"Sapienza" Università di Roma Department of Physics



Neutrino opacity in neutron matter

Andrea Cipollone

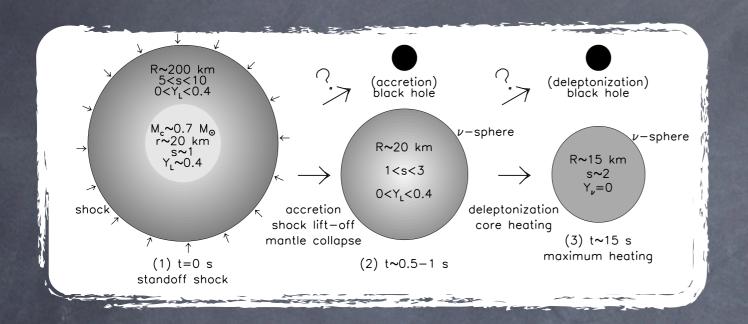
&

Omar Benhar

Collaborator: Andrea Loreti

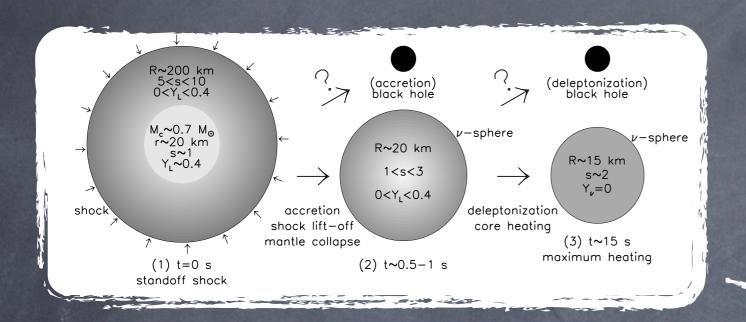
Neutrino opacity of N. M.: an astrophysical scenario

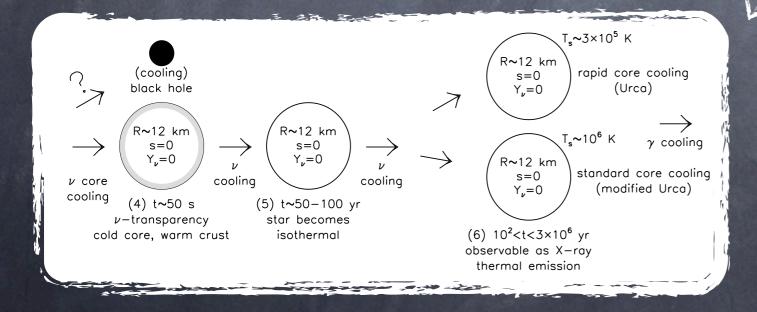
✓ From a qualitative point of view...



(Neutrino opacity of N. M.: an astrophysical scenario

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Neutrino opacity of N. M.: an astrophysical scenario

✓ ...to a quantitative description

Improved models of stellar core collapse and still no explosions: What is missing?

R. Buras, M. Rampp, H.-Th. Janka, and K. Kifonidis¹

¹Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str. 1, D-85741 Garching, Germany (Dated: February 2, 2008)

Two-dimensional hydrodynamic simulations of stellar core-collapse with and without rotation are presented which for the first time were performed by solving the Boltzmann equation for the neutrino transport including a state-of-the-art description of neutrino interactions. Although convection develops below the neutrinosphere and in the neutrino-heated region behind the supernova shock, the models do not explode. This suggests missing physics, possibly with respect to the nuclear equation of state and weak interactions in the subnuclear regime. However, it might also indicate a fundamental problem of the neutrino-driven explosion mechanism.

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equation of state and weak interactions in the subnuclear regime. However, it might also indicate a fundamental problem of the neutrino-driven explosion mechanism.

✓ SN 1987A data*

$$E_{grav} \approx 10^{53} \mathrm{erg}$$
 $E_{\nu} \approx 2.7 \times 10^{53} \mathrm{erg}$ $E_{rad} \approx 10^{49} \mathrm{erg}$ $E_{kin} \approx 10^{51} \mathrm{erg}$

* K. Laganke, nuclear physics school "Raimondo Anni", spring 2011 (Otranto)

✓ Modelling neutrino-matter interaction

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \overbrace{\bar{l}\gamma^{\mu}(1-\gamma_5)\nu_l}^{\mu} (V_{\mu} - A_{\mu})$$

in vacuum

neutral
$$l^0V^0 \sim l^0(x) \left[\psi_n^+(x) \psi_n(x) \right]$$

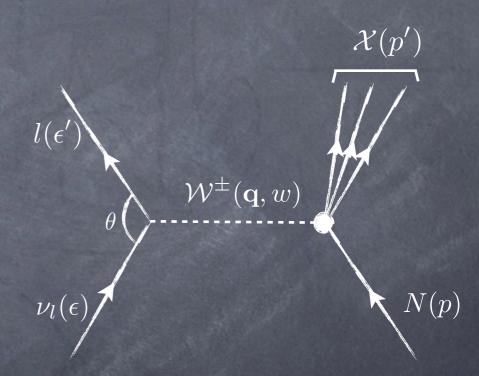
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 $\checkmark \nu_l + N \rightarrow l + X$ cross section

in vacuum



$$\frac{\partial^2 \sigma}{\partial \epsilon' \partial \Omega} = \frac{G_F^2}{(2\pi)^2} \frac{\epsilon' (1 - n_l(\epsilon'))}{8\epsilon} L^{\mu\nu}(\mathbf{q}, w) H_{\mu\nu}(\mathbf{q}, w)$$

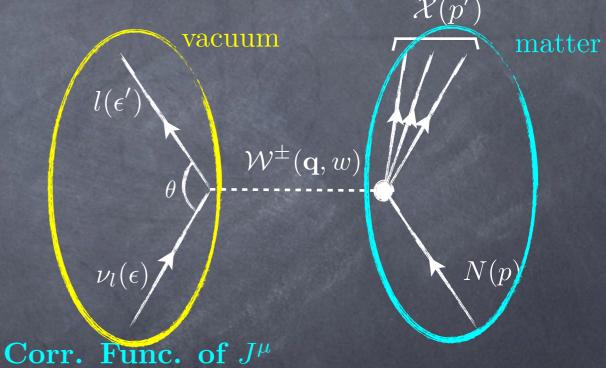
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Lepton Trace

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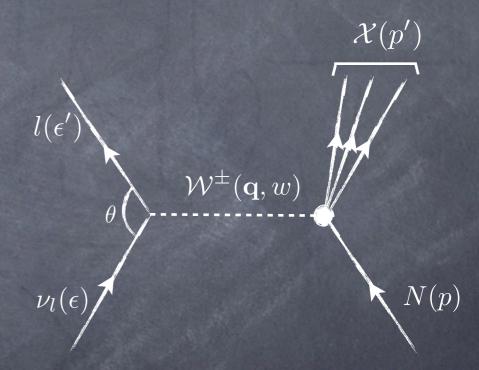
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fluctuation-dissipation theorem

✓
$$p \ll m_N$$
 — non-rel. nucleons

$$\checkmark q \lesssim \text{MeV} \longrightarrow e, \mu, \tau, \nu \text{ (neutral current)}$$

$$L^{\mu\nu}\operatorname{Im}\left[\tilde{H}_{\mu\nu}\right] \to 8\,\epsilon'\epsilon\left[\operatorname{Im}\left[\chi^{\rho\rho}(\mathbf{q},w)\right](1+\cos\theta) + \operatorname{Im}\left[\chi^{\sigma\sigma}(\mathbf{q},w)\right](3-\cos\theta)\right]$$

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density-density response function spin-spin response function

where

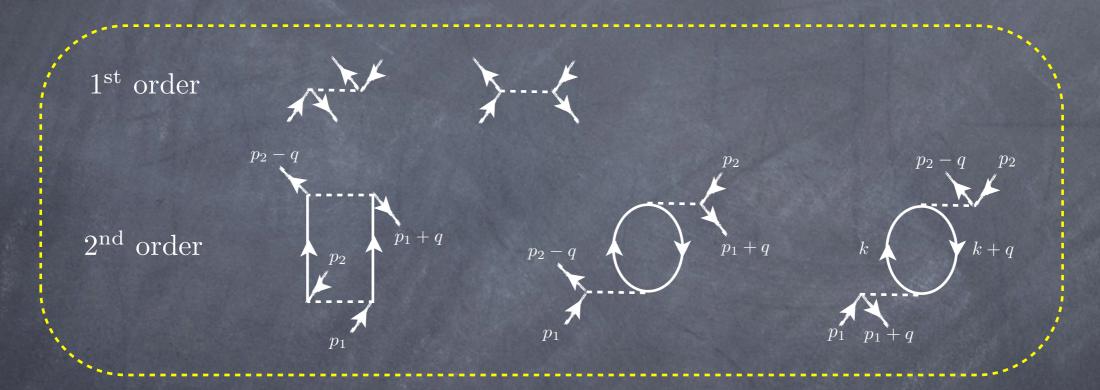
•
$$w = |\mathbf{k}| - |\mathbf{k}'|$$

•
$$\mathbf{q} = \mathbf{k} - \mathbf{k}'$$

Weak interacting system

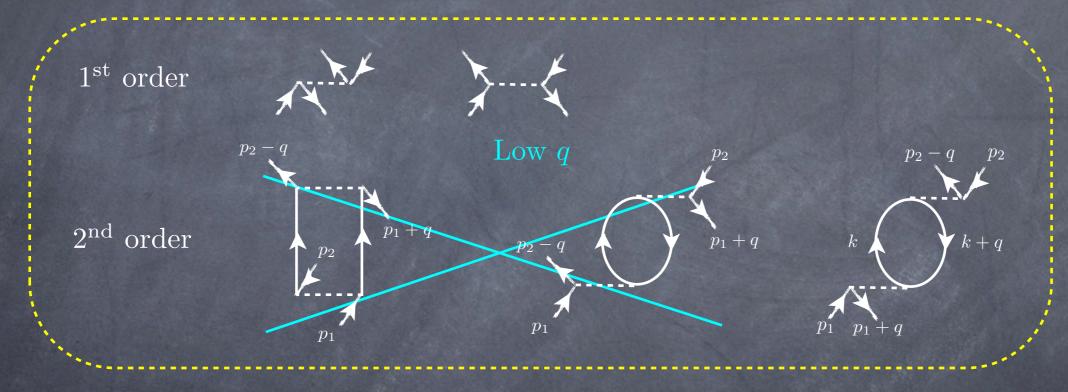
Weak interacting system

- You need just (at least in principle) $\mathcal{Z} = \sum_{N,i} e^{-\beta(E_i^N \mu N)}$
- ✓ Effective interaction between particle and hole



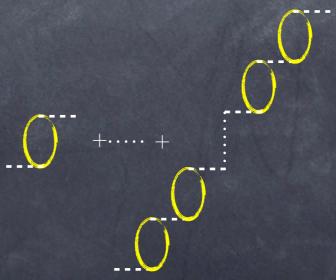
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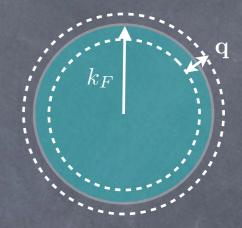


✓ Low-momentum response

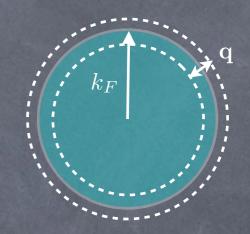
Ring approximation (low q)



 \checkmark Able to treat strong-interacting system but $T \to 0$



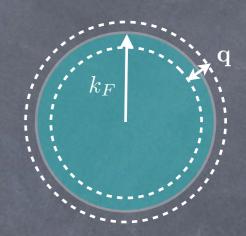
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- ✓ Low-lying energy states obey
 the same principles of the ideal states



✓ States are linear comb. of "elementary (collective) excitations": quasi-particle

$$\frac{1}{\tau} \sim a(\epsilon - \epsilon_F)^2 + bT^2$$

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$$\frac{1}{\tau} \sim a(\epsilon - \epsilon_F)^2 + bT^2$$

$$\delta E = E - E^{0} = \sum_{\mathbf{k},\sigma} \epsilon_{\sigma}^{0}(\mathbf{k}) \delta n_{\sigma}(\mathbf{k}) + \frac{1}{2} \sum_{\mathbf{k},\mathbf{k}',\sigma,\sigma'} f_{\sigma\sigma'}(\mathbf{k},\mathbf{k}') \delta n_{\sigma}(\mathbf{k}) \delta n_{\sigma'}(\mathbf{k}')$$

interaction between q-p

✓ Landau Fermi theory: Dynamic Response (low q)

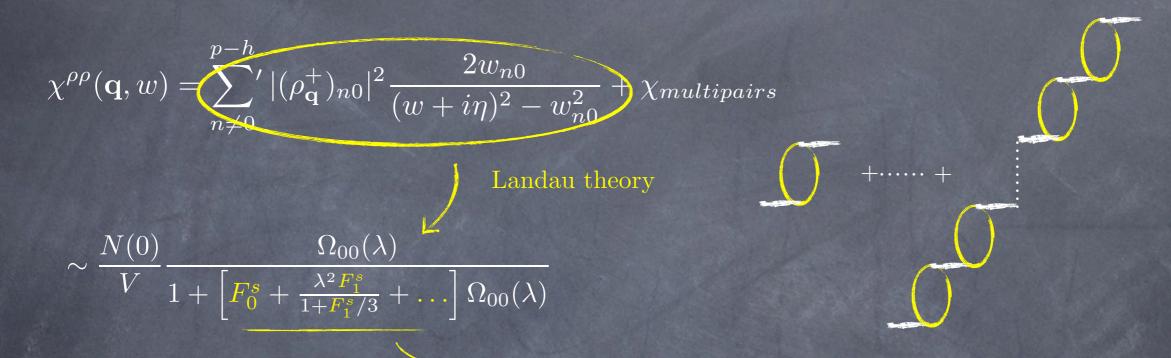
$$\chi^{\rho\rho}(\mathbf{q}, w) = \sum_{n \neq 0}^{p-h} |(\rho_{\mathbf{q}}^+)_{n0}|^2 \frac{2w_{n0}}{(w+i\eta)^2 - w_{n0}^2} + \chi_{multipairs}$$

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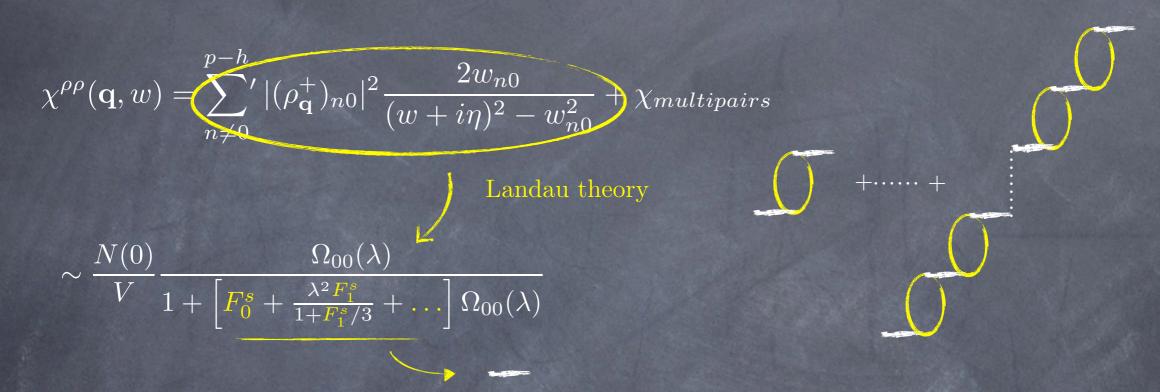
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$$\sim \frac{N(0)}{V} \frac{\Omega_{00}(\lambda)}{1 + \left[F_{0}^{s} + \frac{\lambda^{2}F_{1}^{s}}{1 + F_{1}^{s}/3} + \dots\right] \Omega_{00}(\lambda)}$$

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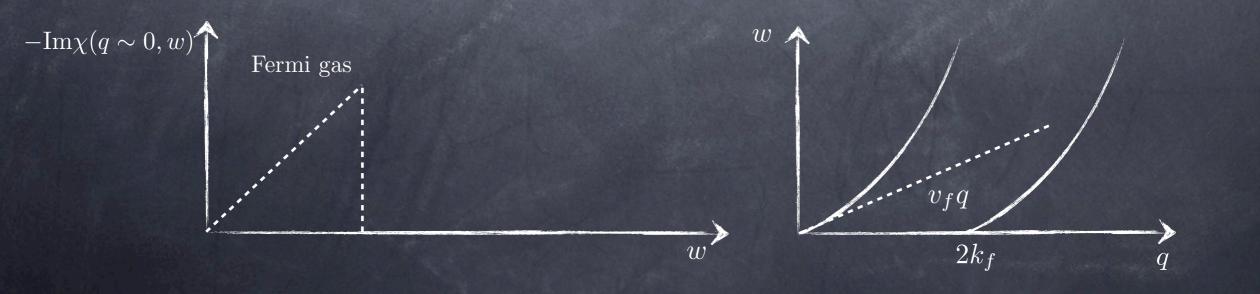


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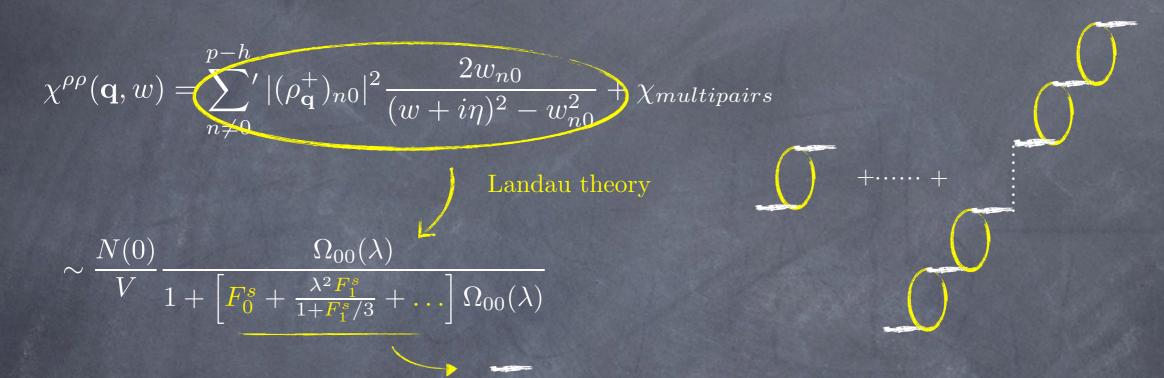


✓ Response Function

✓ Energy Spectrum

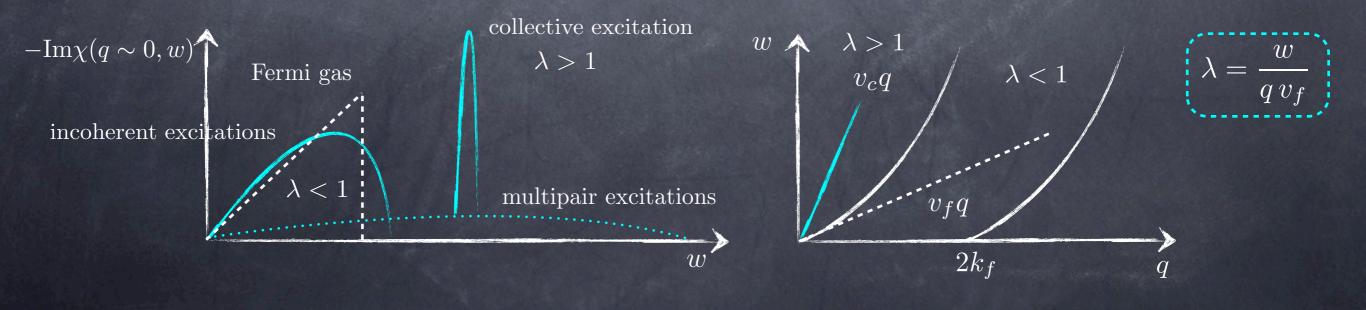


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How can we get these parameters?

- × No experiments available
- ✓ Effective N-N interaction in medium → CBF theory
 (Correlated Basis Function)

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- - → Variational method

$$\langle \hat{H} \rangle = \frac{\int_{V} d^{3N} r \, \Psi^* \hat{H} \Psi}{\int_{V} d^{3N} r \, \Psi^* \Psi}$$

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$$\hat{v}_{12}^{6} = \left[1, (\hat{\sigma}_{1} \cdot \hat{\sigma}_{2}), \hat{S}_{12}(\tilde{\mathbf{r}})\right] \otimes \left[1, (\hat{\tau}_{1} \cdot \hat{\tau}_{2})\right]$$

$$\text{spin} \text{ isospin}$$

$$\Psi \sim \hat{F}(1, \dots, N) \Phi_{FG} = \left[\hat{S} \prod_{i < j}^{N} \hat{f}(\mathbf{r}_{ij})\right] \Phi_{FG}$$

: How can we get these parameters?

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(non-pert. features)

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$$\text{Two-body correlation function} \qquad \hat{f}(\mathbf{r}_{ij}) = \sum_{p=1}^{6} f^{p}(r) \overline{\hat{O}_{ij}^{p}(\tilde{\mathbf{r}})}$$

\bigcirc CBF: looking for the \hat{f}

 $ightharpoonup 2^{nd}$ order (two-body) cluster approximation

$$\langle \hat{H} \rangle_2 = \sum_{i < j}^N \langle \text{FG}, ij | \frac{1}{2} \hat{F}_2 \left[\hat{t}_1 + \hat{t}_2, \hat{F}_2 \right] + \hat{F}_2 \left(\hat{\mathbf{v}}_{12}^6 \right) F_2 | ij, \text{FG} \rangle_a$$

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$$(\hat{w}_{12}^6)^{eff} \xrightarrow[\rho \to 0]{} \hat{v}_{12}^6$$

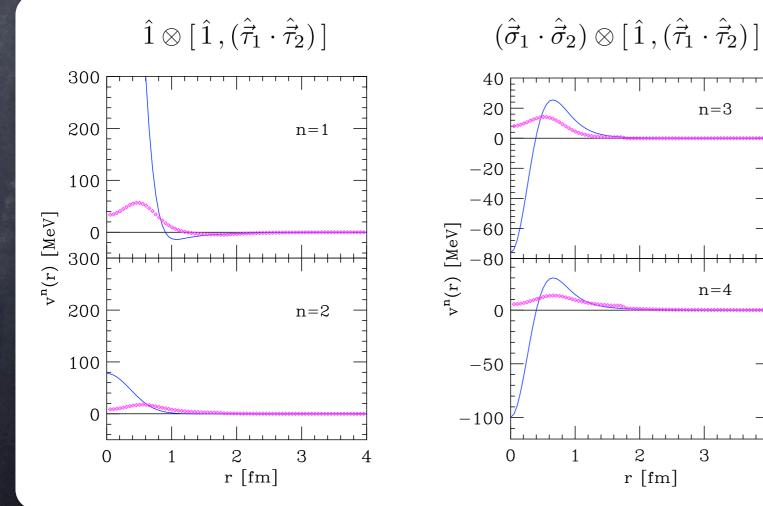
CBF: looking for the \hat{f}

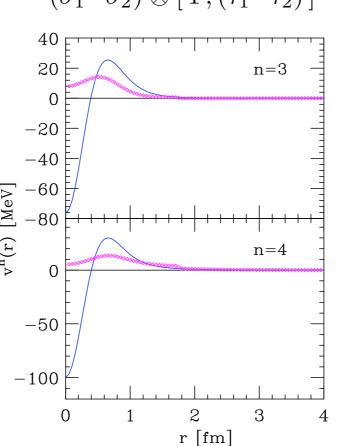
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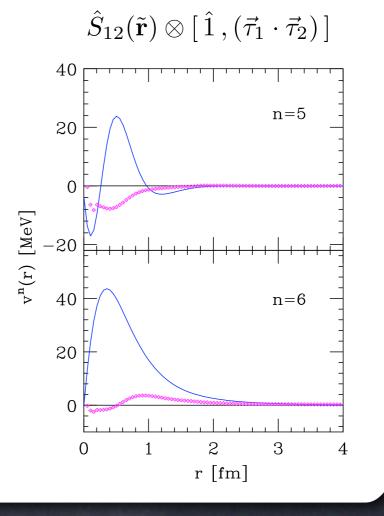
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quenched interaction

$$(\hat{w}_{12}^6)^{eff} \xrightarrow[
ho o 0]{} \hat{v}_{12}^6$$







(Landau-CBF mixing

✓ Energy per nucleon

$$\frac{E}{N} = \frac{1}{N} \langle T_0 \rangle + \frac{1}{2N} \sum_{k_i, k_j} \sum_{i, I, j, J} \frac{1}{L^3} \left[\langle FG, ij | \hat{V}(0) - \hat{V}(\mathbf{k}_i - \mathbf{k}_j) | ij, FG \rangle \right] n_{i,I}(\mathbf{k}_i) n_{j,J}(\mathbf{k}_j)$$

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$$\sum_{p=1}^{6} \int d^3 \mathbf{r} \, \mathbf{w}_p^{eff}(\mathbf{r}) e^{-i(\mathbf{k}_i - \mathbf{k}_j) \cdot \mathbf{r}} \tilde{O}^p(\hat{\mathbf{r}}) P_{\sigma \tau}$$

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f_{ij} = \frac{\delta^2 \mathcal{E}}{\delta n_i \delta n_j} = \frac{1}{L^3} \langle FG, ij | \hat{V}(0) - \hat{V}(\mathbf{k}_i - \mathbf{k}_j) | ij, FG \rangle$$

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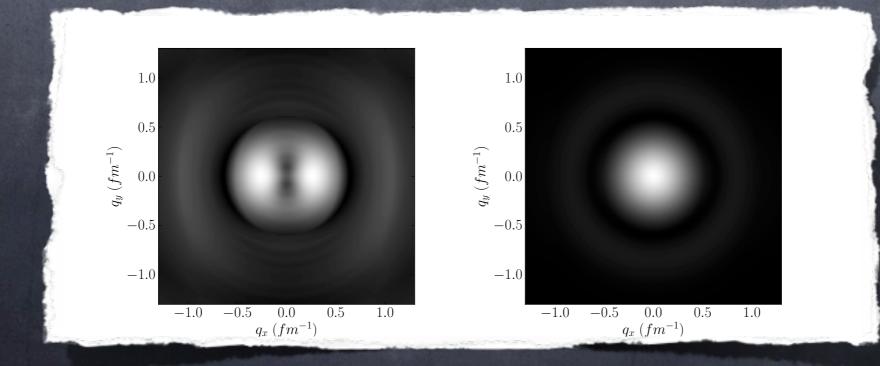
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✓ General adimensional Landau parameters

$$F^{a}(\mathbf{q}) = (F_{\uparrow\uparrow}(\mathbf{q}) - F_{\uparrow\downarrow}(\mathbf{q}))/2$$
 $F^{s}(\mathbf{q}) = (F_{\uparrow\uparrow}(\mathbf{q}) + F_{\uparrow\downarrow}(\mathbf{q}))/2$



 $\overline{\mathbf{q}} = \overline{\mathbf{k}} - \overline{\mathbf{k}}'$

C Landau-CBF mixing

✓ Finally...the Landau parameters

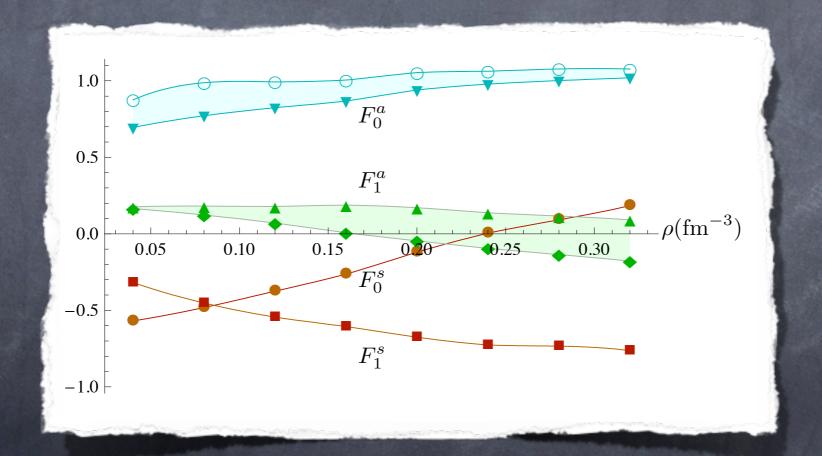
$$F^{s,a}(\cos \xi) = V N(0) f^{s,a}(\cos \xi) = \sum_{l} F_{l}^{s,a} P_{l}(\cos \xi)$$
 $|\mathbf{q}| = 2k_{F} \sin(\xi/2)$

C Landau-CBF mixing

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© CBF and Landau-CBF...are they comparable?

✓ Static properties

$$K^{\rho} = -V \left(\frac{\partial V}{\partial P}\right)_{T}$$

$$\chi_{\alpha\beta}^{\sigma} = \left. \frac{\partial M_{\alpha}}{\partial H_{\beta}} \right|_{H=0}$$

$$\frac{1}{m^{eff}} = \frac{1}{p} \left(\frac{de}{dp} \right)_{p_F}$$

Compressibility

Magnetic susceptibility

Effective mass

single particle spectrum

$$K^{\rho} = \frac{1}{\rho^2} \frac{N(0)}{1 + F_0^s}$$

$$\chi^{\sigma} = \left(\frac{g\mu_B}{2}\right)^2 \frac{N(0)}{1 + F_0^a}$$

$$m^{eff} = m(1 + \frac{F_1^s}{3})$$

CBF and Landau-CBF...are they comparable?

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Compressibility

Magnetic susceptibility

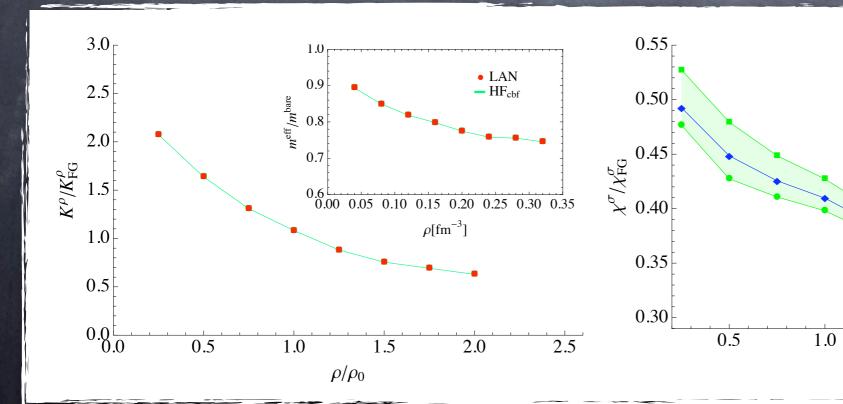
Effective mass

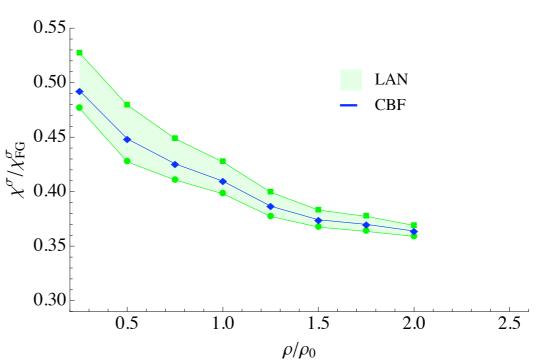
single particle spectrum

$$K^{\rho} = \frac{1}{\rho^2} \frac{N(0)}{1 + F_0^s}$$

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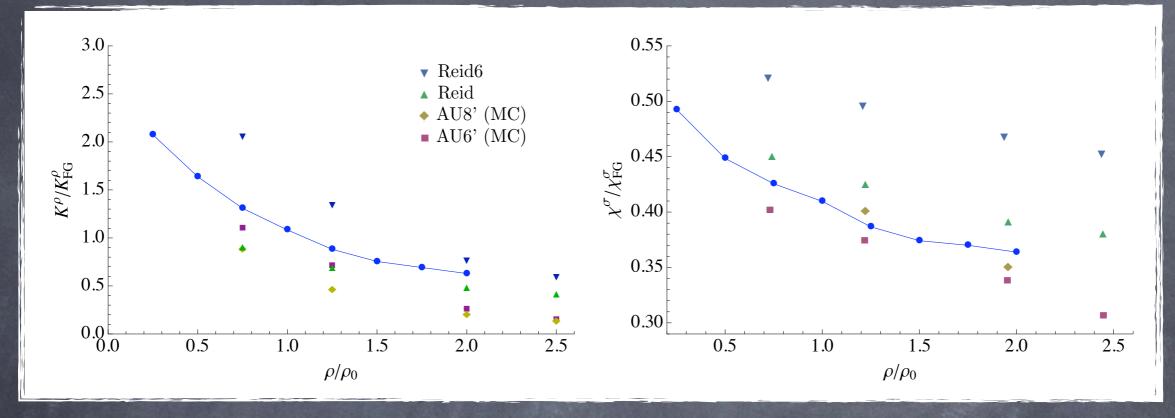
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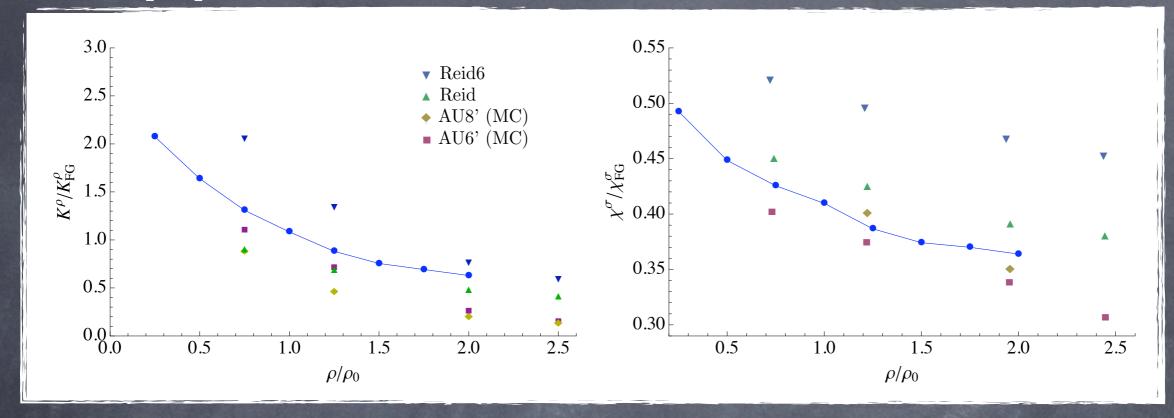
Static properties of matter

✓ Static properties

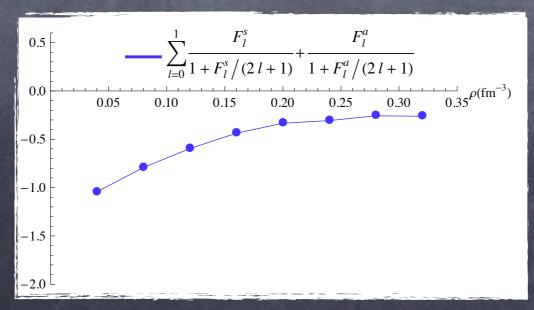


Static properties of matter

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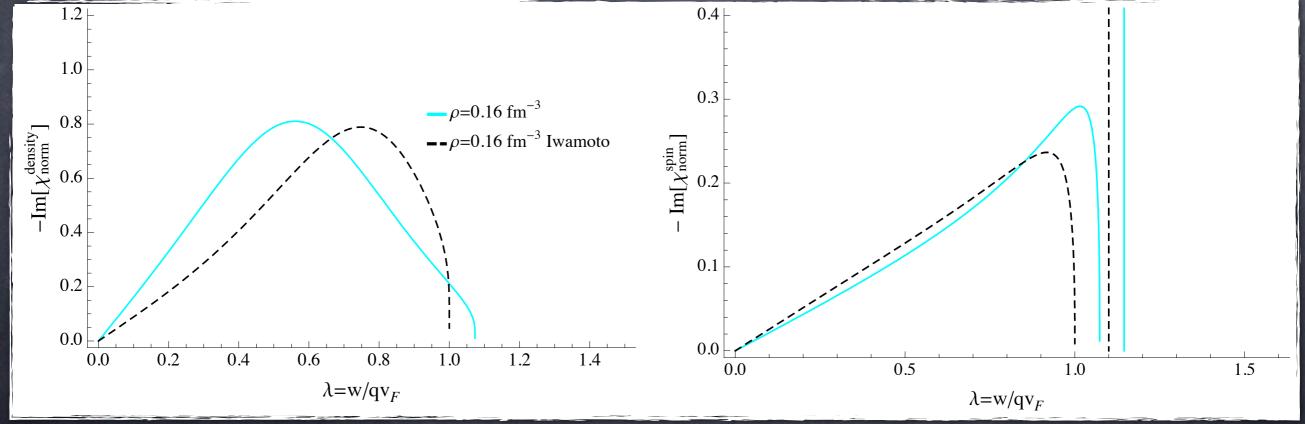


✓ Sum Rule



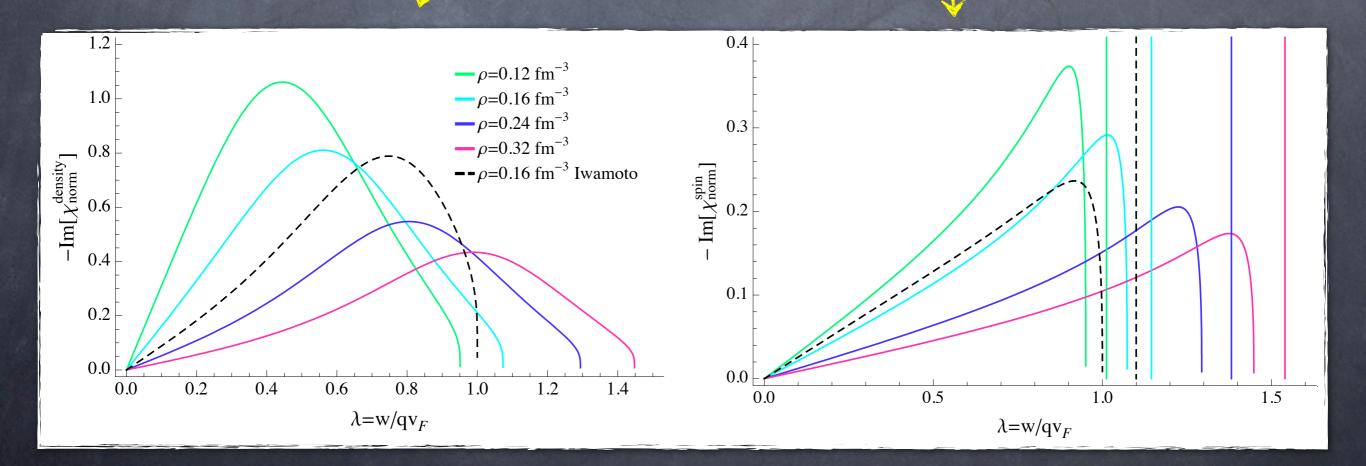
Dynamic Response of neutron matter

$$L^{\mu\nu} \operatorname{Im} \left[\tilde{H}_{\mu\nu} \right] \to 8 \, \epsilon' \epsilon \left[\operatorname{Im} \left[\chi^{\rho\rho} (\mathbf{q}, w) \right] (1 + \cos \theta) + \operatorname{Im} \left[\chi^{\sigma\sigma} (\mathbf{q}, w) \right] (3 - \cos \theta) \right]$$
density-density response function
$$\int$$



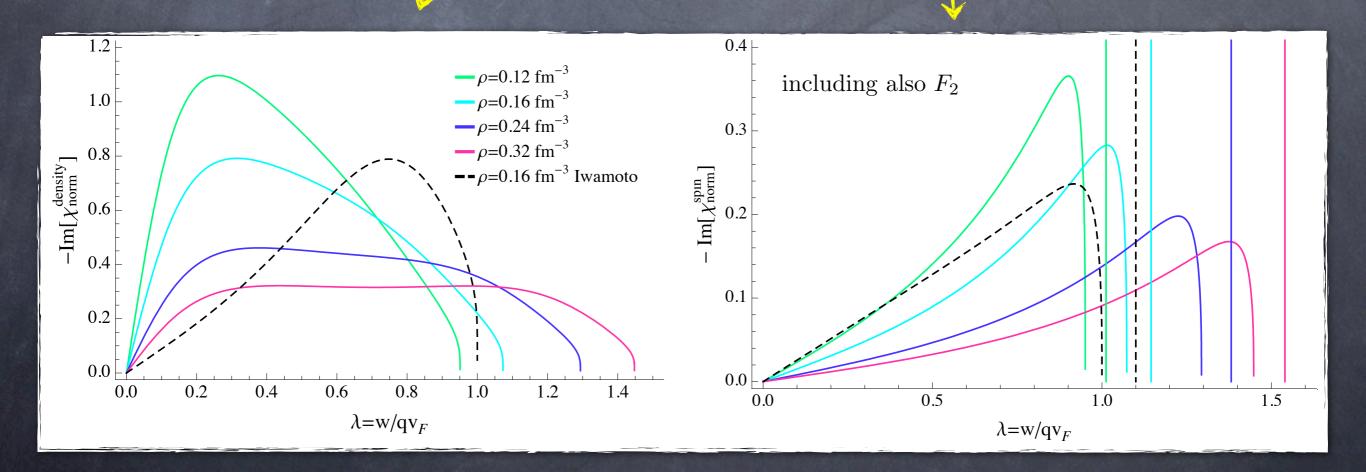
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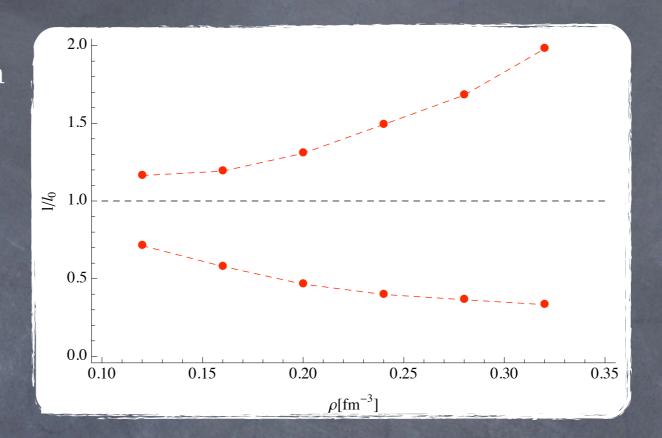
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C An hint of result ...

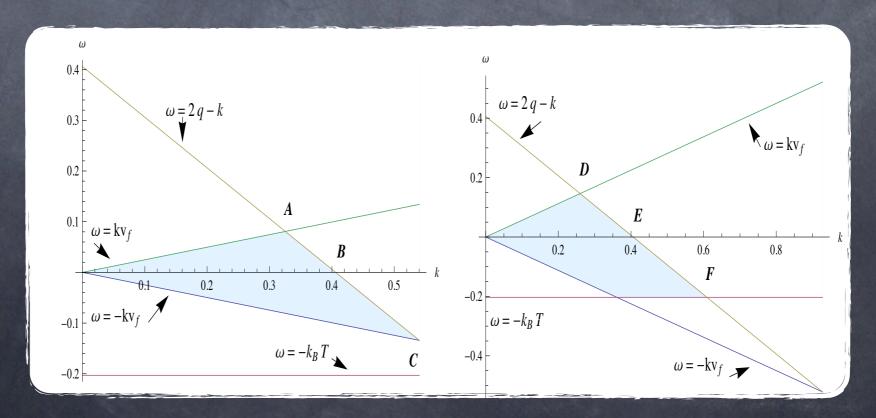
✓ Neutrino mean free path



 $kT \ll qv_F$

 $kT \gg q v_F$

✓ Phase space



Summary and perspective

- ✓ CBF theory is used to model the low-energy, interacting hamiltonian H_{int} in dense matter
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Neutrino mean free path and finite-temperature effects

Extension to asymmetric nuclear matter in β -equilibrium