Effective interaction approach to the Fermi hard-sphere system

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Outline

Motivation & Objectives

- Short range repulsion in many-body interacting systems makes "standard" perturbative calculations not suitable and the introduction of a well-behaved *effective* Hamiltonian essential.
- Effective interactions based on *ab initio* microscopic approaches allow for a consistent calculation of the equilibrium and non equilibrium properties.
- The main problem related to the description of the non-equilibrium properties consists in the calculation of the probability of collisions between quasiparticles in the vicinity of the Fermi surface (Landau-Abrikosov-Khalatnikov formalism).
- Medium modifications of the scattering cross section have been consistently taken into account through an effective interaction obtained from the matrix elements of the bare interaction between correlated states (CBF).

MOTIVATION & OBJECTIVES

- Scattering cross section obtained in CBF effective interaction approach has been tested through comparison with results obtained form G-matrix perturbation theory in neutron matter.
- The calculation of transport coefficients has been carried out using the Hartree Fock approximation for the effective mass.
- The purpose of this work is to investigate concepts and assumptions employed in this procedure through the analysis of a simpler system well known in literature: the hard-sphere case.

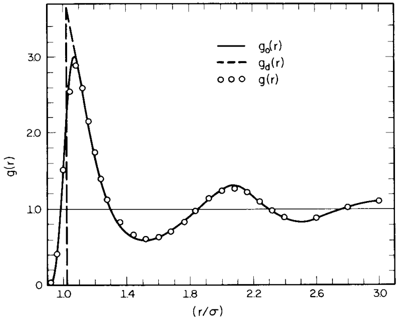
THE HARD-SPHERE MODEL

The details of the repulsive potential are not really relevant to reproduce the main properties of of several systems of fermions (classical and quantum liquids, nuclear matter).

The fermion hard-sphere fluid: a system of point-like spin one-half particles interacting through the potential

$$v(r_{ij}) = \begin{cases} \infty & r_{ij} < a \\ 0 & r_{ij} > a \end{cases}$$

The restriction to purely repulsive potential enables to neglect the possibility of Cooper pairs formation.



Using CBF effective interaction approach for the hard sphere model, we can derive several properties:

- ✓ energy per particle
- ✓ self-energy
- ✓ effective mass
- ✓ momentum distribution

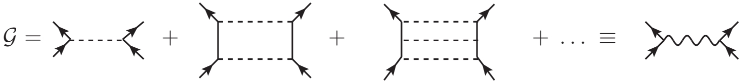
Outline

1. Introduction
2. Equilibrium properties
3. Non equilibrium properties
4. Summary & Prospects

G-matrix

- The problem has been studied by several authors using different methods and employing the usual diagrammatic techniques
- The systematic treatment in many-body perturbation theory of short range repulsion is based on the replacement of the bare interaction potential with the reaction matrix

$$\langle \mathbf{p}' | v | \mathbf{p} \rangle \to \langle \mathbf{p}' | \mathcal{G} | \mathbf{p} \rangle \, .$$



- The sum all (infinite) multiple scattering processes (ladder series) between free particles (t-matrix) or in presence of the Fermi sea background (G-matrix) makes the resulting reaction matrix a well-behaved operator, best suited for perturbative calculations
- t-matrix and G-matrix are distinguished by the different forms of the internal line propagators and the integral equation which defines the reaction matrix

LOW-DENSITY FERMI GAS

The hard-sphere model provides an accurate description of several properties of dilute Fermi systems. Algebraic expressions of the ground state energy, the single-particle energy and the momentum distribution can be written as power series in the parameter

$$c \equiv (k_F a) \;\; , \;\; k_F = (6\pi^2 \rho/\nu)^{1/3} \text{ and } \nu \text{ the degeneracy of the system.}$$

The g.s. energy per particle

$$\frac{E}{N} = \frac{k_F^2}{2m}\left[\frac{3}{5} + \frac{2}{\pi}c + \frac{12}{35\pi^2}\left(11 - 2\,ln\,2\right)c^2 + 0.780c^3 + \frac{32}{9\pi^3}(4\pi - 3\sqrt(3))c^4 \ln c + O(c^4)\right]$$

At the second order in c^2 the effective mass $m^\star$

$$\frac{1}{m^\star} = \frac{1}{k}\frac{de(k)}{dk} \;\; , \;\; \frac{m^\star(k_F)}{m} = 1 + \frac{24}{15\pi^2}(7\ln 2 - 1)c^2$$

[R.F.Bishop,Ann-Phys:11(1973)]

We will use the results as benchmarks to assess the accuracy of the effective interaction approach.

CORRELATED BASIS FUNCTION

The correlated basis ground state is defined by

$$|\Psi_0\rangle \equiv \frac{\hat{F}|\Phi_0\rangle}{\langle\Phi_0|\hat{F}^\dagger\hat{F}|\Phi_0\rangle^{1/2}}$$

The Fermi gas wave function is a Slater determinant of planet waves

$$\Phi_0 = \mathcal{A}\left[\phi_1(x_1)\ldots\phi_A(x_A)\right]$$

The correlation operator reflects the structure of the potential

$$\hat{F} = \mathcal{S}\prod_{j>i}\hat{F}_{ij} \ , \ \ \hat{F}_{ij} = \sum_p \hat{O}^p_{ij} f^p(r_{ij})$$

The variational principle

$$E_V = \langle\Psi_0|H|\Psi_0\rangle = \frac{\langle\Phi_0|\hat{F}^\dagger H\hat{F}|\Phi_0\rangle}{\langle\Phi_0|\hat{F}^\dagger\hat{F}|\Phi_0\rangle} \geq E_0$$

Correlation functions are obtained variationally by minimising E_V

Cluster Expansion Formalism

Clustering property is required for the the many body correlation operator $\hat{F}$

$$\hat{F}(x_1, \ldots, x_N) = \hat{F}(x_1, \ldots, x_p)\hat{F}(x_{p+1}, \ldots, x_N)$$

Factorization of $\hat{F}$ is the basis of the cluster expansion formalism.

Matrix elements of many body operators involve integration over the coordinates of a huge number of particles. They can can be expanded

$$\langle O \rangle = \frac{\langle \Phi | F^{\dagger} O F | \Phi \rangle}{\langle \Psi | \Psi \rangle} = O_0 + \sum_n (\Delta O)_n$$

Each term $(\Delta O)_n$ corresponds to the contribution of an isolate subsystem (*cluster*) involving an increasing number (n) of particles.

This expansion can be represented by generalized Mayer diagrams.

Summation of relevant diagrams(FHNC)

- The cluster decomposition of the $(\Delta O)_n$ is derived in terms of
  - the short range correlation function $h(r_{ij}) = f^2(r_{ij}) - 1$ from the expansion of $F^\dagger F$
  - the Slater function $\ell(x)$ with $x \equiv (k_F r_{ij})$, from expansion of $|\Phi|^2$

$$\ell(x) = \frac{\nu}{N} \sum_{|\mathbf{k}|<k_F} e^{i\mathbf{k}\cdot\mathbf{r}_{12}} = \frac{3}{x^3}(\sin x - x\cos x)$$

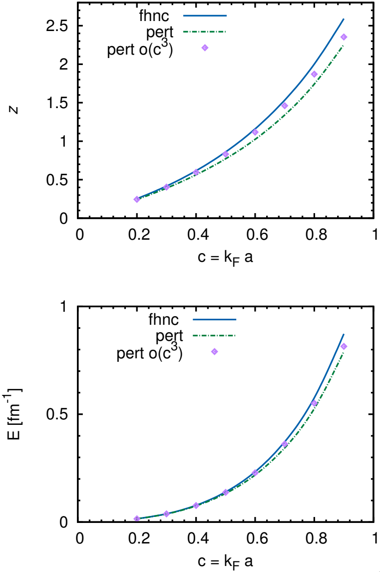
- Diagrams can be classified according to their topological structure.
- Relevant diagrams can be summed up to all orders solving a set of coupled integral equations (FHNC equations)

THE GROUND STATE ENERGY FOR THE HARD-SPHERE SYSTEM

- Within this approach, upper bounds to the g.s. energy of different systems (liquid helium, nuclear & neutron matter, Fermi HS) have been obtained
- The correlation function are determined through the minimisation of $\langle H \rangle$ with boundary condition $f(a) = 0$, $f(d) = 1$ and the additional constraint $f'(d) = 0$
- The correlation range d is the only variational parameter
- Comparison with the low-density results through the dimensionless parameter z

$$E_0 = \frac{3k_F^2}{10m}(1+z)$$

- $a = 1$ fm, $m = 1$ fm^{-1}, $\nu = 4$



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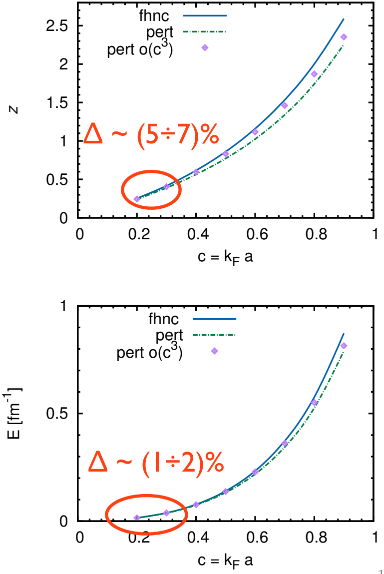


CBF effective interaction

Correlated states are obtained form the non interacting Fermi gas (FG)

$$|n\rangle = F|n_{FG}\rangle \ \ , \ \ F = \prod_{j>i} f(r_{ij})$$

For the hard sphere case

$$f(r_{ij} \leq a) = 0 \quad , \quad \lim_{r_{ij}\to\infty} f(r_{ij}) = 1 \, ,$$

The effective interaction

$$V_{\rm eff} = \sum_{j>i} v_{\rm eff}(r_{ij}) \, ,$$

is defined by the relation

$$\langle H \rangle = \frac{1}{N} \frac{\langle 0|H|0\rangle}{\langle 0|0\rangle} \equiv K_{FG} \ + \ \langle 0_{FG}|V_{\rm eff}|0_{FG}\rangle \, , \text{where} \quad K_{FG} = 3k_F^2/10m$$

- CBF effective interaction is defined in terms of its expectation value on the ground-state

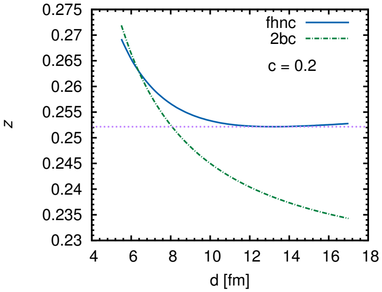
THE DEFINITION OF THE EFFECTIVE INTERACTION

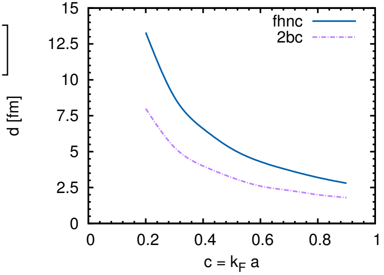
Goal: estimate the ground state energy at fist order in perturbation theory in the Fermi Gas states
Strategy: adjust the range of the correlation function in order to reproduce FHNC result at two-body cluster level

$$E_0^{FHNC} = \frac{3k_F^2}{10m} + (\Delta E)_2$$

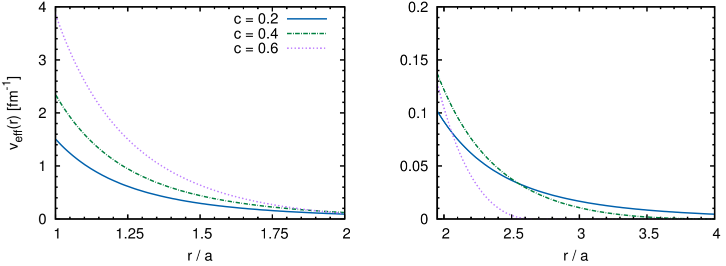
$$(\Delta E)_2 = \frac{\rho}{2} \int d\mathbf{r} \left[\frac{1}{m}|\nabla f|^2 + v(r)\right] \left[1 - \frac{1}{\nu}\ell^2(k_F r)\right]$$

- Two-body cluster approximation understimates FHNC energy
- We need a shorter correlation (stronger effective interaction) to reproduce the FHNC energy





THE EFFECTIVE INTERACTION



$$v_{\text{eff}}(r) = \frac{1}{m}\left[\boldsymbol{\nabla} f(r)\right]^2$$

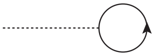
As the Fermi momentum increases

- the correlation range decreases
- the slope increases

V_{eff} contains purely kinetic contributions, deriving from the derivative of the correlation function

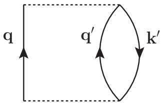
SELF-ENERGY: THE SECOND ORDER

The perturbative expansion $\Sigma(k,E) = \Sigma^{(1)}(k) + \Sigma^{(2)}(k,E) + \dots$



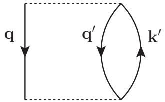
$\Sigma_{HF}(\mathbf{k})$

$$\Sigma_{HF}(k) = \frac{1}{\nu} \sum_{\sigma,\mathbf{k}'\sigma'} n^0_<(k') \langle \mathbf{k}\sigma\ \mathbf{k}'\sigma' | v_{\text{eff}} | \mathbf{k}\sigma\ \mathbf{k}'\sigma' \rangle_a$$



$\Sigma_p(\mathbf{k}, E)$

$$\Sigma_p(k,E) = \frac{m}{\nu} \sum_{\sigma,\mathbf{k}'\sigma',\mathbf{q}\tau,\mathbf{q}'\tau'} \frac{|\langle \mathbf{q}\tau\ \mathbf{q}'\tau' | v_{\text{eff}} | \mathbf{k}\sigma\ \mathbf{k}'\sigma' \rangle_a|^2}{q^2 + q'^2 - k'^2 - 2mE - i\eta} \quad \times n^0_>(q) n^0_>(q') n^0_<(k')$$

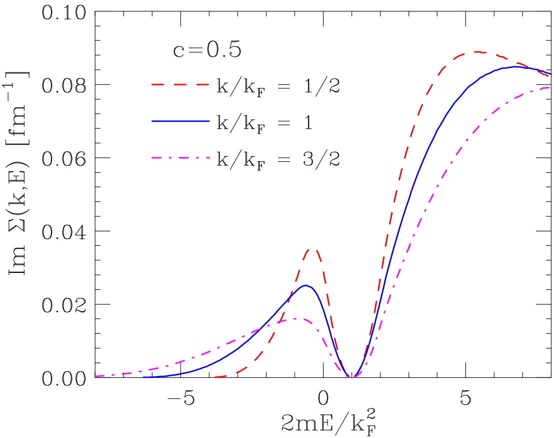


$\Sigma_h(\mathbf{k}, E)$

$$\Sigma_h(k,E) = \frac{m}{\nu} \sum_{\sigma,\mathbf{k}'\sigma',\mathbf{q}\tau,\mathbf{q}'\tau'} \frac{|\langle \mathbf{q}\tau\ \mathbf{q}'\tau' | v_{\text{eff}} | \mathbf{k}\sigma\ \mathbf{k}'\sigma' \rangle_a|^2}{k'^2 - q^2 - q'^2 + 2mE - i\eta} \quad \times n^0_<(q) n^0_<(q') n^0_>(k')$$

$$|lm\rangle_a \equiv \frac{1}{\sqrt{(2)}}(lm\rangle - |ml\rangle)$$
$$n^0_>(k) = \theta(k - k_F)\ ,\ n^0_<(k) = \theta(k_F - k)$$

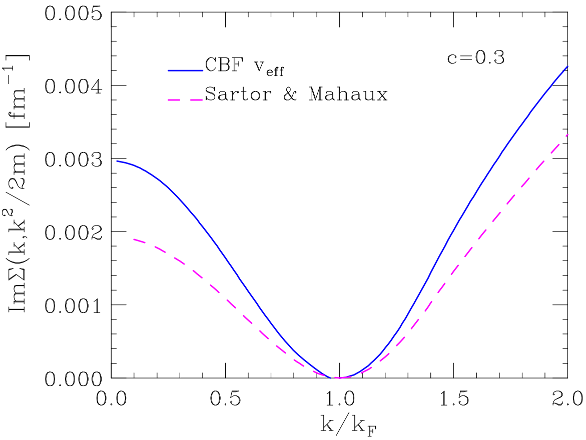
THE IMAGINARY PART



Note that

$$\Sigma_p(k, E < E_F) = 0$$

$$\Sigma_h(k, E > E_F) = 0$$



For comparison, we report $\mathrm{Im}\Sigma(k, k^2/2m)$ obtained from low-density expansion including terms up to order c^2

[R.Sartor and C.Mahaux, Phys.Rev.C21(1980)]

THE ELEMENTARY EXCITATION SPECTRUM

The self energy is responsible for shifting the pole of the Green's function

$$G(k,E) = \frac{1}{E - e_0(k) - \Sigma(k,E)}$$

- The new poles determine energy $e(k)$ and the damping Γ_k of the quasiparticles state
- For small Γ_k, the propagation of quasiparticle states (Landau's Fermi liquid theory) is described in by

$$G(k,E) = \frac{Z_k}{E - e(k) + i\Gamma_k}$$

The energy of quasiparticle

$$e(k) = e_0(k) + \mathrm{Re}\Sigma[k, e(k)]$$

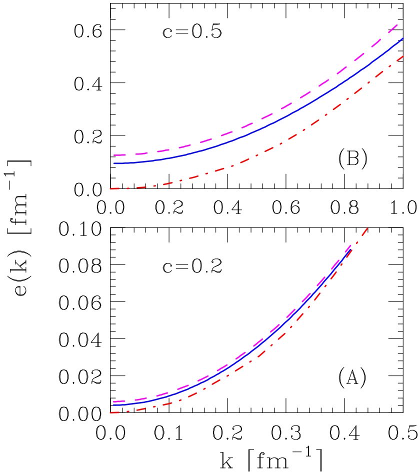
Quasiparticle lifetime

$$\tau_k^{-1} = \Gamma_k = Z_k \mathrm{Im}\Sigma[k, e(k)]$$

The residue of the Green's function

$$Z_k = \left[1 - \frac{\partial}{\partial E}\mathrm{Re}\Sigma[k,E]\right]^{-1}_{E=e(k)}$$

ENERGY OF QUASIPARTICLE



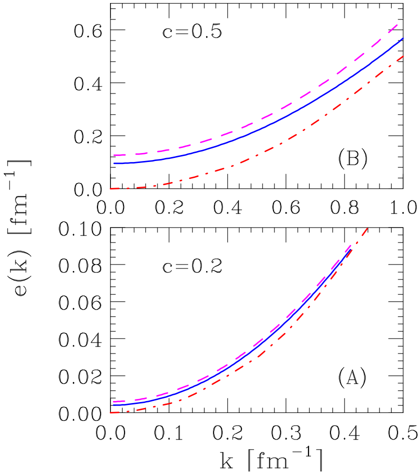
The energy of quasiparticle is obtained solving the equation

$$e(k) = e_0(k) + \mathrm{Re}\Sigma[k, e(k)]$$

The single particle spectrum can be parametrized in term of the effective mass $m^\star$

$$m^\star = \left[\frac{1}{k}\frac{de(k)}{dk}\right]^{-1}$$

Energy of quasiparticle



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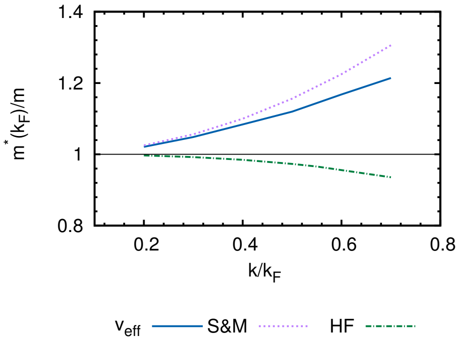
$$m^\star = \left[\frac{1}{k}\frac{de(k)}{dk}\right]^{-1}$$

Since $\mathrm{Re}\,\Sigma(k, E)$ is evaluated at the quasiparticle pole $e(k)$, k and E are non independent variables

The effective mass

$$\frac{de(k)}{dk} = \left[\frac{k}{m} + \frac{\partial}{\partial k}\mathrm{Re}\Sigma\,(k,E)\right]\left[1 - \frac{\partial}{\partial E}\mathrm{Re}\Sigma\,(k,E)\right]^{-1}$$



- Enhancement for $m^\star$ due to the energy-dependent corrections to the self-energy
- At Hartree Fock level $m^\star/m < 1$

Momentum distribution

- Momentum distribution describes the occupation probability of the single-particle of momentum k (see Källén-Lehman representation of $G(k,E)$)

$$G(k,E) = \int_0^\infty dE' \left[\frac{P_p(k,E)}{E - E' - \mu + i\eta} + \frac{P_h(k,E)}{E + E' - \mu - i\eta} \right] \ , \ \mu = e(k_F)$$

- In term of the particle (hole) spectral functions

$$n(k) = \int_0^\infty dE P_h(k,E) = 1 - \int_0^\infty dE P_p(k,E)$$

- In terms of the quasiparticles properties

$$n(k) = Z_k \theta(k_F - k) + \delta n(k)$$

with Z_k from the quasiparticle pole and $\delta n(k)$ a smooth contribution arising from more complex excitations of the system ($k \gtrless k_F$).

MOMENTUM DISTRIBUTION

Exploiting Dyson's equation, $n(k)$ can be determined through the knowledge of the self-energy $\Sigma(k, E)$, computed at the second order

The discontinuity at $k = k_F$ is given by

$$n(k_F - \eta) - n(k_F + \eta) = Z_{k_F} = Z$$

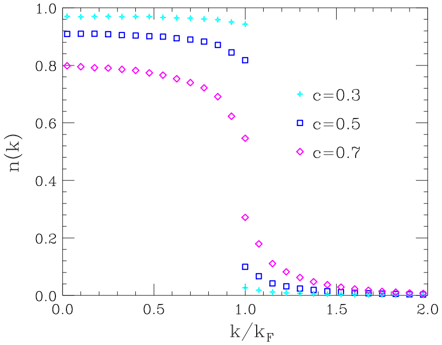
$$n(k) = n_<(k) + n_>(k)$$

with

$$n_<(k > k_F) = n_>(k < k_F) = 0$$

$$n_<(k < k_F) = 1 + \left[\frac{\partial}{\partial E}\text{Re}\Sigma_p(k, E)\right]_{E=e_0(k)}$$

$$n_>(k > k_F) = -\left[\frac{\partial}{\partial E}\text{Re}\Sigma_h(k, E)\right]_{E=e_0(k)}$$



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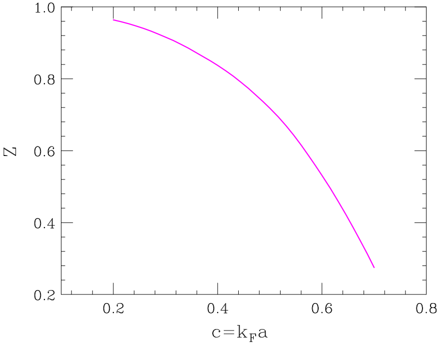
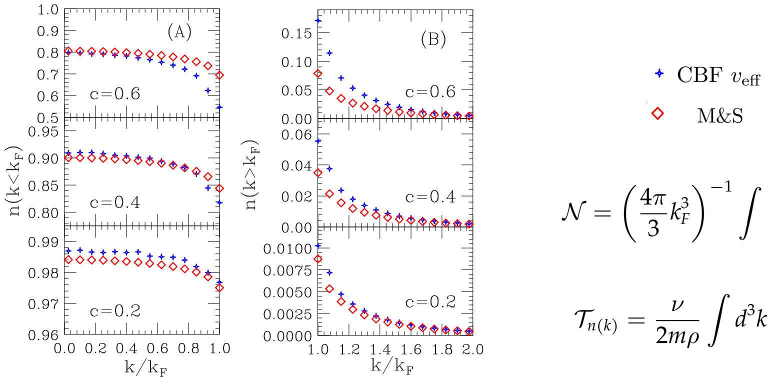


CBF effective interaction in comparison with low-density expansion



$$\mathcal{N} = \left(\frac{4\pi}{3}k_F^3\right)^{-1}\int d^3k\, n(k)$$

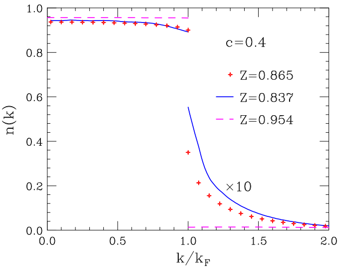
$$\mathcal{T}_{n(k)} = \frac{\nu}{2m\rho}\int d^3k\, n(k)k^2$$

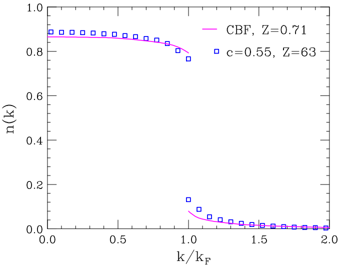
[R.Sartor and C.Mahaux, Phys.Rev.C21(1980)]

k_F	$\mathcal{N}$	$\mathcal{T}[fm^{-1}]$	$\mathcal{T}_{n(k)}[fm^{-1}]$	$\Delta\mathcal{T}(\%)$
0.2	1.003 (0.998)	0.0150	0.0132 (0.0127)	12.5(15.3)
0.4	1.000 (0.983)	0.0775	0.0615 (0.0584)	20.6 (24.6)
0.6	1.001 (0.958)	0.2331	0.1727(0.1533)	25.9 (34.2)

CBF Effective interaction in comparison with CBF variational results

[A.Fabrocini,S. Fantoni,A.Polls,andS.Rosati,NuovoCimentoA56,33(1980)]





[S.Fantoni and V.R. Pandharipande, Nucl.Phys.A427, 473(1984)]

Momentum distribution of the hard sphere system (a = 1 fm , k_F = 0.55) corresponds to nuclear matter at density ρ_{NM} = 0.16 fm^{-3} and k_F = 1.33 fm^{-1}

Nucleons in NM $\sim$ hard spheres of radius $a = 0.55/1.33 \sim$ 0.4 fm

Calculation simpler in the effective interaction approach than in the non-orthogonal CBF perturbation theory

OUTLINE

BOLTZMANN LANDAU'S KINETIC THEORY

Scattering of quasiparticles in a normal Fermi fluid is analysed in the framework of BL's kinetic theory: the kinetic equation for the single QP distribution function $n(\mathbf{p}, \mathbf{r}, t)$ takes the form of a non homogeneus continuity equation

$$\frac{\partial n}{\partial t} + \frac{\partial n}{\partial \mathbf{r}} \cdot \mathbf{v} + \frac{\partial n}{\partial \mathbf{p}} \cdot \mathbf{F} = \mathcal{I}(n)$$

Following the Landau's interpretation of QP energy $e(\mathbf{p}, \mathbf{r}, t)$ as QP Hamiltonian

$$\frac{\partial n}{\partial t} + \frac{\partial n}{\partial \mathbf{r}} \frac{\partial e}{\partial \mathbf{p}} - \frac{\partial n}{\partial \mathbf{p}} \frac{\partial e}{\partial \mathbf{r}} = \mathcal{I}(n)$$

The collision integral $\mathcal{I}(n)$ describes the rate of particles entering into an infinitesimal region of phase space due to two particle collisions.

The scattering probability

At low T, we can consider only binary collisions $1,2 \to 3,4$ (and the inverse process $3,4 \to 1,2$), and define the scattering probability $\mathcal{W}(12;34)$ though the Fermi's golden rule

$$\frac{2\pi}{\hbar}|\langle 3,4|\mathcal{T}|12\rangle|^2 \equiv \frac{1}{V^2}\mathcal{W}(12;34)\delta(p_1+p_2-p_3-p_4)\delta(\sigma_1+\sigma_2,\sigma_3+\sigma_4)$$

$$\mathcal{I} = -\frac{1}{V^2}\sum_2\sum_{3,4}\mathcal{W}(12;34)\delta(p_1+p_2-p_3-p_4)\delta(\sigma_1+\sigma_2,\sigma_3+\sigma_4)$$
$$\times\left[n_1n_2(1-n_3)(1-n_4)-(1-n_1)(1-n_2)n_3n_4\right]$$

In low-T regime only QP states next to the Fermi surface are involved in collision $|\mathbf{p}_i| = p_F$
A-K reference frame: $\mathcal{W}(12;34) = \mathcal{W}(\theta,\phi)$

- θ the angle between the incoming momenta $\mathbf{p}_1$ and $\mathbf{p}_2$
- ϕ the angle between the planes containing $(\mathbf{p}_1,\mathbf{p}_2)$ and $(\mathbf{p}_3,\mathbf{p}_4)$

$$E_{cm} = \frac{p_F^2}{2m}(1-cos\theta) \;\; \text{and} \;\; \theta_{cm} = \phi$$

Medium effects

- Nuclear medium mainly affects the flux of incoming particles and the phase-space available to the final state particles, leaving the transition probability unchanged

$$\mathcal{W}(\theta,\phi) = \frac{16\pi^3}{m^{\star 2}} \left(\frac{d\sigma}{d\Omega}\right)_{vac}$$

- Include the effects of medium-modifications in the scattering amplitude through CBF $v_{\rm eff}$

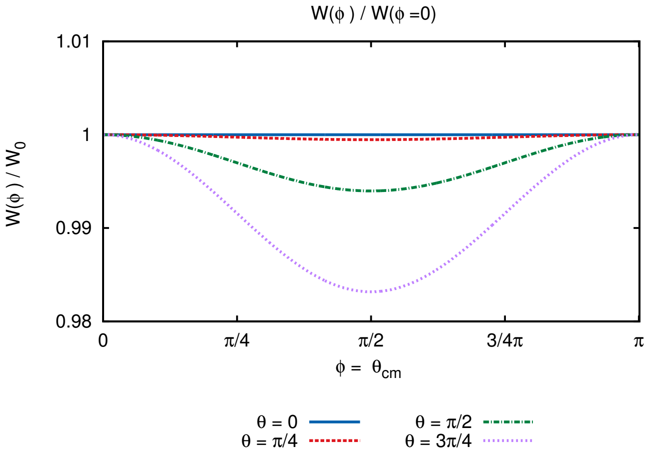
$$\frac{d\sigma}{d\Omega} = \frac{m^{\star 2}}{16\pi^2} |v_{\rm eff}(\mathbf{q})|^2 \ ,$$

- The resulting $\mathcal{W}(\theta,\phi)$ can be used the calculate the QP lifetime & transport coefficients

$$\eta_{AK} = \frac{1}{5}\rho m^\star v_F^2 \tau \frac{2}{\pi^2[1-\lambda_\eta]}$$

where $\tau = \frac{8\pi^4}{m^{\star 3}\langle\mathcal{W}\rangle T^2}$, $\langle\mathcal{W}\rangle \equiv \int \frac{d\Omega}{2\pi}\frac{\mathcal{W}(\theta,\phi)}{\cos(\theta/2)}$ and $\lambda_\eta = \frac{\langle\mathcal{W}[1-3\sin^4(\theta/2)sin^2\phi]\rangle}{\langle\mathcal{W}\rangle}$

$$\mathcal{W}(\theta,\phi) = \pi |v_{\rm eff}(\mathbf{q})|^2 \quad , \quad |\mathbf{q}| = p_F \sqrt{(1-\cos\theta)(1-\cos\phi)}$$



SUMMARY & PROSPECTS

- CBF effective interaction has been employed to compute the self-energy for a hard-sphere system. Calculation of second order terms in V_{eff} has been carried out
- Quasiparticle properties have been obtained (single particle energy, effective mass, momentum distribution), significantly affected by energy dependent second order corrections.
- Comparison with results obtained in low-density expansion:
  - ✓ good agreement for density corresponding to $k_F \gtrsim 0.3\ \text{fm}^{-1}, 0.4\ \text{fm}^{-1}$
  - ✓ discrepancies at higher density, where contributions of higher power of c are not negligible
- The strategy of including the effect of correlations in the definition of the effective potential allows perturbative calculations in the basis of FG: remarkable simplification in comparison with CBF non-orthogonal perturbation theory.
- Comparison with results obtained in different many-body techniques could be performed.

Thank you!

Background slides

Euler-Lagrange equation

From the expression of the energy obtained by the two-body cluster expansion

$$\frac{(\Delta E)_2}{N} = \frac{\rho}{2} \int d\mathbf{r} \frac{1}{m} (\nabla f)^2 g_{FG}(r) \sim \int F[f, f'] dr$$

Correlation functions are obtained solving the Euler-Lagrange equation

$$g'' - \left[\frac{a''(r)}{a(r)} + \lambda \right] g(r) = 0$$

$$a^2(r) = r^2 g_{FG}(r) \ , \ g(r) = a^2(r) f^2(r)$$

with boundary conditions

$$f(a) = 0 \ , \ f(d) = 1 \ , \ f'(d) = 0$$

λ is a Lagrange multiplier introduced to impose the constraint on the derivative.

Remarks

Ambiguity involved in the calculation kinetic term

$$\langle T \rangle_{2b} = -\frac{\rho}{2m} \int d\mathbf{r} f(r) \left[\nabla^2, f(r) \right] \left(1 - \frac{1}{\nu} \ell^2(k_F r) \right)$$

Integrating by parts the kinetic term, the derivatives acting both on the correlation function and $g_{FG}(r)$ can be removed.

$$\langle T \rangle_{2b} = \frac{\rho}{2m} \int d\mathbf{r} \ (\nabla f)^2 \left(1 - \frac{1}{\nu} \ell^2(k_F r) \right)$$

The two-body cluster expansion of the effective potential

$$\langle 0_{FG} | \, V_{\rm eff} \, | 0_{FG} \rangle_{2b} = \frac{\rho}{2} \int d\mathbf{r} \, v_{\rm eff}(r) \left(1 - \frac{1}{\nu} \ell^2(k_F r) \right)$$

The Green's function

With the CBF effective interaction we can obtain the one particle Green's function as expectation value on FG states.

From Dyson's equation

$$G(k,E) = G_0(k,E) + G_0(k,E)\Sigma(k,E)G(k,E)$$

$G_0(k,E)$ is the Green's function of the non interacting system

$$G_0(k,E) = \frac{\theta(k-k_F)}{E-e_0(k)+i\eta} + \frac{\theta(k_F-k)}{E-e_0(k)-i\eta}$$

The proper self-energy $\Sigma(k,E)$ accounts for the effect of interaction

$$G(k,E) = \frac{1}{E-e_0(k)-\Sigma(k,E)}$$

The calculation of $\Sigma(k,E)$ can be carried out perturbatively in the effective potential V_{eff} and using the basis of non interacting FG.

DISPERSION RELATIONS

$$\text{Im}\left[\Sigma_{\text{pol}}(\mathbf{k},\omega)\right] = \frac{1}{2}\pi \sum_{q,q',k'} \left|\langle q,q',k'|v_{\text{eff}}|k\rangle\right|^2 \delta\left(\frac{\mathbf{q}^2}{2m}+\frac{\mathbf{q}'^2}{2m}-\frac{\mathbf{k}'^2}{2m}-\omega\right) n_>(q)n_>(q')n_<(k')$$

$$\text{Im}\left[\Sigma_{\text{cor}}(\mathbf{k},\omega)\right] = \frac{1}{2}\pi \sum_{q,q',k'} \left|\langle q,q',k'|v_{\text{eff}}|k\rangle\right|^2 \delta\left(\frac{\mathbf{k}'^2}{2m}-\frac{\mathbf{q}^2}{2m}-\frac{\mathbf{q}'^2}{2m}+\omega\right) n_<(q)n_<(q')n_>(k')$$

$$\text{Re}[\Sigma(\mathbf{k},\omega)] = \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty} \frac{\text{Im}[\Sigma(\mathbf{k},\omega')]}{\omega-\omega'}d\omega'$$

Neutron Matter