

Relativistic Models for Neutrino-Nucleus Scattering

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Collaboration

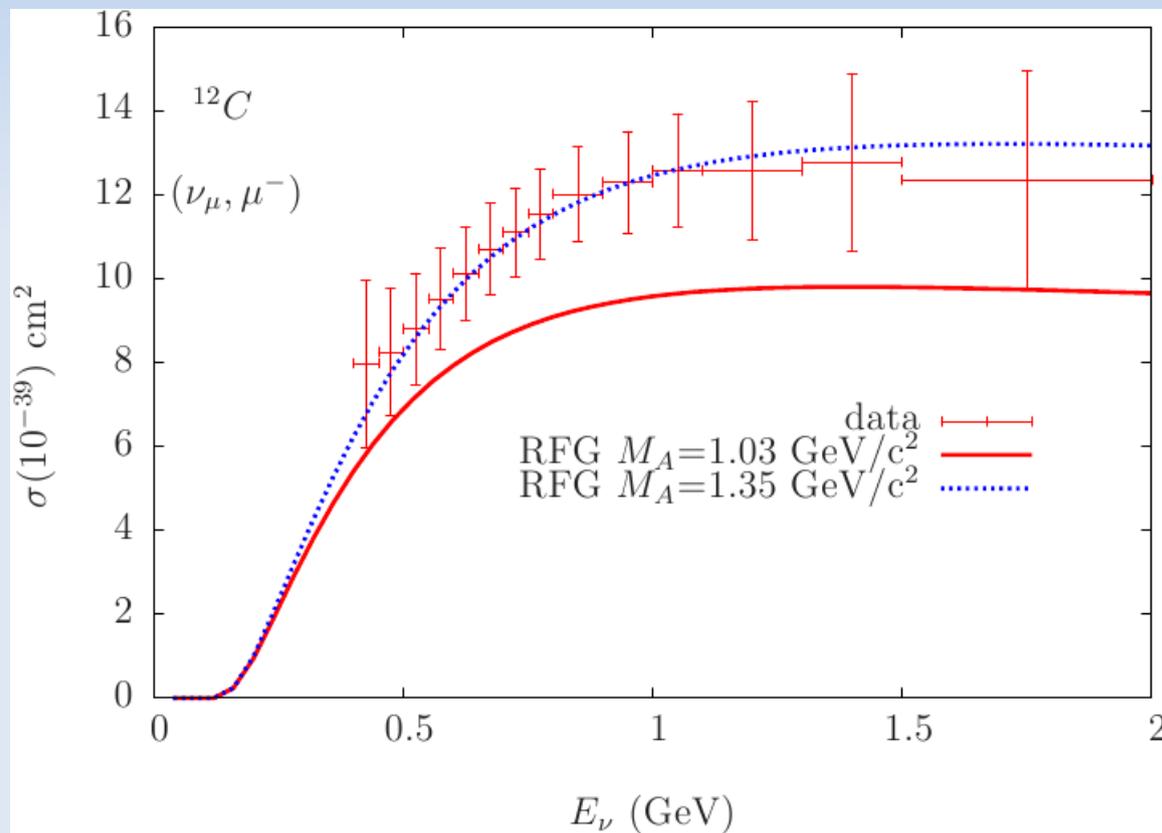
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- J.A. Caballero (Sevilla)
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Motivation

Current and future experiments searching for neutrino oscillations use complex nuclei as targets and rely on the precise prediction of neutrino-nucleus cross sections for their analysis

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The Relativistic Fermi Gas underpredicts the MiniBooNE total cross section by about 20%

unless a very high axial mass $M_A=1.35 \text{ GeV}/c^2$ is used

More realistic nuclear models are needed in order to understand this result

MiniBooNE data, Phys. Rev. D81, 092005 (2010)

Outline

Charged-Current neutrino-nucleus scattering formalism

- Quasielastic peak
- Delta-resonance region

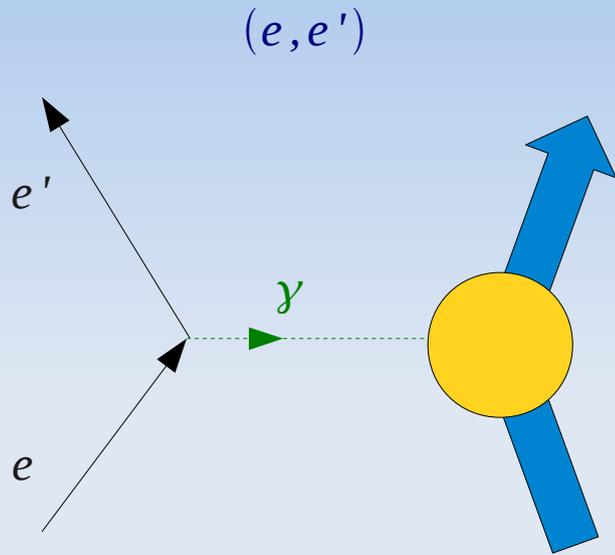
Relativistic models:

1. Relativistic Fermi Gas
2. Phenomenological approach based on SuperScaling analysis of (e,e')
3. Relativistic Mean Field Model
4. Semi-relativistic Shell Model

Results: comparison with MiniBooNE quasielastic differential cross sections

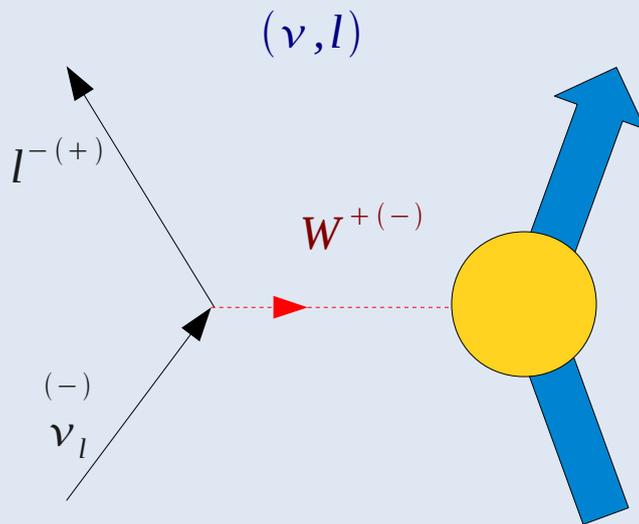
Summary and Conclusions

(l,l') inclusive scattering



$$\frac{d^2\sigma}{d\epsilon' d\Omega} = \sigma_{Mott} (v_L R_L + v_T R_T)$$

2 electromagnetic response functions



$$\frac{d^2\sigma}{d\epsilon' d\Omega} = \sigma_0 (V_{CC} R_{CC} + 2V_{CL} R_{CL} + V_{LL} R_{LL} + V_T R_T \pm 2V_{T'} R_{T'})$$

5 weak response functions

$l = \mu, e, \tau$

CC neutrino reaction formalism

$$\nu_l + A \rightarrow l^- + B$$

$$\bar{\nu}_l + A \rightarrow l^+ + B$$

$$\left[\frac{d^2 \sigma}{d\Omega dk'} \right]_X = \sigma_0 F_X^2$$

← nuclear response function

← outgoing lepton

← elementary cross section

$$\sigma_0 = \frac{(G \cos \theta_c)^2}{2\pi^2} [k' \cos \tilde{\theta}/2]^2$$

$$\tan \tilde{\theta}/2 = \frac{|Q^2|}{4\epsilon\epsilon' - |Q^2|} \quad \text{effective angle}$$

Generalized Rosenbluth separation (different components of the hadronic tensor $W^{\mu\nu}$)

$$F_X^2 = V_{CC} R_{CC} + 2V_{CL} R_{CL} + V_{LL} R_{LL} + V_T R_T + 2\chi V_{T'} R_{T'}$$

← transverse-axial(12)

V_K kinematical factors

Weak current $j_\mu = j_\mu^V - j_\mu^A \rightarrow F_X^2 = X_L^{VV} + X_{C/L}^{AA} + X_T^{VV+AA} + \chi X_{T'}^{VA}$

← Vector ← Axial-vector

$$\chi = \begin{matrix} +1 & \nu \\ -1 & \bar{\nu} \end{matrix}$$

← opposite sign for neutrino and antineutrino

Quasi-elastic peak

$$\nu_l + n \rightarrow l^- + p, \quad \bar{\nu}_l + p \rightarrow l^+ + n$$

- Single nucleon current: $j^\mu = j_V^\mu - j_A^\mu$

$$j_V^\mu = \bar{u}(P') \left(F_1 \gamma^\mu + \frac{i}{2m_N} F_2 \sigma^{\mu\nu} Q_\nu \right) u(P) \rightarrow R_L^{VV} = \frac{\kappa^2}{\tau} [G_E^{(1)}]^2, \quad R_T^{VV} = 2\tau [G_M^{(1)}]^2$$

$$j_A^\mu = \bar{u}(P') \left(G_A \gamma^\mu + \frac{1}{2m_N} G_P Q^\mu \right) \gamma_5 u(P) \rightarrow R_{LL}^{AA} = \frac{\kappa^2}{\lambda^2} R_{CC}^{AA} = \frac{-\kappa}{\lambda} R_{CL}^{AA} = \frac{\kappa^2}{\tau} [G_A^{(1)} - \tau G_P^{(1)}]^2$$

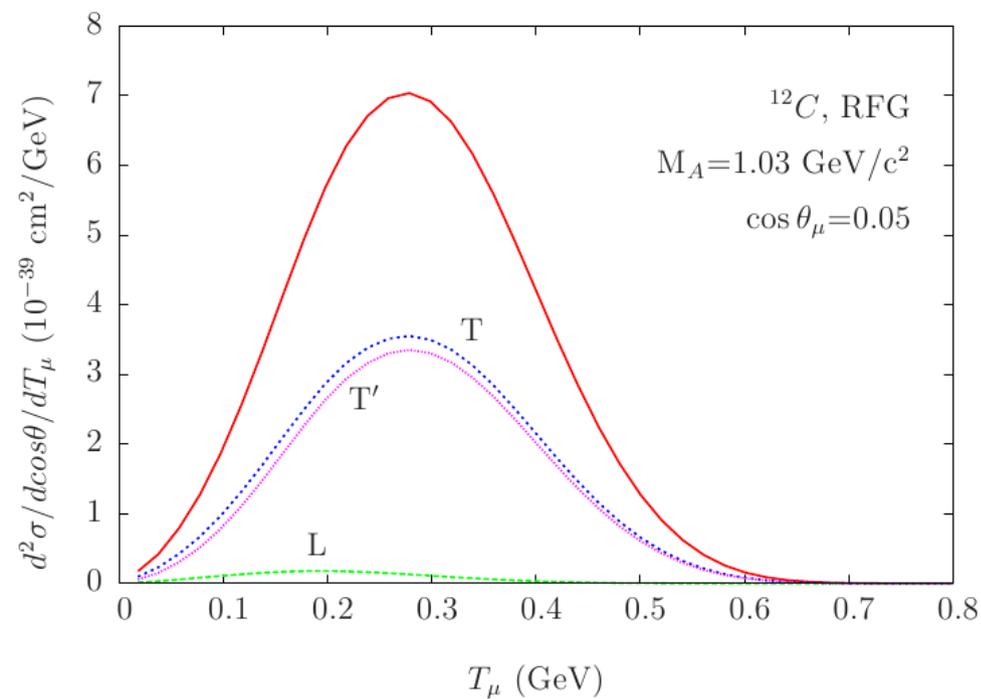
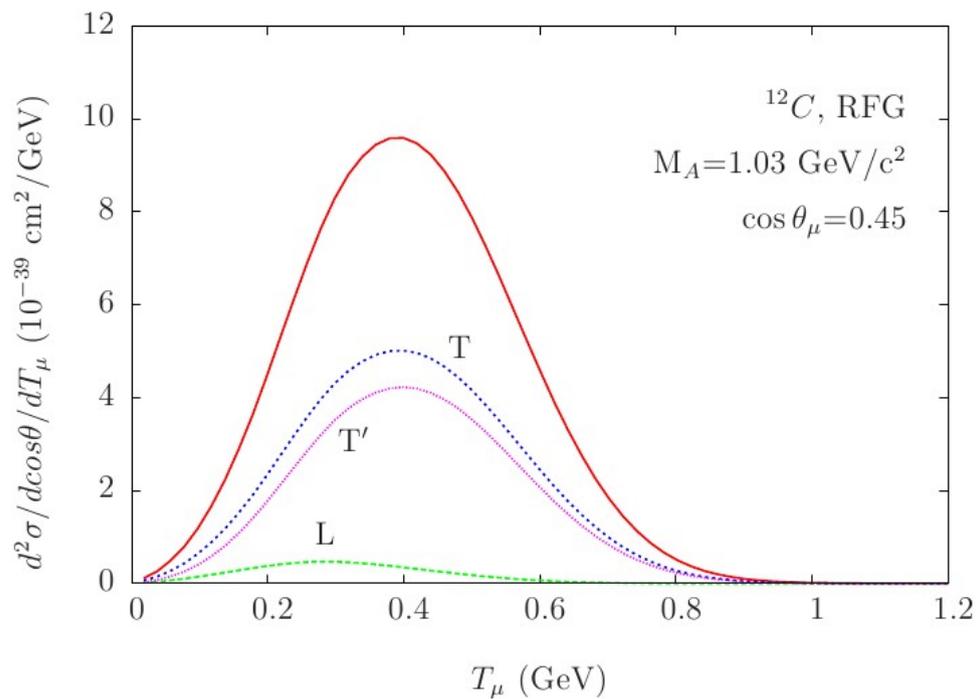
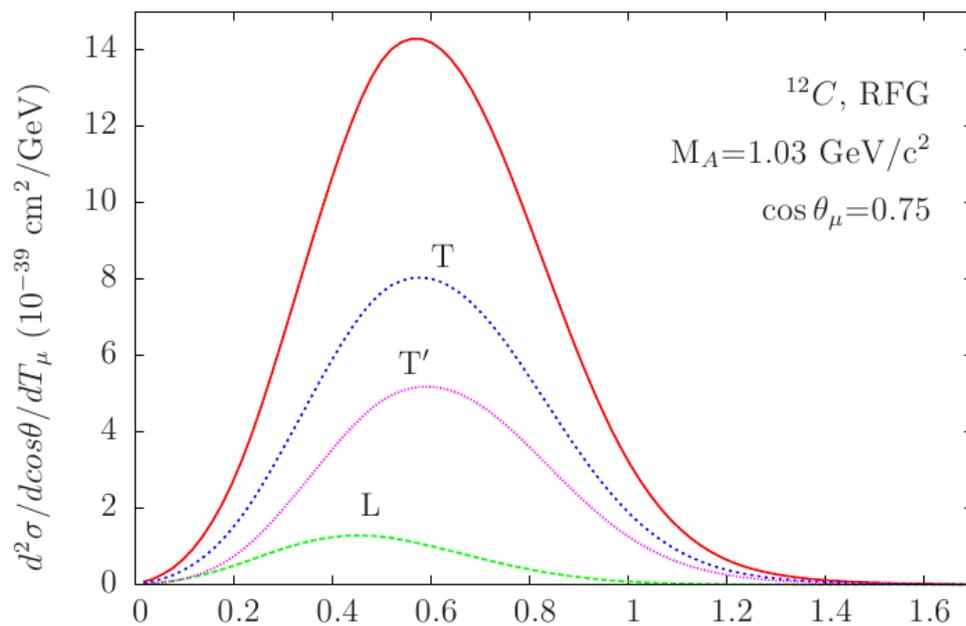
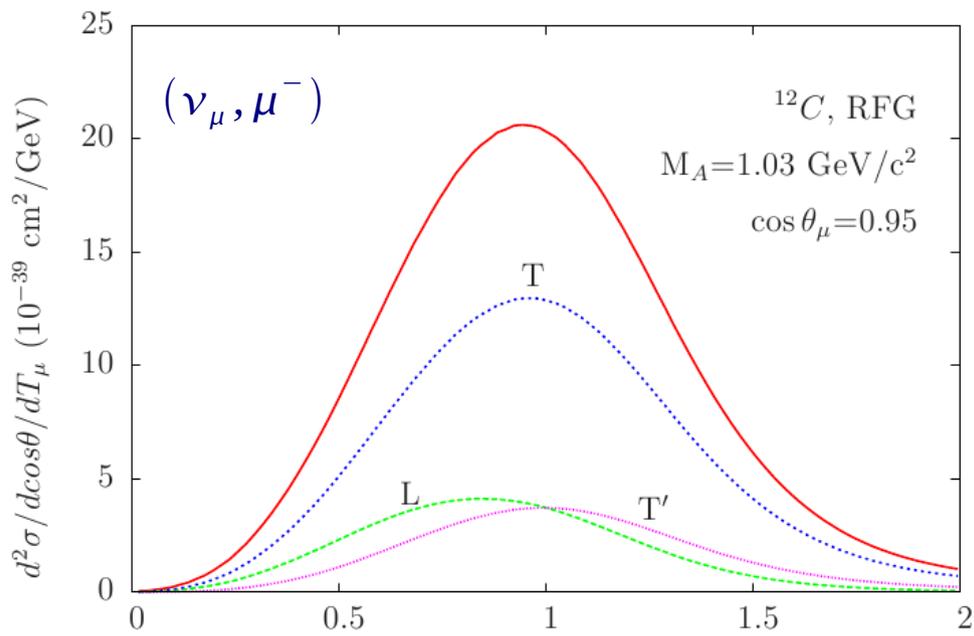
$$R_T^{AA} = 2(1+\tau) [G_A^{(1)}]^2, \quad R_{T'}^{VA} = 2\sqrt{\tau(1+\tau)} G_M^{(1)} G_A^{(1)}$$

$$\kappa = q/(2m_N), \quad \lambda = \omega/(2m_N), \quad \tau = \kappa^2 - \lambda^2 \quad \text{dimensionless variables}$$

- The **nuclear responses** are purely **isovector**, typically **transverse** and have vector-vector (**VV**), axial-axial (**AA**) and vector-axial (**VA**) contributions

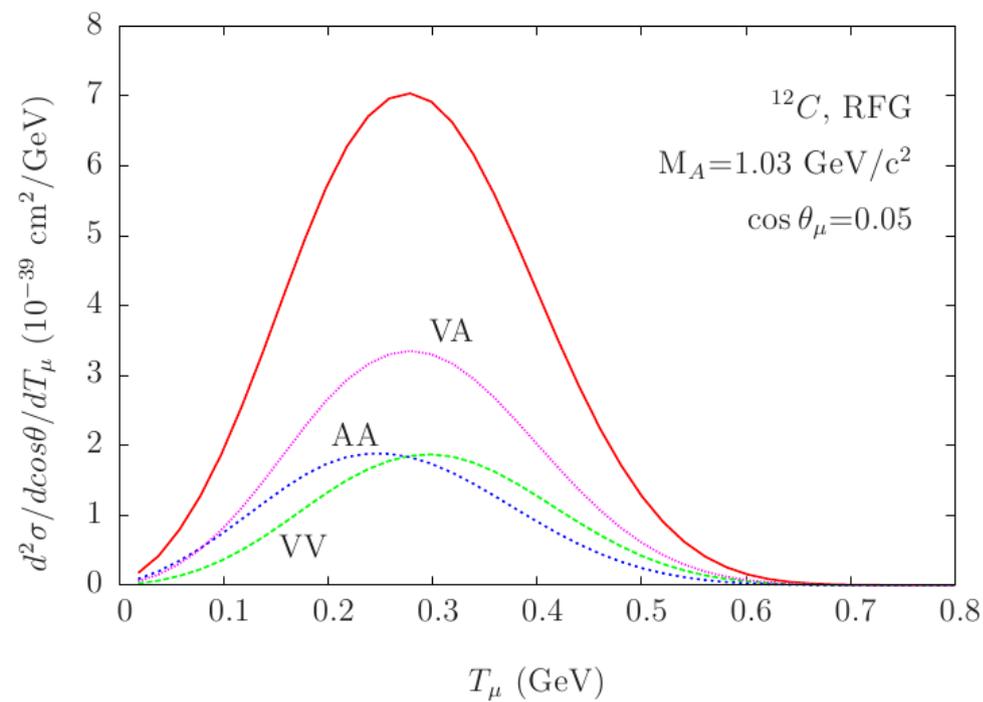
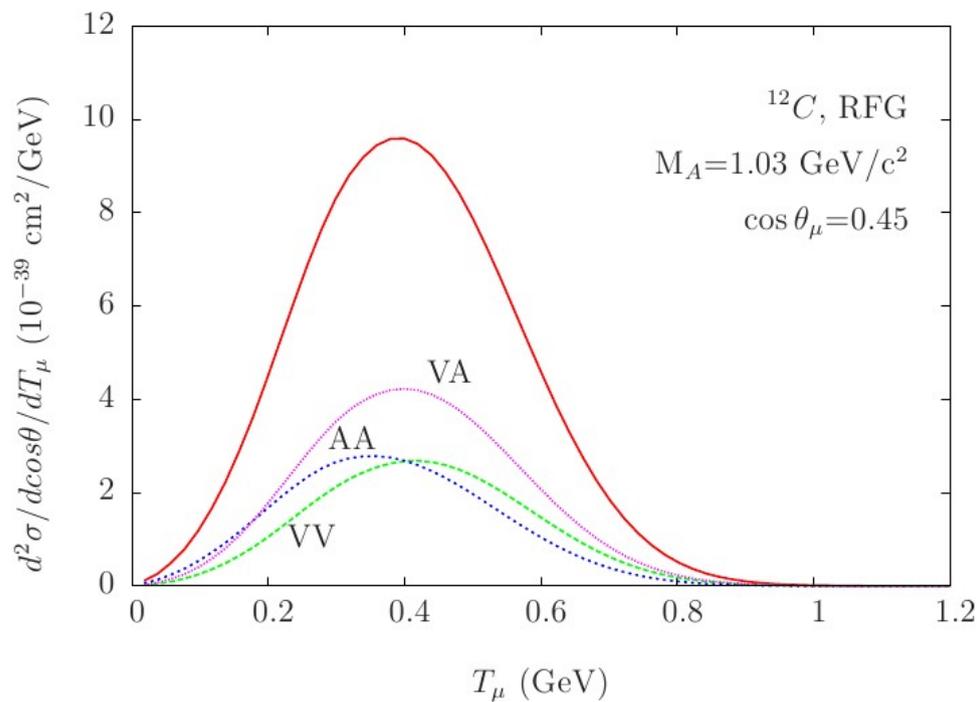
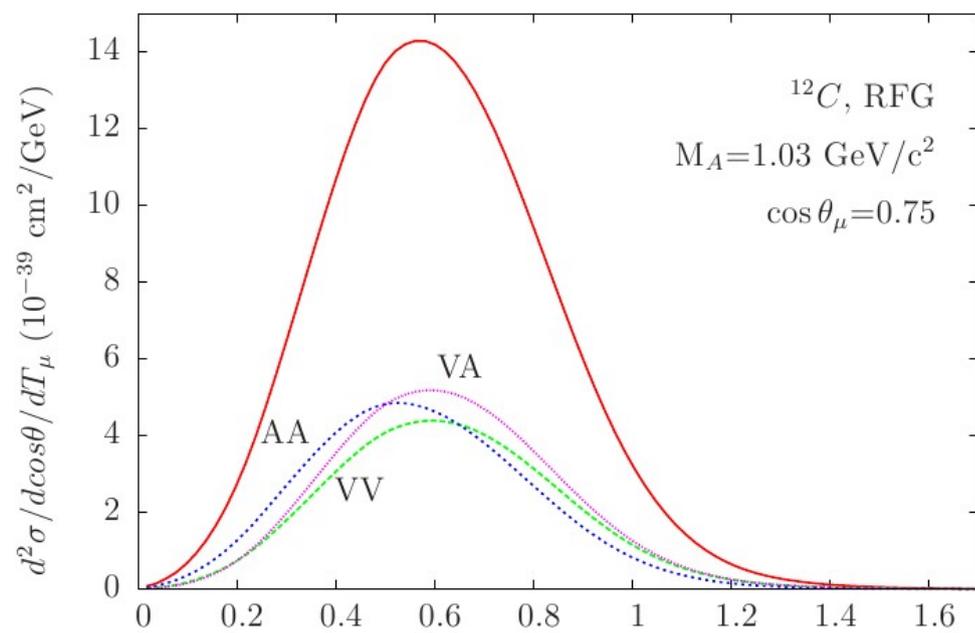
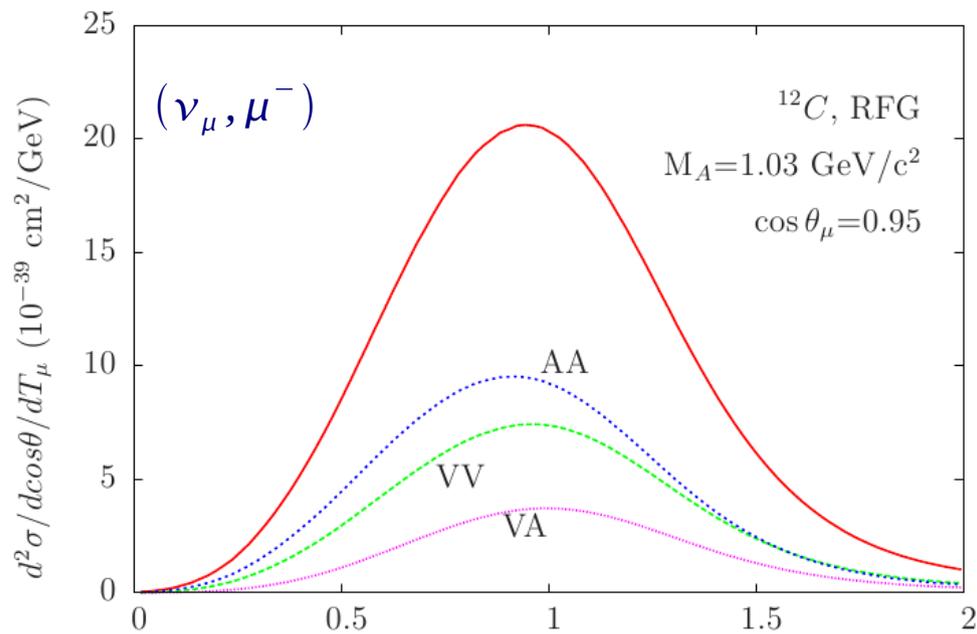
L-T-T' separation

MiniBooNE
kinematics



VV-AA-VA separation

MiniBooNE
kinematics



Delta-resonance excitation

$$\begin{aligned} \nu_\mu + n &\rightarrow \mu^- + \Delta^+, & \nu_\mu + p &\rightarrow \mu^- + \Delta^{++} \\ \bar{\nu}_\mu + n &\rightarrow \mu^+ + \Delta^-, & \bar{\nu}_\mu + p &\rightarrow \mu^+ + \Delta^0 \end{aligned}$$

$$j_\Delta^\mu = T \bar{u}_\alpha^{(\Delta)}(P', s') \Gamma^{\alpha\mu} u(P, s)$$

isospin factor

Rarita-Schwinger spinor

$$\Gamma^{\alpha\mu} = \Gamma_V^{\alpha\mu} (C_3^V, C_4^V, C_5^V, C_6^V) \gamma_5 + \Gamma_A^{\alpha\mu} (C_3^A, C_4^A, C_5^A, C_6^A)$$

vector

axial-vector

vertex tensor

8 form factors

$$CVC \rightarrow C_6^V = 0$$

$$PCAC \rightarrow C_6^A = C_5^A / (\mu_\pi^2 + 4\tau)$$

6 form factors :

$$C_3^V(\tau) = 2.05(1 + |Q^2|/0.54 \text{ GeV}^2)^{-2}, \quad C_4^V(\tau) = \frac{-m_N}{m_\Delta} C_3^V(\tau), \quad C_5^V(\tau) = 0$$

$$C_3^A(\tau) = 0, \quad C_4^A(\tau) = -0.3 \frac{1 - 1.21|Q^2|/(2 \text{ GeV}^2 + |Q^2|)}{[1 + |Q^2|/(1.28)^2 \text{ GeV}^2]^2}, \quad C_5^A(\tau) = -4 C_4^A(\tau)$$

Relativistic Fermi Gas

- The nucleus is a collection of free nucleons described by Dirac spinors $u(p,s)$
- The only correlations between nucleons are the Pauli correlations
- Fully Lorentz covariant

Response functions:

$$R_K(\kappa, \lambda) = G_K(\kappa, \lambda) f_{RFG}(\psi)$$

$$\lambda = \frac{\omega}{2m_N}, \kappa = \frac{q}{2m_N}, \tau = \kappa^2 - \lambda^2$$

dimensionless variables

$$\psi(\kappa, \lambda) = \frac{1}{\sqrt{\xi_F}} \frac{\lambda - \tau}{\sqrt{\tau(1+\lambda) + \kappa(1+\tau)}}$$

RFG scaling variable

$$\xi_F = T_F / m_N \quad \text{Fermi kinetic energy}$$

↑
single-nucleon
functions

←
“super-scaling” function
(universal)

$$f_{RFG}(\psi) = \frac{3}{4} (1 - \psi^2) \theta(1 - \psi^2)$$

- 2 parameters: k_F Fermi momentum

$$E_s \quad \text{energy shift (typically } \sim 20 \text{ MeV)}: \quad \begin{aligned} \omega &\rightarrow \omega' = \omega - E_s \\ \psi &\rightarrow \psi' = \psi(\lambda', \tau') \end{aligned}$$

Super-Scaling Analysis of (e,e')

[Day,McCarthy,Donnelly,Sick,Ann.Rev.Nucl.Part.Sci.40(1990); Donnelly & Sick, PRC60(1999),PRL82(1999)]

- Plot the reduced cross section

$$F(q, y) = \frac{d\sigma/d\Omega d\omega}{\sigma_{Mott}(v_L G_L + v_T G_T)}$$

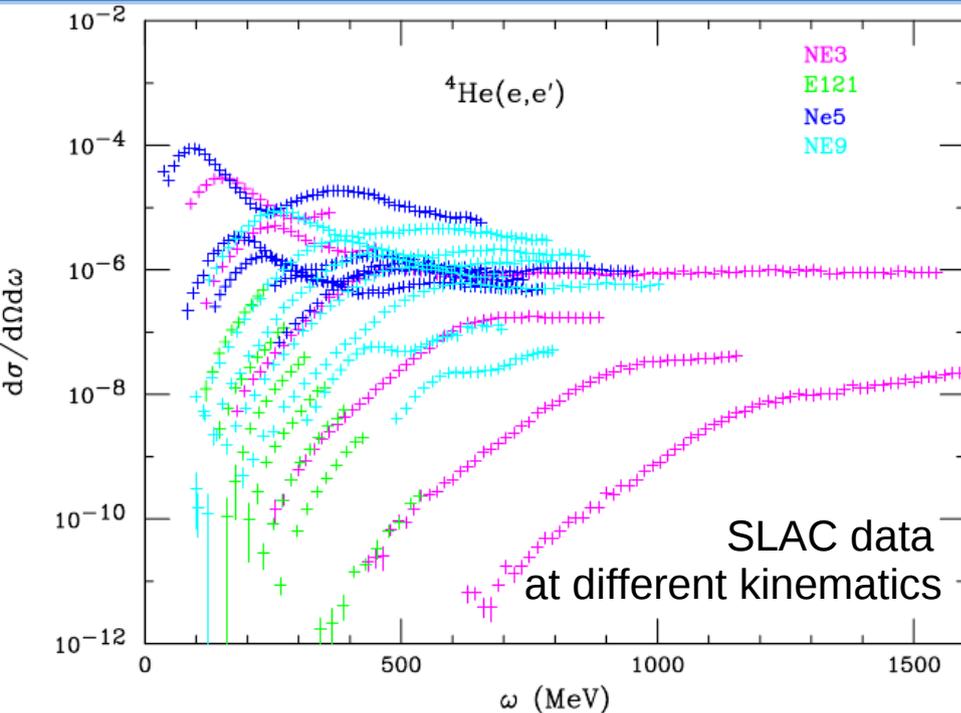
as a function of the scaling variable

- ψ (from RFG) or
- y (from non-relativistic scaling analysis)
for different kinematics and nuclei

$$(y \simeq k_F \psi)$$

- No q -dependence \rightarrow first kind scaling
 - No A -dependence \rightarrow second kind scaling
- } super-scaling

Scaling of first kind

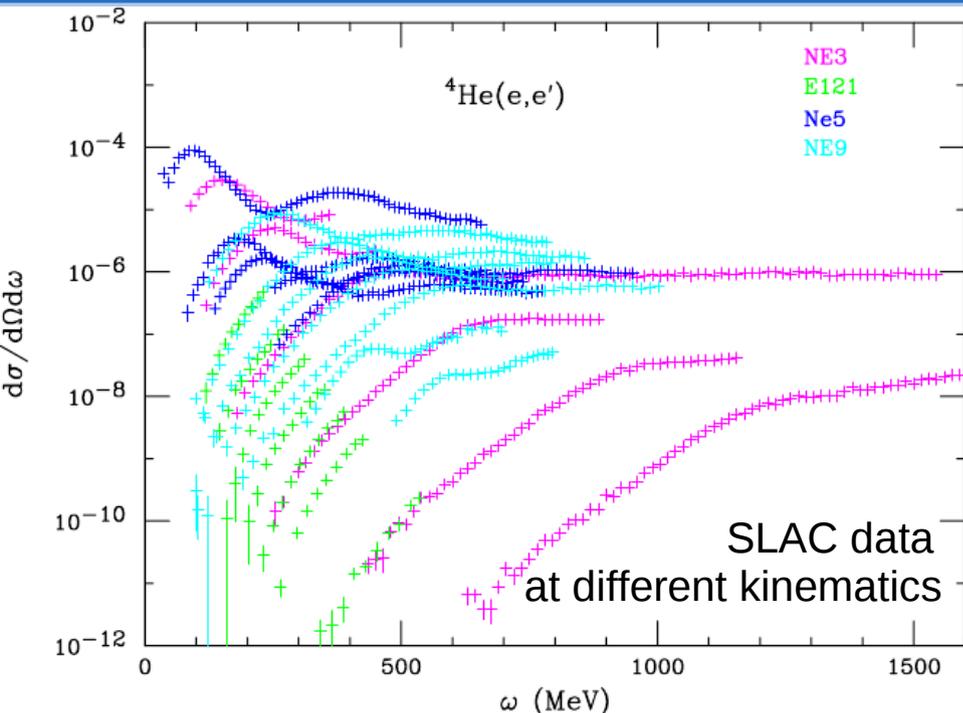


Reduced cross section:

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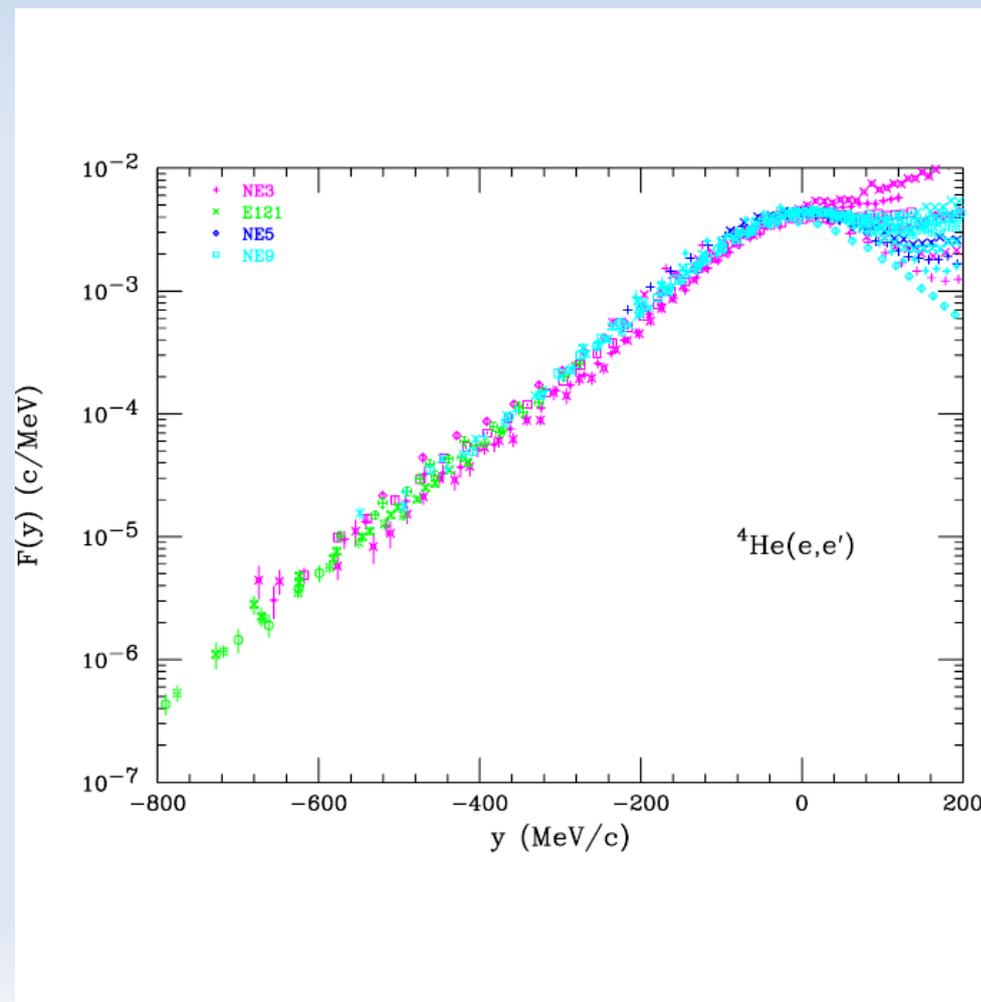
Plot $F(q, y)$ versus y
for different q \longrightarrow

Scaling of first kind



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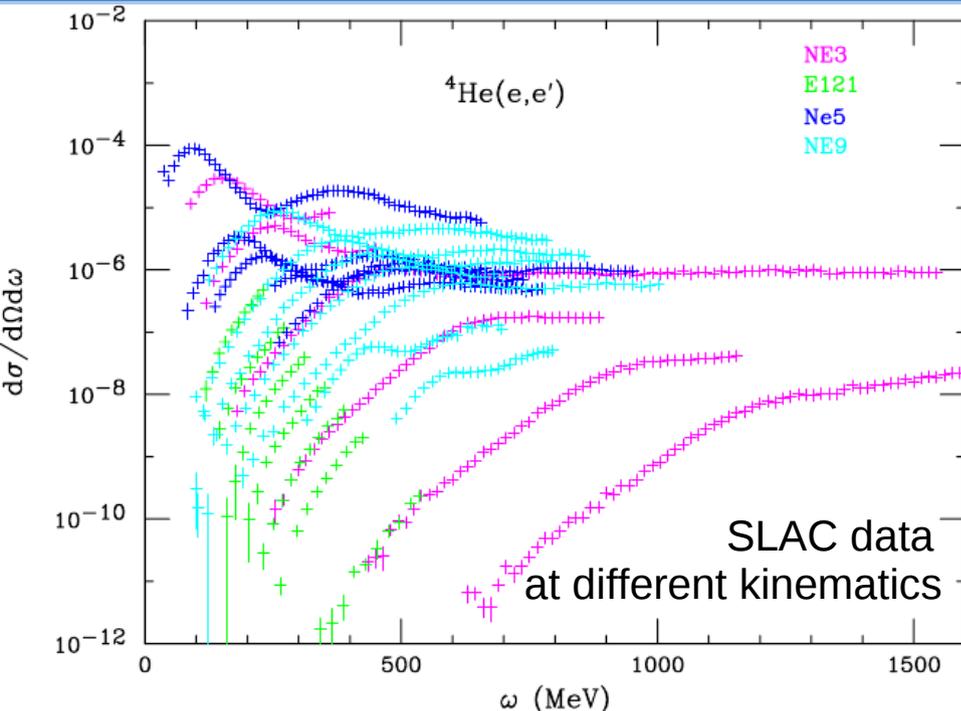
Plot $F(q, y)$ versus y
for different q

$$F(q, y) \rightarrow F(y) \text{ for } q \rightarrow \infty$$

y-scaling

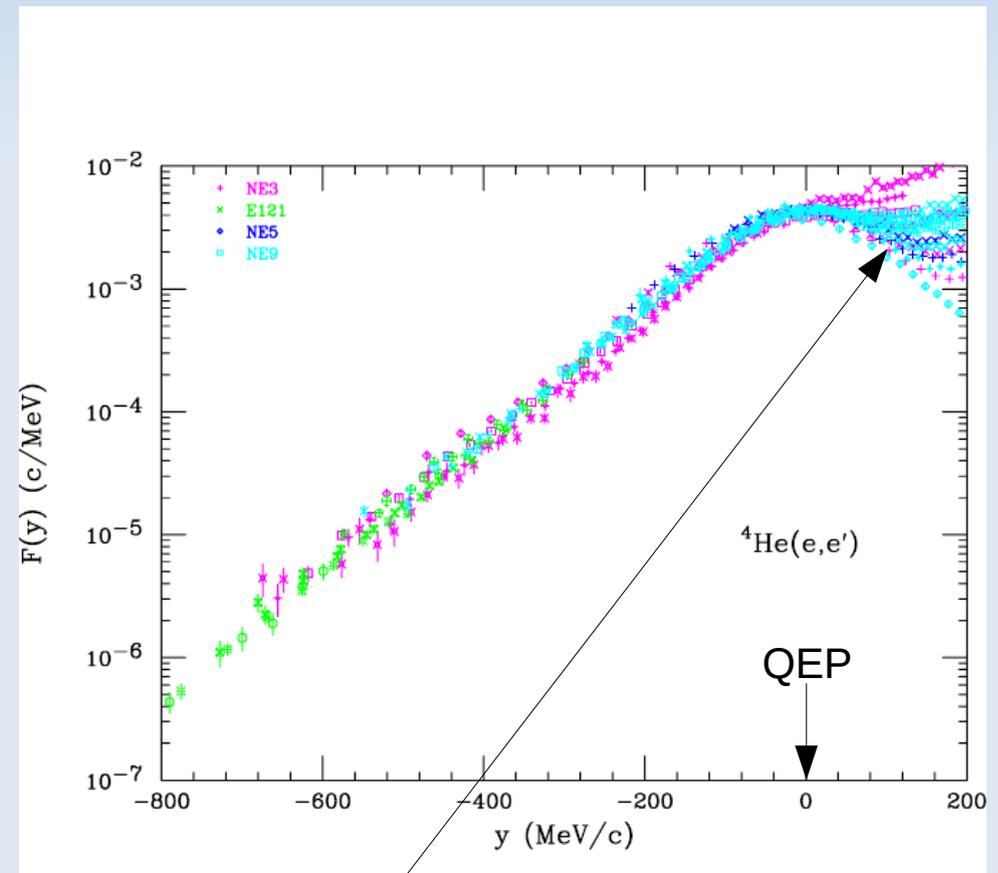
scaling function

Scaling of first kind



Reduced cross section:

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Plot $F(q, y)$ versus y
for different q

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y-scaling

scaling function

Note that at $y > 0$ the scaling is not good, due to the presence of resonances, meson production, etc.

Scaling of second kind

Donnelly & Sick, PRC60, 065502 (1999): define a super-scaling function

$$f(q, \psi) = k_F \times F(q, \psi)$$

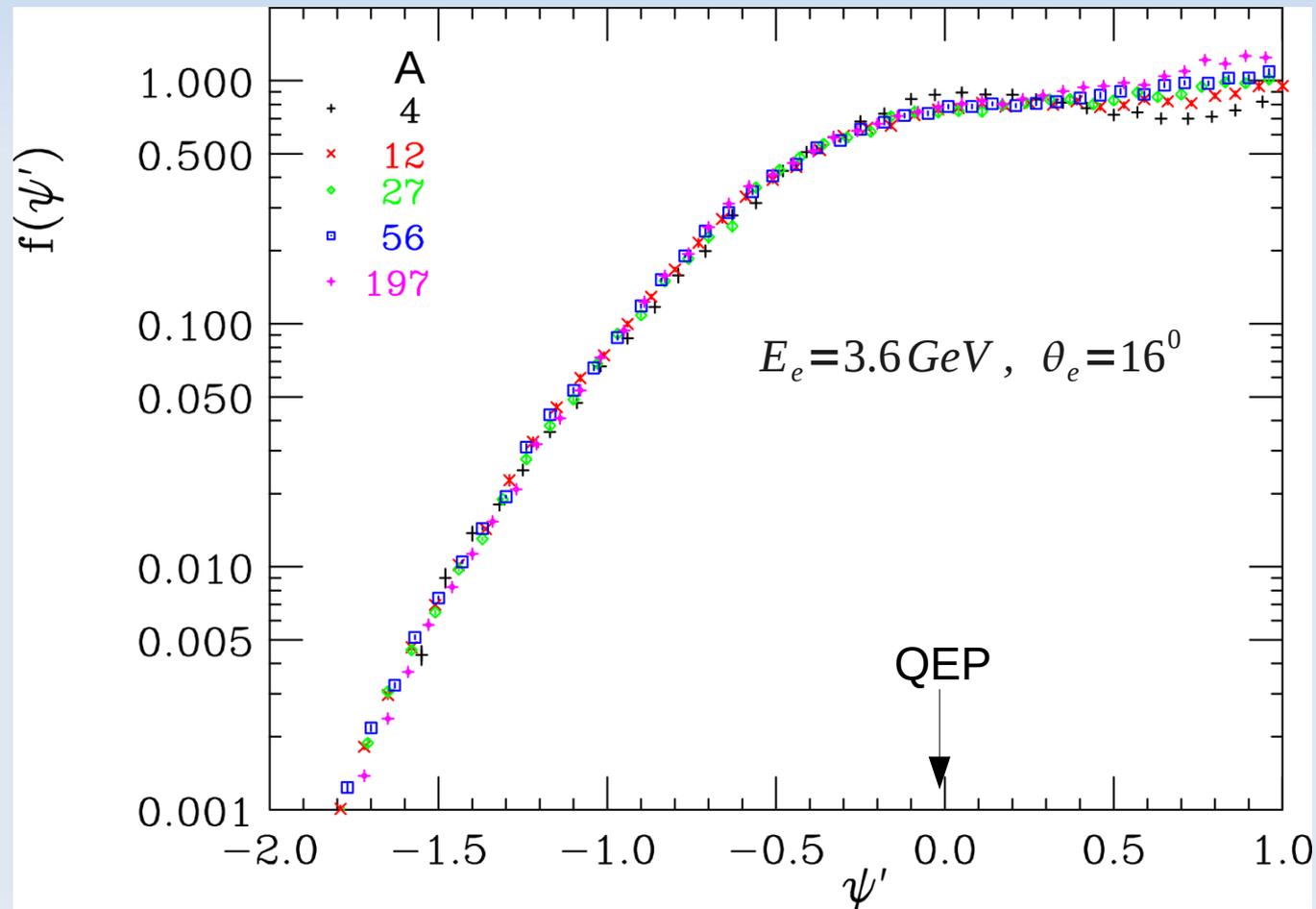
and plot it versus ψ for a variety of nuclei at the same kinematics

In the scaling region ($\psi' < 0$)
a universal behavior is seen,
with very little dependence on
the nuclear species



Scaling of II kind

In the region above $\psi' = 0$ where
resonances, meson production
and the start of DIS enter the
2nd-kind scaling is not as good



Scaling of 0-th kind ($f_L=f_T$)?

Although the amount of available data separated into longitudinal (L) and transverse (T) responses is small, one can attempt a scaling analysis with what does exist:

$$f_L(q, \psi) = \frac{k_F \times R_L(q, \omega)}{A \sum_{eN}^L / \sigma_{Mott} v_L}$$
$$f_T(q, \psi) = \frac{k_F \times R_T(q, \omega)}{A \sum_{eN}^T / \sigma_{Mott} v_T}$$

L and **T** scaling functions: *Is $f_L=f_T$?*

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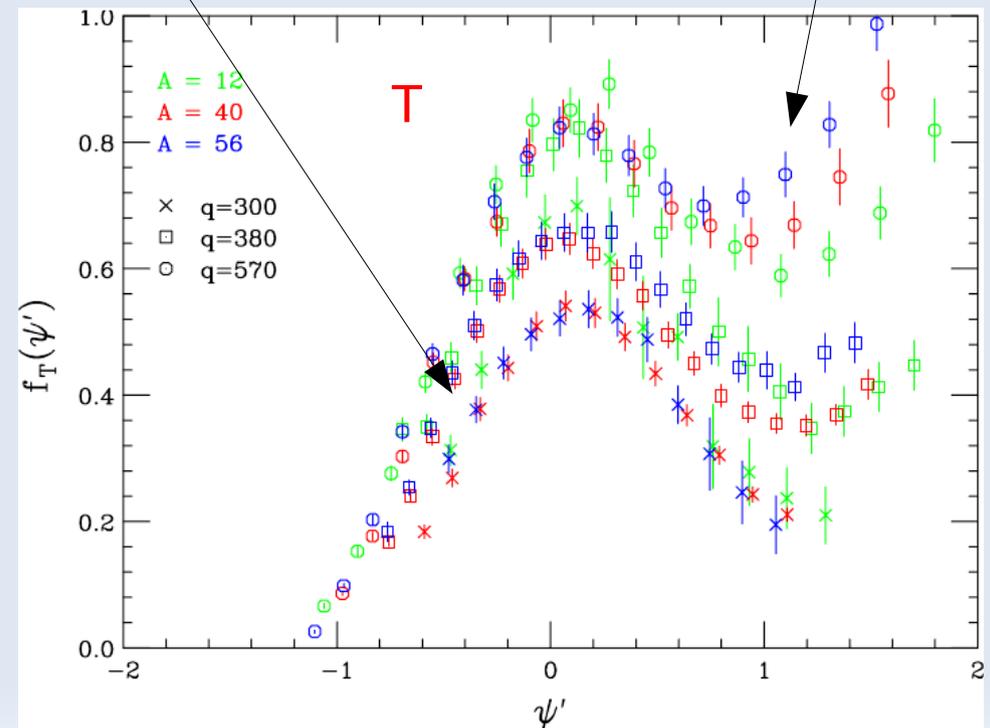
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L and T scaling functions: $Is f_L = f_T?$

Inelastic contributions

Some violation below the QEP



Scaling of 0-th kind?

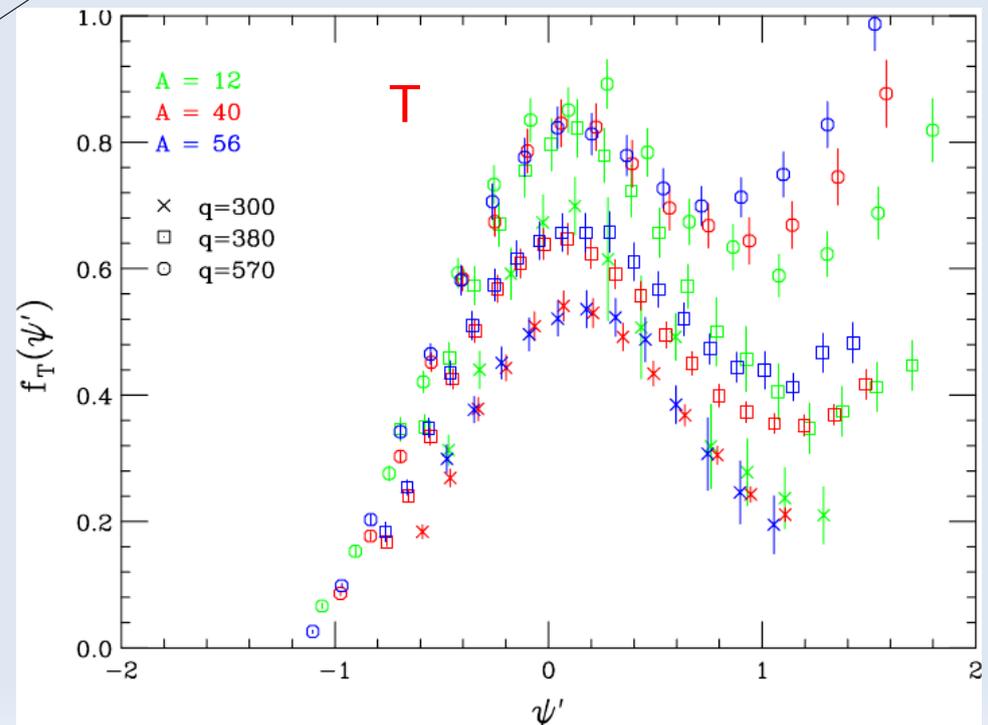
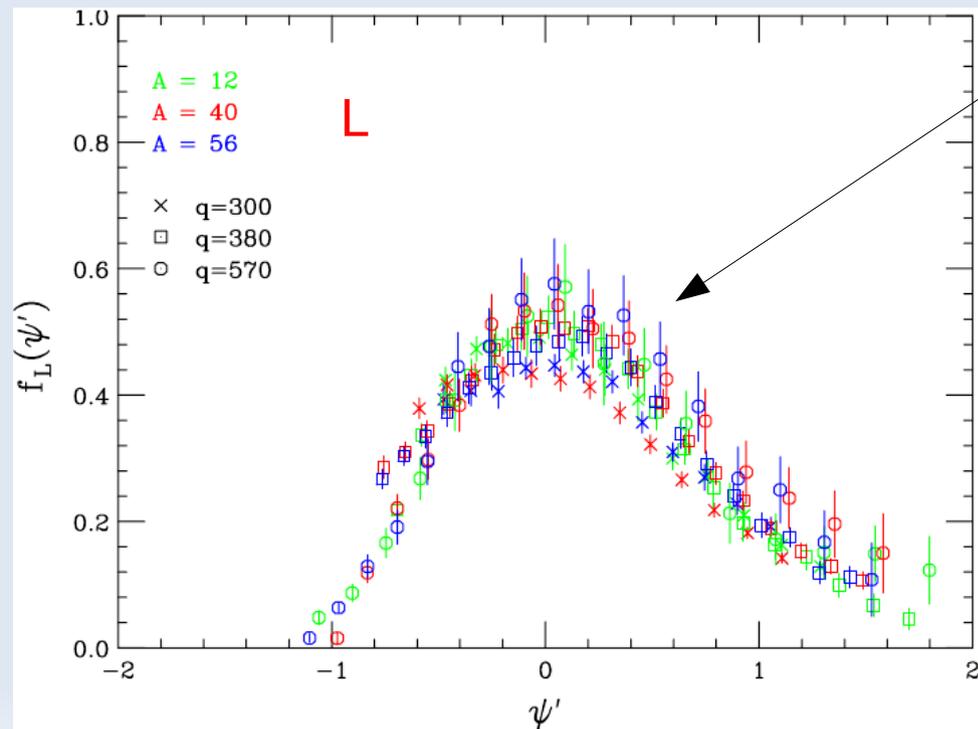
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L and T scaling functions: *Is* $f_L = f_T$?

In contrast, the L results show a **universal behavior**

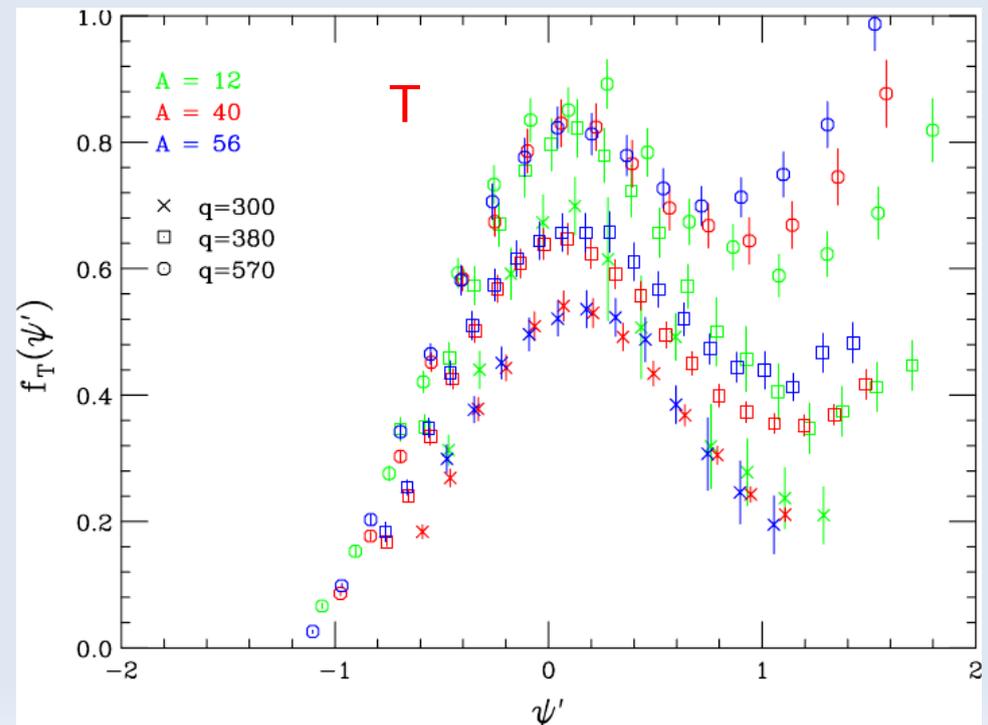
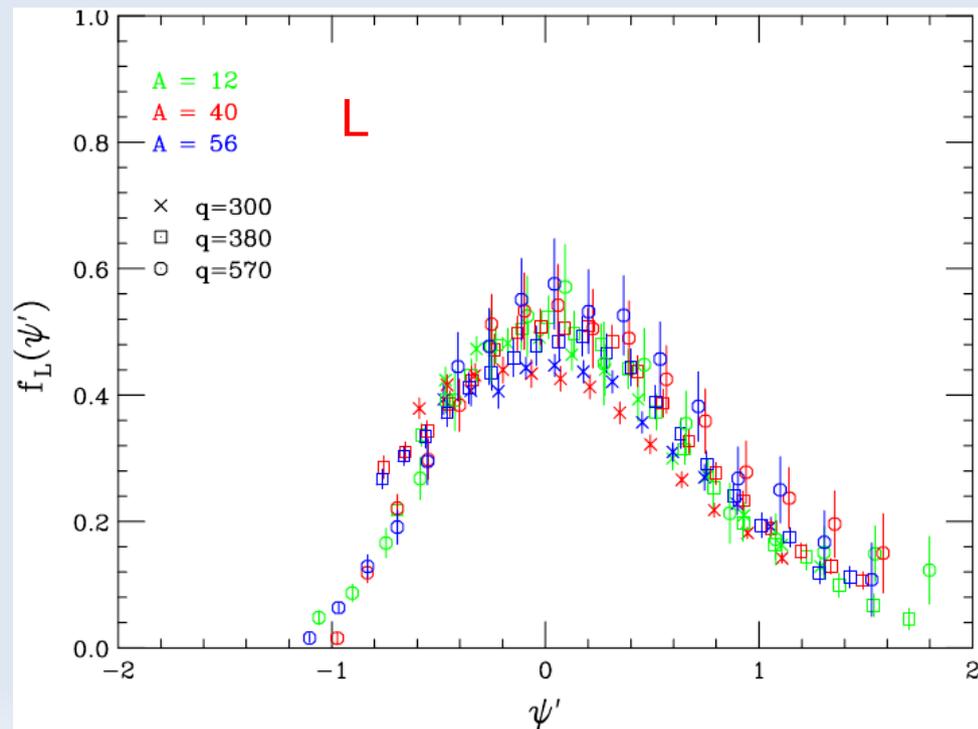


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$$f_T(q, \psi) = \frac{k_F \times R_T(q, \omega)}{A \sum_{eN}^T / \sigma_{Mott} v_T}$$

However: $q < 570$ MeV/c
New Rosenbluth separated JLab data
are needed!



Scaling in the QEP: summary

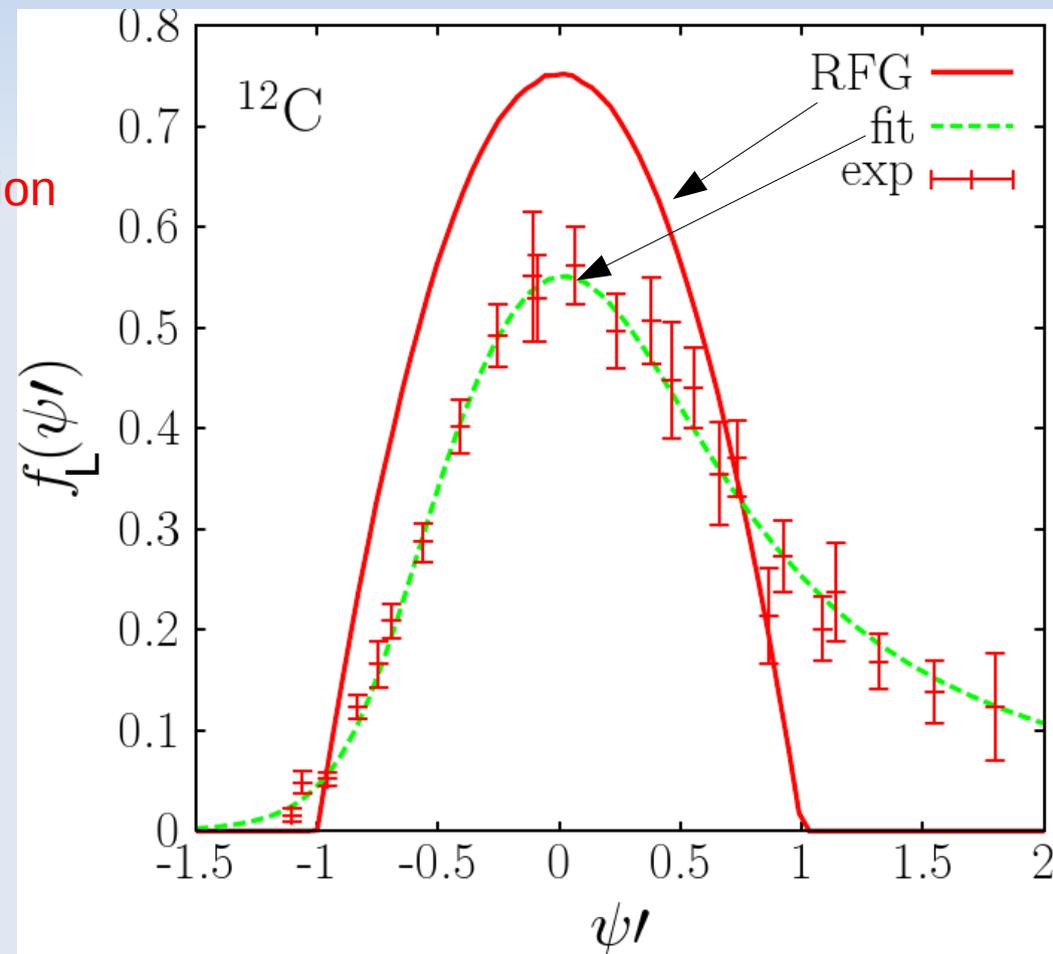
- Scaling of I kind is reasonably good below the QE peak ($\psi < 0$)
- Scaling of II kind is excellent in the same region
- Violations of scaling, particularly of I kind, occur above the QEP and reside mainly in the **transverse** response
- The **longitudinal** response super-scales

Scaling in the QEP: summary

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An **experimental longitudinal super-scaling function** has been extracted from (e,e') data [Jourdan, NPA603, 117 ('96)]

1. **Asymmetric** shape: long tail at high energy transfer
2. Only **4 parameters** for all kinematics and all nuclei
3. Represents a **strong constraint on nuclear models**



Scaling in the Delta region

- 1) subtract the QE contribution obtained from Superscaling hypothesis from the total experimental cross section

$$\left[\frac{d^2 \sigma}{d\omega d\Omega} \right]_{\Delta'} = \left[\frac{d^2 \sigma}{d\omega d\Omega} \right]_{\text{exp}} - \left[\frac{d^2 \sigma}{d\omega d\Omega} \right]_{\text{QE}}$$

- 2) divide by the elementary $N \rightarrow \Delta$ c.s.

$$F_{\Delta'} = \frac{\left[\frac{d^2 \sigma}{d\omega d\Omega} \right]_{\Delta'}}{\sigma_M (v_L G_L^\Delta + v_T G_T^\Delta)}$$

- 3) multiply by the Fermi momentum

$$f_{\Delta'} = k_F F_{\Delta'}$$

- 4) plot versus the appropriate scaling variable

$$\psi_\Delta = \psi(q\rho, \omega\rho)$$

$$\rho = 1 + \frac{1}{4\tau} (m_\Delta^2/m_N^2 - 1) \quad \text{inelasticity}$$

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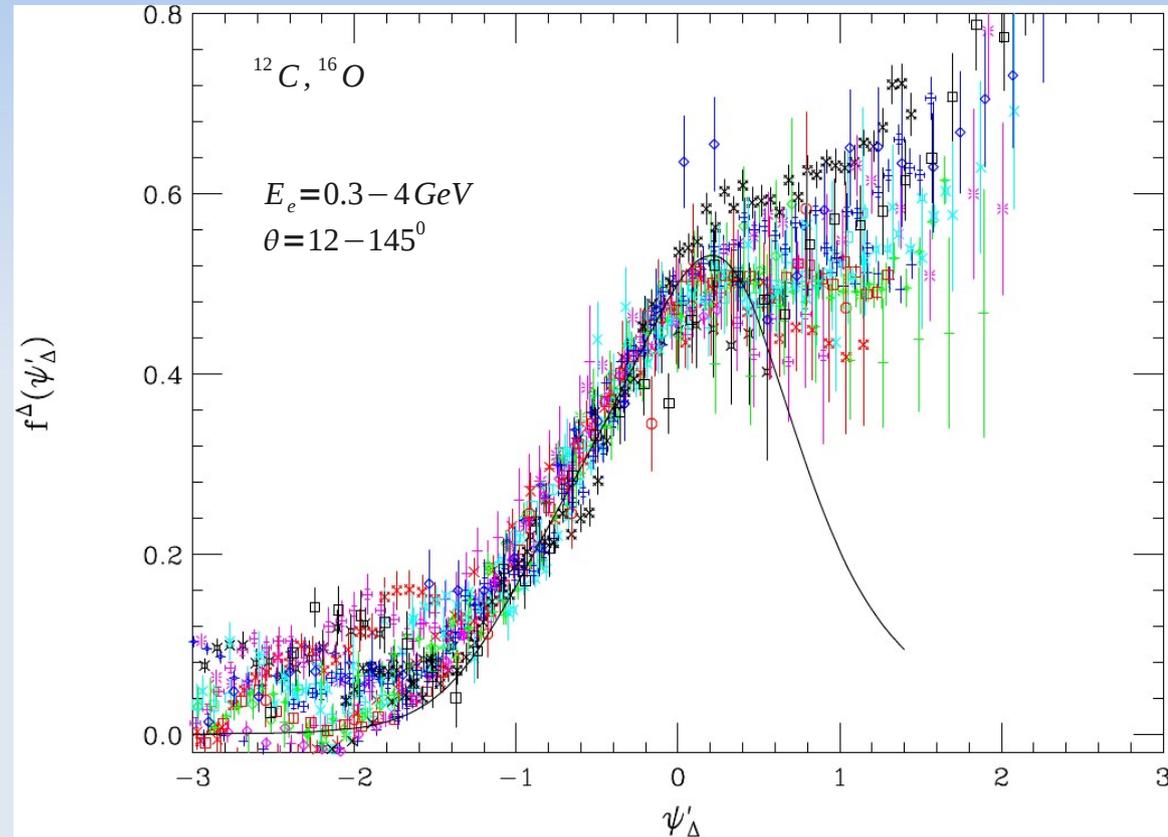
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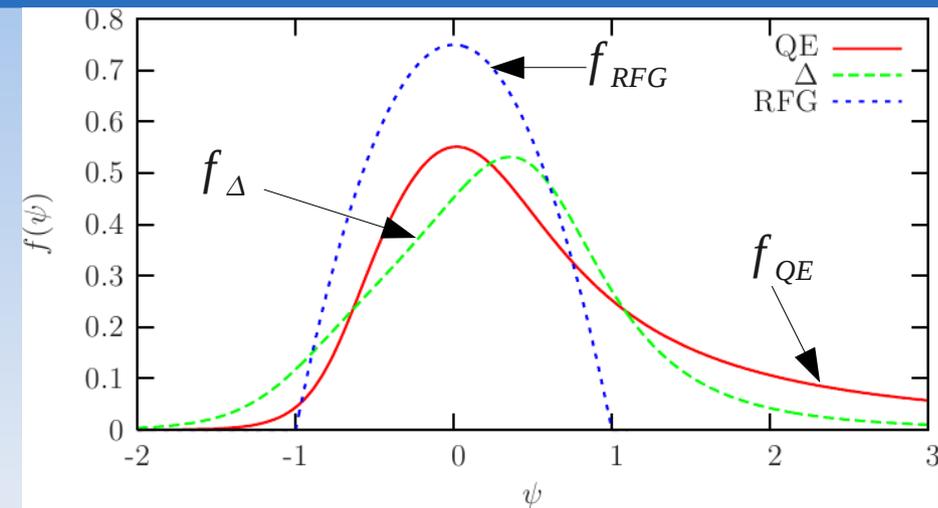
$$\rho = 1 + \frac{1}{4\tau} (m_\Delta^2/m_N^2 - 1) \quad \text{inelasticity}$$

Amaro, Barbaro, Caballero, Donnelly, Molinari, Sick, PRC71 (2005)



This approach can work only at $\Psi_\Delta < 0$, since at $\Psi_\Delta > 0$ other resonances and the tail of DIS contribute

Test of superscaling



$$R_L(q, \omega) = G_L(q, \omega) f_{QE}(\psi) + G_L^{\Delta}(q, \omega) f_{\Delta}(\psi_{\Delta})$$

$$R_T(q, \omega) = G_T(q, \omega) f_{QE}(\psi) + G_T^{\Delta}(q, \omega) f_{\Delta}(\psi_{\Delta})$$

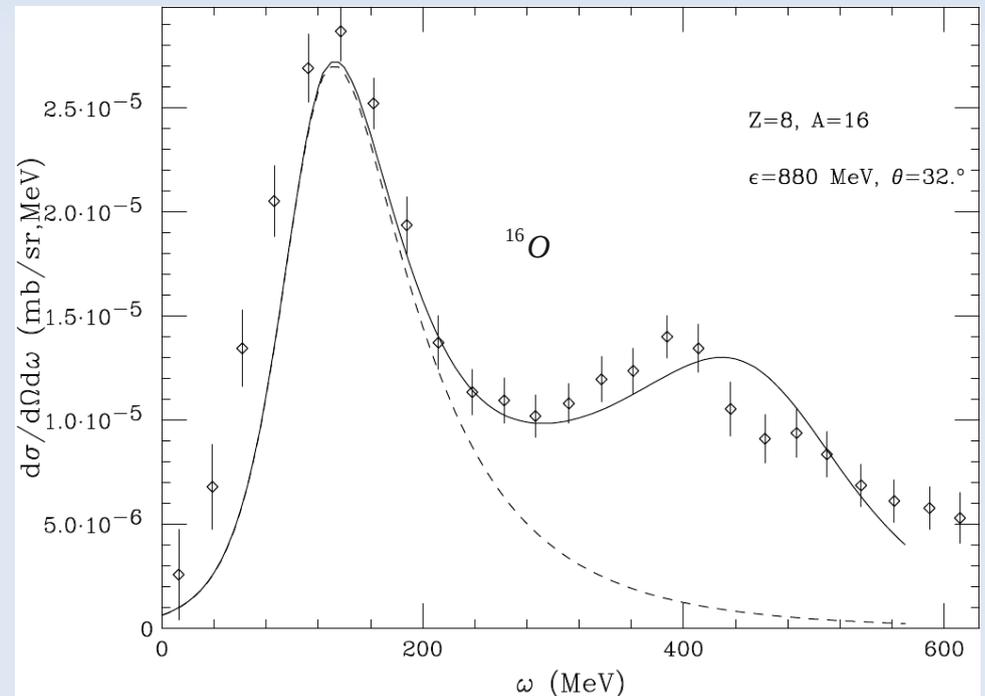
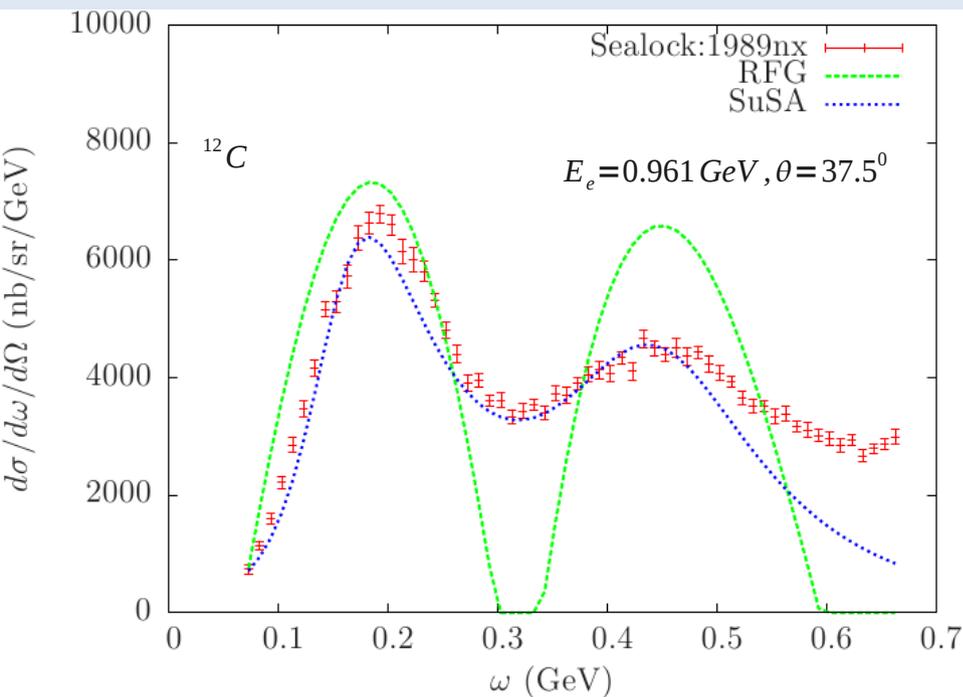
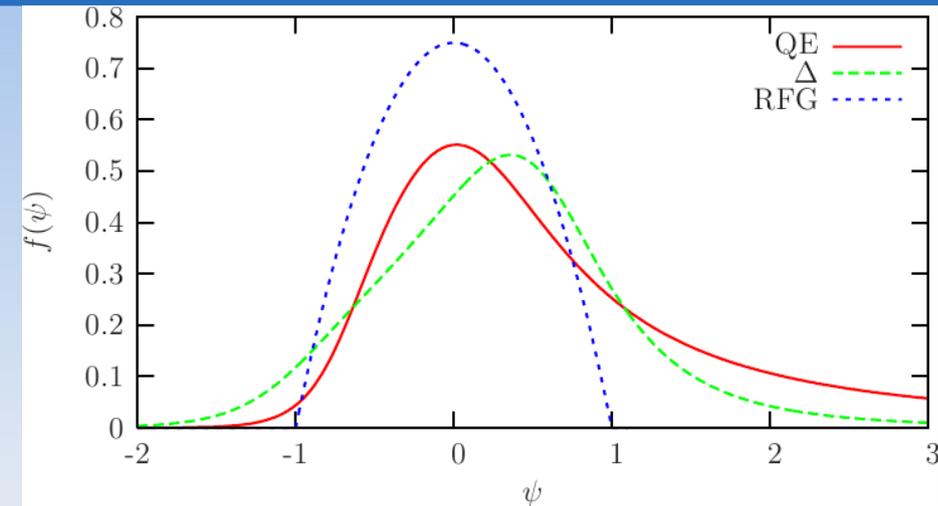
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$$\frac{d^2\sigma}{d\omega d\Omega} = \sigma_{Mott} (v_L R_L + v_T R_T)$$



Amaro et al., PRC71, 015501 (2005)

Application to neutrino scattering

Just as for the electron scattering reactions in the QE and Δ regions, we use the scaling functions determined above, but now multiply by the corresponding charged-current neutrino reaction cross sections for the Z protons and N neutrons in the nucleus

-----► SuperScaling Approximation (SuSA)

Limits of the approach:

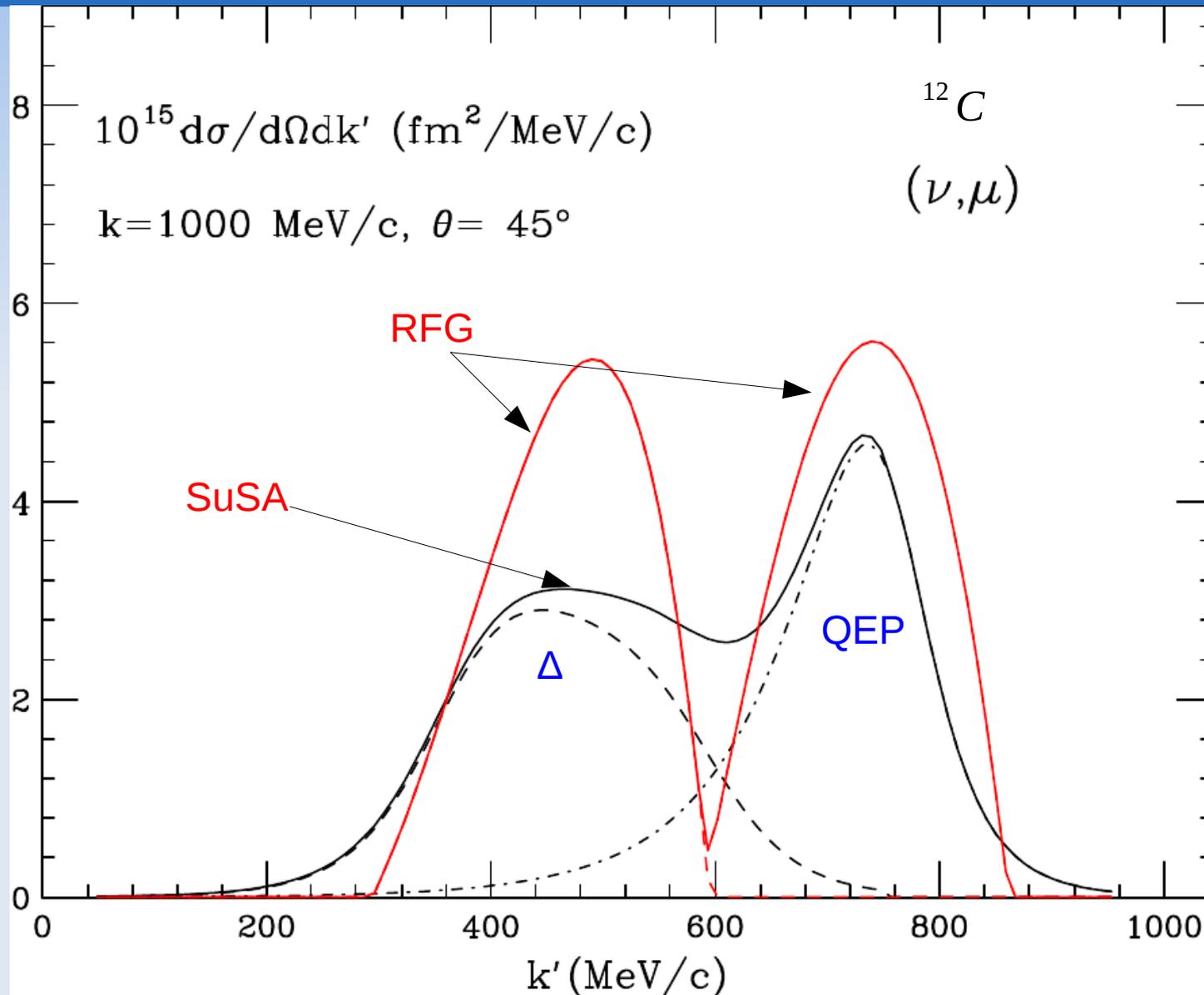
1. it can be applied only at high enough momentum transfers, where the scaling ideas make sense: at low q and ω collective effects (like giant resonances) become important
2. CC neutrino reactions are isovector only, whereas electron scattering contains both isoscalar and isovector contributions (the transverse EM response is, in fact, predominantly isovector at high energy).

Thus, in going from electron scattering, where the universal scaling function came from the L response (essentially 50% isoscalar and 50% isovector) to CC neutrino reactions we have had to invoke

Scaling of the 3rd Kind : $f(T=0) = f(T=1)$

where the isospin nature of the scaling functions is assumed to be universal.

CC neutrino cross section



Relativistic Impulse Approximation

In order to understand the microscopic origin of superscaling and test the validity of the superscaling approach to neutrino scattering we have explored other relativistic nuclear models:

- 1) Relativistic Mean Field Model - RMF
- 2) Semi-relativistic Shell Model (with FSI) - RSM

Both models are based on RIA:

Scattering off a nucleus \Rightarrow Incoherent sum of single-nucleon scattering processes

Nuclear current \Rightarrow One-body operator

$$J_N^\mu(\omega, q) = \int d\vec{p} \bar{\Psi}_F(\vec{p} + \vec{q}) J_N^\mu \Psi_B(\vec{p})$$

1) RMF model: basic ingredients

Ψ_B : bound nucleon w.f. \Rightarrow self-consistent Dirac-Hartree solutions derived within a RMF approach using a relativistic Lagrangian with scalar and vector potentials
[Horowitz&Serot, NPA368(1981),PLB86(1979); Serot&Walecka, Adv.Nucl.Phys.16 (1986)]

Ψ_F : outgoing nucleon w.f. \Rightarrow different treatments of Final State Interactions (**FSI**):

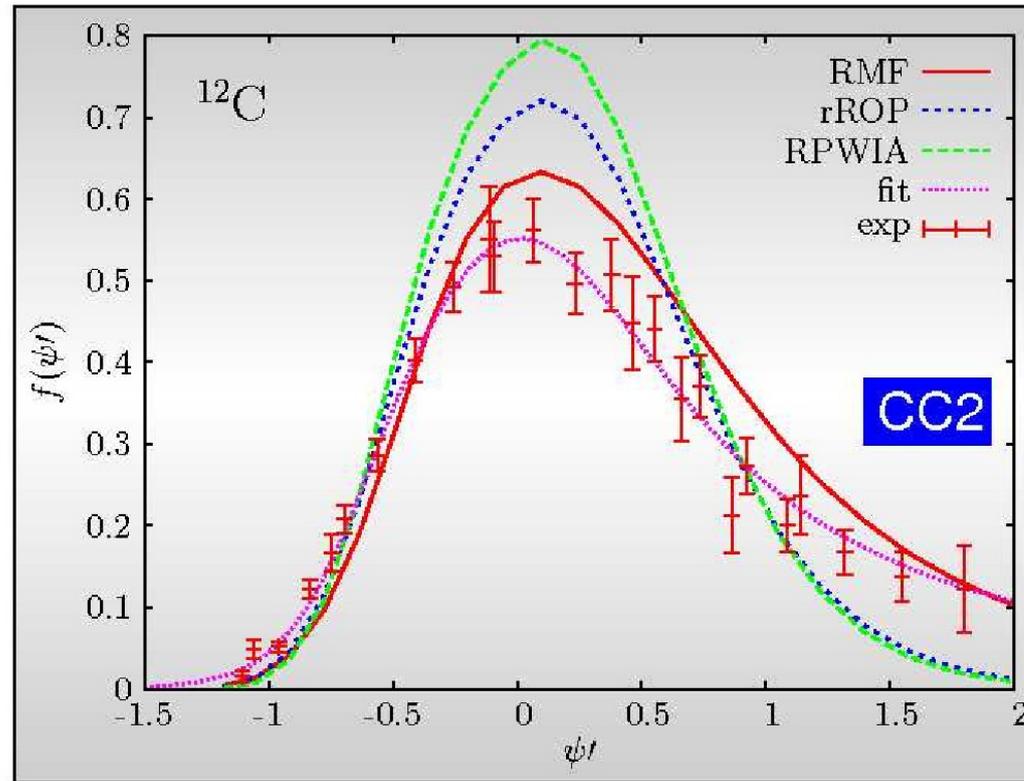
1. **RPWIA** (no FSI)

2. **RDWIA** solutions of Dirac equation containing relativistic potentials:

a. ROP relativistic optical potential, successfully applied to exclusive (e,e'p) reactions [Udias et al., PRC48 (1993), PRC51 (1995), PRC64 (2001)], where the imaginary part represents the loss of flux into inelastic channels. For inclusive reactions, (e,e'), we simply use the real part of the optical potential (energy-dependent, A-indep. parameterizations EDAIC,EDAIO,EDAICa, Clark et al., PRC47 (1993)) \Rightarrow **rROP**

b. distorted waves obtained with the same mean field as in the bound state \Rightarrow **RMF**

RMF Super-scaling Function

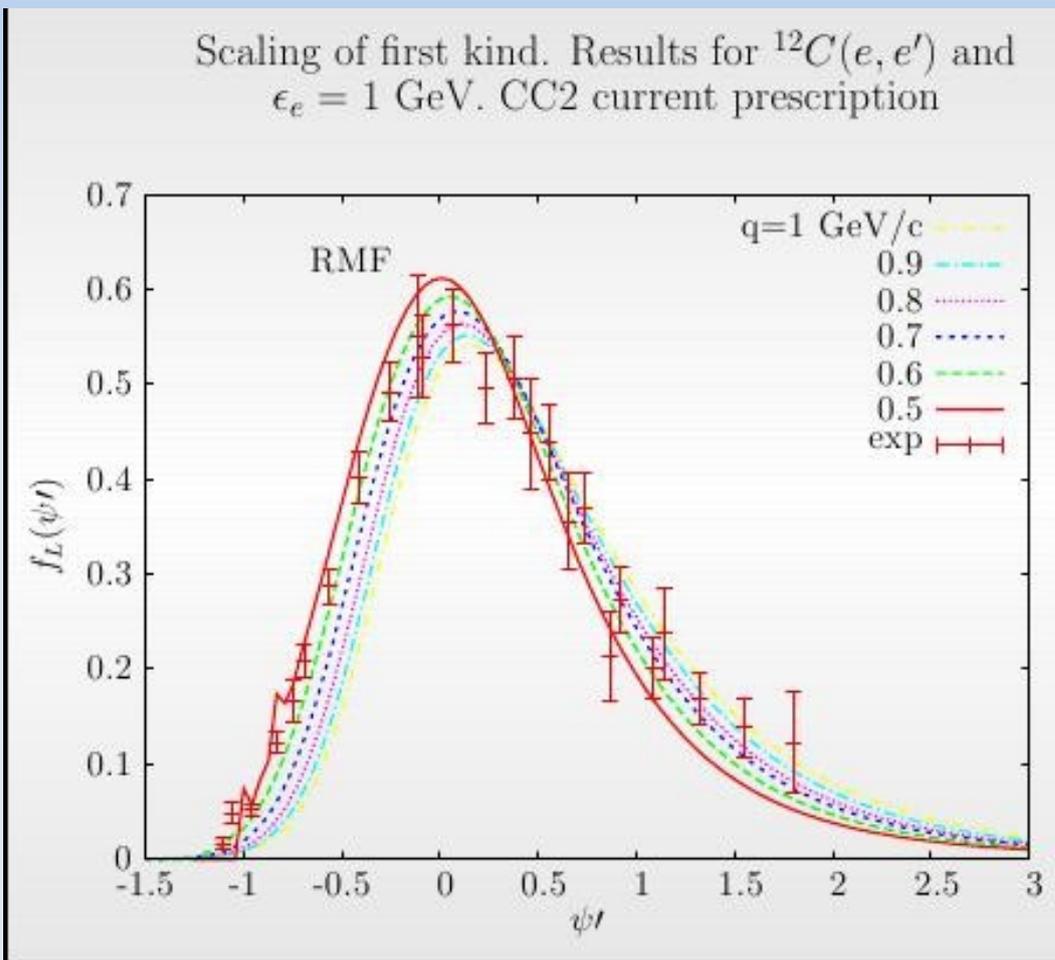


Only the description of FSI provided by RMF leads to an asymmetric scaling function in agreement with the behavior of the data

Test of SuSA in the RMF model

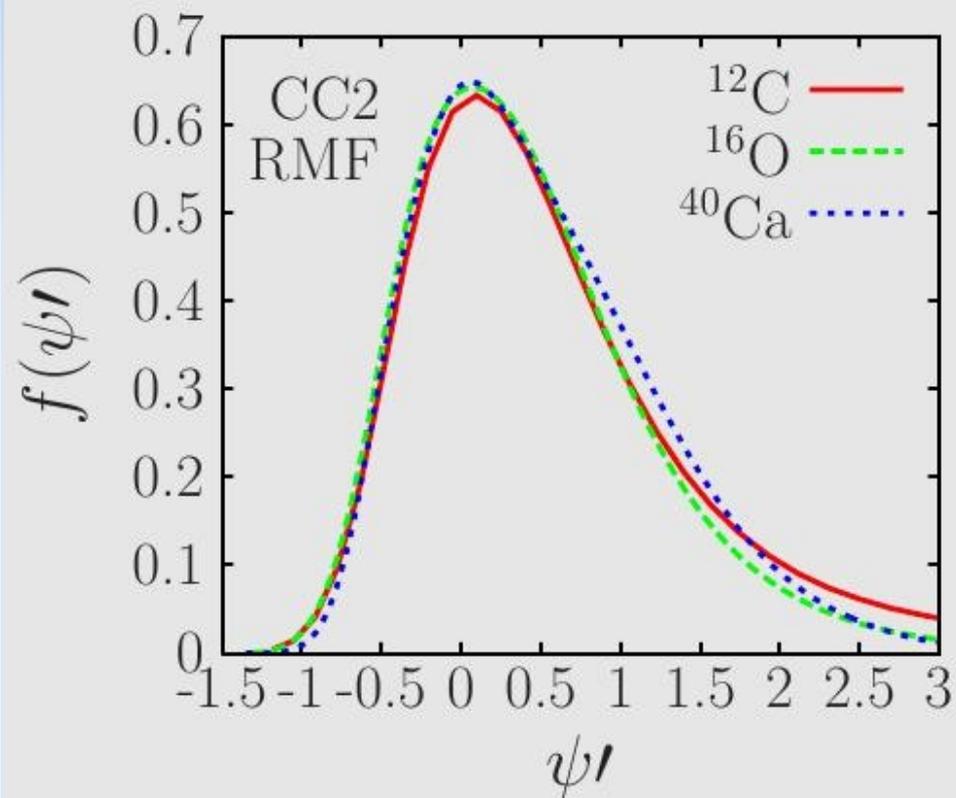
I kind

Scaling of first kind. Results for $^{12}\text{C}(e, e')$ and $\epsilon_e = 1 \text{ GeV}$. CC2 current prescription



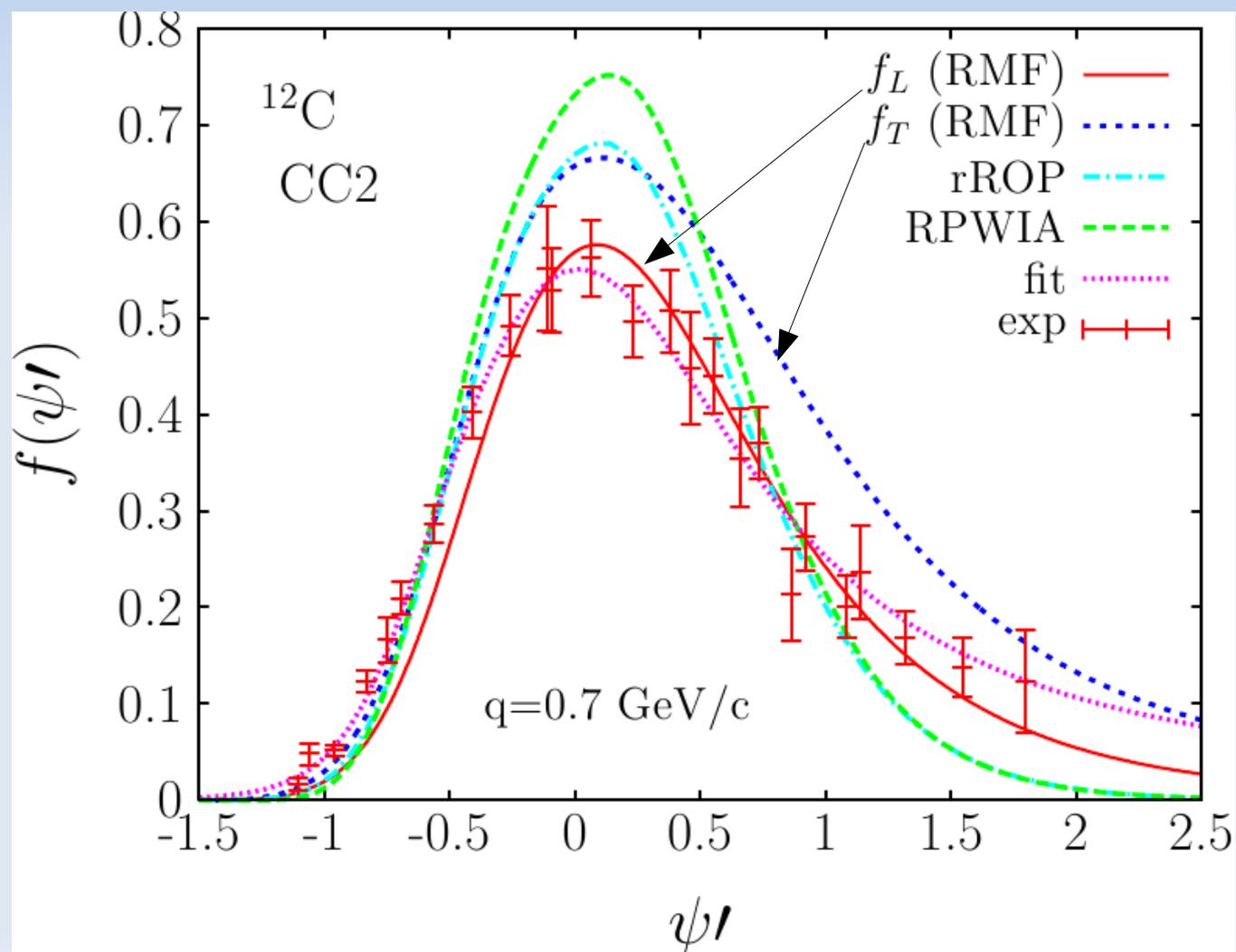
Shift at $\psi' < 0$ and breakdown of I-kind scaling of $\sim 25\text{-}30\%$ at $\psi' > 0$ (compatible with data) for $q > 0.5 \text{ GeV}/c$

II kind



Good scaling of II kind (but dependent on the prescription for the current, not so good with CC1)

Test of scaling of 0th kind



In the RMF model

$$f_T > f_L$$

Scaling of 0-th kind
is violated.

2) Semi-relativistic Shell Model

- Suited to describe closed shell nuclei
- Initial state $|i\rangle$: Slater determinant with all shells occupied
- **Impulse approximation**: final states are particle-hole excitations coupled to total angular momentum

$$|f\rangle = |(ph^{-1})J\rangle$$
- Single particle and hole wave functions are obtained by solving the Schrödinger equation with a **Woods-Saxon potential** including central, spin-orbit and Coulomb terms
- **Relativistic kinematics** are taken into account by the substitution

$$\epsilon_p \rightarrow \epsilon_p (1 + \epsilon_p / 2m_N)$$

as the eigenvalue of the Schrödinger equation for the outgoing nucleon

- **Semi-relativistic currents** involve an expansion of the relativistic nucleon currents $J^\mu = \bar{u}(\vec{p}', s') \Gamma^\mu u(\vec{p}, s)$ in the bound nucleon momentum $\vec{\eta} = \vec{p}/m_N$ to first order

q, ω can be large

$$J_V^0 = \xi_0 + i\xi'_0 (\vec{k} \times \vec{\eta}) \cdot \sigma$$

$$J_V^T = \xi_1 \eta^T + i\xi'_1 \vec{\sigma} \times \vec{k}$$

Vector

$$J_A^0 = \xi'_0 \vec{k} \cdot \vec{\sigma} + \xi''_0 \vec{\eta}^T \cdot \vec{\sigma}$$

$$J_A^z = \xi'_3 \vec{k} \cdot \vec{\sigma}^T$$

$$J_A^T = \xi''_1 \sigma + \xi''_3 \vec{\eta}^T \cdot \vec{\sigma}$$

Axial

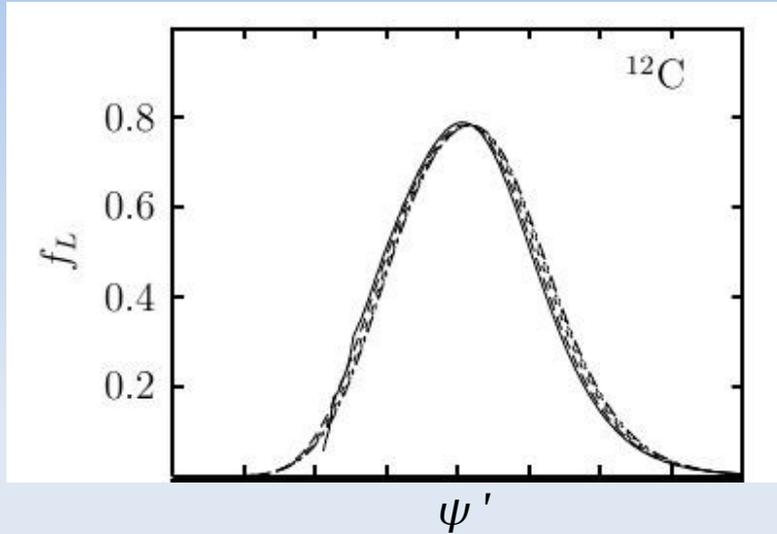
$$\xi_0 = 2G_E^V \frac{K}{\sqrt{\tau}}, \quad \xi'_0 = \frac{2G_M^V - G_E^V}{\sqrt{1+\tau}}$$

$$\xi_1 = 2G_E^V \frac{\sqrt{\tau}}{K}, \quad \xi'_1 = 2G_M^V \frac{\sqrt{\tau}}{K}$$

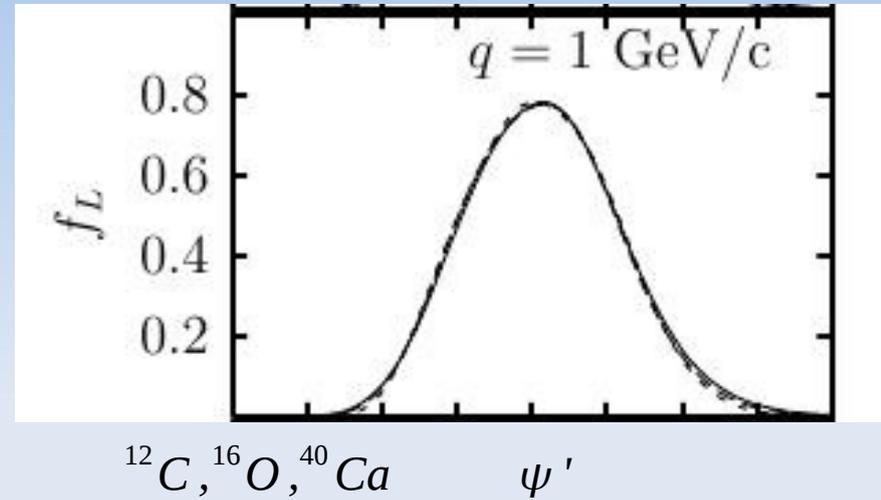
etc., provide the required relativistic behavior

Semi-relativistic Shell Model (SRSM)

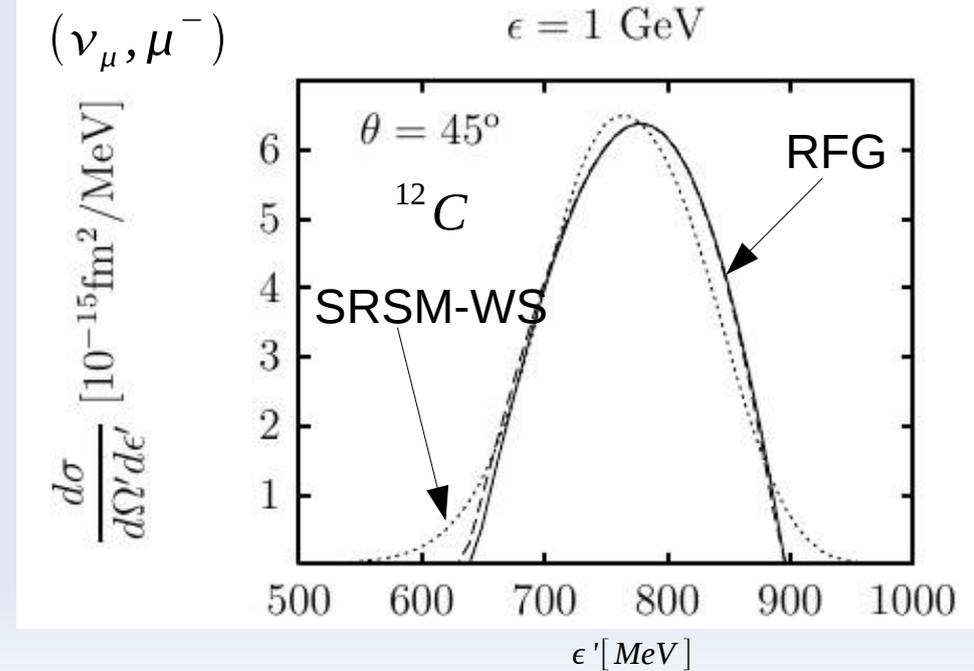
The model superscales



$q=0.5,0.7,1,1.3,1.5\text{ GeV}/c$



and yields essentially the same results as the RFG (symmetric scaling function) but for extended tails outside the RFG response region.



Relativistic Shell Model with FSI

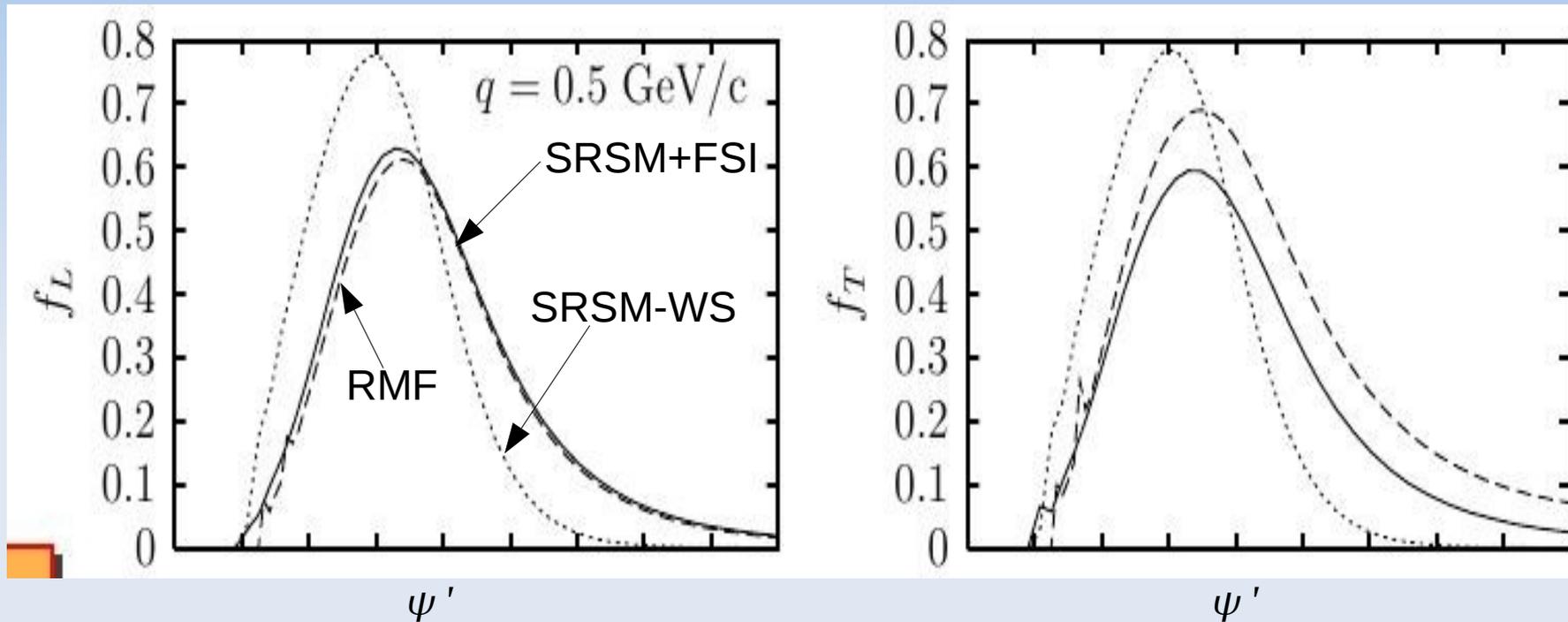
Improve the **FSI**:

1. Re-write the Dirac equation as a second order equation for the up component ψ_{up}
2. Include the Darwin term $\psi_{up}(\vec{r}) = K(r, E)\Phi(\vec{r})$
3. The function $\Phi(\vec{r})$ satisfies the Schrödinger equation

$$\left[\frac{-\Delta}{2m_N} + U_{DEB}(r, E) \right] \Phi(\vec{r}) = \frac{E^2 - m_N^2}{2m_N} \Phi(\vec{r})$$

Dirac Equation Based potential (energy dependent)

Superscaling Functions in SRSM+FSI

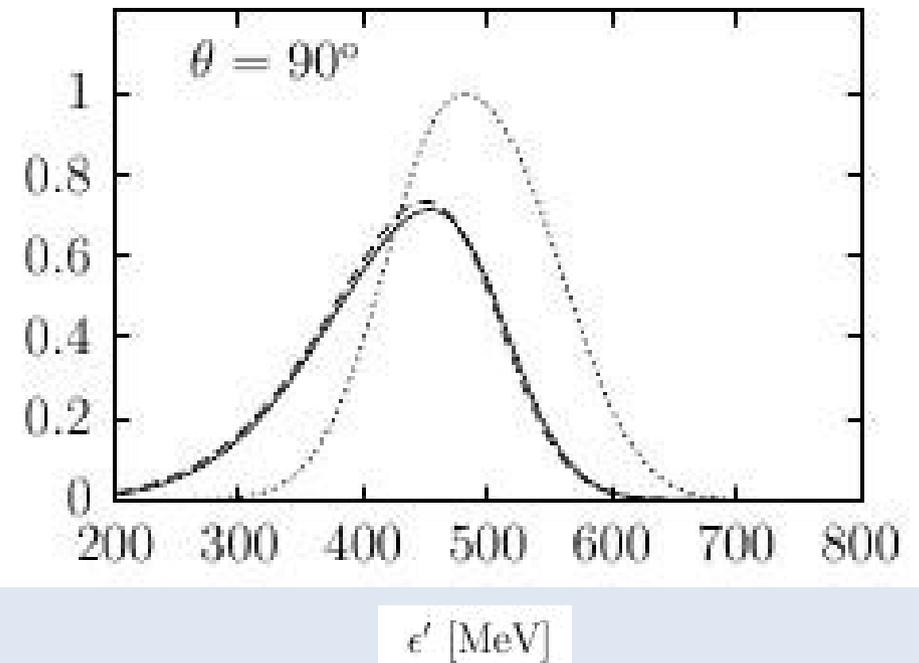
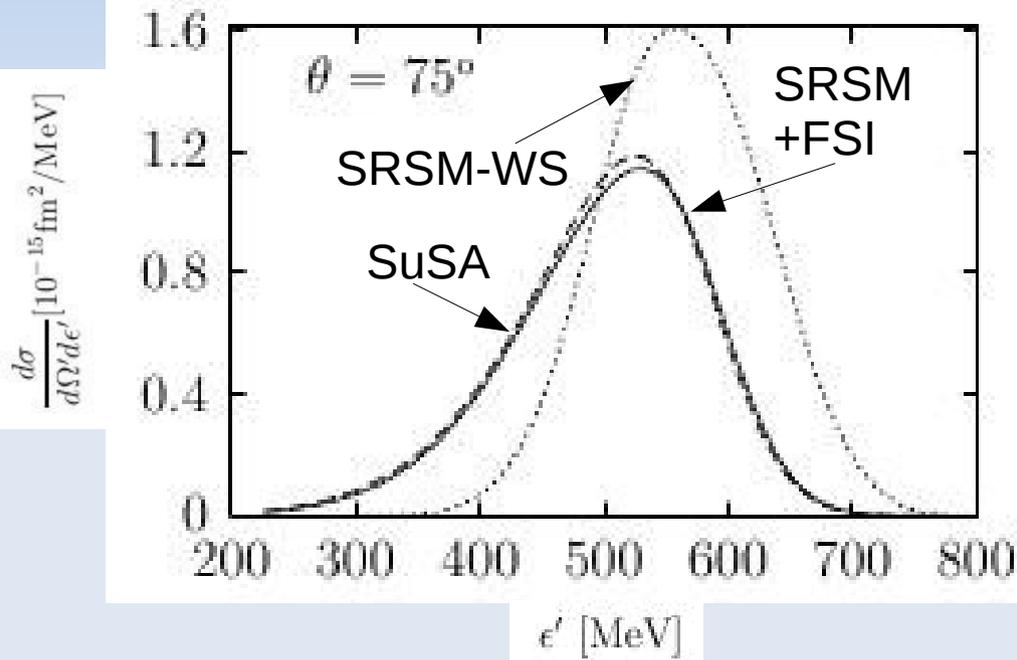


The SRSM with FSI is essentially equivalent to the RMF in the longitudinal channel, but it does not contain the L-T scaling violations

Results for neutrino reactions

$(\nu_\mu, \mu^-) \ ^{12}\text{C}$

$E_\nu = 1\text{ GeV}$

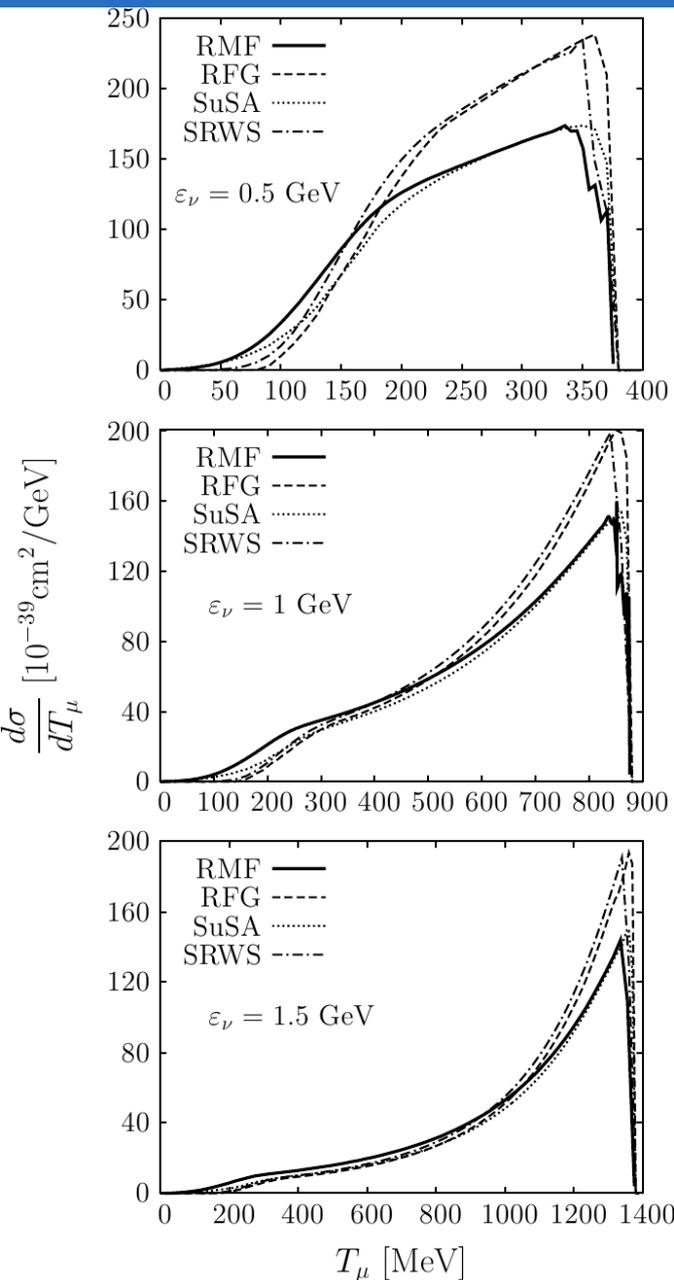


Integrated cross sections

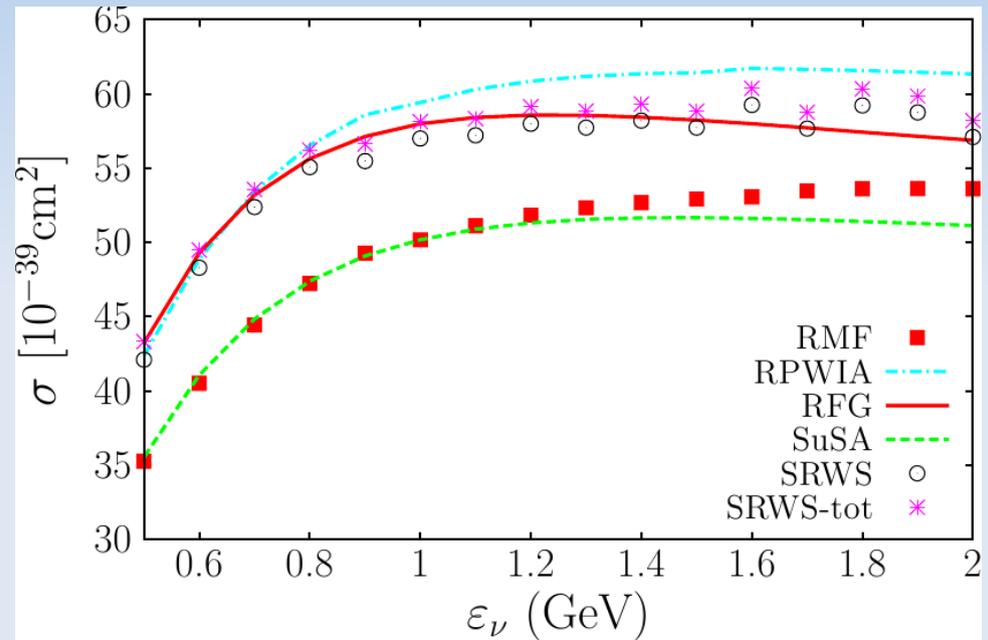
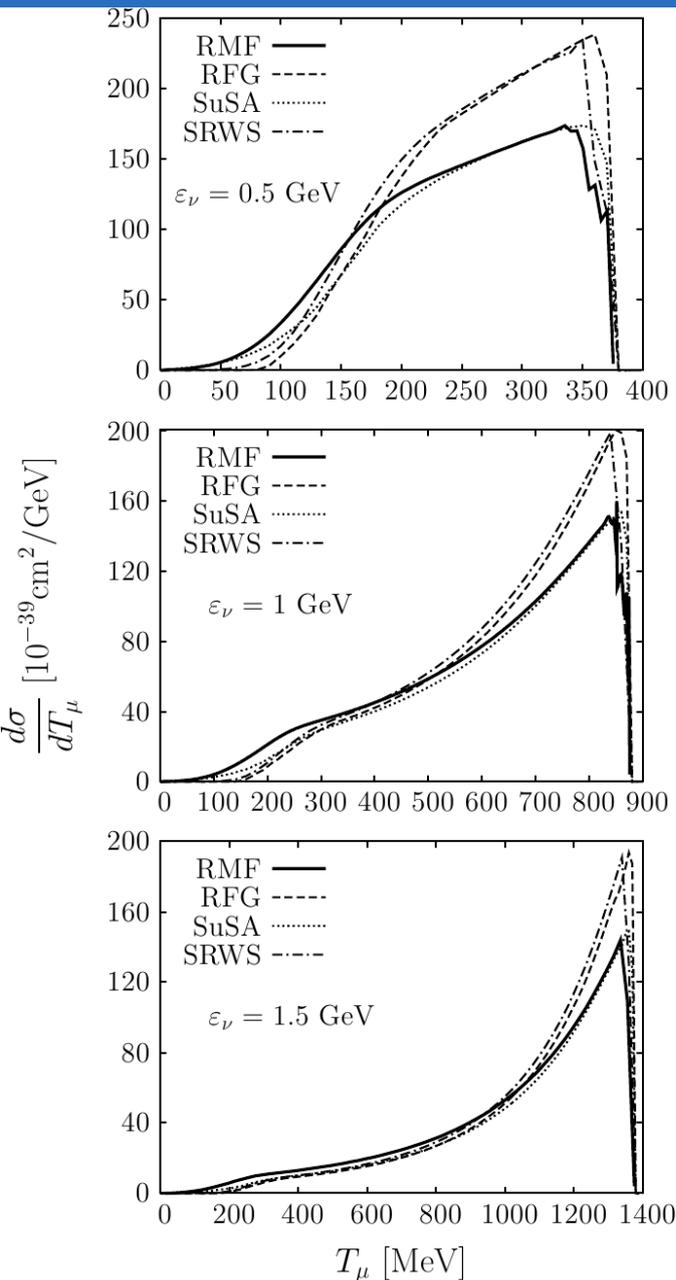
[Amaro et al., Phys. Rev. Lett. 98, 242501 (2007)]

Relativistic Mean Field \approx SuperScaling Approach

Relativistic Fermi Gas \approx Semi-relativistic Shell Model
(with WS potential)



Integrated cross sections



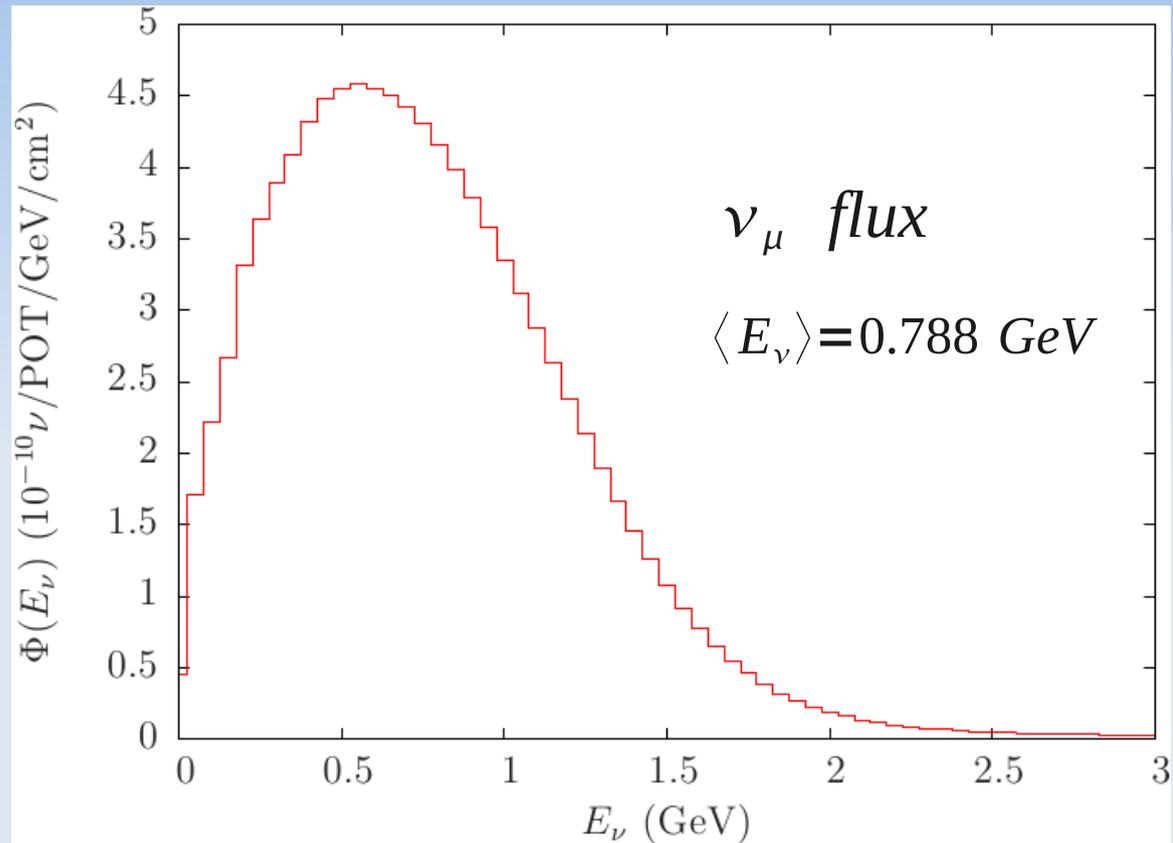
[Amaro et al., PRL 98, 242501 (2007)]

Comparison with MiniBooNE

We average the double differential cross section

$$\frac{d^2 \sigma}{d\cos\theta dT_\mu}$$

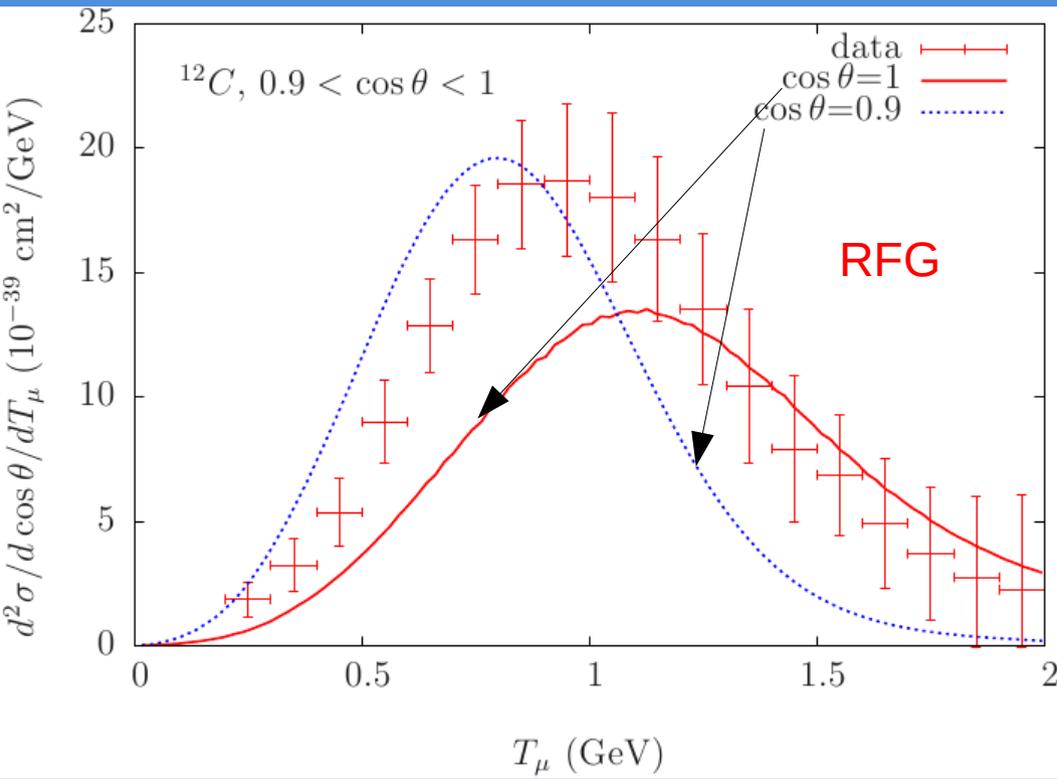
over the MiniBooNE Neutrino flux:



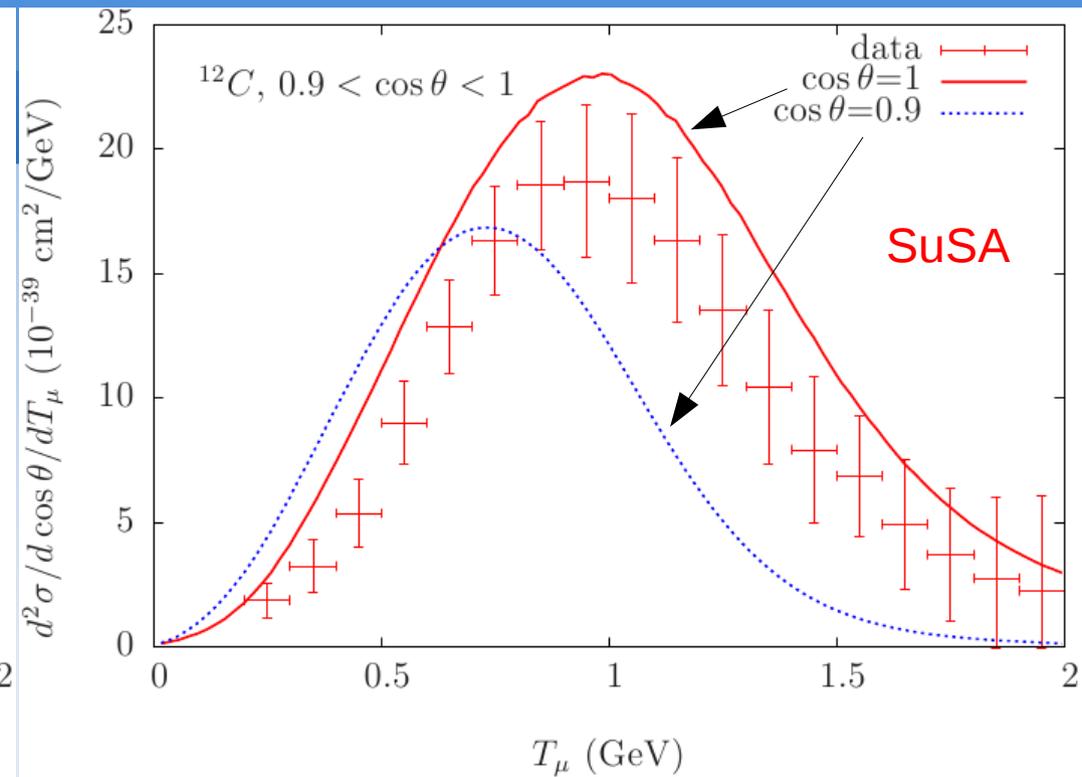
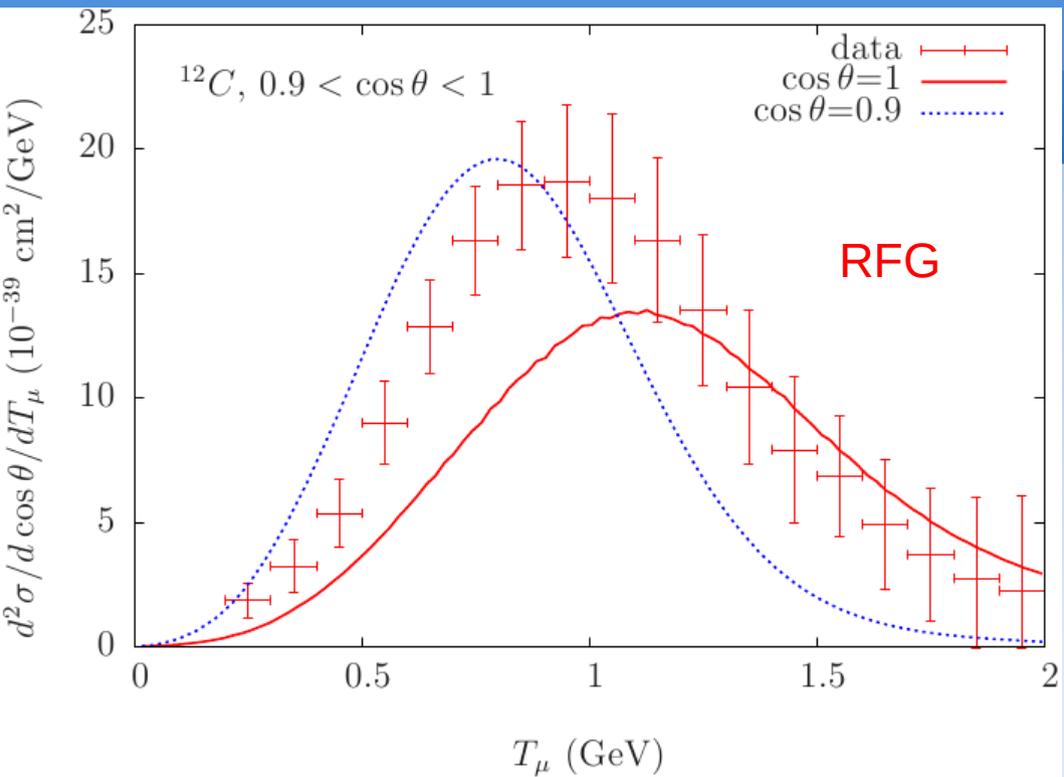
[PRD81,092005 (2010)]

Data are given in 0.1 GeV bins of T_μ and 0.1 bins of $\cos\theta_\mu$

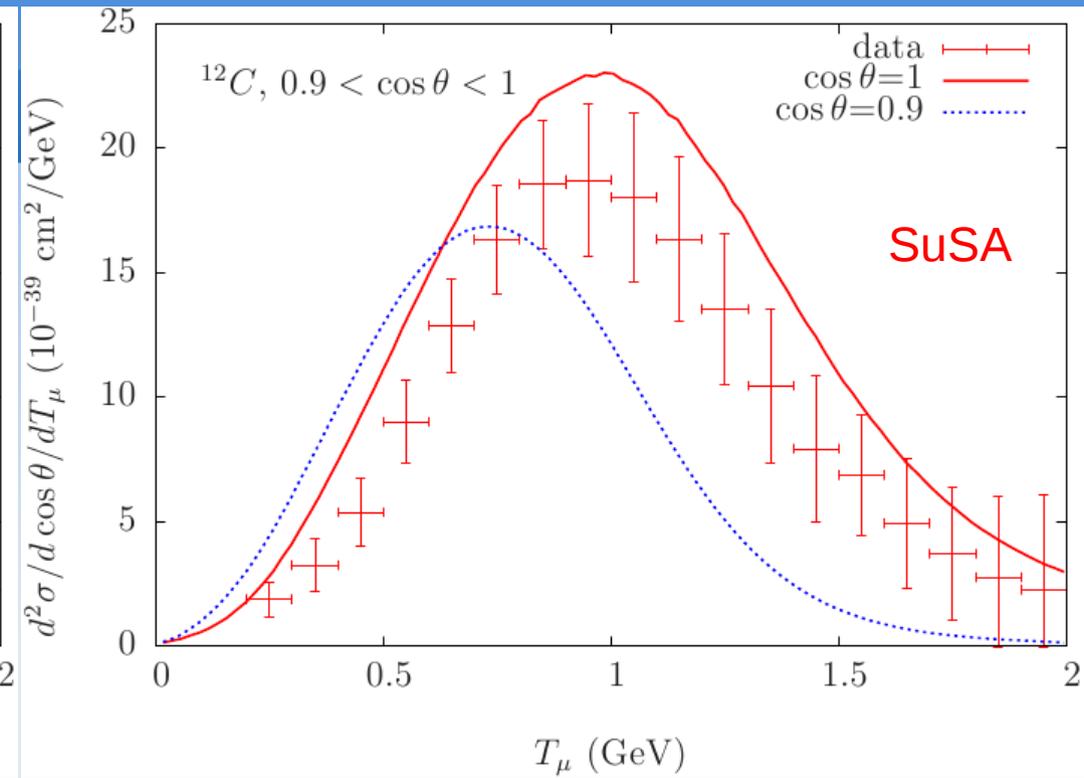
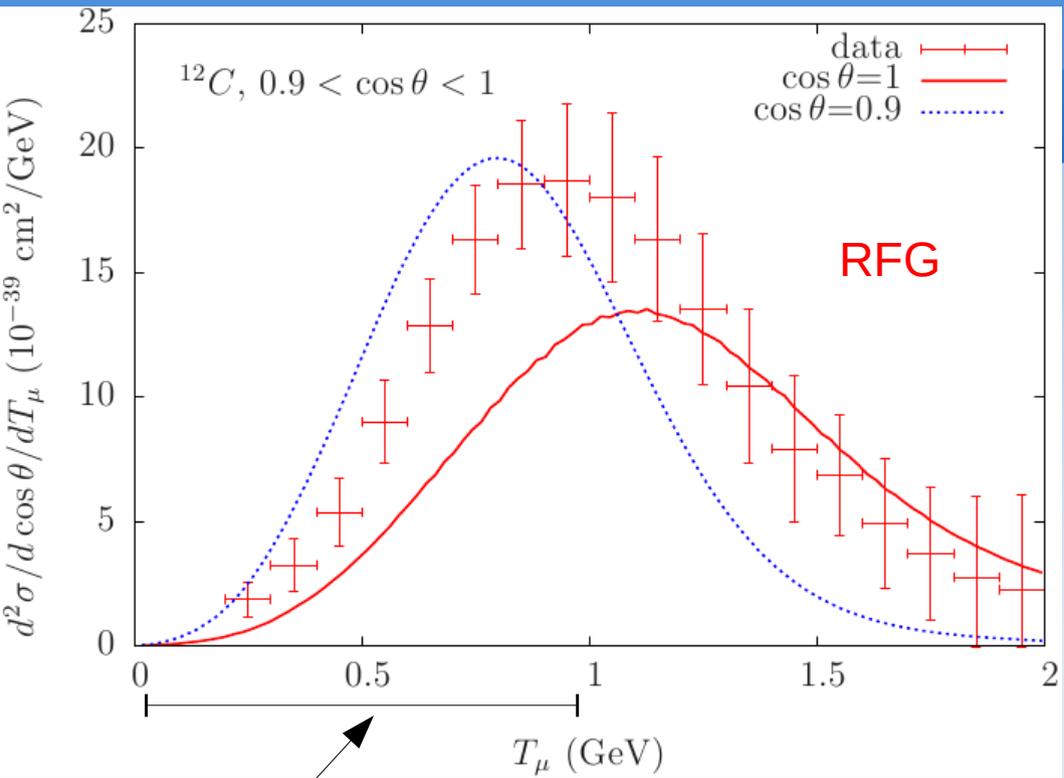
Comparison with MiniBooNE data at $0.9 < \cos\theta < 1$



Comparison with MiniBooNE data at $0.9 < \cos\theta < 1$



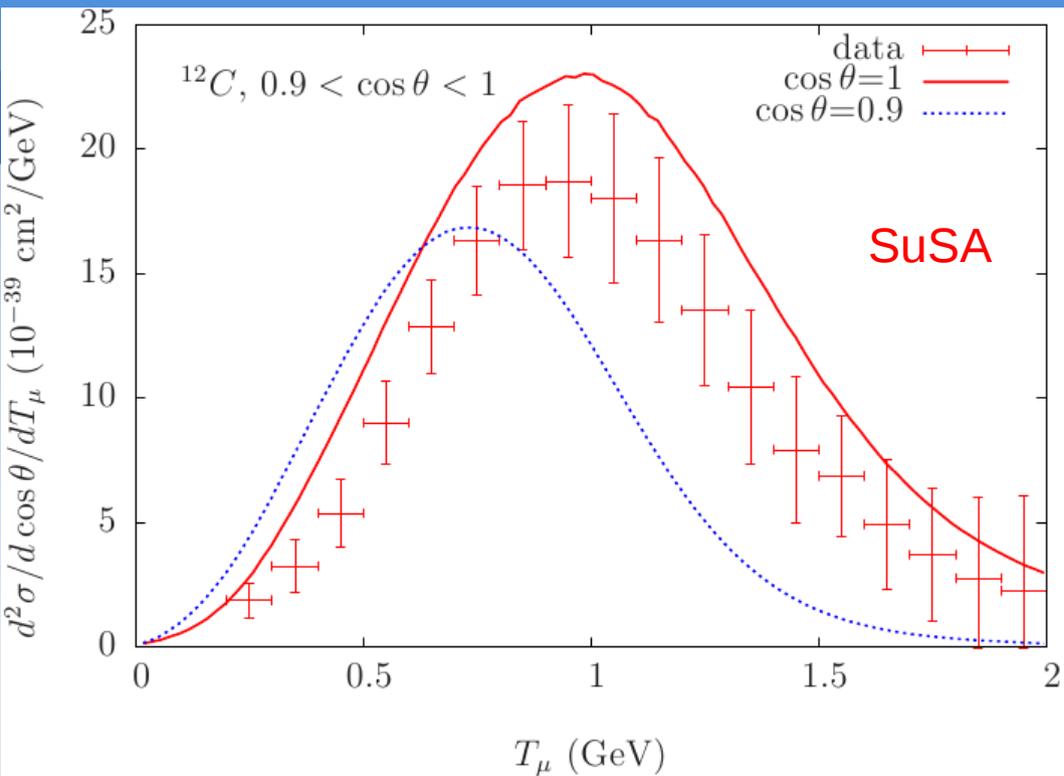
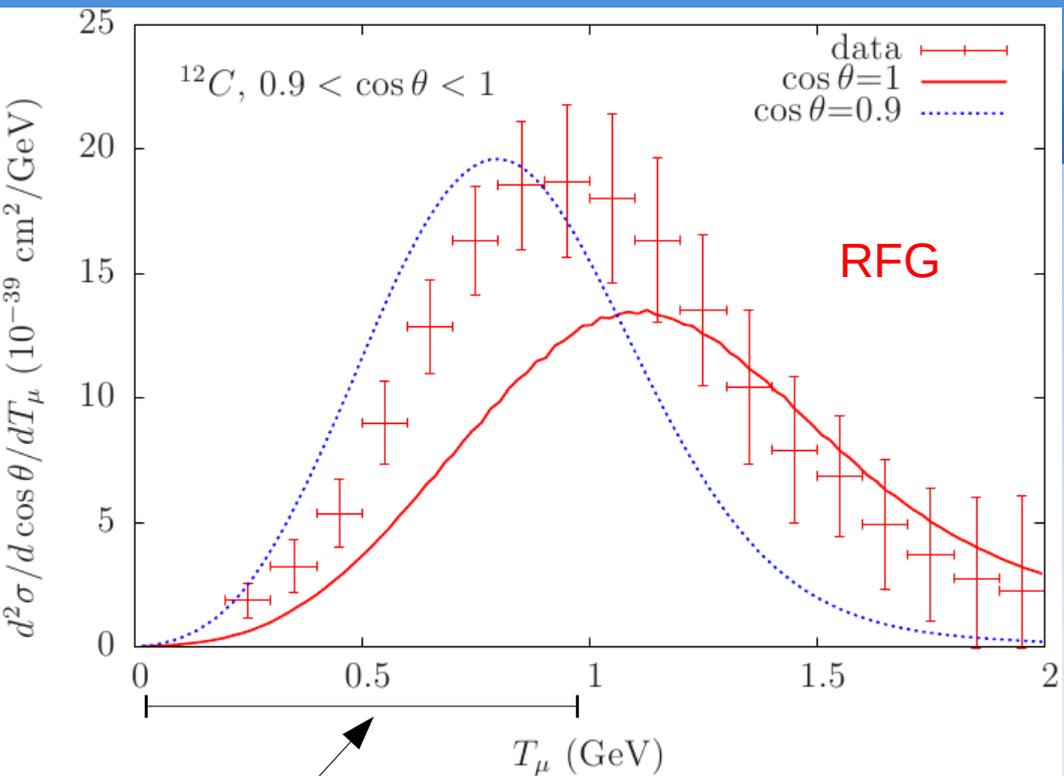
Comparison with MiniBooNE data at $0.9 < \cos\theta < 1$



Pauli blocking is active in this region (low momentum transfers, $q \leq 0.4$ GeV/c): this explains the huge difference between the RFG (where PB is included by definition) and the SuSA (which has no PB) results.

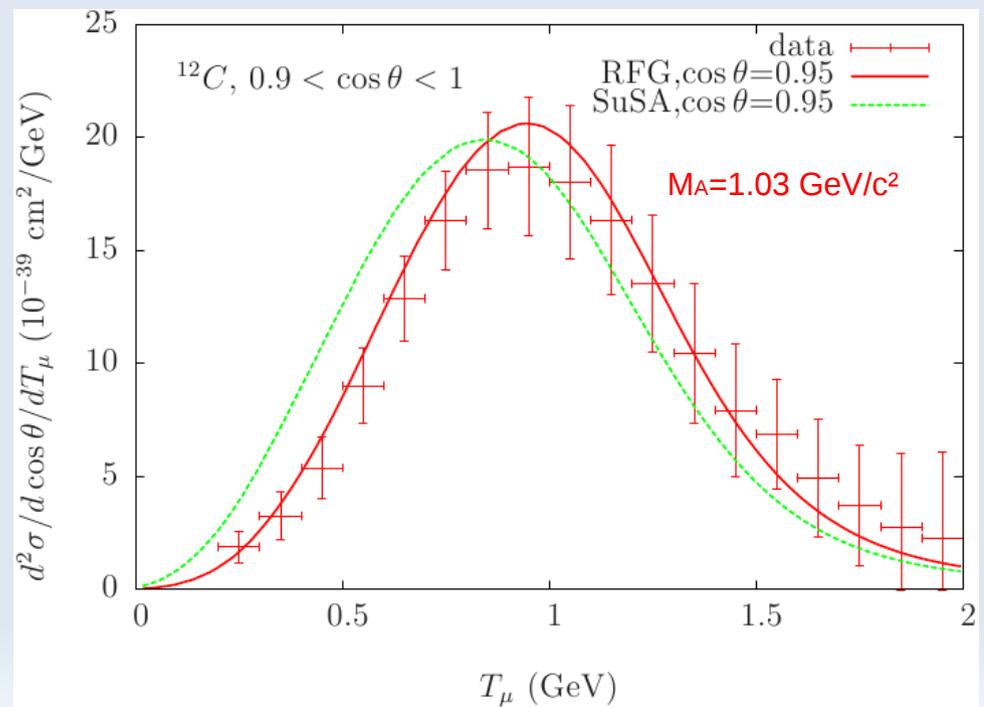
At very low angles both RFG and SuSA are compatible with the data, except for the Pauli-blocked region, where super-scaling ideas are not applicable.

Comparison with MiniBooNE data at $0.9 < \cos\theta < 1$

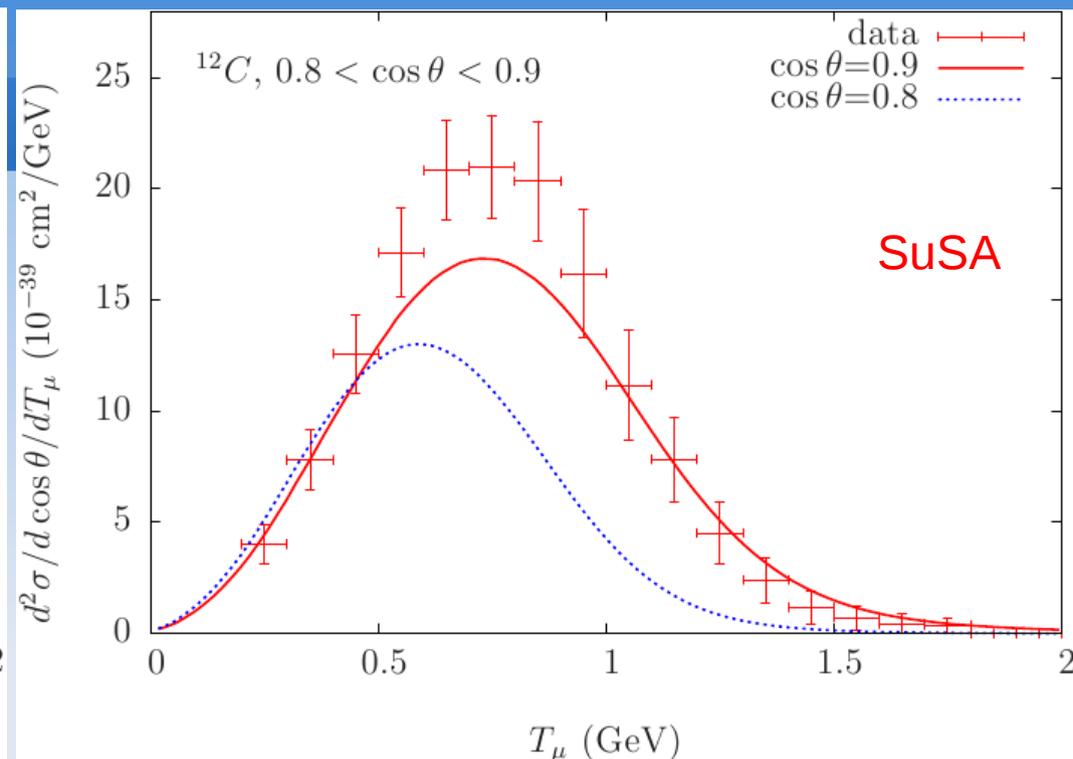
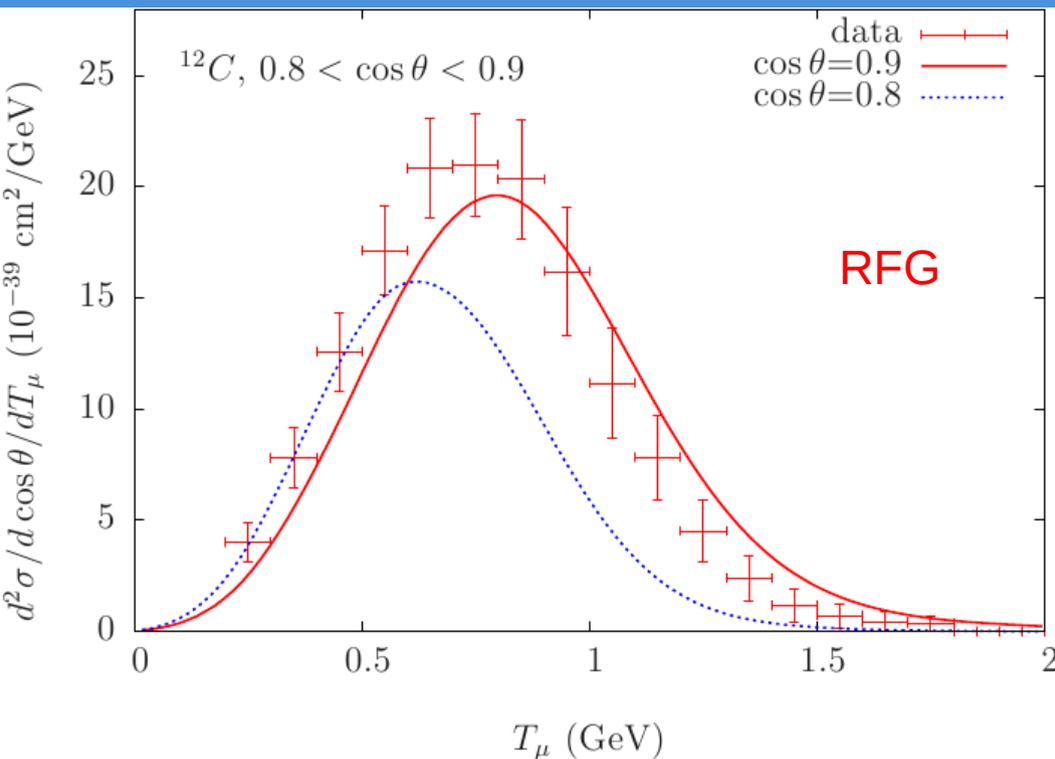


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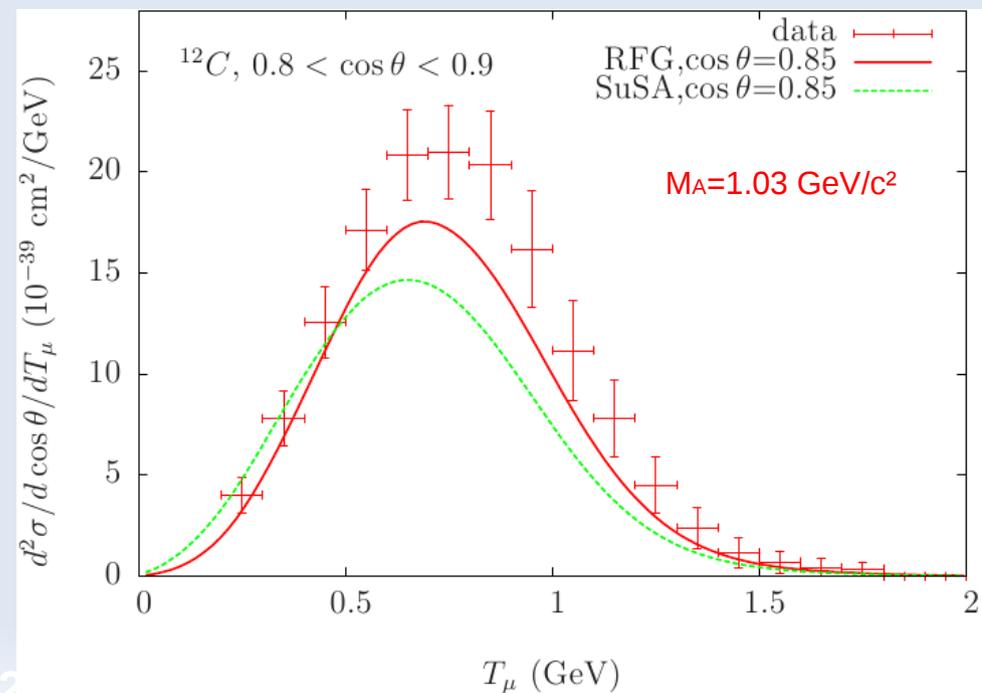
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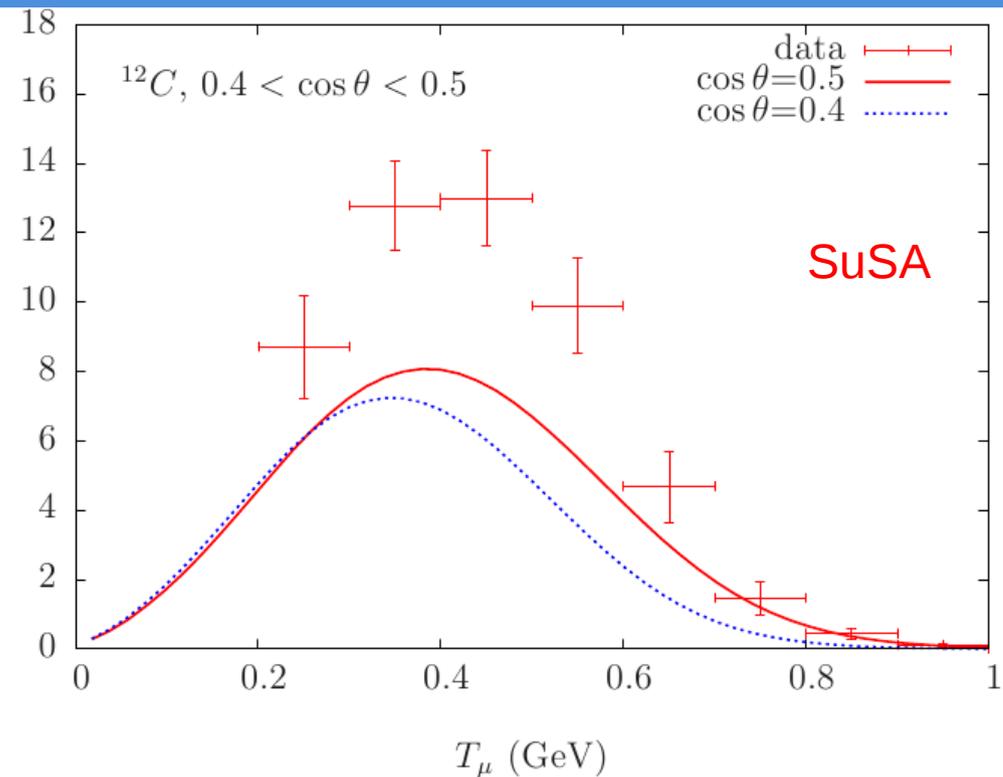
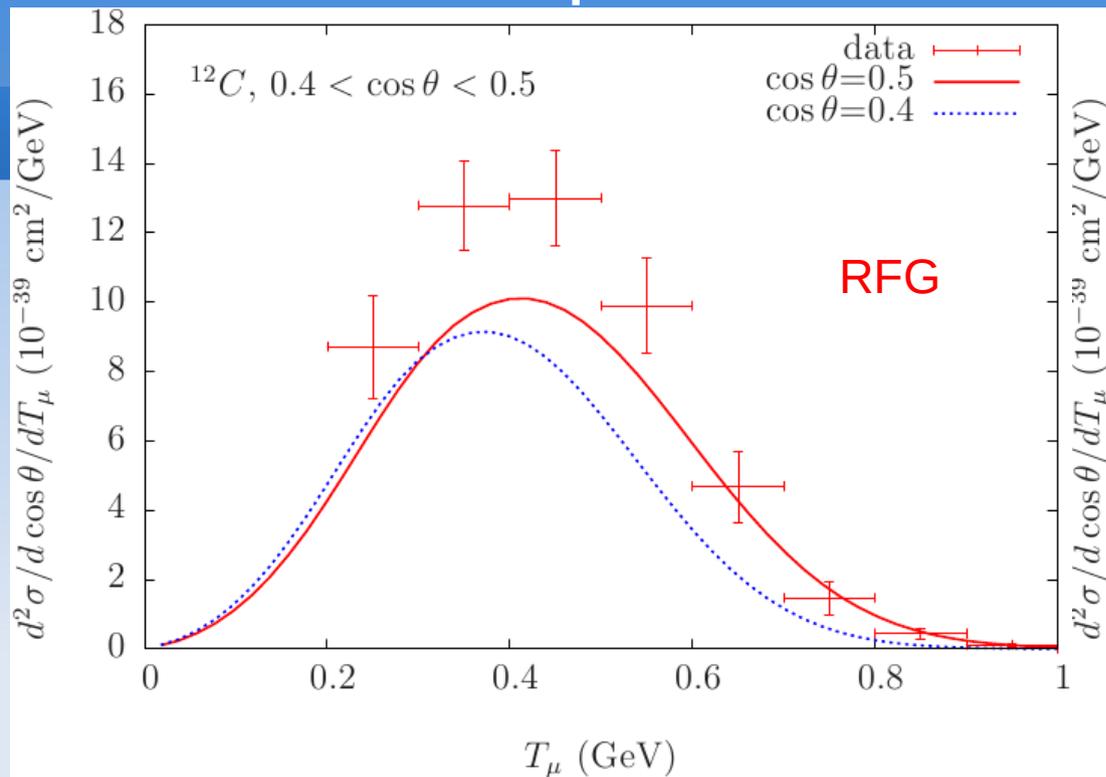
Comparison with MiniBooNE data at $0.8 < \cos\theta < 0.9$



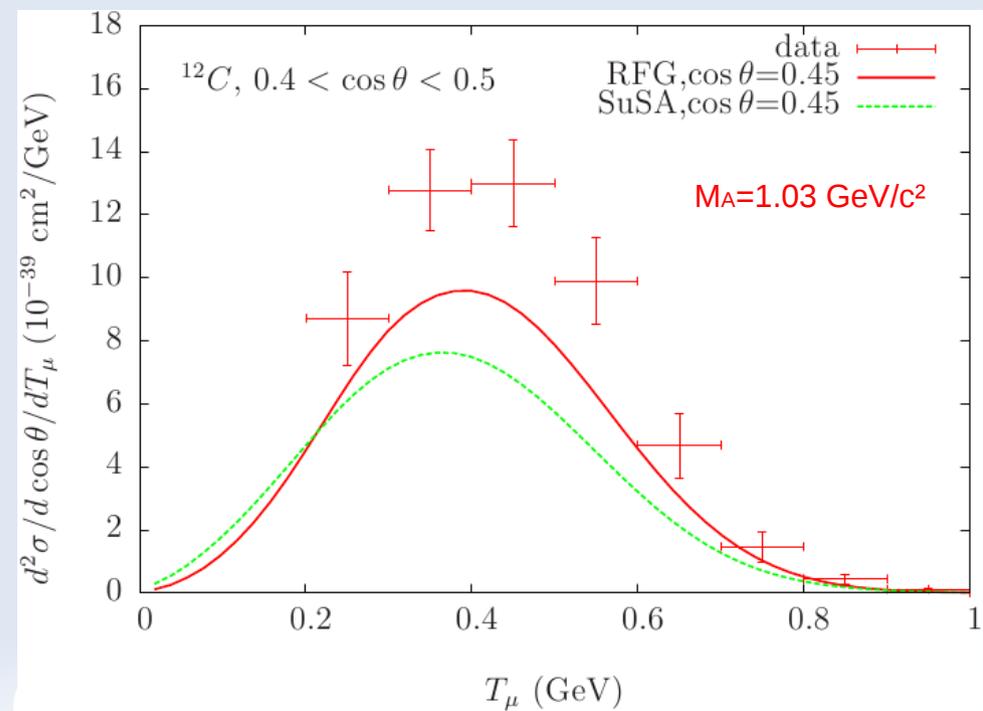
For $\theta \sim 30^\circ$ the RFG is still compatible with the data, the SuSA result is too low at the QEP.



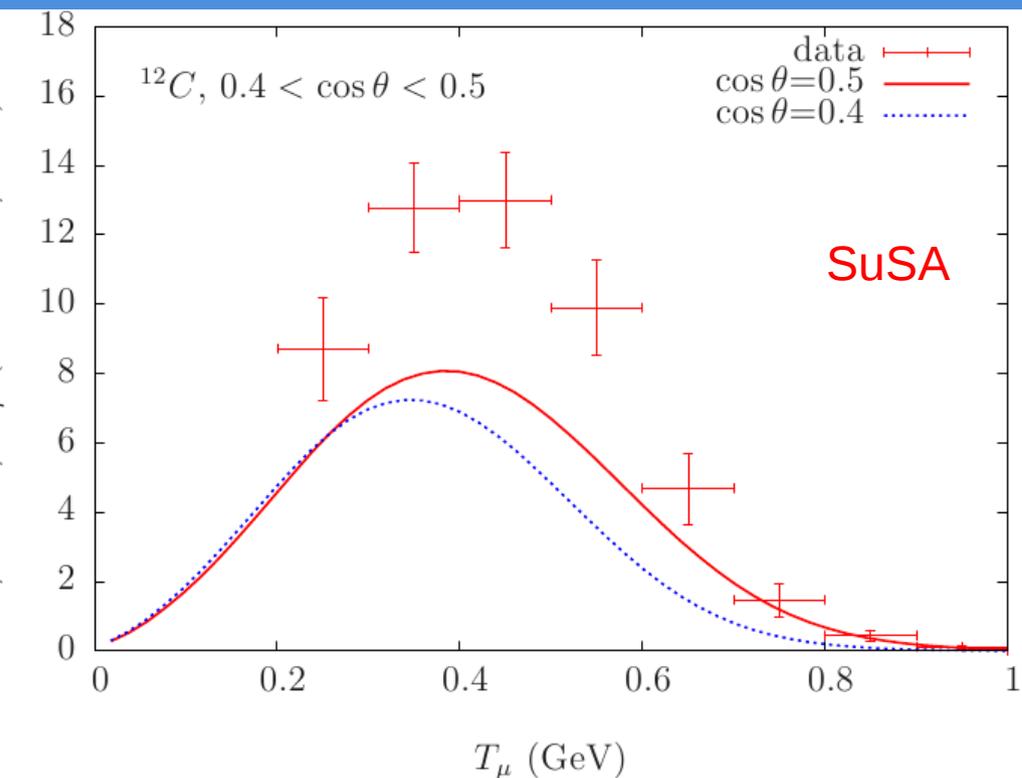
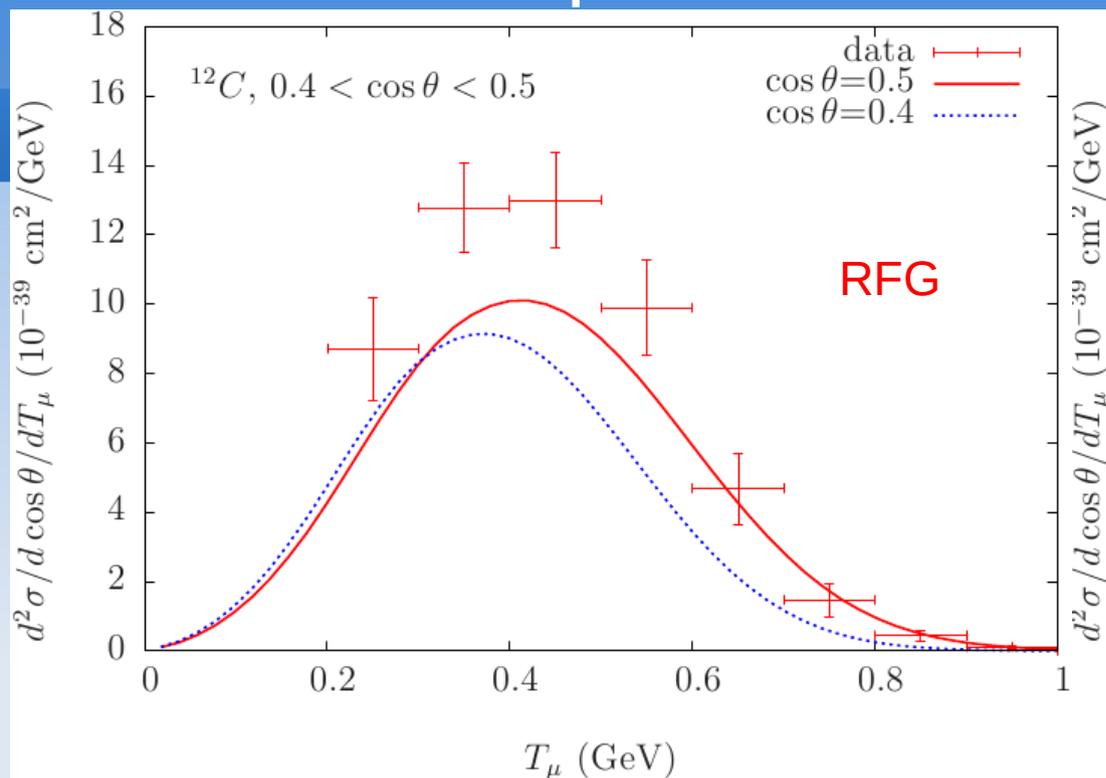
Comparison with MiniBooNE data at $0.4 < \cos\theta < 0.5$



For $\theta \sim 60^\circ$ both RFG and SuSA underestimate the experimental data.

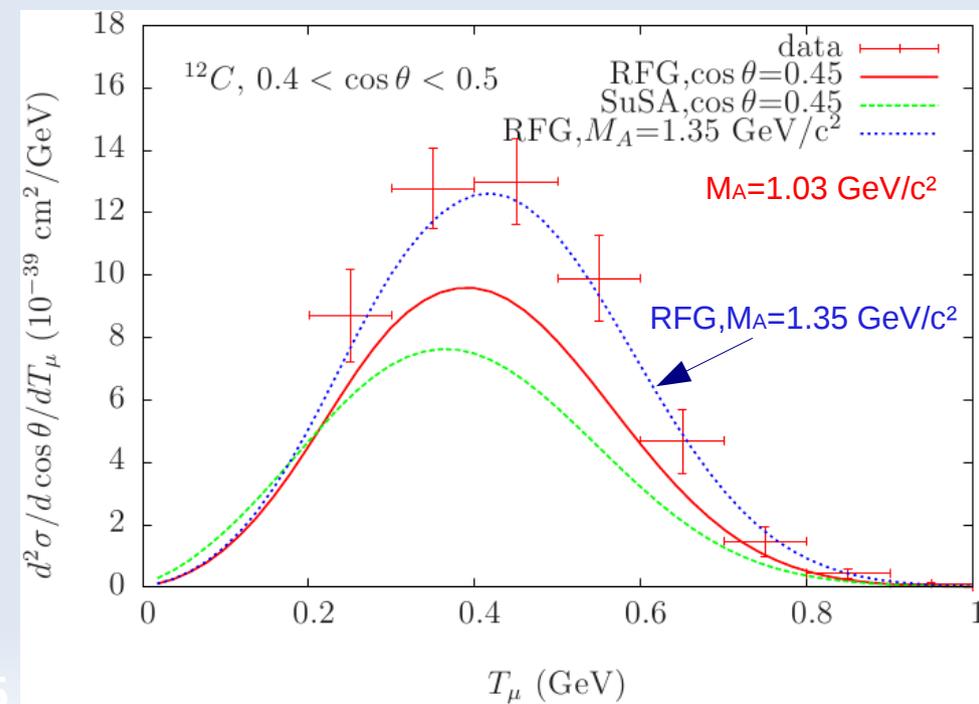


Comparison with MiniBooNE data at $0.4 < \cos\theta < 0.5$

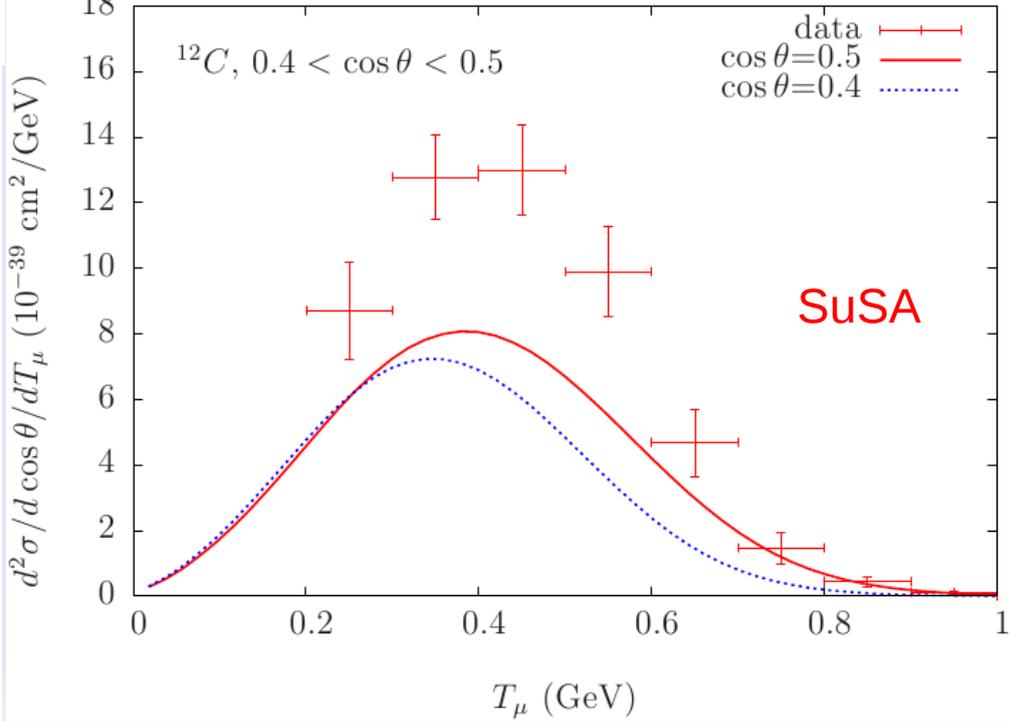
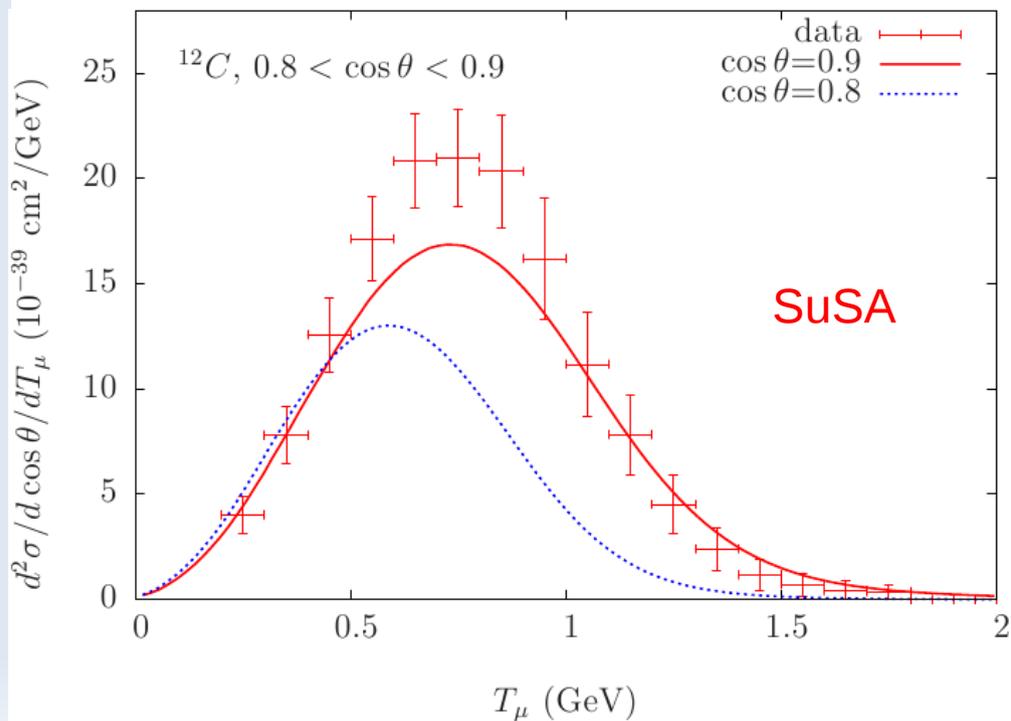
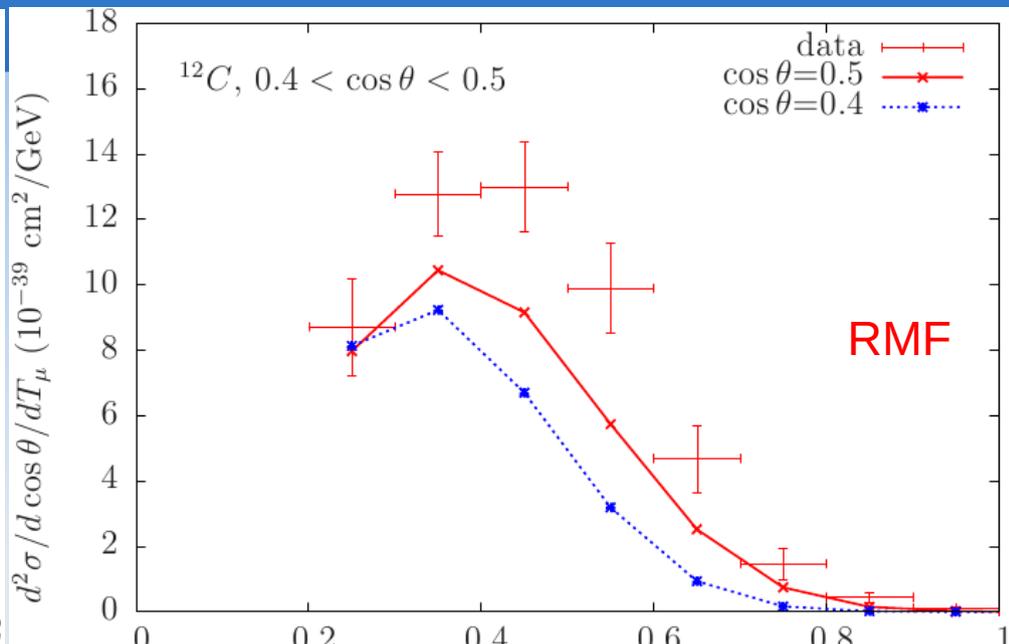
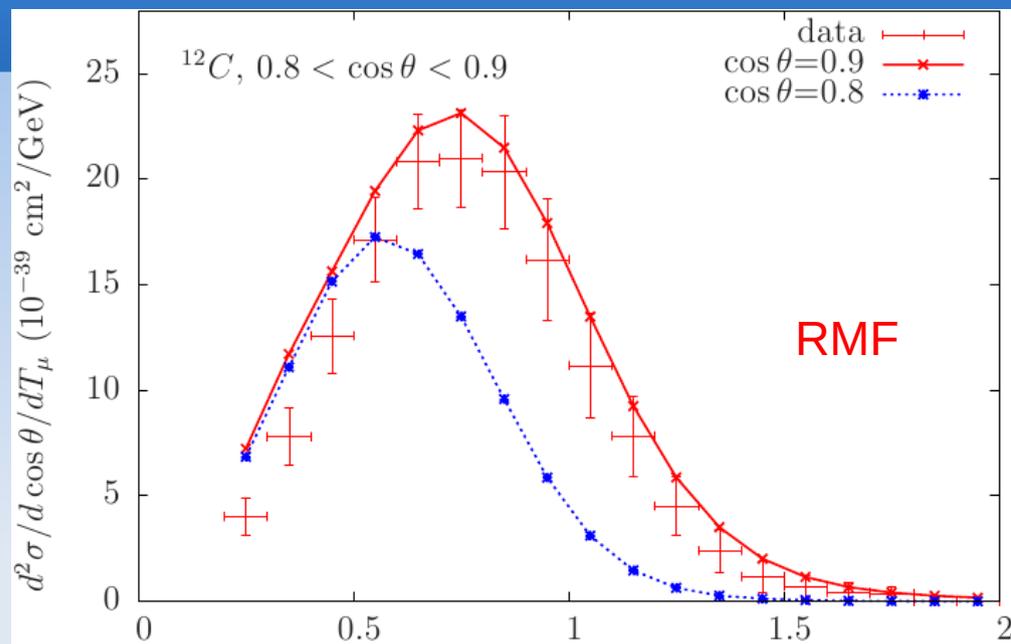


For $\theta \sim 60^\circ$ both RFG and SuSA underestimate the experimental data.

A RFG with high axial mass fits the data: what is the physics?

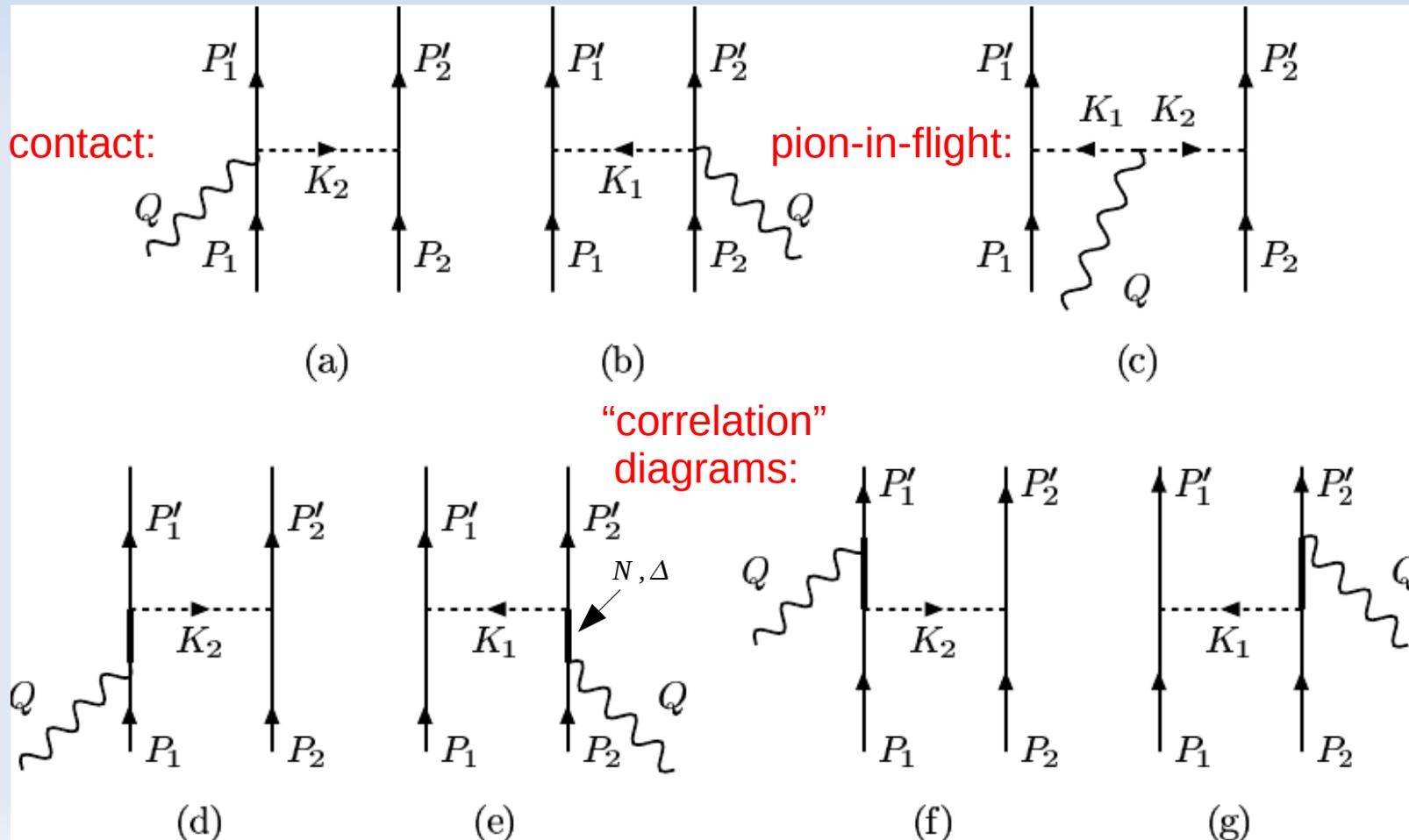


RMF versus SuSA (preliminary)



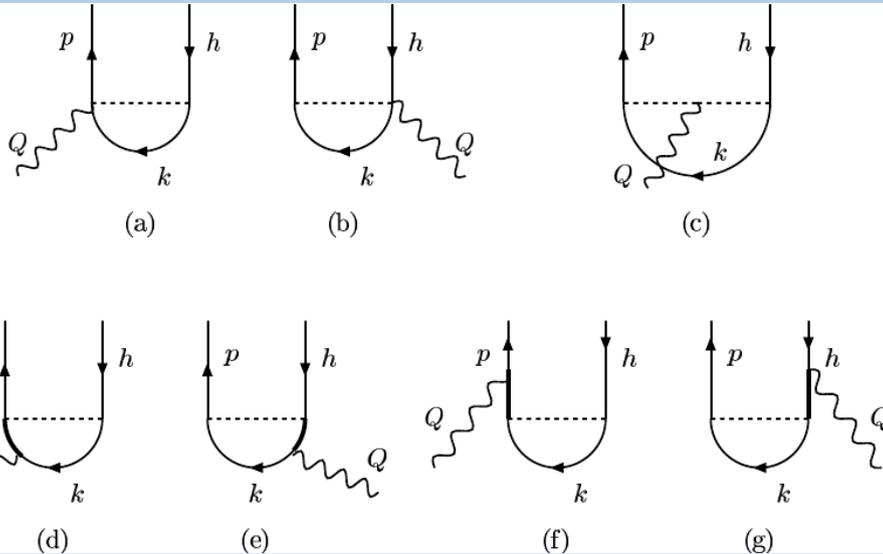
Beyond Impulse Approximation: two-body currents

At the kinematics where meson-exchange currents (MEC) play a significant role **superscaling violations** may occur in the transverse channel.

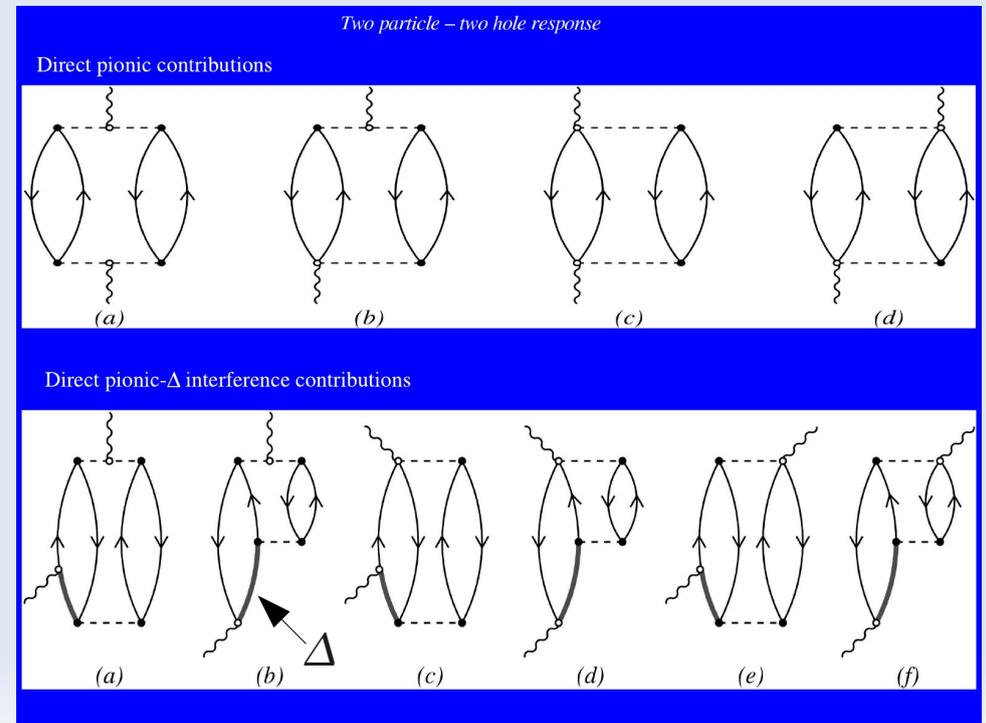


MEC

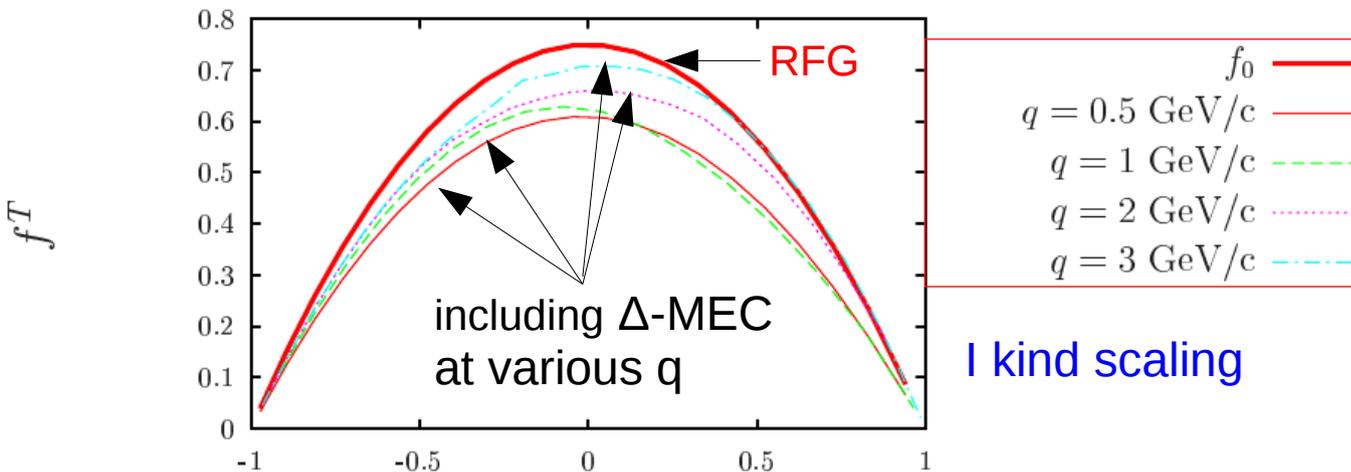
Meson Exchange Currents contribute to the 1p-1h sector:



and to the 2p-2h one:
(just some of the many diagrams)



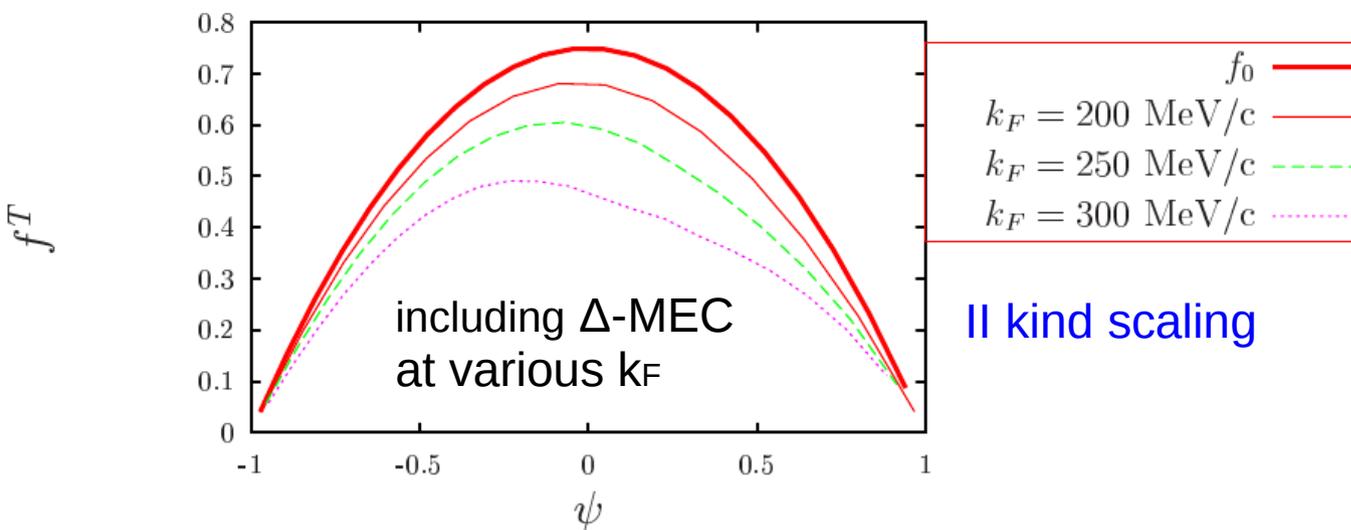
1p-1h MEC



I kind scaling

The response is calculated on the **RFG basis** and is **mainly transverse** (although relativistically there is a small L contribution)

The **Δ -MEC** diagrams give the dominant contribution

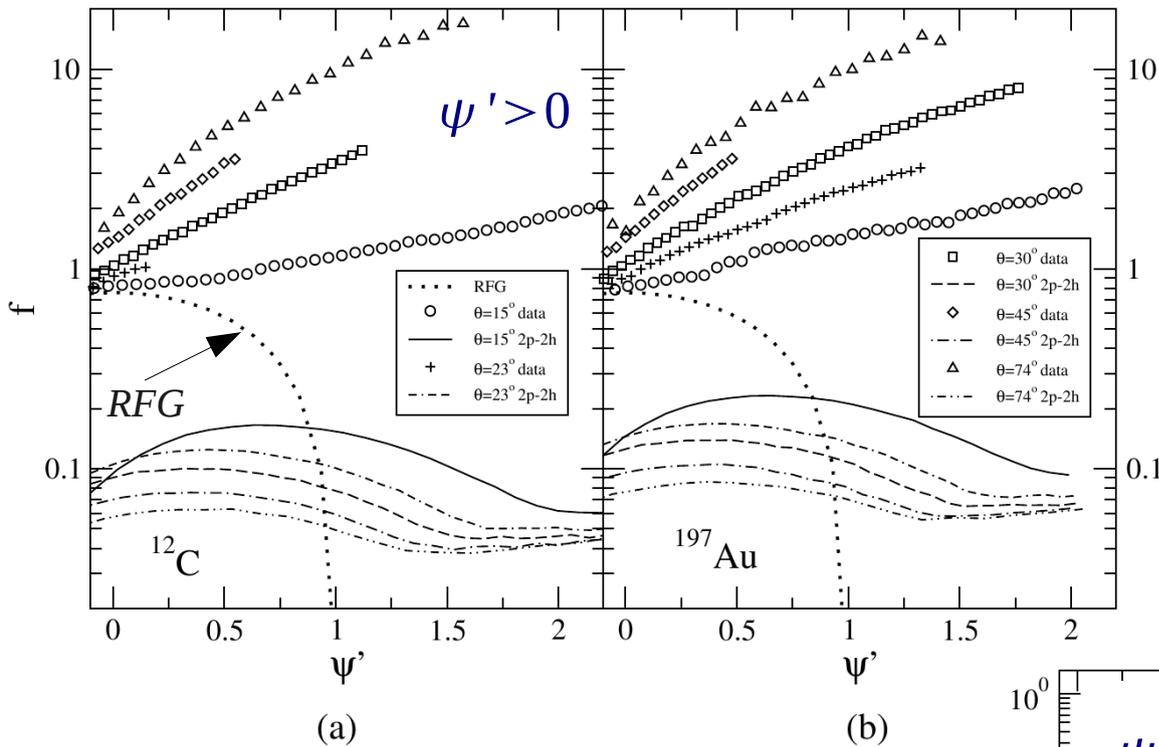


II kind scaling

Both kinds of scaling are violated

1p-1h MEC interfere destructively with the 1-body current, **lowering the cross section**

2p-2h MEC



They contribute outside the RFG response region $-1 < \psi < 1$

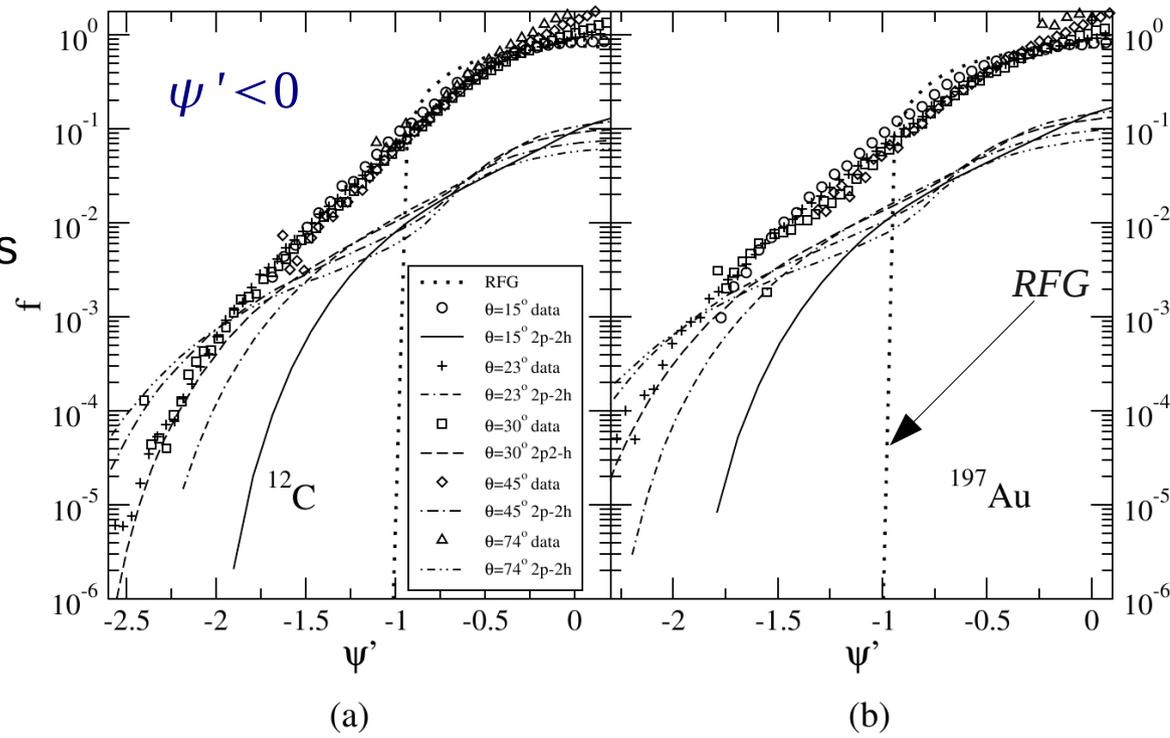
Scaling is broken both below and above the QEP

They give a **positive** contribution of about 10-15% at the QEP

[De Pace et al., NPA741, 249 (2004)]

However, the calculation is not complete because it does not include the correlations required by gauge invariance

Very demanding calculation, even in RFG

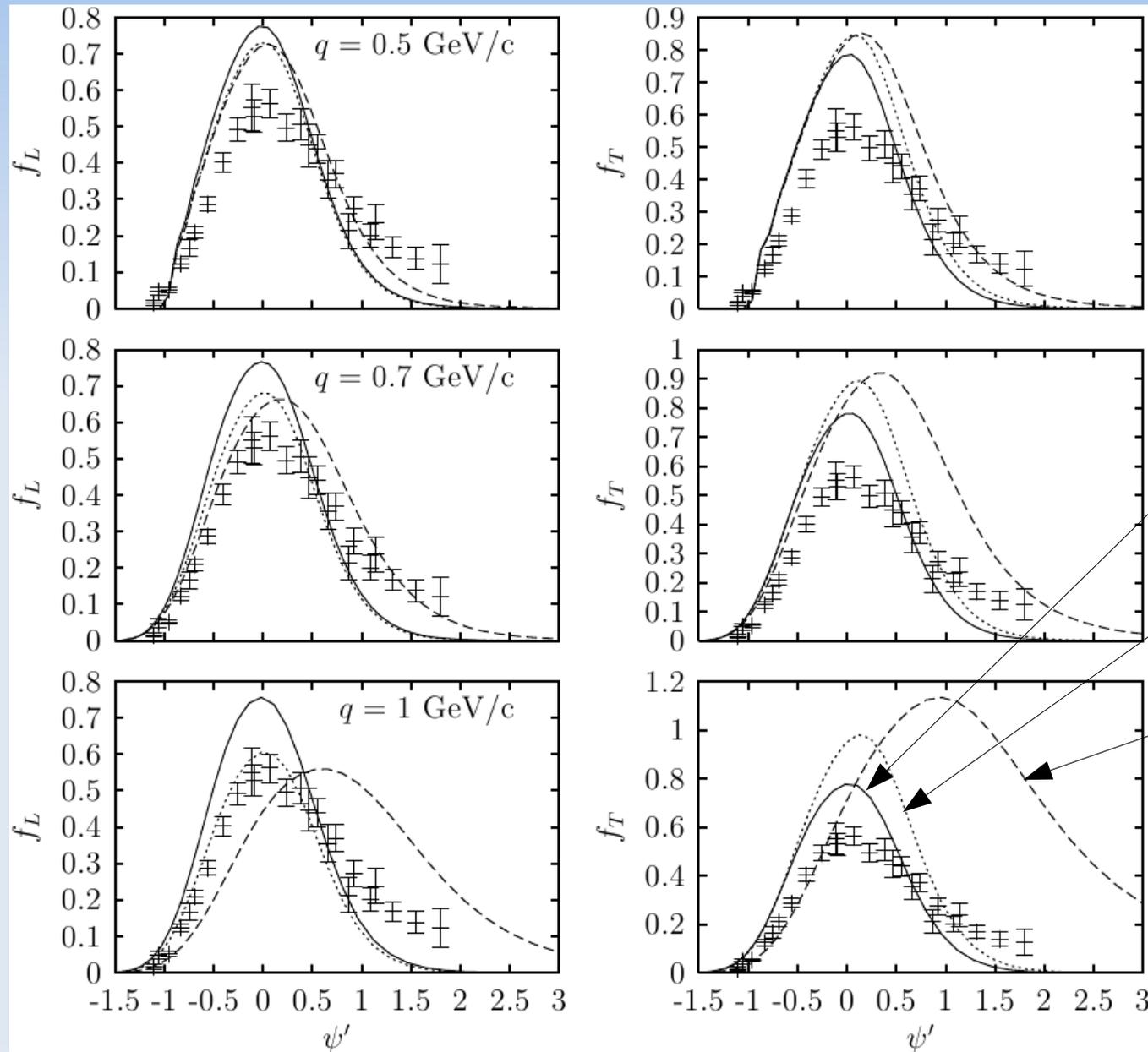


Summary and Conclusions

- Puzzle: the Relativistic Fermi Gas overestimates the (e,e') data and underestimates CC neutrino cross section
- The SuSA model, based on super-scaling and designed to fit inclusive electron scattering data, gives even worse agreement with the experiment
- The Relativistic Mean Field model is in qualitative agreement with the SuSA predictions and also fails to reproduce the MiniBooNE data at high scattering angles
- The Relativistic Shell model, including Final State Interactions through a Dirac equation based potential, is in good agreement with the RMF results
- The SuSA approach relies on scaling of 0th kind [$f_L=f_T$] and of 3rd kind [$f(T=0)=f(T=1)$]: a better understanding of scale-violating contributions, such as MEC, is needed in order to use (e,e') data to predict neutrino cross sections
- Future (present) work:
 1. Complete the calculation of MEC in the 2p-2h sector
 2. Implement scaling violations (of 0, I and II kind) in the SuSA approach
 3. Compare results with other models in the same conditions: Relativistic Green's Function (Pavia group), Coherent Density Fluctuation model (Sofia group), anybody is welcome!

Thank You

Relativistic Effects



Relativistic

Rel. kinematics
Non-rel. currents

Non-relativistic

DEB potential

