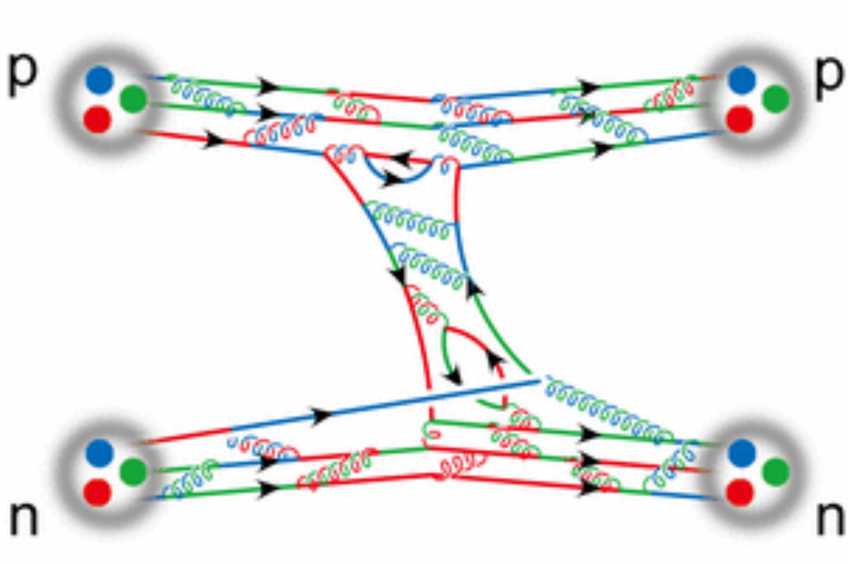
# Hadron-Hadron Interactions from Lattice QCD

Sinya AOKI

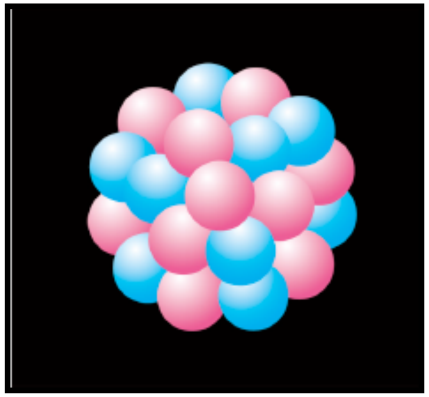


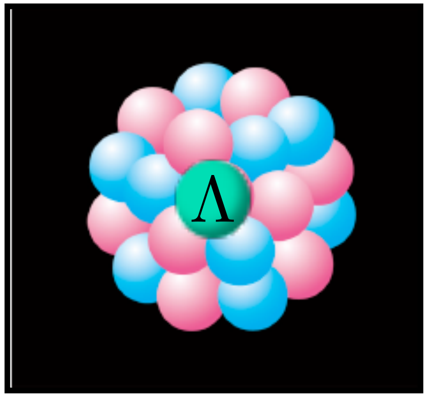

Elba XI Workshop “Electron-Nucleus Scattering XI”
June 21-25,2010, Elba, Italy

# 1. Introduction

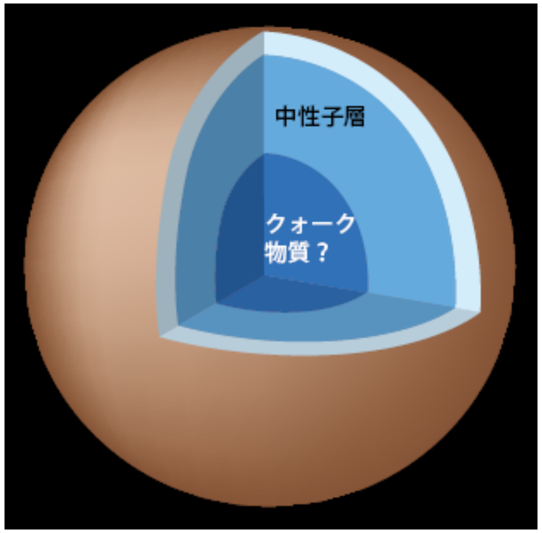
# Nuclear force is a basis for understanding ...

- structure of ordinary and hyper nuclei





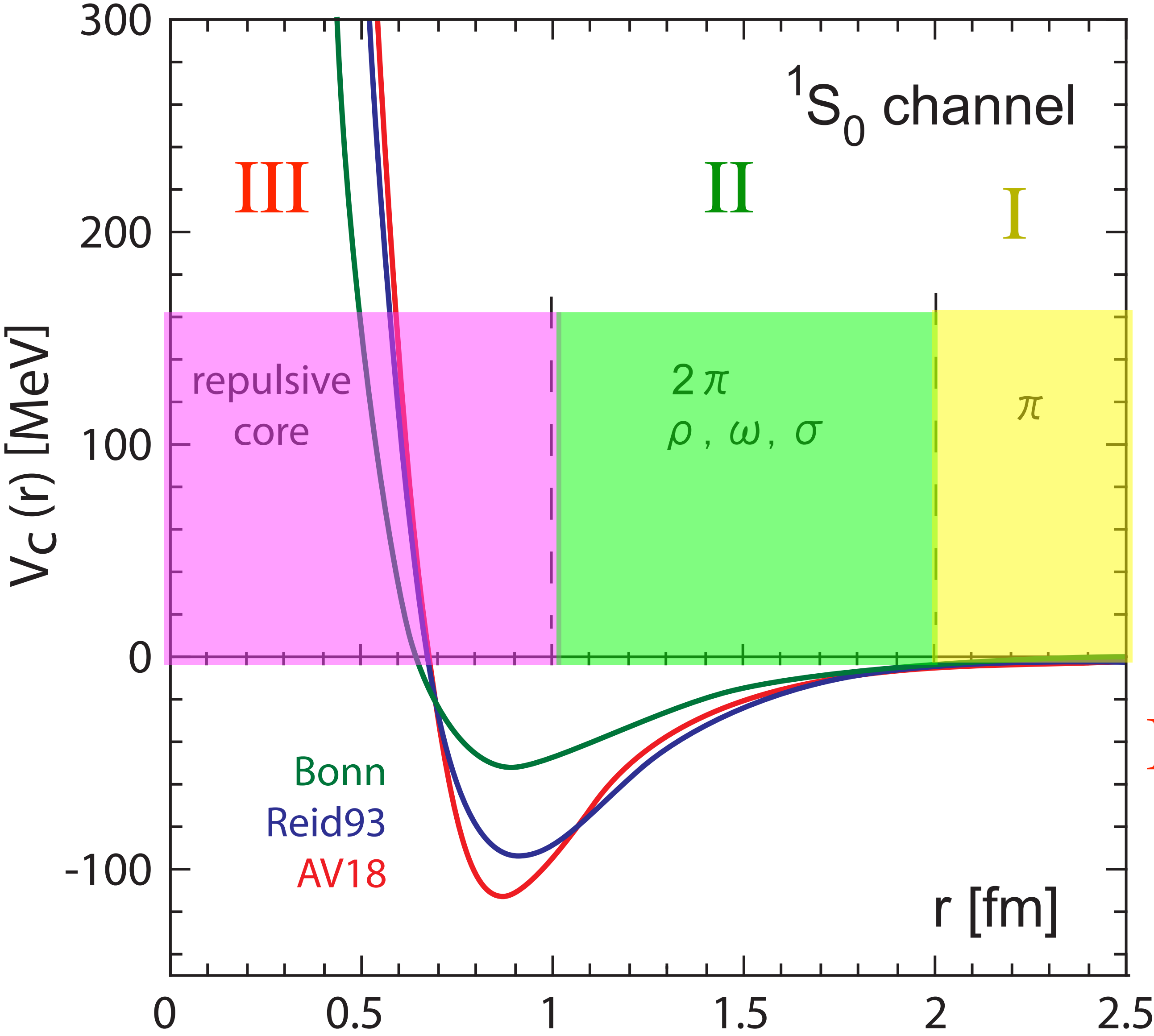

- Structure of neutron star




- Ignition of Type II supernova

# Phenomenological NN potential

(~40 parameters to fit 5000 phase shift data)




I One-pion exchange

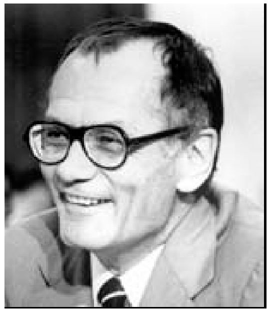
Yiukawa(1935)

II Multi-pions

Taketani et al.(1951)

III Repulsive core

Jastrow(1951)

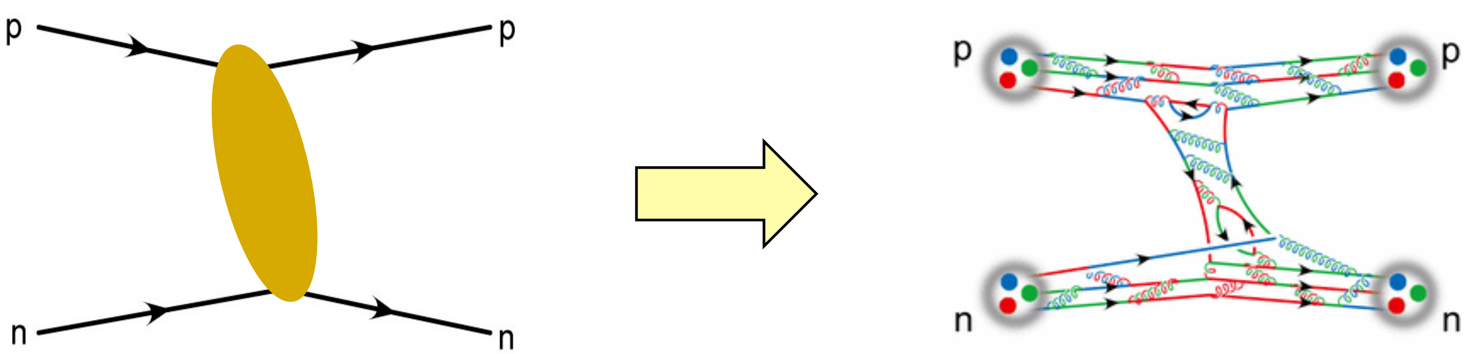


# Plan of my talk

1. Introduction
2. Strategy in (Lattice) QCD
3. Recent Developments
   1. Tensor potential
   2. Full QCD calculation
4. YN and YY interactions in lattice QCD
   1. S=-1 System
   2. S=-2 System
   3. BB interactions in an SU(3) symmetric world
   4. S=-2 Inelastic scattering
   5. H dibaryon
5. Conclusion

# 2. Strategy in (Lattice) QCD

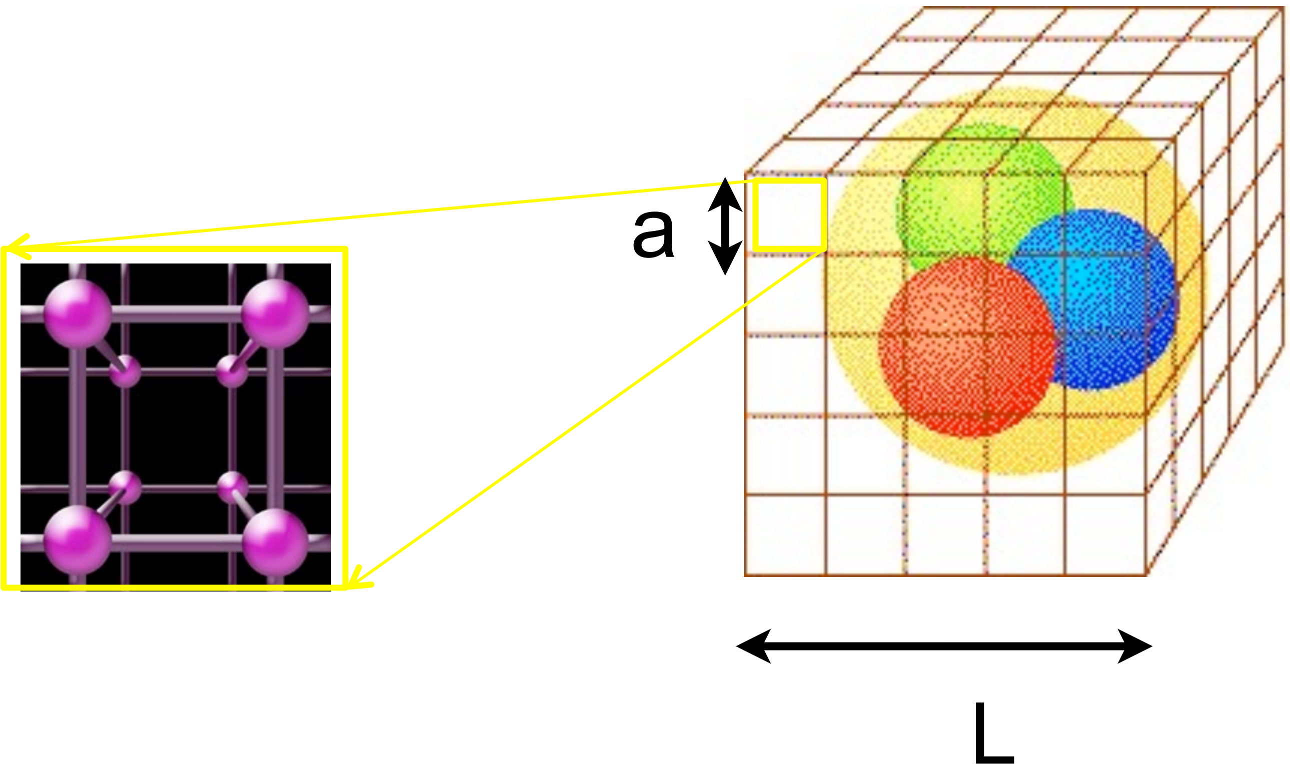
## From Phenomenology to First Principle




Y. Nambu, “Quarks：Frontiers in Elementary Particle Physics”,  World Scientific  (1985)

**“Even now, it is impossible to completely describe nuclear forces beginning with a fundamental equation.  But since we know that nucleons themselves are not elementary, this is like asking if one can exactly deduce the characteristics of a very complex molecule starting from Schroedinger equation, a practically impossible task.”**

# Lattice QCD




- well-defined statistical system (finite a and L)
- gauge invarinat
- fully non-perturbative

→ Monte-Calro simulations

Quenched QCD : neglects creation-anihilation of quark-anitiquak pair
Full QCD : includes creation-anihilation of quark-anitiquak pair

# How to extract NN potentials in (lattice) QCD

Y. Nambu
“Force Potentials in Quantum Field Theory”
Prog. Theor. Phys. 5 (1950) 614.

K. Nishijima
“Formulation of Field Theories for Composite Particles”
Phys. Rev. 111 (1958) 995.

**HAL QCD Collaboration**

Sinya Aoki, Takumi Doi, Tetsuo Hatsuda,
Youichi Ikeda, Takashi Inoue, Noriyoshi Ishii,
Keiko Murano, Hidekatu Nemura, Kenji Sasaki

# Quantum Field Theoretical consideration

- S-matrix below inelastic threshold. Unitarity gives

$$S = e^{2i\delta}$$

- Nambu-Bethe-Salpeter (NBS) Wave function

$$\varphi_E(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r},0) N(\mathbf{x},0) | 6q, E \rangle$$

6 quark QCD eigen-state with energy E

$N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x)$: local operator

Asymptotic behavior

$$r = |\mathbf{r}| \to \infty$$

$$\varphi_E^l(r) \longrightarrow A_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} \qquad E = \frac{k^2}{2\mu_N} = \frac{k^2}{m_N}$$

partial wave

$\delta_l(k)$ is the scattering phase shift

# Systemtic procedure to define the NN potential in lattice QCD

Full details: Aoki, Hatsuda & Ishii, PTP123(2010)89 (arXiv0909.5585)

1. Choose your favarite opetator: e.g. $N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x)$
   - observables do not depend on the choice
   - yet the local operator is useful

   Nishijima,Haag,Zimmermann(1958)

2. Measure the NBS amplitude: $\varphi_E(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r}, 0) N(\mathbf{x}, 0) | 6q, E \rangle$

3. Define the non-local potential: $[E - H_0]\varphi_E(\mathbf{x}) = \int d^3y\, U(\mathbf{x}, \mathbf{y}) \varphi_E(\mathbf{y})$

4. Velocity expansion: $U(\mathbf{x}, \mathbf{y}) = V(\mathbf{x}, \nabla) \delta^3(\mathbf{x} - \mathbf{y})$

$$V(\mathbf{x}, \nabla) = V_C(r) + V_T(r) S_{12} + V_{\rm LS}(r) \mathbf{L} \cdot \mathbf{S} + \{V_D(r), \nabla^2\} + \cdots$$

LO LO NLO NNLO

$$S_{12} = \frac{3}{r^2}(\sigma_1 \cdot \mathbf{x})(\sigma_2 \cdot \mathbf{x}) - (\sigma_1 \cdot \sigma_2)$$

Okubo-Marshak (1958), Tamagaki-Watari(1967)

5. Calculate observables: phase shift, binding energy etc.

# Key Channels in NN Scattering

$^{2s+1}L_J$

LO LO NLO NNLO

$$V(\mathbf{x}, \nabla) = V_C(r) + V_T(r)S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + \{V_D(r), \nabla^2\} + \cdots$$

central tensor spin-orbit

$^1S_0$ Central Force ⟺ nuclear BCS pairing

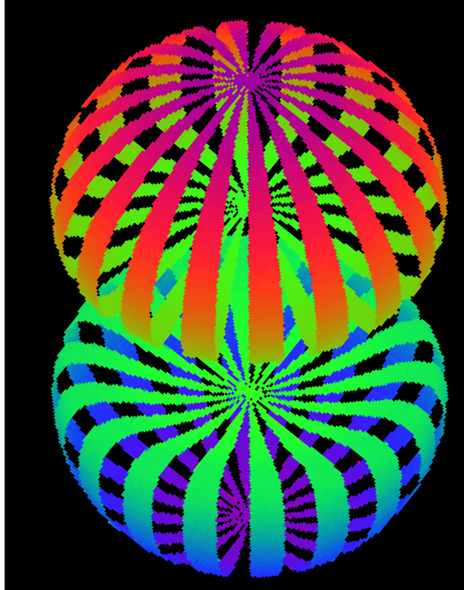
Bohr, Mottelson, Pines, Phys. Rev. 110 (1958)

$^3S_1 - {}^3D_1$ Tensor Force ⟺ deuteron binding

Pandharipande et al., Phys. Rev. C54 (1996)

$^3P_2 - {}^3F_2$ LS Force ⟺ neutron superfluidity in neutron stars

Tamagaki, Prog. Theor. Phys. 44 (1970)

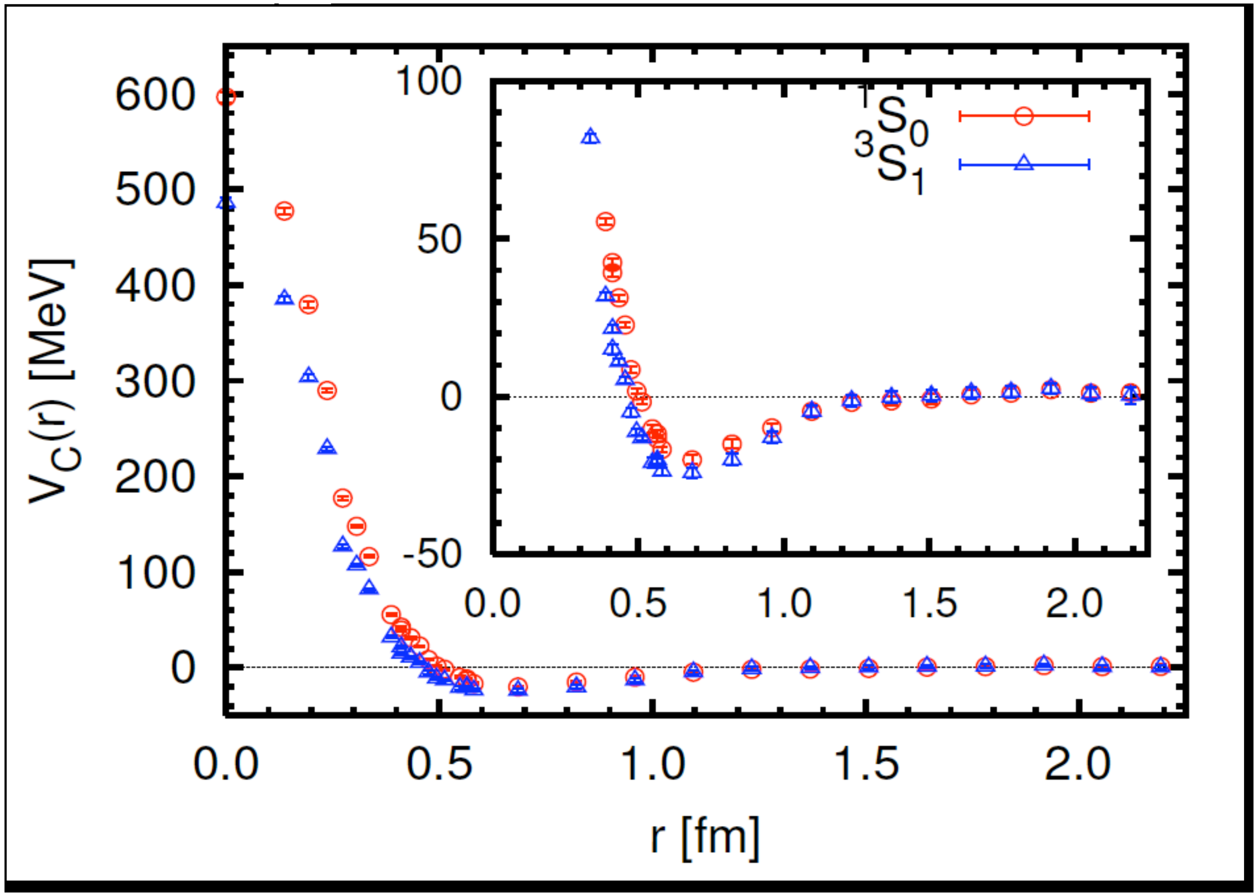


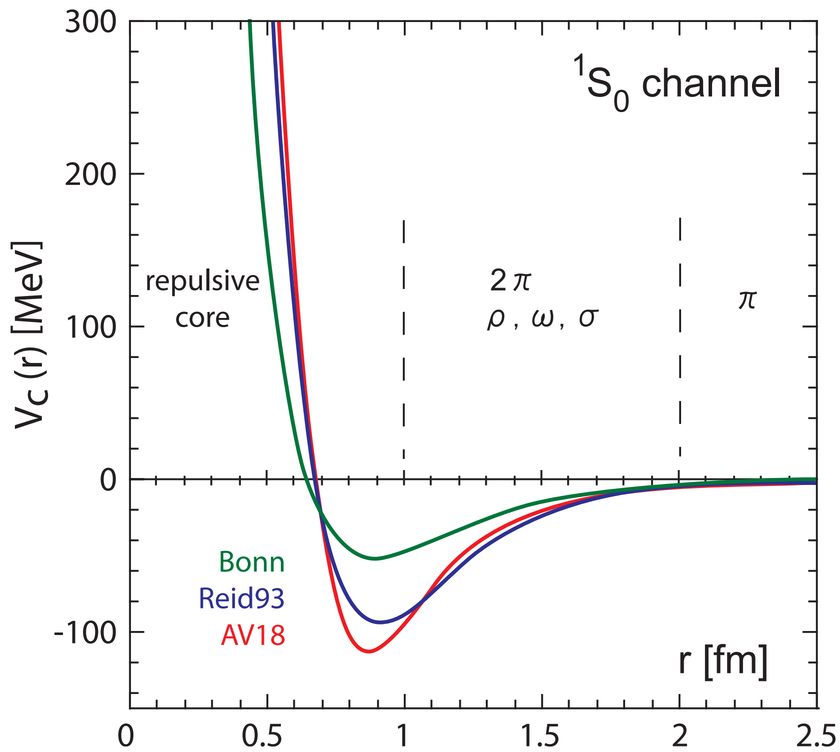
Density profile of the deuteron with $S_z=\pm1$

# First (quenched) results

Central Potential

$^1S_0, \ ^3S_1$ $E \simeq 0$ $m_\pi \simeq 0.53$ GeV




Qualitative features of NN potential are reproduced !

Ishii-Aoki-Hatsuda, PRL90(2007)0022001

This paper has been selected as one of 21 papers in
Nature Research Highlights 2007

"The achievement is both a computational *tour de force* and a triumph for theory."

# Frequently Asked Questions

[Q1] Operator dependence of the potential

[Q2] Energy dependence of the potential

[A1] (N(x), U(x,y) ) is a combination to define ovservables

- remember,

  QM: ($\Phi$, U) $\rightarrow$ observables

  QFT: (asymptotic field, vertices) $\rightarrow$ observables

  EFT: (choice of field, vertices) $\rightarrow$ observables
- local operator = convenient choice for reduction formula

[A2] U(x,y) is E-independent by construction

- non-locality can be determined order by order in velocity expansion

  ( c.f. ChPT)

Question 3

# How good is the velocity expansion of V ?

Leading Order

$$V_C(r) = \frac{(E - H_0)\varphi_E(\mathbf{x})}{\varphi_E(\mathbf{x})}$$

Local potential approximation

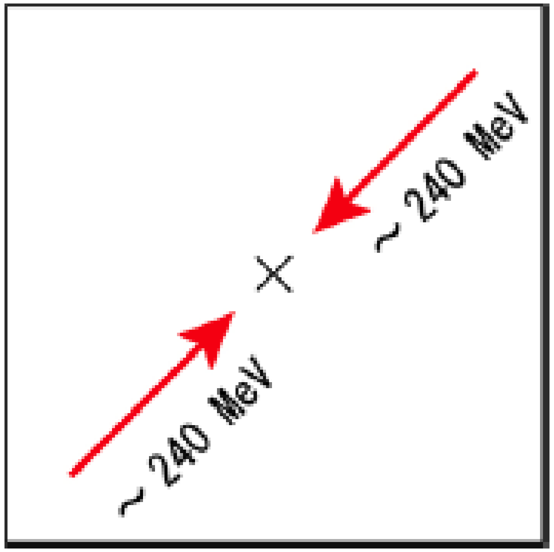
The local potential obtained at given energy E may depend on E.

If the energy dependence of the potential is weak, the local potential approximation is good.

Furthermore one may determine the higher order terms by comparing results among different energies.

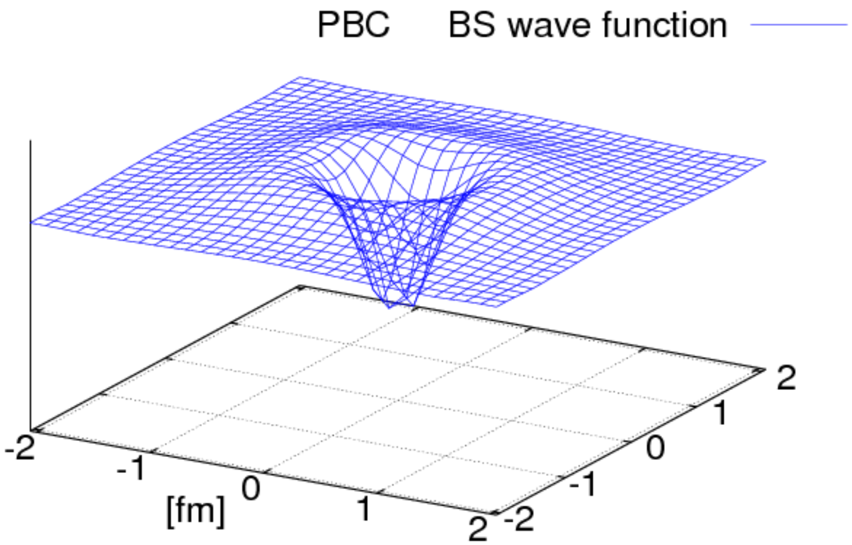
$$V(\mathbf{x}, \nabla) = V_C(r) + V_T(r) S_{12} + V_{\mathrm{LS}}(r) \mathbf{L} \cdot \mathbf{S} + \{V_D(r), \nabla^2\} + \cdots$$

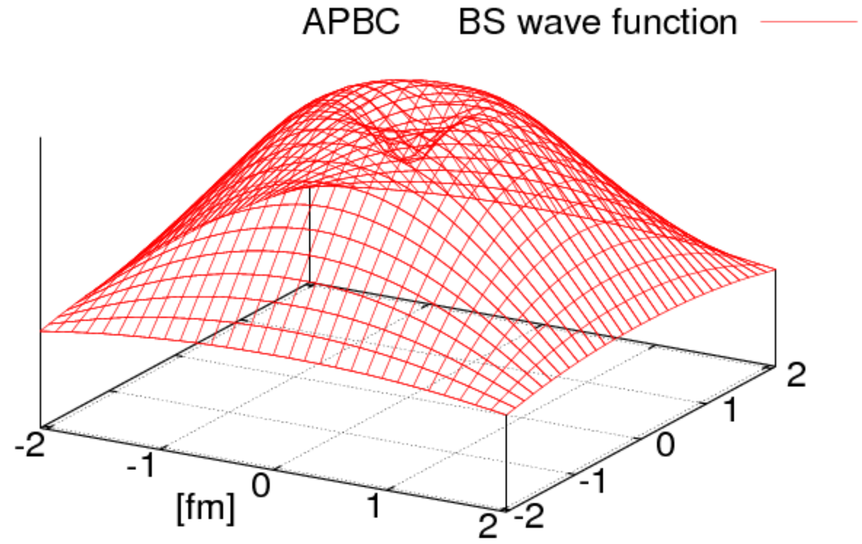
Numerical check in quenched QCD $m_\pi \simeq 0.53$ GeV

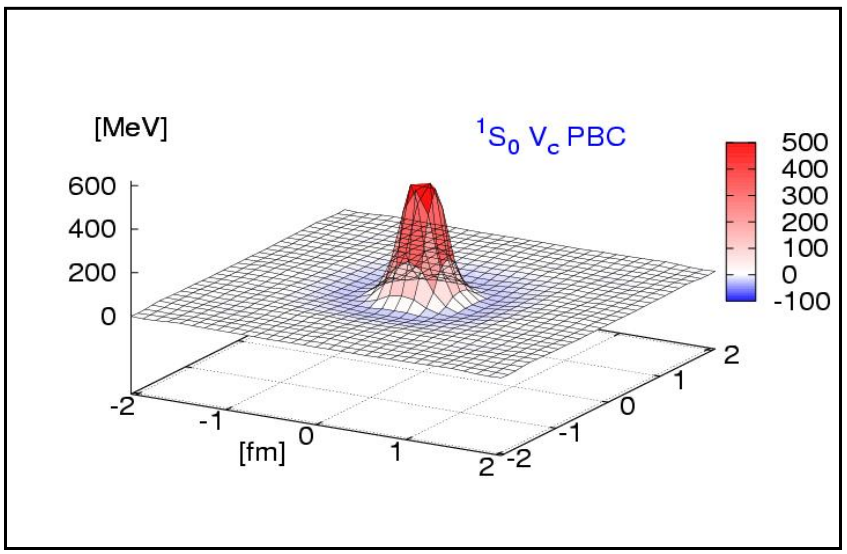


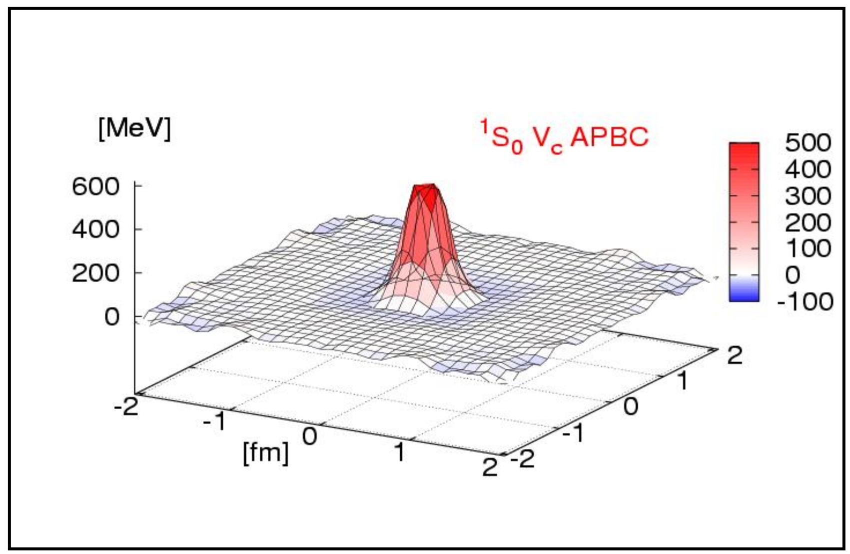

K. Murano, S. Aoki, T. Hatsuda, N. Ishii, H. Nemura

● PBC (E～0 MeV) ● APBC (E～46 MeV)

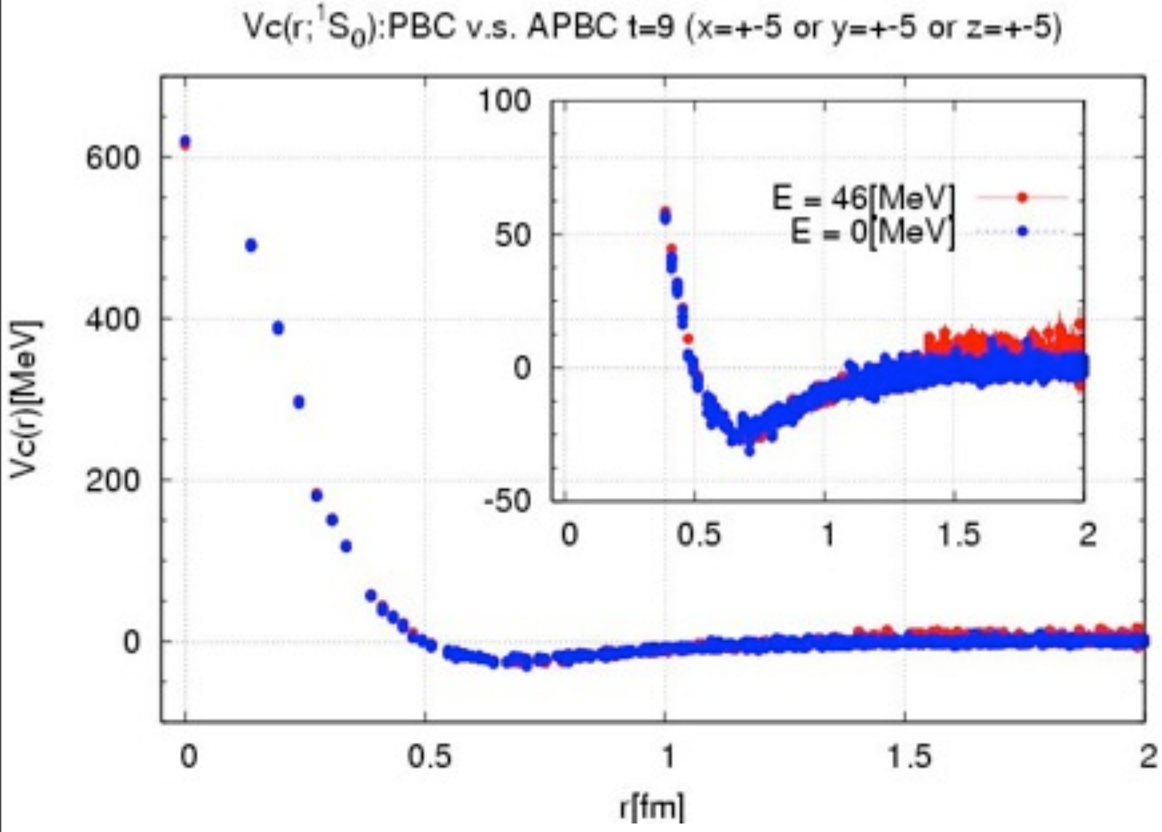


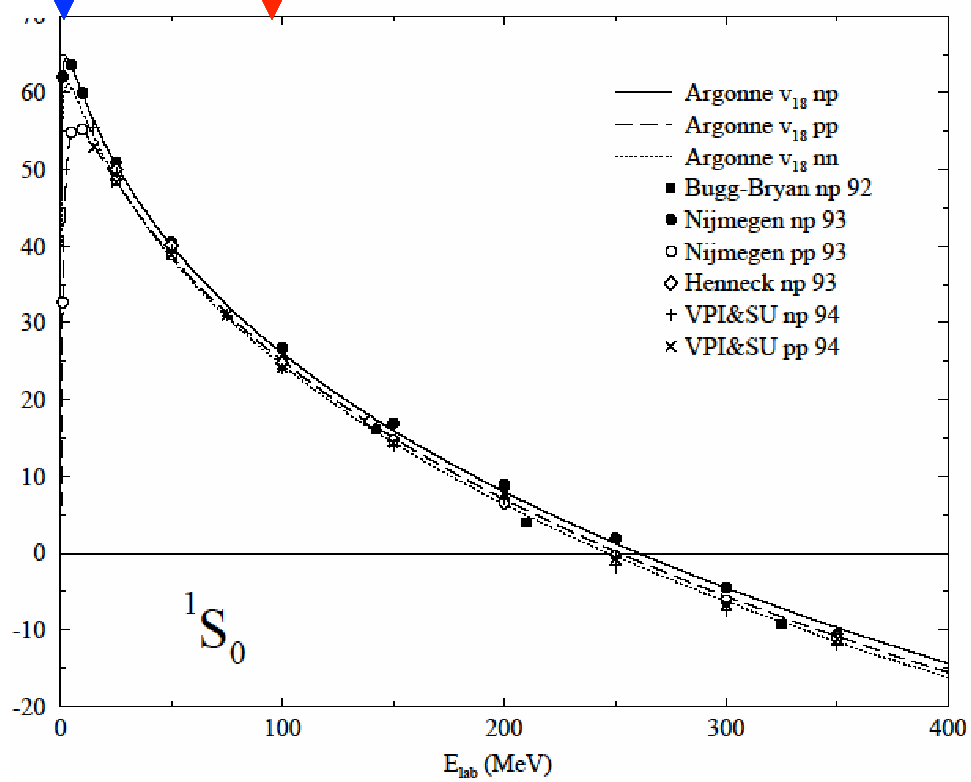



$m_\pi \simeq 0.53$ GeV




PBC APBC




E dependence of the local potential turns out to be very small at low energy in our choice of wave function.

# 3. Recent developments

# 3-1. Tensor potential

$$(H_0 + V_C + V_T S_{12})|\phi\rangle = E|\phi\rangle$$

mixing between ${}^3S_1$ and ${}^3D_1$ through the tensor force

$$|\phi\rangle = |\phi_S\rangle + |\phi_D\rangle$$

$$|\phi_S\rangle = P|\phi\rangle = \frac{1}{24}\sum_{R\in\mathcal{O}} R|\phi\rangle$$ "projection" to L=0 ${}^3S_1$

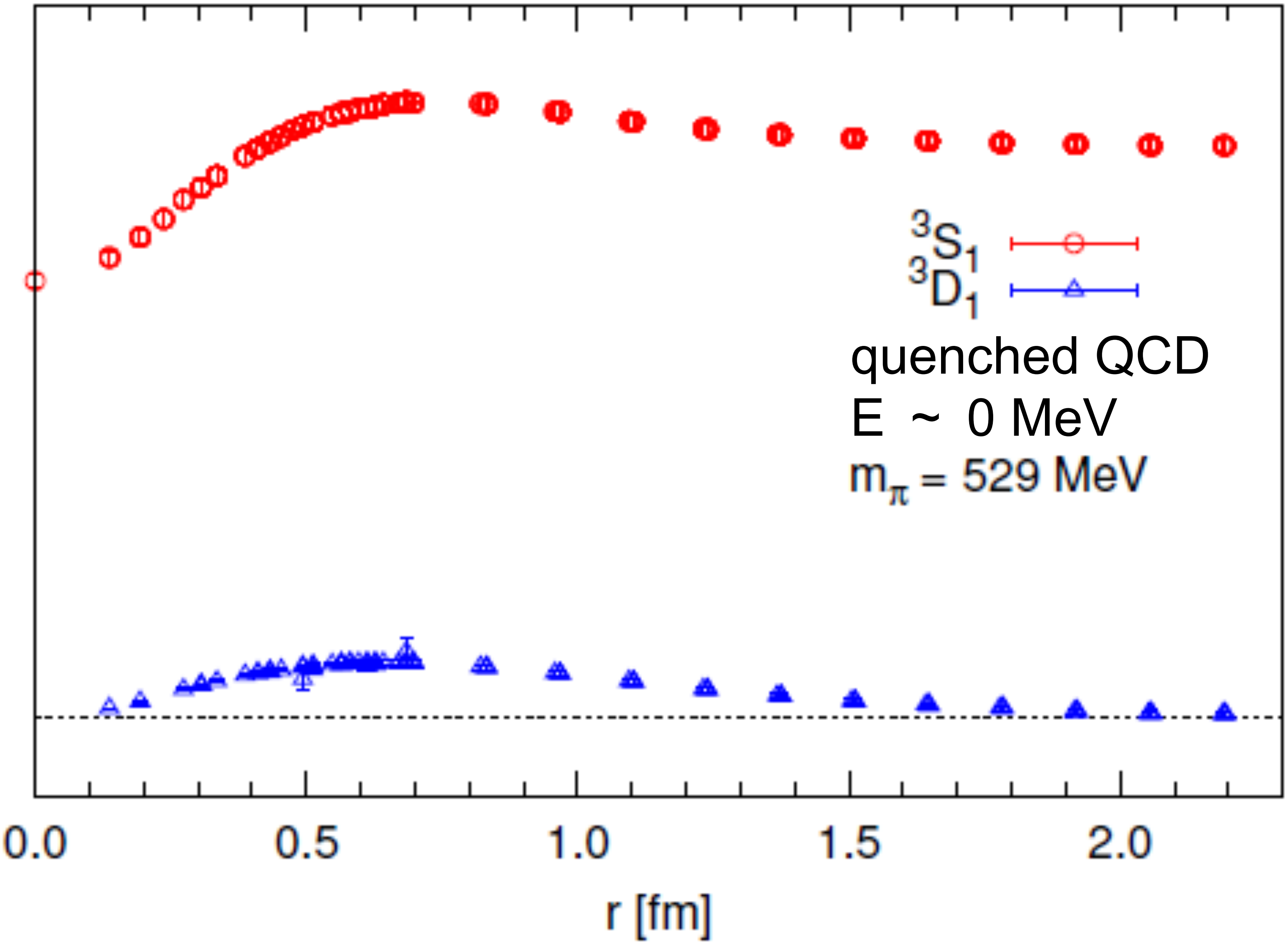
$$|\phi_D\rangle = Q|\phi\rangle = (1-P)|\phi\rangle$$ "projection" to L=2 ${}^3D_1$

$$\begin{aligned} P(H_0 + V_C + V_T S_{12})|\phi\rangle &= EP|\phi\rangle \\ Q(H_0 + V_C + V_T S_{12})|\phi\rangle &= EQ|\phi\rangle \end{aligned}$$

# Wave functions

Quenched

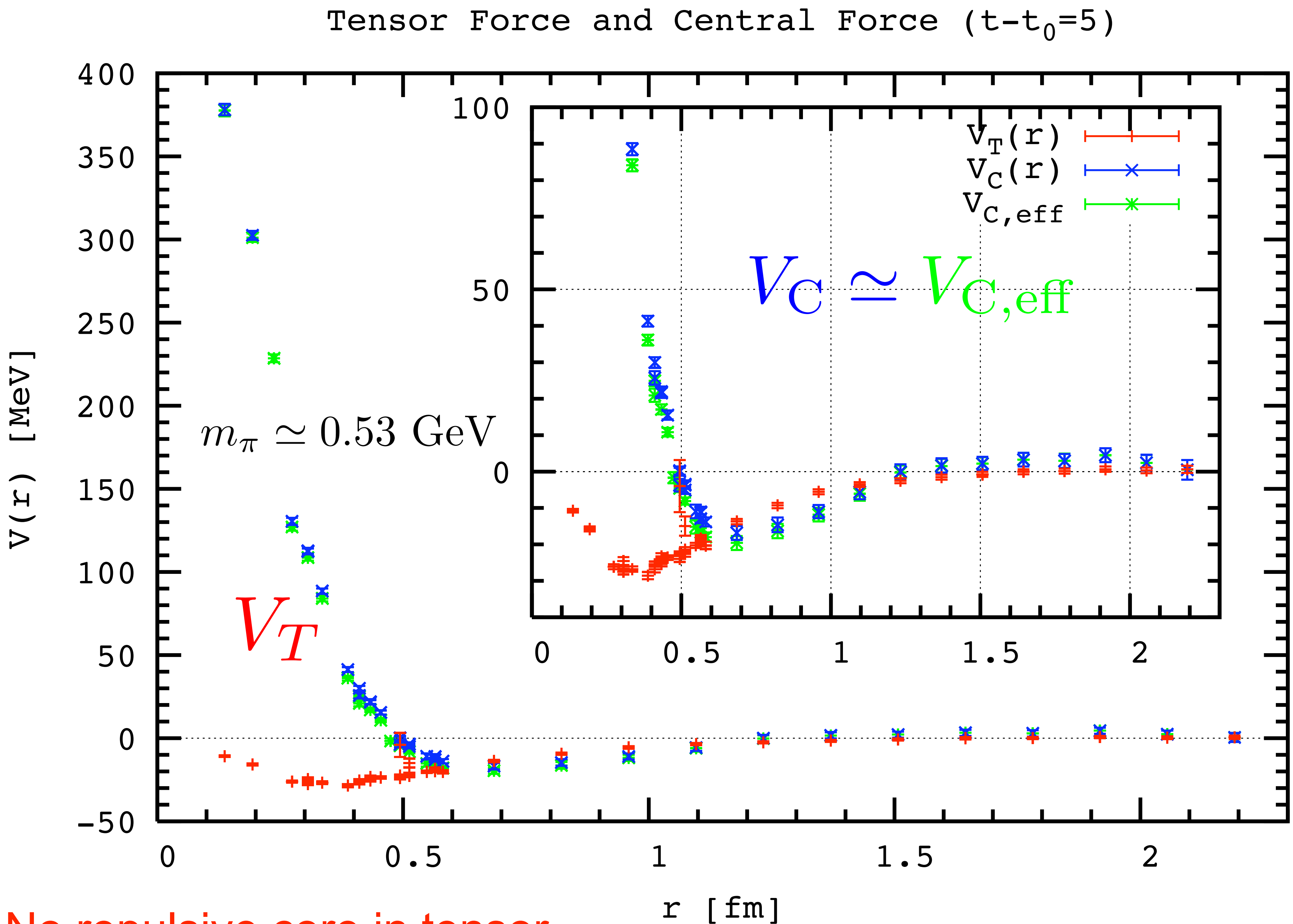
Aoki, Hatsuda, Ishii, PTP 123 (2010)89
arXiv:0909.5585



# Potentials

Quenched

Aoki, Hatsuda, Ishii, PTP 123 (2010)89
arXiv:0909.5585

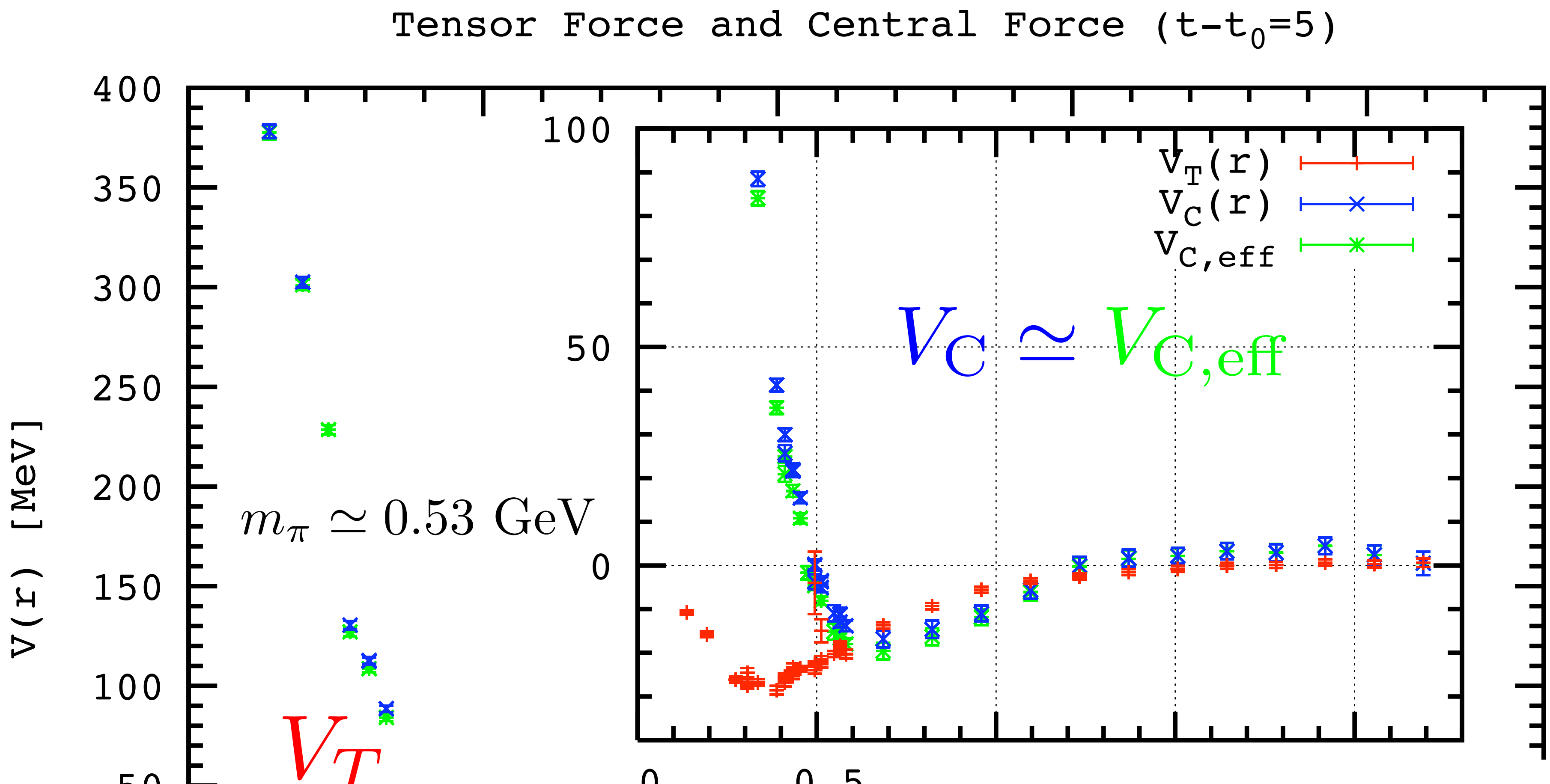



No repulsive core in tensor

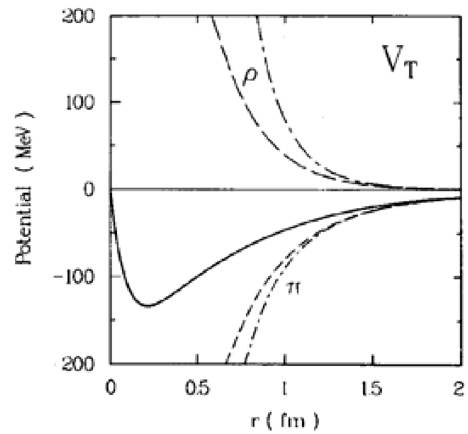
# Potentials

Quenched

Aoki, Hatsuda, Ishii, PTP 123 (2010)89
arXiv:0909.5585




No repulsive core in tensor

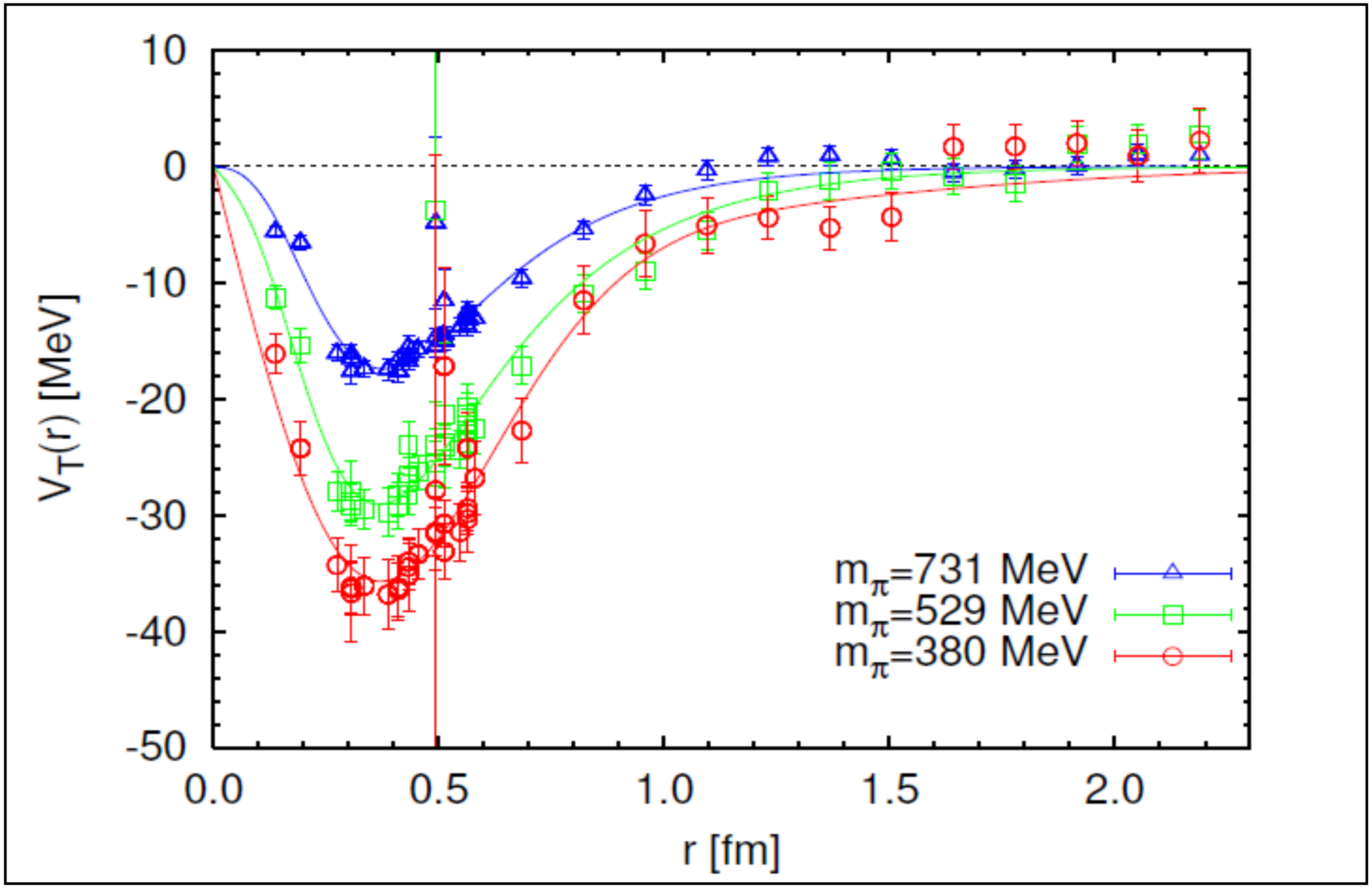



from
R.Machleidt,
Adv.Nucl.Phys.**19**

**Fig. 3.7.** The contributions from $\pi$ and $\rho$ (dashed) to the $T = 0$ tensor potential. The solid line is the full potential. The dash-dot lines are obtained when the cutoff is omitted.

# Quark mass dependence

Quenched

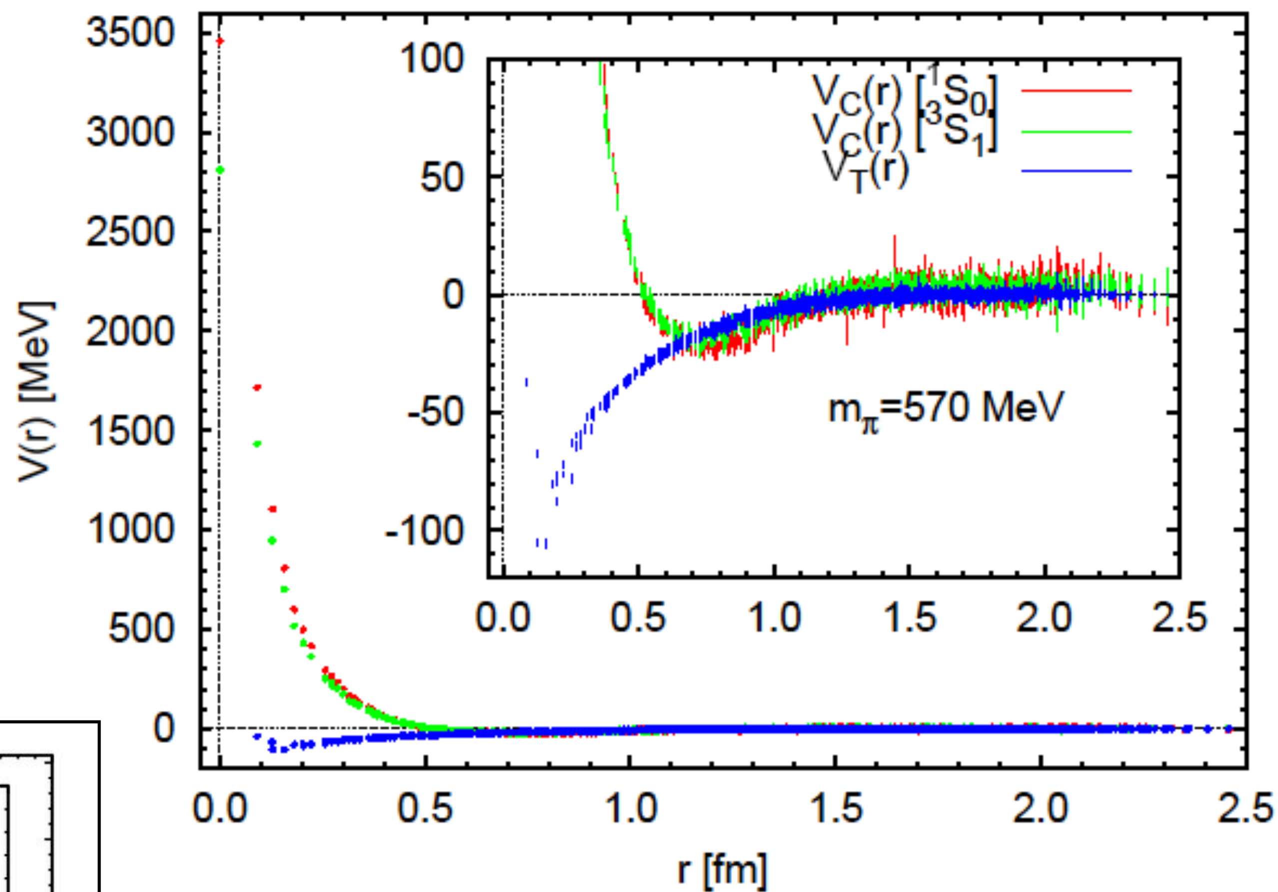



Fit function

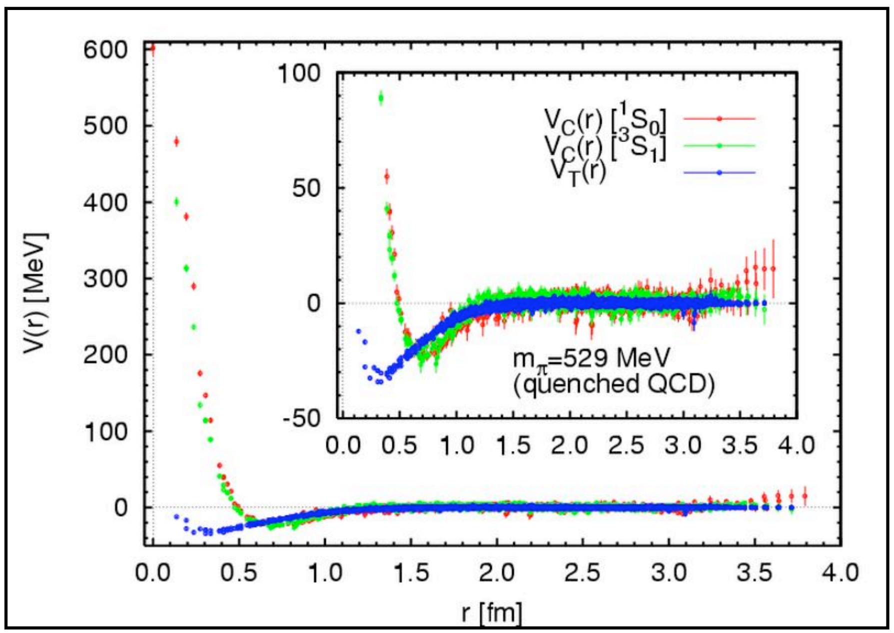
$$V_T(r) = b_1(1 - e^{-b_2 r^2})^2 \left(1 + \frac{3}{m_\rho r} + \frac{3}{(m_\rho r)^2}\right) \frac{e^{-m_\rho r}}{r}$$
$$+ b_3(1 - e^{-b_4 r^2})^2 \left(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right) \frac{e^{-m_\pi r}}{r},$$

- Rapid quark mass dependence of tensor potential
- Evidence of one-pion exchange

# 3-2. Full QCD Calculation



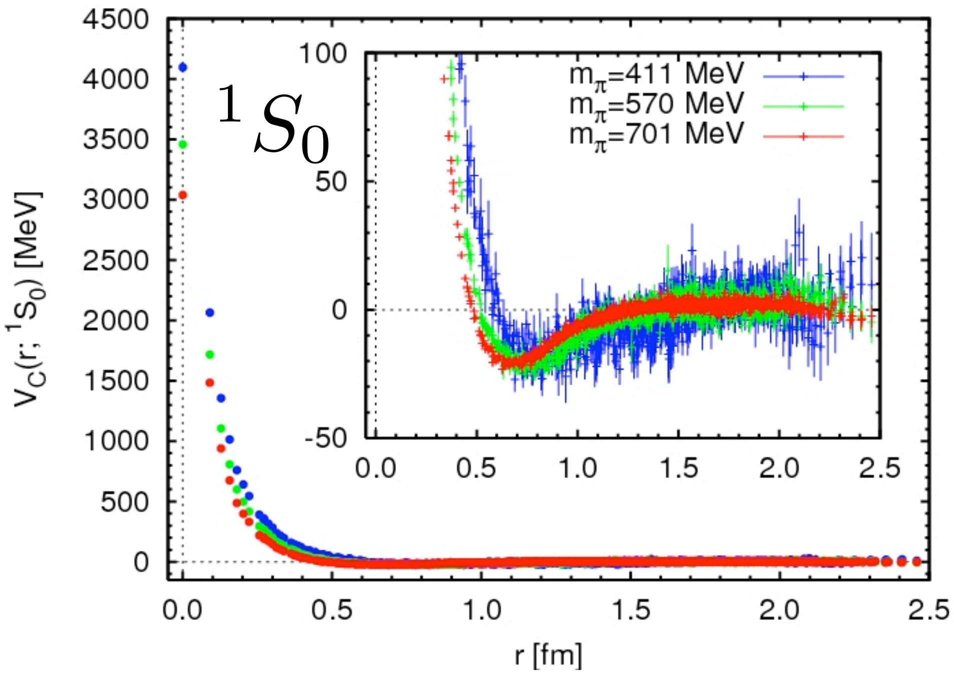

**Quenched QCD**

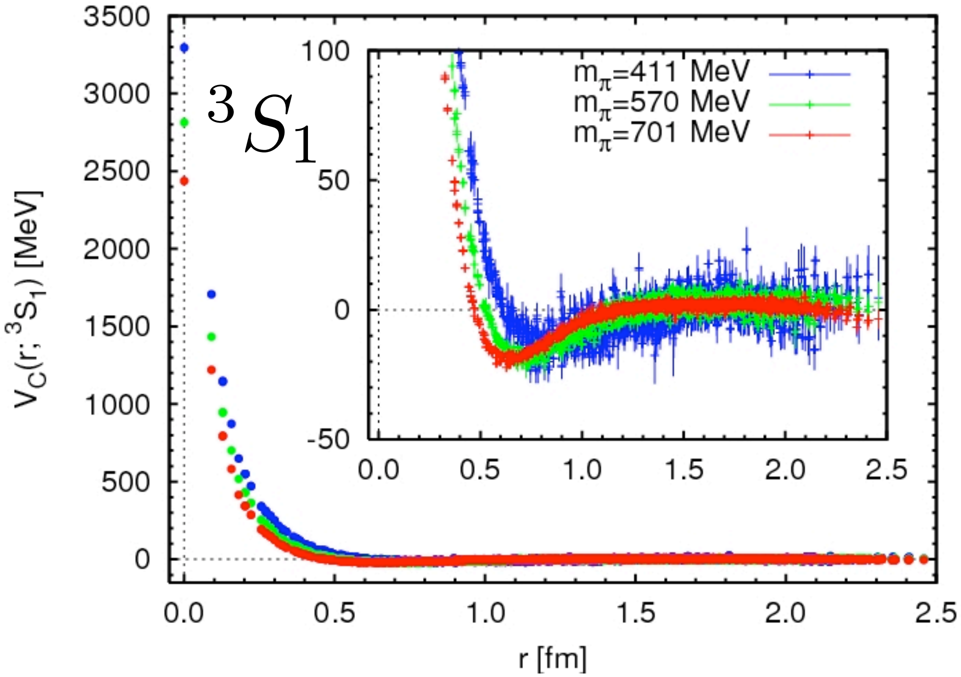


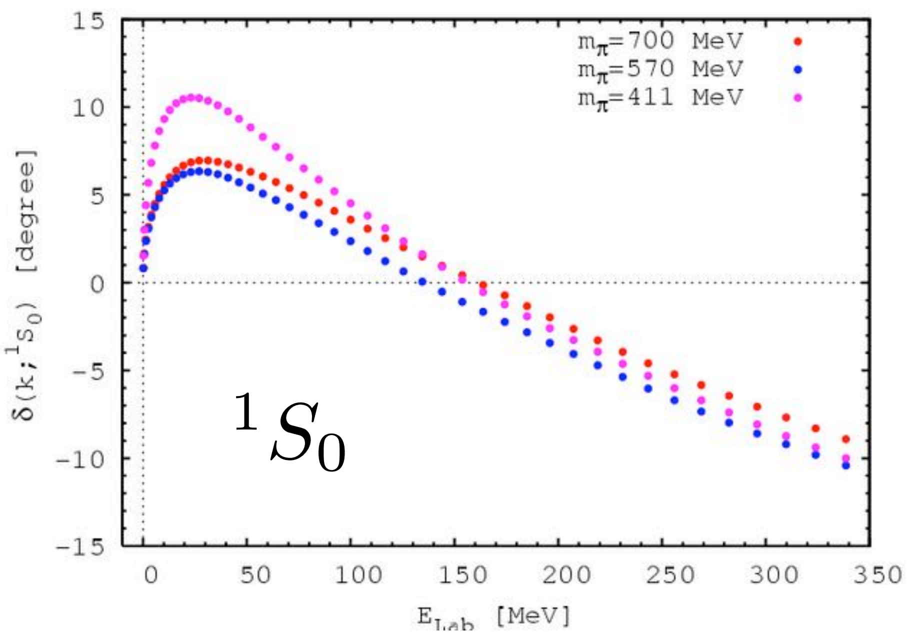

**Full QCD**

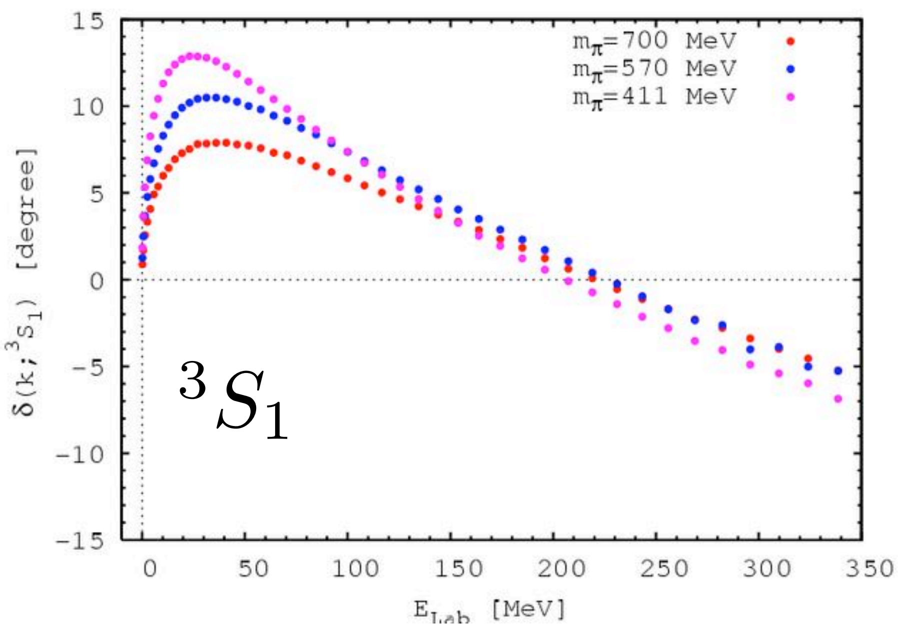
$$m_\pi = 570 \text{ MeV}, \; L = 2.9 \text{ fm}$$

# Phase shift from V(r) in full QCD

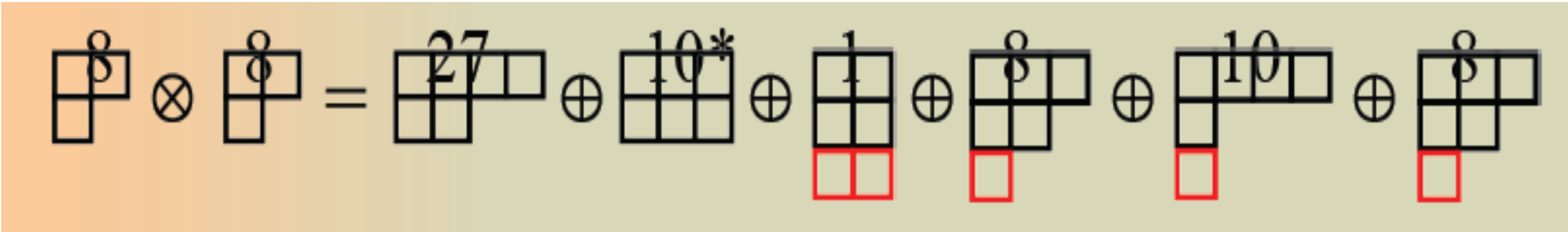


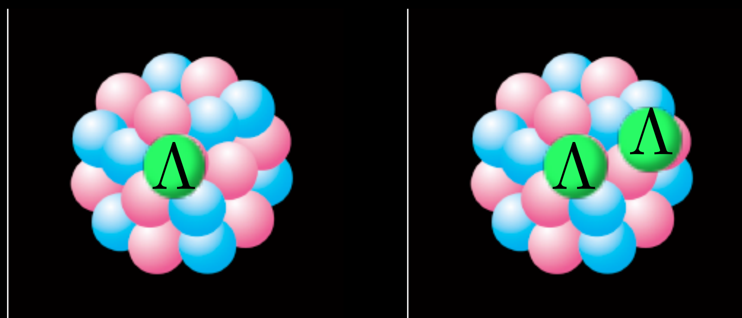



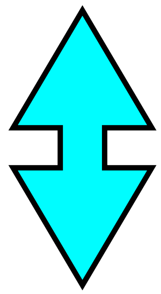



# 4. YN and YY interactions in lattice QCD



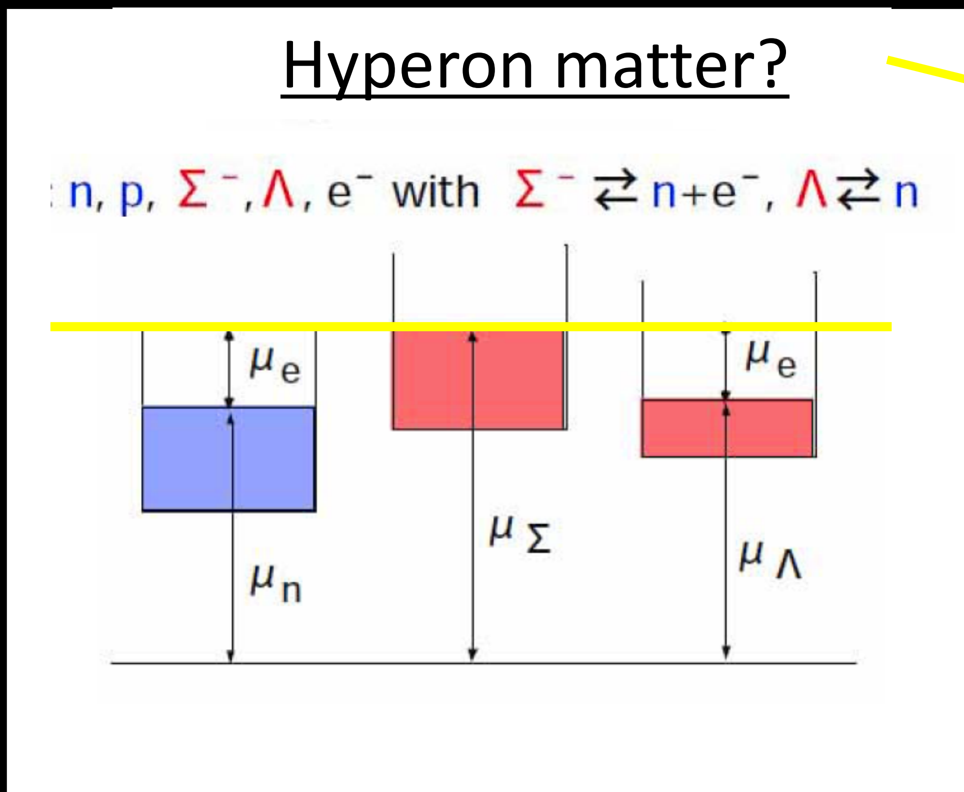

- no phase shift available for YN and YY scattering
- plenty of hyper-nucleus data will be soon available at J-PARC

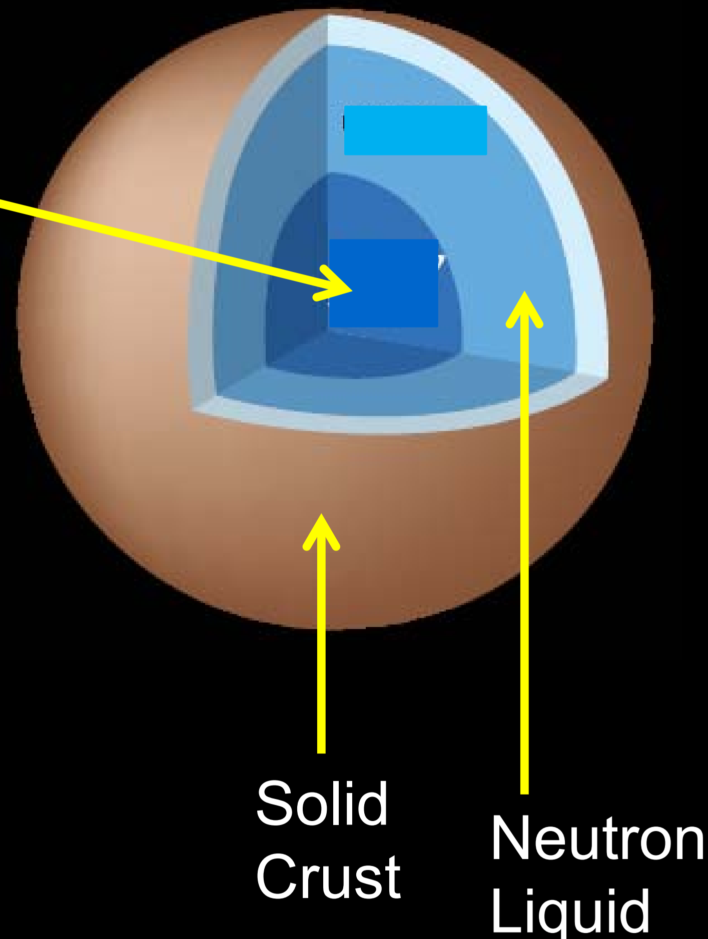



- prediction from lattice QCD
- difference between NN and YN ?

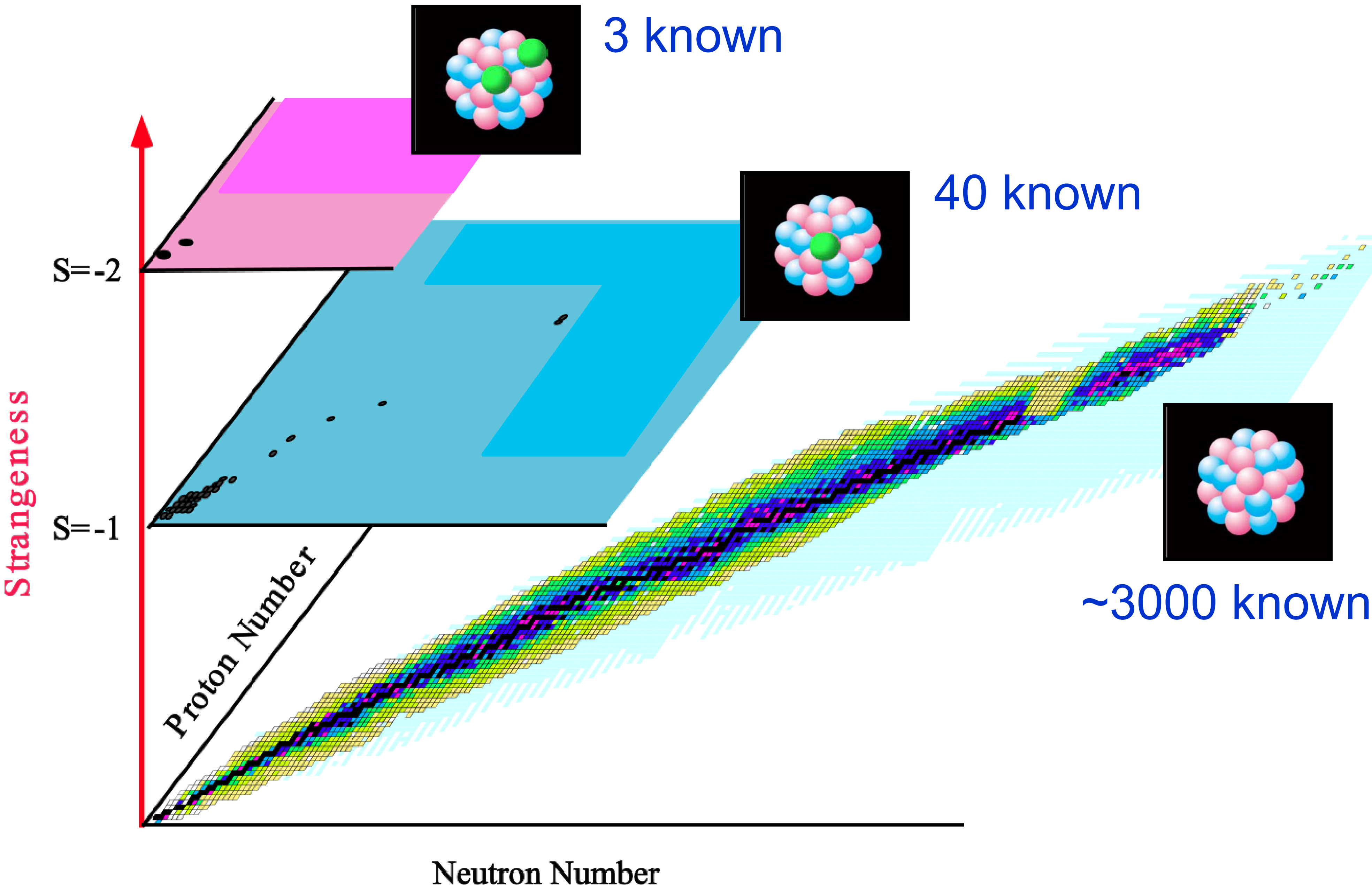
# Hyperon Core of Neutron Stars

Radius ～ 10 km
Mass ～ solar mass
Central density ～ $10^{12}$ kg/cm$^3$



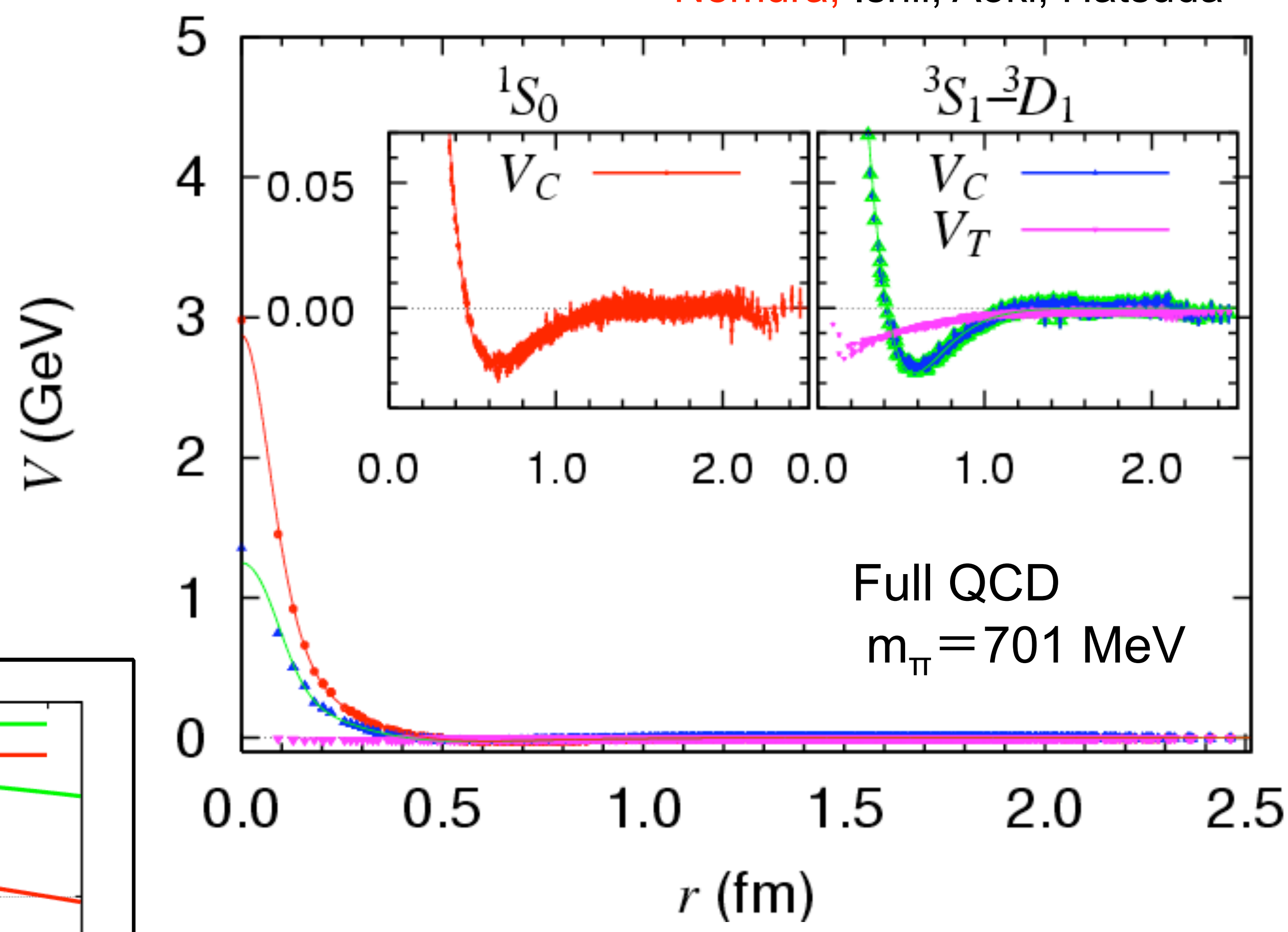



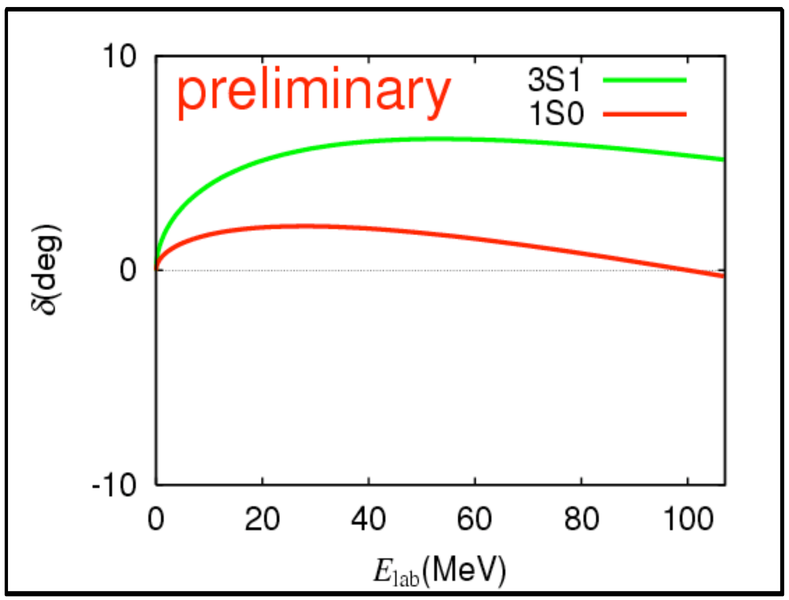
# 3D Nuclear chart



# 4-1.S= -1 System: ΛN interaction(I=1/2) in full QCD

Nemura, Ishii, Aoki, Hatsuda



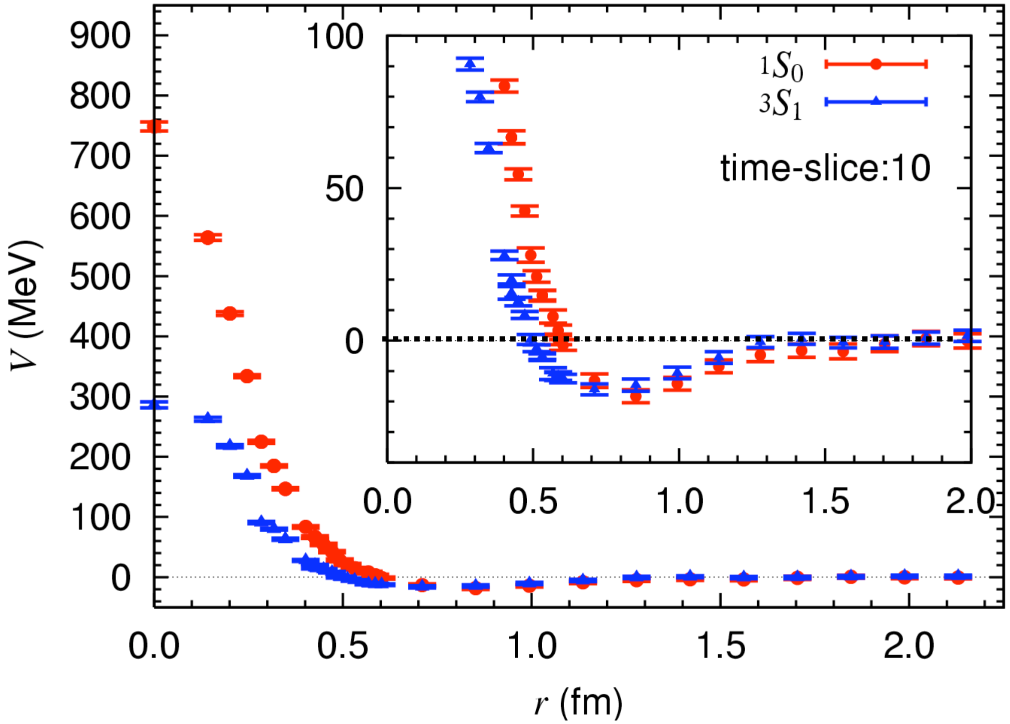




1. repulsive core + attractive well
2. Large spin dependence
3. Overall attraction

# 4-2.S= -2 System

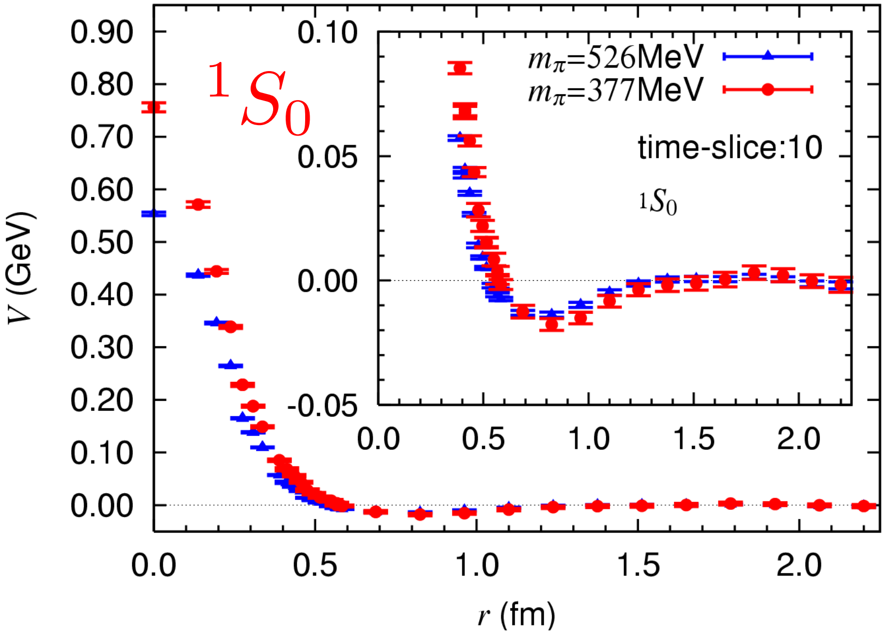
## $\Xi N$ (I=1) potential

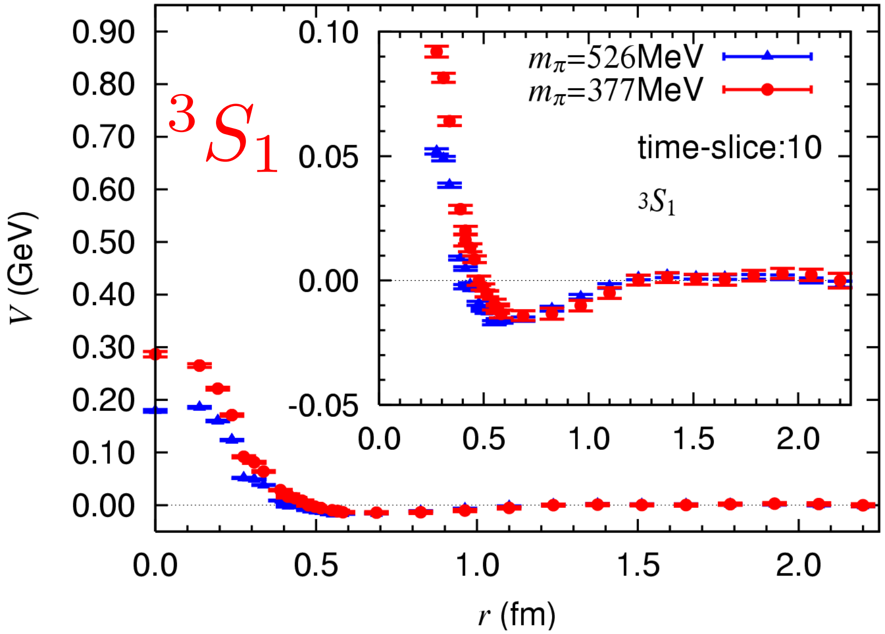
Quenched




Nemura, Ishii, Aoki, Hatsuda,
Phys.Lett.B673 (2009)136

1. repulsive core + attractive well
2. Large spin dependence
3. weaker quark mass dependence

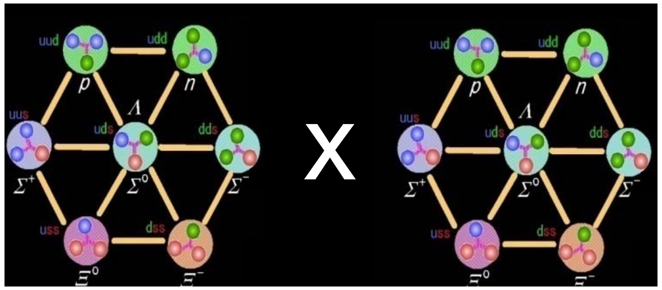


## 4-3. BB interactions in an SU(3) symmetric wrold

$m_u = m_d = m_s$

1. First setp to predict YN, YY interactions not accessible in exp.
2. Origin of the repulsive core (universal or not)



$$8 \times 8 = \underline{27 + 8s + 1} + \underline{10^* + 10 + 8a}$$

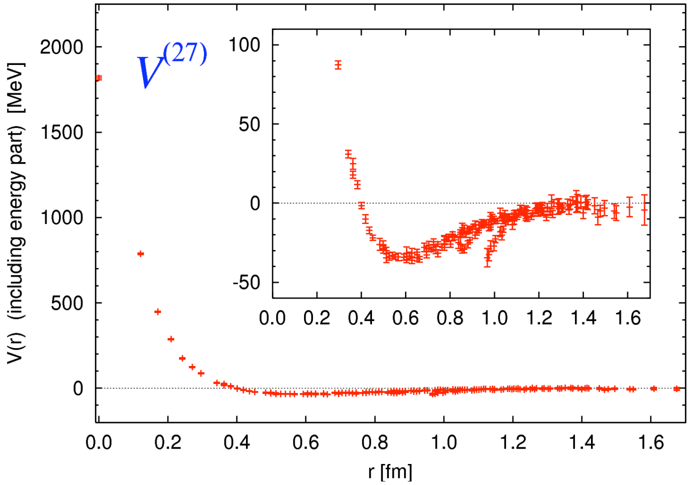
Symmetric　　Anti-symmetric

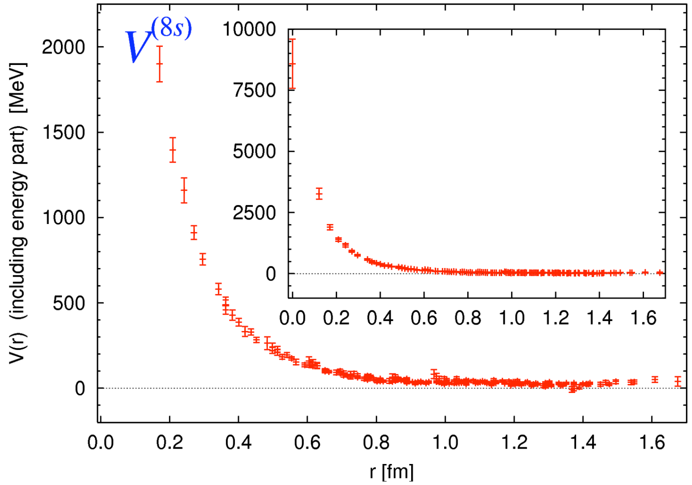
6 independent potential in flavor-basis

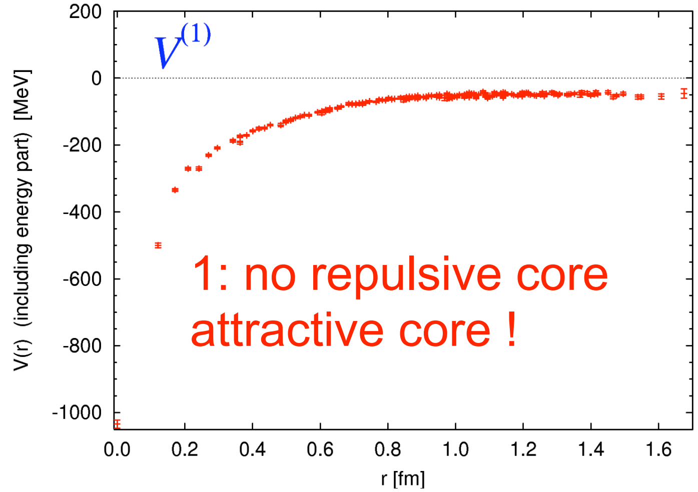
$$V^{(27)}(r),\ V^{(8s)}(r),\ V^{(1)}(r) \longleftarrow {}^1S_0$$

$$V^{(10^*)}(r),\ V^{(10)}(r),\ V^{(8a)}(r) \longleftarrow {}^3S_1$$

# Potentials

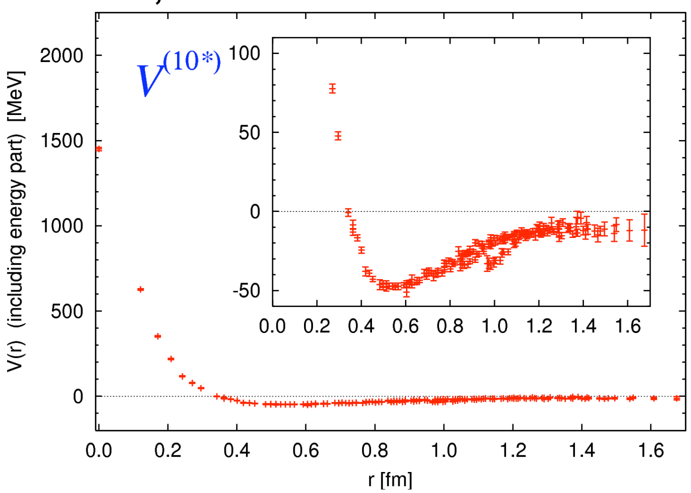


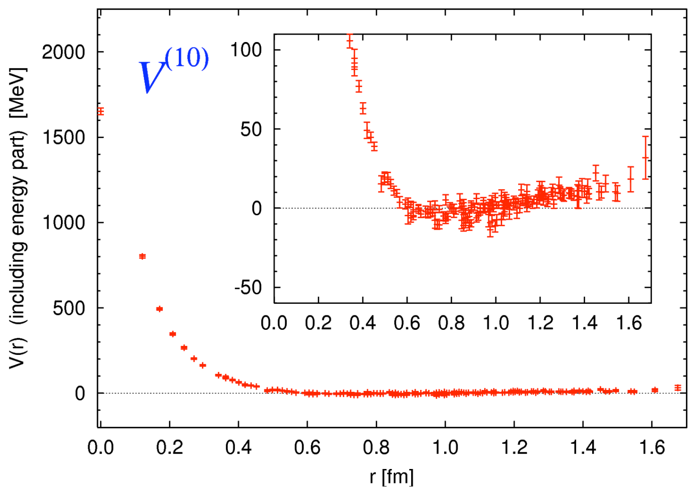



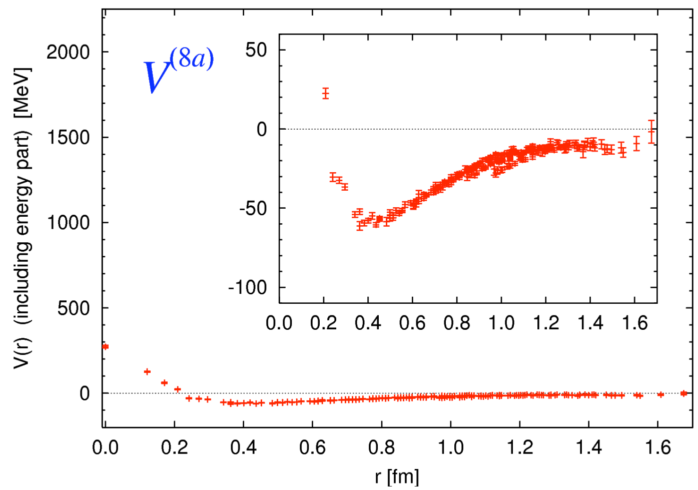




1: no repulsive core
attractive core !

27, 10*: same as before

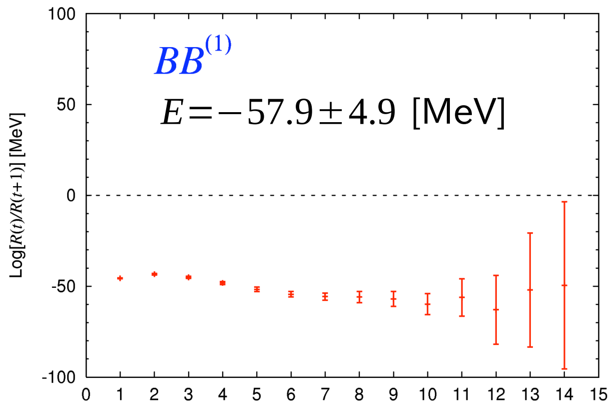



8s, 10: strong repulsive core

8a: week repulsive core,
deep attractive pocket




Bound state in 1(singlet) channel ?
H-dibaryon ?

Inoue for HAL QCD Collaboration

# 4-4. S=-2 In-elastic scattering

$$m_N = 939 \text{ MeV}, m_\Lambda = 1116 \text{ MeV}, m_\Sigma = 1193 \text{ MeV}, m_\Xi = 1318 \text{ MeV}$$

## S=-2 System

$$M_{\Lambda\Lambda} = 2232 \text{ MeV} < M_{N\Xi} = 2257 \text{ MeV} < M_{\Sigma\Sigma} = 2386 \text{ MeV}$$

They are so close, the eigen-state of QCD in the finite box is a mixture of them:

$$|S=-2, E\rangle^{\text{lattice}} = c_1 |\Lambda\Lambda, E\rangle_{\text{in}} + c_2 |\Xi N, E\rangle_{\text{in}} + c_3 |\Sigma\Sigma, E\rangle_{\text{in}}$$

$$E = 2\sqrt{m_\Lambda^2 + \mathbf{p}_1^2} = \sqrt{m_\Xi^2 + \mathbf{p}_2^2} + \sqrt{m_N^2 + \mathbf{p}_2^2} = 2\sqrt{m_\Sigma^2 + \mathbf{p}_3^2}$$

In this situation, we can not directly extract the scattering phase shift in lattice QCD.

# HAL's proposal

Let us consider 2-channel problem for simplicity.

NBS wave functions for 2 channels at 2 values of energy:

$$\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = \langle 0|\Lambda(\mathbf{x})\Lambda(\mathbf{0})|E_\alpha\rangle$$

$$\Psi_\alpha^{\Xi N}(\mathbf{x}) = \langle 0|\Xi(\mathbf{x})N(\mathbf{0})|E_\alpha\rangle$$

$$\alpha = 1, 2$$

They satisfy

$$(\nabla^2 + \mathbf{p}_\alpha^2)\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = 0$$

$$(\nabla^2 + \mathbf{q}_\alpha^2)\Psi_\alpha^{\Xi N}(\mathbf{x}) = 0$$

$$|\mathbf{x}| \to \infty$$

We define the "potential" from the coupled channel Schroedinger equation:

$$\left(\frac{\nabla^2}{2\mu_{\Lambda\Lambda}}+\frac{\mathbf{p}_\alpha^2}{2\mu_{\Lambda\Lambda}}\right)\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = V^{\Lambda\Lambda\leftarrow\Lambda\Lambda}(\mathbf{x})\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) + V^{\Lambda\Lambda\leftarrow\Xi N}(\mathbf{x})\Psi_\alpha^{\Xi N}(\mathbf{x})$$

diagonal　　　　off-diagonal

$$\left(\frac{\nabla^2}{2\mu_{\Xi N}}+\frac{\mathbf{q}_\alpha^2}{2\mu_{\Xi N}}\right)\Psi_\alpha^{\Xi N}(\mathbf{x}) = V^{\Xi N\leftarrow\Lambda\Lambda}(\mathbf{x})\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) + V^{\Xi N\leftarrow\Xi N}(\mathbf{x})\Psi_\alpha^{\Xi N}(\mathbf{x})$$

$\mu$: reduced mass

$$\begin{pmatrix} (E_1 - H_0^X)\Psi_1^X(\mathbf{x}) \\ (E_2 - H_0^X)\Psi_2^X(\mathbf{x}) \end{pmatrix} = \begin{pmatrix} \Psi_1^X(\mathbf{x}) & \Psi_1^Y(\mathbf{x}) \\ \Psi_2^X(\mathbf{x}) & \Psi_2^Y(\mathbf{x}) \end{pmatrix} \begin{pmatrix} V^{X\leftarrow X}(\mathbf{x}) \\ V^{X\leftarrow Y}(\mathbf{x}) \end{pmatrix}$$

$X \neq Y$

$X, Y = \Lambda\Lambda$ or $\Xi N$

$$\begin{pmatrix} V^{X\leftarrow X}(\mathbf{x}) \\ V^{X\leftarrow Y}(\mathbf{x}) \end{pmatrix} = \begin{pmatrix} \Psi_1^X(\mathbf{x}) & \Psi_1^Y(\mathbf{x}) \\ \Psi_2^X(\mathbf{x}) & \Psi_2^Y(\mathbf{x}) \end{pmatrix}^{-1} \begin{pmatrix} (E_1 - H_0^X)\Psi_1^X(\mathbf{x}) \\ (E_2 - H_0^X)\Psi_2^X(\mathbf{x}) \end{pmatrix}$$

Using the potentials:

$$\begin{pmatrix} V^{\Lambda\Lambda\leftarrow\Lambda\Lambda}(\mathbf{x}) & V^{\Xi N\leftarrow\Lambda\Lambda}(\mathbf{x}) \\ V^{\Lambda\Lambda\leftarrow\Xi N}(\mathbf{x}) & V^{\Xi N\leftarrow\Xi N}(\mathbf{x}) \end{pmatrix}$$

we solve the coupled channel Schroedinger equation with appropriate boundary conditions.

For example, we take the incomming $\Lambda\Lambda$ state by hand.

In this way, we can avoid the mixture of several “in”-states.

$$|S=-2,E\rangle^{\text{lattice}} = c_1|\Lambda\Lambda,E\rangle_{\text{in}} + c_2|\Xi N,E\rangle_{\text{in}} + c_3|\Sigma\Sigma,E\rangle_{\text{in}}$$

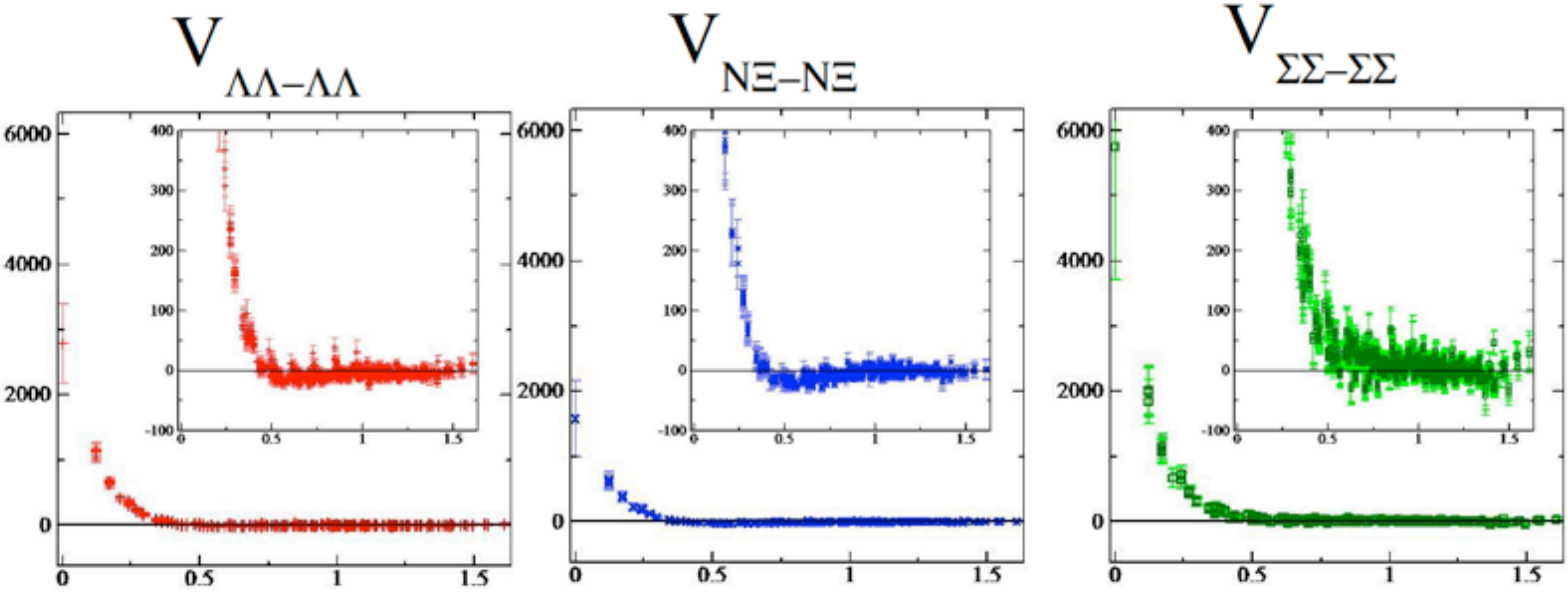
Lattice is a tool to extract the interaction kernel (“T-matrix” or “potential”).

# Preliminary results from HAL QCD Collaboration

Sasaki for HAL QCD Collaboration

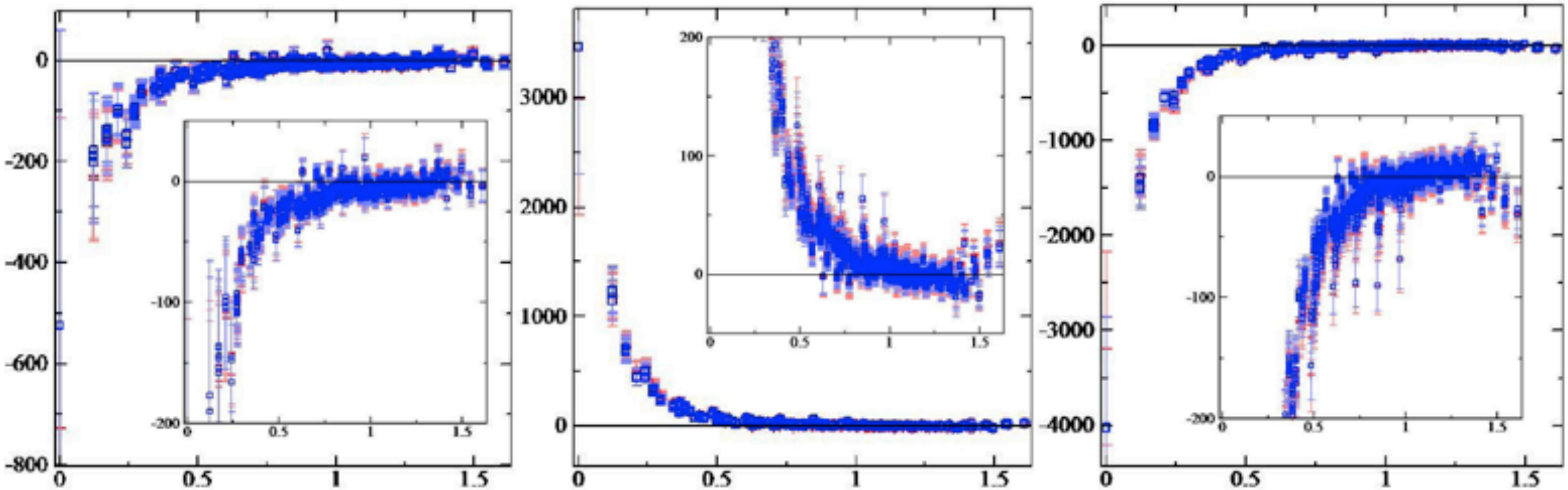
2+1 flavor full QCD

Diagonal part of potential matrix



# Non-diagonal part of potential matrix

$V_{\Lambda\Lambda-N\Xi}$ $V_{\Lambda\Lambda-\Sigma\Sigma}$ $V_{N\Xi-\Sigma\Sigma}$



$$V_{A-B} \simeq V_{B-A}$$

Hermiticity

# 4-5. Possible scenario for H-dibaryon

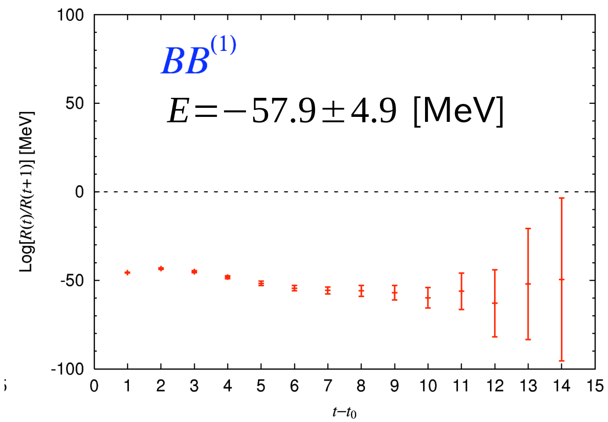
1. S=-2 singlet state become the bound state in flavor SU(3) limit.

2. In the real world (s is heavier than u,d), some resonance appears above ΛΛ but below ΞN threshold.

3. We can check this scenarion using the lattice QCD.

   3.1. The potential in SU(3) limit

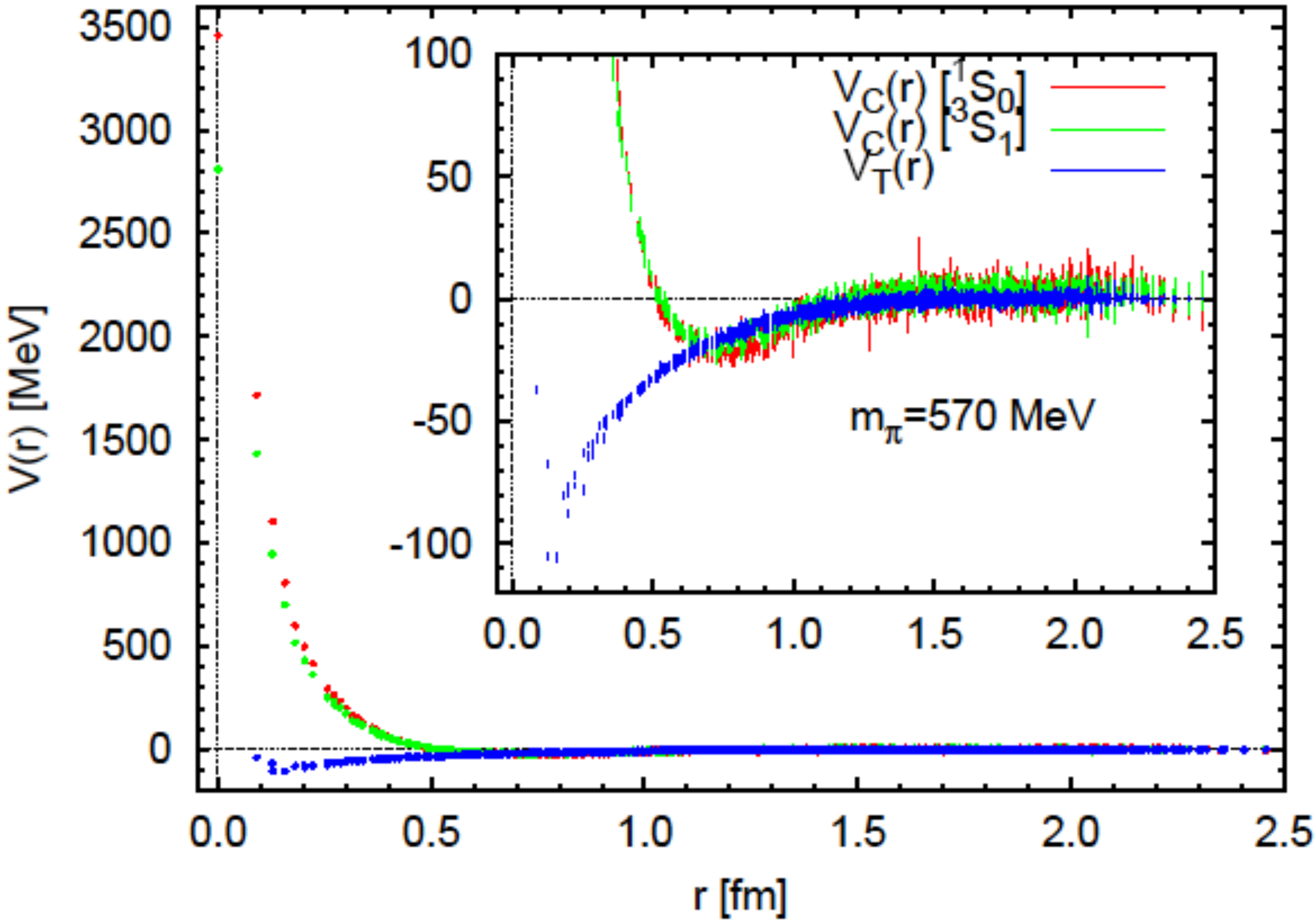
   3.2. The 3 x 3 potential matrix in real world




4. We may use this type of analysis for other systems such as penta-quark state.

# 5. Conclusion

# QCD meets Nuclei !




*Thank you for your attention !*

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Question 4

## Scheme-dependence of the potential ?

- the potential depends on the definition of the wave function, in particular, on the choice of the nucleon operator N(x). (Scheme-dependence)
- Moreover, the potential itself is NOT a physical observable. Therefore it is NOT unique and is naturally scheme-dependent.
  - Observables: scattering phase shift of NN, binding energy of deuteron
- Is the scheme-dependent potential useful ? Yes !
  - useful to understand/describe physics
  - a similar example: running coupling
    - Although the running coupling is scheme-dependent, it is useful to understand the deep inelastic scattering data (asymptotic freedom).
  - “good” scheme ?
    - good convergence of the perturbative expansion for the running coupling.
    - good convergence of the derivative expansion for the potential ?
      - completely local and energy-independent one is the best and must be unique. (Inverse scattering method)