PROGRESS IN THE DEVELOPMENT OF A REALISTIC EQUATION OF STATE OF NUCLEAR AND NEUTRON MATTER

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EOS OF HADRONIC MATTER

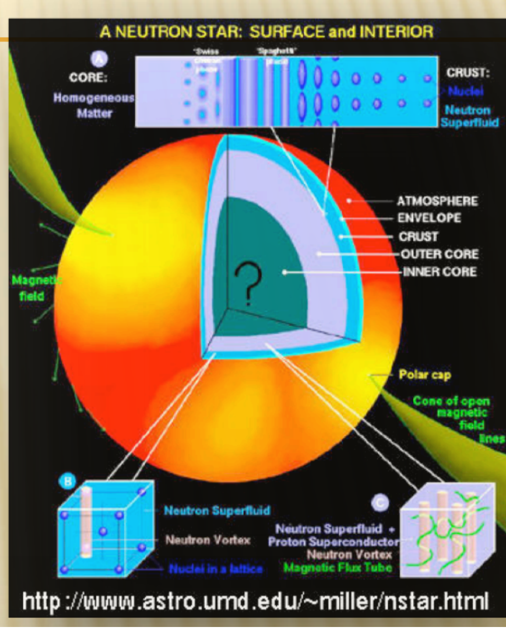
Relevance:

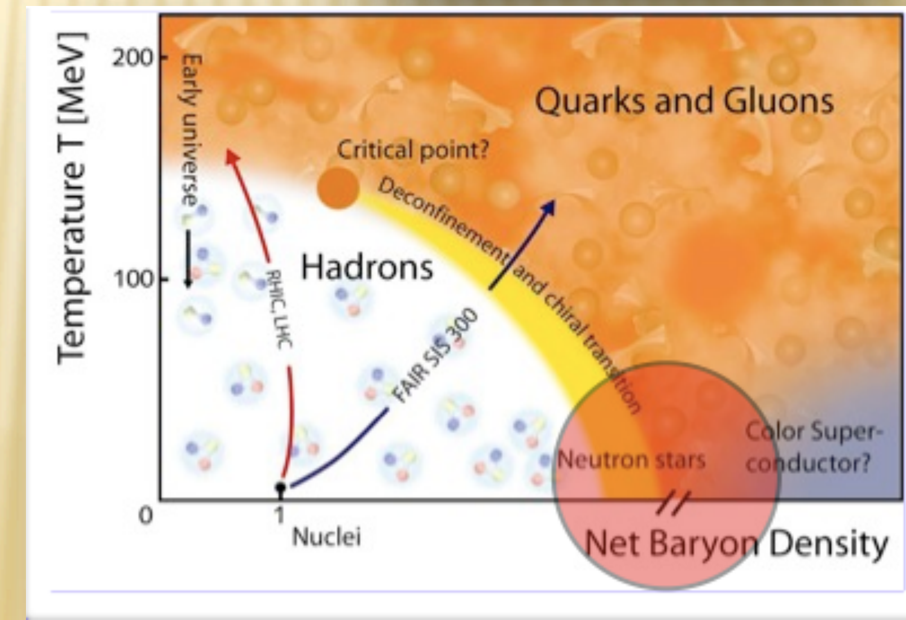
1)PROPERTIES OF COMPACT STARS
2)BENCHMARK OF THE NUCLEAR FORCES
3)PHASE TRANSITIONS

WHAT IS THE ROLE OF MANY-BODY INERACTIONS?

CAN WE IMMEDIATELY EXPORT THE KNOWLEDGE WE HAVE ABOUT NUCLEI TO NUCLEAR MATTER?

WHERE ARE PHASE TRANSITIONS LOCATED IN NUCLEAR MATTER? WHAT IS THE ROLE OF HYPERONS?





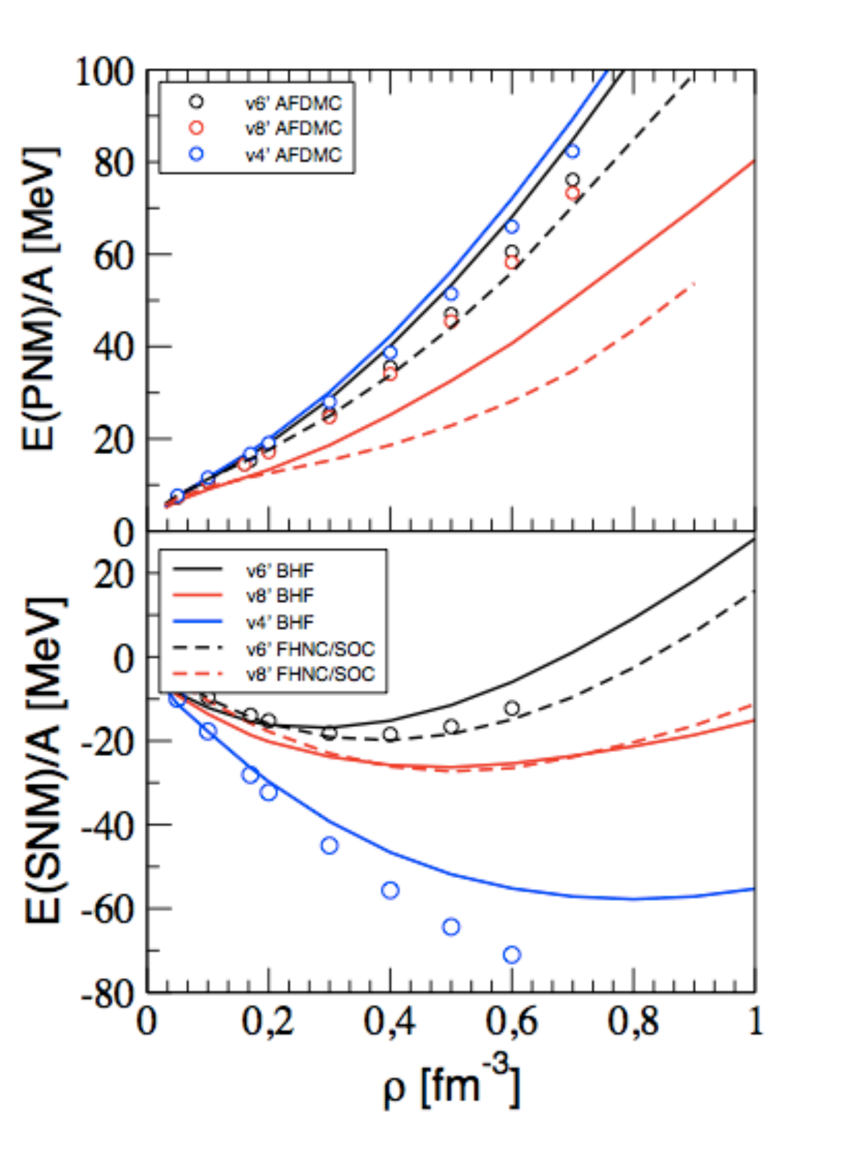
EOS OF HADRONIC MATTER

A satisfactory theoretical framework in which the phenomenology can be consistently explained is still far from being developed.

MAIN REASONS

1)The complex nature of the nuclear medium prevents from determining in a univocal way the relevant degrees of freedom and the corresponding forces (compare for instance with quantum chemistry or electronic structure calculations…)
2)The use of approximate methods combined with approximate model Hamiltonians generated a very wide spectrum of theoretical predictions. Can we make more definite assessments about the relationship between nuclear forces at microscopic level and the properties of huge massive objects like neutron stars?

NEUTRON MATTER BENCHMARK

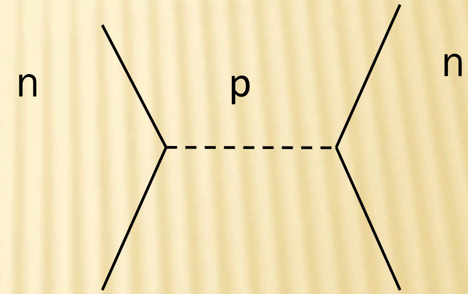


Recently a detailed analysis of the results obtained with different theoretical methods (BHF, SCGF, FHNC, AFDMC) (Benhar, Polls, Vidaña, Rios, Balso, Schulze, Illarionov, Fantoni, Gandolfi, Pederiva...)

NUCLEAR HAMILTONIANS

The nucleon-nucleon interaction is still not completely understood. Many different models are used, and fitted to *reproduce NN scattering data in different channels* ➔ No proper "ab-initio" description.

E.g.: one pion exchange is a major ingredient of nucleon-nucleon interaction, but more complex processes occur.



Argonne AVX potentials (two-body)

$$V(\vec{r}_1,\vec{r}_2,\vec{\sigma}_1,\vec{\sigma}_2,\vec{\tau}_1,\vec{\tau}_2) = \sum_{p=1}^{X} V^p(\vec{r}_1,\vec{r})\hat{O}^p$$

$$O^{p=1\ldots6} = (\mathbf{1},\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j,S_{ij})\otimes(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j) \quad \text{where} \quad S_{ij} = 3(\mathbf{r}_{ij}\cdot\boldsymbol{\sigma}_i)(\mathbf{r}_{ij}\cdot\boldsymbol{\sigma}_j) - \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j$$

METHODOLOGY: QUANTUM MONTE CARLO

Variational Monte Carlo (VMC)

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

Expectation values can be efficiently computed with stochastic methods:

$$E_T = \frac{\langle\psi_T|H|\psi_T\rangle}{\langle\psi_T|\psi_T\rangle} \geq E_0$$

Wave function with operatorial Jastrow 2 body and 3 body correlations should be used to impose correct analytic behavior at short range in each channel:

$$|\psi_T\rangle = \left[S\prod_{ij}\left(v_c(r_{ij}) + v_\sigma(r_{ij})\vec{\sigma}_i\cdot\vec{\sigma}_j + \ldots\right)\right]|\phi\rangle$$

[Wiringa, et al. PRC 43, 1585, (1991)]
[Wiringa, et al. NPA, 543, (1992)]

QUANTUM MONTE CARLO

Green's Function Monte Carlo

Project the ground state using the standard proagator:

$$\psi(R,t) = e^{-(H-E_T)t}\psi(R,0)$$

$$\psi(R,t) = \int dR' \, G(R,R',t)\psi(R',0)$$

Approximate with a finite sum of eigenstates of the position ("walkers")

the propagator can be broken in the usual way using the Trotter-Suzuki formula:

Diffuses the walker

$$G(R,R',\tau) = \left[\frac{1}{2\pi\hbar^2/m\Delta\tau}\right]^{\frac{3A}{2}} e^{-\frac{(R-R')^2}{2\hbar^2/m\Delta\tau}} e^{-\left[\frac{\hat{V}(R)+\hat{V}(R')}{2}-E_T\right]\Delta\tau}$$

Weights the walker, and produces zero, one or more copies

- ✓ In order to be able to perform a DMC calculations the potential should be local. One-body spin-orbit is still treatable. Three-body potentials are also treatable.
- ✓ The propagator acts in a non trivial way on the spin-isospin variables.

QUANTUM MONTE CARLO

Wave Functions

The presence of quadratic spin/isospin operators introduces a heavy complication both in VMC and GFMC calculations. In fact, the Green's function $e^{-u^{(p)}(r_{ij})O^{(p)}}$, and the operatorial components of the correlations mix the spin/isospin two body states. This implies the use of

MULTICOMPONENT WAVEFUNCTIONS!

The number of components grow as $\approx \frac{A!}{Z!(A-Z)!}2^A$

exponential scaling with A

GFMC limit is (now) A=12

	A	Pairs	Spin $\times$ Isospin
^{4}He	4	6	8×2
^{6}Li	6	15	32×5
^{7}Li	7	21	128×14
^{8}Be	8	28	128×14
^{9}Be	9	36	512×42
^{10}Be	10	45	512×90
^{11}B	11	55	2048×132
^{12}C	12	66	2048×132
^{16}O	16	120	32768×1430
^{40}Ca	40	780	$3.6 \times 10^{21} \times 6.6 \times 10^9$
^{8}n	8	28	128×1
^{14}n	14	91	8192×1

[Carlson, PRC 36(5),2026, (1987)]

[Pudliner, Pandharipande, Carlson, Pieper,Wiringa, PRC 56(4), 1720 , (1997)]

[Pieper, NPA, 751, (2005)]

AUXILIARY FIELD DIFFUSION MONTE CARLO

Hubbard Stratonovich transform

$$e^{-\sum_{i=1}^{3N}\lambda_i O_i^2 \Delta\tau} \approx \prod_{i=1}^{3N} e^{-\lambda_i O_i^2 \Delta\tau} \propto \prod_{i=1}^{3N} \int dx\, e^{-\frac{x^2}{2}+\sqrt{-\lambda_i \Delta\tau}\, O_i}$$

Appro. order Dt

2 body operators → $\int$ Auxiliary Field x + 1 body operator

AFDMC = DMC + HS-transform

Other issues:

- ✓Sign problem "treated" with Fixed-Phase approximation.
- ✓Wave functions simplified to a central Jastrow (no operatorial dependence) times an antisymmetrized product of spinors.

[Schmidt & Fantoni, PLB 446, 99 (1999)]

[Fantoni,Sarsa ,Schmidt,Prog.Part.Nuc.Phys. 44 2000)

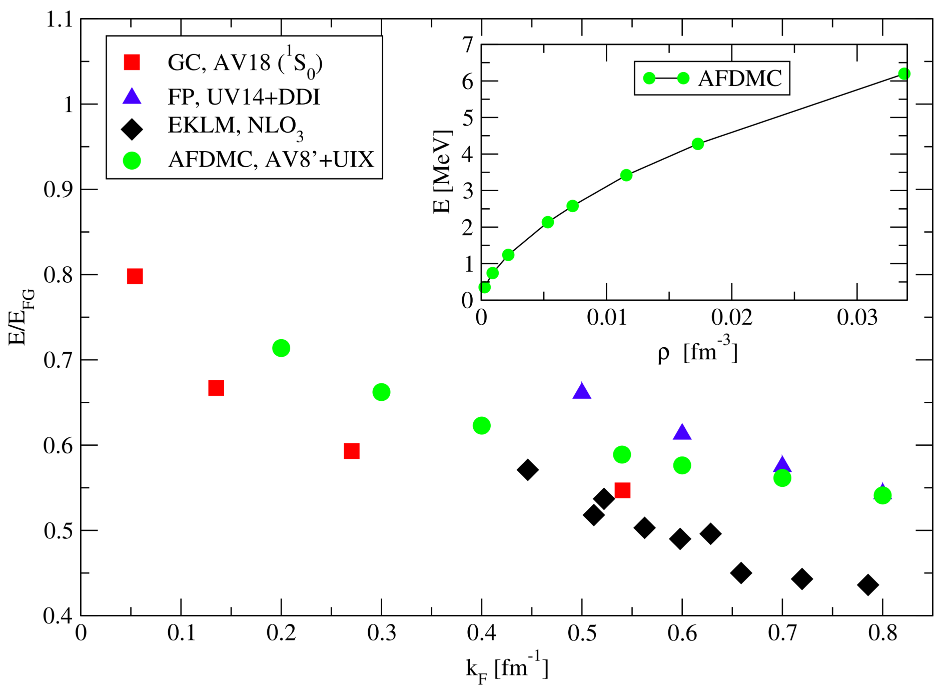
[S. Gandolfi , F. Pederiva, S. Fantoni, K. E. Schmidt, Phys. Rev. Lett. 98, 102503 (2007) PRL 99,022507(2007)]

NEUTRON MATTER

Early work: A. Sarsa, S. Fantoni, K.E. Schmidt, F.P., Phys. Rev. C 68, 024308 (2003)

Low density

S. Gandolfi, A. Yu. Illarionov, F. Pederiva, K. E. Schmidt, and S. FantoniPhys. Rev. C 80, 045802 (2009)



At very low densities the equation of state should be not very sensitive to the interaction used. However already above 0.1fm^{-1} some effects are visible.

AFDMC results refer to a realistic NN interaction including spin-orbit and three-body forces.

NEUTRON MATTER

AFDMC allows for an accurate estimate of the gap in superfluid neutron matter.

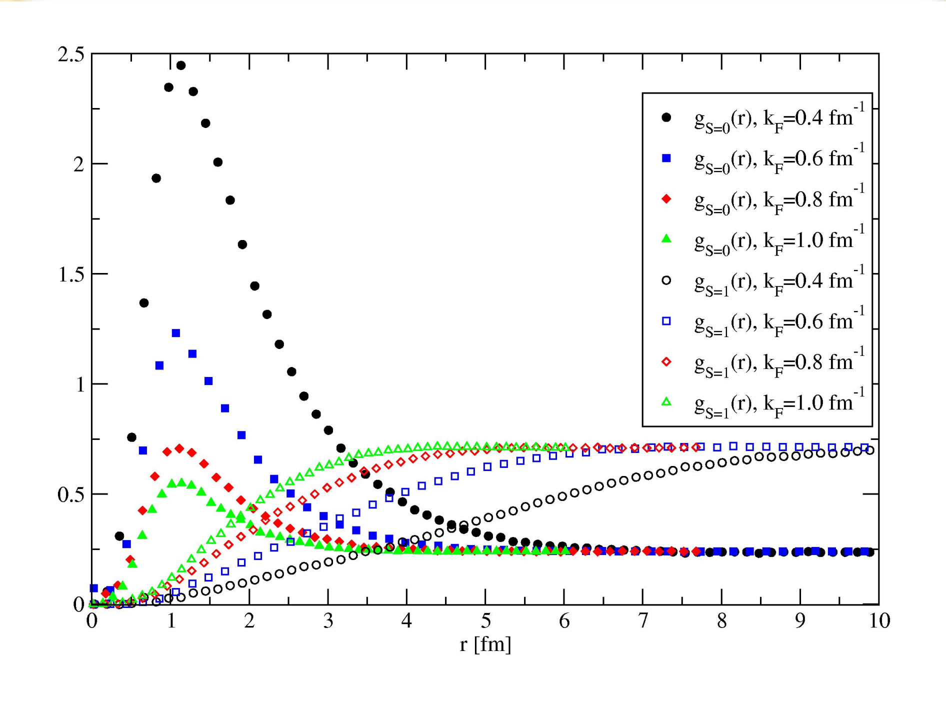
INGREDIENT NEEDED: A “SUPERFLUID” WAVEFUNCTION.

Nodes and phase in the superfluid are better described by a **Jastrow-BCS wavefunction**

$$\Psi_T(R) = \left[\prod_{i<j} f_J(r_{ij})\right] \phi_{BCS}(R,S)$$

$$\phi(\mathbf{r}_{ij}, s_i, s_j) = \sum_a \frac{v_{k_a}}{u_{k_a}} e^{-i\mathbf{k}\cdot\mathbf{r}_{ij}} \chi(s_i, s_j)$$

Coefficients from CBF calculations.



Gandolfi S., Illarionov A., Fantoni S., P.F., Schmidt K., PRL 101, 132501 (2008)

The gap is estimated by the even-odd energy difference at fixed density:

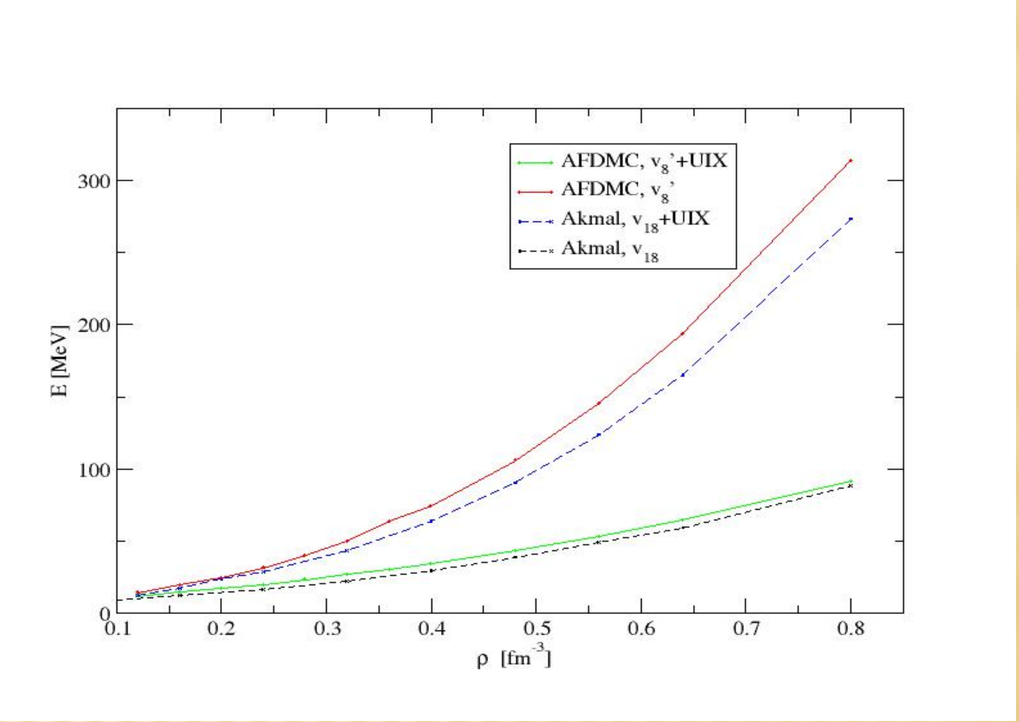
$$\Delta(N) = E(N) - \frac{1}{2}\left[E(N+1) - E(N-1)\right]$$

Calculations have been performed around two different values of N, i.e. N=66 and N=14

NEUTRON MATTER

High density (supersaturation)

S. Gandolfi, A. Yu. Illarionov, K. E. Schmidt, F. Pederiva, and S. FantoniPhys. Rev. C 79, 054005 (2009)



We revised the computations made on Neutron Matter to check the effect of the use of the fixed-phase approximation.

Results are *more stable*, and some of the issues that were not cleared in the previous AFDMC work are now under control.

In particular the *energy per nucleon* computed with the AV8' potential in PNM with A=14 neutrons in a periodic box is now *17.586(6) MeV*, which compares very well with the GFMC-UC calculations of J. Carlson et al. which give *17.00(27) MeV*. The previous published AFDMC result was *20.32(6) MeV*.

CONSTRAINTS

EOS can be tested against available data from astrophysical observations.

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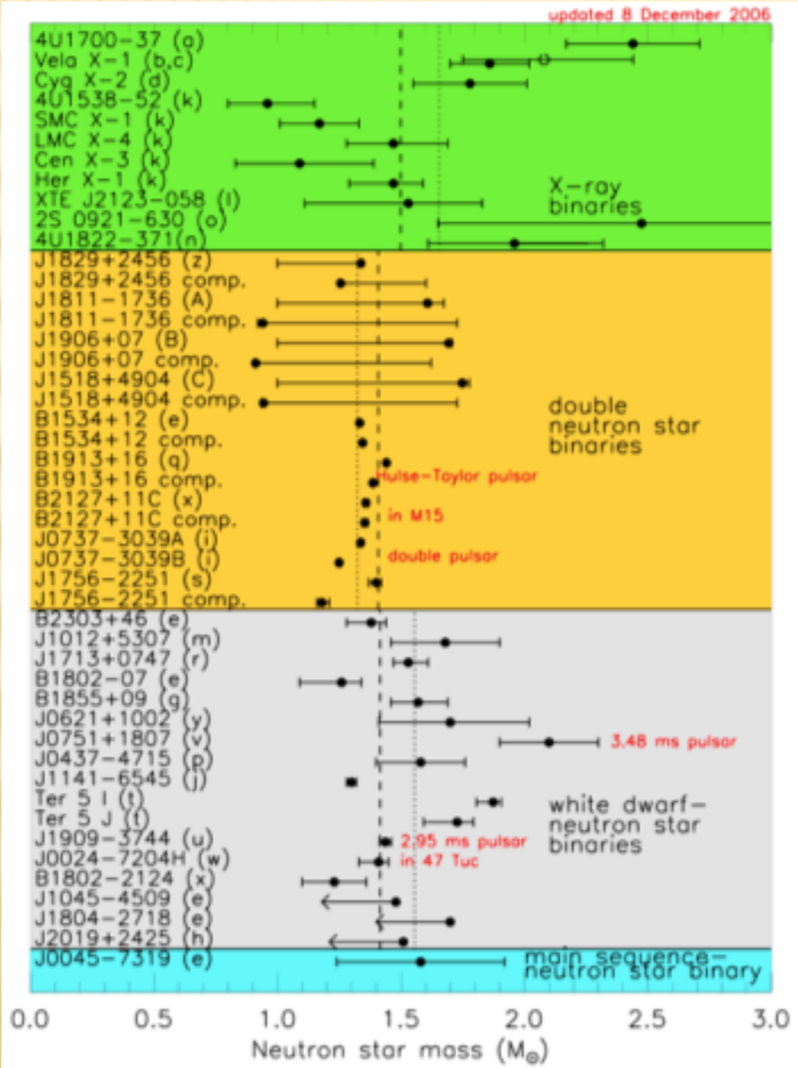
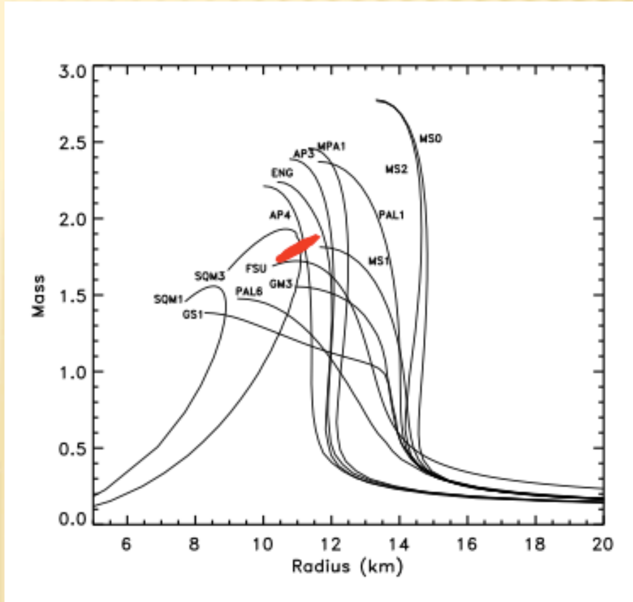
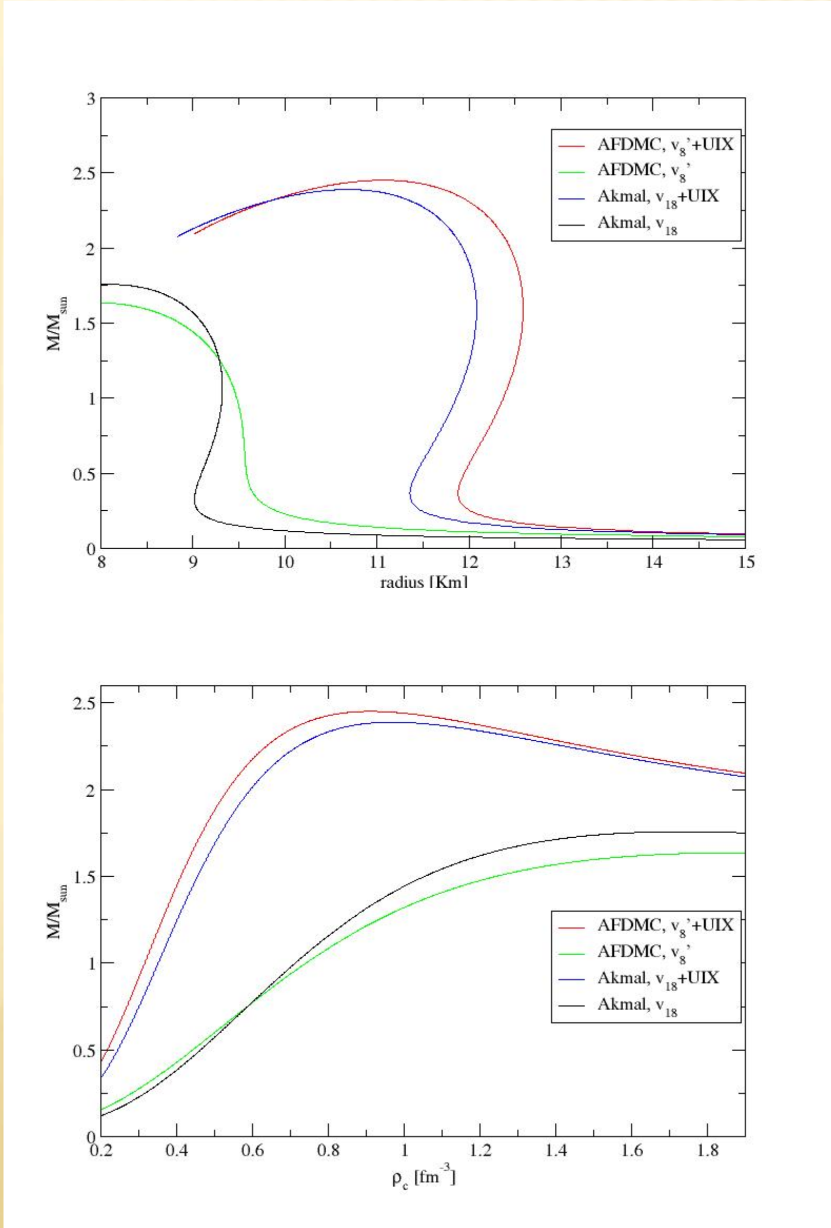




Mass-radius relation can be computed by solving TOV equation. Experimental data have been obtained from measurement of X-ray bursts from accreting neutron stars (F. Özel et. al, 2009 – 2010)

TOV EQUATIONS

The equation of state is the main ingredient used to compute structural properties of neutron stars. One of the best known relations is the Tolman-Oppenheimer-Volkov equation implementing the condition of hydrostatic equilibrium.



$$\frac{dP(r)}{dr} = -\frac{G\varepsilon m(r)}{r^2}\left(1+\frac{P(r)}{c^2\varepsilon(r)}\right)\left(1+\frac{4\pi r^3 P(r)}{m(r)c^2}\right)\left(1-\frac{2Gm(r)}{c^2 r}\right)$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \varepsilon(r)$$

$$P \equiv P(\rho) = -\rho^2 \frac{1}{A}\frac{d^2E(\rho)}{d\rho^2} \qquad \varepsilon = \rho\left[m_N + \frac{E}{c^2 A}(\rho)\right]$$

PROBLEM

- The solution of the TOV equation shows that the three body interaction used to fit the properties of light nuclei does not describe correctly the properties of bulk matter.
- Some new scheme has to be searched. In the meantime it is possible to turn back to an effective description in the attempt of providing useful information to astrophysicists.

DENSITY DEPENDENT INTERACTION

Following Lagaris and Pandharipande *(Nucl. Phys. A359, 349 (1981))*, it is possible to redifine the interaction with density dependent parameters:

$$H = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 + \sum_{i<j} v_{ij} + TNR + TNA$$

The density dependent repulsive part is a modification of the intermediate part of AV6':

$$v_{ij} + TNR = v_\pi + e^{-\gamma_1\rho} v_I + v_R$$

The attractive part is completely phenomenological, and it is written as:

$$TNA = \gamma_2 \rho^2 e^{-\gamma_3\rho}\left[3 - 2\left(\frac{\rho_n - \rho_p}{\rho}\right)^2\right]$$

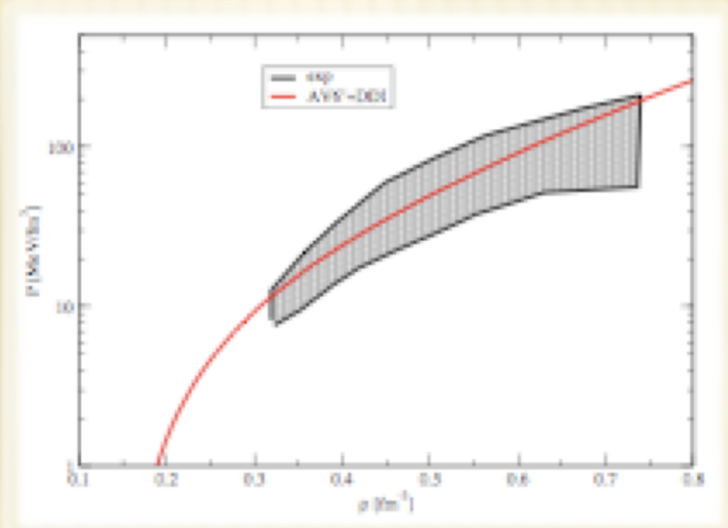
DENSITY DEPENDENT POTENTIAL

The parameters are fitted in order to reproduce the saturation density, the energy at the saturation density and the compressiblity of symmetric nuclear matter. The values are

$$\gamma_1 = 0.10 \text{ fm}^3$$

$$\gamma_2 = \text{-}750 \text{ MeV} \cdot \text{fm}^6$$

$$\gamma_3 = 13.9 \text{ fm}^3$$



The equation of state of nuclear matter fitted with these values of the parameters is:

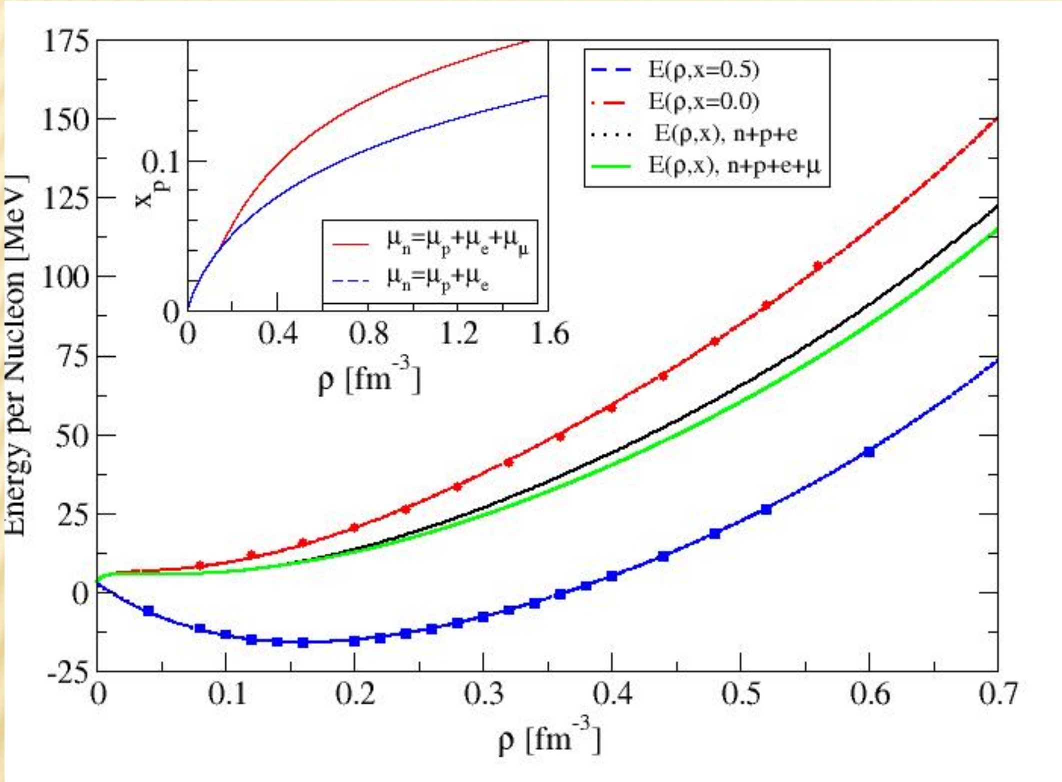
$$\frac{E_{SNM}[\rho]}{N} = E_0 + b(\rho - \rho_0)^2 + c(\rho - \rho_0)^3 e^{\gamma(\rho - \rho_0)}$$

with:

$$E_0 = -16.0(1) \text{ MeV} \qquad \rho_0 = 0.160 \text{ fm}^{-3} \qquad b = 520.0 \text{ MeV} \cdot \text{fm}^6$$

$$c = -1297.4 \text{ MeV} \cdot \text{fm}^9 \qquad \gamma = -2.213 \text{ fm}^3$$

DENSITY DEPENDENT POTENTIAL



$$E(\rho, x_p) = E_{SNM}(\rho) + C_s \left(\frac{\rho}{\rho_0}\right)^{\gamma_s} (1 - 2x_p)^2$$

$C_S = 31.3$ $\gamma_S = 0.64$

With the density dependent interaction we computed the equation of state not only for SNM or PNM.

Given the symmetry energy, it is then possible to compute the energy for an arbitrary proton fraction x_p, and as a consequence the proton fraction corresponding to the b-equilibrium, considering both *electrons* and *muons*.

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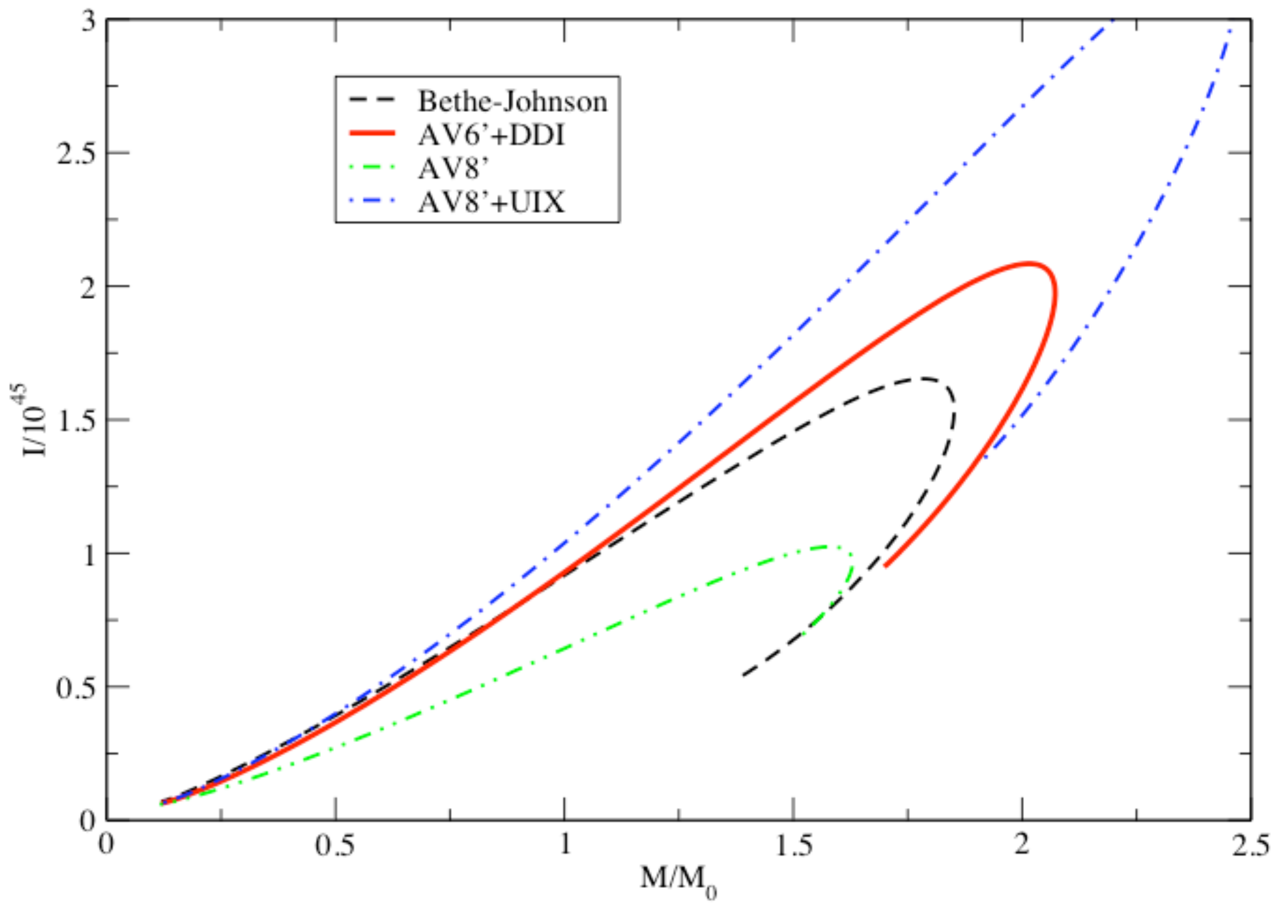
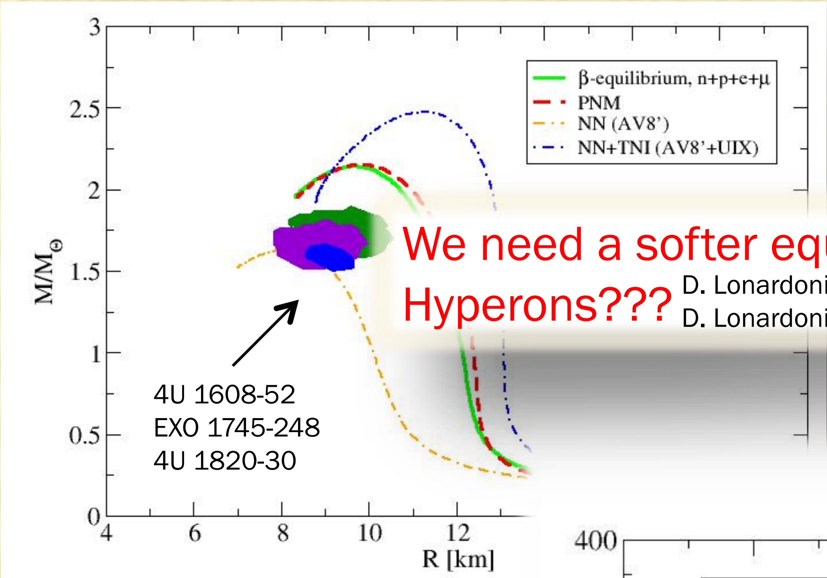
MOMENTUM OF INERTIA



DENSITY DEPENDENT POTENTIAL



S. Gandolfi, A. Yu. Illarionov, S. Fantoni, J. C. Mille… Schmidt, MNRAS **404** (2010) L35

Mass-radius relation computed from AFDMC results for PNM with pure two-nucleon (AV8')

(AV8'+UIX), and with the DDI (PNM). Results

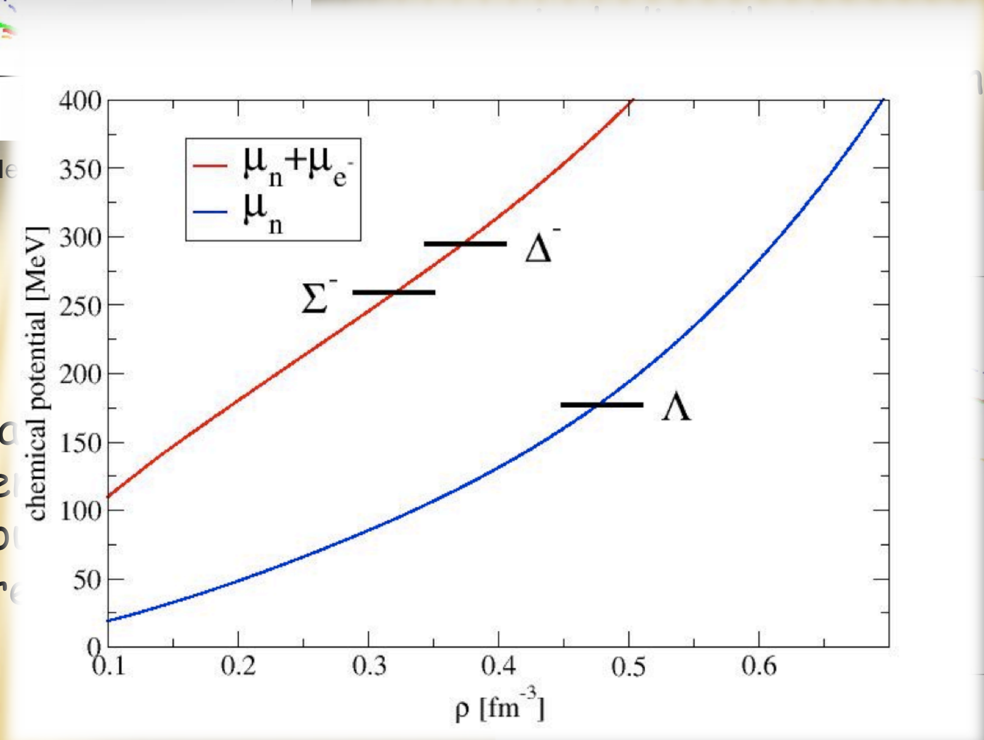
…s

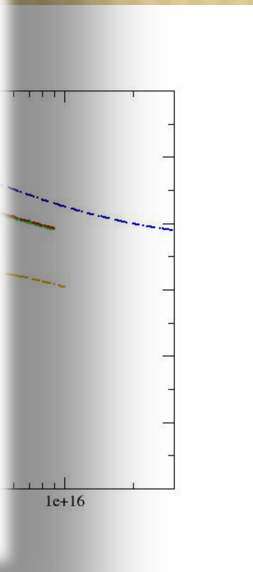
We need a softer equation of state.

Hyperons???

D. Lonardoni, Master thesis

D. Lonardoni, P. Armani, A. Yu. Illarionov, S. Gandolfi, F. Pederiva, in progress





Mass of the star a… function of the ce… density for the fo… cases of the figure above.

NUCLEON-NUCLEON INTERACTION FROM CHIRAL P.T.

The idea!

- Auxiliary fields of HS transform ⟺ pion fields
- 3-body potential term are generated (also) by 2nucleon-pion EFT terms

- Fundamental EFT Hamiltonian used instead of phenomenological potentials
- Include explicitly pion fields regularized on a lattice, assume a fixed nucleon number
- Original 3-nucleon forces could be treated with AFDMC as 2nucleon-pion terms
- Hamiltonian can be improved systematically by adding higher terms of the EFT chiral expansion, and eventually other degrees of freedom (e.g. Δ baryon)

Ph.D. thesis of P. Armani

LAGRANGIAN

$$\begin{aligned}
\mathscr{L}_0 = &- \frac{1}{2}\left[(\vec{\nabla}\pi_i)^2 - (\partial_0\pi_i)^2 + m_\pi^2\pi_i^2\right] \\
&+ N^\dagger\left[i\partial_0 - \frac{1}{2f_\pi}\epsilon_{ijk}\tau_i\pi_j\partial_0\pi_k - M_0 + \frac{\nabla^2}{2M_0}\right]N \\
&- \frac{g_A}{2f_\pi}N^\dagger\tau_i\sigma_j\nabla_j\pi_i N \\
&- \frac{1}{2}C\left(N^\dagger N\right)\left(N^\dagger N\right) \\
&- \frac{1}{2}C_I\left(N^\dagger\tau_i N\right)\left(N^\dagger\tau_i N\right)
\end{aligned}$$

Higher order, but we keep it to maintain the DMC scheme

AFDMC AND CHIRAL P.T. HAMILTONIANS

Problems (partially solved)

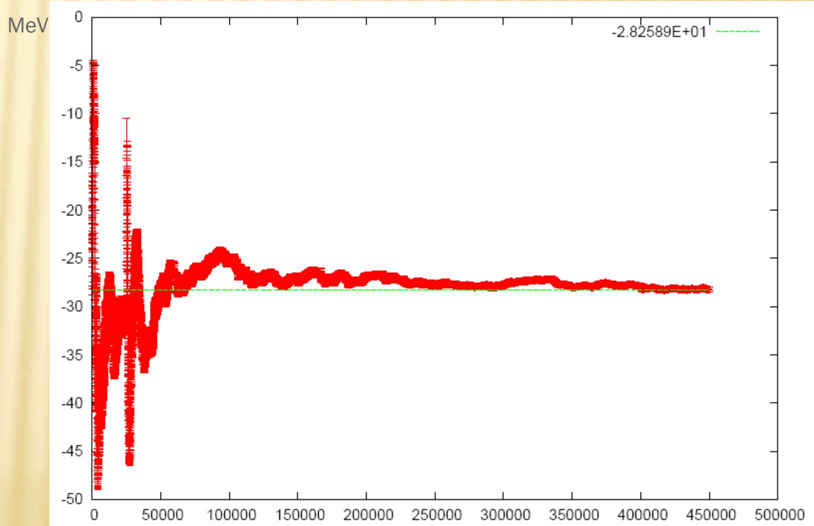
- Pion vacuum energy ~ 20 GeV
- Nucleon eigenenergy ~ 40 MeV

We need an accurate wavefunction(als) including nucleon-nucleon, pion-pion, and pion-nucleon correlations in order to make the variance of the energy as small as possible

Because the Hamiltonian is regularizaion-dependent, we also need to fit the coefficient over some data. At present the good candidates are the binding energy of ^{4}He and Tritium (np, nn, and pp present some unexpected (?) problem)

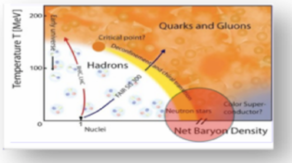
Preliminary results

- Nucleon bare mass computed
- Algorithm scales linearly with nucleon number ⇨ medium size nuclei (^{16}O, ^{40}Ca) study is feasible [^{4}He example run of only 48 cpu hours]

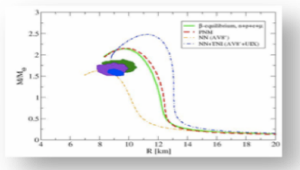


- Inclusion of higher Hamiltonian terms does not change the scalability

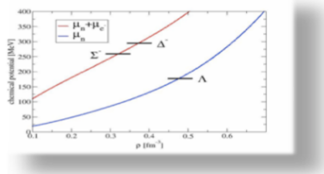
CONCLUSIONS



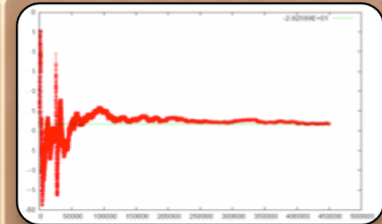
The AFDMC technique can be successfully used to study bulk nuclear and neutron matter.



A phenomenological density dependent interaction has been devised, which better approaches the constraints from terrestrial and astrophysical observations.



The EOS might still be too stiff (hyperons?)



It is worth rethinking the approach to many nucleon calculations including explicitly pion d.o.f.