Quantifying short-range correlations in nuclei

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i)PRC **84**, 031302(R) (2011)
ii) arXiv:1206.5151

Outline

Some theory concepts

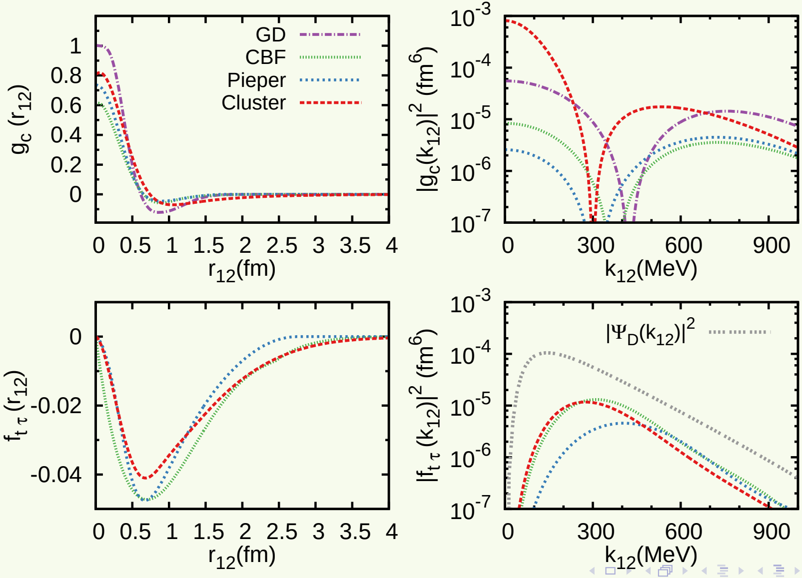
- With proton charge radius of $\sim 0.9 fm$ one gets a nuclear packing fraction of 42 %.
- The nucleus is quite a dense quantum liquid.
- Mean-field approaches have been very successful but nucleus is more than the sum of A nucleons.
- A time-honored method to account for the effect of correlations (classical and quantum systems): **Correlation functions**
- Realistic wave functions $\mid \overline{\Psi} \rangle$ after applying a many-body correlation operator to a Slater determinant $\mid \Psi_{MF} \rangle$

$$\mid \overline{\Psi} \rangle = \frac{1}{\sqrt{\langle \Psi \mid \widehat{\mathcal{G}}^{\dagger} \widehat{\mathcal{G}} \mid \Psi \rangle}} \widehat{\mathcal{G}} \mid \Psi_{MF} \rangle \, .$$

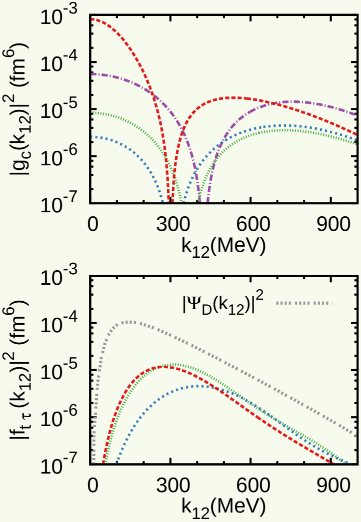
- The nuclear $\widehat{\mathcal{G}}$ is complicated but is dominated by the central and tensor correlations

$$\widehat{\mathcal{G}} \approx \widehat{\mathcal{S}} \left[\prod_{i<j=1}^{A} \left(1 - g_c(r_{ij}) + f_{t\tau}(r_{ij}) \widehat{S_{ij}} \vec{\tau}_i . \vec{\tau}_j \right) \right]$$

Correlation functions



Correlation functions



- very high relative pair momenta: central correlations
- moderate relative pair momenta: tensor correlations
- $|f_{t\tau}(k_{12})|^2$ is well constrained! (*D*-state deuteron wave function)
- the $g_C\,(k_{12})$ looks like the correlation function of a monoatomic classical liquid (reflects finite-size effects)
- the $g_c\,(k_{12})$ are ill constrained!

Some Theory Concepts (2)

- Correlations are dynamically generated by operating with $\widehat{\mathcal{G}}$ on IPM wave functions (UCOM etc.)
- In practice: perturbative (cluster, virial) expansions are required
- Nucleon-nucleon correlations are highly local which naturally truncates the expansions (2N $\gg$ 3N)
- Part of the mean-field wave function with strength at $r = 0$ (equiv. to relative S-wave!) receives largest corrections.
- Effective one-body operator receives two-body etc. contributions through the correlation operators.

$$\widehat{\Omega}^{eff} = \widehat{\mathcal{G}}^{\dagger}\,\widehat{\Omega}\,\widehat{\mathcal{G}} \approx \widehat{\Omega} + \sum_{i<j=1}^{A}\left(\left[\widehat{\Omega}^{[1]}(i) + \widehat{\Omega}^{[1]}(j)\right] \times \left[-g_c\left(r_{ij}\right) + \widehat{t}\,(i,j)\right] + h.c.\right).$$

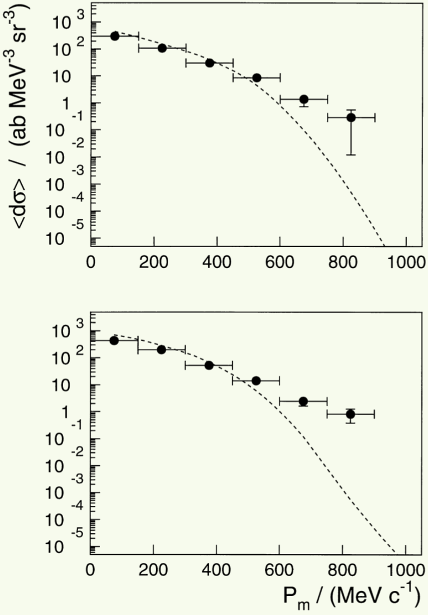
Some Theory concepts (3)

- Two-body contributions reflect the correlated part of the spectral function and induce two-nucleon knockout!
- The corresponding cross sections do NOT scale according to $K\sigma_{eN}\rho^{(D)}(\vec{p}_m)$
- FACTORIZED CROSS SECTION FOR 2N KNOCKOUT:
  J. Ryckebusch et al. PLB 383 (1996) 1

$$\frac{d^8\sigma}{d\epsilon' d\Omega_{\epsilon'} d\Omega_1 d\Omega_2 dT_{p_2}}(e, e' N_1 N_2) = E_1 p_1 E_2 p_2 f_{rec}^{-1} \times \sigma_{eN_1N_2}(k_+, k_-, q) F_{h_1,h_2}(P)$$

  - Factorization requires relative S states!
  - $F_{h_1,h_2}(P)$: Probability to find a dinucleon with c.o.m. momentum P
  - $\sigma_{eN_1N_2}(k_+, k_-, q)$: Probability to have an electromagnetic interaction with a dinucleon with relative momentum $k_\pm$

What do experiments say?

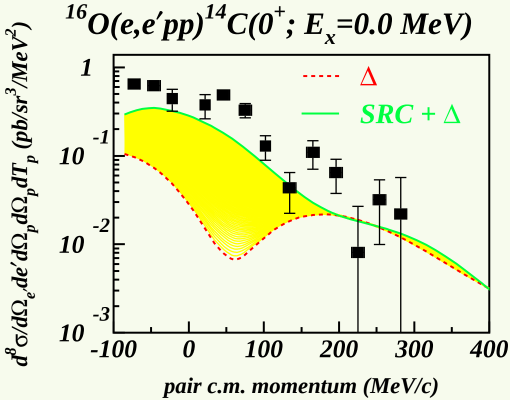


$^{12}C(e, e'pp)$ @ MAMI (Mainz) (Physics Letters B **421** (1998) 71.)

- Up to **P** $=$ **0.5 GeV** c.o.m. motion in ^{12}C is mean-field (Gaussian) like
- Data agree with the factorization in terms of $F(P)$!
- Largest at $P_m = 0 \rightarrow$ back-to-back

What do experiments say (2)

Triple coincidence measurements $A(e, e'pp)$ at low Q^2 determined the quantum number of the correlated pairs!



Unfactorized theory (MEC, IC, central + tensor correlations) EPJA 20 (2004) 435

- High resolution $^{16}O(e, e'pp)$ studies (MAMI)
- Ground-state transition: $^{16}O\ (0^+) \rightarrow {}^{14}C\ (0^+)$
- Quantum numbers of the active diproton [relative (c.m.)]: $^1S_0(\Lambda = 0)$ (lower P) and $^3P_1(\Lambda = 1)$ (higher P)
- only $^1S_0(\Lambda = 0)$ diprotons are subject to SRC

Two-nucleon correlations (1)

- Suggestion: number of relative S states is a measure for the ammount of 2N correlated pairs in $A(N, Z)$
- Requires transformation from $(\vec{r}_1, \vec{r}_2)$ to $\left(\vec{r}_{12} = \frac{\vec{r}_1 - \vec{r}_2}{\sqrt{2}}, \vec{R}_{12} = \frac{\vec{r}_1 + \vec{r}_2}{\sqrt{2}}\right)$
- In a HO basis a normalized and antisymmetrized two-body state reads ($\alpha_a = (n_a l_a j_a t_a)$) [Moshinsky transformation]

$$
\begin{aligned}
|\alpha_a \alpha_b; J_R M_R\rangle_{nas} = & \sum_{LM_L} \sum_{n\mathcal{L}} \sum_{N\Lambda} \sum_{SM_S} \sum_{TM_T} \frac{1}{\sqrt{2\left(1 + \delta_{\alpha_a \alpha_b}\right)}} \\
& \times \left[1 - (-1)^{\mathcal{L}+S+T}\right] \\
& \times \mathcal{C}\left(\alpha_a \alpha_b J_R M_R; (n\mathcal{L}N\Lambda) LM_L SM_S TM_T\right) \\
& \times \left| \left[n\mathcal{L}\left(\vec{r}_{12}\right), N\Lambda\left(\vec{R}_{12}\right)\right] LM_L, \left(\frac{1}{2}\frac{1}{2}\right) SM_S, \left(\frac{1}{2}\frac{1}{2}\right) TM_T \right\rangle \ ,
\end{aligned}
$$

$\left|n\mathcal{L}\left(\vec{r}_{12}\right)\right\rangle$ ($\left|N\Lambda\left(\vec{R}_{12}\right)\right\rangle$) is the relative (c.m.) pair wave function

Two-nucleon correlations (2)

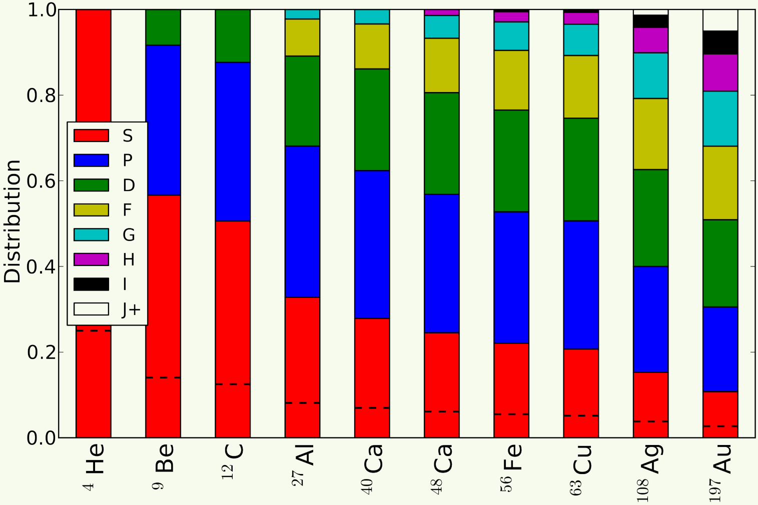
- Normalization: one has

$$\sum_{J_R M_R} \sum_{\alpha_a \leq \alpha_F^p} \sum_{\alpha_b \leq \alpha_F^n} \langle \alpha_a \alpha_b; J_R M_R \,|\alpha_a \alpha_b; J_R M_R \rangle_{nas} = NZ \;, \qquad (1)$$

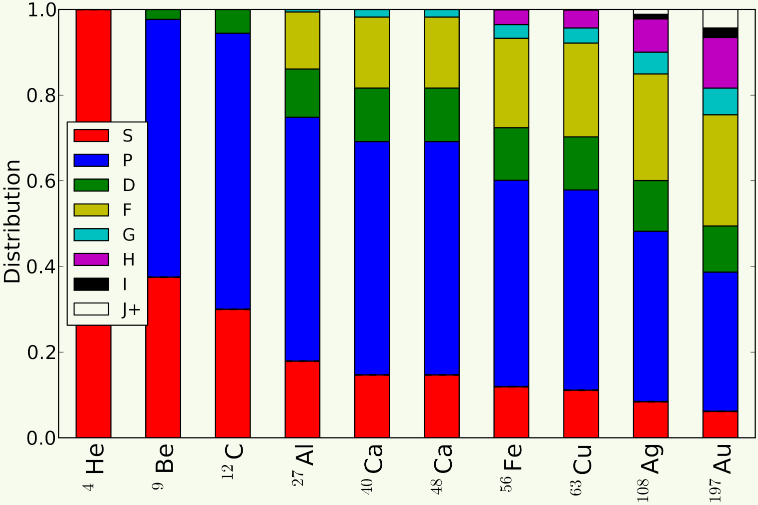
  Fermi level for the proton and neutron: α_F^p and α_F^n.

- Similar expressions for the number of proton-proton $\left(\frac{Z(Z-1)}{2}\right)$ and neutron-neutron pairs $\left(\frac{N(N-1)}{2}\right)$
- One can compute how much $|(n\mathcal{L}, N\Lambda)LM_L, SM_S, TM_T\rangle$ contributes for each IPM pair $|\alpha_a m_a \alpha_b m_b\rangle$
- IPM pairs in a relative $\left|n = 0 \mathcal{L} = 0 \left(\vec{r}_{12}\right)\right\rangle$ are prone to SRC ("central" + "tensor")!

Distribution of the relative quantum numbers $\mathcal{L} = S, P, D, F, G, H, I, \geq J$ for *pn* pairs



Distribution of the relative quantum numbers $\mathcal{L} = S, P, D, F, G, H, I, \geq J$ for pp pairs



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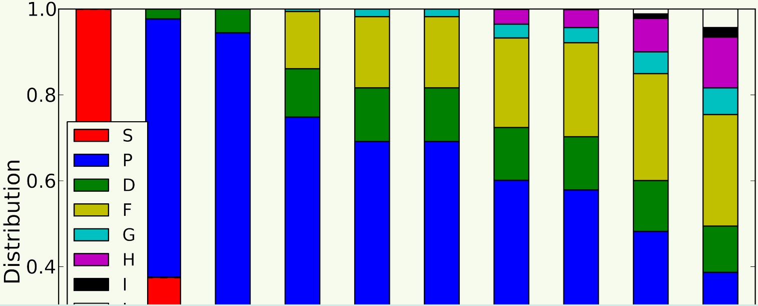
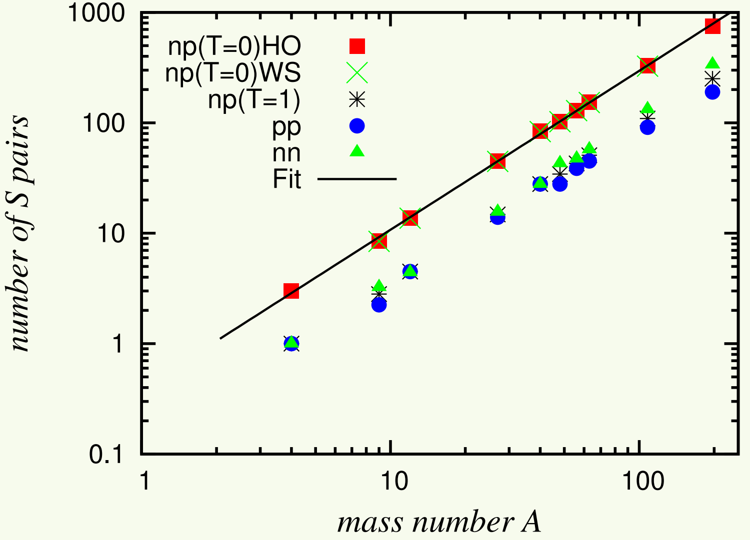


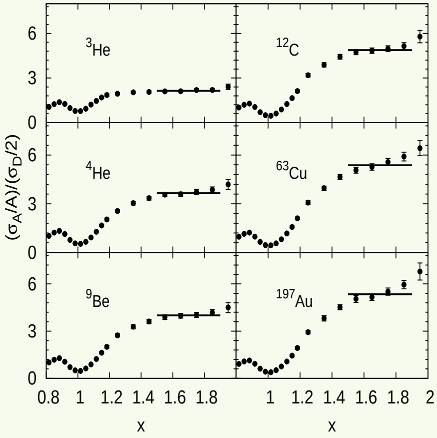
1. with increasing *A*: a smaller fraction of the pairs reside in a relative *S* state
2. strong isospin dependence: fraction of the *pn* pairs residing in a relative *S* state is substantially larger than for proton-proton and neutron-neutron pairs.

Number of *pp*, *nn* and *pn* pairs with $\mathcal{L} = 0$



- number of pairs prone to SRC effects: a power law $\sim A^{1.44\pm0.01}$
- the power law is very robust

$A(e,e')$ for $1.4 < x_{BJ}$ and 2N SRC's



Hall-C, arXiv:1107.3583

- quantify scaling behavior:

$$a_2\left(\frac{A}{D}\right) = \frac{2}{A}\frac{\sigma^A(x_B,Q^2)}{\sigma^D(x_B,Q^2)},$$

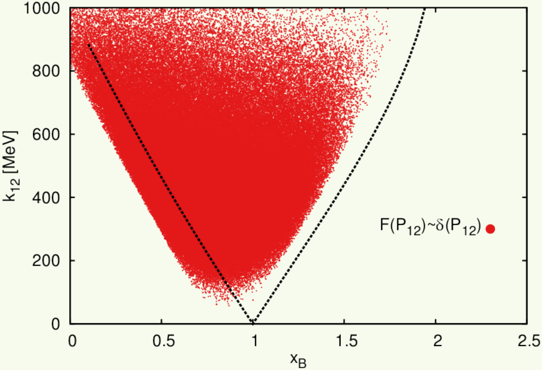
- very naive counting: all *pn* pairs contribute $a_2 \sim A$
- $\sigma^A(x_B,Q^2) \approx B^{np}_{l=0}(A)F(P)\sigma_D(x_B,Q^2)$
- suggestion: $a_2(A/D) \sim \frac{2}{A}B^{np}_{l=0}(A)$ (number of *pn* pairs in a relative $|n=0 \updownarrow = 0\rangle$ state)

Corrections?

Corrections to the ratio's of cross-section data which affect the extracted value of $a_2(A/D)$: unlike the deuteron

- $A-2$ fragment can be left with excitation energy
- Pairs have c.o.m. motion
- Final-state interactions on the ejected two nucleons (?)
- Contribution of the pp and nn correlations (small)

Effect of $A-2$ excitation energy



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simulations of breakup of 2N correlated pairs in ^{12}C for $\epsilon = 5.766$ GeV and $\langle Q^2 \rangle$=2.7 GeV^2

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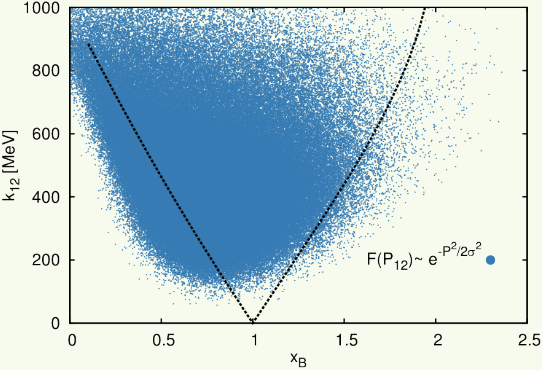
Effect of c.m. motion of pn pairs



MC simulations of breakup of 2N correlated pairs in ^{12}C for $\epsilon = 5.766$ GeV and $\langle Q^2 \rangle$=2.7 GeV^2

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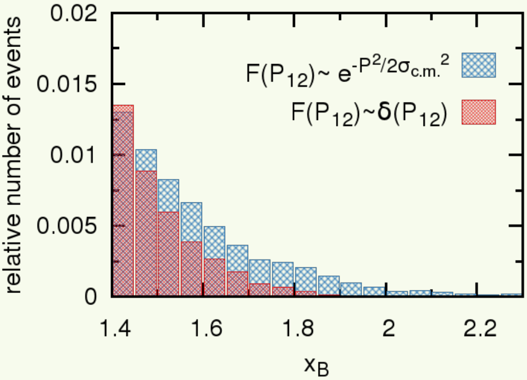
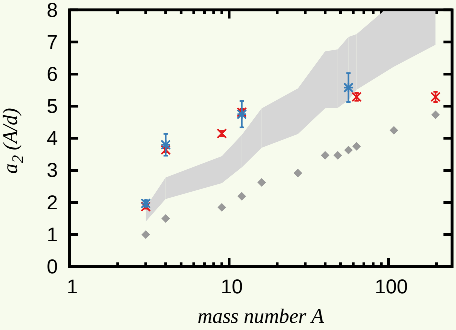


A	$\sigma_{c.m.}$	c.m. correction factor
^{12}C	115 MeV	1.64 ± 0.23
^{56}Fe	128 MeV	1.70 ± 0.27
^{208}Pb	141 MeV	1.71 ± 0.29

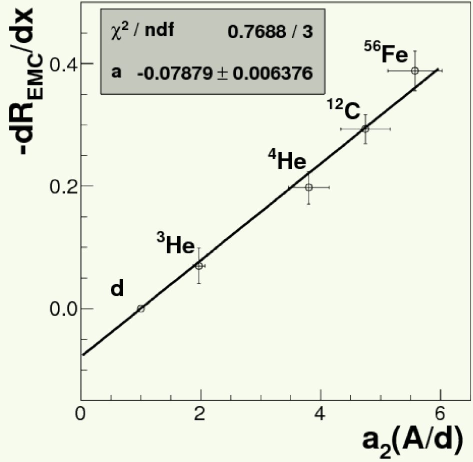
$A(e,e')$ for $1.4 < x_{BJ}$ and 2N SRC's



Data: CLAS **PRL96** 082501 (2006), N.Fomin et al. **PRL108** 092502 (2012)

- Correction of the c.m. motion applied to the computed values of $a_2(A/D)$
- Prediction: $a_2(^{40}\text{Ca}) \approx a_2(^{48}\text{Ca})$
- Missing strength at low A due to clustering?
- overestimation at high A.

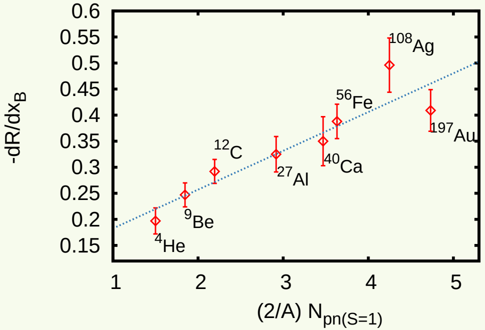
Experimental Observations III: EMC vs SRC



- Recent observation that $a_2(A)$ and EMC slope show a linear correlation.
- Suggests that both phenomena might be driven by local density fluctuations.
- Number of relative S-pairs per nucleon shows linear correlation with EMC slopes.

L.B.Weinstein et al. **PRL106** 052301 (2011)

Experimental Observations III: EMC vs SRC



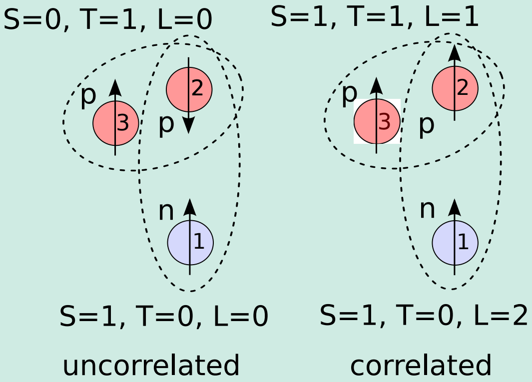
$$-\frac{dR}{dx_B} = (0.108 \pm 0.028) + \frac{2}{A} N_{pn(S=1)} \cdot (0.074 \pm 0.010).$$

- Recent observation that $a_2(A)$ and EMC slope show a linear correlation.
- Suggests that both phenomena might be driven by local density fluctuations.
- Number of relative *S*-pairs per nucleon shows linear correlation with EMC slopes.

Can one quantify the number of 3*N* correlations? (1)

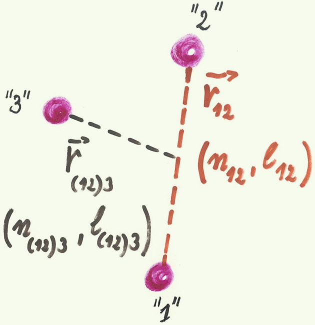
Three-body correlations induced by tensor correlations

S=1,T=1 pairs originate from 3N correlations



Feldmeier, Horiuchi, Neff, Suzuki: arXiv:1107.4956

Can one quantify the number of 3N correlations? (2)



- seek for those wave-function components where all three nucleons are "close"
- requires transformation from $(\vec{r}_1, \vec{r}_2, \vec{r}_3)$ to $\left(\vec{r}_{12}, \vec{r}_{(12)3}, \vec{R}_{123}\right)$ (Jacobi coordinates)

$$\vec{r}_{12} = \frac{\vec{r}_1 - \vec{r}_2}{\sqrt{2}}, \quad \vec{R}_{12} = \frac{\vec{r}_1 + \vec{r}_2}{\sqrt{2}}$$

$$\vec{r}_{(12)3} = \frac{\vec{R}_{12} - \sqrt{2}\vec{r}_3}{\sqrt{3}},$$

$$\vec{R}_{123} = \frac{\sqrt{2}\vec{R}_{12} + \vec{r}_3}{\sqrt{3}},$$

Can one quantify the number of $3N$ correlations? (3)

In a HO basis one can perform the transformation from $(\vec{r}_1, \vec{r}_2, \vec{r}_3)$ to $\left(\vec{r}_{12}, \vec{r}_{(12)3}, \vec{R}_{123}\right)$

$$\left| n_1 l_1 \left(\vec{r}_1\right), n_2 l_2 \left(\vec{r}_2\right), n_3 l_3 \left(\vec{r}_3\right) \right\rangle$$

$$\left| n_{12} l_{12} \left(\vec{r}_{12}\right), N_{12} \Lambda_{12} \left(\vec{R}_{12}\right), n_3 l_3 \left(\vec{r}_3\right) \right\rangle$$

$$\left| n_{12} l_{12} \left(\vec{r}_{12}\right), n_{(12)3} l_{(12)3} \left(\vec{r}_{(12)3}\right), N_{123} \Lambda_{123} \left(\vec{R}_{123}\right) \right\rangle$$

by making use of Moshinsky Brackets and Standard Transformation Brackets (STB)

Can one quantify the number of $3N$ correlations? (3)

- Antisymmetrized three-nucleon states

$$
\begin{aligned}
|\alpha_a m_a, \alpha_b m_b \alpha_c m_c\rangle_{as} &= [1 - P_{12}] \left|\alpha_a m_a (\vec{r}_1)\, \alpha_b m_b (\vec{r}_2)\, \alpha_c m_c (\vec{r}_3)\right\rangle \\
&+ [1 - P_{12}] \left|\alpha_b m_b (\vec{r}_1)\, \alpha_c m_c (\vec{r}_2)\, \alpha_a m_a (\vec{r}_3)\right\rangle \\
&+ [1 - P_{12}] \left|\alpha_c m_c (\vec{r}_1)\, \alpha_a m_a (\vec{r}_2)\, \alpha_b m_b (\vec{r}_3)\right\rangle
\end{aligned}
$$

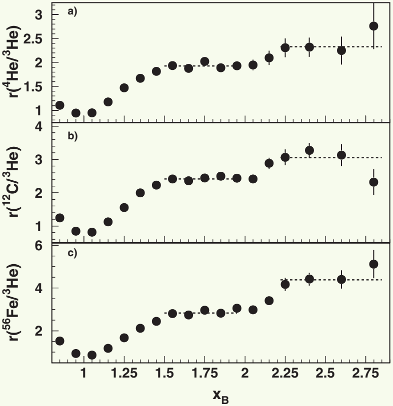
- One has

$$
\sum_{\alpha_a,\alpha_b \leq \alpha_F^p} \sum_{\alpha_c \leq \alpha_F^n} \sum_{m_a m_b m_c \mathrm{nas}} \langle \alpha_a m_a \alpha_b m_b \alpha_c m_c \left| \alpha_a m_a \alpha_b m_b \alpha_c m_c \right\rangle_{\mathrm{nas}}
$$

$$
= \quad N \frac{Z(Z-1)}{2} \, .
$$

- For given $A(N,Z)$ the antisymmetrized (ppn) states with $(n_{12} = 0 \; l_{12} = 0 n_{(12)3} = 0 \; l_{(12)3} = 0)$ are prone to SRC

$A(e,e')$ for $2.2 < x_{BJ}$ and 3N SRC's



Hall-B, PRL **96**, 082501 (2006)

- ratio of raw data

$$R(A,^3He) = \frac{3}{A}\frac{\sigma^A(x_B,Q^2)}{\sigma^{^3He}(x_B,Q^2)} \,,$$

- acceptance and $\sigma_{ep} \neq \sigma_{en}$ corrections: $r(A,^3He)$
- very naive counting: all *ppn* pairs contribute $r(A,^3He) \sim A^2$

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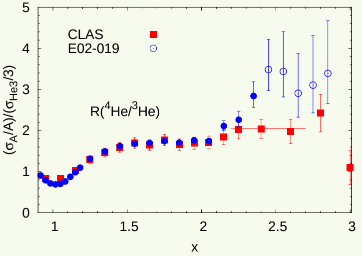

Hall-C, arXiv:1107.3583

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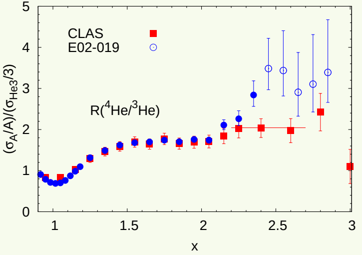
Suggestion: $a_3(A/^3He) = R(A,^3He) \approx \frac{3}{A} B^{ppn}_{l_{12}=0,l_{(12)3}=0}(A)$ (number of *ppn* pairs in a $|n_{12}=0\ l_{12}=0, n_{(12)3}=0\ l_{(12)3}=0\rangle$ state)

$A(e,e')$ for $2.2 < x_{BJ}$ and 3N SRC's



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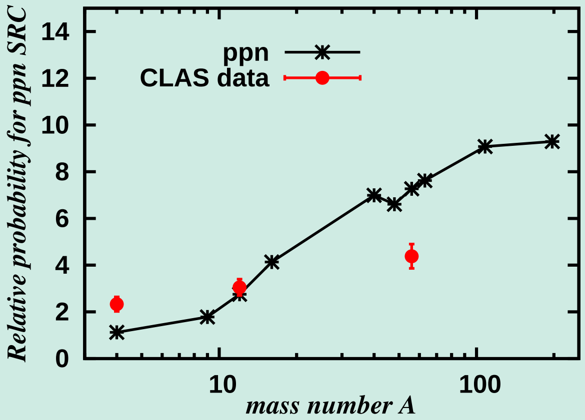
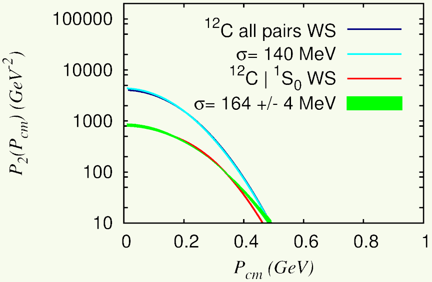
$A(e,e')$ for $2.2 \lesssim x_B$ and 3N SRC's II

$a_3(A/^3He)$ as a measure of the per-nucleon probability of ppn SRC relative to ^{3}He *(calculations are NOT corrected for c.m. motion, FSI, ...)*



C.o.m. pair momentum distribution from correlated *pp* events in data mining



- Compare width of pair c.o.m. mom. distribution for all possible *pp* pairs and only pairs with relative *S* quantum numbers
- Robust results for HO and WS wave functions
- Significant difference in width between the two, $^1S_0$ pairs give bigger width
- $^1S_0$ widths agree very nicely with extracted values from the data mining

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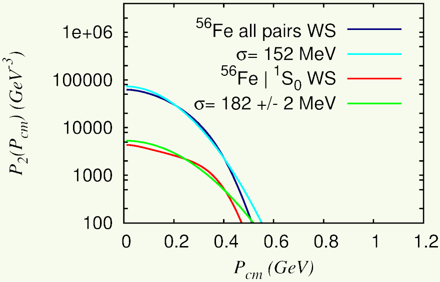

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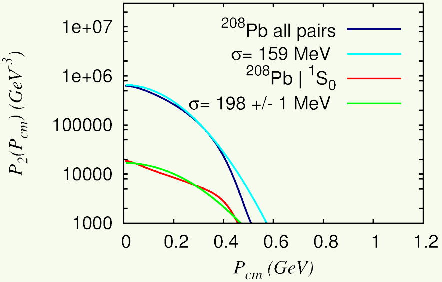

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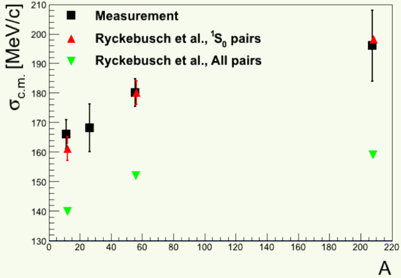

Plot from Or Hen

- Compare width of pair c.o.m. mom. distribution for all possible *pp* pairs and only pairs with relative *S* quantum numbers
- Robust results for HO and WS wave functions
- Significant difference in width between the two, $^1S_0$ pairs give bigger width
- $^1S_0$ widths agree very nicely with extracted values from the data mining

Conclusions

- Suggestion that the number of correlated nucleon pairs in a nucleus $A(N, Z)$is proportional with the number of relative S states *(nucleon pairs are prone to correlations when they are "close")*
- The number of relative $l = 0$ states follows a power law $d \times A^{1.44 \pm 0.01}$
- Power law is robust: independent of choices for mean-field wave functions
- The computed number of $(\mathcal{L} = 0, T = 0)$ *pn* can be used to predict the $a_2 \left(\frac{A}{D}\right)$.
- Influence of c.o.m. motion quantified.
- Predictions for a_2 are not inconsistent with trends and magnitude of the data

Conclusions (2)

- Technique can be extended to count the number of correlated nucleon triples in a nucleus
- Tag correlated pairs by looking at the width of c.o.m. momentum distribution, good agreement with data mining values
- Future!: reaction model for two nucleon knockout at high energies with FSI, delta d.o.f., etc.

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Thanks!