Quantum Monte Carlo calculations for neutrino-nucleus scattering

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NUCLEI
Nuclear Computational Low-Energy Initiative

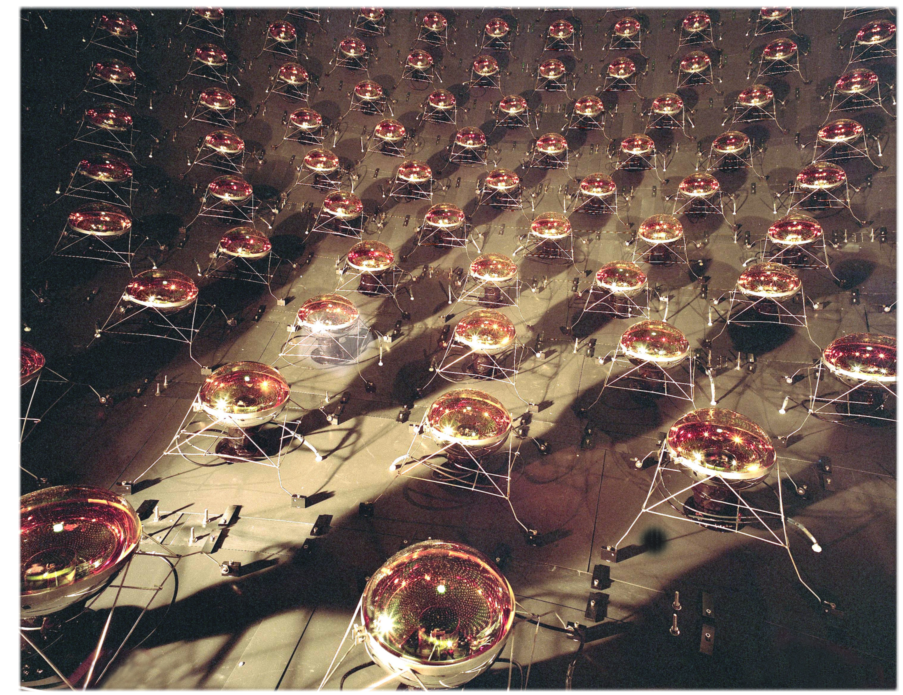
Introduction

- The electroweak response is a fundamental ingredient to describe neutrino - ^{12}C scattering.
- Excess, at relatively low energy, of measured cross section relative to oversimplified theoretical calculations.

Neutrino experimental communities need accurate theoretical calculations

- We have first studied the electromagnetic response of ^{12}C for which precise experimental data are available.

A model unable to describe electron-nucleus scattering is unlikely to describe neutrino-nucleus scattering.



First step: electron-nucleus scattering

The electromagnetic inclusive cross section of the process

$$e + ^{12}\mathrm{C} \rightarrow e' + X$$

where the target final state is undetected, can be written as

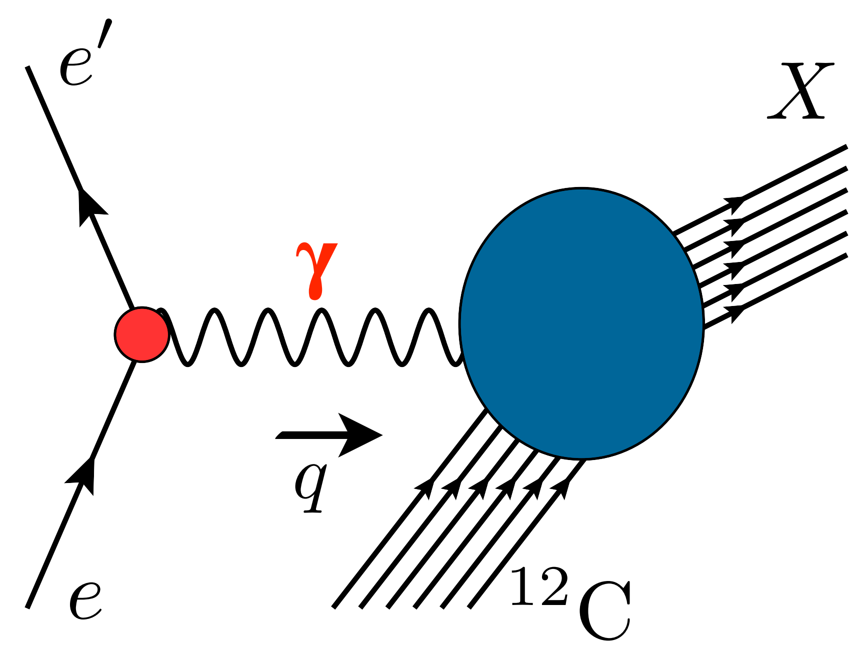
$$\frac{d^2\sigma}{d\Omega_{e'} dE_{e'}} = -\frac{\alpha^2}{q^4}\frac{E_{e'}}{E_e} L^{EM}_{\mu\nu} W^{\mu\nu}_{EM} \ ,$$

The leptonic tensor is fully specified by the measured electron kinematic variables

$$L^{EM}_{\mu\nu} = 2[k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu}(kk')]$$

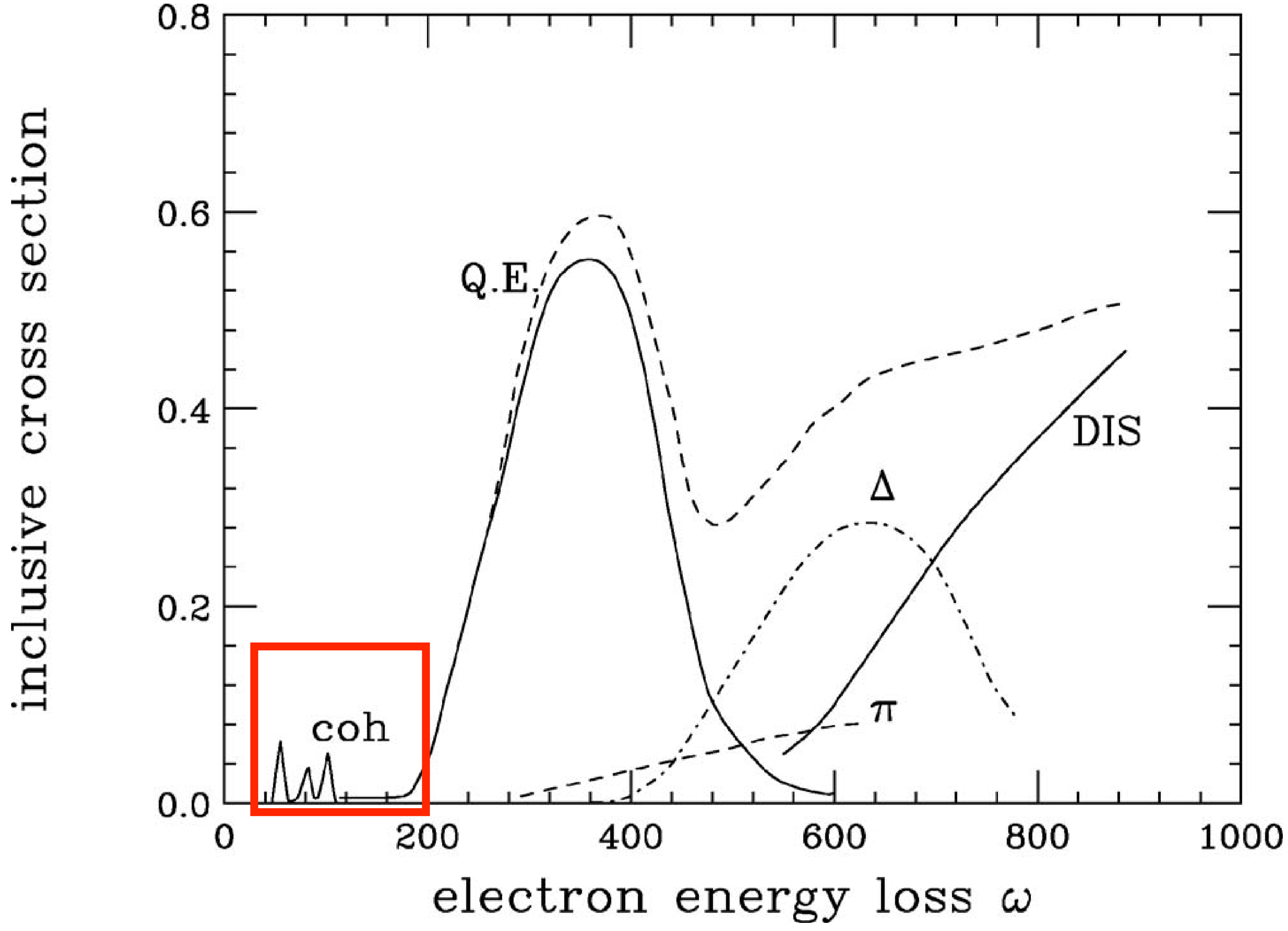
The Hadronic tensor contains all the information on target structure.

$$W^{\mu\nu}_{EM} = \sum_X \langle \Psi_0 | J^{\mu\,\dagger}_{EM} | \Psi_X \rangle \langle \Psi_X | J^\nu_{EM} | \Psi_0 \rangle \delta^{(4)}(p_0 + q - p_X)$$



Electron-nucleus scattering

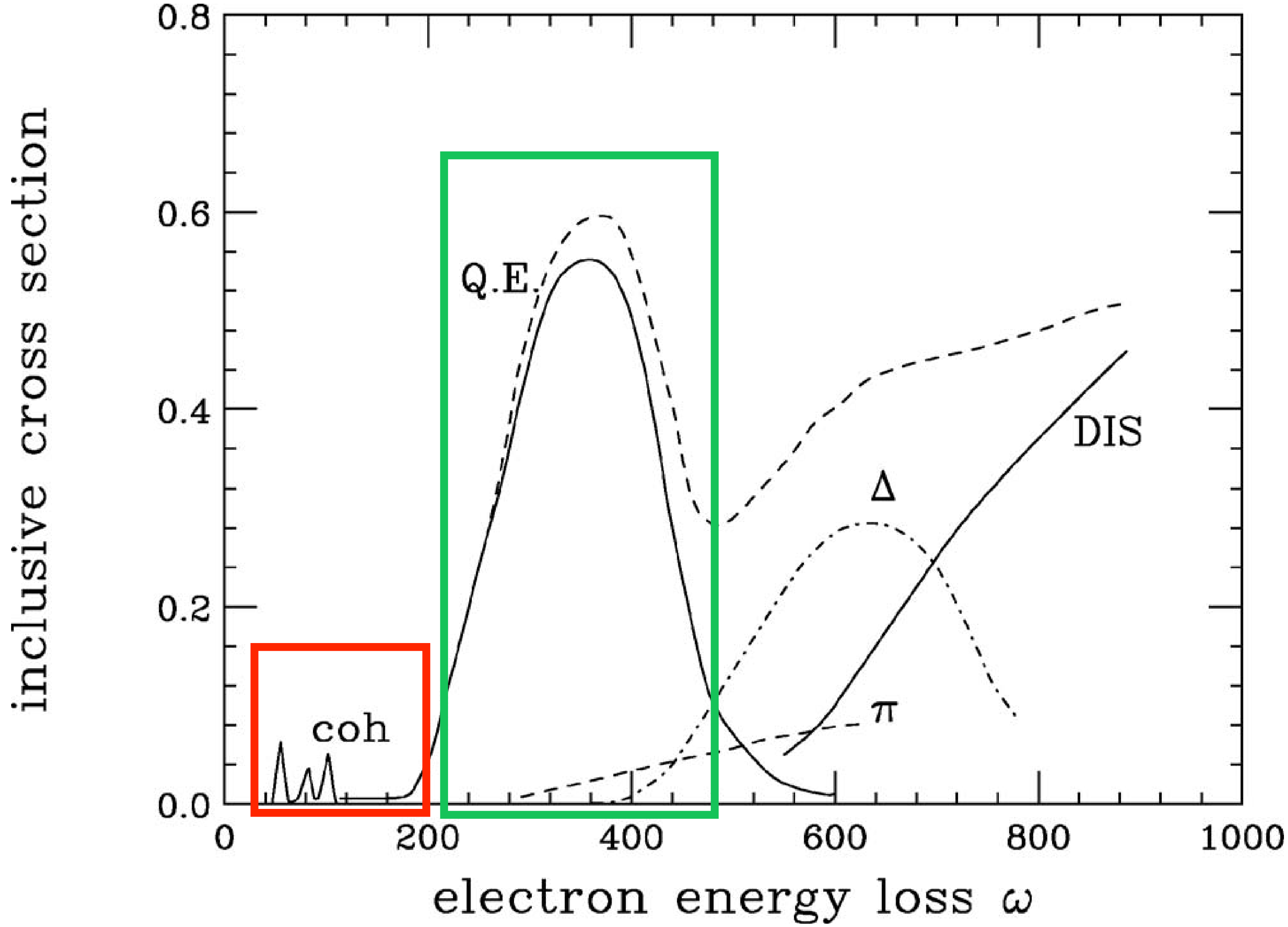
<u>Schematic</u> representation of the inclusive cross section as a function of the energy loss.



- Elastic scattering and inelastic excitation of discrete nuclear states

Electron-nucleus scattering

Schematic representation of the inclusive cross section as a function of the energy loss.



- Elastic scattering and inelastic excitation of discrete nuclear states.
- Broad peak due to quasi-elastic electron-nucleon scattering.

Electron-nucleus scattering

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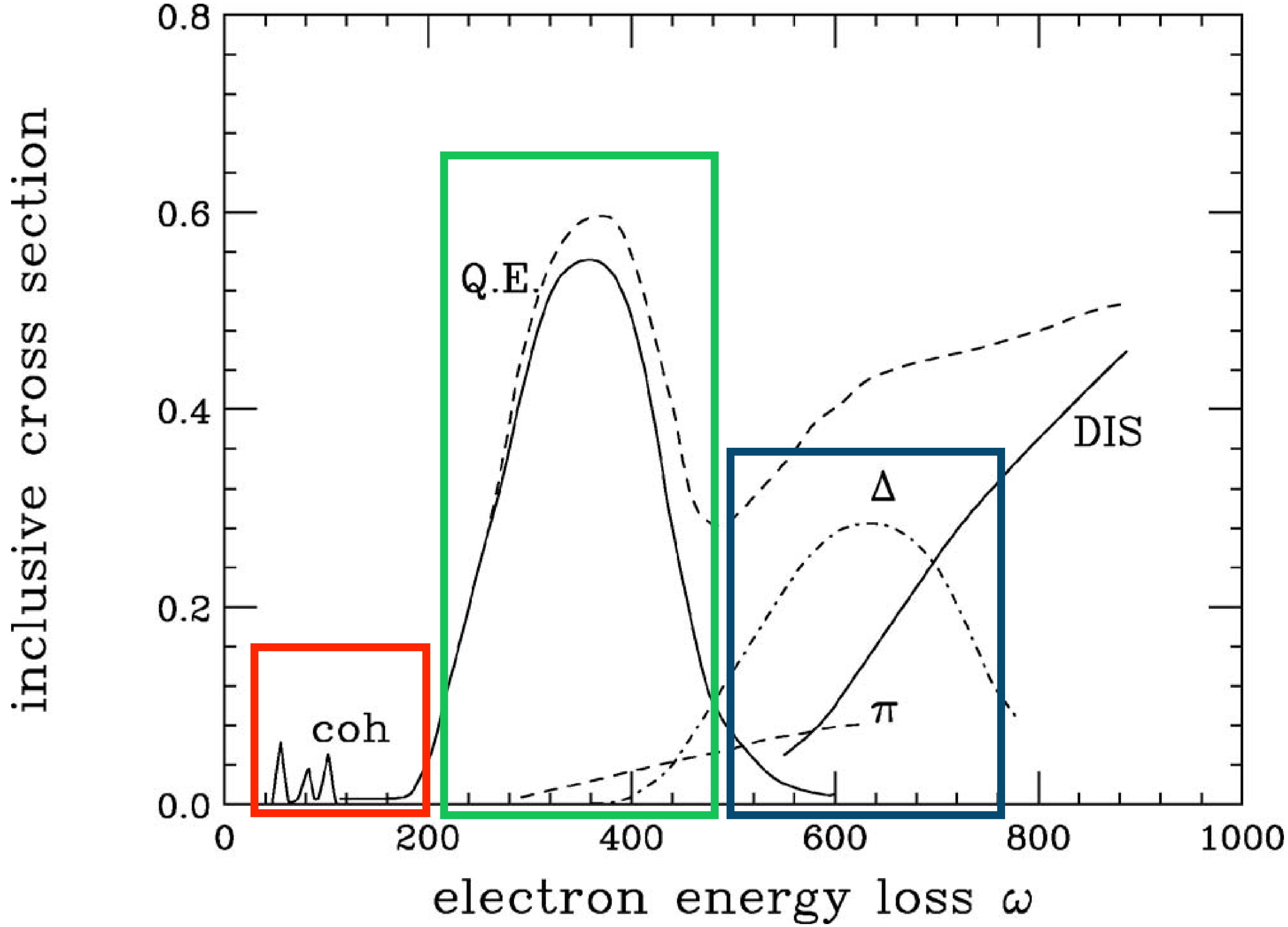


- Elastic scattering and inelastic excitation of discrete nuclear states.
- Broad peak due to quasi-elastic electron-nucleon scattering.
- Excitation of the nucleon to distinct resonances (like the Δ) and pion production.

Neutrino-nucleus scattering

The neutral current inclusive cross section of the process

$$\nu_\ell + A \to \nu_{\ell'} + X$$

where the target final state is undetected, can be written as

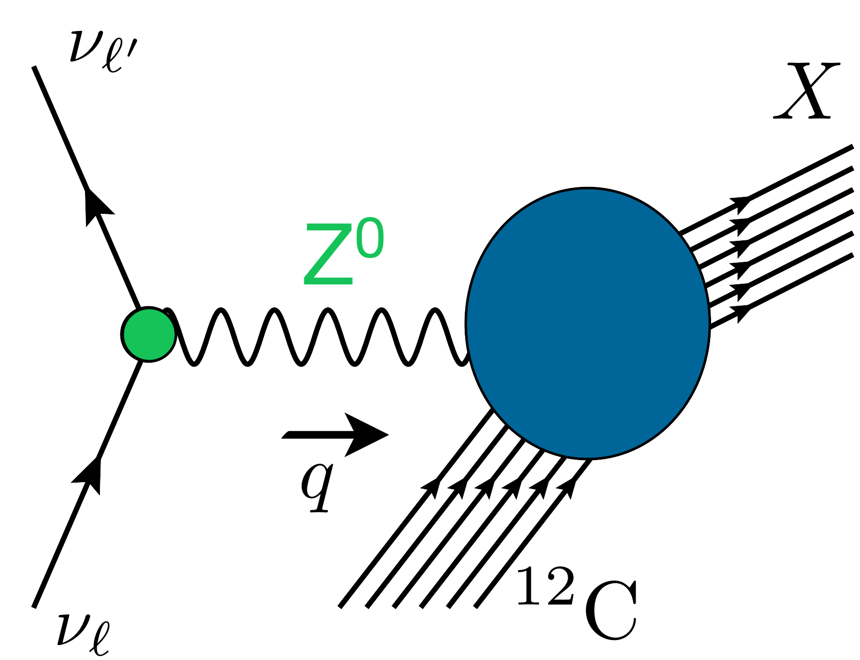
$$\frac{d^2\sigma}{d\Omega_{\nu'} dE_{\nu'}} = \frac{G_F^2}{4\pi^2} \frac{|\mathbf{k}'|}{|\mathbf{k}|} L_{\mu\nu}^{NC} W^{\mu\nu}_{NC}$$

The leptonic tensor is fully specified by the measured neutrino kinematic variables

$$L_{\mu\nu}^{NC} = 8\,\left[k'_\mu\, k_\nu + k'_\nu\, k_\mu - g_{\mu\nu}(k \cdot k') - i\,\varepsilon_{\mu\nu\alpha\beta}\, k'^\beta\, k^\alpha\right]$$

The Hadronic tensor contains all the information on target structure.

$$W^{\mu\nu}_{NC} = \sum_X \langle \Psi_0 | J^{\mu\,\dagger}_{NC} | \Psi_X \rangle \langle \Psi_X | J^\nu_{NC} | \Psi_0 \rangle \delta^{(4)}(p_0 + q - p_X)$$



Neutrino-nucleus scattering

The neutral current operator can be written as

$$J^{\mu}_{NC} = -2\sin^2\theta_W J^{\mu}_{\gamma,S} + (1-2\sin^2\theta_W) J^{\mu}_{\gamma,z} + J^{\mu\,5}_{z}$$

- Weinberg angle $\sin^2\theta_W = 0.2312$
- Isoscalar and isovector terms of the electromagnetic current.

$$J^{\mu}_{EM} = J^{\mu}_{\gamma,S} + J^{\mu}_{\gamma,z}$$

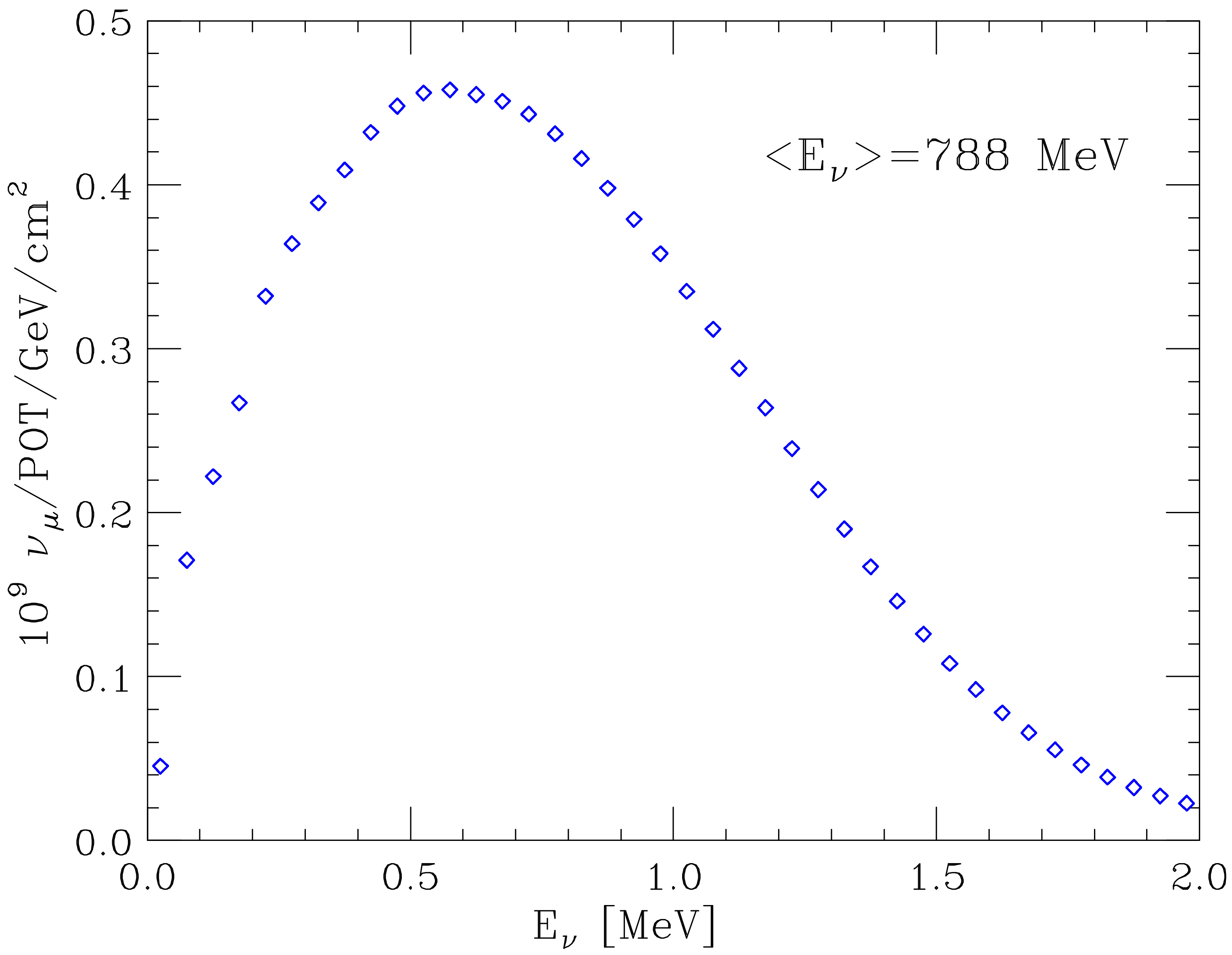
- Isovector term of the axial current, the one-body contributions of which are proportional to the axial form factor, often written in the simple dipole form

$$J^{\mu\,5}_{z} \propto G_A(Q^2) = \frac{g_A}{(1+Q^2/\Lambda_A^2)^2}$$

The value of the axial mass obtained on neutrino-deuteron and neutrino-proton scattering data is $\Lambda_A \sim 1.03\,\mathrm{GeV}$.

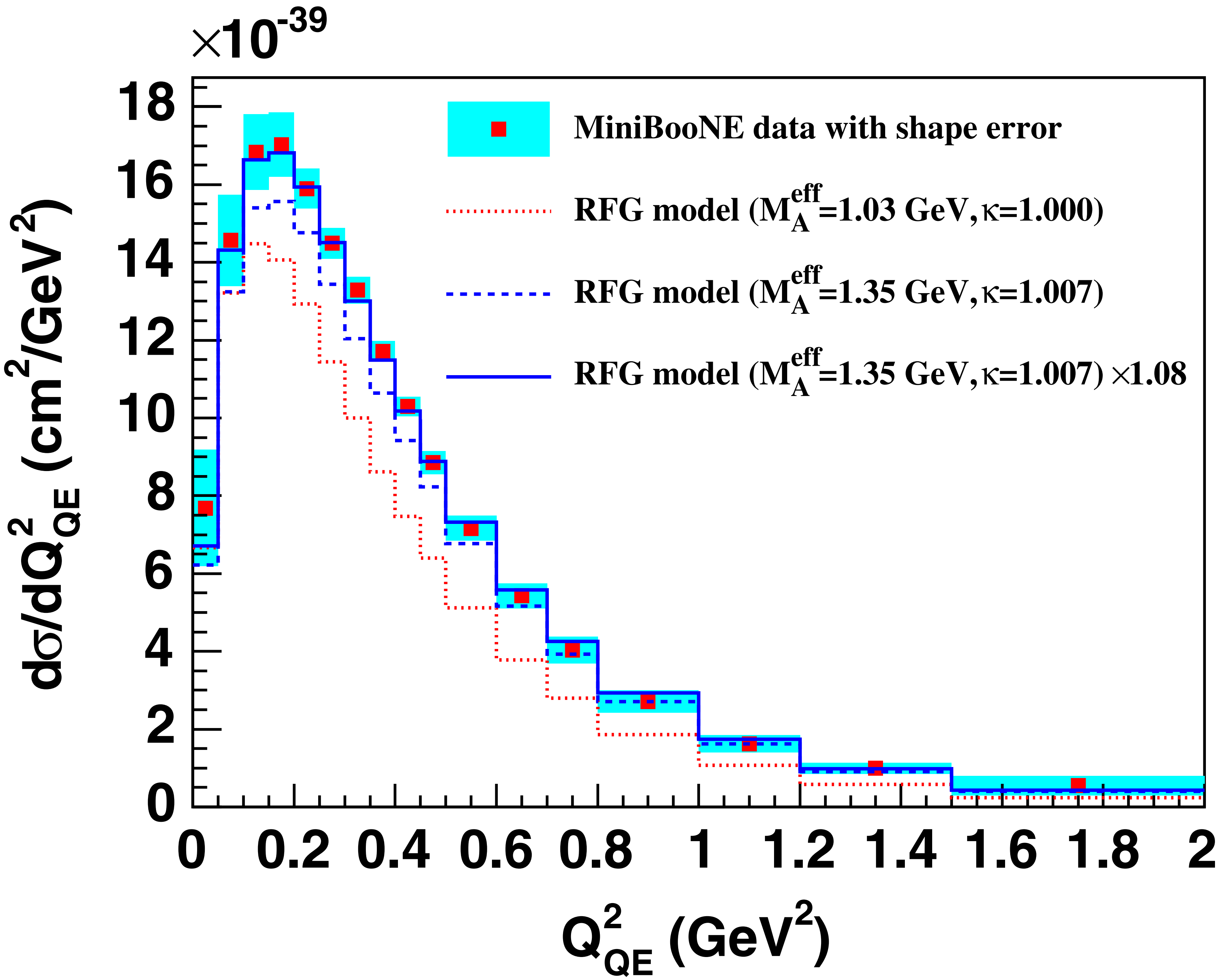
Neutrino-nucleus scattering

Because neutrino beams are always produced as secondary decay products, their energy is not sharply defined, but broadly distributed.



Neutral current response

Relativistic Fermi gas calculations require an artificially large nucleon axial mass to reproduce the data.



- Two-body currents?
- Nuclear correlations?

Two-body MEC currents and correlations are fully accounted for in our GFMC calculations of response functions and sum rules

Nuclear hamiltonian

- Within the nonrelativistic many-body approach, nucleons are point like particles. The two-body potential

Argonne v_{18}: $$v_{18}(r_{12}) = \sum_{p=1}^{18} v^p(r_{12})\hat{O}^p_{12}$$

is controlled by ~4300 np and pp scattering data below 350 MeV of the Nijmegen database.

- Static part $\hat{O}^{p=1-6}_{ij} = (1, \sigma_{ij}, S_{ij}) \otimes (1, \tau_{ij})$ Deuteron, S and D wave phase shifts

- Spin-orbit $\hat{O}^{p=7-8}_{ij} = \mathbf{L}_{ij} \cdot \mathbf{S}_{ij} \otimes (1, \tau_{ij})$ P wave phase shifts

$$\mathbf{L}_{ij} = \frac{1}{2i}(\mathbf{r}_i - \mathbf{r}_j) \times (\boldsymbol{\nabla}_i - \boldsymbol{\nabla}_j) \quad \longleftrightarrow \quad \text{Angular momentum}$$

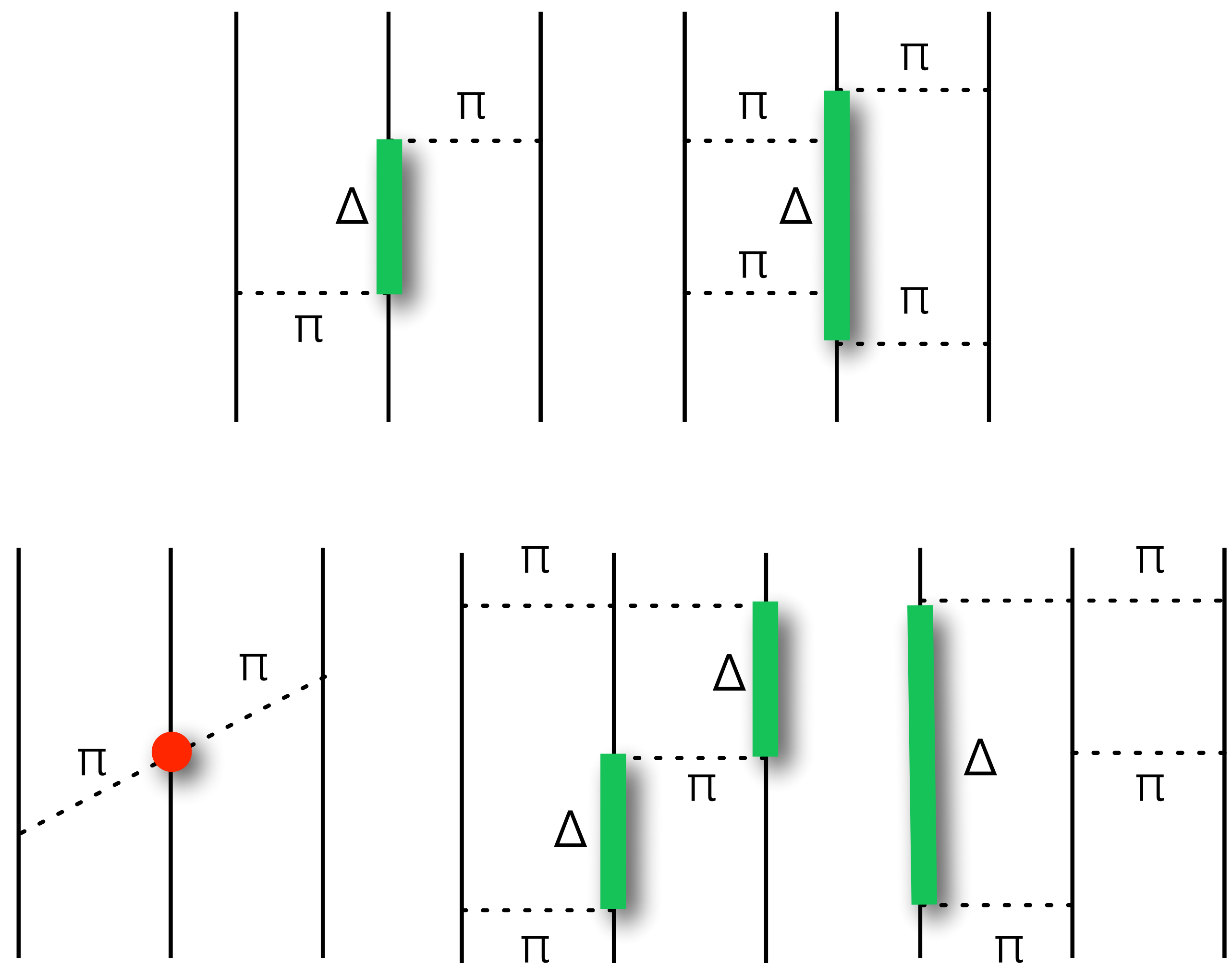
$$\mathbf{S}_{ij} = \frac{1}{2}(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \quad \longleftrightarrow \quad \text{Total spin of the pair}$$

The remaining operators, associated to quadratic spin-orbit interaction and charge symmetry breaking effectsare needed to achieve the description of the Nijmegen scattering data with $\chi^2 \simeq 1$.

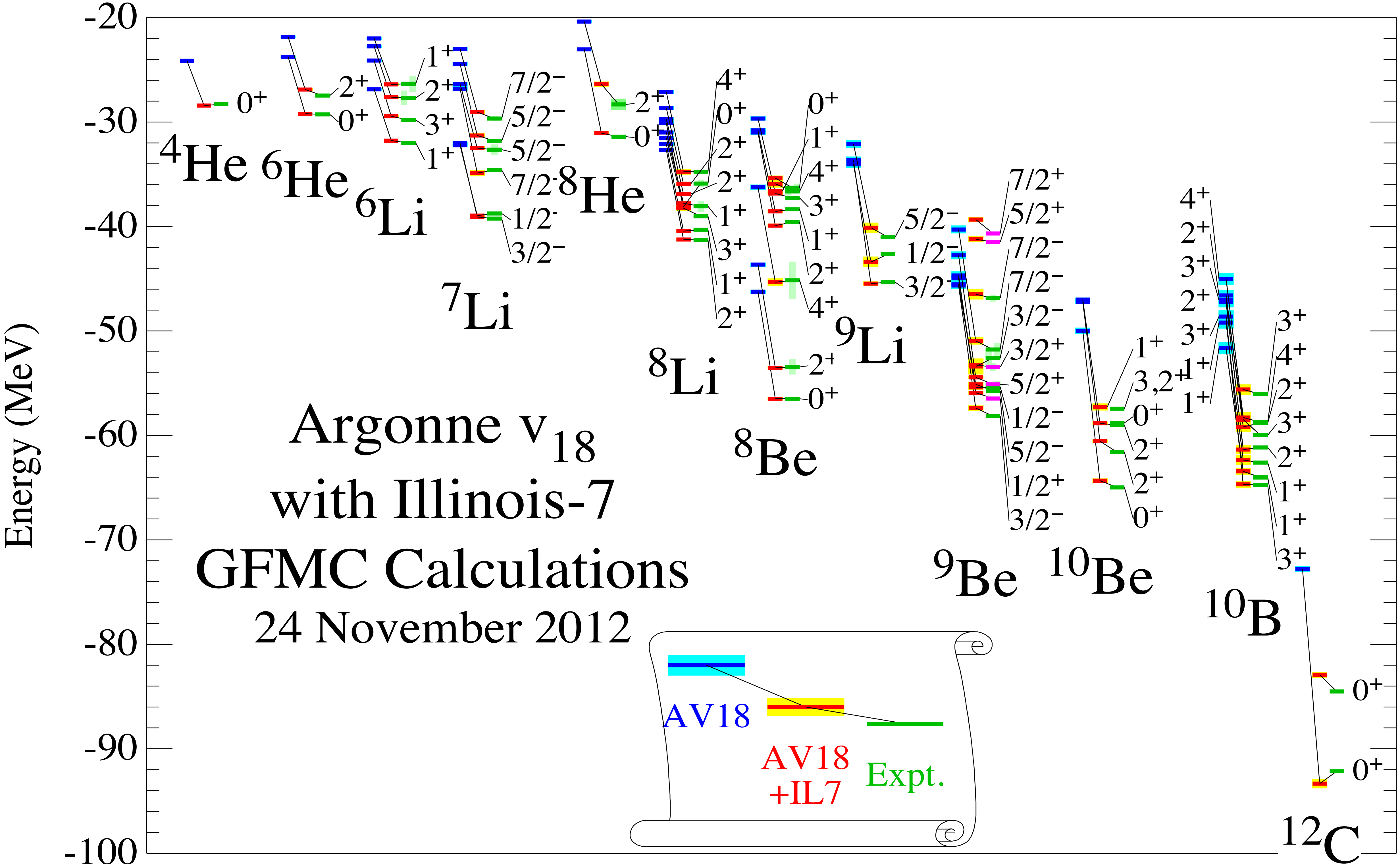
Nuclear hamiltonian

- In order to accurately reproduce the energy spectrum of light nuclei three body potential has to be introduced.

Illinois 7
contains the attractive Fujita and Miyazawa two-pion exchange interaction, a phenomenological repulsive contribution, the two-pion S-wave contribution and terms originating from three-pion exchange diagrams



Nuclear hamiltonian



Two-body currents

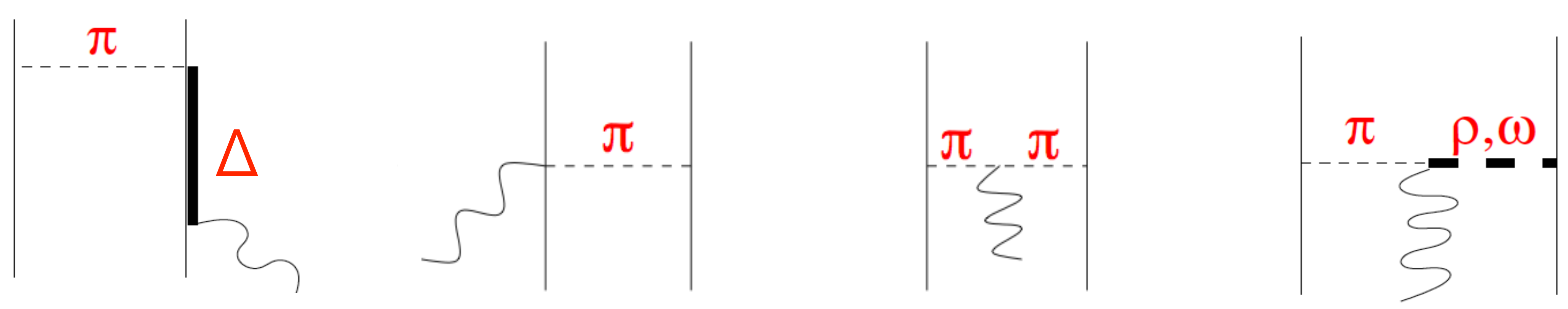
At moderate momentum transfer, the inclusive cross section of the process $\ell +^{12} C \rightarrow \ell' + X$ can be written in terms of the response functions

$$R_{\alpha\beta}(q,\omega) = \sum_f \langle\Psi_0|J^{\dagger\,\alpha}(\mathbf{q},\omega)|\Psi_f\rangle\langle\Psi_f|J^\beta(\mathbf{q},\omega)|\Psi_0\rangle\delta(\omega + E_0 - E_f),$$

Nuclear current includes one-and two-nucleon contributions

$$J^\alpha = \sum_i j_i^\alpha + \sum_{i<j} j_{ij}^\alpha$$

- j_i^α describes interactions involving a single nucleon,
- j_{ij}^α accounts for processes in which the vector boson couples to the currents arising from meson exchange between two interacting nucleons.



Moderate momentum-transfer regime

- At moderate momentum transfer, both initial and final states are eigenstates of the nonrelativistic nuclear hamiltonian

$$\hat{H}|\Psi_0\rangle = E_X|\Psi_0\rangle \qquad \hat{H}|\Psi_X\rangle = E_X|\Psi_X\rangle$$

- In the electron scattering on ^{12}C among the possible states there are

$$|\Psi_X\rangle = |^{11}B, p\rangle\,,\ |^{11}C, n\rangle\,,\ |^{10}B, pn\rangle\,,\ |^{10}Be, pp\rangle\ \ldots$$

- Relativistic corrections are included in the current operators and in the nucleon form factors.

- GFMC allows for "exactly" solving the nonrelativistic many-body Schrödinger equation for nuclei as large as ^{12}C.

- GFMC also allows for extracting dynamical observables from ground-state properties.

Sum rules of the response functions

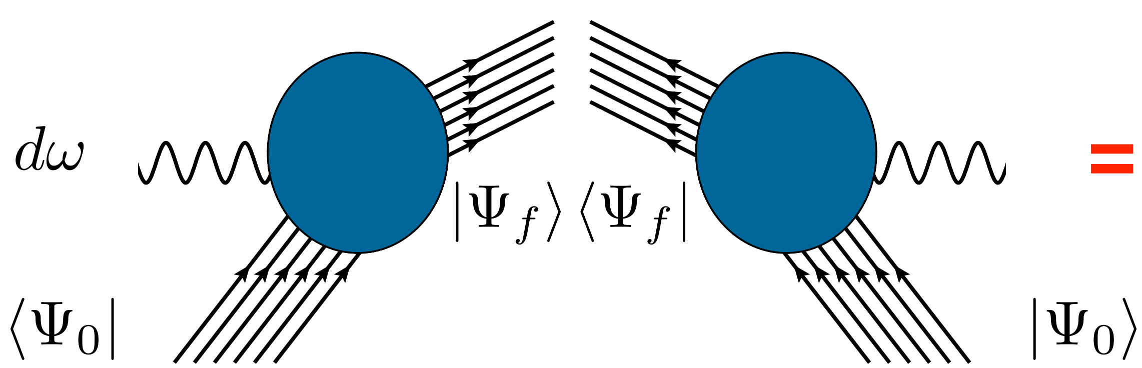
- The direct calculation of the response requires the knowledge of all the transition amplitudes: $\langle \Psi_f | J^\alpha(\mathbf{q}, \omega) | \Psi_0 \rangle$.

- The <u>sum rules</u> provide an useful tool for studying integral properties of the neutrino-nucleus scattering.

$$S_{\alpha\beta}(\mathbf{q}) = C_{\alpha\beta}(q) \int_{\omega_{el}}^{\infty} d\omega R_{\alpha\beta}(\mathbf{q}, \omega)$$

- Using the completeness relation, they can be expressed as <u>ground-state expectation values of the charge and current operators</u>.

$S_{\alpha\beta}(\mathbf{q})$ = $\sum_f \int d\omega$



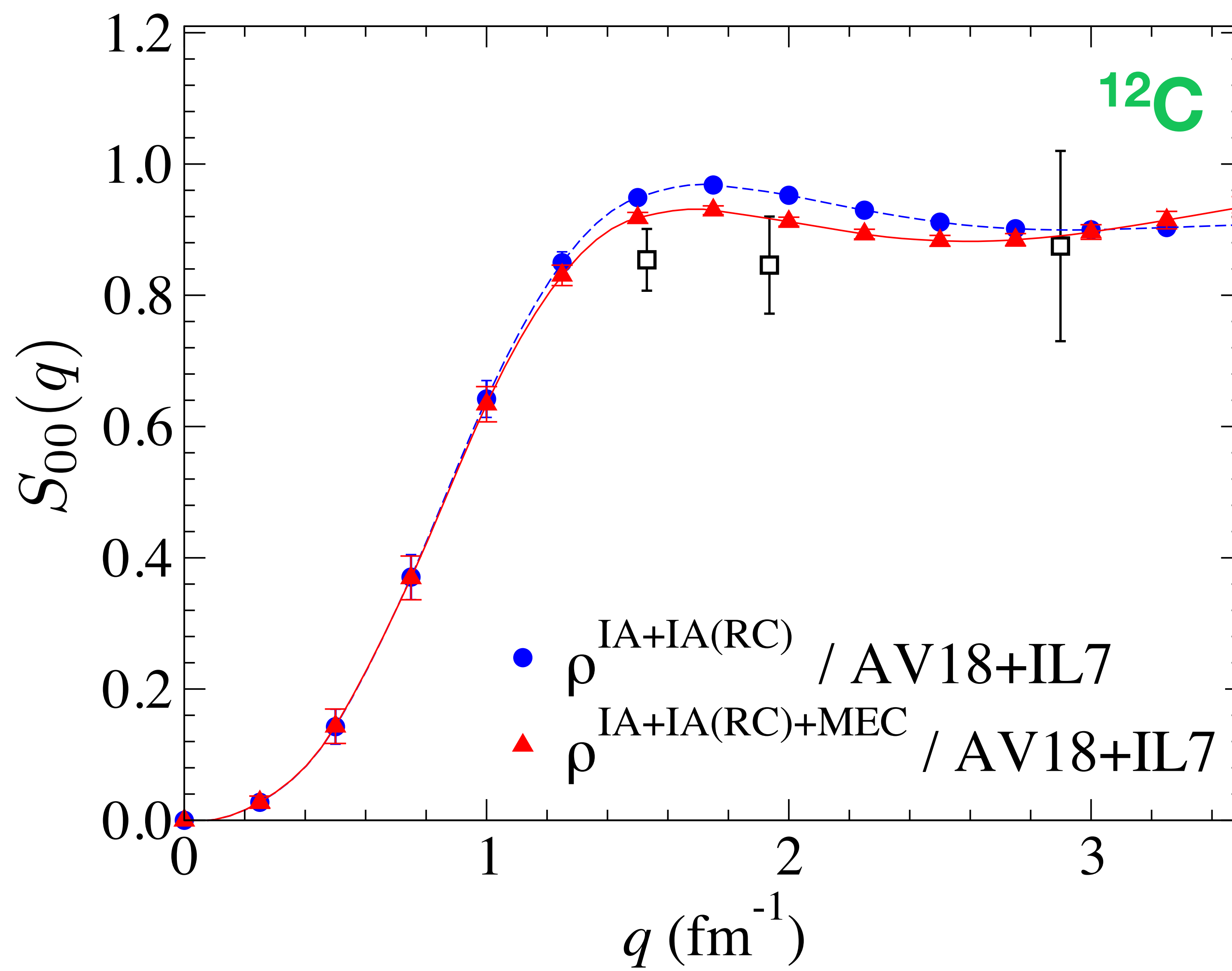
= $\langle \Psi_0 | J_\alpha^\dagger(\mathbf{q}, \omega) J_\beta(\mathbf{q}, \omega) | \Psi_0 \rangle$

Electromagnetic longitudinal sum rule of ^{12}C

$$S_{00} = C_{00} \langle \Psi_0 | \rho^\dagger(\mathbf{q}, \omega_{qe}) \rho(\mathbf{q}, \omega_{qe}) | \Psi_0 \rangle \qquad C_{00} = \frac{1}{G_E^{p\,2} Z}$$

- S_{00} vanishes quadratically at small momentum transfer.

- Satisfactory agreement with the experimental values.

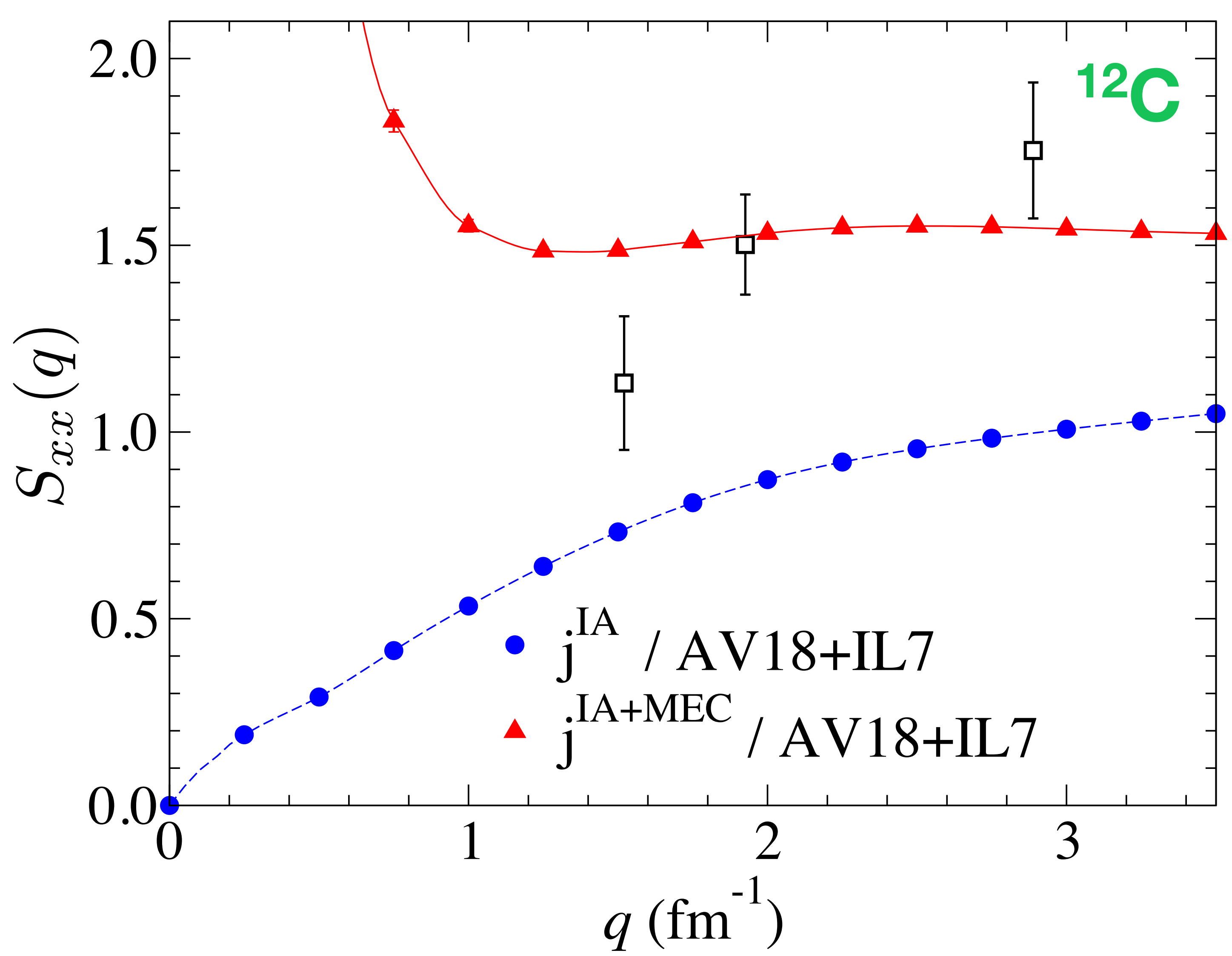


Electromagnetic transverse sum rule of ^{12}C

$$S_{xx} = C_{xx} \langle \Psi_0 | J_x^\dagger(\mathbf{q}, \omega_{qe}) J_x(\mathbf{q}, \omega_{qe}) | \Psi_0 \rangle \qquad C_{xx} = \frac{2}{{G_E^p}^2 (Z\mu_p^2 + N\mu_n^2)} \frac{m^2}{q^2}$$

- <u>Large two-body contribution needed for a better agreement with experimental data.</u>

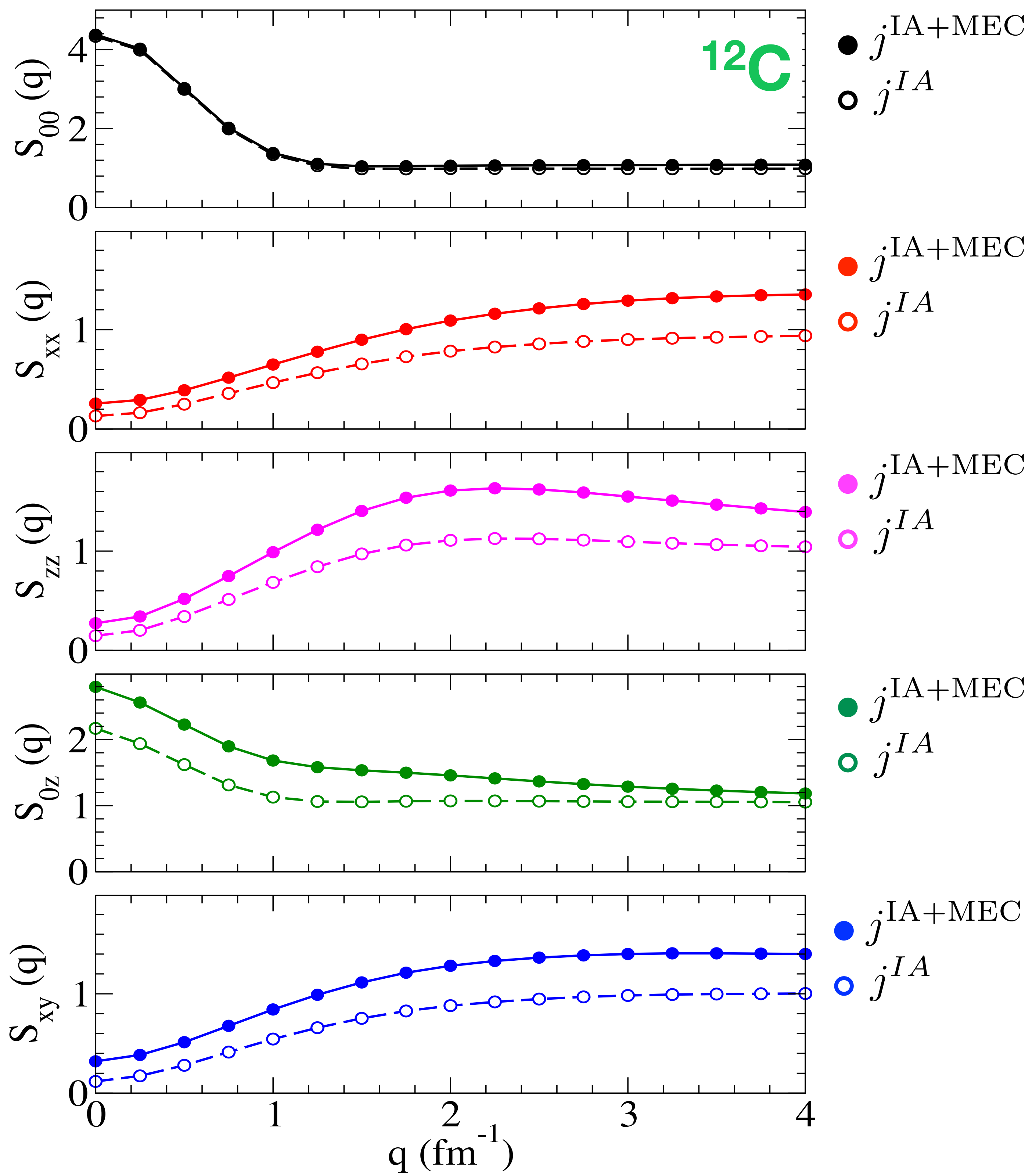
- Comparison with experimental data made difficult by the Δ peak.



Neutral-current sum rules of ^{12}C

- Except for, the $S_{00}(q)$ case, the sum rules of the response functions of ^{12}C exhibit a sizable enhancement due to two-body currents.

- A direct calculation of the response functions is needed to determine how this excess strength is distributed in energy transfer.



Euclidean response function

- Euclidean neutral-current response calculation

$$E_{\alpha\beta}(\tau,\mathbf{q}) = C_{\alpha\beta}(q)\int_{\omega_{el}}^{\infty} d\omega e^{-\omega\tau} R_{\alpha\beta}(\mathbf{q},\omega)$$

allows us to make a more direct comparison with data. Its implementation in quantum Monte Carlo algorithms consists in the evaluation of

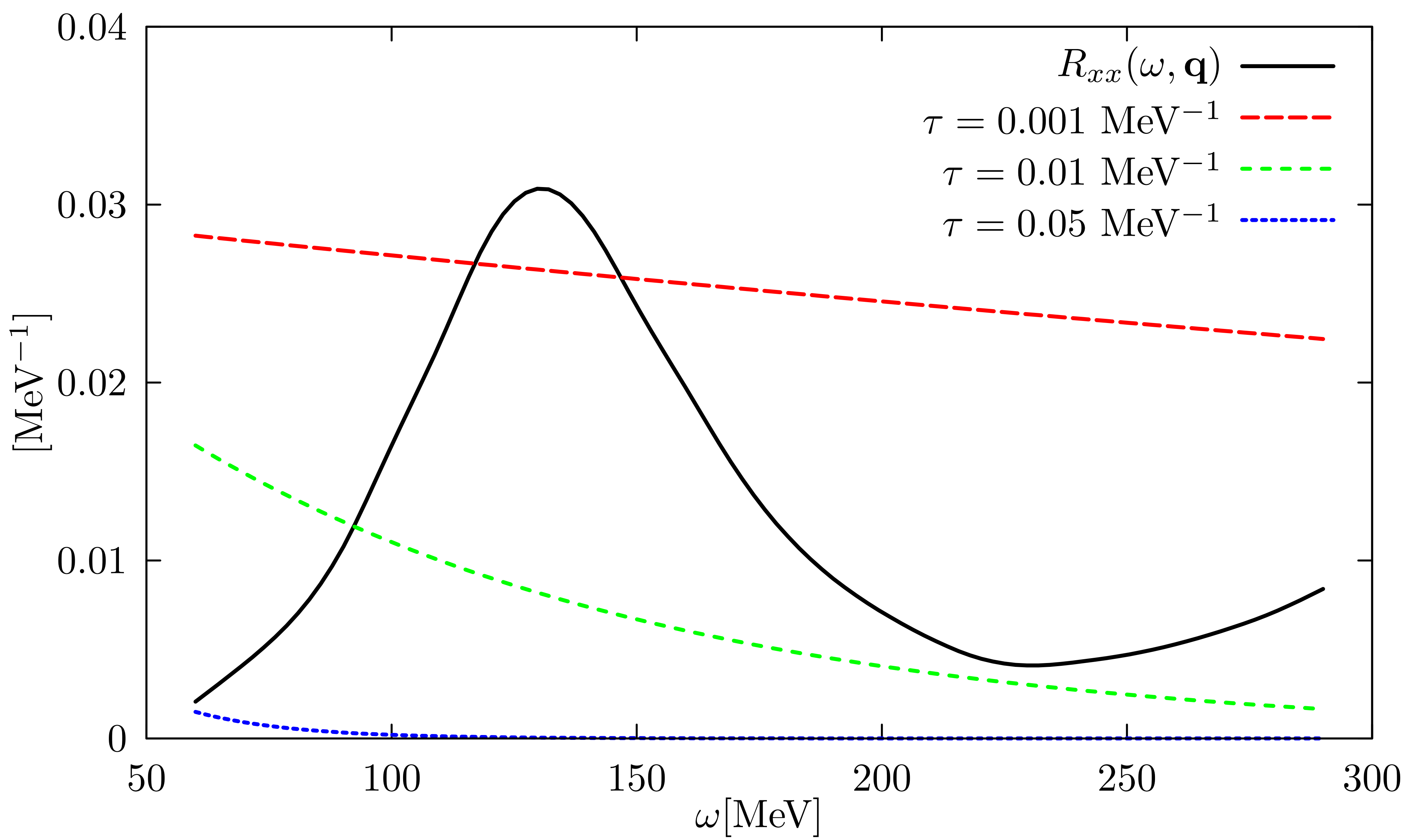
$$E_{\alpha\beta}(\tau,\mathbf{q}) = \frac{\langle\Psi_0|J_\alpha^\dagger(\mathbf{q})e^{-(H-E_0)\tau}J_\beta(\mathbf{q})|\Psi_0\rangle}{\langle\Psi_0|e^{-(H-E_0)\tau}|\Psi_0\rangle}$$

The algorithm:

- The "history" of a standard imaginary time propagation has to be saved.
- The same path has to be followed by $e^{-(H-E_0)\tau}J_\beta(\mathbf{q})|\Psi_0\rangle$
- The matrix element $\langle\Psi_0|J_\alpha^\dagger(\mathbf{q})e^{-(H-E_0)\tau}J_\beta(\mathbf{q})|\Psi_0\rangle$ has to be evaluated.

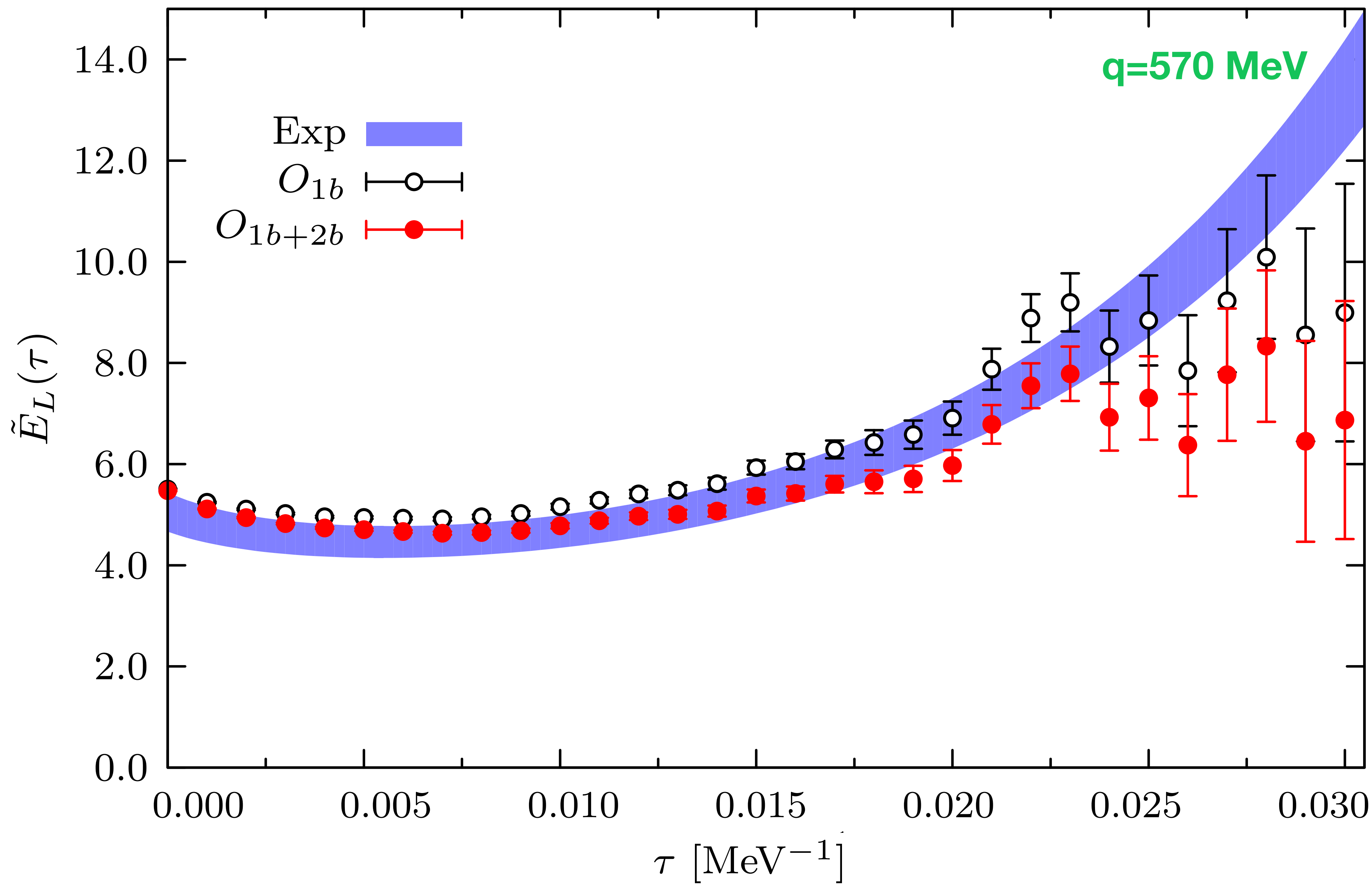
Euclidean response function

The Euclidean response at finite imaginary time very quickly suppresses the contribution from large energy transfer.



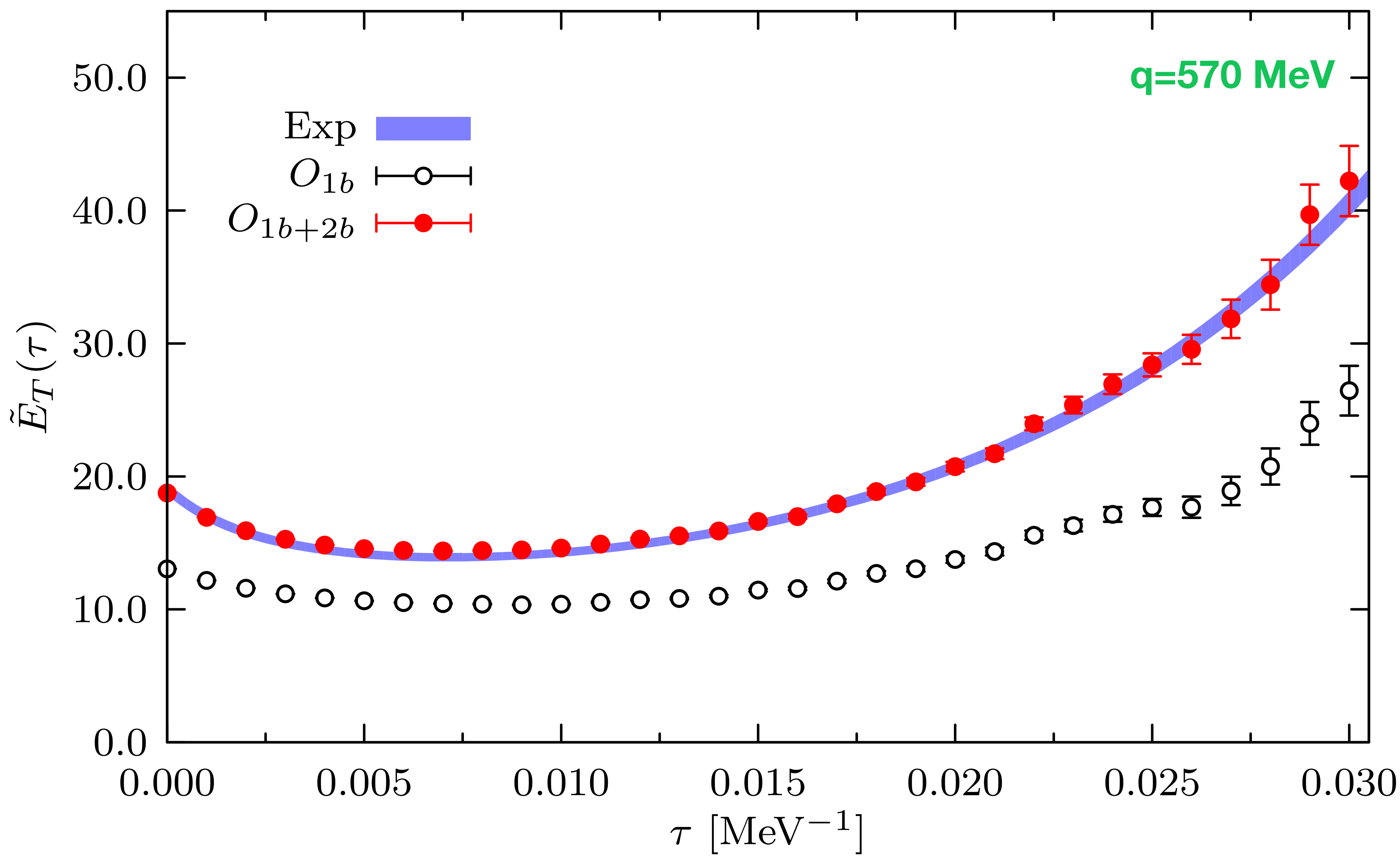
^{12}C electromagnetic Euclidean response

In the electromagnetic longitudinal case, destructive interference between the matrix elements of the one- and two-body charge operators reduces, albeit slightly, the one-body response.



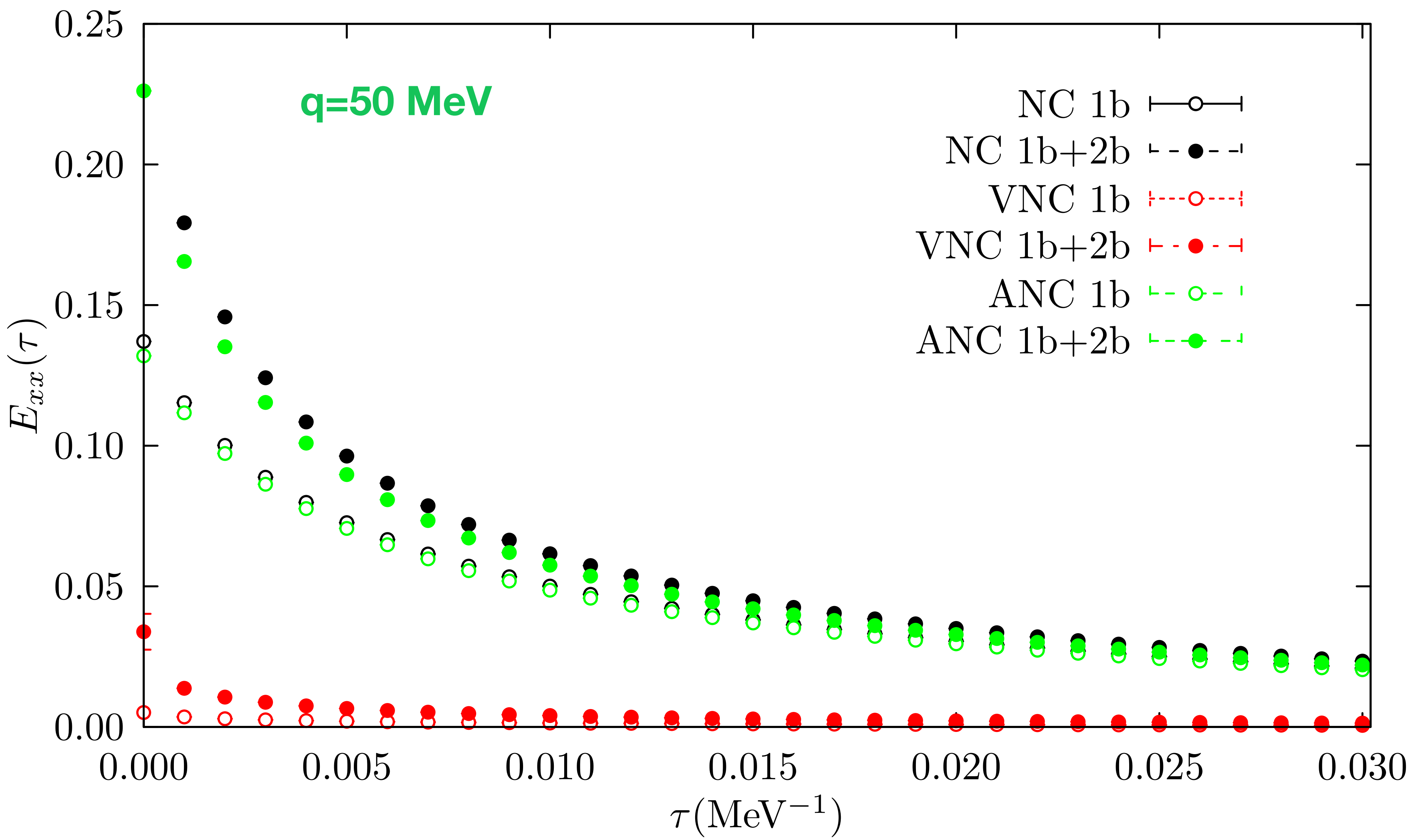
^{12}C electromagnetic Euclidean response

In the electromagnetic transverse case, two-body current contributions substantially increase the one-body response. This enhancement is effective over the whole imaginary-time region we have considered.



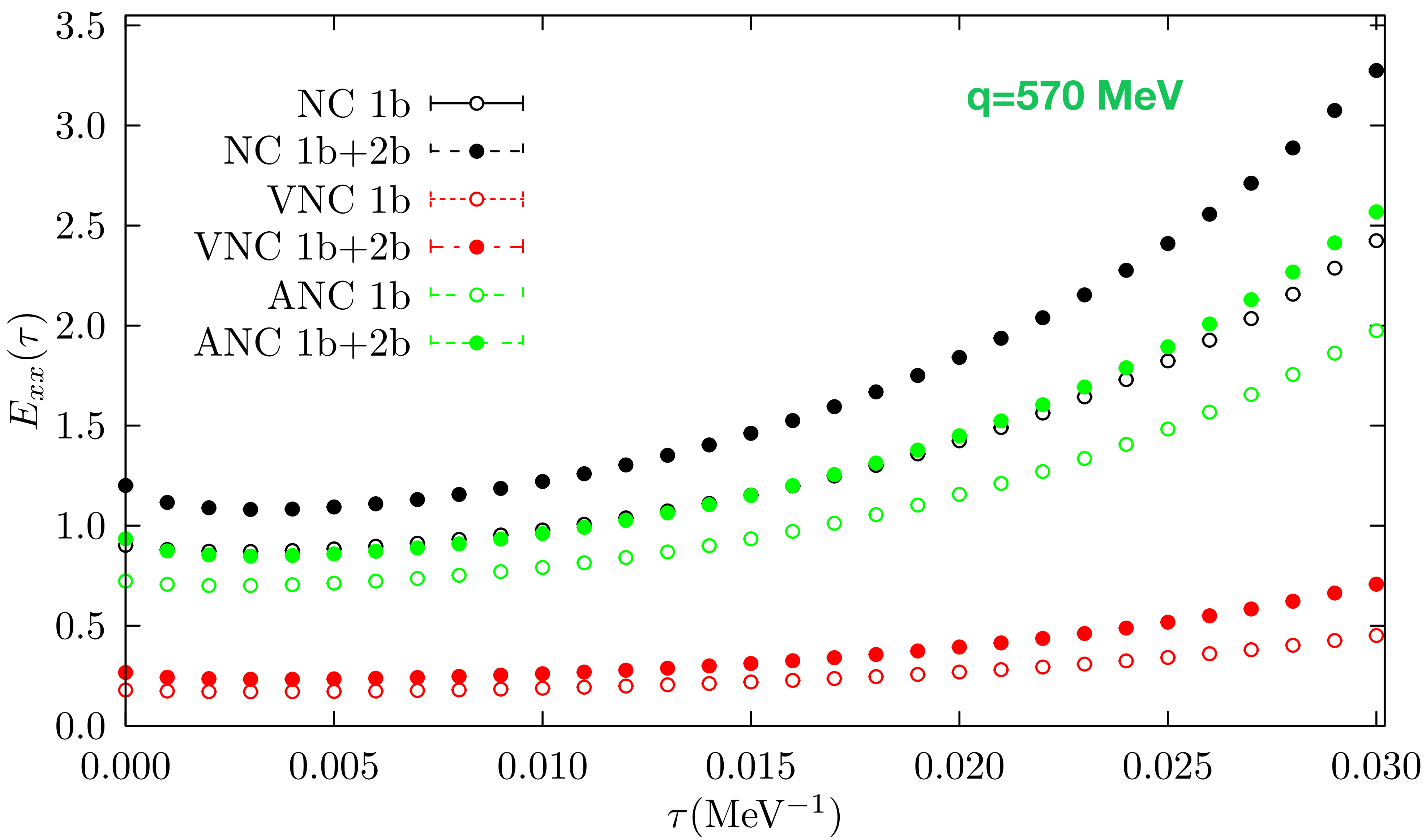
^{4}He neutral-current Euclidean response

At lower momentum transfer, our calculations indicate that the enhancement is limited to the high-energy transfer region



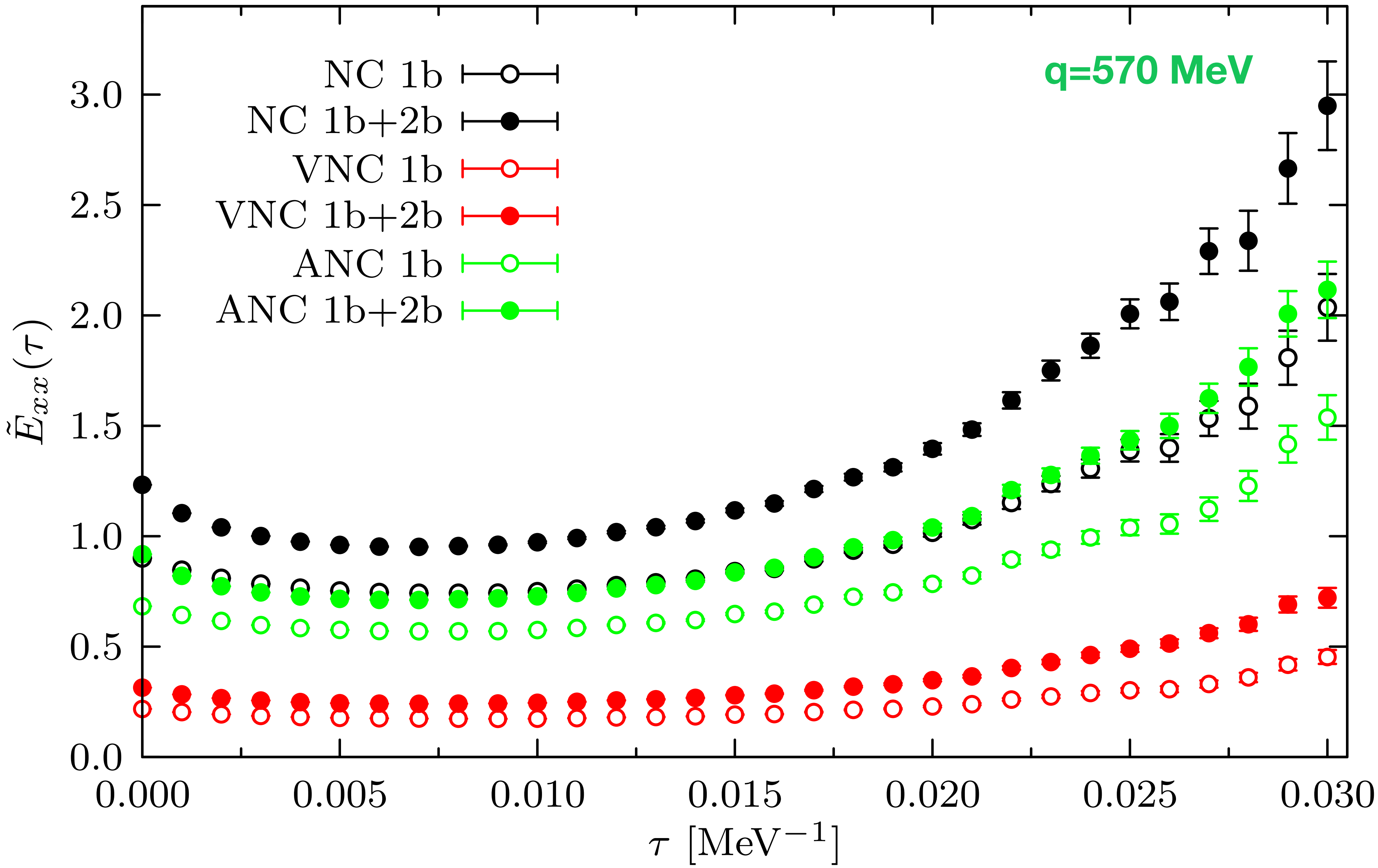
^{4}He neutral-current Euclidean response

Two-body currents enhance the transverse response function over the entire energy transfer region, and not only in the “dip region”.



^{12}C neutral-current Euclidean response

Both the vector neutral current and the axial neutral current transverse responses are substantially enhanced over the entire imaginary-time region we considered.



Inversion of the Euclidean response

The Euclidean response formalism allows one to extract dynamical properties of the system from its ground-state.

- Best suited for Quantum Monte Carlo approaches
- Wide range of applicability: atomic physics, cold atoms, neutrino scattering, neutron star cooling…

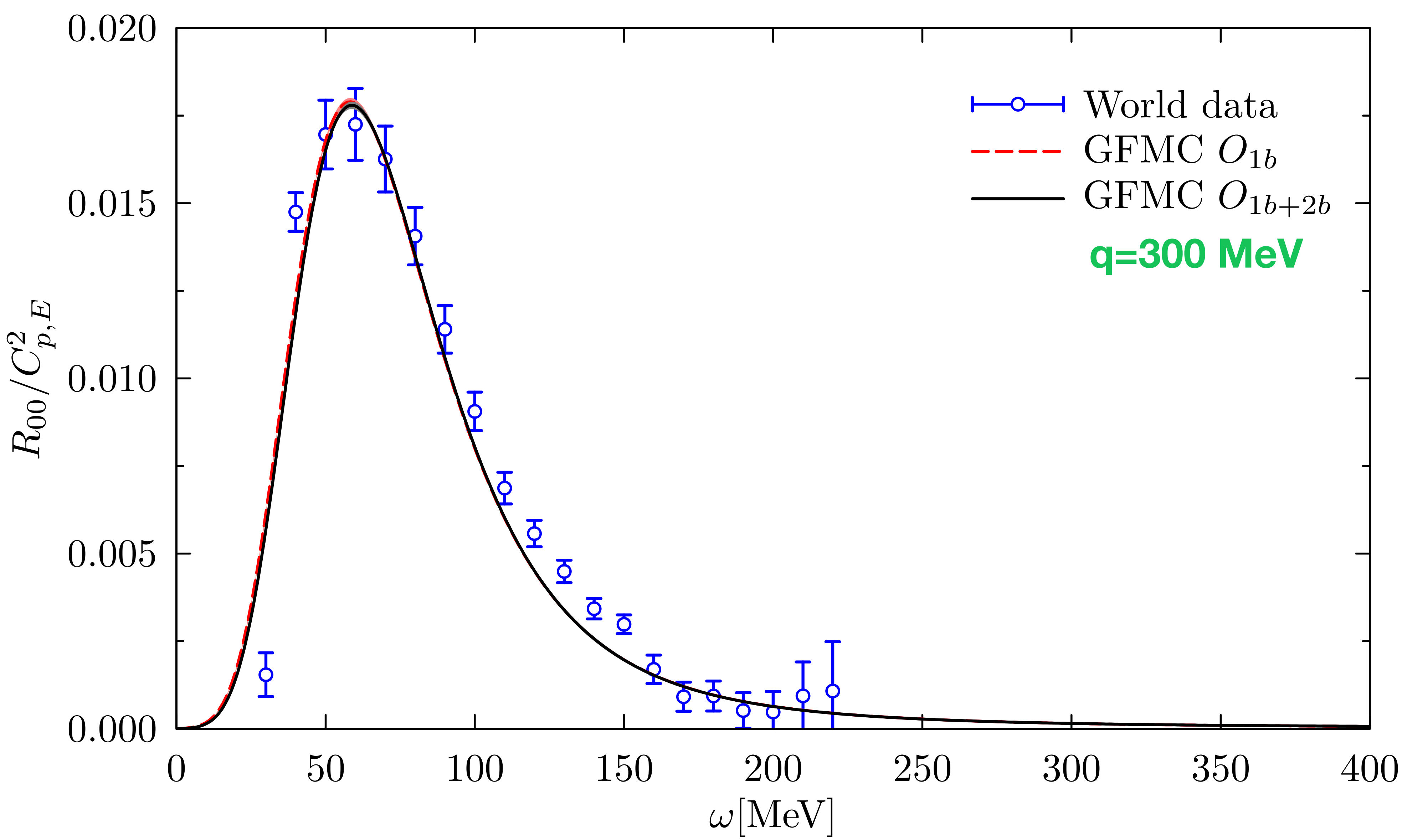
Inverting the Euclidean response is an ill posed problem: any set of observations is limited and noisy and the situation is even worse since the kernel is a smoothing operator.

$$E_{\alpha\beta}(\tau, \mathbf{q}) \longrightarrow R_{\alpha\beta}(\omega, \mathbf{q})$$

We found **historic maximum entropy** to be simple to implement and adequate for our purposes.

^{4}He electromagnetic response

Preliminary results indicate that the two-body currents do not provide significant changes in the longitudinal response.



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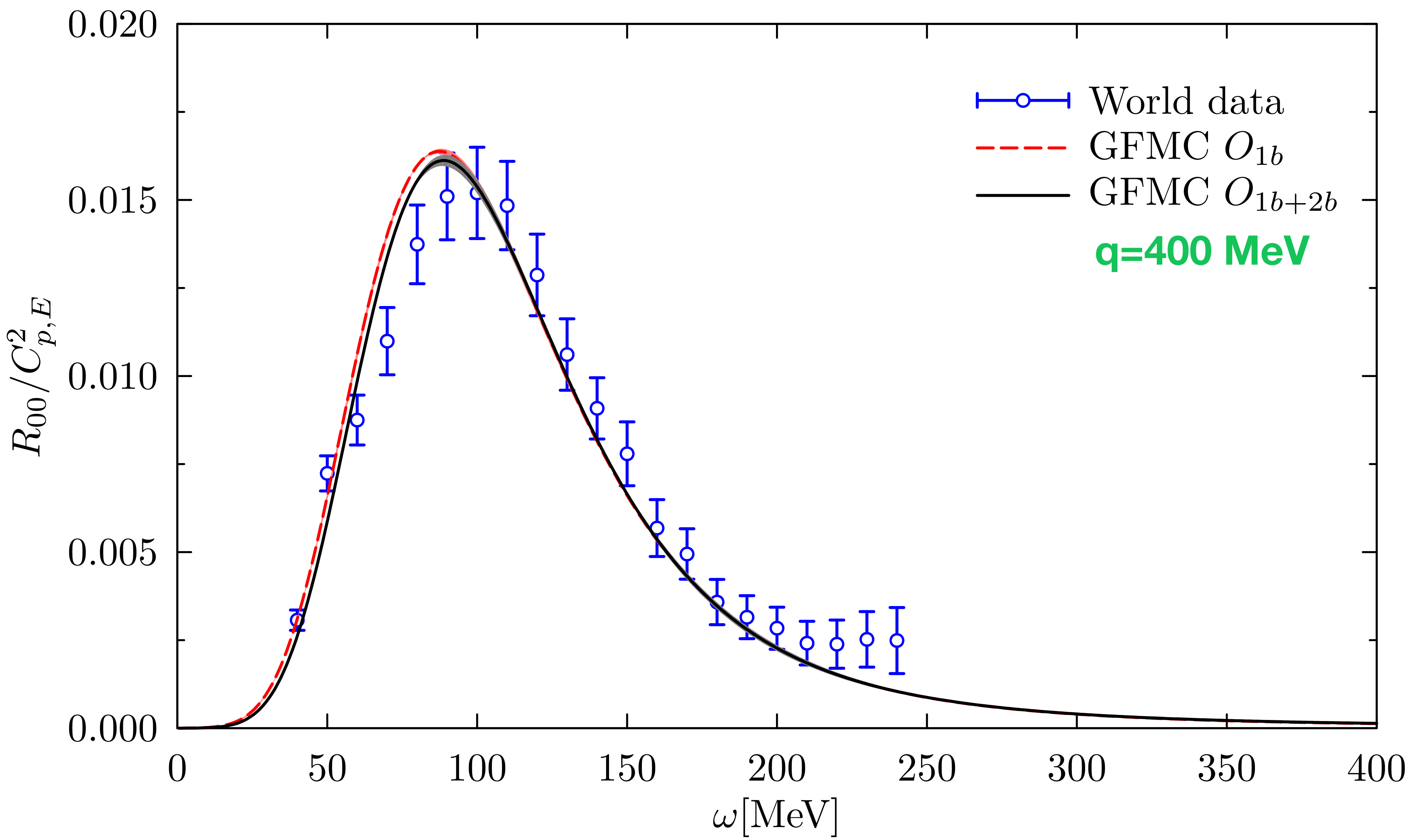
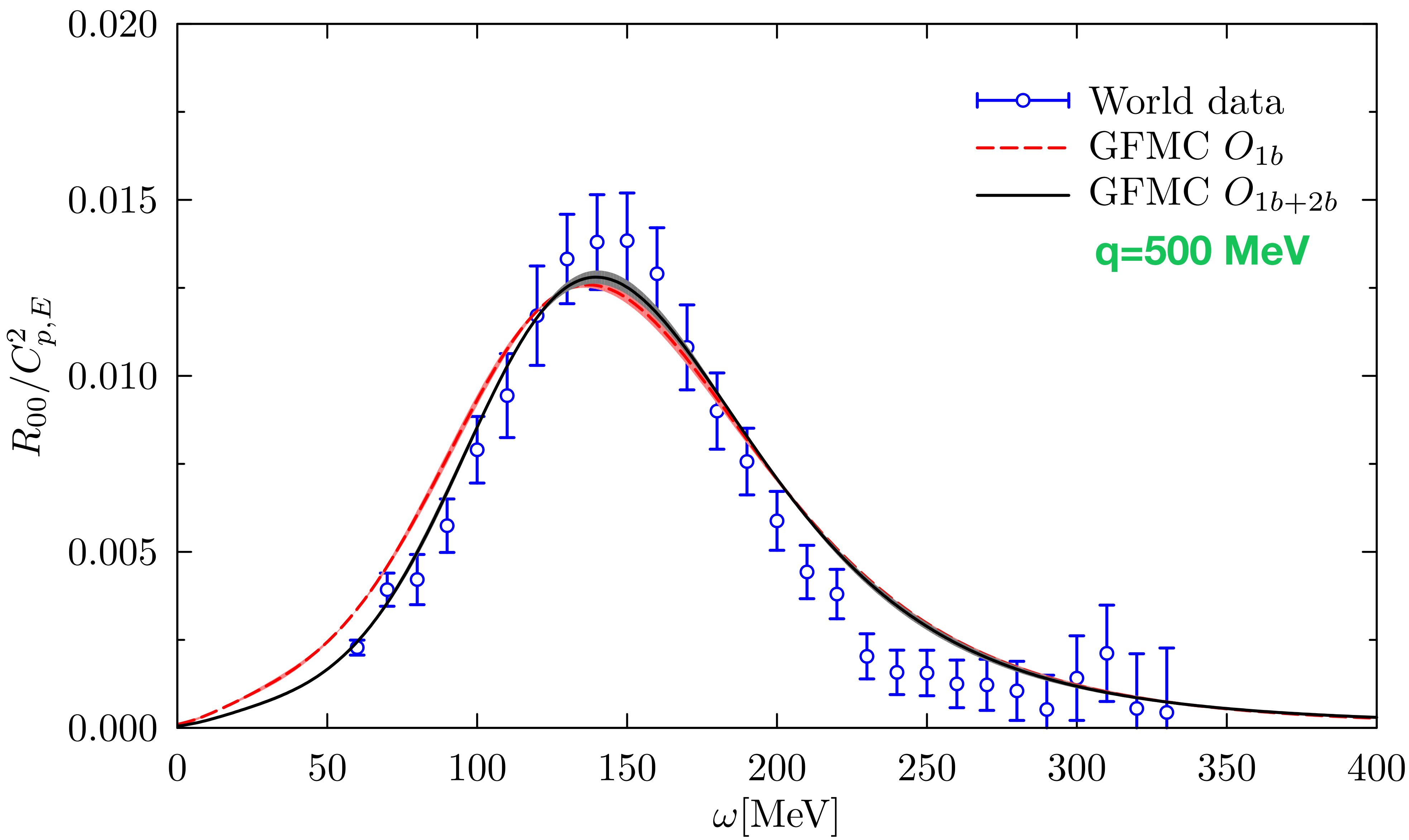


^{4}He electromagnetic response

Two-body currents do not provide significant changes in the longitudinal response.
The agreement with experimental data appears to be remarkably good.



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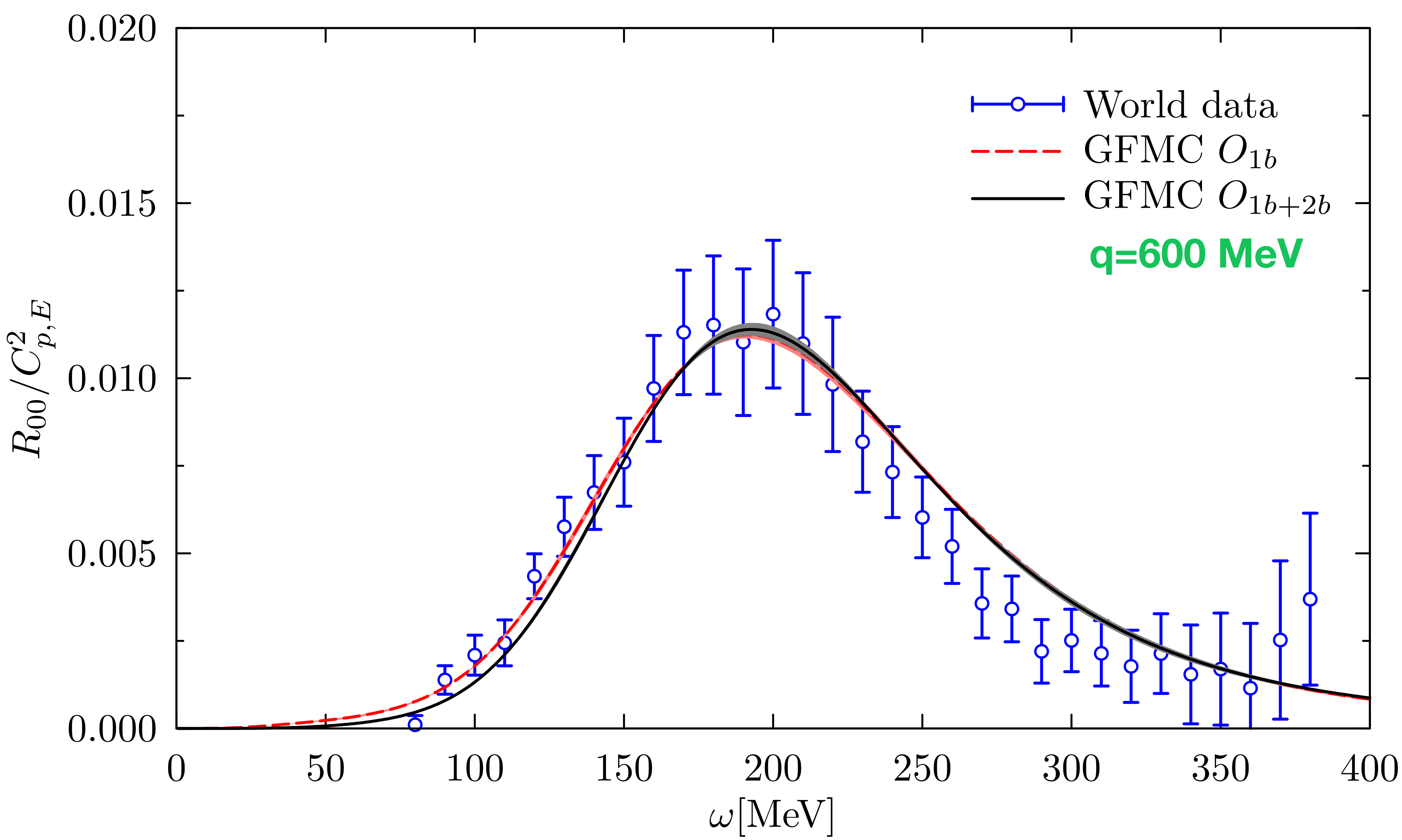

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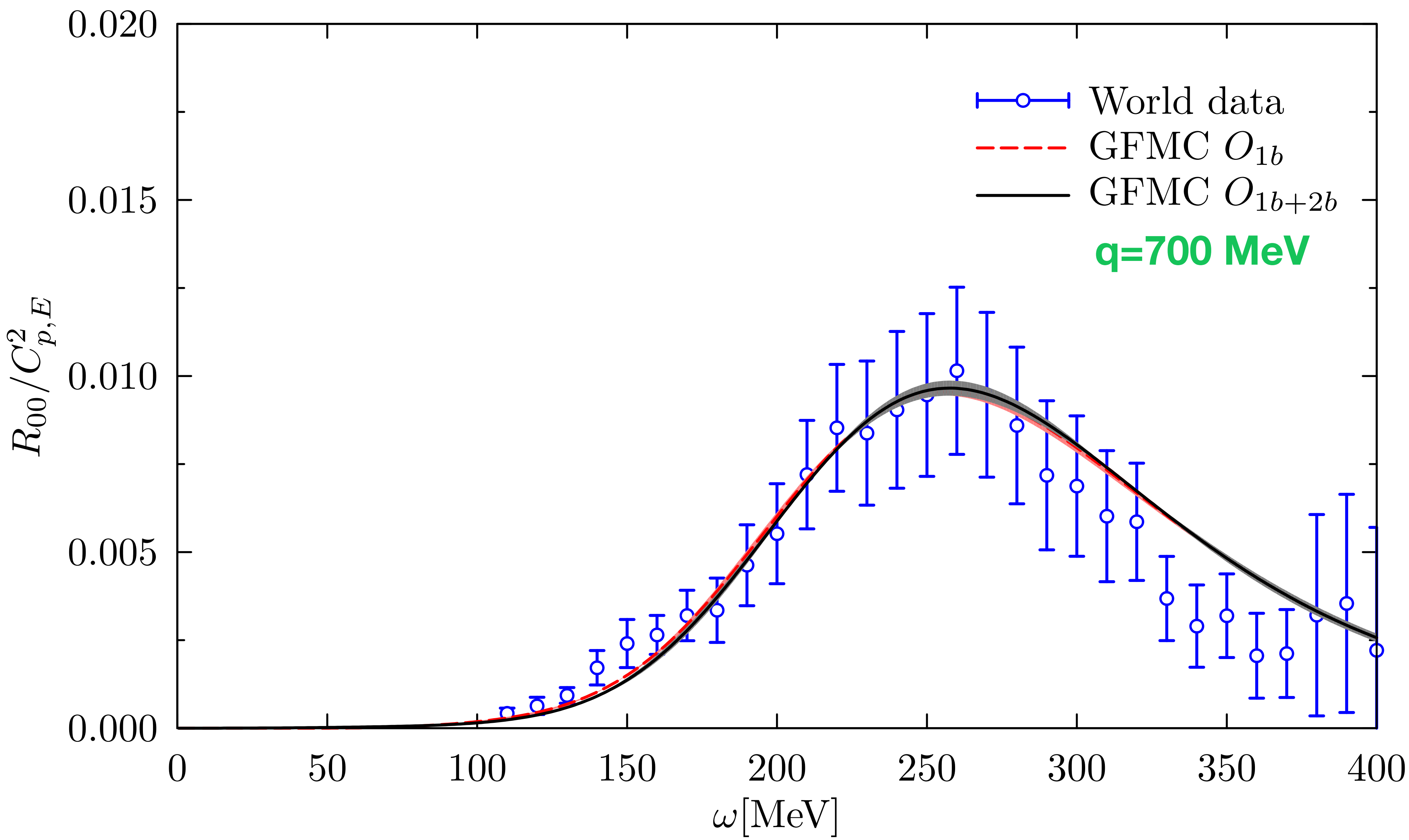
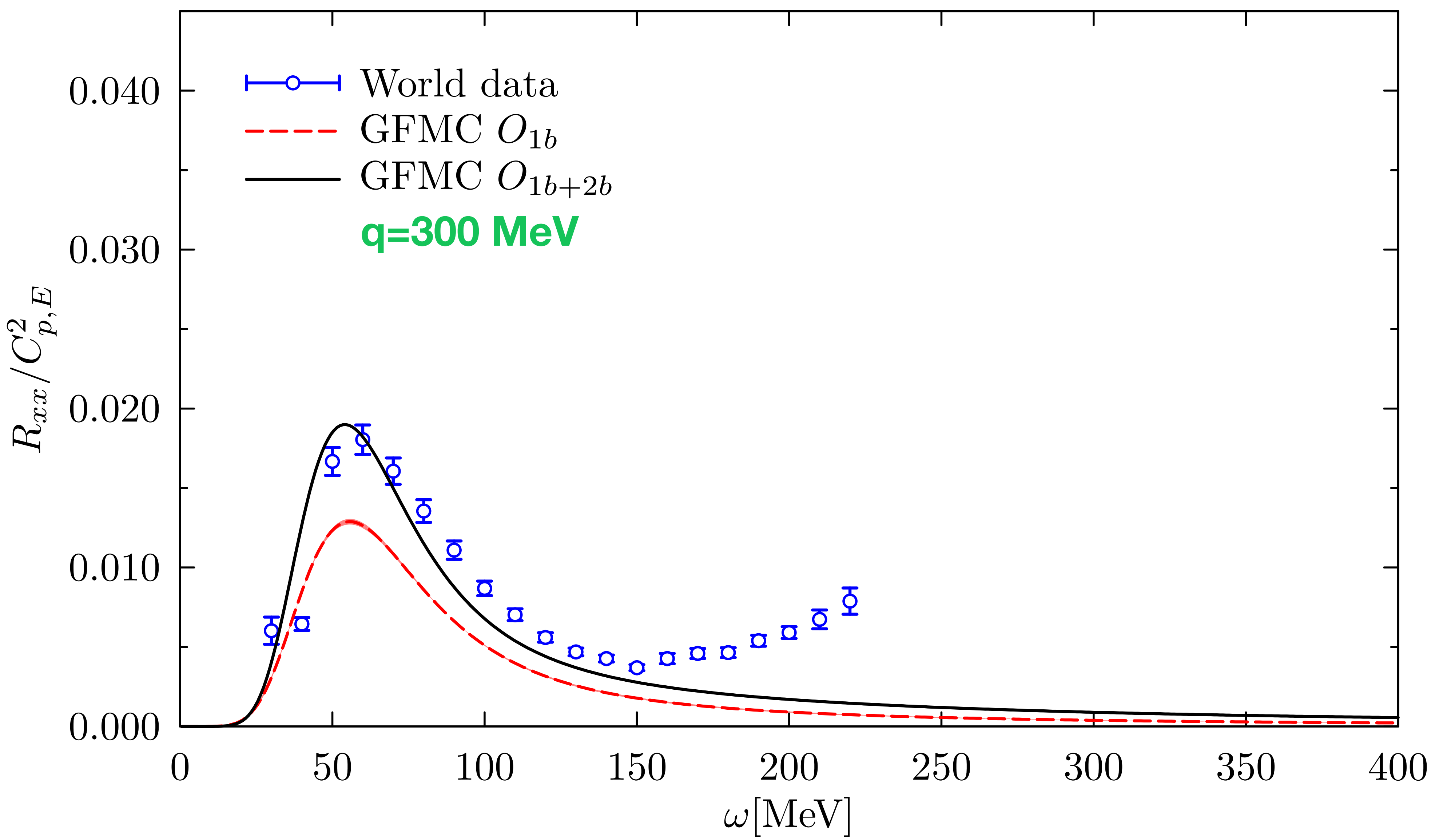


^{4}He electromagnetic response

Two-body currents significantly enhance the transverse response function, not only in the dip region, but also in the quasielastic peak and threshold regions.



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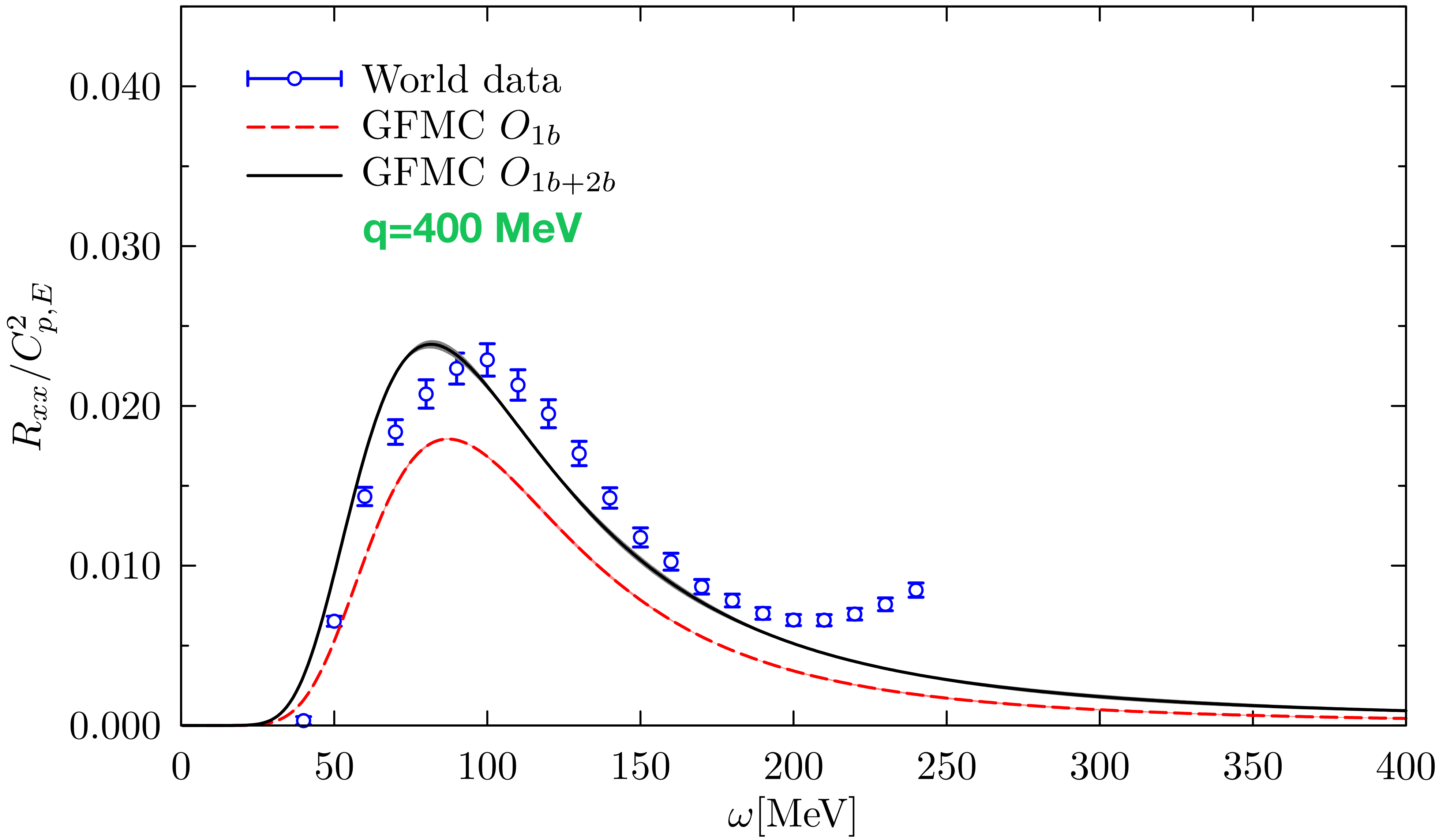

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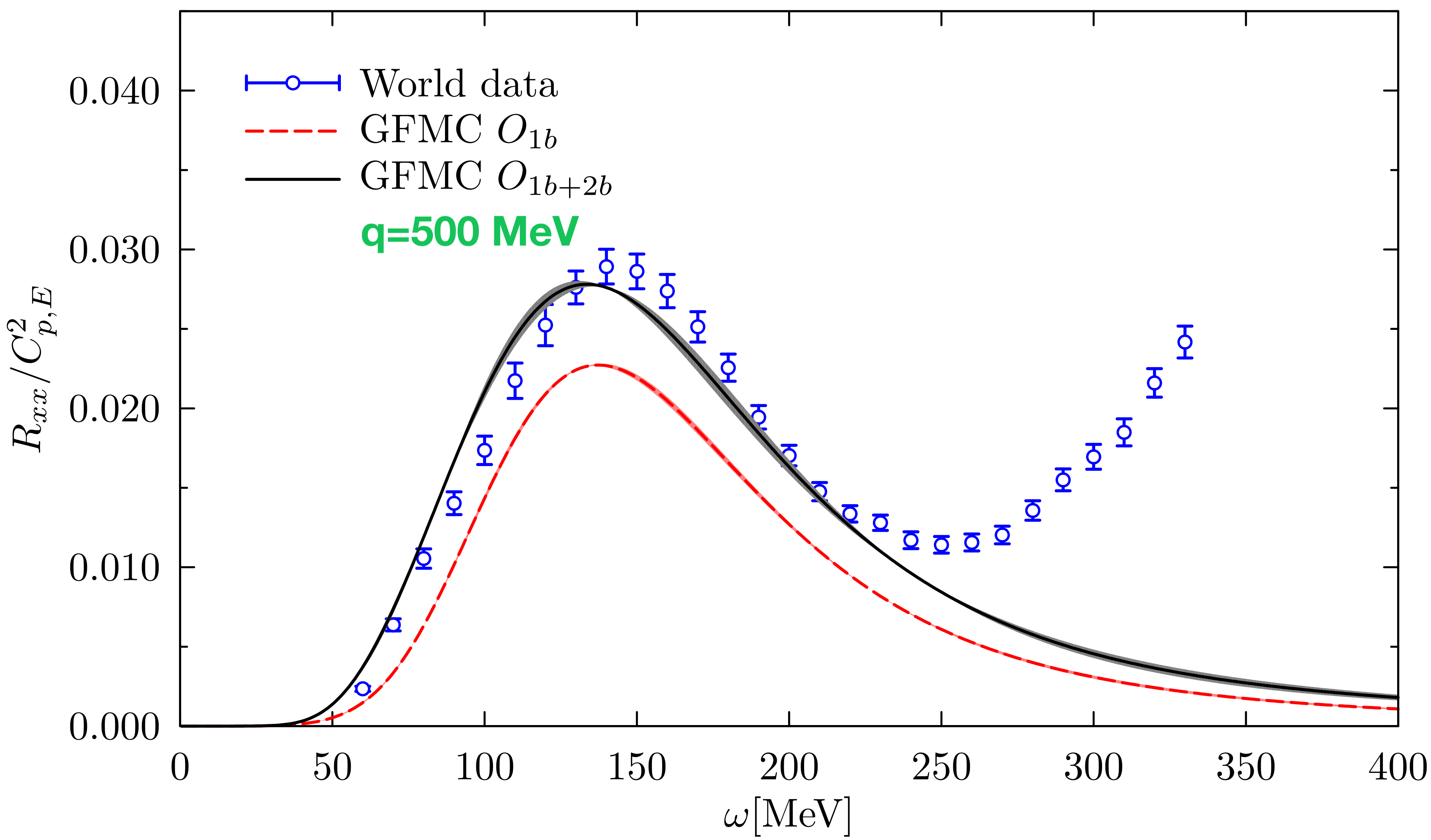

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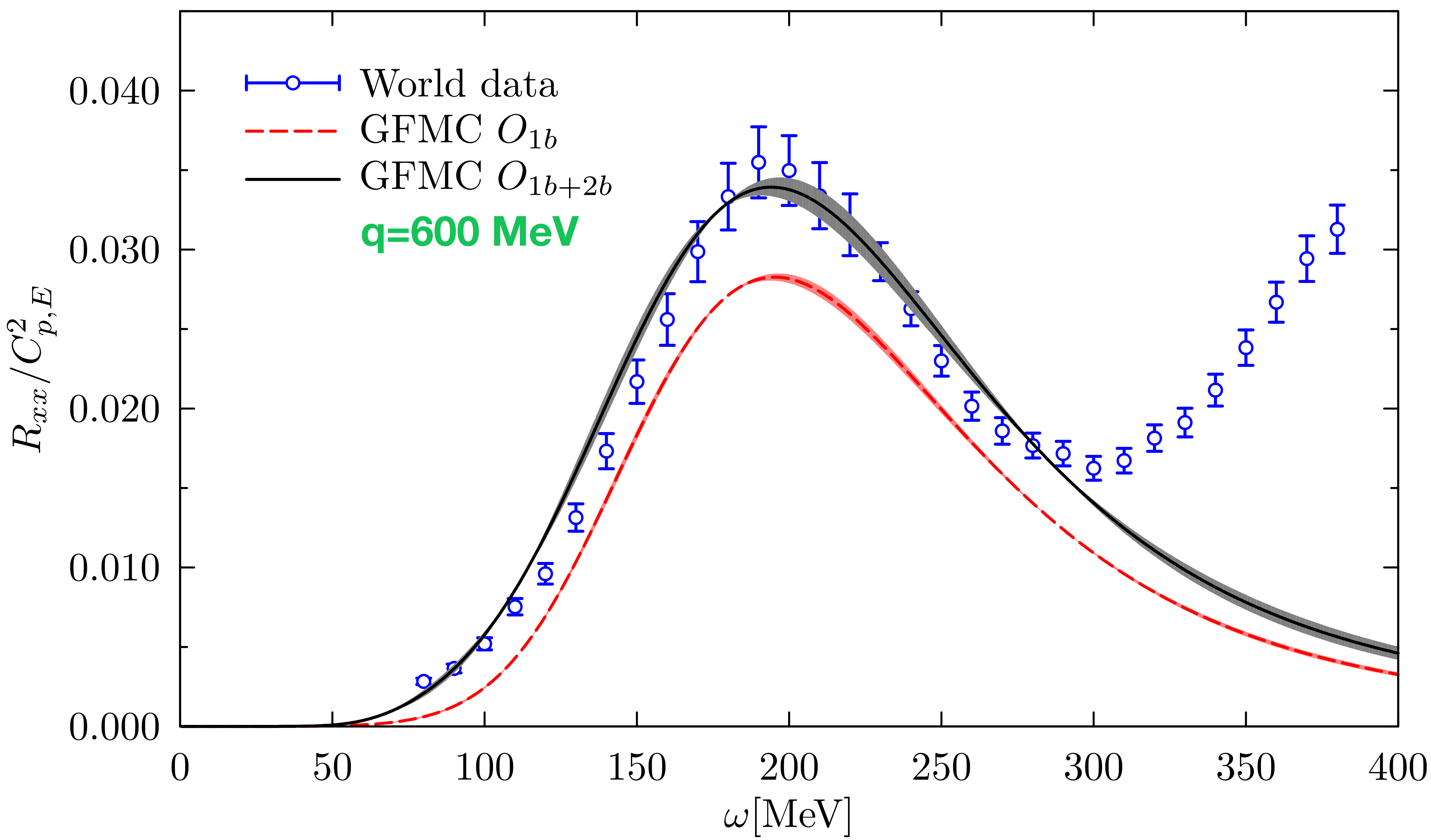

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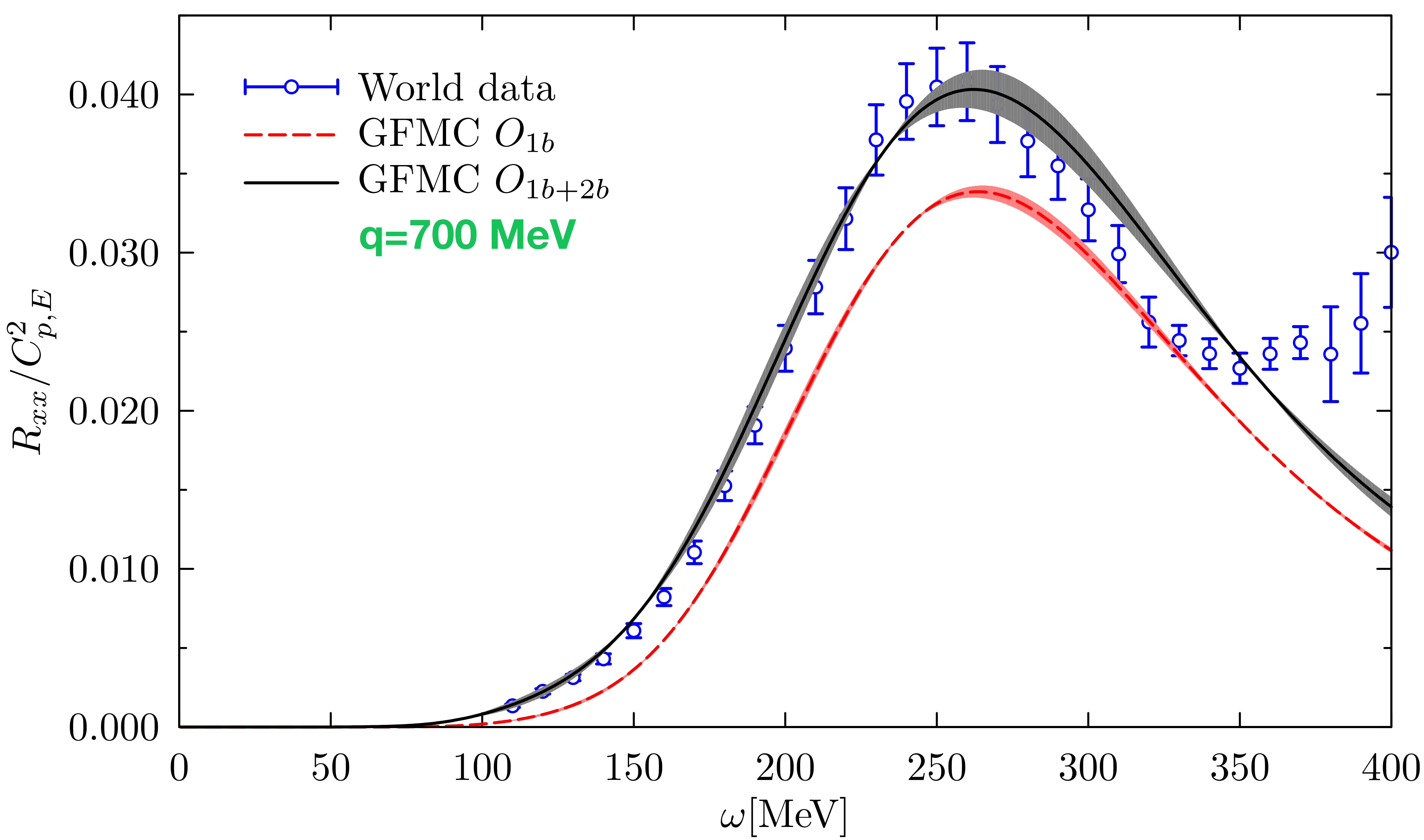

Conclusions

- For relatively large momentum transfer, the two-body currents enhancement is effective in the entire energy transfer domain.

- For small momentum transfer, two-body currents enhancement is limited to the high energy transfer region.

- ^{4}He results for the electromagnetic response obtained using Maximum Entropy technique are in very good agreement with experimental data.

- We have computed the electromagnetic and neutral-current Euclidean response of ^{12}C. Its inversion requires massive computing time ~25 million core-hours per q-value.

- The extension of the factorization scheme underlying the IA is a viable option for the development of a unified treatment of processes involving one- and two-nucleon currents in the region of large momentum transfer.

Future goals

- The chief drawback of the present GFMC method is the exponential growth in computational requirements with the number of nucleons. This limits the applicability of the method to A ≤ 12 nuclei at present.

- To deal with larger systems we have developed auxiliary-field diffusion Monte Carlo method (AFDMC). AFDMC calculations of ground-state energies of nuclei as large as ^{40}Ca have already been carried out.

- Both GFMC and AFDMC approaches provides momentum distributions that are useful for the spectral function approach, which allows to fully account for relativistic effects.

- An interplay between Quantum Monte Carlo and spectral function approaches, which rely on the same dynamical model, will be extremely beneficial.

Thank you