Many-body aspects of the two-site Bose-Hubbard model

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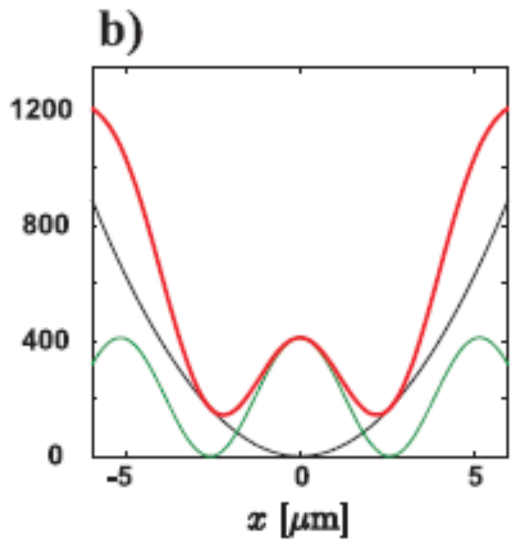
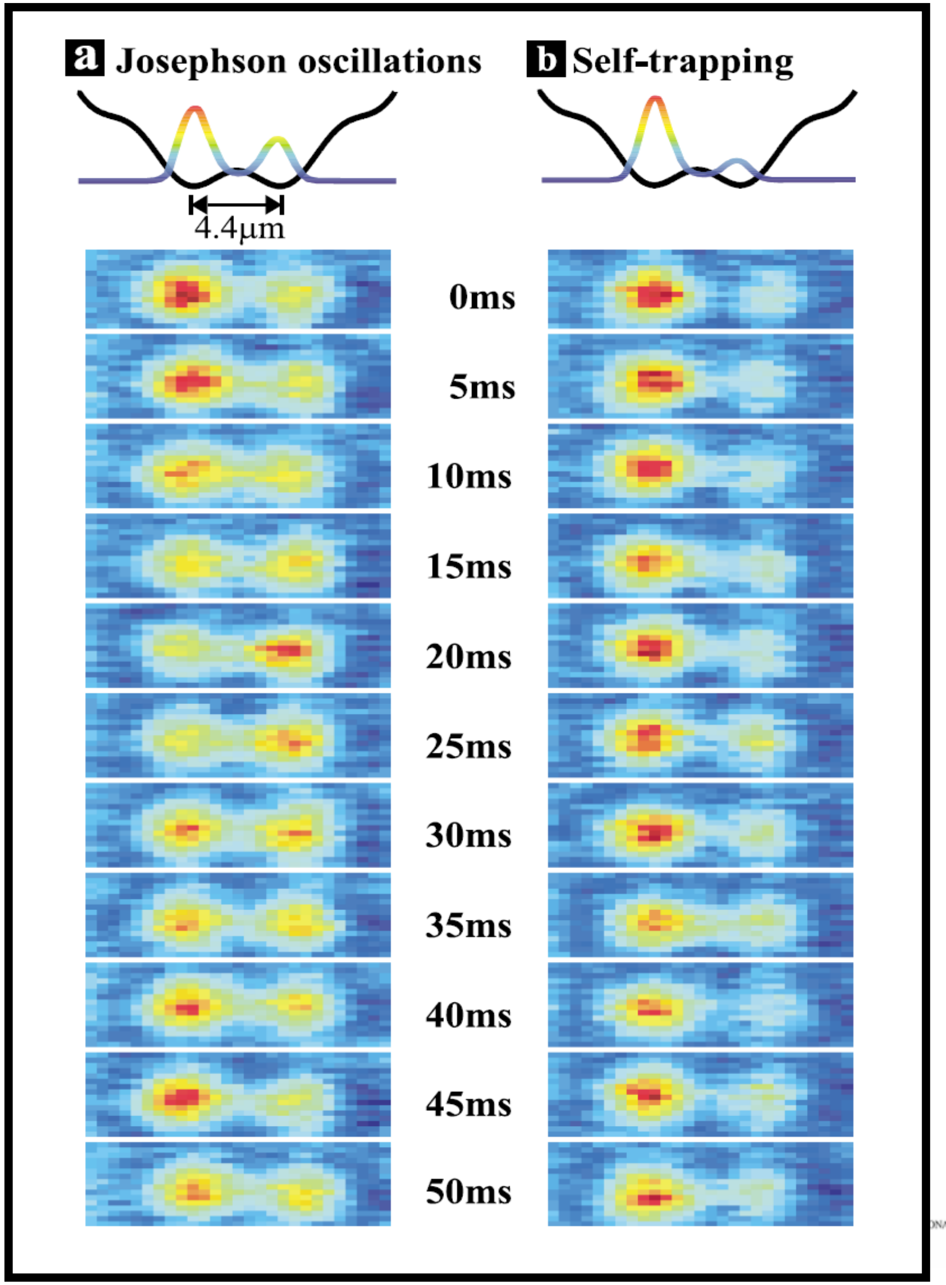
Experimentally…

Albiez et al. Phys. Rev. Lett. 95,010402 (2005)





Summary

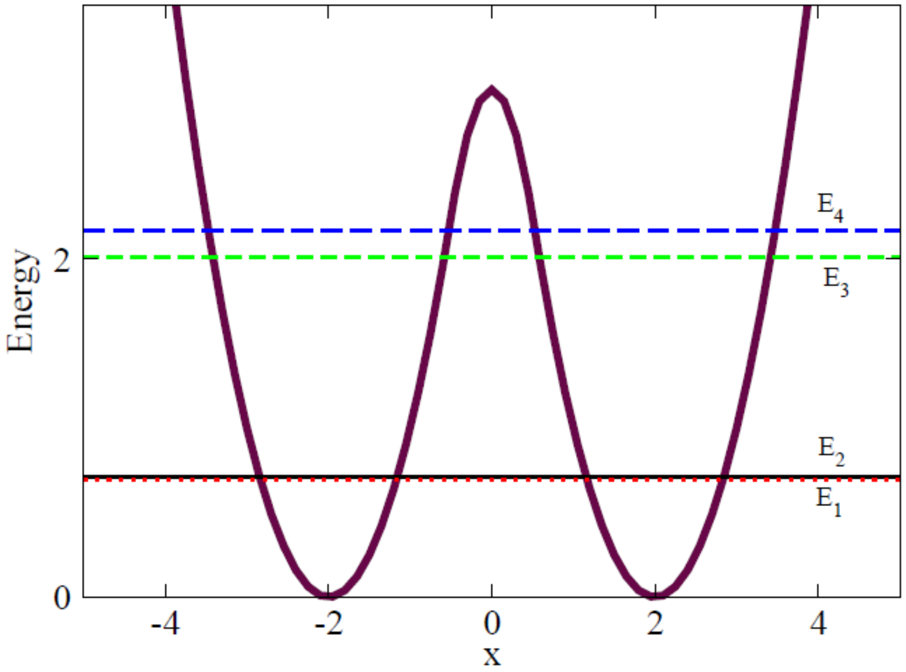
1. Introduction. Josephson's oscillations in a bosonic junction.
2. Single component case. GP.
3. Static properties of the two-site Bose-Hubbard Hamiltonian
4. Mean-field vs exact dynamics
5. Beyond standard two-mode dynamics.

Josephson in BECs

- Lets consider a cigar-shaped cloud of ultracold bosons trapped by a double-well potential along the x-direction

- The atom-atom scattering is assumed to be well represented by a contact interaction

- The single particle hamiltonian (kinetic + external double-well) has a quasidegenerate doublet (1,2) and two more states below the barrier (3,4)



Mean field description

- For large enough number of atoms (>1000) a mean-field approach describes the relevant physics (time dependent **Gross-Pitaevskii** equation)

$$i\hbar\frac{\partial\Psi(\mathbf{r};t)}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) + gN|\Psi(\mathbf{r};t)|^2\right]\Psi(\mathbf{r};t)$$

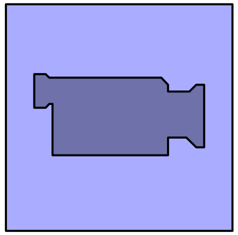
- **Ψ**: wave function shared by all atoms in the cloud (normalized to 1)
- **N**: Total number of atoms (assumed constant)
- **g**: coupling constant measuring the strength of the atom-atom contact interaction (proportional to the s-wave scattering length)
- **V(r)**, external trapping potential (double-well)

One simulation

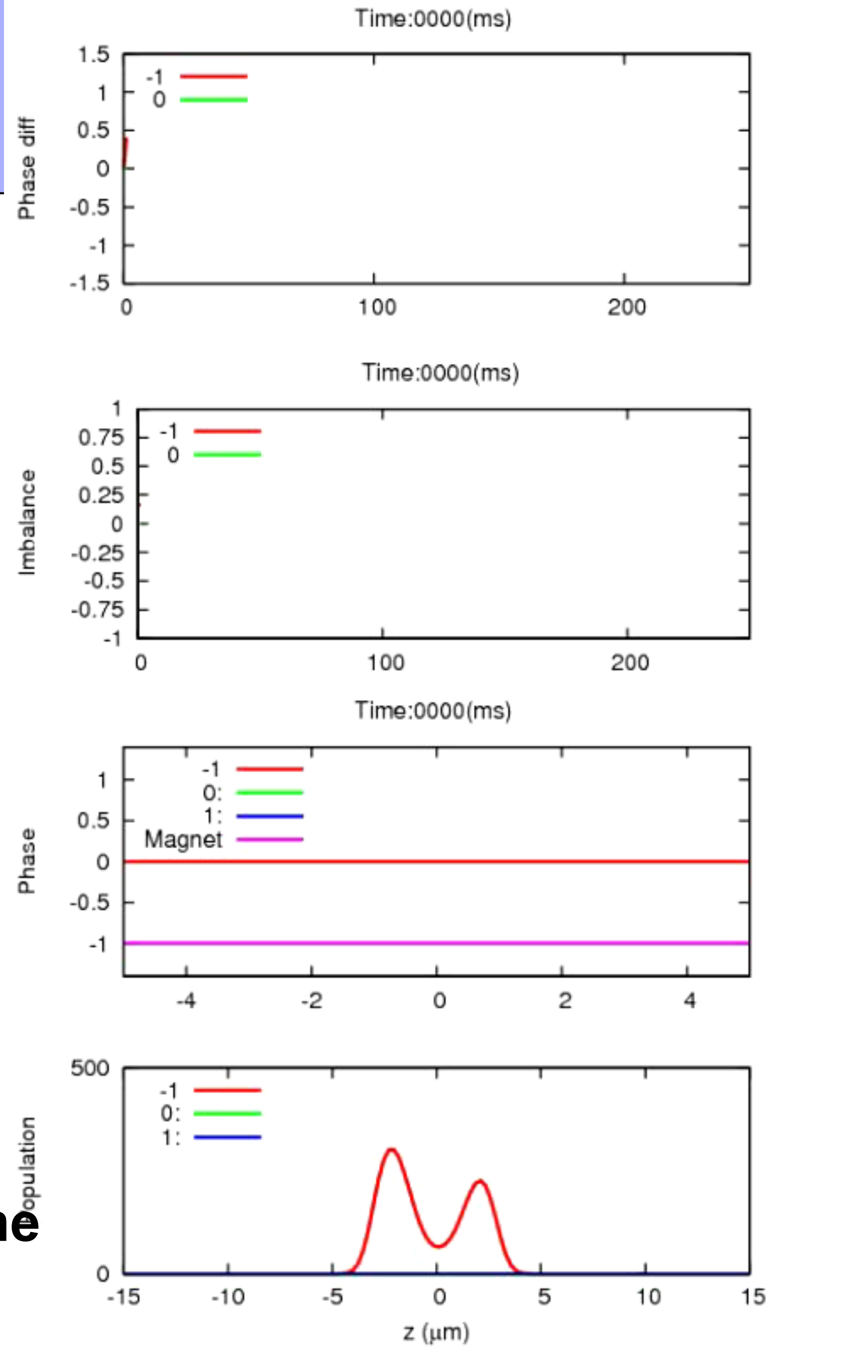
- N~1000 atoms ^{87}Rb, trap conditions as Heidelberg experiments (Albiez 2005)

$$i\hbar\frac{\partial\Psi(\mathbf{r};t)}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) + gN|\Psi(\mathbf{r};t)|^2\right]\Psi(\mathbf{r};t)$$

- Definitions (usual),
  - $Z(t) = (N_{left}(t)-N_{right}(t))/N_{total}$
  - Phase difference= $\delta\phi=\phi_{right}-\phi_{left}$
- Note:
  - Phase coherence at each side
  - Clear coupling between $\delta\phi$ and $Z(t)$



The dynamics in this regime is essentially bimodal!!

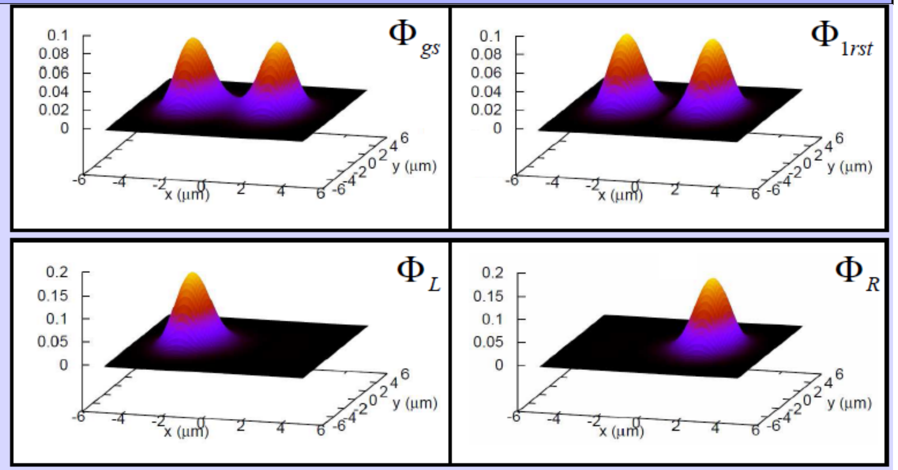


□Usual two mode ansatz

Introduce the following ansatz,

$$\Psi(\mathbf{r};t) = \Psi_L(t)\Phi_L(\mathbf{r}) + \Psi_R(t)\Phi_R(\mathbf{r}).$$

$$\Psi_j(t) = \sqrt{N_j(t)}\, e^{i\phi_j(t)}$$



One gets (neglecting certain overlaps) a coupled system:

$$i\hbar\frac{\partial\Psi_L(t)}{\partial t} = E_L^0\,\Psi_L(t) + U|\Psi_L(t)|^2\Psi_L(t) - K\Psi_R(t)$$

$$i\hbar\frac{\partial\Psi_R(t)}{\partial t} = E_R^0\,\Psi_R(t) + U|\Psi_R(t)|^2\Psi_R(t) - K\Psi_L(t)$$

$E^0{}_L=E^0{}_R$ $$E^0_{L(R)} = \int d\mathbf{r}\left[\frac{\hbar^2}{2m}|\nabla\Phi_{L(R)}(\mathbf{r})|^2 + \Phi^2_{L(R)}(\mathbf{r})\,V(\mathbf{r})\right],$$

Tunneling $$K = -\int d\mathbf{r}\left[\frac{\hbar^2}{2m}\nabla\Phi_L(\mathbf{r})\cdot\nabla\Phi_R(\mathbf{r}) + \Phi_L(\mathbf{r})\,V(\mathbf{r})\,\Phi_R(\mathbf{r})\right],$$

$U=U_L=U_R$ $$U_{L(R)} = g\int d\mathbf{r}\,\Phi^4_{L(R)}(\mathbf{r}).$$

Atom-atom interaction

Smerzi et al. (1997)
Raghavan et al (1998)
Zapata et al (1998)
See review by Leggett (2001)

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□Usual two mode ansatz

$$\dot{z}(t) = -\sqrt{1 - z^2(t)}\ \sin\delta\phi(t),$$

$$\delta\dot{\phi}(t) = \Lambda z(t) + \frac{z(t)}{\sqrt{1 - z^2(t)}}\ \cos\delta\phi(t).$$

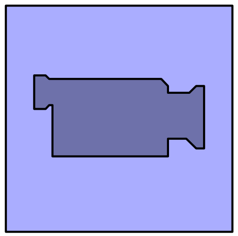
z(t) :	population imbalance, $(N_L(t)-N_R(t))/N$
$\delta\phi(t)$:	phase difference, $\phi_R-\phi_L$
$\Lambda=NU/(2K)$	Ratio between the interaction term and Rabi term

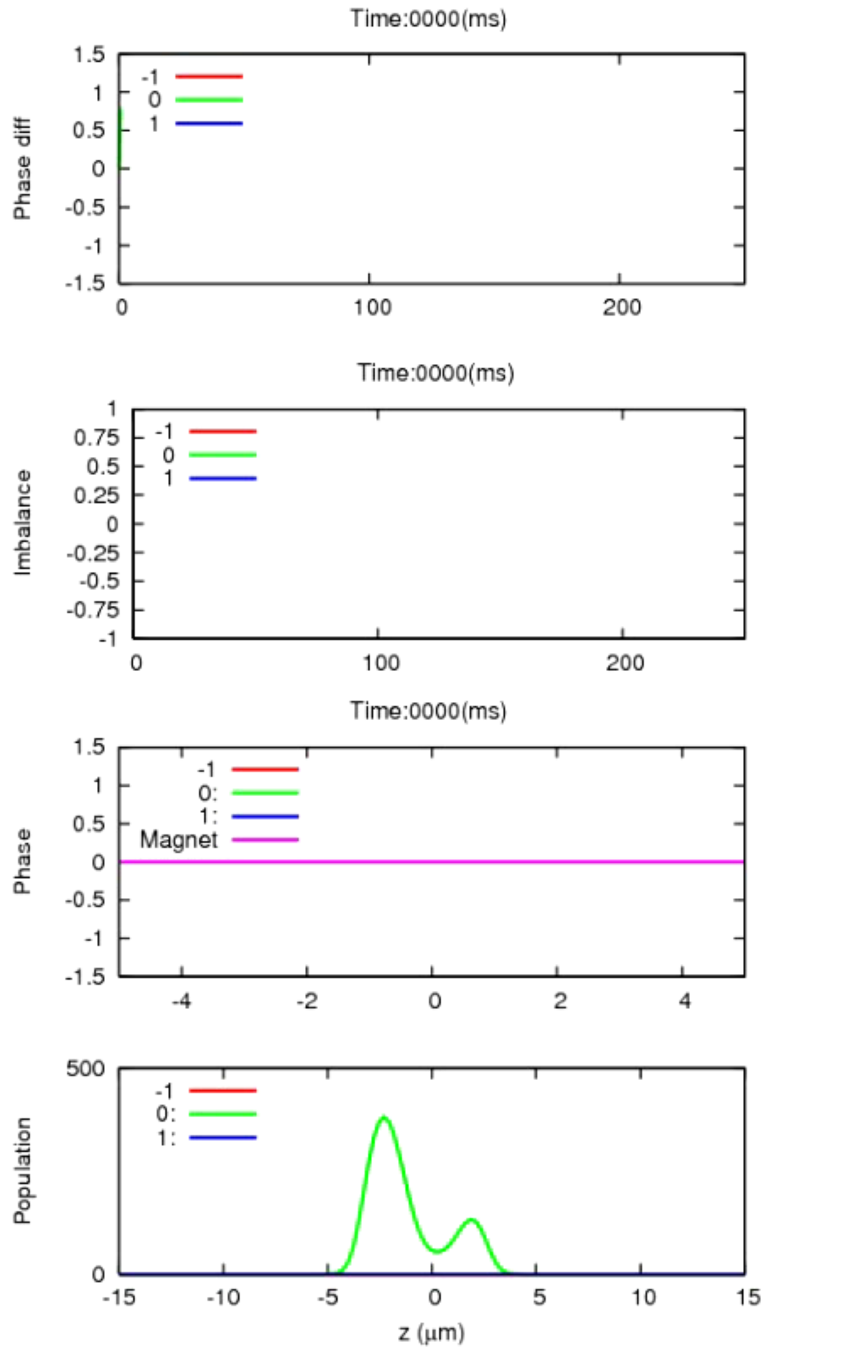
□There are different regimes depending on the value of Λ , and the initial values of the population imbalance and phase difference

Self trapping (GP)

- 1150 atoms, trap conditions as before
- If the initial imbalance is large enough, no Josephson oscillation occurs. Instead a self trapping regime appears

Smerzi et al. (1997).





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Experimentally…

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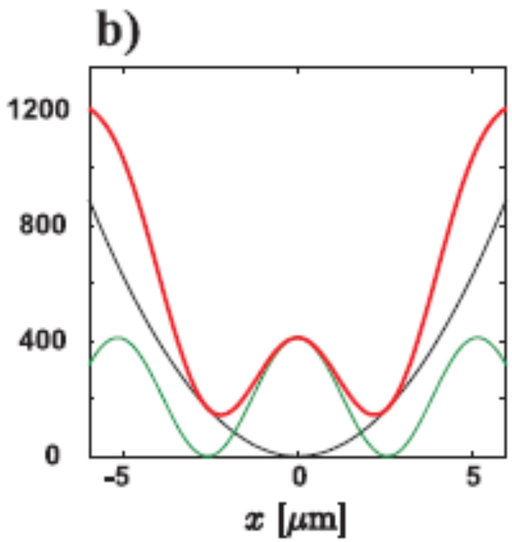
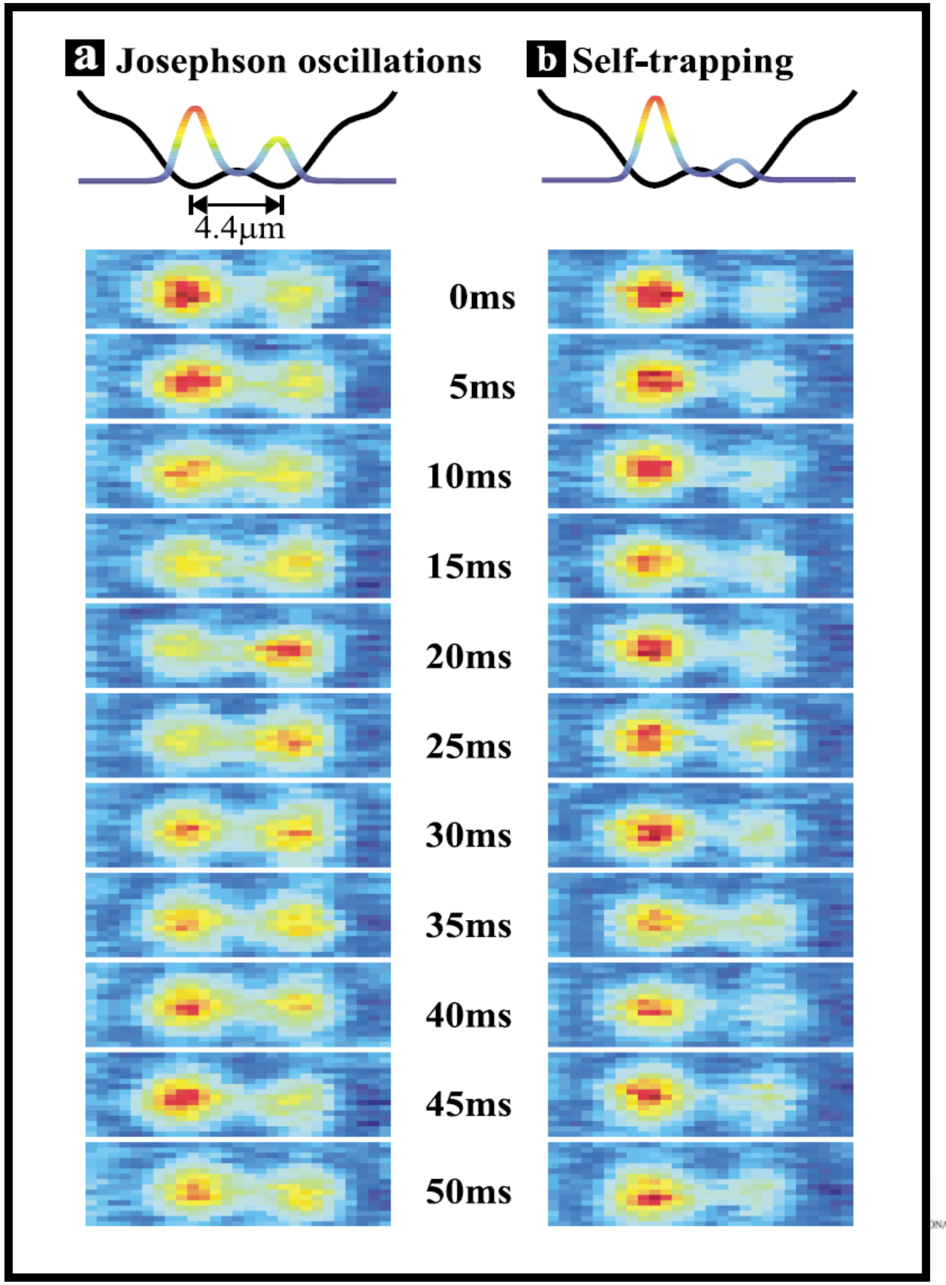
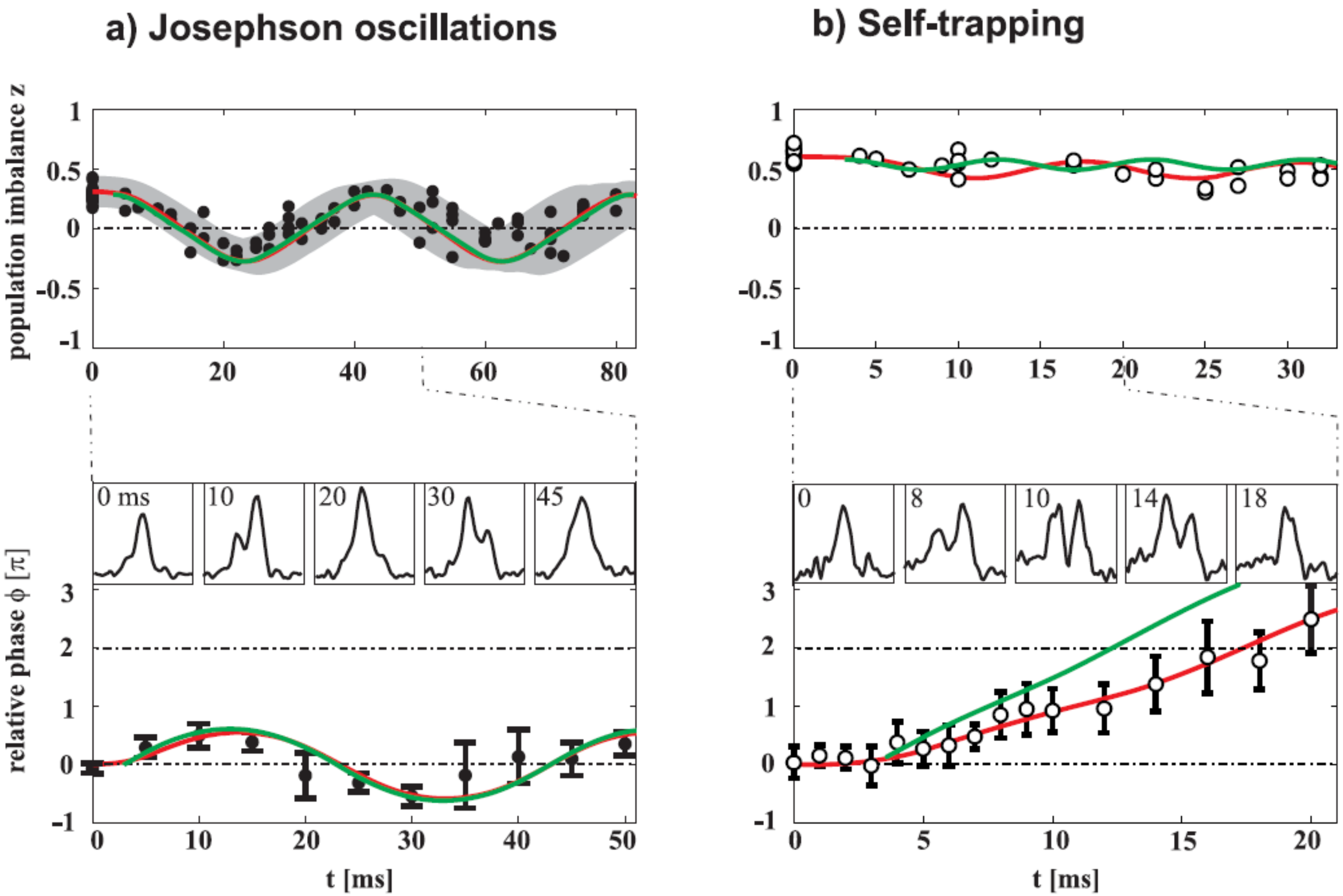






Albiez Thesis

A simple, but many-body H

Lets consider the following two-site Bose-Hubbard model:

$$\begin{aligned} H &= \frac{-U}{2}\left(\hat{n}_L(\hat{n}_L - 1) + \hat{n}_R(\hat{n}_R - 1)\right) \\ &- J\left(a_R^\dagger a_L + a_L^\dagger a_R\right) - \epsilon(\hat{n}_L - \hat{n}_R) \end{aligned}$$

J: hopping parameter >0

U: atom-atom interaction >0 (proportional to **g**)(attractive)

Epsilon: Bias>0, promotes the left well

The bias is here taken very small, Epsilon<<J

It is customary to define, Λ=NU/J

Milburn et al (1997)

semiclassics

The semiclassics is governed by the well known:

$$\frac{\dot{z}(t)}{2J} = -\sqrt{1-z^2}\sin\varphi$$
$$\frac{\dot{\varphi}(t)}{2J} = -\frac{\Lambda}{2}z + \frac{z}{\sqrt{1-z^2}}\cos\varphi\,;$$

$$a_{L(R)} = \sqrt{n_{L(R)}}e^{\imath\varphi_{L(R)}}$$

Heisenberg equations of motion

z: population imbalance, $(N_L-N_R)/N$

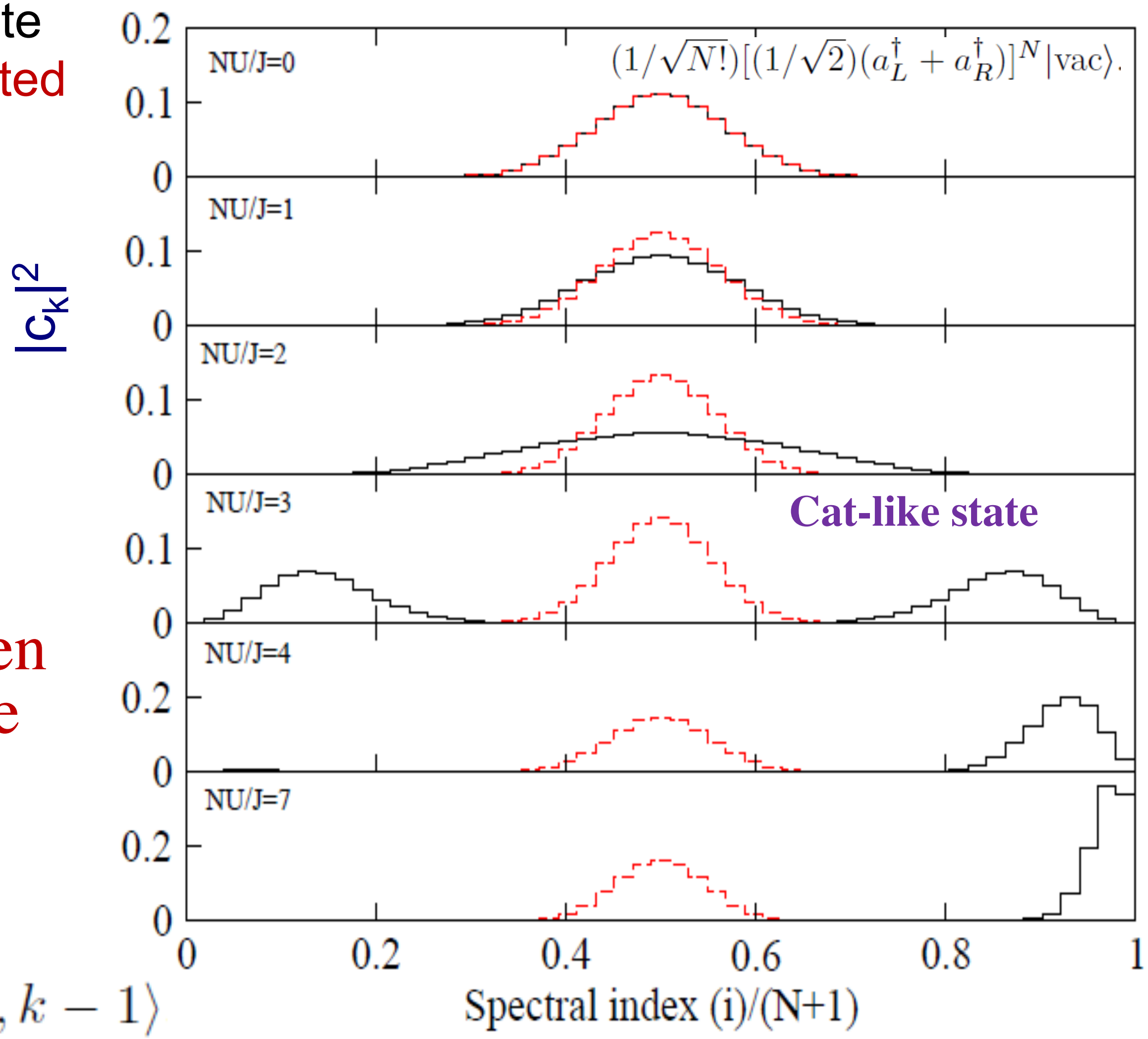
φ: phase difference, $\varphi_R-\varphi_L$

2J: Rabi time (the time it takes for the atoms to go from left to right and back in absence of atom-atom interactions)

Smerzi et al. (1997) (Assuming a two mode ansatz for the Gross Pitaevskii equation)

Ground and highest excited state

Black, ground state
Red, highest excited



With the usual base:

$|N_L,N_R\rangle = \{|N-k,k\rangle\} =$
$\{ |N,0\rangle, |N-1,1\rangle, \ldots, |0,N\rangle \}$

The hamiltonian can be written as an N+1 square matrix (here 50+1)

Any N particle vector can be

$|\Psi\rangle = \sum_{k=1,N+1} c(k)|N+1-k, k-1\rangle$

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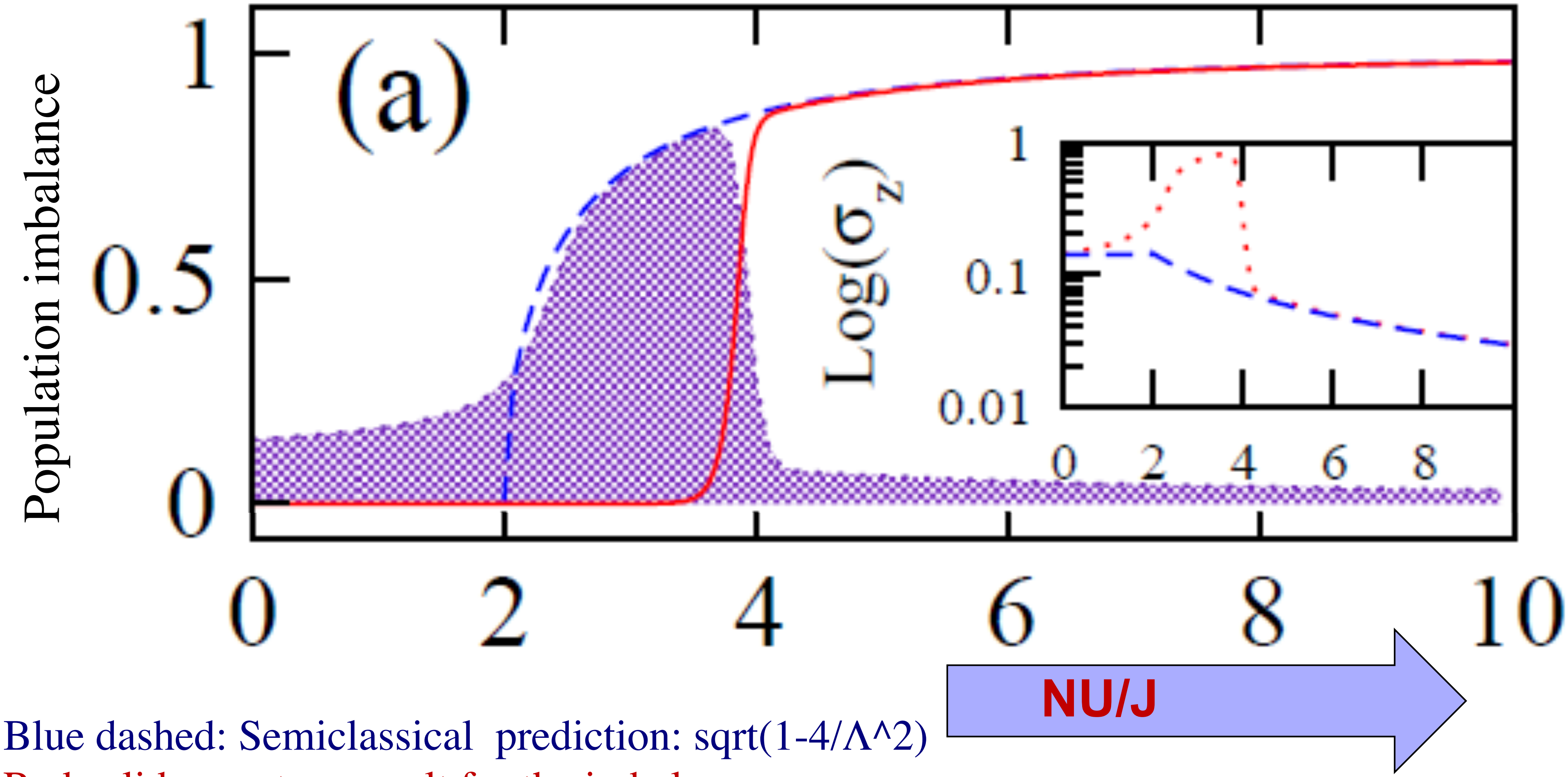
Ground state: imbalance



Blue dashed: Semiclassical prediction: sqrt(1-4/Λ^2)

Red solid: quantum result for the imbalance

Band: dispersion of the imbalance

N=50, bias=J/10^10

One body density matrix

The one body density matrix reads,

$$\rho = \frac{1}{N}\begin{pmatrix} a_L^+ a_L & a_L^+ a_R \\ a_R^+ a_L & a_R^+ a_R \end{pmatrix}$$

Eigenvalues, $n_1 + n_2 = 1$

If the system is fully condensed, $\Psi_{\mathrm{MF}} = [|\Psi_1(\theta,\phi)\rangle]^{\otimes N}$
then the eigenvalues are 1 and 0.
The eigenvector corresponding to 1 is, $|\Psi_1(\theta,\phi)\rangle$

Departure from 0,1 indicates the system is fragmented

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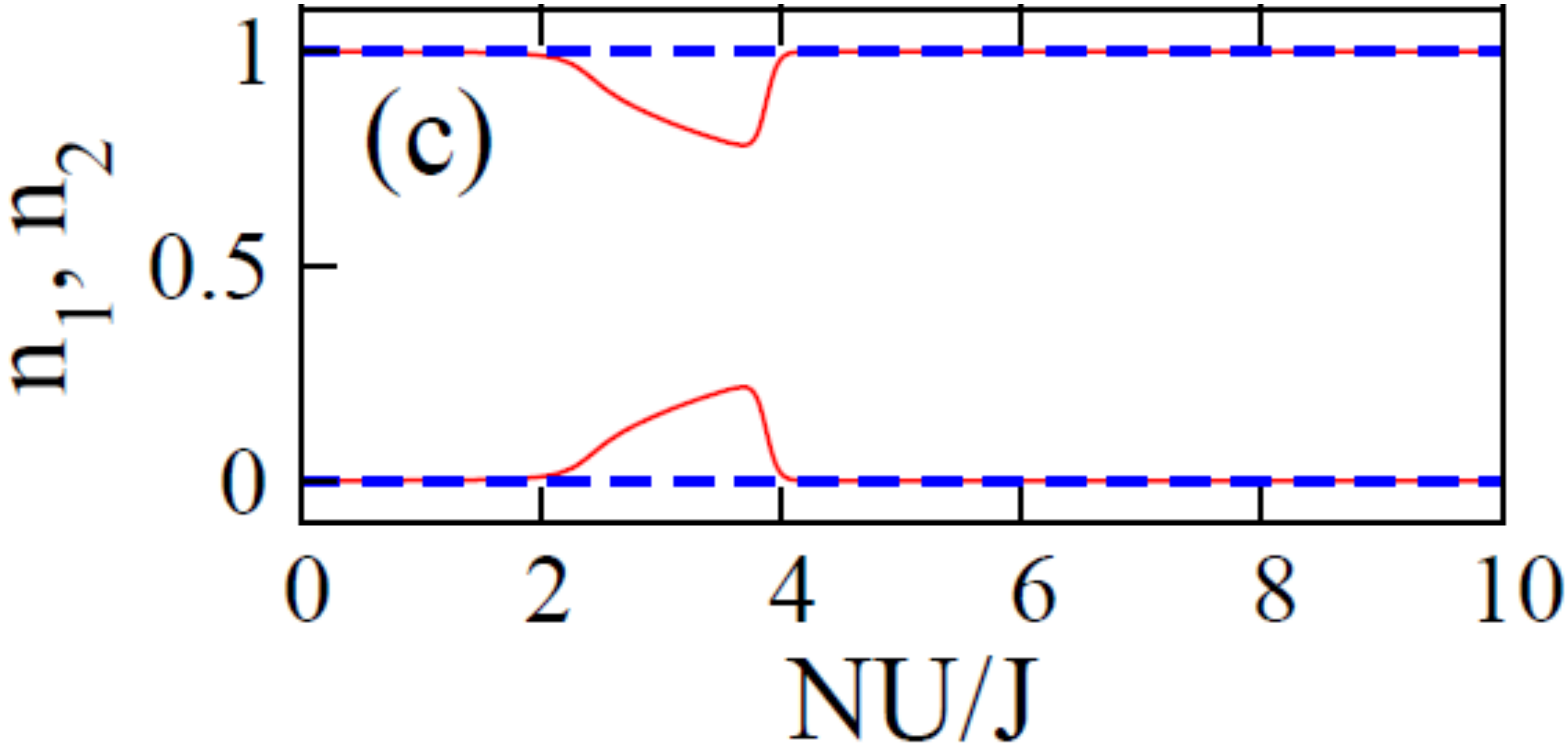
Occupations of the orbitals

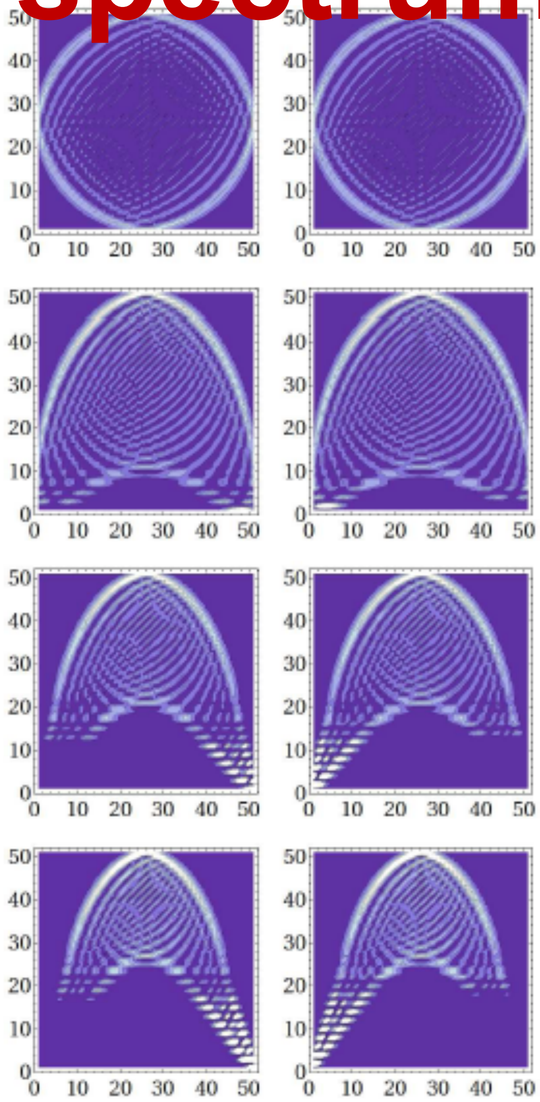


Blue dashed: Semiclassical prediction → 1,0

Red solid: quantum result for the eigenvalues of the one body density matrix

N=50, ε=J/10^10

Properties of the whole spectrum



$\Lambda=0$

GS: binomial

$\Lambda=4$

GS: Cat-like

$\Lambda=8$

GS: Trapped

$\Lambda=12$

GS: Trapped

In the plot,

x-axis: k index

y-axis: eigenvector index

1, ground,

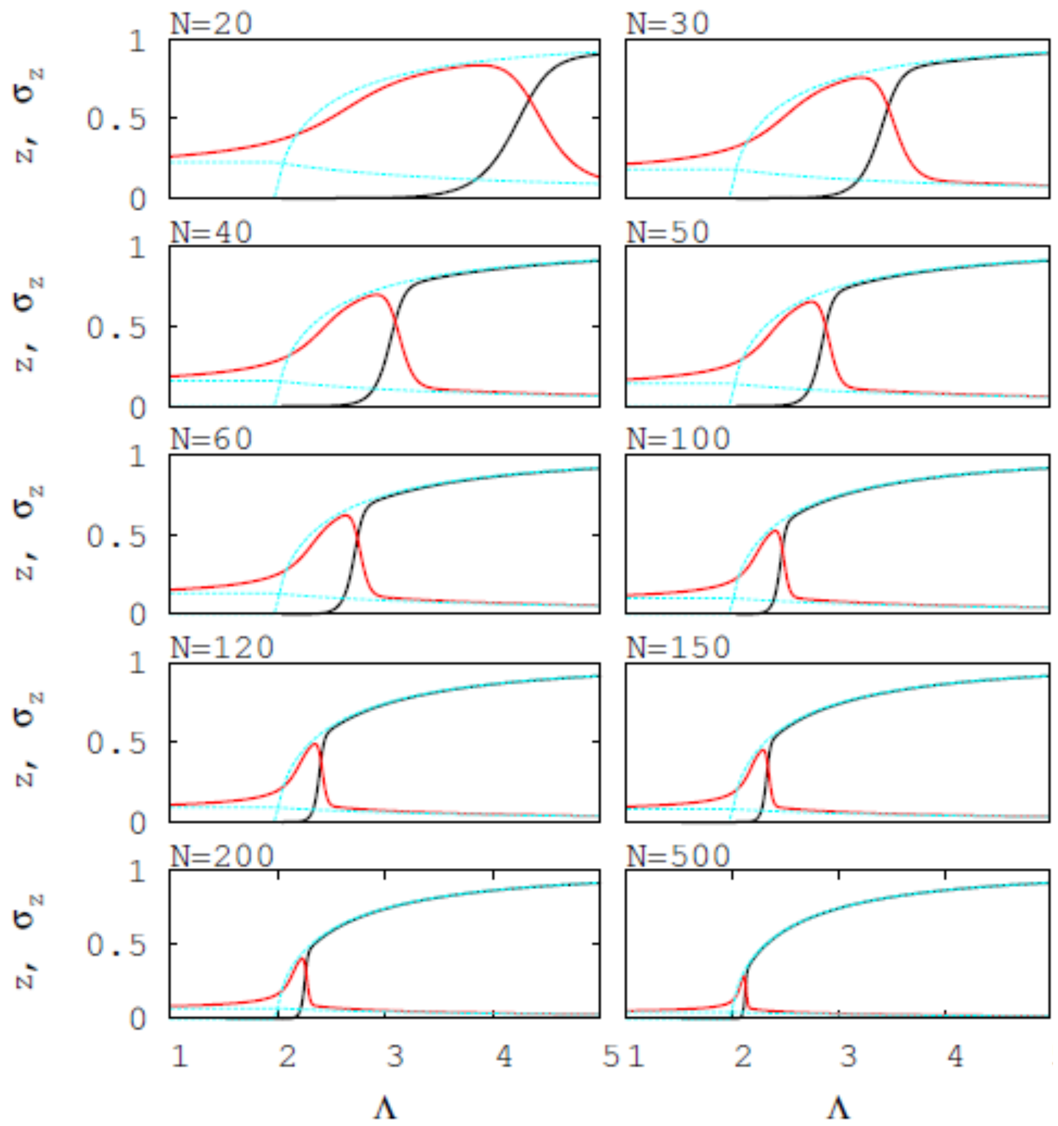
N+1 highest excited

Blue shading corresponds to zero

And white to the maximum value of $|c_k|^2$

N=50, bias=J/10^10

Variation with N



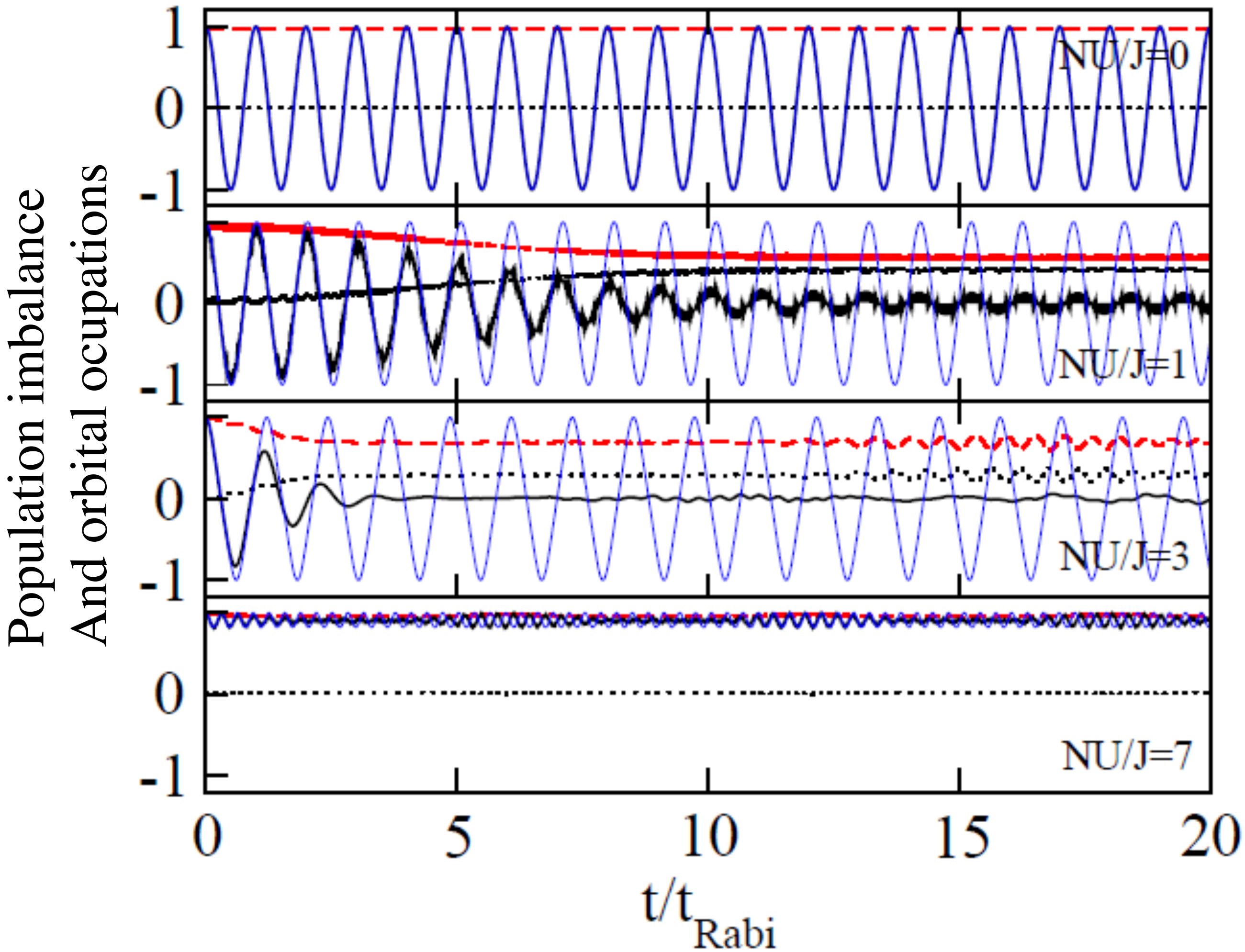
The semiclassical behavior is the same in all cases (the bias is taken the same)

The size of the highly disperse region decreases as N is increased

Time evolution of |N,0>

For fixed N and starting from a 'mean-field' like state:

- The smaller the interaction, the better the mean-field describes the exact result.
- Fragmentation builds up with time



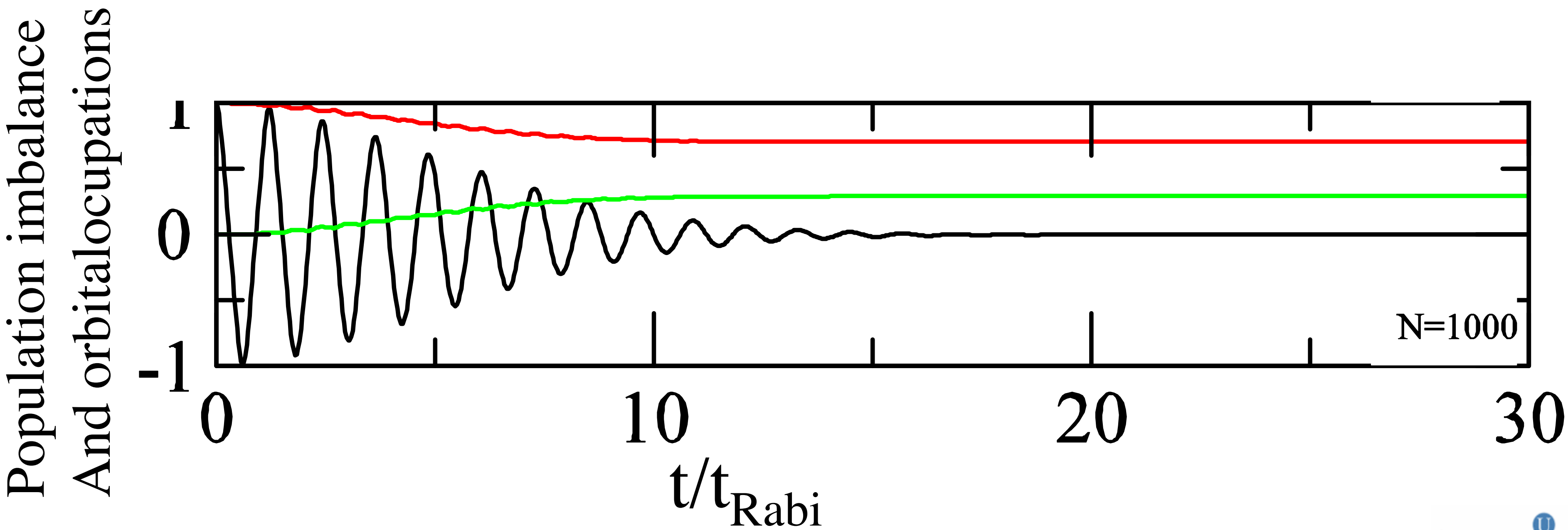
Blue solid: Semiclassical
Black solid: quantum for the imbalance
Red dashed: n_1, black dotted, n_2
N=50, epsilon=J/10^10

Self-trapping due to the large overlap of |N,0> with the g.s of H

Time evolution of |N,0>

When starting from a ‘mean-field state’:

- For large N (here 1000), the mean field provides an excellent account of the full dynamics during long times (here almost two Rabi periods)
- The cloud, thus, remains condensed for a while.



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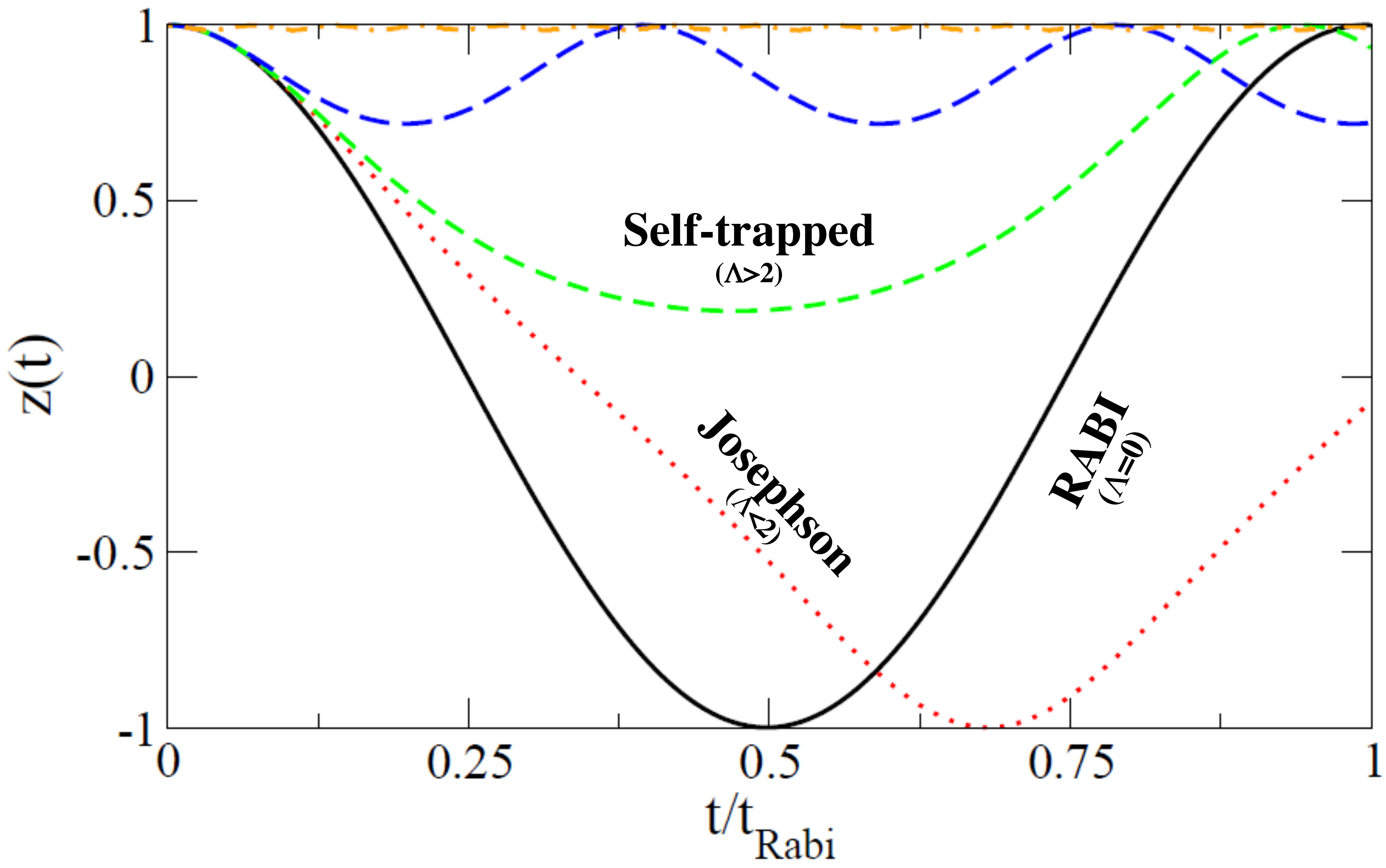
□Usual two mode ansatz



□Beyond the usual two-modes

- Lets still consider a large enough number of atoms (>1000), so that the mean field description remains valid

- Consider in all cases the same initial condition
  - All atoms are on the left well at t=0

- Study the dynamics as we increase the non-linear term (g_{1D}). Either by increasing **N** or **g**

- The time dependent GP ,

$$\imath \frac{\partial}{\partial t}\psi(x,t) = -\frac{1}{2}\frac{\partial^2}{\partial x^2}\psi(x,t) + V_{\text{eff}}[\psi(x,t)]\psi(x,t).$$

$$V_{\text{eff}}[\psi(x,t)] = V(x) + \lambda_0 N|\psi(x,t)|^2 \qquad g_{1D} \equiv \lambda_0 N,$$

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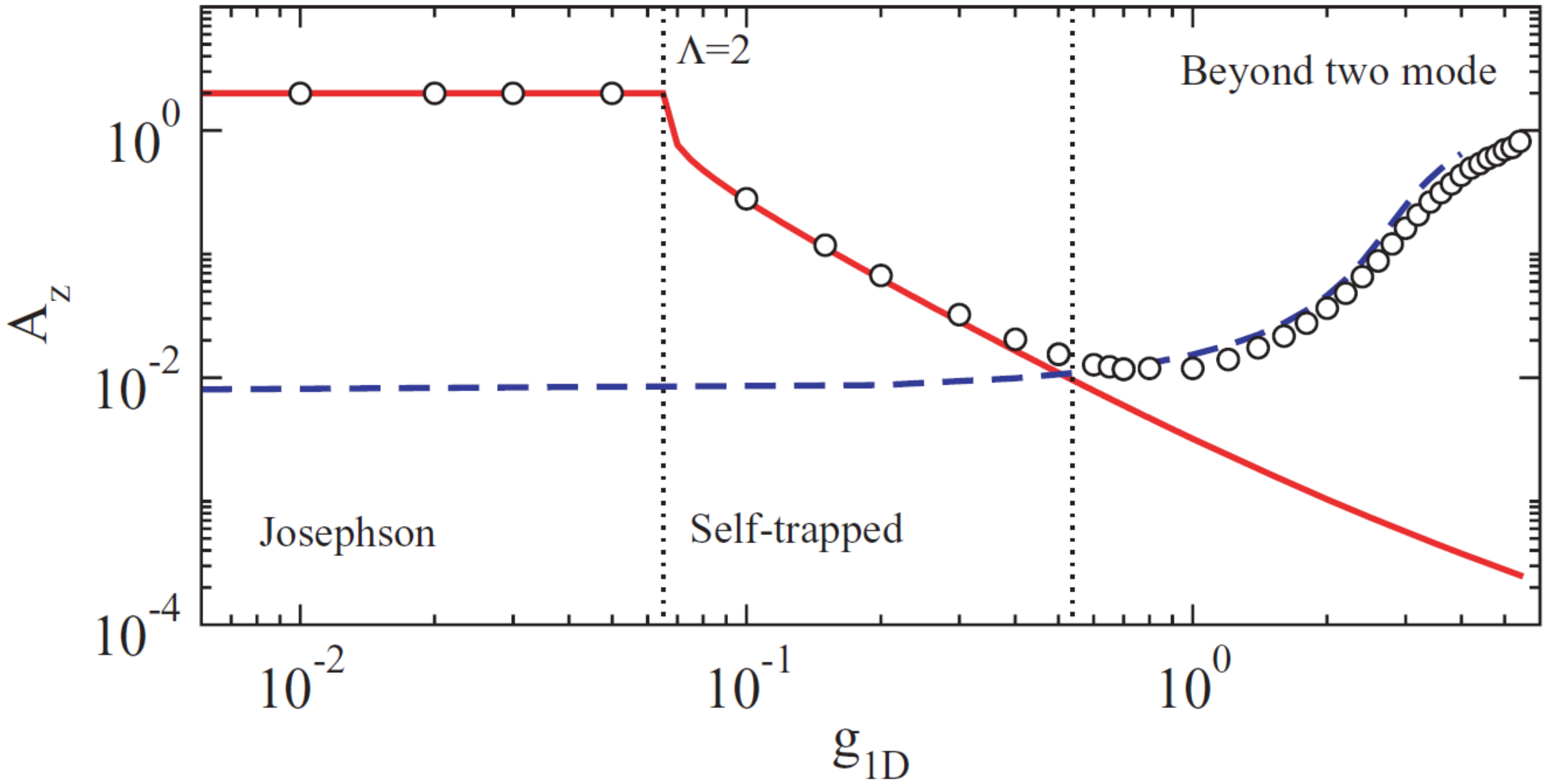
□Different regimes

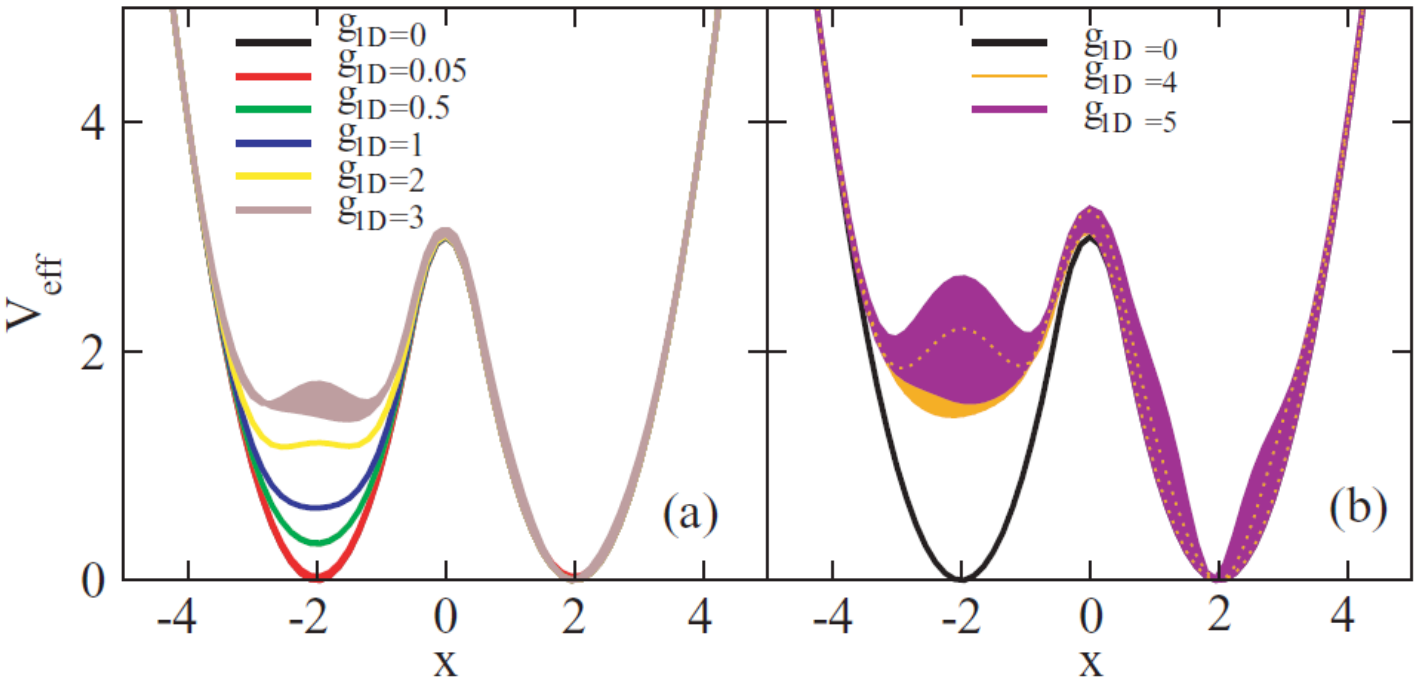


A_z: Amplitude of the oscillation
$A_z=2$, maximum possible

Usual two mode (1,2)

Two mode (2,3)

□Effective potential



The dynamics is essentially 'quasi-self-trapped'

- i.e. the effective potential (containing the non-linearity) remains 'almost time independent' during the time evolution
- (Fig) the bands are generated by plotting V_{eff} for a full Rabi period

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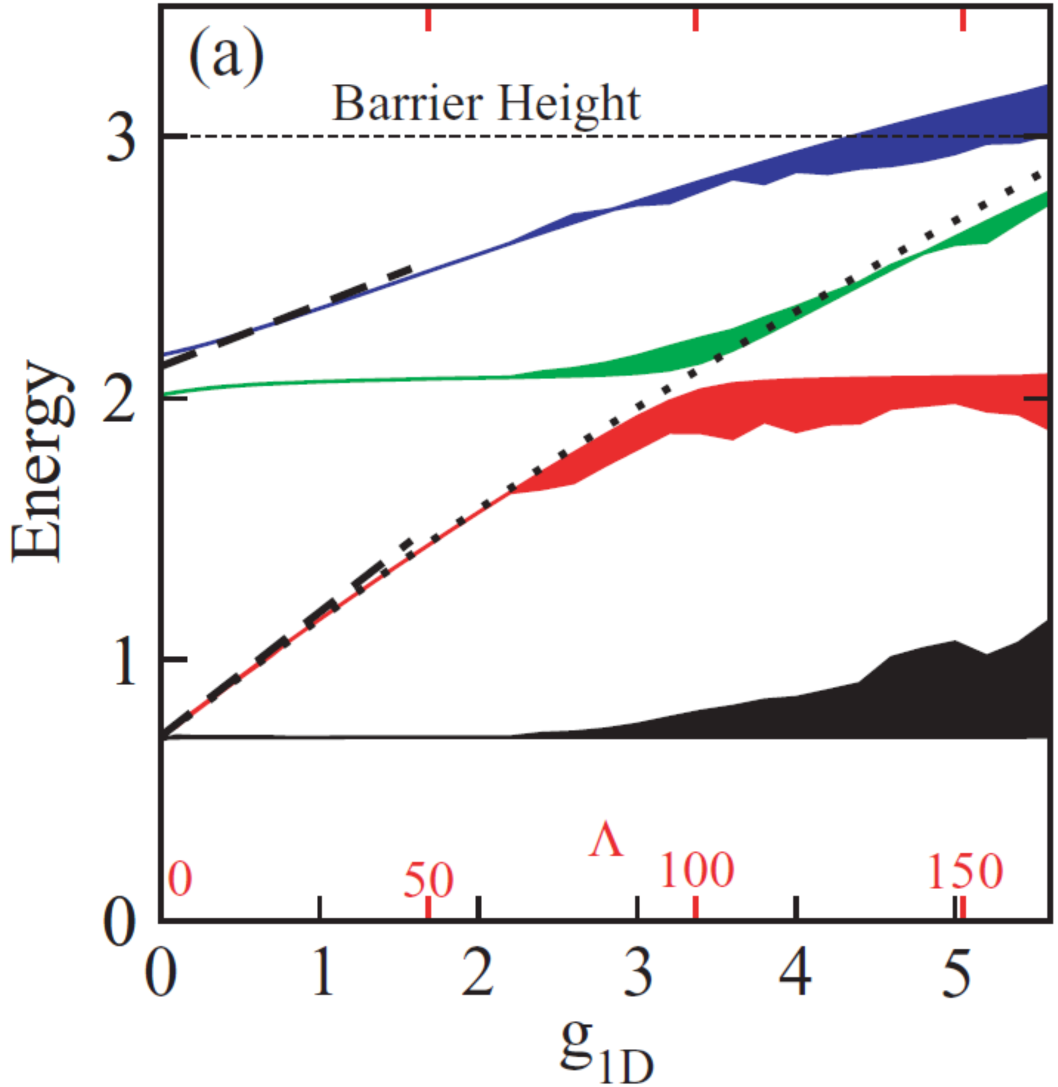
From usual, (1,2), to beyond, (2,3)

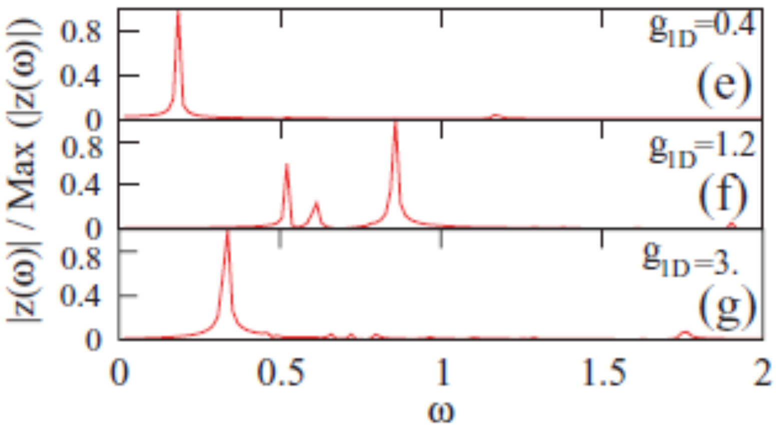
In the figure:

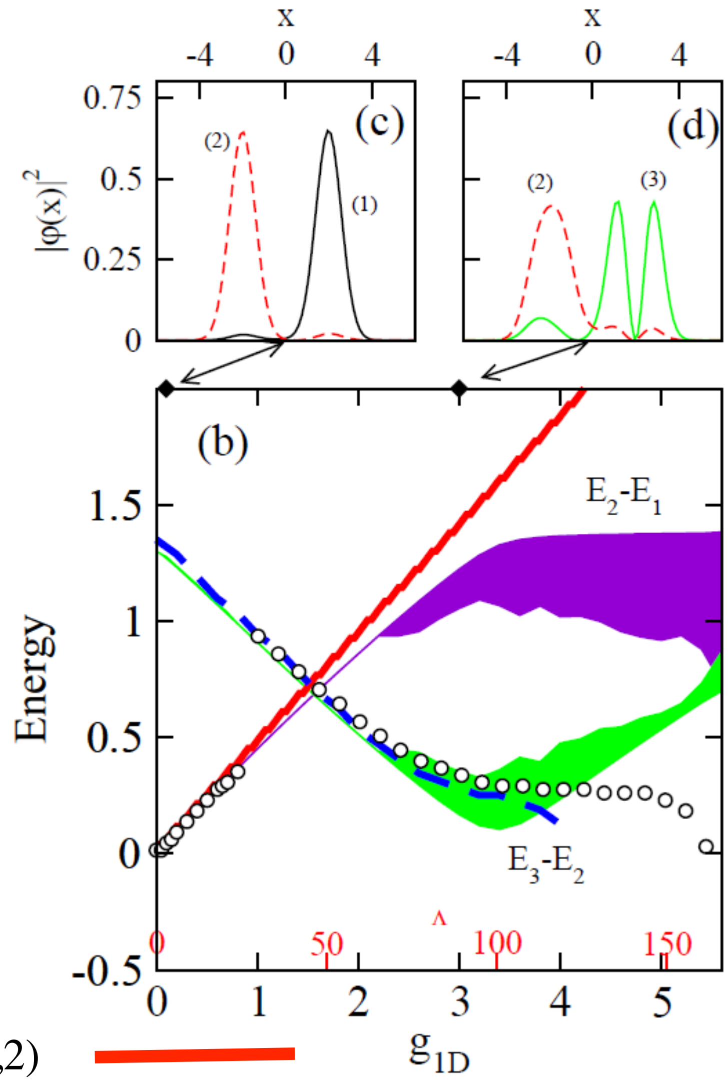
- Evolution of the first four eigenvalues of the single particle hamiltonian using the $V_{eff.}$ as a function of the nonlinearity.
- The bands correspond to using the two extremes of the potential on a Rabi period.



From usual, (1,2), to beyond, (2,3)

- We analyze the signal of the population imbalance during several Rabi periods, z(t), and extract the main frequencies.
- These are compared to the frequencies corresponding to the transitions (1,2) and (2,3) of the single particle hamiltonian (with V_{eff})





Usual two mode (1,2)

Two mode (2,3)

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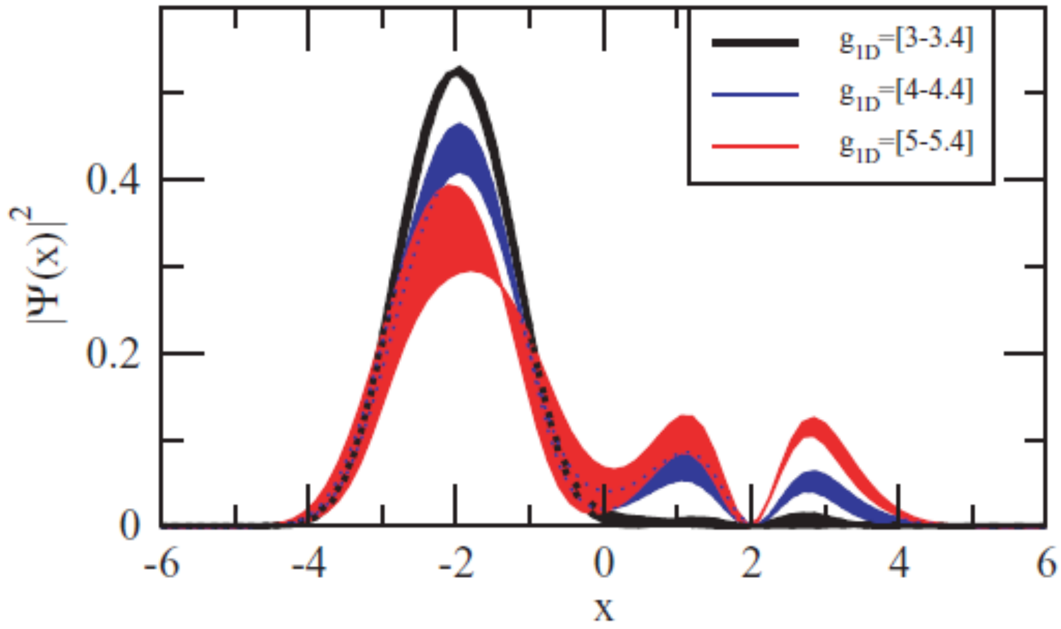
□Signature on the atomic clouds

From the experimental point of view a trace of entering the (2,3) regime would be the appearance of a node in the center of the cloud on the less populated well



Summary

- Static properties of the Bose-Hubbard Hamiltonian with small bias. Beyond mean field.
- Existence of strongly correlated 'cat-like' ground states for attractive interactions
- Relation of the self-trapping to the properties of the spectrum.
- Squeezing

Juliá-Diaz, Dagnino, Lewenstein, Martorell, Polls, PRA A 81, 023615 (2010)

Juliá-Diaz, Martorell, Polls, PRA81, 063625 (2010)

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