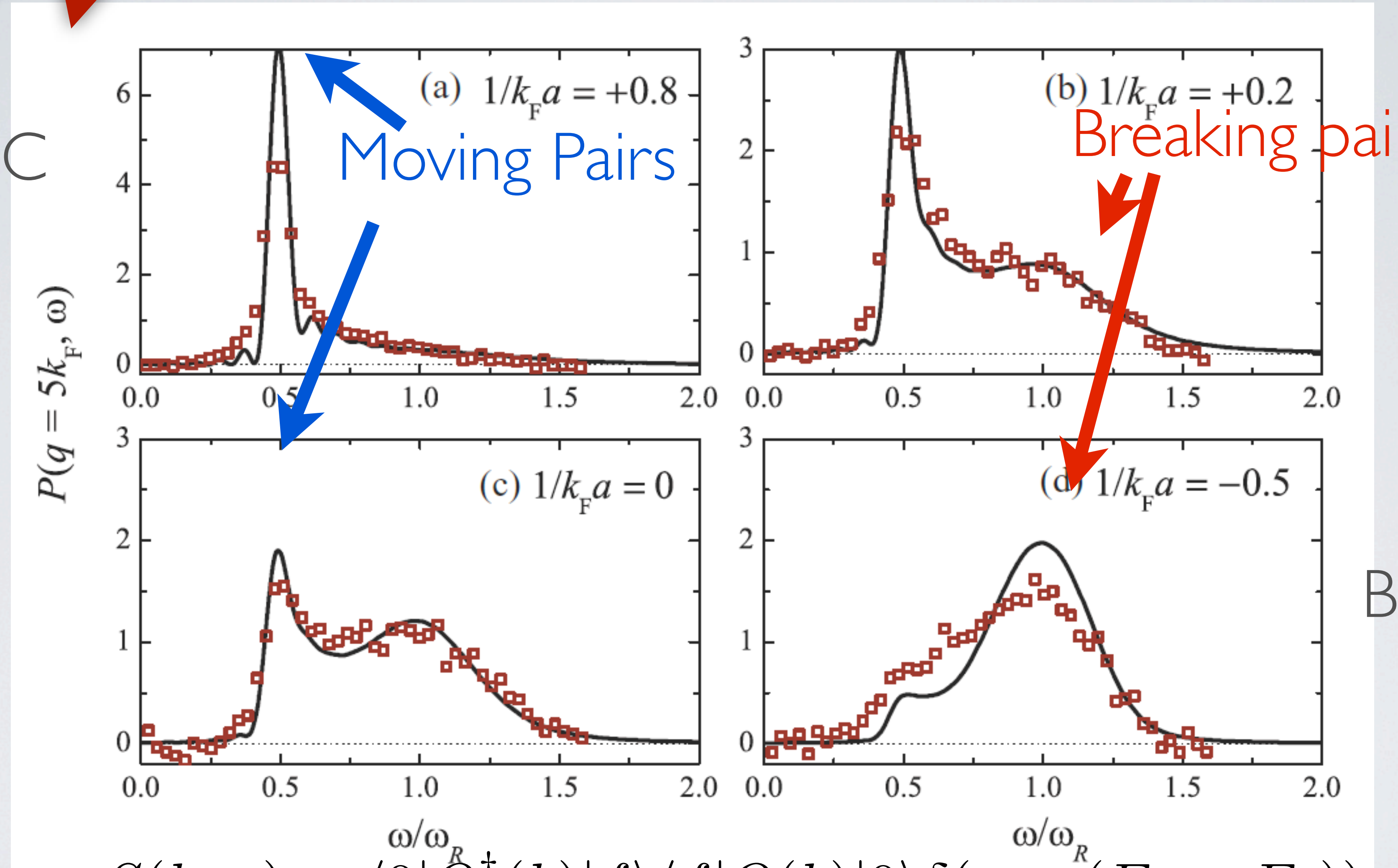
Spin response (large q): Flip atom between states
and give 'kick' of momentum q

Density response (large q): Give atom `kick' of
momentum q

Response sensitive to pairing gap, `contact', and more

# Density Response: Bragg Spectroscopy at large momentum transfer

BEC



Moving Pairs

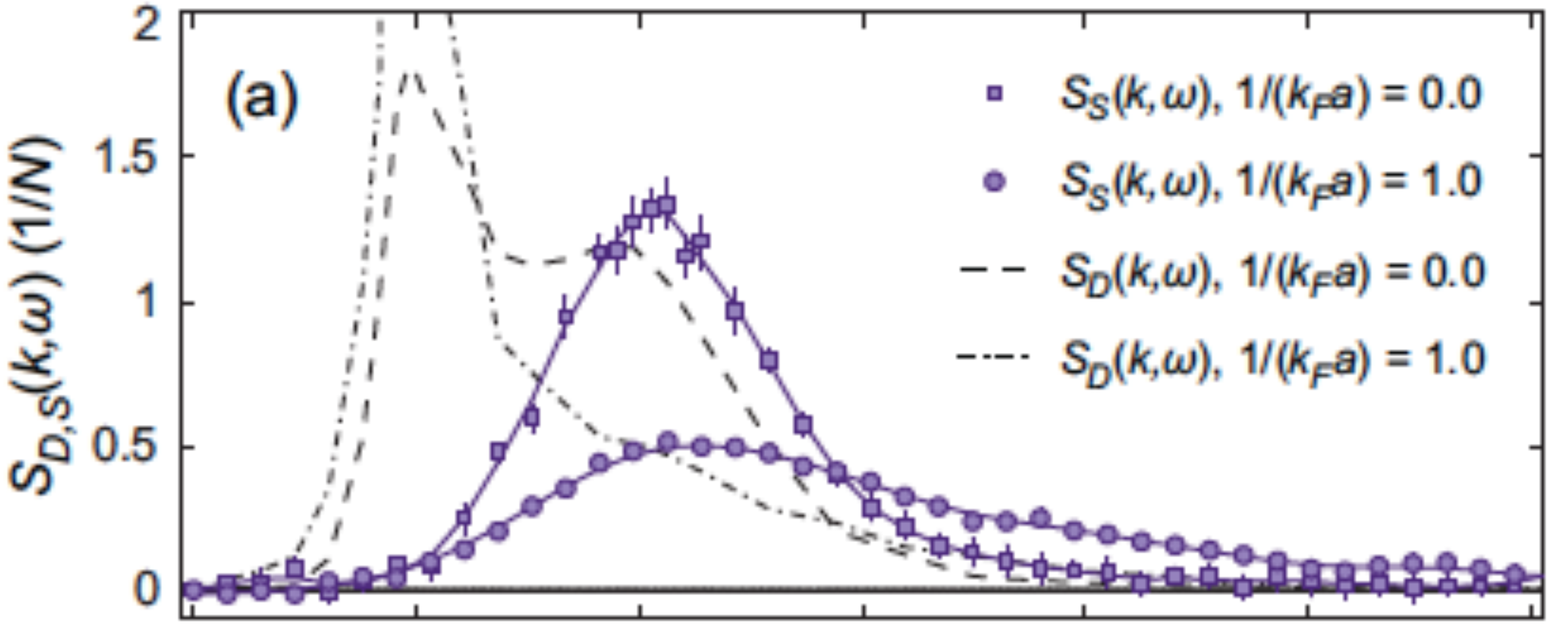
Breaking pairs

BCS

$$S(k,\omega) = \langle 0|O^\dagger(k)|f\rangle\langle f|O(k)|0\rangle\delta(\omega - (E_f - E_0))$$

$$O = \sum_i \exp[ik\cdot r]$$

# Spin Response from Bragg Spectroscopy

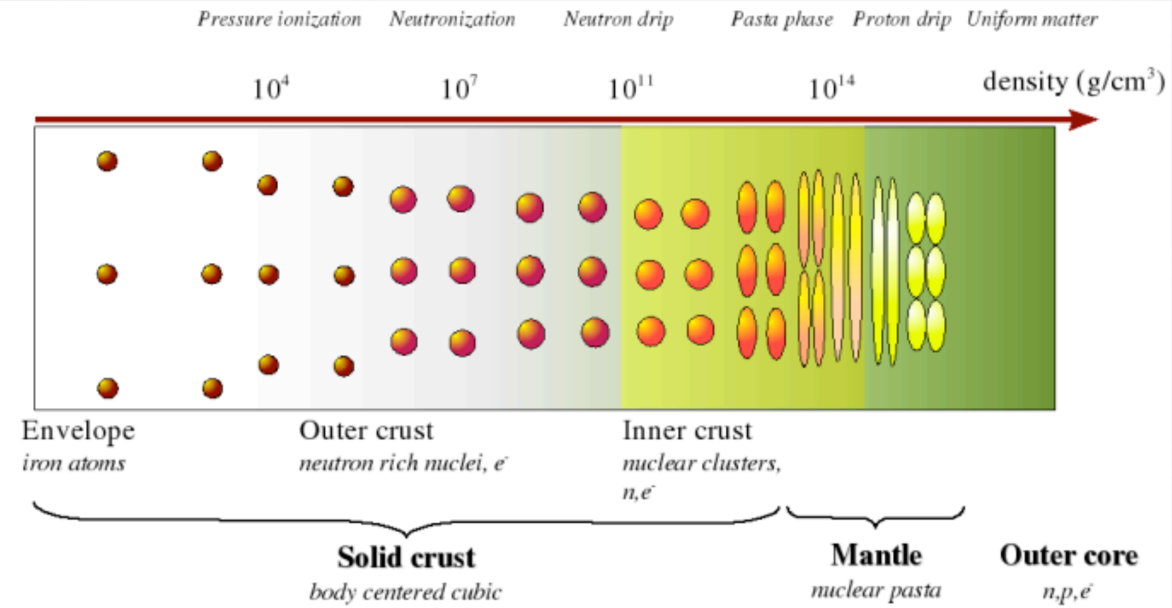


S. Hoinka, M. Lingham, M. Delehaye, and C. J. Vale, arXiv 1203.4657

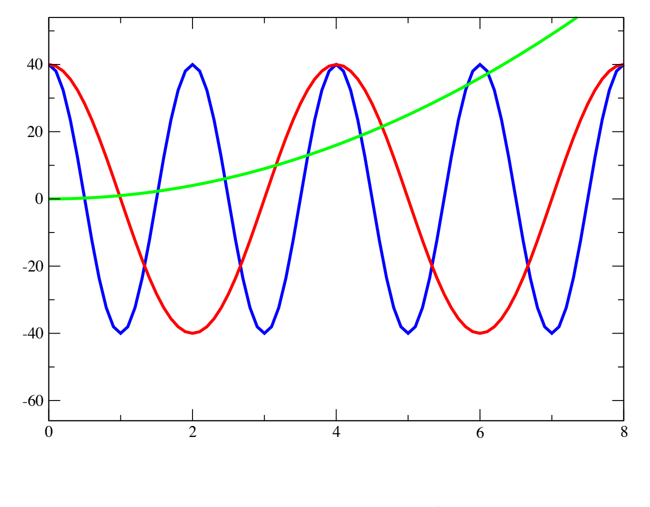
Large momentum transfer: sensitive to contact, gap,...

Spin response in neutron matter critical for neutrinos
low q important, depends upon L.S, tensor interactions

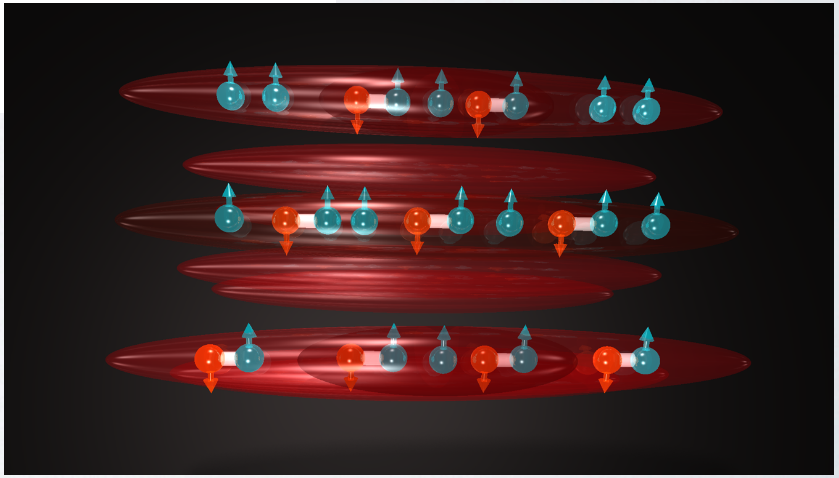
# Inhomogeneous Matter



Neutron Star Matter (Chamel)



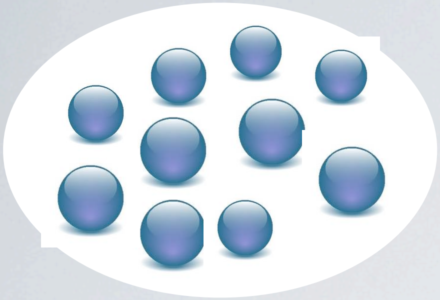
external potentials



Atoms: nearly 1D

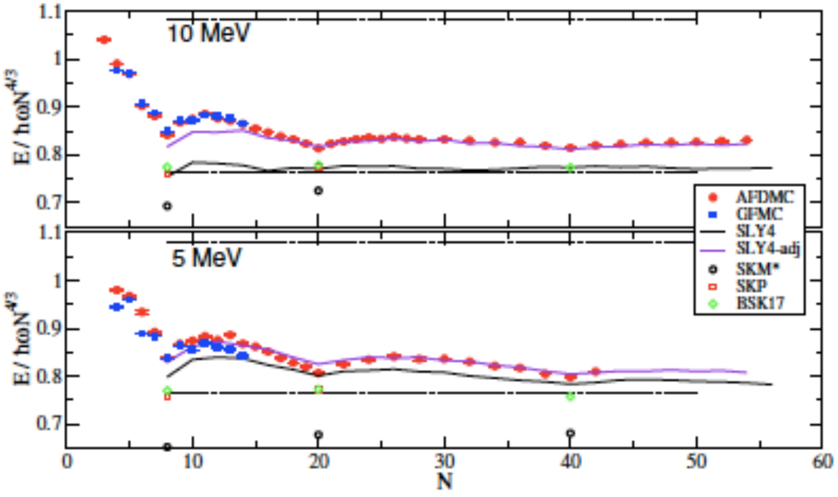
Hulet

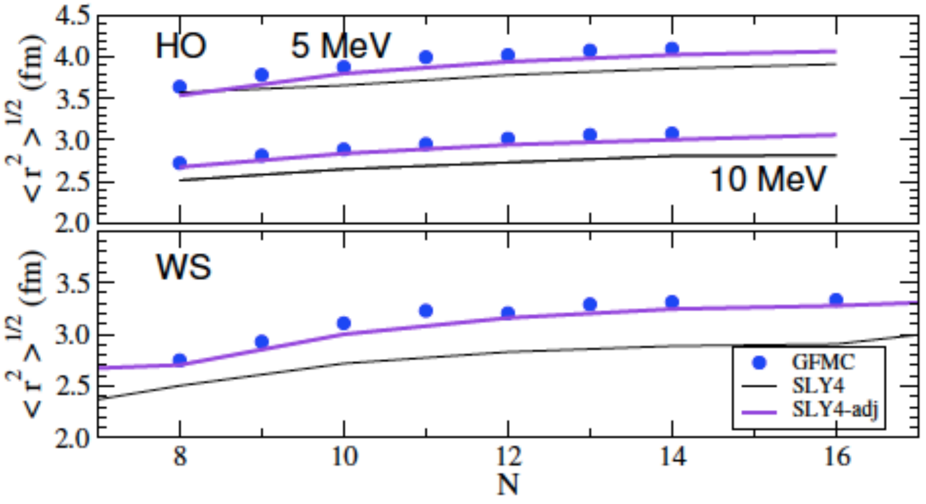
# Inhomogeneous Neutron Matter



N = 6 to 50 Neutrons
Harmonic Oscillator and
Wood-Saxon external wells

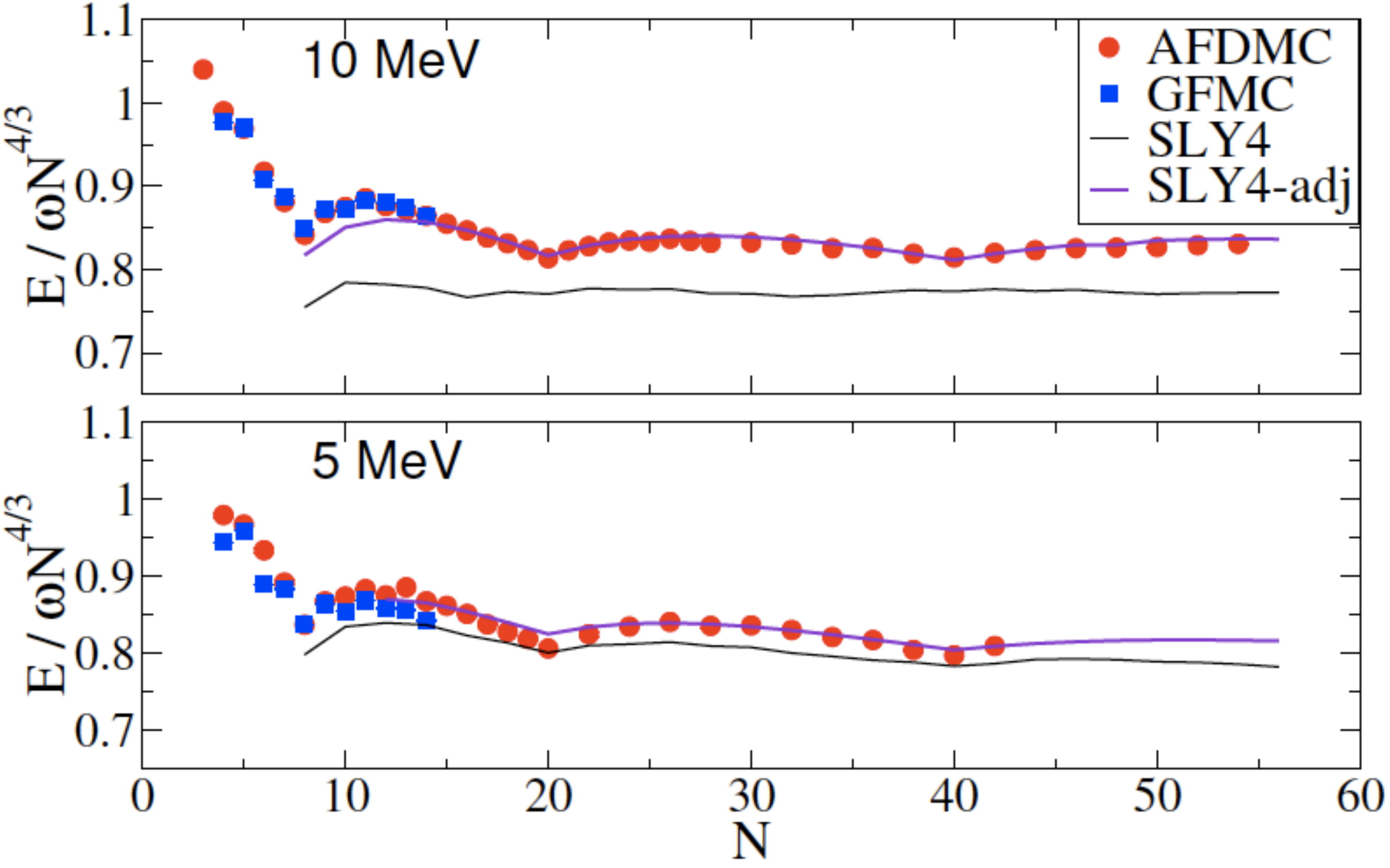
UNEDF SCIDAC





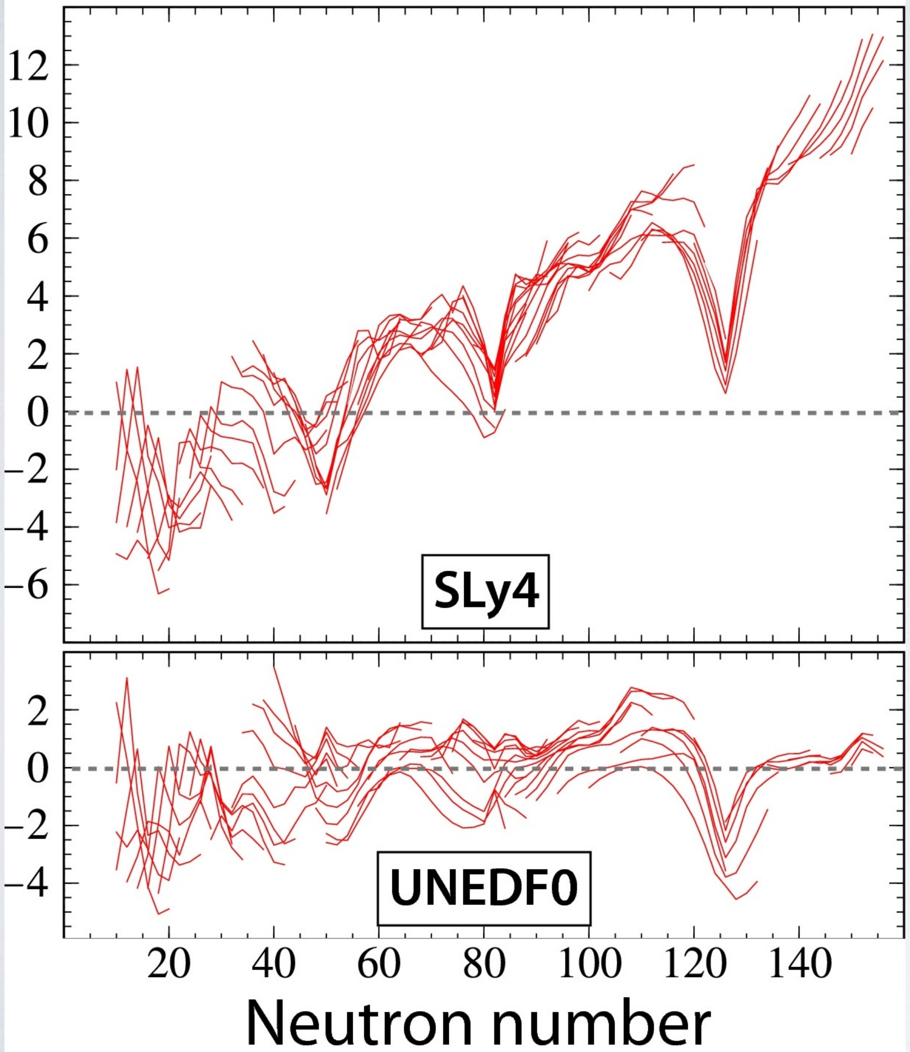
Gandolfi, et al, PRL 2011

Explore very large isospin limit of the density functional.
Examine gradient, spin-orbit, and pairing terms



Repulsive gradient terms required to fit neutron drops
also smaller spin-orbit, pairing interactions

# Improved Density Functionals
# Neutron Drops, Masses, Fission,...



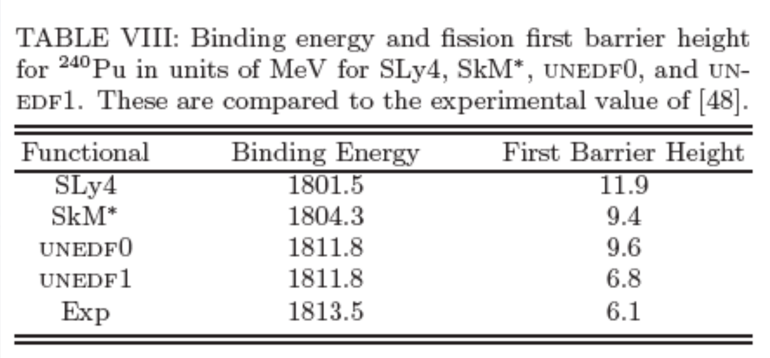
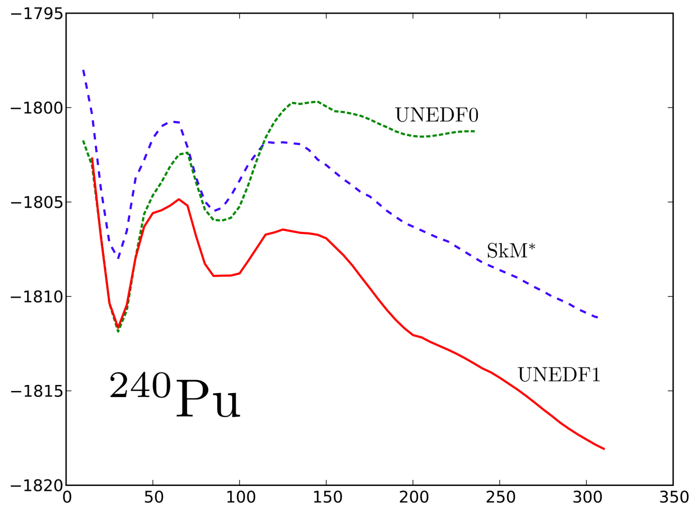
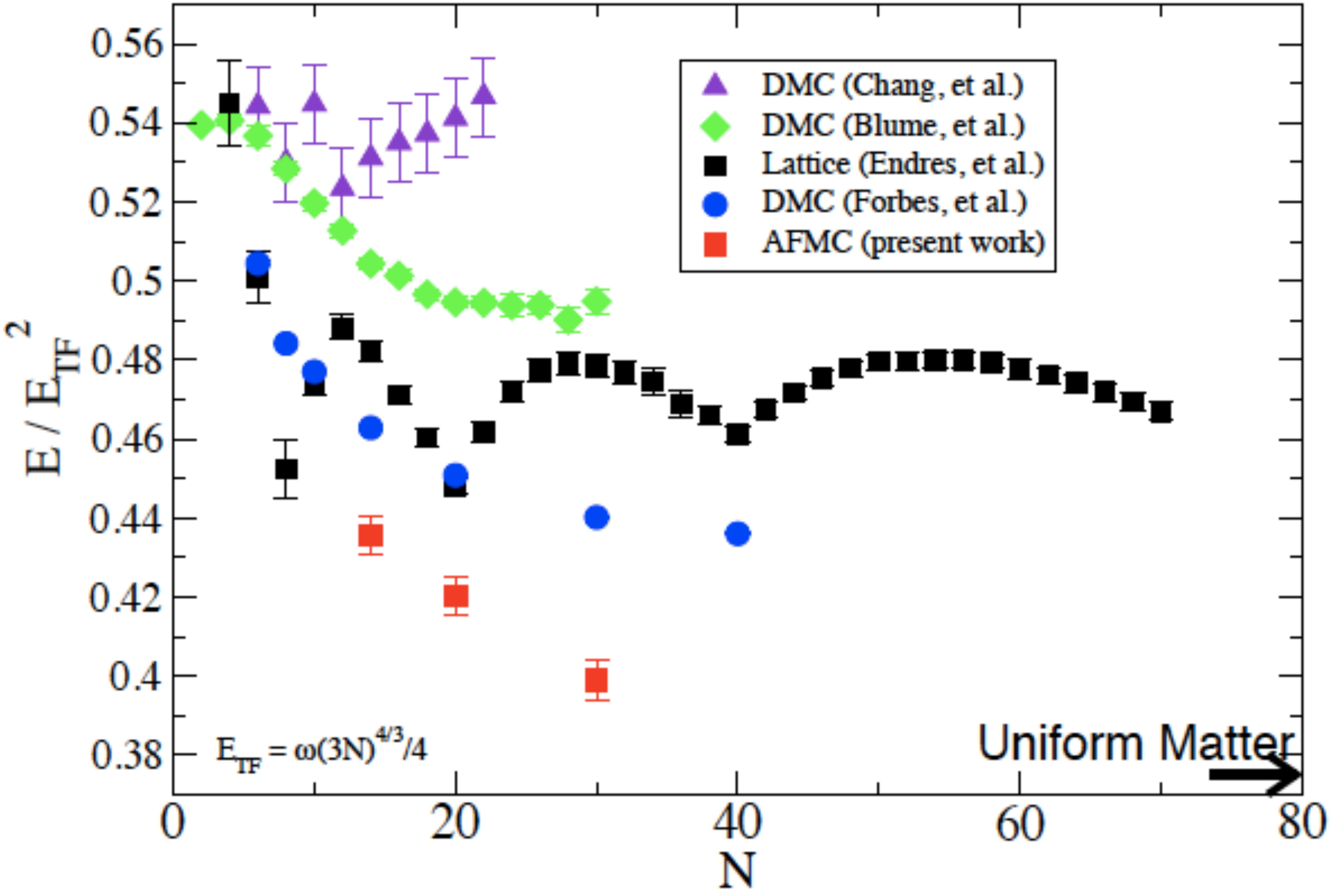


TABLE VIII: Binding energy and fission first barrier height for $^{240}$Pu in units of MeV for SLy4, SkM*, UNEDF0, and UNEDF1. These are compared to the experimental value of [48].

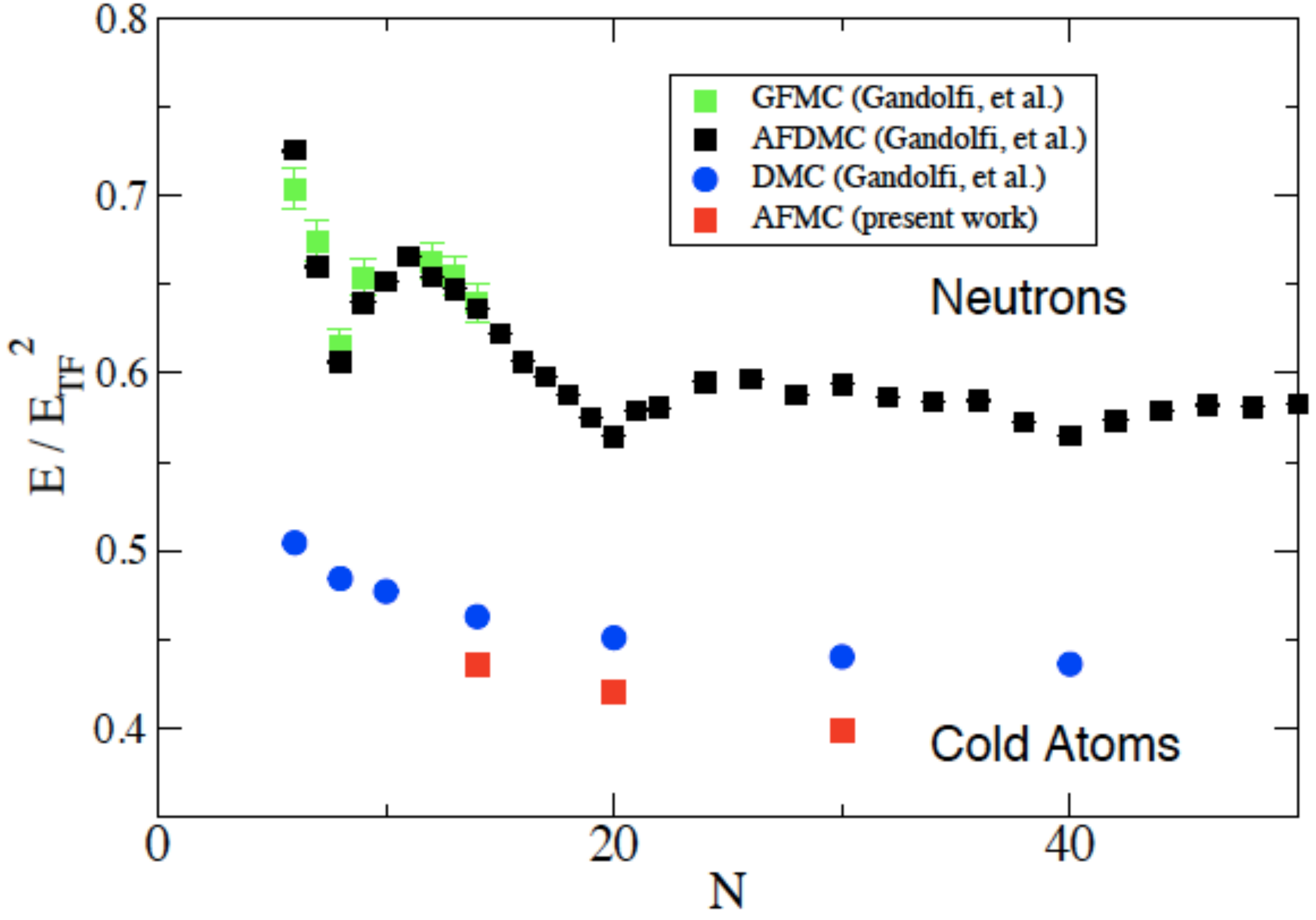
| Functional | Binding Energy | First Barrier Height |
|---|---|---|
| SLy4 | 1801.5 | 11.9 |
| SkM* | 1804.3 | 9.4 |
| UNEDF0 | 1811.8 | 9.6 |
| UNEDF1 | 1811.8 | 6.8 |
| Exp | 1813.5 | 6.1 |



# Results for Trapped Cold Atoms at Unitarity



# Trapped Neutrons and Unitary Fermi Gas



neutrons: Gandolfi, Pieper, JC, PRL 2011
atoms: Gandolfi, Gezerlis, Forbes, preliminary

Effective range impacts: EOS, shell structure, pairing gaps

# Future

Transition from 3D to 2D in cold atom systems

Pairing in inhomogeneous systems in strong interaction regime

Spin/Density response at small/moderate q

Spin response in neutron matter

Additional response: viscosity,...

Low-energy excitations