Hadron-Hadron Interactions from Lattice QCD

Sinya AOKI

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1. Introduction
Nuclear force is a basis for understanding ...

- Structure of ordinary and hyper nuclei
- Structure of neutron star
- Ignition of Type II supernova
Phenomenological NN potential
(~40 parameters to fit 5000 phase shift data)

I One-pion exchange
   Yiukawa (1935)

II Multi-pions
   Taketani et al. (1951)

III Repulsive core
   Jastrow (1951)
Plan of my talk

1. Introduction

2. Strategy in (Lattice) QCD

3. Recent Developments
   1. Tensor potential
   2. Full QCD calculation

4. YN and YY interactions in lattice QCD
   1. S=-1 System
   2. S=-2 System
   3. BB interactions in an SU(3) symmetric world
   4. S=-2 Inelastic scattering
   5. H dibaryon

5. Conclusion
2. Strategy in (Lattice) QCD
   From Phenomenology to First Principle

"Even now, it is impossible to completely describe nuclear forces beginning with a fundamental equation. But since we know that nucleons themselves are not elementary, this is like asking if one can exactly deduce the characteristics of a very complex molecule starting from Schroedinger equation, a practically impossible task."

• well-defined statistical system (finite a and L)
• gauge invariant
• fully non-perturbative

Quenched QCD: neglects creation-annihilation of quark-antiquark pair
Full QCD: includes creation-annihilation of quark-antiquark pair
How to extract NN potentials in (lattice) QCD

Y. Nambu
“Force Potentials in Quantum Field Theory”

K. Nishijima
“Formulation of Field Theories for Composite Particles”

HAL QCD Collaboration

Sinya Aoki, Takumi Doi, Tetsuo Hatsuda,
Youichi Ikeda, Takashi Inoue, Noriyoshi Ishii,
Keiko Murano, Hidekatsu Nemura, Kenji Sasaki
Quantum Field Theoretical consideration

- S-matrix below inelastic threshold. Unitarity gives
  \[ S = e^{2i\delta} \]

- Nambu-Bethe-Salpeter (NBS) Wave function
  \[ \varphi_E(r) = \langle 0 | N(x + r, 0) N(x, 0) | 6q, E \rangle \]
  6 quark QCD eigen-state with energy \( E \)

  \( N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x) \): local operator

Asymptotic behavior

\[ r = |r| \rightarrow \infty \]

\[ \varphi^l_E(r) \rightarrow A_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} \]

partial wave

\[ \delta_l(k) \] is the scattering phase shift

\[ E = \frac{k^2}{2\mu_N} = \frac{k^2}{m_N} \]
Systematic procedure to define the NN potential in lattice QCD

1. Choose your favorite operator: e.g.
   \[ N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x) \]
   - observables do not depend on the choice
   - yet the local operator is useful

2. Measure the NBS amplitude:
   \[ \varphi_E(r) = \langle 0| N(x + r, 0) N(x, 0)|6q, E \rangle \]

3. Define the non-local potential:
   \[ [E - H_0] \varphi_E(x) = \int d^3 y U(x, y) \varphi_E(y) \]

4. Velocity expansion:
   \[ U(x, y) = V(x, \nabla) \delta^3(x - y) \]
   \[ V(x, \nabla) = V_C(r) + V_T(r) S_{12} + V_{LS}(r) L \cdot S + \{V_D(r), \nabla^2\} + \cdots \]

   \[ S_{12} = \frac{3}{r^2} (\sigma_1 \cdot x)(\sigma_2 \cdot x) - (\sigma_1 \cdot \sigma_2) \]

5. Calculate observables: phase shift, binding energy etc.

Full details: Aoki, Hatsuda & Ishii, PTP123(2010)89 (arXiv0909.5585)

Nishijima, Haag, Zimmermann (1958)

Okubo-Marshak (1958), Tamagaki-Watari (1967)
Key Channels in NN Scattering

\[ V(x, \nabla) = V_C(r) + V_T(r)S_{12} + V_{LS}(r)L \cdot S + \{V_D(r), \nabla^2\} + \cdots \]

1\(^1\)S\(_0\)  Central Force  nuclear BCS pairing

Bohr, Mottelson, Pines, Phys. Rev. 110 (1958)

3\(^3\)S\(_1\) - 3\(^3\)D\(_1\)  Tensor Force  deuteron binding


3\(^3\)P\(_2\) - 3\(^3\)F\(_2\)  LS Force  neutron superfluidity in neutron stars

First (quenched) results

Central Potential

\[ ^1S_0, ^3S_1 \]

\[ \frac{1}{2} \pi \sim 0.53 \text{ GeV} \]

\[ E \sim 0 \]

Qualitative features of NN potential are reproduced!


This paper has been selected as one of 21 papers in Nature Research Highlights 2007

“The achievement is both a computational tour de force and a triumph for theory.”
Frequently Asked Questions

Q1: Operator dependence of the potential
Q2: Energy dependence of the potential

A1: \((N(x), U(x,y))\) is a combination to define observables

- remember, QM: \((\Phi, U)\) → observables
  QFT: (asymptotic field, vertices) → observables
  EFT: (choice of field, vertices) → observables
- local operator = convenient choice for reduction formula

A2: \(U(x,y)\) is E-independent by construction

- non-locality can be determined order by order in velocity expansion
  (c.f. ChPT)
Question 3

How good is the velocity expansion of \( V \)?

**Leading Order**

\[
V_C(r) = \frac{(E - H_0)\varphi_E(x)}{\varphi_E(x)}
\]

**Local potential approximation**

The local potential obtained at given energy \( E \) may depend on \( E \).

If the energy dependence of the potential is weak, the local potential approximation is good.

Furthermore one may determine the higher order terms by comparing results among different energies.

\[
V(x, \nabla) = V_C(r) + V_T(r)S_{12} + V_{LS}(r)L \cdot S + \{V_D(r), \nabla^2\} + \cdots
\]

**Numerical check in quenched QCD**

\( m_\pi \approx 0.53 \text{ GeV} \)

K. Murano, S. Aoki, T. Hatsuda, N. Ishii, H. Nemura
1. Use phase shifts for $T_{lab} = 0 \sim 350$ MeV ($\sim 4500$ data points) & deuteron properties

2. Fits with $\chi^2$/dof $\sim 1$ by e.g. 18 parameters (Argonne V18)

$E$ dependence of the local potential turns out to be very small at low energy in our choice of wave function.

$m_\pi \simeq 0.53$ GeV
3. Recent developments
3-1. Tensor potential

\[
(H_0 + V_C + V_T S_{12})|\phi\rangle = E|\phi\rangle
\]

mixing between \( ^3S_1 \) and \( ^3D_1 \) through the tensor force

\[
|\phi\rangle = |\phi_S\rangle + |\phi_D\rangle
\]

\[
|\phi_S\rangle = P|\phi\rangle = \frac{1}{24} \sum_{R \in \mathcal{O}} R|\phi\rangle \quad \text{“projection” to L=0} \quad ^3S_1
\]

\[
|\phi_D\rangle = Q|\phi\rangle = (1 - P)|\phi\rangle \quad \text{“projection” to L=2} \quad ^3D_1
\]
Wave functions

Aoki, Hatsuda, Ishii, PTP 123 (2010)89
arXiv:0909.5585

Quenched

quenched QCD
E $\sim$ 0 MeV
$m_\pi = 529$ MeV
Tensor Force and Central Force ($t-t_0=5$)

$m_\pi \simeq 0.53$ GeV

No repulsive core in tensor

Aoki, Hatsuda, Ishii, PTP 123 (2010)89
arXiv:0909.5585
No repulsive core in tensor
Quark mass dependence

- Rapid quark mass dependence of tensor potential
- Evidence of one-pion exchange
3-2. Full QCD Calculation

$m_\pi = 570$ MeV, $L = 2.9$ fm
Phase shift from $V(r)$ in full QCD

$1S_0$

$3S_1$

$V_C(r, 1S_0)$ [MeV]

$V_C(r, 3S_1)$ [MeV]

$r$ [fm]

$E_{\text{lab}}$ [MeV]

$\delta(k; 1S_0)$ [degree]

$\delta(k; 3S_1)$ [degree]

$m_\pi=411$ MeV

$m_\pi=570$ MeV

$m_\pi=701$ MeV

2010年6月23日水曜日
4. YN and YY interactions in lattice QCD

- no phase shift available for YN and YY scattering
- plenty of hyper-nucleus data will be soon available at J-PARC

- prediction from lattice QCD
- difference between NN and YN?
Hyperon Core of Neutron Stars

Radius $\sim 10$ km
Mass $\sim$ solar mass
Central density $\sim 10^{12}$ kg/cm$^3$

Hyperon matter?

\[ n, p, \Sigma^-, \Lambda, e^- \text{ with } \Sigma^- \leftrightarrow n + e^-, \Lambda \leftrightarrow n \]

Solid Crust
Neutron Liquid
3D Nuclear chart

S=-2

3 known

S=-1

40 known

~3000 known
4-1. S = -1 System: ΛN interaction (l=1/2) in full QCD

1. repulsive core + attractive well
2. Large spin dependence
3. Overall attraction
4-2. $S = -2$ System

\[ \Xi N \text{ (I=1) potential} \]

1. repulsive core + attractive well
2. Large spin dependence
3. weaker quark mass dependence

4-3. BB interactions in an SU(3) symmetric world

1. First step to predict YN, YY interactions not accessible in exp.
2. Origin of the repulsive core (universal or not)

\[
8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8a
\]

Symmetric \hspace{2cm} Anti-symmetric

6 independent potential in flavor-basis

\[
V^{(27)}(r), \ V^{(8s)}(r), \ V^{(1)}(r) \hspace{2cm} \begin{array}{c} 1S_0 \\ 3S_1 \end{array}
\]

\[
V^{(10^*)}(r), \ V^{(10)}(r), \ V^{(8a)}(r)
\]

\[m_u = m_d = m_s\]
Inoue et al.

Potentials

$V^{(27)}$, 1: no repulsive core
attractive core!

27, 10*: same as before

$V^{(10s)}$, 8s, 10: strong repulsive core

$V^{(8a)}$, 8a: weak repulsive core,
deep attractive pocket

Bound state in 1(singlet) channel? 
H-dibaryon?

Inoue for HAL QCD Collaboration

2010年6月23日水曜日
4-4. S=-2 In-elastic scattering

\[ m_N = 939 \text{ MeV}, \ m_\Lambda = 1116 \text{ MeV}, \ m_\Sigma = 1193 \text{ MeV}, \ m_\Xi = 1318 \text{ MeV} \]

**S=-2 System**

\[ M_{\Lambda\Lambda} = 2232 \text{ MeV} < M_{N\Xi} = 2257 \text{ MeV} < M_{\Sigma\Sigma} = 2386 \text{ MeV} \]

They are so close, the eigen-state of QCD in the finite box is a mixture of them:

\[ |S = -2, E\rangle^{\text{lattice}} = c_1 |\Lambda\Lambda, E\rangle_{\text{in}} + c_2 |\Xi N, E\rangle_{\text{in}} + c_3 |\Sigma\Sigma, E\rangle_{\text{in}} \]

\[ E = 2 \sqrt{m_\Lambda^2 + p_1^2} = \sqrt{m_\Xi^2 + p_2^2} + \sqrt{m_N^2 + p_2^2} = 2 \sqrt{m_\Sigma^2 + p_3^2} \]

In this situation, we can not directly extract the scattering phase shift in lattice QCD.
HAL’s proposal

Let us consider 2-channel problem for simplicity.

NBS wave functions for 2 channels at 2 values of energy:

\[ \Psi_{\alpha}^{\Lambda \Lambda}(x) = \langle 0 | \Lambda(x) \Lambda(0) | E_\alpha \rangle \]
\[ \Psi_{\alpha}^{\Xi N}(x) = \langle 0 | \Xi(x) N(0) | E_\alpha \rangle \] \( \alpha = 1, 2 \)

They satisfy

\[ (\nabla^2 + p_\alpha^2) \Psi_{\alpha}^{\Lambda \Lambda}(x) = 0 \]
\[ (\nabla^2 + q_\alpha^2) \Psi_{\alpha}^{\Xi N}(x) = 0 \] \(|x| \to \infty\)
We define the “potential” from the coupled channel Schrödinger equation:

\[
\left( \frac{\nabla^2}{2\mu_{\Lambda\Lambda}} + \frac{p_\alpha^2}{2\mu_{\Lambda\Lambda}} \right) \Psi_{\alpha}^{\Lambda\Lambda}(x) = V^{\Lambda\Lambda\leftarrow\Lambda\Lambda}(x) \Psi_{\alpha}^{\Lambda\Lambda}(x) + V^{\Lambda\Lambda\leftarrow\Xi N}(x) \Psi_{\alpha}^{\Xi N}(x)
\]

\[
\text{diagonal}
\]

\[
\text{off-diagonal}
\]

\[
\left( \frac{\nabla^2}{2\mu_{\Xi N}} + \frac{q_\alpha^2}{2\mu_{\Xi N}} \right) \Psi_{\alpha}^{\Xi N}(x) = V^{\Xi N\leftarrow\Lambda\Lambda}(x) \Psi_{\alpha}^{\Lambda\Lambda}(x) + V^{\Xi N\leftarrow\Xi N}(x) \Psi_{\alpha}^{\Xi N}(x)
\]

\[
\mu: \text{reduced mass}
\]

\[
\left( \begin{array}{c} (E_1 - H_{0}^{X}) \Psi_{1}^{X}(x) \\ (E_2 - H_{0}^{X}) \Psi_{2}^{X}(x) \end{array} \right) = \left( \begin{array}{cc} \Psi_{1}^{X}(x) & \Psi_{1}^{Y}(x) \\ \Psi_{2}^{X}(x) & \Psi_{2}^{Y}(x) \end{array} \right) \left( \begin{array}{c} V^{X\leftarrow X}(x) \\ V^{X\leftarrow Y}(x) \end{array} \right)
\]

\[
X \neq Y
\]

\[
X, Y = \Lambda\Lambda \text{ or } \Xi N
\]

\[
\left( \begin{array}{c} V^{X\leftarrow X}(x) \\ V^{X\leftarrow Y}(x) \end{array} \right) = \left( \begin{array}{cc} \Psi_{1}^{X}(x) & \Psi_{1}^{Y}(x) \\ \Psi_{2}^{X}(x) & \Psi_{2}^{Y}(x) \end{array} \right)^{-1} \left( \begin{array}{c} (E_1 - H_{0}^{X}) \Psi_{1}^{X}(x) \\ (E_2 - H_{0}^{X}) \Psi_{2}^{X}(x) \end{array} \right)
\]
Using the potentials:

\[
\begin{pmatrix}
V_{\Lambda\Lambda \leftarrow \Lambda\Lambda}(x) & V_{\Xi N \leftarrow \Lambda\Lambda}(x) \\
V_{\Lambda\Lambda \leftarrow \Xi N}(x) & V_{\Xi N \leftarrow \Xi N}(x)
\end{pmatrix}
\]

we solve the coupled channel Schroedinger equation with appropriate boundary conditions.

For example, we take the incoming $\Lambda\Lambda$ state by hand.

In this way, we can avoid the mixture of several “in”-states.

\[
|S = -2, E\rangle^{\text{lattice}} = c_1|\Lambda\Lambda, E\rangle_{\text{in}} + c_2|\Xi N, E\rangle_{\text{in}} + c_3|\Sigma\Sigma, E\rangle_{\text{in}}
\]

Lattice is a tool to extract the interaction kernel (“T-matrix” or “potential”).
Preliminary results from HAL QCD Collaboration

2+1 flavor full QCD

Diagonal part of potential matrix
Non-diagonal part of potential matrix

\[
V_{\Lambda\Lambda-N\Xi} \quad V_{\Lambda\Lambda-S\Sigma} \quad V_{N\Xi-S\Sigma}
\]

\[
V_{A-B} \simeq V_{B-A}
\]

Hermiticity
4-5. Possible scenario for H-dibaryon

1. S=-2 singlet state become the bound state in flavor SU(3) limit.

2. In the real world (s is heavier than u,d), some resonance appears above $\Lambda\Lambda$ but below $\Xi N$ threshold.

3. We can check this scenario using the lattice QCD.
   3.1. The potential in SU(3) limit
   3.2. The 3 x 3 potential matrix in real world

4. We may use this type of analysis for other systems such as penta-quark state.
5. Conclusion
QCD meets Nuclei!

Thank you for your attention!
backup slide
Question 4

Scheme-dependence of the potential?

- The potential depends on the definition of the wave function, in particular, on the choice of the nucleon operator $N(x)$. (Scheme-dependence)

- Moreover, the potential itself is NOT a physical observable. Therefore it is NOT unique and is naturally scheme-dependent.
  - Observables: scattering phase shift of NN, binding energy of deuteron

- Is the scheme-dependent potential useful? Yes!
  - Useful to understand/describe physics
  - A similar example: running coupling
    - Although the running coupling is scheme-dependent, it is useful to understand the deep inelastic scattering data (asymptotic freedom).

- "Good" scheme?
  - Good convergence of the perturbative expansion for the running coupling.
  - Good convergence of the derivative expansion for the potential?
    - Completely local and energy-independent one is the best and must be unique. (Inverse scattering method)