Two-boson ($\gamma\gamma$+γZ) exchange corrections and parity-violating electron scattering

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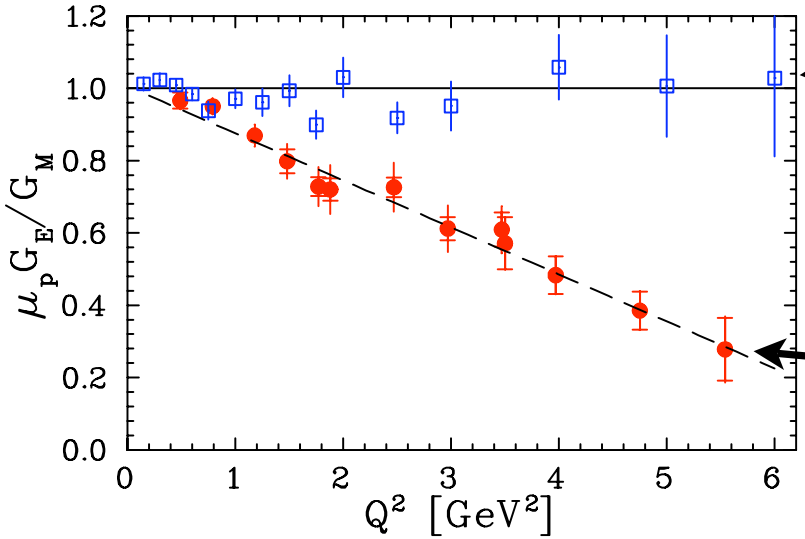
University of Manitoba

Workshop on Electron-Nucleus Scattering XI

Elba, June 22, 2010

Proton G_E/G_M Ratio



Rosenbluth (Longitudinal-Transverse) Separation

Polarization Transfer

LT method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

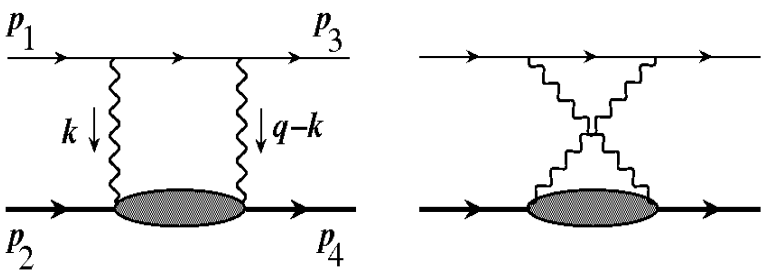
→ G_E from slope in ε plot

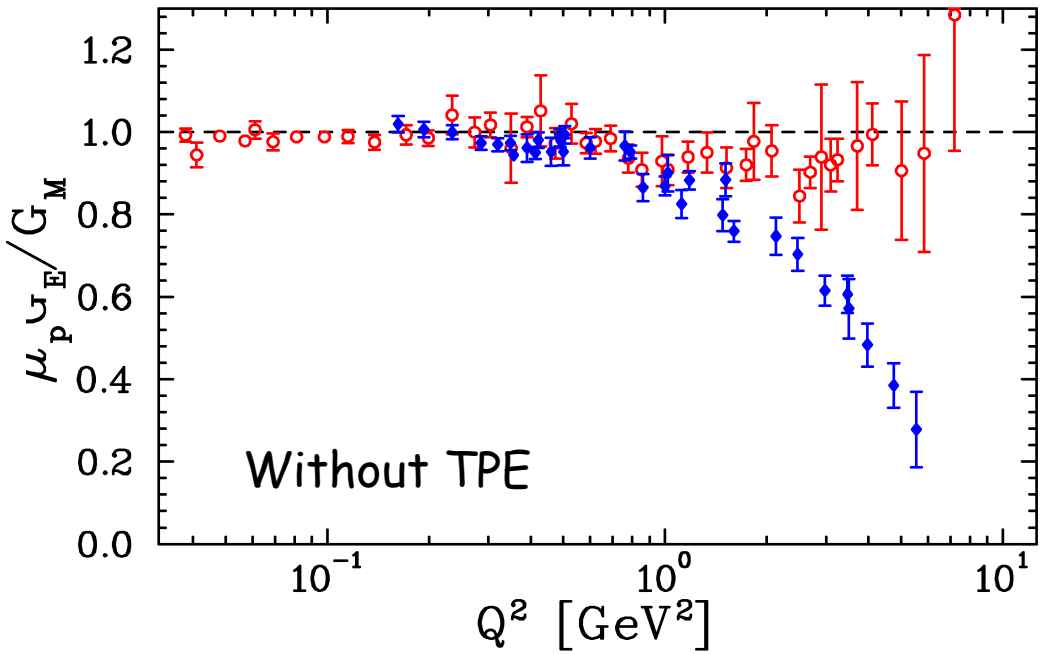
→ suppressed at large Q^2

PT method

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}\frac{P_T}{P_L}$$

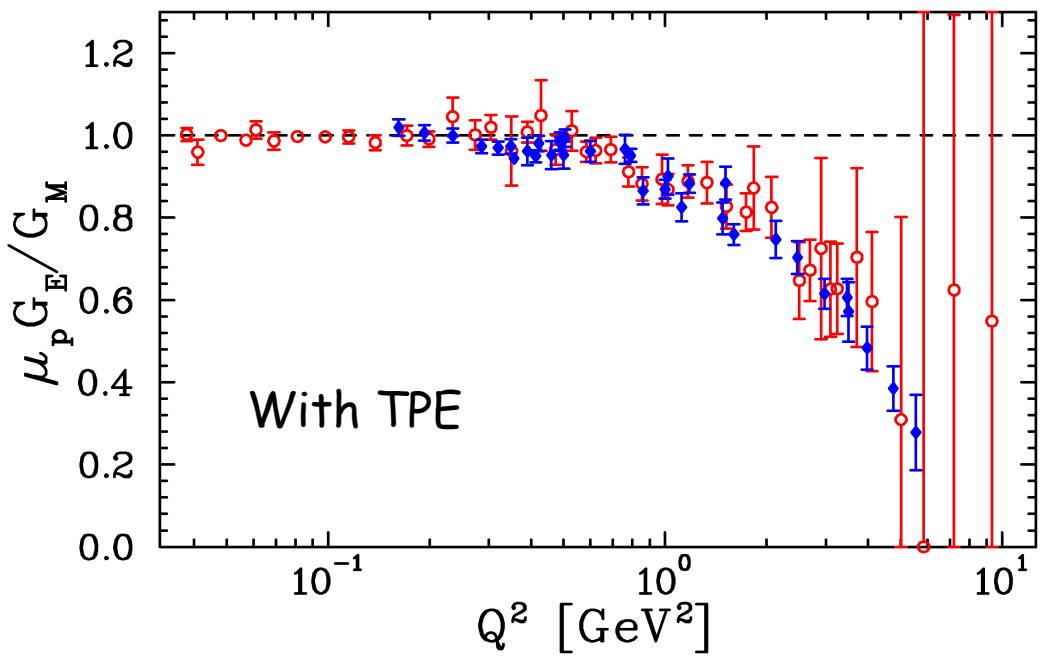
→ $P_{T,L}$ recoil proton polarization in $\vec{e}\, p \to e\, \vec{p}$





Correction relative to OPE

$$\delta_{2\gamma} = \frac{2\Re\left\{M_\gamma^\dagger M_{2\gamma}\right\}}{|M_\gamma|^2}$$



Various Approaches

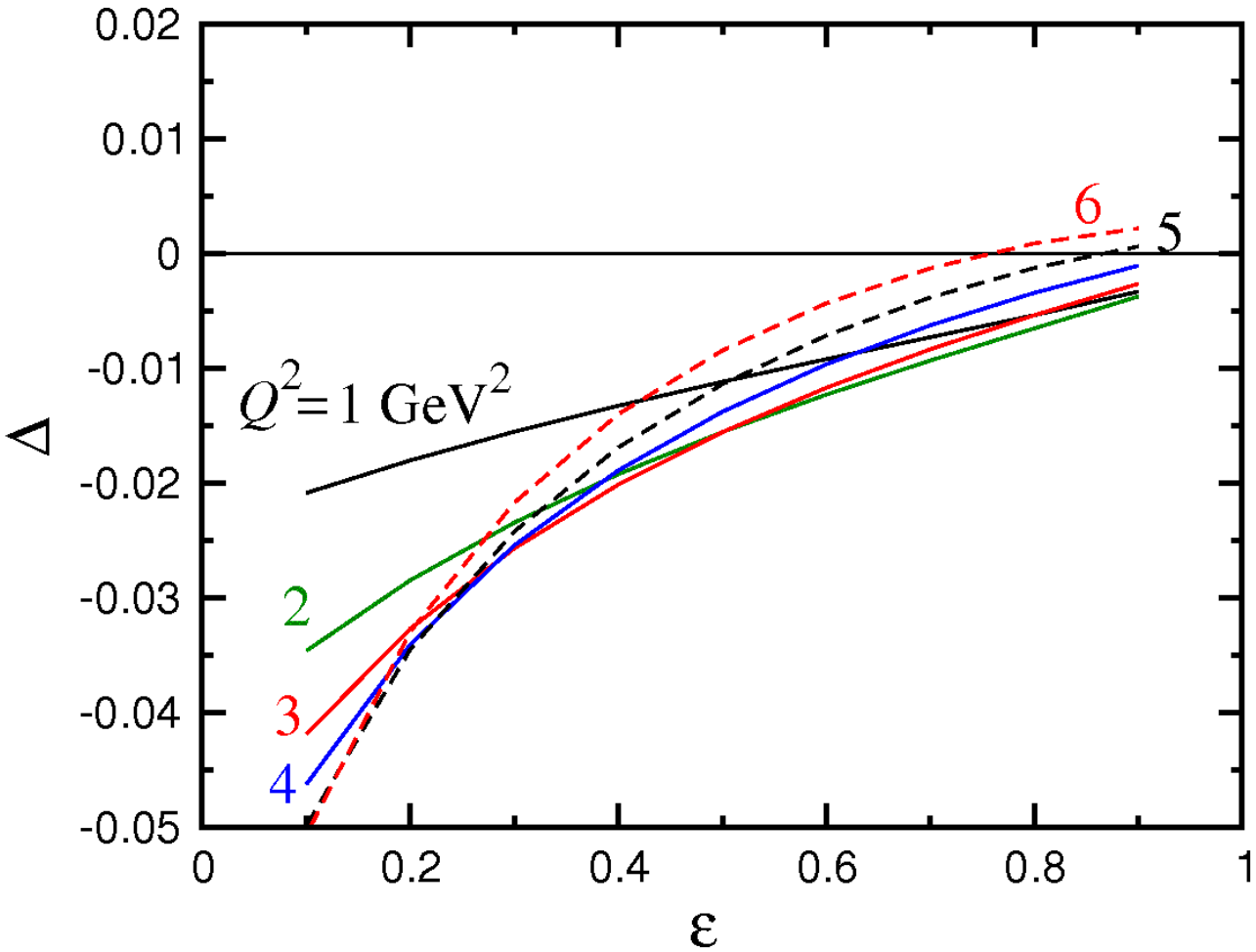
Rely on Models

- Low to moderate Q^2: hadronic: N + Δ + N* etc.
  - more and more parameters, less and less reliable
- Moderate to high Q^2:
  - GPD approach: assumption of 1 active quark
    - Valid only in certain kinematic range
  - pQCD: recent work indicates 2 active quarks dominate

Rely on data

- Use dispersion integrals to relate Real and Imaginary parts. Imaginary parts fixed by cross section data
  - Valid at forward angles: must use models to extrapolate
  - Incomplete: not all data is available (e.g. axial hadron coupling and isospin dependence in γZ diagrams

Corrections to unpolarized cross sections for Q^2=1 to 6 GeV^2



Effect largest at small ε (backward angles)

Vanishes as ε ->1

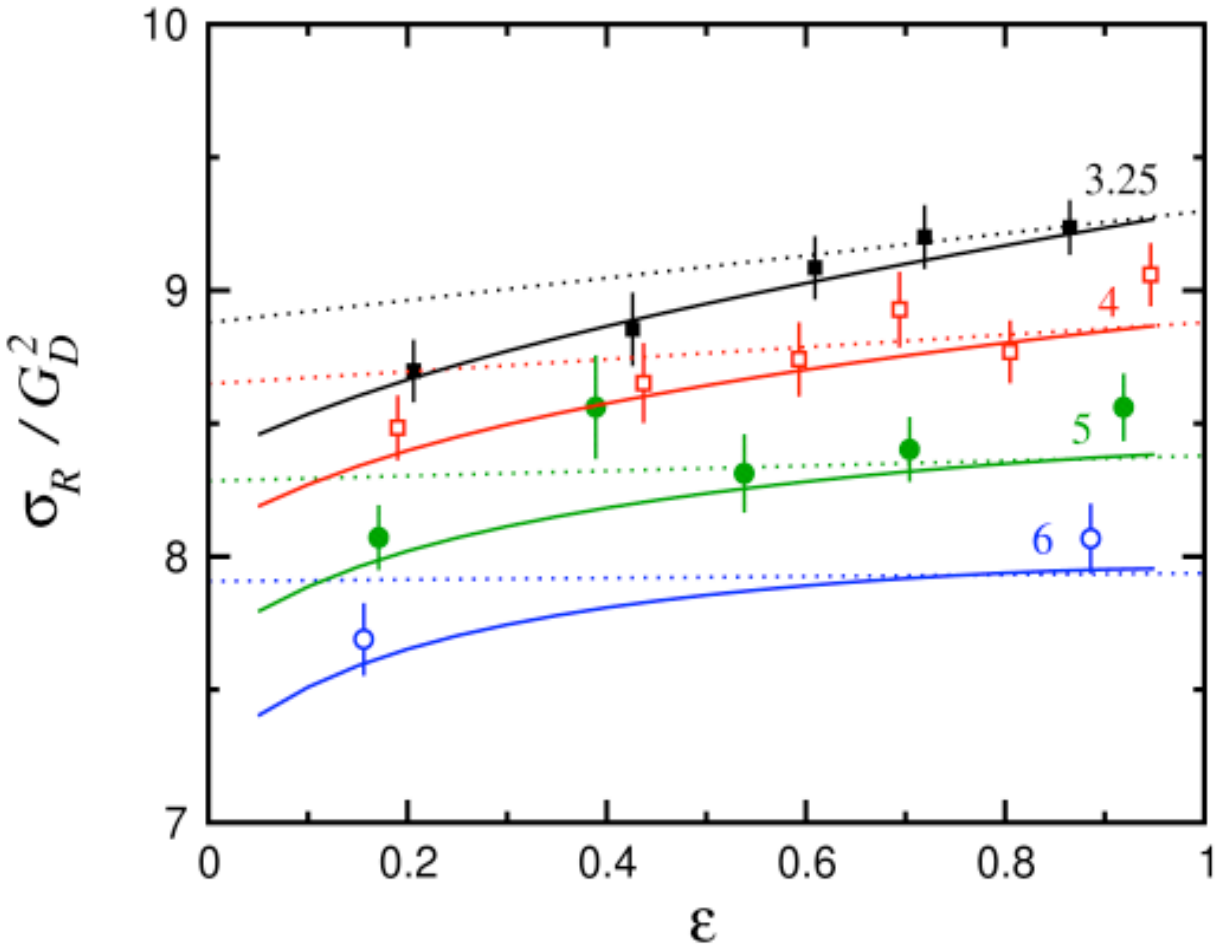
Nonlinearity grows with Q^2

POSITIVE slope for Q^2 > 0.5 GeV^2

NEGATIVE slope for Q^2 < 0.5 GeV^2 (pointlike limit)

Effect on SLAC reduced cross sections at different Q^2

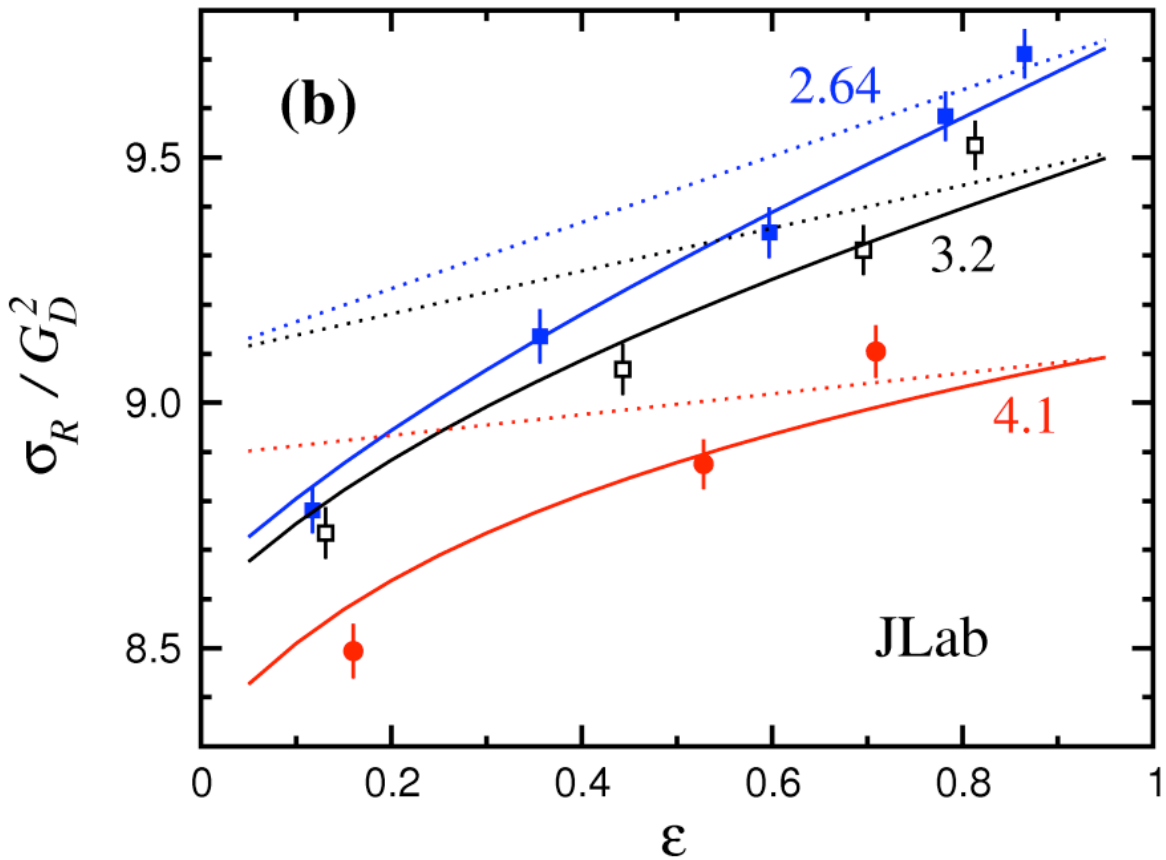
(normalized to dipole G_D^2)



Nonlinearity in ε is displayed here

JLAB experiments to measure nonlinearity

SuperRosenbluth (JLAB) data



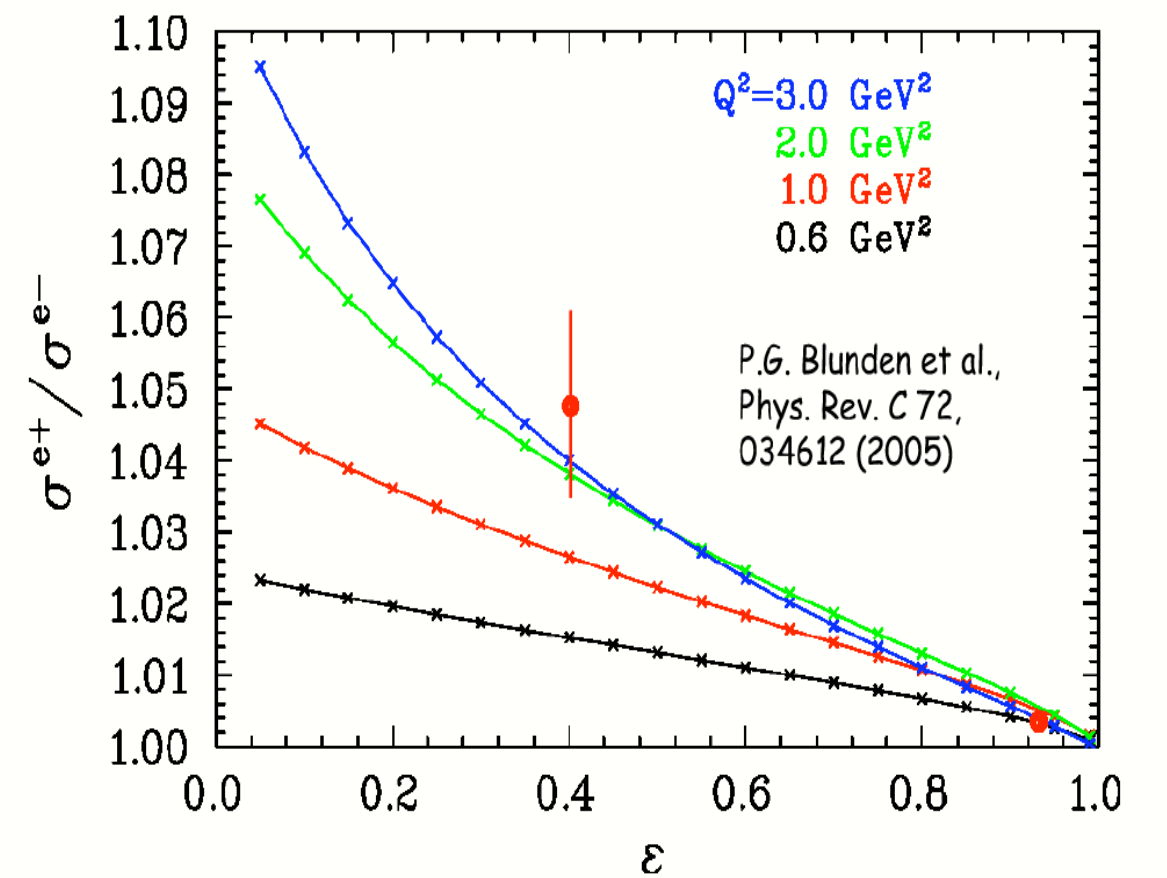
Curves shifted by

+1.0%	2.64
+2.1%	3.20
+3.0%	4.10

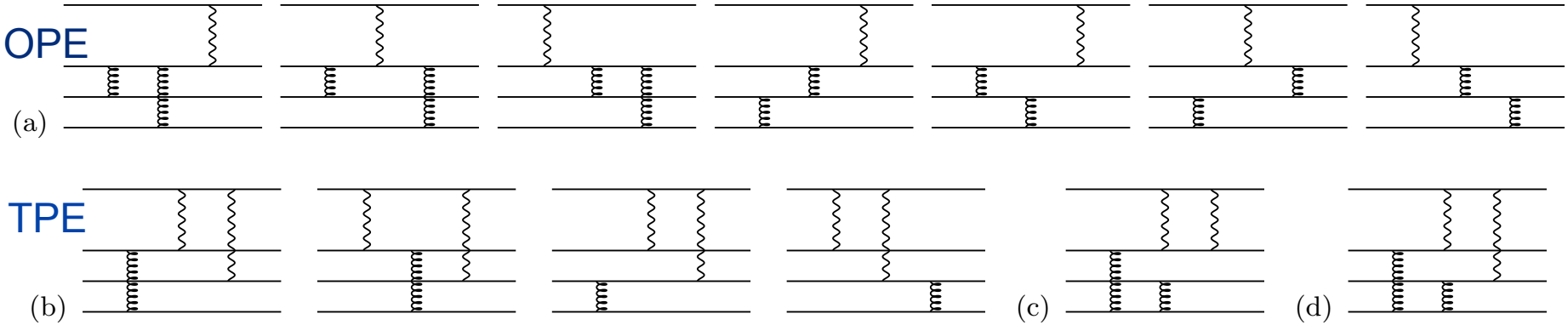
(Effect on determination of G_M)

Very preliminary Novosibirsk data

e^+-p/e^-- p cross section ratio



J. Arrington, L. M. Barkov, V. F. Dmitriev, V. V. Gauzshtajn, R. A Golovin, A. V. Gramolinv
R. J. Holt, V. V. Kaminsky, B. A. Lazarenko, S. I . Mishnev, N. Yu. Muchnoi, D. M. Nikolenk
A. V. Osipov, I. A. Rachek, R. Sh. Sadykov, Yu. V. Shestakov, V. N. Stibunov, H. de Vries,
S. A. Zevakov, V. N. Zhilich ----- *ANL, BINP, INP TPU, NIKHEF*



Recent pQCD calculation: Borisyuk & Kobushkin, PRD **79**, 2009

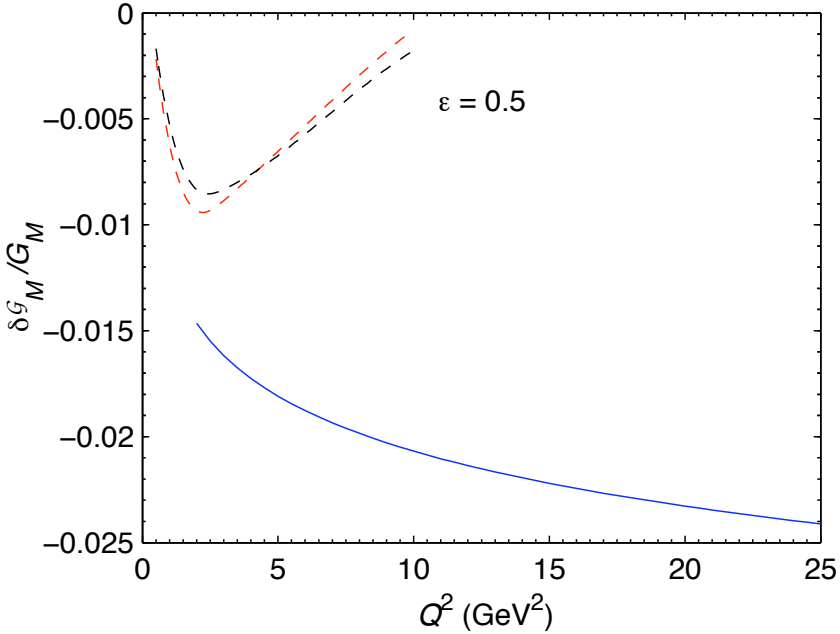
(a) one-photon exchange: need 2 hard gluons to turn momentum of all 3 quarks

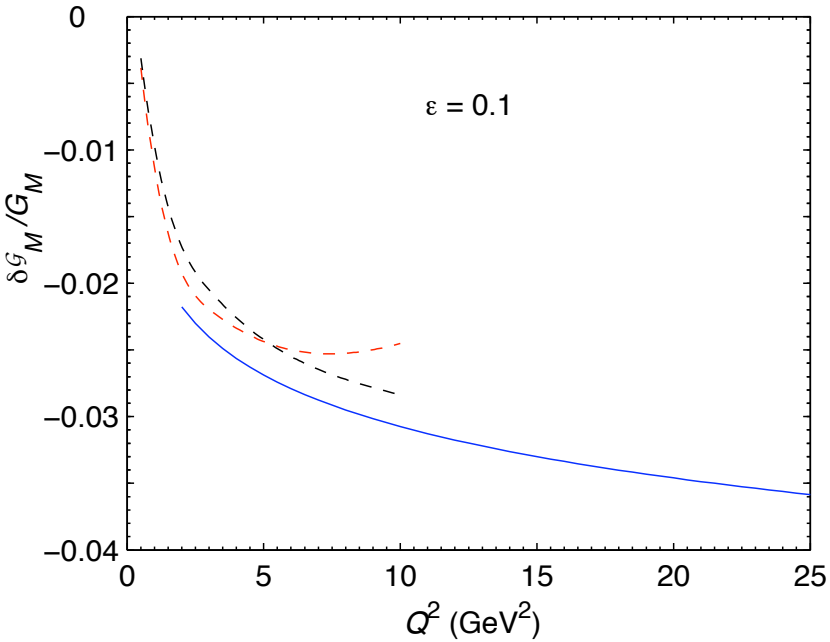
$$\alpha\alpha_s^2/Q^6$$

(b) two-photon exchange:
leading order needs 1 hard gluon

$$\alpha^2\alpha_s/Q^6 \quad \text{TPE/OPE} \sim \alpha/\alpha^s$$

subleading order (both photons on one quark) requires 2 hard gluons



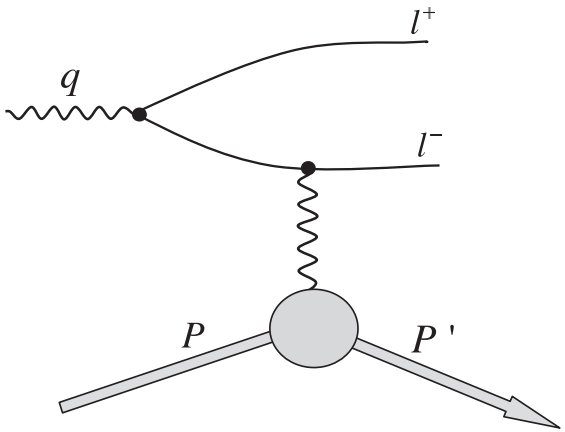


Comparison of hadronic and pQCD results

Connect smoothly around $Q^2 = 3$ GeV^2 ??

Lepton-antilepton photoproduction using real photons

(Pervez Hoodbhoy, PRD 2006)



PHYSICAL REVIEW D **73,** 054027 (2006)

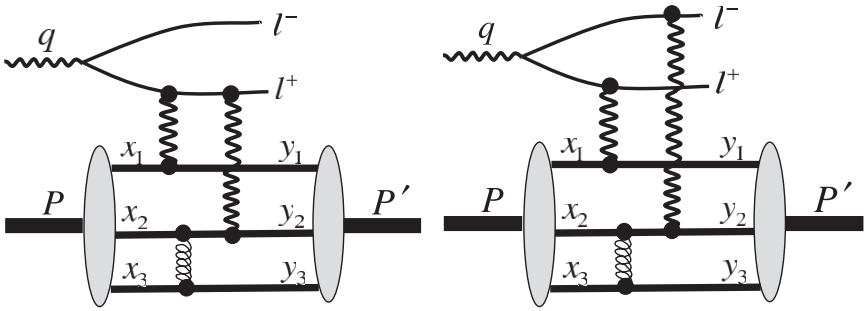


FIG. 6. Typical diagrams for lepton pair production from a 3-quark proton.

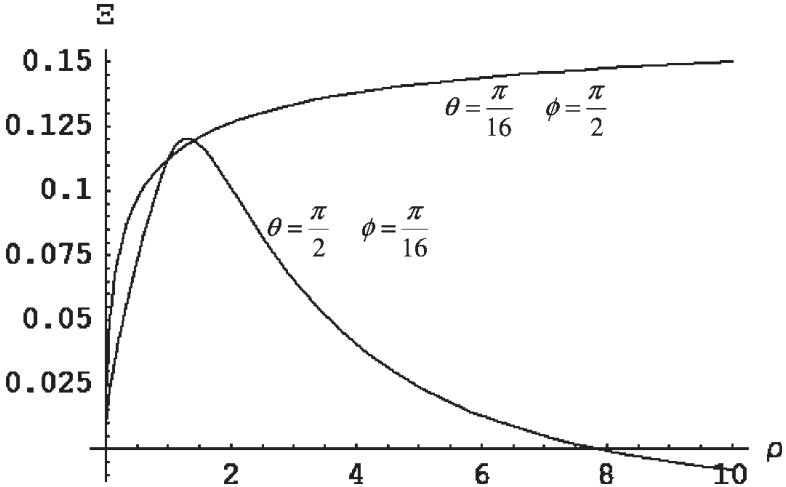
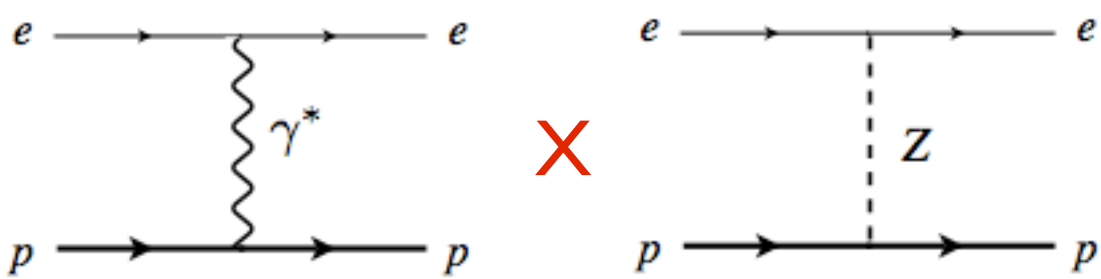


FIG. 7. Lepton pair asymmetry from a proton target.

Parity-violating electron scattering

right-left polarization asymmetry in $\vec{e} + p \to e + p$ scattering

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = \frac{2\Re\{\mathcal{M}_\gamma^\dagger \mathcal{M}_Z\}}{|\mathcal{M}_\gamma|^2}$$



$$j_\mu^Z = \bar{u}\left(g_V^e \gamma_\mu + g_A^e \gamma_\mu \gamma_5\right) u$$

$$J_Z^\mu(q) = \bar{U}\left(F_1^Z \gamma^\mu + F_2^Z i \frac{\sigma^{\mu\nu} q_\nu}{2M} + G_A^Z \gamma^\mu \gamma_5\right) U$$

$$g_V^e = -(1 - 4s_W^2)$$
$$g_A^e = +1$$
$$F_i^Z = (1 - 4s_W^2) F_i^p - F_i^n - F_i^s$$
$$G_A^Z = -G_A \tau_3 + G_A^s$$

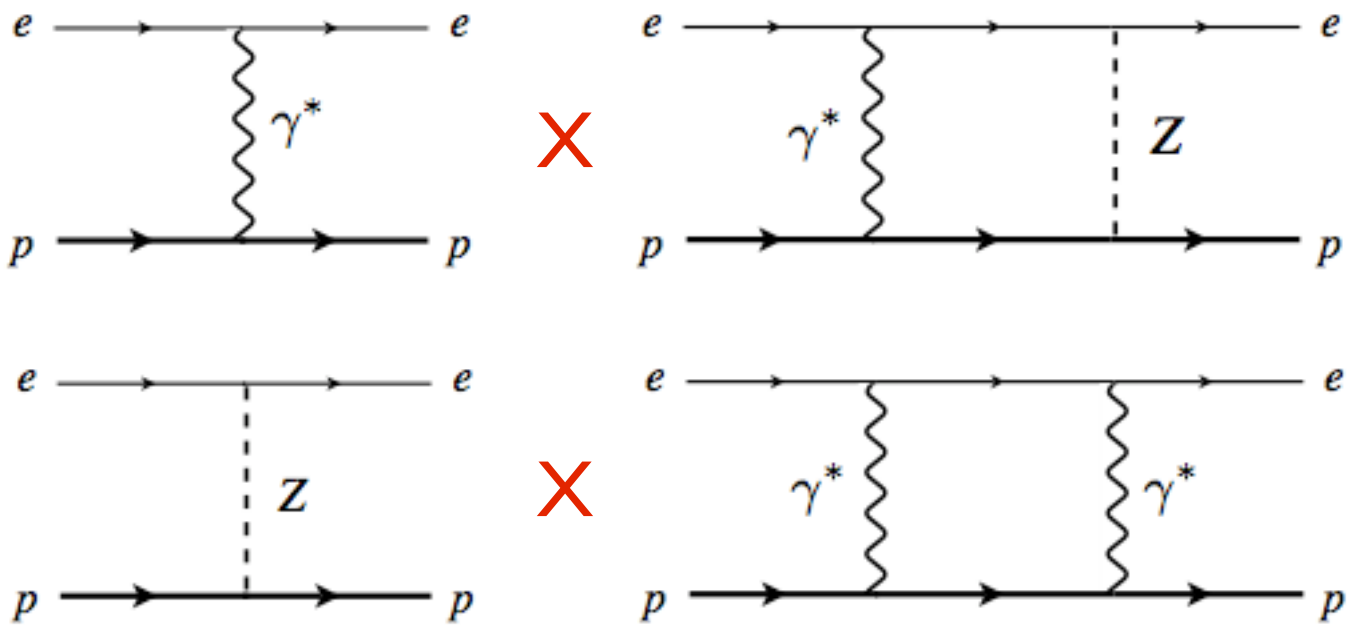
using relation between weak and EM form factors

One boson exchange

$$\mathcal{A}_{PV} = -\frac{G_F Q^2}{e^2\sqrt{2}} \frac{g_A^e\left(\epsilon G_E\, G_E^Z + \tau G_M\, G_M^Z \qquad\qquad\right) + g_V^e\left(\epsilon' G_M \qquad\qquad\right) G_A^Z}{\epsilon G_E^2 + \tau G_M^2}$$

$$G_{E,M}^Z = (1 - 4\sin^2\theta_W) G_{E,M} - G_{E,M}^n$$

Two-boson exchange corrections



"$\gamma(Z\gamma)$"

"$Z(\gamma\gamma)$"

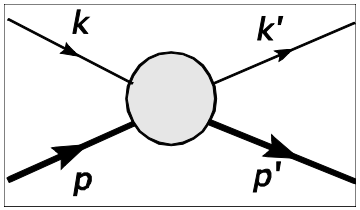
Electromagnetic radiative corrections interfere with M_Z ($M_\gamma \rightarrow M_\gamma + M_{\gamma\gamma}$)

plus weak radiative corrections interfere with M_γ ($M_Z \rightarrow M_Z + M_{\gamma Z}$)

plus two-photon exchange "$\gamma(\gamma\gamma)$" in denominator

Zhou, Kao & Yang, PRL 2007; Tjon & Melnitchouk, PRL 2008;
Tjon, Melnitchouk & Blunden, PRC 2009

$\gamma\gamma$ phenomenology: Generalized form factors



$$T = (\gamma_\mu)^{(e)} \otimes \left(F_1' \gamma^\mu + i \frac{F_2'}{2M} \sigma^{\mu\nu} q_\nu \right)^{(p)} + (\gamma_\mu \gamma_5)^{(e)} \otimes (G_A' \gamma^\mu \gamma_5)^{(p)}$$

In general, 16 independent amplitudes:

parity 16 $\rightarrow$ 8; time reversal 8 $\rightarrow$ 6; helicity conservation (m_e=0) 6 $\rightarrow$ 3

Generalized (complex) form factors

$$F_1'(\nu, Q^2) = F_1(Q^2) + \delta F_1'(\nu, Q^2) \qquad \nu = k \cdot p + q^2/4$$
$$F_2'(\nu, Q^2) = F_2(Q^2) + \delta F_2'(\nu, Q^2)$$
$$G_A'(\nu, Q^2)$$

Dispersion relations $\Re\mathrm{e} F_i' \leftrightarrow \Im\mathrm{m} F_i'$

One boson exchange

$$\mathcal{A}_{PV} = -\frac{G_F Q^2}{e^2\sqrt{2}} \frac{g_A^e\left(\epsilon G_E\, G_E^Z + \tau G_M\, G_M^Z\right) + g_V^e\left(\epsilon' G_M\right) G_A^Z}{\epsilon G_E^2 + \tau G_M^2}$$

+ γγ exchange $G \to G'$

Afanasev and Carlson (PRL 2005)

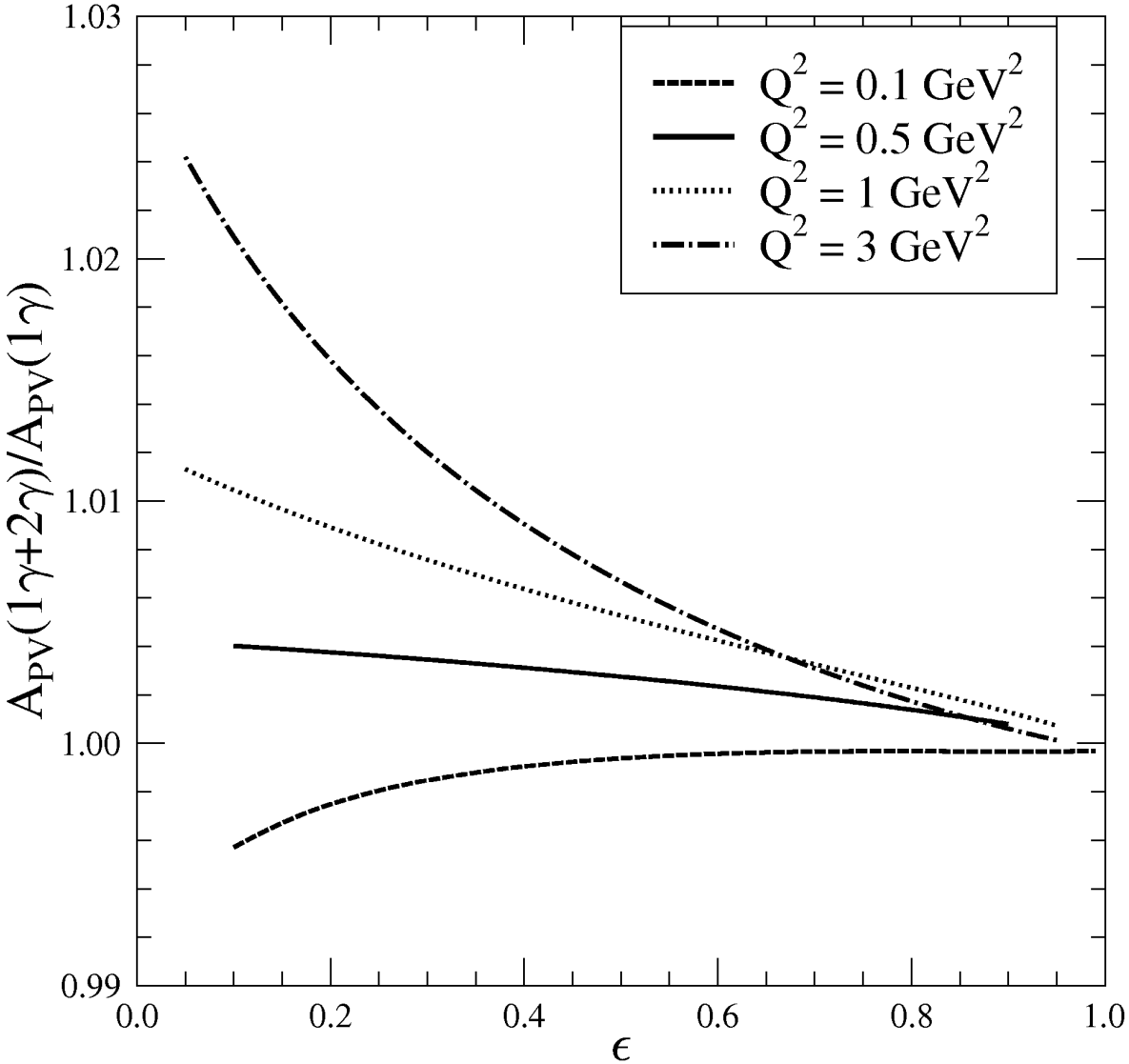
$$\mathcal{A}_{PV} = -\frac{G_F Q^2}{e^2\sqrt{2}} \frac{g_A^e\left(\epsilon G'_E G_E^Z + \tau G'_M G_M^Z + \epsilon' G'_A G_M^Z\right) + g_V^e\left(\epsilon' G_M + (1+\tau) G'_A\right) G_A^Z}{\epsilon G'^2_E + \tau G'^2_M}$$

$$1 + \delta = \frac{(1 + \delta_{Z(\gamma\gamma)})}{(1 + \delta_{\gamma(\gamma\gamma)})} \approx 1 + \delta_{Z(\gamma\gamma)} - \delta_{\gamma(\gamma\gamma)}$$

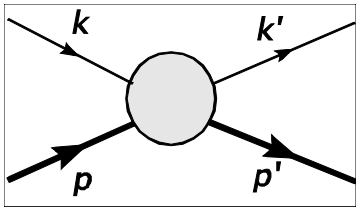
$$\to 1 - \delta_{\gamma(\gamma\gamma)}/2$$

at backward angles where G_M dominates

A_{PV} vs. ε for Q^2 = 0.1, 0.5, 1.0, 3.0 GeV^2 (TPE only)



γZ phenomenology: Generalized form factors



$$T = (\gamma_\mu\gamma_5)^{(e)} \otimes \left(\tilde{F}_1\gamma^\mu + i\tilde{F}_2\frac{\sigma^{\mu\nu}q_\nu}{2M}\right)^{(p)} + (\gamma_\mu)^{(e)} \otimes \left(\tilde{G}_A\gamma^\mu\gamma_5\right)^{(p)}$$

In general, 16 independent amplitudes:

parity NC 16 → 8; time reversal 8 → 6; helicity conservation (m_e=0) 6 → 3

Generalized (complex) form factors

$$\begin{aligned}
\tilde{F}_1(\nu,Q^2) &= F_1^Z(Q^2) + \delta\tilde{F}_1(\nu,Q^2) \\
\tilde{F}_2(\nu,Q^2) &= F_2^Z(Q^2) + \delta\tilde{F}_2(\nu,Q^2) \\
\tilde{G}_A(\nu,Q^2) &= G_A^Z(Q^2) + \delta\tilde{G}_A(\nu,Q^2)
\end{aligned}$$

At $Q^2 = 0$ only 2 needed: related to C_1^p and C_2^p of Marciano-Sirlin

No new terms arise in Afanasev-Carlson expression

One boson exchange

$$\mathcal{A}_{PV} = -\frac{G_F Q^2}{e^2\sqrt{2}} \frac{g_A^e\left(\epsilon G_E\, G_E^Z + \tau G_M\, G_M^Z \qquad\qquad\right) + g_V^e\left(\epsilon' G_M \qquad\qquad\right) G_A^Z}{\epsilon G_E^2 + \tau G_M^2}$$

+ γγ exchange $G \to G'$

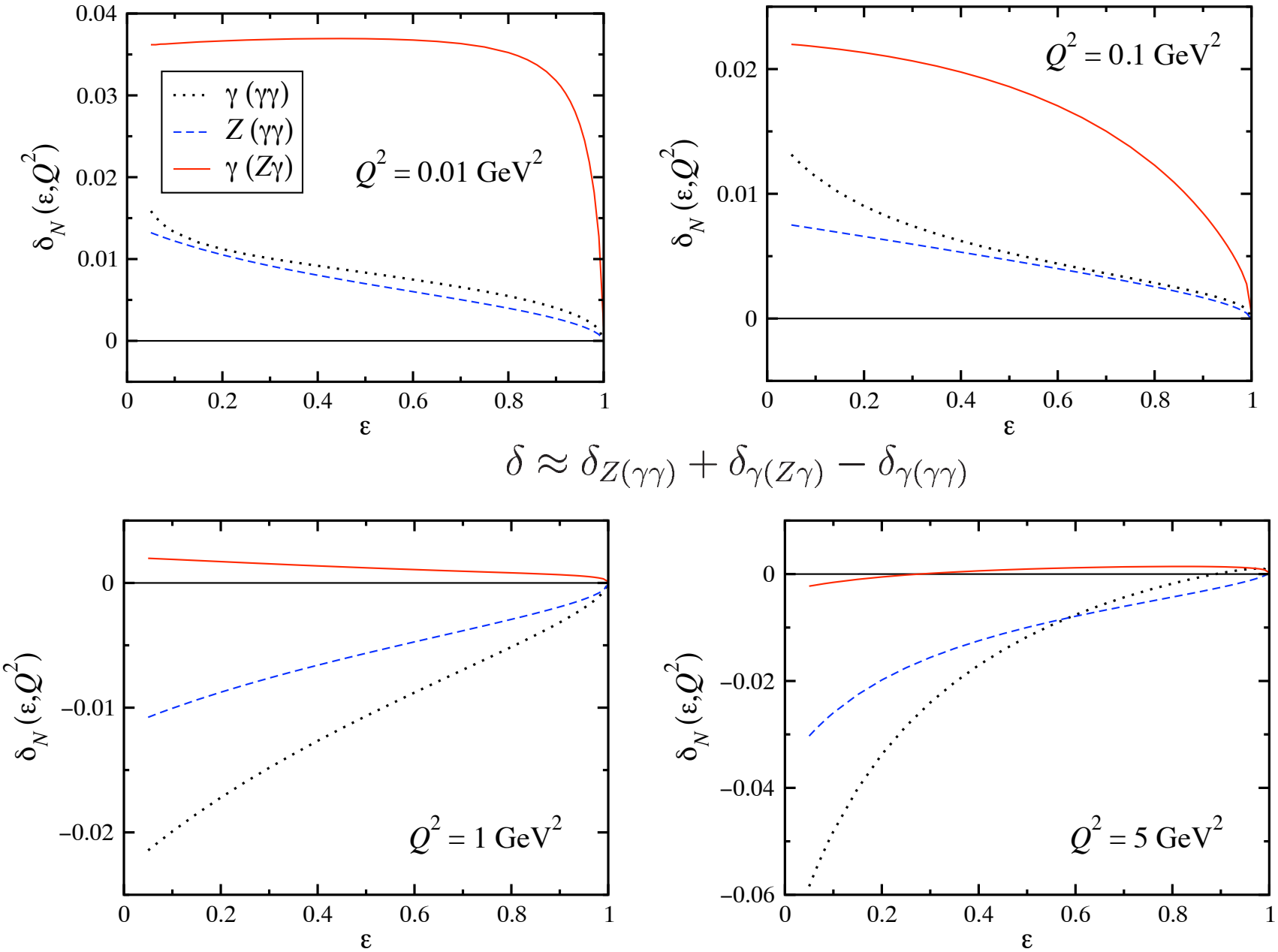
$$\mathcal{A}_{PV} = -\frac{G_F Q^2}{e^2\sqrt{2}} \frac{g_A^e\left(\epsilon G'_E G_E^Z + \tau G'_M G_M^Z + \epsilon' G'_A G_M^Z\right) + g_V^e\left(\epsilon' G_M + (1+\tau) G'_A\right) G_A^Z}{\epsilon G_E'^2 + \tau G_M'^2}$$

+ γZ exchange $G^Z \to \tilde{G}^Z$

$$\mathcal{A}_{PV} = -\frac{G_F Q^2}{e^2\sqrt{2}} \frac{g_A^e\left(\epsilon G'_E \tilde{G}_E^Z + \tau G'_M \tilde{G}_M^Z + \epsilon' G'_A G_M^Z\right) + g_V^e\left(\epsilon' G_M + (1+\tau) G'_A\right) \tilde{G}_A^Z}{\epsilon G_E'^2 + \tau G_M'^2}$$

$$1 + \delta = \frac{\left(1 + \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)}\right)}{\left(1 + \delta_{\gamma(\gamma\gamma)}\right)} \approx 1 + \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)} - \delta_{\gamma(\gamma\gamma)}$$

Tjon, Blunden & Melnitchouk, PRC (2009)



$$\delta \approx \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)} - \delta_{\gamma(\gamma\gamma)}$$

Marciano-Sirlin (PV in atoms)

$$H = \frac{G_F}{2\sqrt{2}}\left(C_1^p \bar{u}_e \gamma_\mu \gamma_5 u_e \bar{U}_p \gamma^\mu U \quad + \quad C_2^p \bar{u}_e \gamma_\mu u_e \bar{U}_p \gamma^\mu \gamma_5 U_p\right)$$

$$C_{1p} = \frac{1}{2}\rho(1 - 4\kappa s_w^2) + \frac{5}{2}\Delta$$

$$= \frac{1}{2}\rho'(1 - 4\kappa' s_w^2)$$

Perturbative (free quark result) $\Delta_{\text{quark}} = \frac{\alpha}{2\pi}(1 - 4s^2)\left(\ln\frac{M_Z^2}{\mu^2} + \frac{3}{2}\right)$

Nonperturbative

$$\Delta = \frac{\alpha}{2\pi}(1 - 4s^2)\left(K + \frac{4}{5}(\xi_1)_B^p\right)$$

$$K = M_Z^2 \int_{\mu^2}^{\infty} \frac{du}{u(u + M_Z^2)}\left[1 - \frac{\alpha_s(u)}{\pi}\right]$$

$K = 8.58$ for $\mu = 1$ GeV, and $(\xi_1)_B^p = 2.55$ using dipole proton form factors, showing that the quark contribution dominates.

Delta resonance contribution

Vector coupling

CVC and isospin symmetry relate $\gamma N\Delta$ to $ZN\Delta$ form factors

$$g_i^V = 2(1-2s_w^2)g_i$$

For N: $g_i^Z = 2(1-2s_w^2)g_i^{(1)} - 2s_w^2 g_i^{(0)} = (1-4s_w^2)g_i^p - g_i^n$

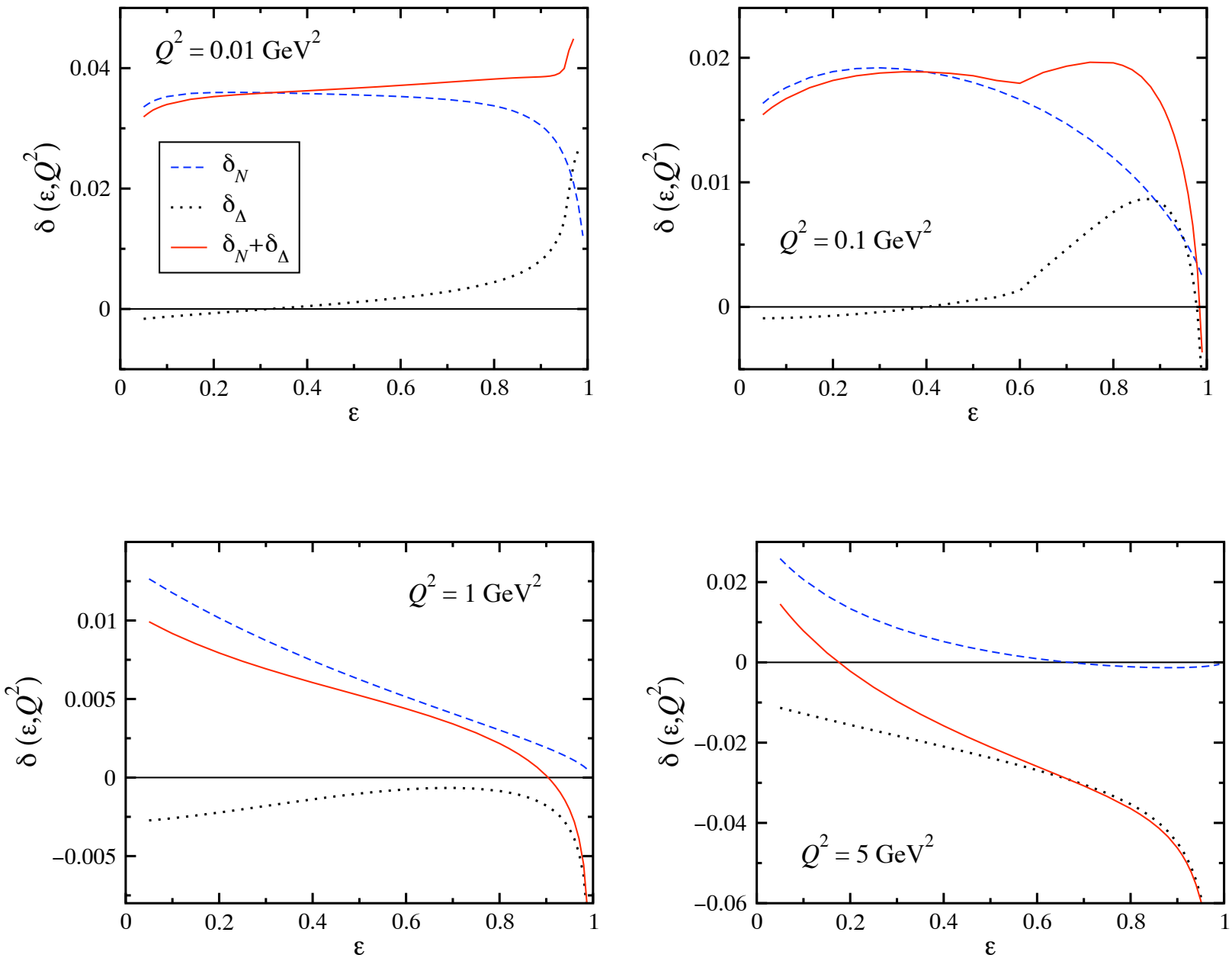
Axial vector coupling

Take from neutrino scattering parametrization of Lalakulich & Paschos

$\delta_{\gamma(\gamma Z)}$ Δ contribution enhanced at forward angles and low Q^2

enhancement: $g_A^e 2(1-2s_w^2)/(1-4s_w^2) = (1+Q_w^p)/Q_w^p \approx 14$

Nucleon and Delta contribution



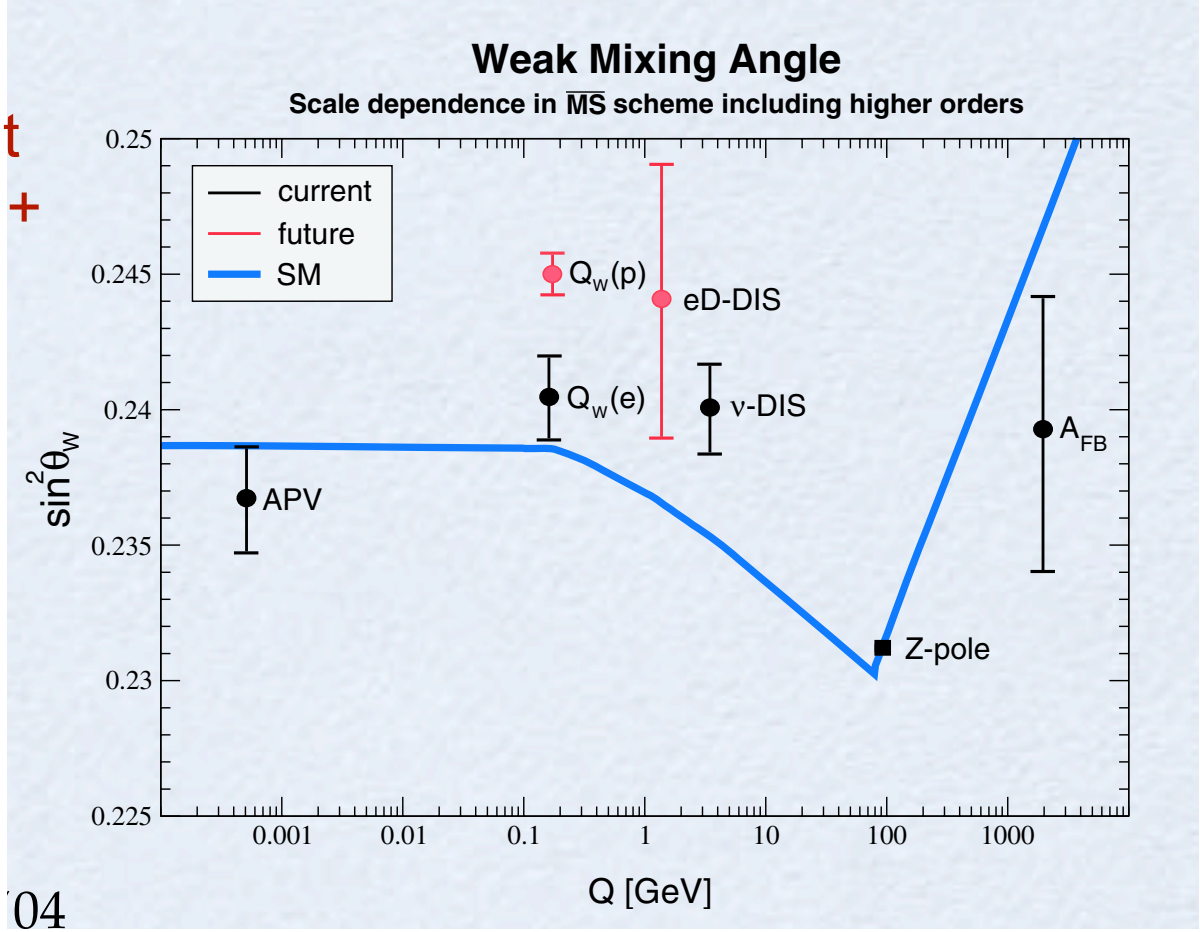
Weak charge of the proton: $Q_W^p = 1 - 4\sin^2\theta_W \approx 0.072$

$$\sin^2\theta_W(M_Z^2) = 0.23113 \pm 0.00015$$ (PDG)

$$\sin^2\theta_W(0) = 0.23807 \pm 0.00017$$ (Erler et al., 2004)

Qweak: 4% measurement of weak charge (2% expt + 2% theory)

0.3% measurement of weak mixing angle



$$A^{\rm PV} \equiv \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \to \frac{G_F}{4\pi\alpha\sqrt{2}}\, t\, Q_W^p$$

Measure at forward angles where $t \to 0$

- At tree level proton's weak charge given by

$$Q_{\rm w}^p = 1 - 4\sin^2\theta_W$$

- At higher orders $Q_{\rm w}^p$ receives corrections from electroweak quantum fluctuations

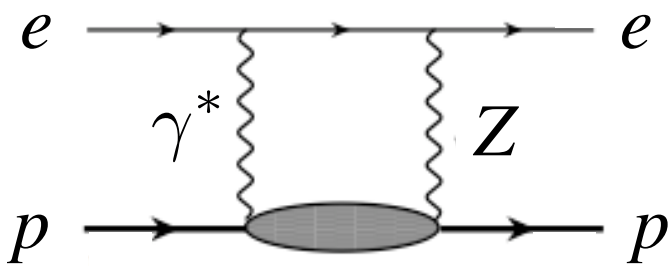
$$\begin{aligned} Q_W^p &= (1 + \Delta\rho + \Delta_e)(1 - 4\sin^2\theta_W(0) + \Delta_e') \\ &\quad + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}(0) \end{aligned}$$

$$Q_W^p = 0.0713(8)$$

Allow for E-dependence

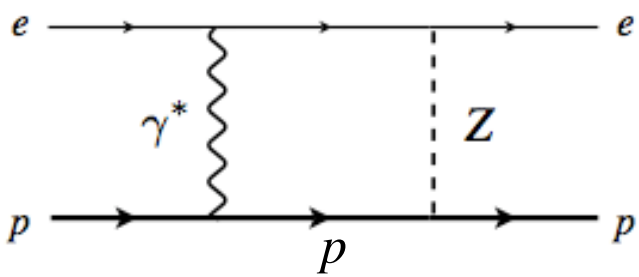
- QED and short-distance corrections under control

  $\longrightarrow$ most uncertain is γZ box contribution, sensitive to *long-distance* physics



- First estimates of γZ boxes by Marciano & Sirlin (MS) in atomic parity violation (APV)

  $\longrightarrow$ low-energy part approximated by Born contribution (elastic intermediate state)



  $\longrightarrow$ high-energy part (above scale $\Lambda \sim 1\,\mathrm{GeV}$) computed in terms of scattering from free quarks

Marciano, Sirlin, PRD ***29****, 75 (1984)*

Two parity-violating contributions

$$\Box_{\gamma Z}(E) = \Box^{A}_{\gamma Z}(E) + \Box^{V}_{\gamma Z}(E)$$

$\boxed{V_e \times A_h}$ $\longrightarrow$ computed by MS $\quad \Box^{A}_{\gamma Z}(E)$

$$\delta^{(\gamma Z)} Q^p_{\mathrm{w}} = \frac{5\alpha}{2\pi}(1 - 4\sin^2\theta_W)\left[\ln\frac{M_Z^2}{\Lambda^2} + C_{\gamma Z}(\Lambda)\right]$$

high-energy $\quad$ low-energy

$\longrightarrow$ estimated uncertainty $\Delta Q^p_{\mathrm{w}} = 0.65\%$

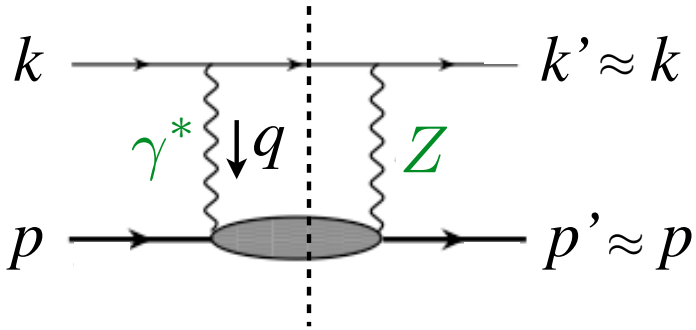
Erler et al., PRD ***68****, 016006 (2003)*

$\boxed{A_e \times V_h}$ $\longrightarrow$ small at low E_e $\quad \Box^{V}_{\gamma Z}(E)$

$\longrightarrow$ neglected in APV, but is it small at GeV energies?

- A_e x V_h term recently computed in forward limit within dispersion relation (DR) approach

Gorchtein, Horowitz, PRL ***102****, 091806 (2009)*



forward limit

$$t = (k - k')^2 \to 0$$

$$s = (k + p)^2 = M(M + 2E)$$

$$\Re e\, \Box^V_{\gamma Z}(E) = \frac{1}{\pi} P \int_{-\infty}^{\infty} dE' \frac{\Im m\, \Box^V_{\gamma Z}(E')}{E' - E}$$

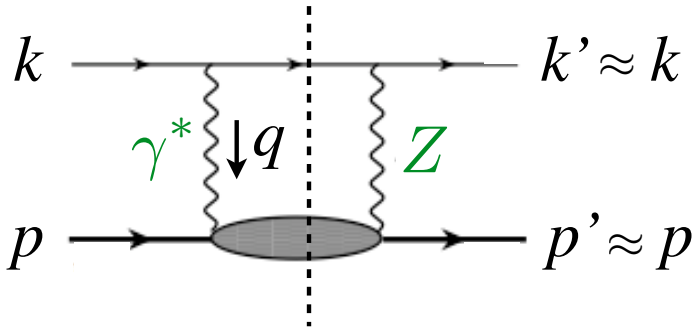
Negative energy corresponds to crossed box

By optical theorem

$$2\Im m\, \mathcal{M}_{fi} = \int d\rho \sum_n \mathcal{M}^*_{nf} \mathcal{M}_{ni}.$$

- A_e x V_h term recently computed in forward limit within dispersion relation (DR) approach

Gorchtein, Horowitz, PRL ***102****, 091806 (2009)*



forward limit

$$t = (k - k')^2 \to 0$$

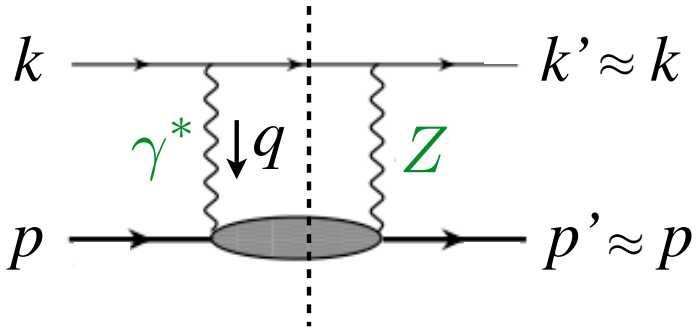
$$s = (k + p)^2 = M(M + 2E)$$

$$2\,\Im m\,\mathcal{M}^{\rm PV}_{\gamma Z} = 4\pi M \int \frac{d^3k'}{(2\pi)^3 2E_{k'}} \left(\frac{4\pi\alpha}{Q^2}\right)\left(\frac{-2G_F}{\sqrt{2}}\right) \times \frac{1}{1+Q^2/M_Z^2} L^{\gamma Z}_{\mu\nu} W^{\mu\nu}_{\gamma Z}$$

$$W^{\mu\nu}_{\gamma Z} = \frac{1}{4\pi M}\int d^4x\, e^{iq\cdot x} \times \langle p \,|\, [J^\mu_\gamma(x) J^\nu_Z(0) + J^\mu_Z(x) J^\nu_\gamma(0)] \,|\, p\rangle = \frac{1}{M}\left[-F_1^{\gamma Z} g^{\mu\nu} + F_2^{\gamma Z}\frac{p^\mu p^\nu}{p\cdot q}\right]$$

- A_e x V_h term recently computed in forward limit within dispersion relation (DR) approach

Gorchtein, Horowitz, PRL ***102****, 091806 (2009)*



forward limit

$$t = (k - k')^2 \to 0$$

$$s = (k + p)^2 = M(M + 2E)$$

$$\Im m \, \Box^V_{\gamma Z}(E) = \frac{\alpha}{(s - M^2)^2} \int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{\max}} \frac{dQ^2}{1 + Q^2/M_Z^2} \times \left[F_1^{\gamma Z} + F_2^{\gamma Z} \frac{s\left(Q^2_{\max} - Q^2\right)}{Q^2\left(W^2 - M^2 + Q^2\right)} \right]$$

Check: elastic (nucleon) intermediate state

$$
\begin{aligned}
F_1^{\gamma Z} &= 2M^2 \tau G_M G_M^Z \delta(W^2 - M^2) \\
F_2^{\gamma Z} &= 4M^2 \tau \frac{G_E G_E^Z \tau G_M G_M^Z}{1+\tau} \delta(W^2 - M^2) \\
F_3^{\gamma Z} &= -2M^2 \tau G_M G_A^Z \delta(W^2 - M^2)
\end{aligned}
$$

- Gives same result as direct loop calculation
  - Standard MS result in E=0 limit

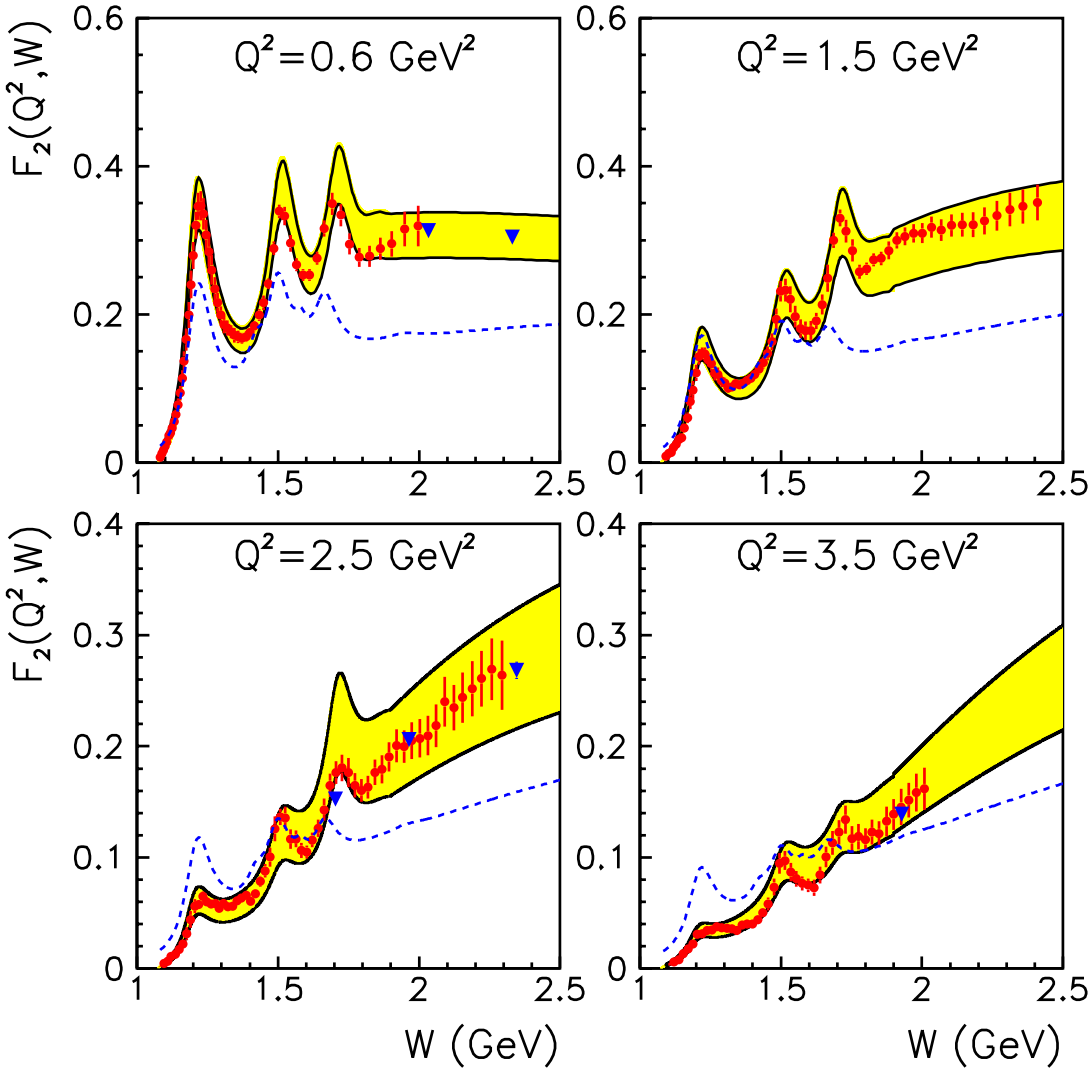
Sibirtsev, Blunden, Melnitchouk & Thomas (submitted to PRD)

Resonance region (W<2.5 GeV)

- For isospin 3/2 states, CVC and isospin symmetry imply

$$F^{\gamma Z} = (1 + Q_W^p)\, F^{\gamma}$$

- For isospin 1/2 states, transition couplings with few percent
- Use phenomenological input
- Take P33 (1232), D13 (1520), F15(1680), F37 (1950) plus background



Deep inelastic region

- Approximate interference structure functions

$$F_{1,2}^{\gamma Z} \approx F_{1,2}^{\gamma} \equiv F_{1,2}$$

$$F_2^{\gamma Z} = x \sum_q 2 e_q g_V^q \, (q + \bar{q})$$

$$F_2^{\gamma} = x \sum_q e_q^2 \, (q + \bar{q})$$

$\longrightarrow$ good approximation at low x

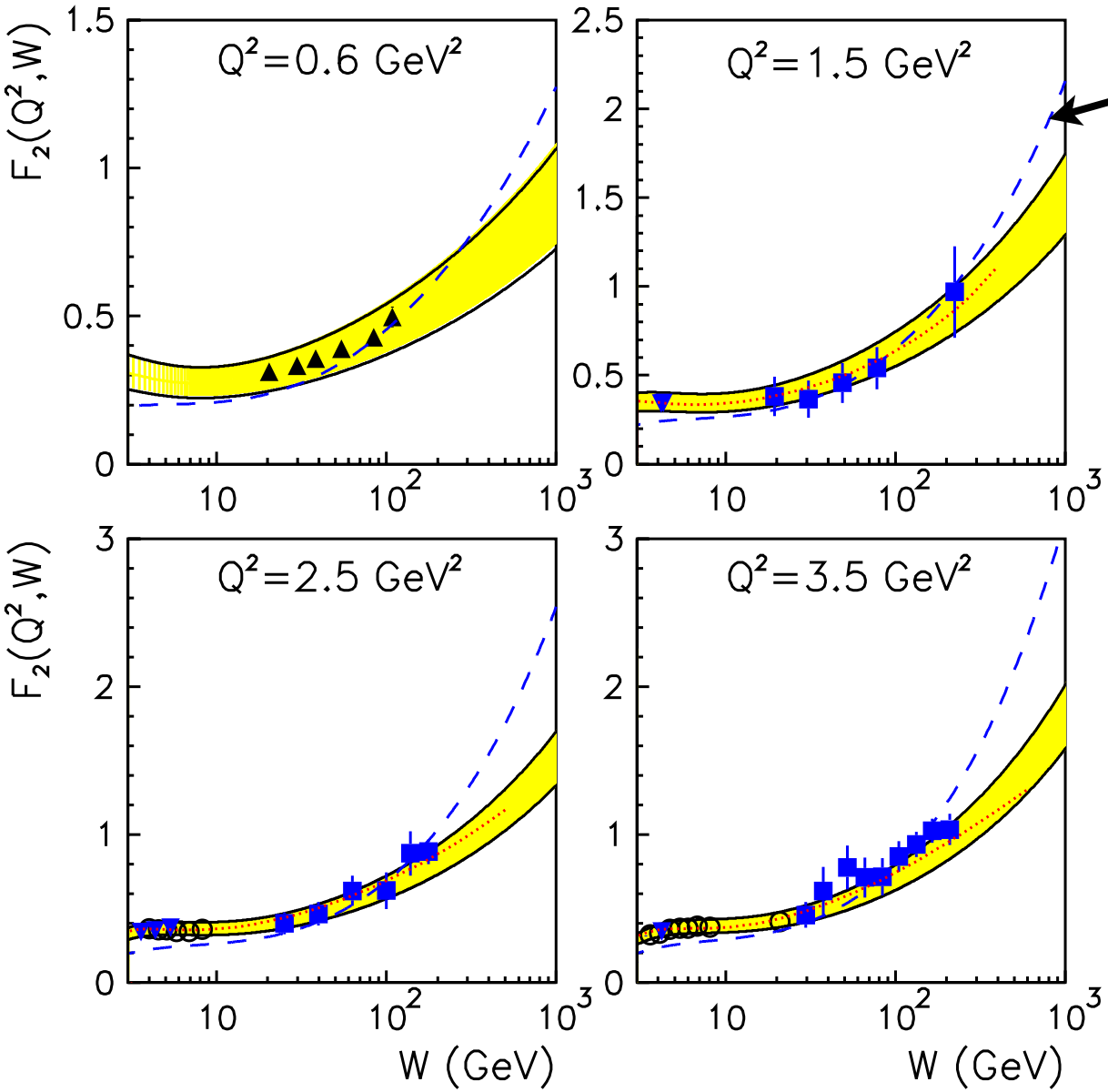
$\longrightarrow$ provides upper limit at larger x

- F_2 parametrization motivated by Regge theory; valid at both low and high Q^2
- Pomeron (related to sea quark), and Reggeon (related to valence quark) components

$$F_2(x, Q^2) = A_P \, x^{-\Delta} (1-x)^{n+4} \left[\frac{Q^2}{Q^2 + \Lambda_P^2} \right]^{1+\Delta} + A_R \, x^{1-\alpha_R} (1-x)^n \left[\frac{Q^2}{Q^2 + \Lambda_R^2} \right]^{\alpha_R}$$

Integral dominated by low x and F_2

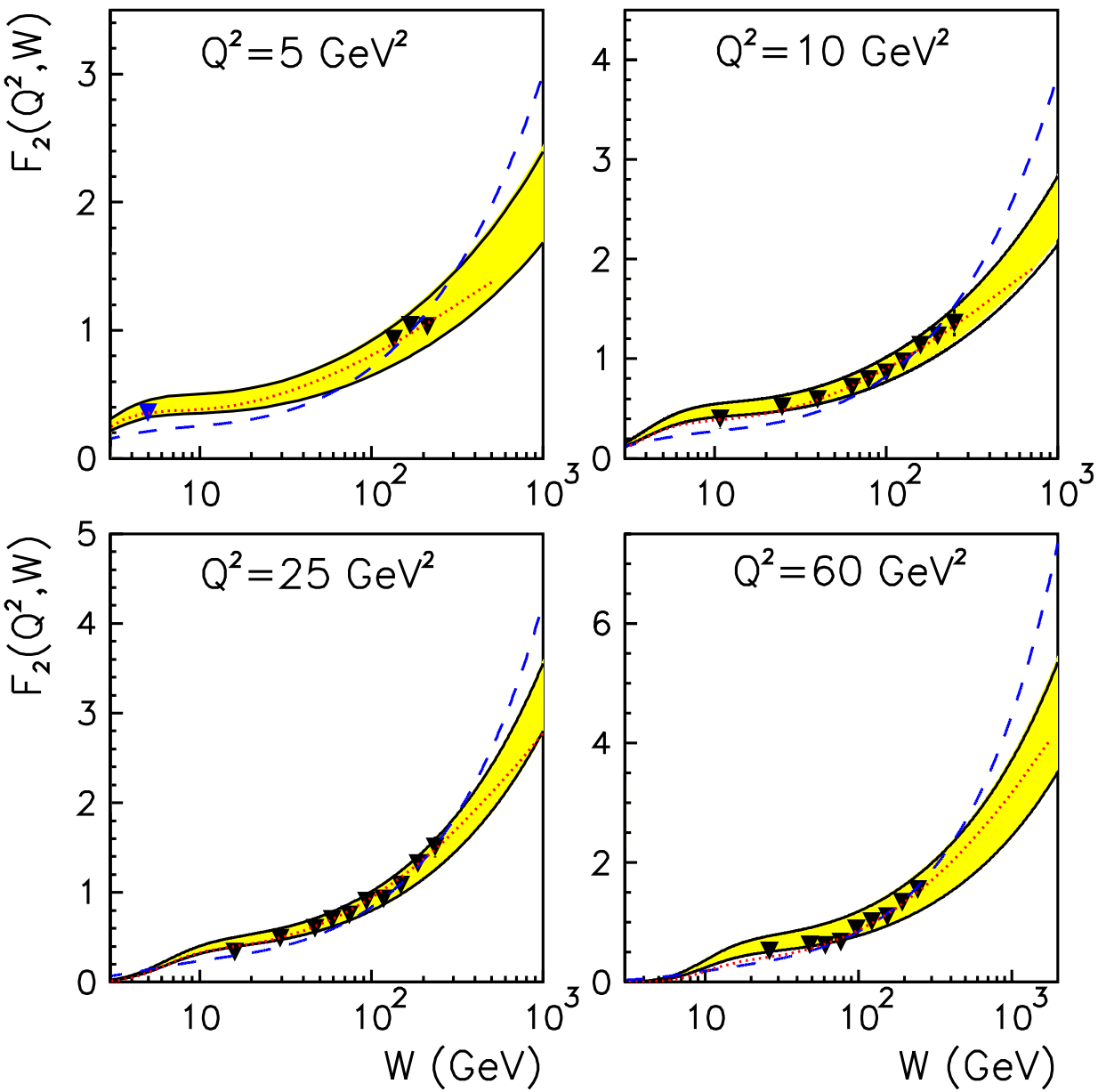
Large W, low Q^2



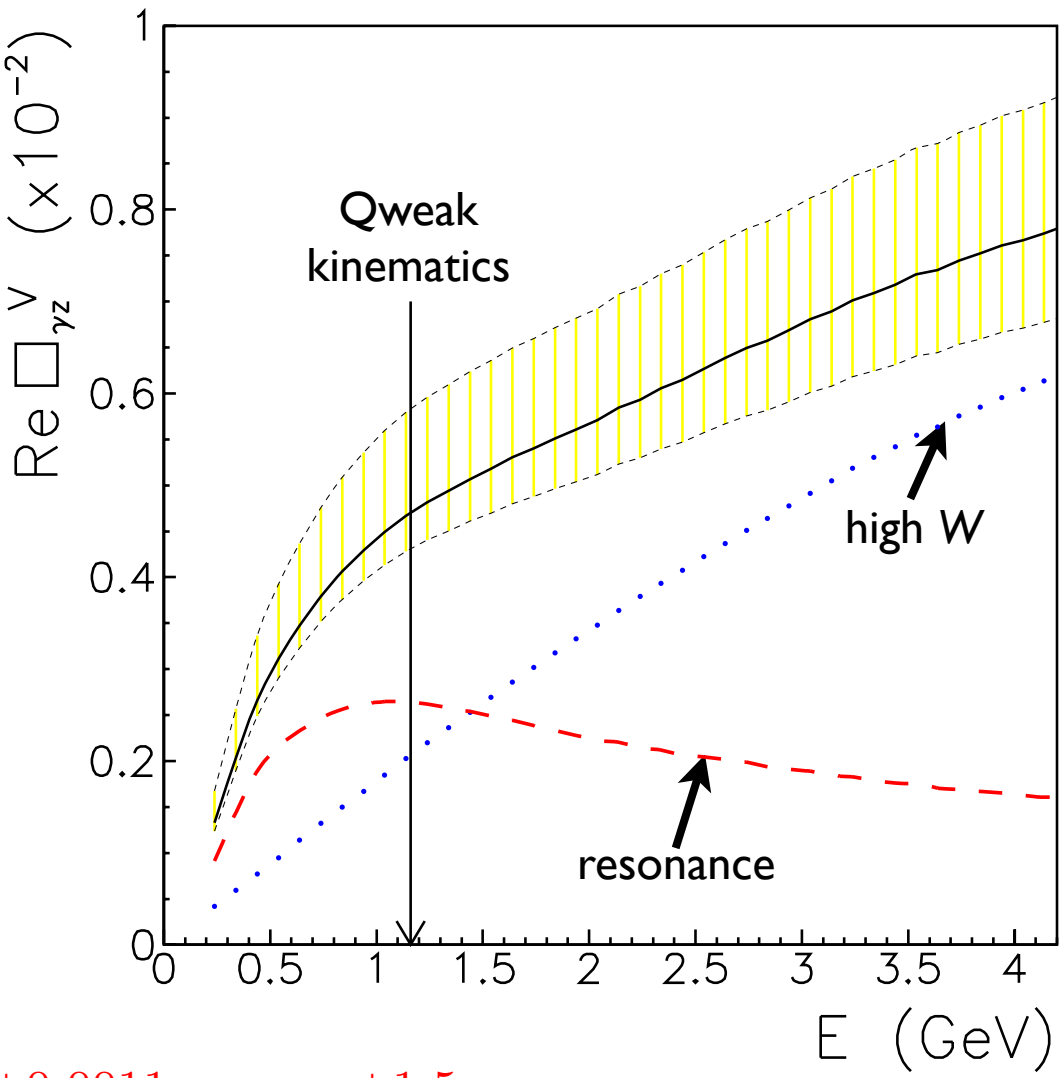
Cvetic et al, EPJC ***20****, 77 (2001)*

used as input in GH model

Large W, large Q^2



Performing dispersion integral: real part of correction



$0.0047^{+0.0011}_{-0.0004}$ or $6.6^{+1.5}_{-0.5}\%$

Summary

- Qweak correction large, but uncertainty under control
- Uncertainty in Qweak may be reduced further with measurements of γZ interference structure functions in PVDIS
- 50% larger than GH result
- Dispersion relations that use cross section data are useful at forward angles, however still need for models to extrapolate (not all data is available, e.g. γZ interference, axial part)

Collaborators: Sibirtsev, Melnitchouk, Thomas; Tjon, Kondratyuk