Two-boson ($\gamma\gamma + \gamma Z$) exchange corrections and parity-violating electron scattering

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Proton $G_E/G_M$ Ratio

\[ \sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2) \]

\[ \frac{G_E}{G_M} = -\sqrt{\frac{\tau(1 + \varepsilon)}{2\varepsilon}} \frac{P_T}{P_L} \]

**LT method**

- $G_E$ from slope in $\varepsilon$ plot
- suppressed at large $Q^2$

**PT method**

- $P_{T,L}$ recoil proton polarization in $\vec{e} p \rightarrow e \vec{p}$
Correction relative to OPE

\[ \delta_{2\gamma} = \frac{2 \Re \left\{ M_{\gamma}^\dagger M_{2\gamma} \right\}}{|M_{\gamma}|^2} \]
Various Approaches

Rely on Models

• Low to moderate $Q^2$: hadronic: $N + \Delta + N^*$ etc.
  • more and more parameters, less and less reliable
• Moderate to high $Q^2$:
  • GPD approach: assumption of 1 active quark
    • Valid only in certain kinematic range
  • pQCD: recent work indicates 2 active quarks dominate

Rely on data

• Use dispersion integrals to relate Real and Imaginary parts. Imaginary parts fixed by cross section data
  • Valid at forward angles: must use models to extrapolate
  • Incomplete: not all data is available (e.g. axial hadron coupling and isospin dependence in $\gamma Z$ diagrams)
 Corrections to unpolarized cross sections for $Q^2=1$ to $6$ GeV$^2$

Effect largest at small $\varepsilon$ (backward angles)
Vanishes as $\varepsilon \to 1$
Nonlinearity grows with $Q^2$
POSITIVE slope for $Q^2 > 0.5$ GeV$^2$
NEGATIVE slope for $Q^2 < 0.5$ GeV$^2$ (pointlike limit)
Effect on SLAC reduced cross sections at different $Q^2$

(normalized to dipole $G_D^2$)

Nonlinearity in $\varepsilon$ is displayed here

JLAB experiments to measure nonlinearity
SuperRosenbluth (JLAB) data

Curves shifted by
+1.0% 2.64
+2.1% 3.20
+3.0% 4.10

(Effect on determination of $G_M$)
Very preliminary Novosibirsk data

e\textsuperscript{+}-p/e\textsuperscript{-} p cross section ratio

\[ \frac{\sigma_{e^+}}{\sigma_{e^-}} \]

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig.png}
\caption{\(Q^2=3.0 \text{ GeV}^2\)}
\end{figure}


----- ANL, BINP, INP TPU, NIKHEF
Recent pQCD calculation: Borisyuk & Kobushkin, PRD 79, 2009

(a) one-photon exchange: need 2 hard gluons to turn momentum of all 3 quarks

\[ \alpha \alpha_s^2 / Q^6 \]

(b) two-photon exchange:
leading order needs 1 hard gluon

\[ \alpha^2 \alpha_s / Q^6 \quad \text{TPE/OPE} \sim \alpha / \alpha_s \]

subleading order (both photons on one quark) requires 2 hard gluons
Comparison of hadronic and pQCD results

Connect smoothly around $Q^2 = 3 \text{ GeV}^2$
Lepton-antilepton photoproduction using real photons
(Pervez Hoodbhoy, PRD 2006)

TWO-PHOTON EFFECTS IN LEPTON-ANTILEPTON ...

FIG. 6. Typical diagrams for lepton pair production from a 3-quark proton.

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FIG. 7. Lepton pair asymmetry from a proton target.
Parity-violating electron scattering

right-left polarization asymmetry in $\vec{e} + p \rightarrow e + p$ scattering

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = \frac{2\Re\{M_γ^\dagger M_Z\}}{|M_γ|^2}$$

$$j_{\mu}^Z = \bar{u} (g_V^e γ_\mu + g_A^e γ_\mu γ_5) u$$

$$J_{\mu}^Z(q) = \bar{U} \left( F_1^Z γ_\mu + F_2^Z i \frac{σ^{μν} q_ν}{2M} + G_A^Z γ_μ γ_5 \right) U$$

$$g_V^e = -(1 - 4s_W^2)$$
$$g_A^e = +1$$
$$F_i^Z = (1 - 4s_W^2) F_i^p - F_i^n - F_i^s$$
$$G_A^Z = -G_A τ_3 + G_A^s$$

using relation between weak and EM form factors
One boson exchange

\[ \mathcal{A}_{PV} = - \frac{G_F Q^2}{e^2 \sqrt{2}} g_a^e \left( \varepsilon G_E G_E^Z + \tau G_M G_M^Z \right) + g_V^e \left( \varepsilon' G_M \right) G_A^Z \]

\[ G_{E,M}^Z = (1 - 4 \sin^2 \theta_W) G_{E,M} - G_{E,M}^n \]
Two-boson exchange corrections

Electromagnetic radiative corrections interfere with $M_Z (M_\gamma \rightarrow M_\gamma + M_{\gamma\gamma})$

plus weak radiative corrections interfere with $M_\gamma (M_Z \rightarrow M_Z + M_{\gamma Z})$

plus two-photon exchange "$\gamma(\gamma\gamma)$” in denominator

Zhou, Kao & Yang, PRL 2007; Tjon & Melnitchouk, PRL 2008; Tjon, Melnitchouk & Blunden, PRC 2009
\[ T = (\gamma_{\mu})^{(e)} \otimes \left( F_1' \gamma_{\mu} + i \frac{F_2'}{2M} \sigma^{\mu\nu} q_{\nu} \right)^{(p)} + (\gamma_{\mu} \gamma_5)^{(e)} \otimes (G_A' \gamma_{\mu} \gamma_5)^{(p)} \]

In general, 16 independent amplitudes:

parity 16 \rightarrow 8; \text{ time reversal } 8 \rightarrow 6; \text{ helicity conservation } (m_e=0) 6 \rightarrow 3

**Generalized (complex) form factors**

\[
\begin{align*}
F_1'(\nu, Q^2) &= F_1(Q^2) + \delta F_1'(\nu, Q^2) & \nu = k \cdot p + q^2 / 4 \\
F_2'(\nu, Q^2) &= F_2(Q^2) + \delta F_2'(\nu, Q^2) \\
G_A'(\nu, Q^2) &
\end{align*}
\]

Dispersion relations \[ \Re F_i' \leftrightarrow \Im F_i' \]
One boson exchange

\[ A_{PV} = -\frac{G_F Q^2}{e^2 \sqrt{2}} \left( g_A^e \left( \epsilon G_E G_E^Z + \tau G_M G_M^Z \right) + g_V^e \left( \epsilon' G_M \right) \right) \frac{\epsilon G_E^2 + \tau G_M^2}{\epsilon G_E^2 + \tau G_M^2} \]

+ $\gamma\gamma$ exchange \hspace{1cm} $G \rightarrow G'$ \hspace{1cm} Afanasev and Carlson (PRL 2005)

\[ A_{PV} = -\frac{G_F Q^2}{e^2 \sqrt{2}} \left( g_A^e \left( \epsilon G_E^\prime G_E^Z + \tau G_M^\prime G_M^Z + \epsilon A G_A^Z \right) + g_V^e \left( \epsilon' G_M + (1 + \tau) G_A^\prime \right) \right) \frac{\epsilon G_E^\prime 2 + \tau G_M^\prime 2}{\epsilon G_E^\prime 2 + \tau G_M^\prime 2} \]

\[ 1 + \delta = \frac{(1 + \delta_{Z(\gamma\gamma)})}{(1 + \delta_{\gamma(\gamma\gamma)})} \approx 1 + \delta_{Z(\gamma\gamma)} - \delta_{\gamma(\gamma\gamma)} \]

\[ \rightarrow 1 - \delta_{\gamma(\gamma\gamma)}/2 \] \hspace{1cm} at backward angles where $G_M$ dominates
$A_{PV}$ vs. $\varepsilon$ for $Q^2 = 0.1, 0.5, 1.0, 3.0 \text{ GeV}^2$ (TPE only)
\[ T = (\gamma_\mu \gamma_5)^{(e)} \otimes \left( \tilde{F}_1 \gamma^\mu + i \tilde{F}_2 \frac{\sigma^{\mu\nu} q_\nu}{2M} \right)^{(p)} \]

\[ + (\gamma_\mu)^{(e)} \otimes (\tilde{G}_A \gamma^\mu \gamma_5)^{(p)} \]

In general, 16 independent amplitudes:

parity NC 16 \rightarrow 8; time reversal 8 \rightarrow 6; helicity conservation \((m_e=0)\) 6 \rightarrow 3

**Generalized (complex) form factors**

\[ \tilde{F}_1(\nu, Q^2) = F_1^Z(Q^2) + \delta \tilde{F}_1(\nu, Q^2) \]

\[ \tilde{F}_2(\nu, Q^2) = F_2^Z(Q^2) + \delta \tilde{F}_2(\nu, Q^2) \]

\[ \tilde{G}_A(\nu, Q^2) = G_A^Z(Q^2) + \delta \tilde{G}_A(\nu, Q^2) \]

At \(Q^2 = 0\) only 2 needed: related to \(C_1^p\) and \(C_2^p\) of Marciano-Sirlin

No new terms arise in Afanasev-Carlson expression
One boson exchange

\[ A_{PV} = \left(-\frac{G_F Q^2}{e^2 \sqrt{2}}\right) \frac{g^e_A \left(\epsilon G_E G^Z_E + \tau G_M G^Z_M\right)}{\epsilon G^2_E + \tau G^2_M} \] + \gamma\gamma \text{ exchange} \quad G \rightarrow G'

\[ A_{PV} = \left(-\frac{G_F Q^2}{e^2 \sqrt{2}}\right) \frac{g^e_A \left(\epsilon G'_E G^Z_E + \tau G'_M G^Z_M + \epsilon' G'_A G^Z_M\right) + g^e_V \left(\epsilon' G_M + (1 + \tau) G'_A\right)}{\epsilon G'^2_E + \tau G'^2_M} \] + \gamma Z \text{ exchange} \quad G^Z \rightarrow \tilde{G}^Z

\[ A_{PV} = \left(-\frac{G_F Q^2}{e^2 \sqrt{2}}\right) \frac{g^e_A \left(\epsilon G'_E \tilde{G}^Z_E + \tau G'_M \tilde{G}^Z_M + \epsilon' G'_A \tilde{G}^Z_M\right) + g^e_V \left(\epsilon' G_M + (1 + \tau) G'_A\right) \tilde{G}^Z_A}{\epsilon G'^2_E + \tau G'^2_M} \]

\[ 1 + \delta = \frac{(1 + \delta_Z(\gamma\gamma) + \delta_Z(\gamma Z))}{(1 + \delta_\gamma(\gamma\gamma))} \approx 1 + \delta_Z(\gamma\gamma) + \delta_\gamma(Z\gamma) - \delta_\gamma(\gamma\gamma) \]
\[ \delta \approx \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)} - \delta_{\gamma(\gamma\gamma)} \]
Marciano-Sirlin (PV in atoms)

\[ H = \frac{G_F}{2\sqrt{2}} \left( C_1^p \bar{u}_e \gamma_\mu \gamma_5 u_e \bar{U}_p \gamma^\mu U + C_2^p \bar{u}_e \gamma_\mu u_e \bar{U}_p \gamma^\mu \gamma_5 U_p \right) \]

\[ C_1^p = \frac{1}{2} \rho(1 - 4\kappa s_w^2) + \frac{5}{2} \Delta \]

\[ = \frac{1}{2} \rho'(1 - 4\kappa' s_w^2) \]

Perturbative (free quark result) \[ \Delta_{\text{quark}} = \frac{\alpha}{2\pi} (1 - 4s^2) \left( \ln \frac{M_Z^2}{\mu^2} + \frac{3}{2} \right) \]

Nonperturbative \[ \Delta = \frac{\alpha}{2\pi} (1 - 4s^2) \left( K + \frac{4}{5} (\xi_1)_B^p \right) \]

\[ K = M_Z^2 \int_{\mu^2}^{\infty} \frac{du}{u(u + M_Z^2)} \left[ 1 - \frac{\alpha_s(u)}{\pi} \right] \]

\[ K = 8.58 \text{ for } \mu = 1 \text{ GeV, and } (\xi_1)_B^p = 2.55 \text{ using dipole proton form factors, showing that the quark contribution dominates.} \]
Delta resonance contribution

Vector coupling

CVC and isospin symmetry relate $\gamma N\Delta$ to $ZN\Delta$ form factors

$$g_i^V = 2(1 - 2s_w^2)g_i$$

For N: $g_i^Z = 2(1 - 2s_w^2)g_i^{(1)} - 2s_w^2g_i^{(0)} = (1 - 4s_w^2)g_i^p - g_i^n$

Axial vector coupling

Take from neutrino scattering parametrization of Lalakulich & Paschos

$$\delta_{\gamma(\gamma Z)} \quad \Delta \text{ contribution enhanced at forward angles and low } Q^2$$

enhancement: $$g_A^e 2(1 - 2s_w^2)/(1 - 4s_w^2) = (1 + Q_w^p)/Q_w^p \approx 14$$
Nucleon and Delta contribution

\[ Q^2 = 0.01 \text{ GeV}^2 \]
\[ Q^2 = 0.1 \text{ GeV}^2 \]
\[ Q^2 = 1 \text{ GeV}^2 \]
\[ Q^2 = 5 \text{ GeV}^2 \]
Weak charge of the proton: \( Q_w^p = 1 - 4 \sin^2 \theta_W \approx 0.072 \)

\[
\begin{align*}
\sin^2 \theta_W (M_Z^2) & = 0.23113 \pm 0.00015 \quad \text{(PDG)} \\
\sin^2 \theta_W (0) & = 0.23807 \pm 0.00017 \quad \text{(Erler et al., 2004)}
\end{align*}
\]

\( Q\text{weak} \): 4% measurement of weak charge (2% expt + 2% theory)

0.3% measurement of weak mixing angle
At tree level proton’s weak charge given by

\[ Q^p_W = 1 - 4 \sin^2 \theta_W \]

At higher orders \( Q^p_W \) receives corrections from electroweak quantum fluctuations

\[ Q^p_W = (1 + \Delta \rho + \Delta_e)(1 - 4 \sin^2 \theta_W (0) + \Delta'_e) + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}(0) \]

\[ Q^p_W = 0.0713(8) \]
QED and short-distance corrections under control

most uncertain is $\gamma Z$ box contribution, sensitive to long-distance physics
First estimates of $\gamma Z$ boxes by Marciano & Sirlin (MS) in atomic parity violation (APV)

→ low-energy part approximated by Born contribution (elastic intermediate state)

→ high-energy part (above scale $\Lambda \sim 1\,\text{GeV}$) computed in terms of scattering from free quarks

Two parity-violating contributions

\[ \Box_{\gamma Z}(E) = \Box^A_{\gamma Z}(E) + \Box^V_{\gamma Z}(E) \]

\( V_e \times A_h \rightarrow \) computed by MS \( \Box^A_{\gamma Z}(E) \)

\[ \delta(\gamma Z)Q^p_w = \frac{5\alpha}{2\pi} (1 - 4\sin^2 \theta_W) \left[ \ln \frac{M^2_Z}{\Lambda^2} + C_{\gamma Z}(\Lambda) \right] \]

\( A_e \times V_h \rightarrow \) small at low \( E_e \) \( \Box^V_{\gamma Z}(E) \)

\( \rightarrow \) estimated uncertainty \( \Delta Q^p_w = 0.65\% \)

Erler et al., PRD 68, 016006 (2003)

\( \rightarrow \) neglected in APV, but is it small at GeV energies?
\[ A_e \times V_h \text{ term recently computed in forward limit} \]

within dispersion relation (DR) approach

\[ \Re \Box^V_{\gamma Z}(E) = \frac{1}{\pi} P \int_{-\infty}^{\infty} dE' \frac{\Im m \Box^V_{\gamma Z}(E')}{E' - E} \]

Negative energy corresponds to crossed box

By optical theorem

\[ 2\Im m M_{fi} = \int d\rho \sum_n M_{nf}^* M_{ni}. \]
$A_e \times V_h$ term recently computed in forward limit within dispersion relation (DR) approach

\[ \Gamma^{PV} = 4\pi m \int \frac{d^3 k'}{(2\pi)^3 2E_{k'}} \left( \frac{4\pi \alpha}{Q^2} \right) \left( \frac{-2G_F}{\sqrt{2}} \right) \times \frac{1}{1 + Q^2/M_Z^2} L^{\gamma Z}_{\mu\nu} W^{\mu\nu}_{\gamma Z} \]

\[ W^{\mu\nu}_{\gamma Z} = \frac{1}{4\pi M} \int d^4 x \ e^{ip \cdot x} \times \langle p \left| [ J^\mu_\gamma(x) J^\nu_Z(0) + J^\mu_Z(x) J^\nu_\gamma(0) ] \right| p \rangle \]

\[ = \frac{1}{M} \left[ -F_1^{\gamma Z} g^{\mu\nu} + F_2^{\gamma Z} \frac{p^\mu p^\nu}{p \cdot q} \right] \]

**Gorchtein, Horowitz, PRL 102, 091806 (2009)**
\[ \mathcal{A}_e \times V_h \text{ term recently computed in forward limit} \]

within dispersion relation (DR) approach

\[ k \xrightarrow[\gamma^* q]{} k' \approx k \]
forward limit
\[ t = (k - k')^2 \to 0 \]
s = (k + p)^2
\[ s = M(M + 2E) \]

\[ \text{Gorchtein, Horowitz, PRL 102, 091806 (2009)} \]

\[ \Im m \square^V \gamma Z(E) = \frac{\alpha}{(s - M^2)^2} \int\limits_{W^2}^s dW^2 \int\limits_0^{Q^2_{\text{max}}} \frac{dQ^2}{1 + Q^2/M^2_Z} \]

\[ \times \left[ F^\gamma Z_1 + F^\gamma Z_2 \frac{s}{Q^2} \frac{(Q^2_{\text{max}} - Q^2)}{(W^2 - M^2 + Q^2)} \right] \]
Check: elastic (nucleon) intermediate state

\[
F_1^{\gamma Z} = 2M^2 \tau G_M G_M^Z \delta(W^2 - M^2)
\]

\[
F_2^{\gamma Z} = 4M^2 \tau \frac{G_E G_E^Z \tau G_M G_M^Z}{1 + \tau} \delta(W^2 - M^2)
\]

\[
F_3^{\gamma Z} = -2M^2 \tau G_M G_A^Z \delta(W^2 - M^2)
\]

• Gives same result as direct loop calculation
  • Standard MS result in E=0 limit
Resonance region ($W < 2.5$ GeV)

- For isospin 3/2 states, CVC and isospin symmetry imply
  \[ F_{\gamma Z} = (1 + Q^p_W) F_{\gamma} \]

- For isospin 1/2 states, transition couplings with few percent
- Use phenomenological input
- Take P33 (1232), D13 (1520), F15(1680), F37 (1950) plus background
Deep inelastic region

- Approximate interference structure functions
  \[ F_{1,2}^{\gamma Z} \approx F_{1,2}^{\gamma} = F_{1,2} \]
  \[ F_{2}^{\gamma Z} = x \sum_{q} 2e_{q}g_{V}^{q} (q + \bar{q}) \]
  \[ F_{2}^{\gamma} = x \sum_{q} e_{q}^{2} (q + \bar{q}) \]
  → good approximation at low \( x \)
  → provides upper limit at larger \( x \)

- \( F_{2} \) parametrization motivated by Regge theory; valid at both low and high \( Q^{2} \)
- Pomeron (related to sea quark), and Reggeon (related to valence quark) components

\[
F_{2}(x, Q^{2}) = A_{P} x^{-\Delta} (1 - x)^{n+4} \left[ \frac{Q^{2}}{Q^{2} + \Lambda_{P}^{2}} \right]^{1+\Delta} + A_{R} x^{1-\alpha_{R}} (1 - x)^{n} \left[ \frac{Q^{2}}{Q^{2} + \Lambda_{R}^{2}} \right]^{\alpha_{R}}
\]

Integral dominated by low \( x \) and \( F_{2} \)
Large $W$, low $Q^2$

Approximate interference structure functions provide a good approximation at low $x$ and are used as input in the GH model. Cvetic et al., EPJC 20, 77 (2001) used their results in the fit for $R$. $F_{\gamma Z} \approx F_{\gamma 1,2} \equiv F_{1,2}$.
Large $W$, large $Q^2$
Performing dispersion integral: real part of correction

0.0047$^{+0.0011}_{-0.0004}$ or 6.6$^{+1.5}_{-0.5}$%
Summary

- Qweak correction large, but uncertainty under control
- Uncertainty in Qweak may be reduced further with measurements of $\gamma Z$ interference structure functions in PVDIS
- 50% larger than GH result
- Dispersion relations that use cross section data are useful at forward angles, however still need for models to extrapolate (not all data is available, e.g. $\gamma Z$ interference, axial part)

Collaborators: Sibirtsev, Melnitchouk, Thomas; Tjon, Kondratyuk