Three-nucleon potentials in nuclear matter

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ma per seguir virtute e canoscenza
SISSA

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Outline

- Ab initio many body method
- Nuclear Hamiltonian: 2- and 3- body potentials
- Density dependent potential from UIX interaction
- Beyond UIX: chiral NNN potential and its local form
- Cutoff dependence of the local chiral NNN contact term
- Comparative study of local three-body potential in nuclear matter
- Conclusions

Ab initio many body method

Our aim is to perform *ab initio* calculations in nuclear matter

The interaction must not to be affected by uncertanties involved in many body techniques

Ab initio many body calculations

- Fully predictive
- Their approximations can be estimated
- Provide a test for the interaction itself

"Ab initio" concept is related to the **energy scale** of the system one wants to study.

A "Realistic" nuclear hamiltonian

The non relativistic Hamiltonian describing nuclear matter is

$$H = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{j>i=1}^{A} v_{ij} + \dots$$

Some of the **realistic** nucleon-nucleon (NN) potentials are

- **Argonne v_{18}, v_8'**
  - Mainly phenomenological
  - Local in coordinate space
- **CD-BONN**
  - Meson-exchange based
  - Nonlocal
- **Chiral N^3LO**
  - From Chiral Lagrangians
  - Nonlocal

The parameters of these potentials have been obtained

Fitting the ~ 4300 nn and np Nijmegen scattering data below 350 MeV with $^2 \approx 1$.

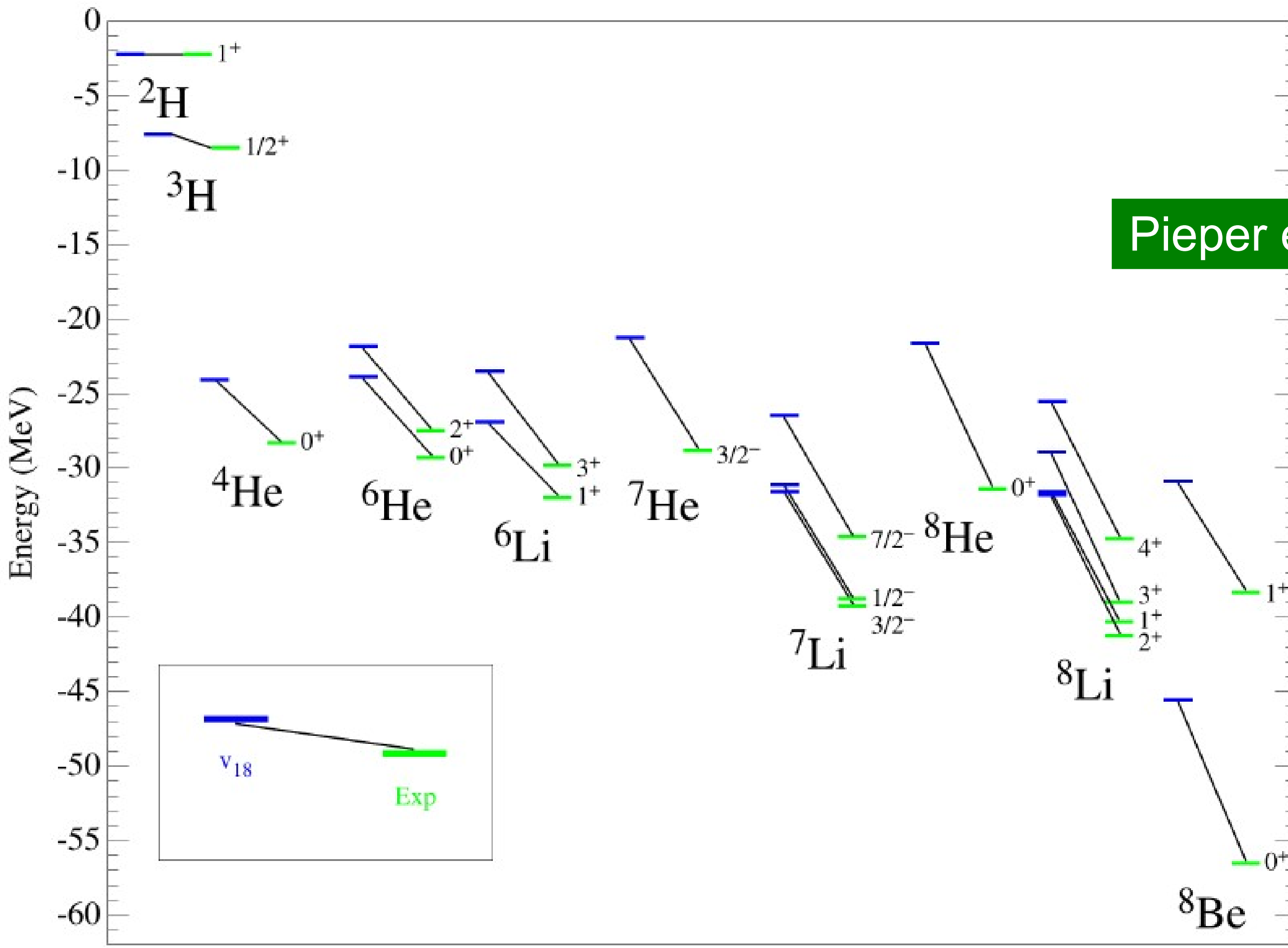
Fitting the binding energy of the deuteron (v_{18} & CD-BONN)

No many-body methods are needed for the fit ⟹ Potentials with high predictive power, suitable for "ab initio" calculations.

NN potential is not enough

When two body potential only is considered:

- The description of three- and four- nucleon bound and scattering states gives a ^{2} per datum much larger than 1.



Pieper et al 2001

NN potential is not enough

When two body potential only is considered:

- The *nd* scattering length is very different from the experimental value.

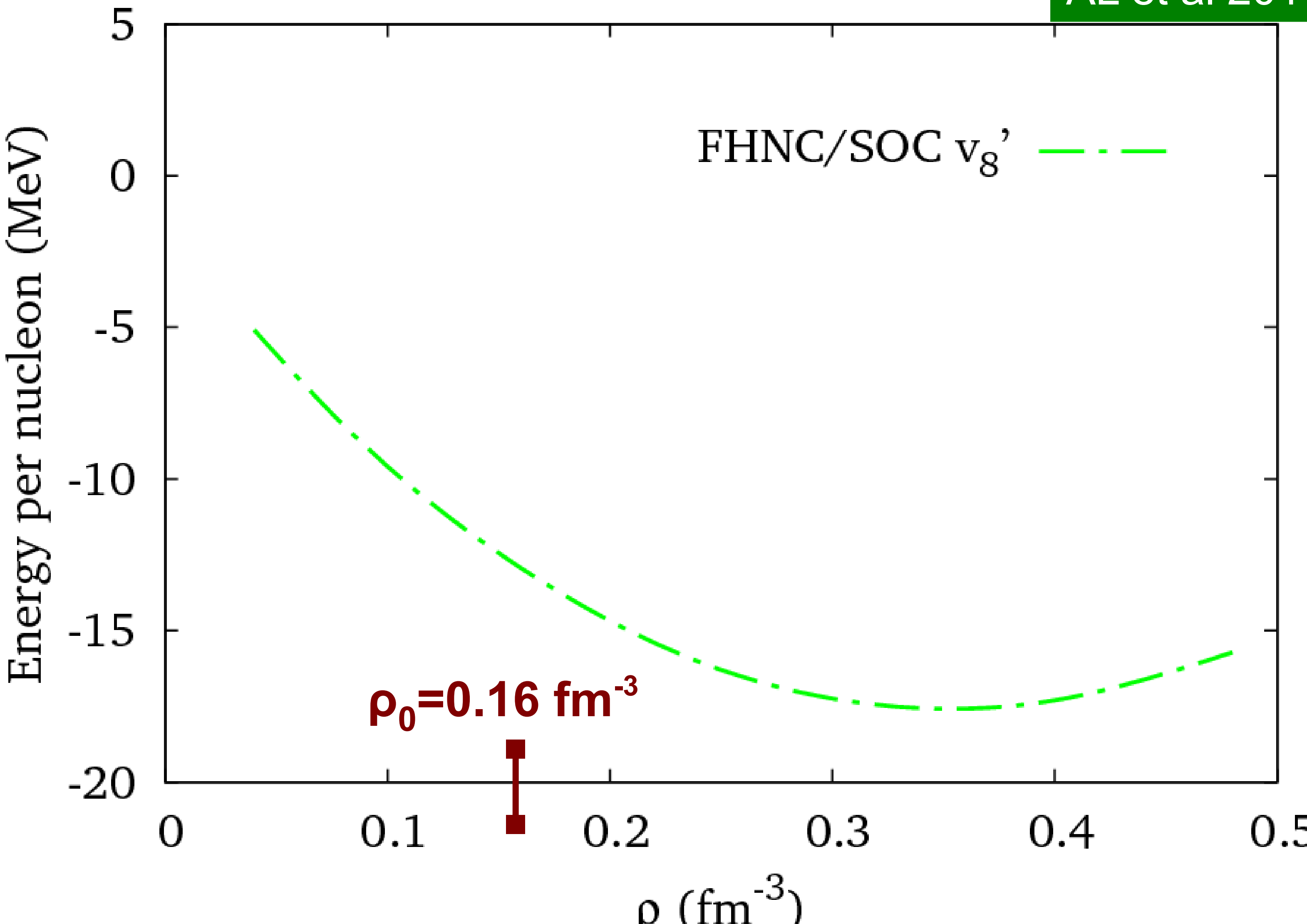
$$^{2}a_{nd} = \begin{cases} 1.258\ \mathrm{fm} & \rightarrow \ V_{18} \\ 0.645 \pm 0.003 \pm 0.007\ \mathrm{fm} & \rightarrow \ \text{Experiment} \end{cases}$$

NN potential is not enough

When two body potential only is considered:

- The equilibrium density ρ_0 of Symmetric Nuclear Matter (SNM) is overestimated.

AL et al 2011

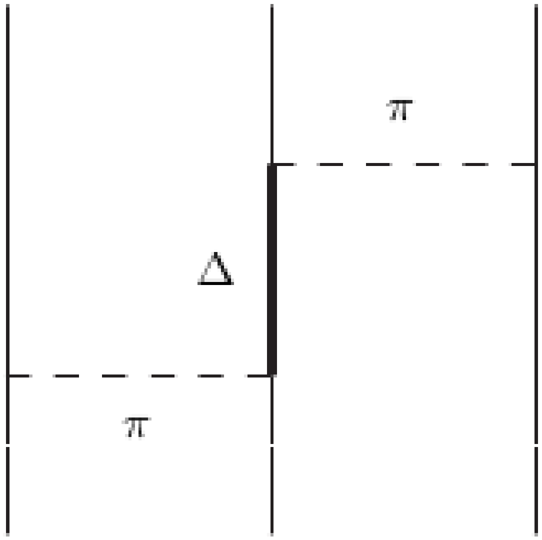


Urbana IX three body potential

UIX potential consists of **two contributions**

$V^{2\pi}$ Fujita Myiazawa

Two pions are exchanged among nucleons and a Δ resonance is excited in the intermediate state.
It solves the underbinding of light nuclei, but makes nuclear matter even more overbound.



Cyclic sum of three permutations

$$V^{2\pi} = A^{2\pi}(O^{2\pi}_{123} + O^{2\pi}_{231} + O^{2\pi}_{312})$$

$$O^{2\pi}_{123} = \left(\{\hat{X}_{12}, \hat{X}_{23}\}\{\tau_{12}, \tau_{13}\} + \frac{1}{4}[\hat{X}_{12}, \hat{X}_{23}][\tau_{12}, \tau_{23}]\right)$$

$$\hat{X}_{ij} = Y(m_\pi r)\sigma_{ij} + T(m_\pi r)S_{ij}$$

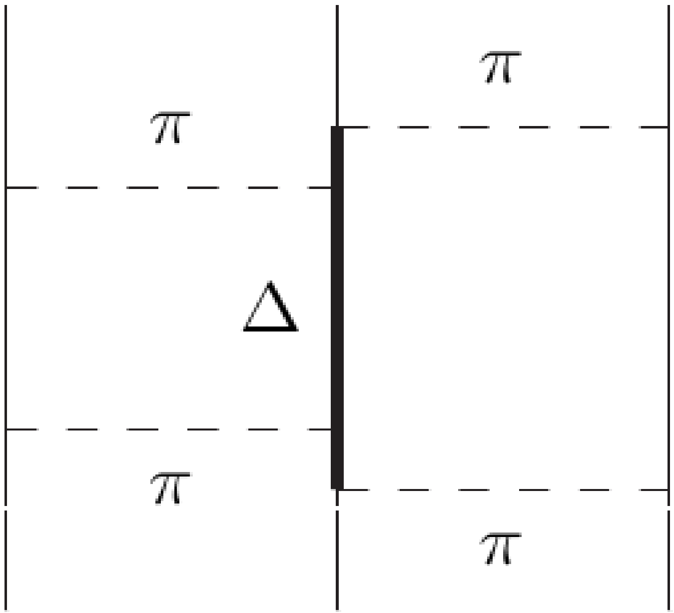
Cutoff functions

Urbana IX three body potential

UIX potential consists of **two contributions**

V^R Phenomenological scalar repulsive term

It has been introduced by Lagaris and Pandharipande in order for FHNC/SOC calculation to reproduce the correct binding energy of SNM.



$$V^R = U_0 \sum_{cycl} T^2(m_\pi r_{12}) T^2(m_\pi r_{23})$$

Same cutoff functions of $V^{2\pi}$

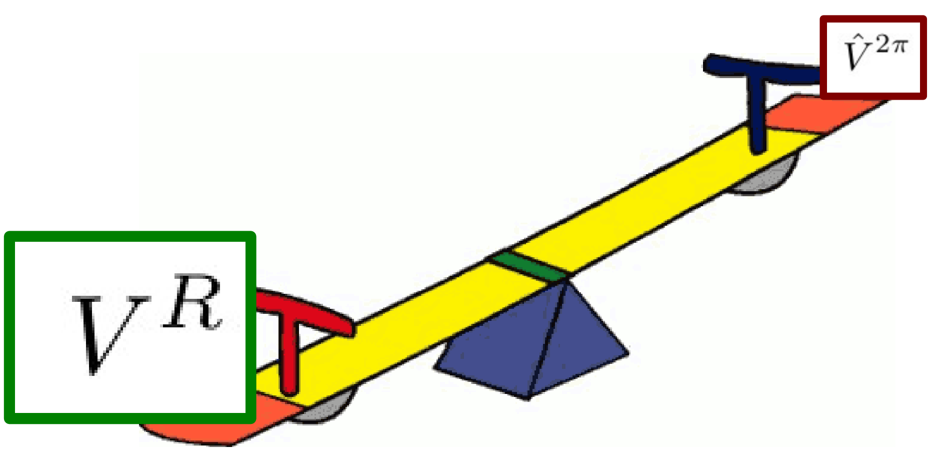
Urbana IX three body potential

UIX potential has two parameters

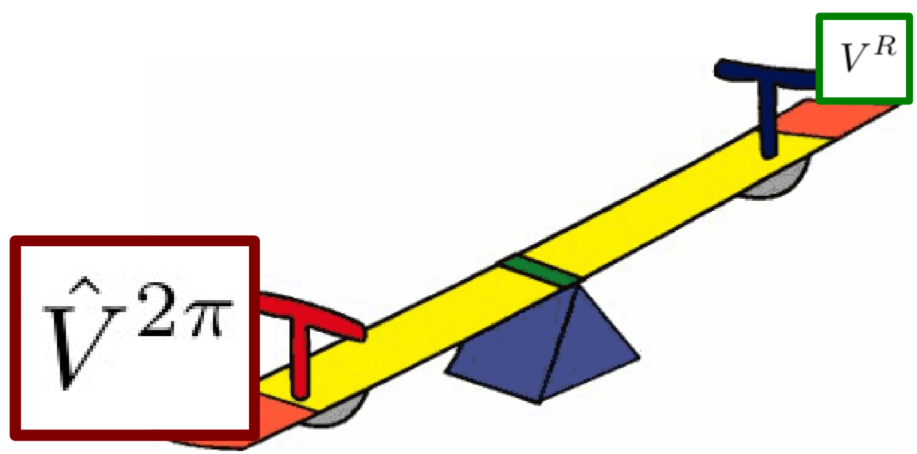
- **$A^{2\pi}$** adjusted to reproduce the observed binding energies of ^{3}H.
- **U_0** tuned in order for FHNC/SOC calculations to reproduce the empirical equilibrium density of SNM ρ_0=0.16 fm^{-3}.

Lagaris and Pandharipande argued that, because of correlations, the relative weight of the contribution depends upon the density of the system:

High density



Low density



Urbana IX three body potential

Theoretical issues

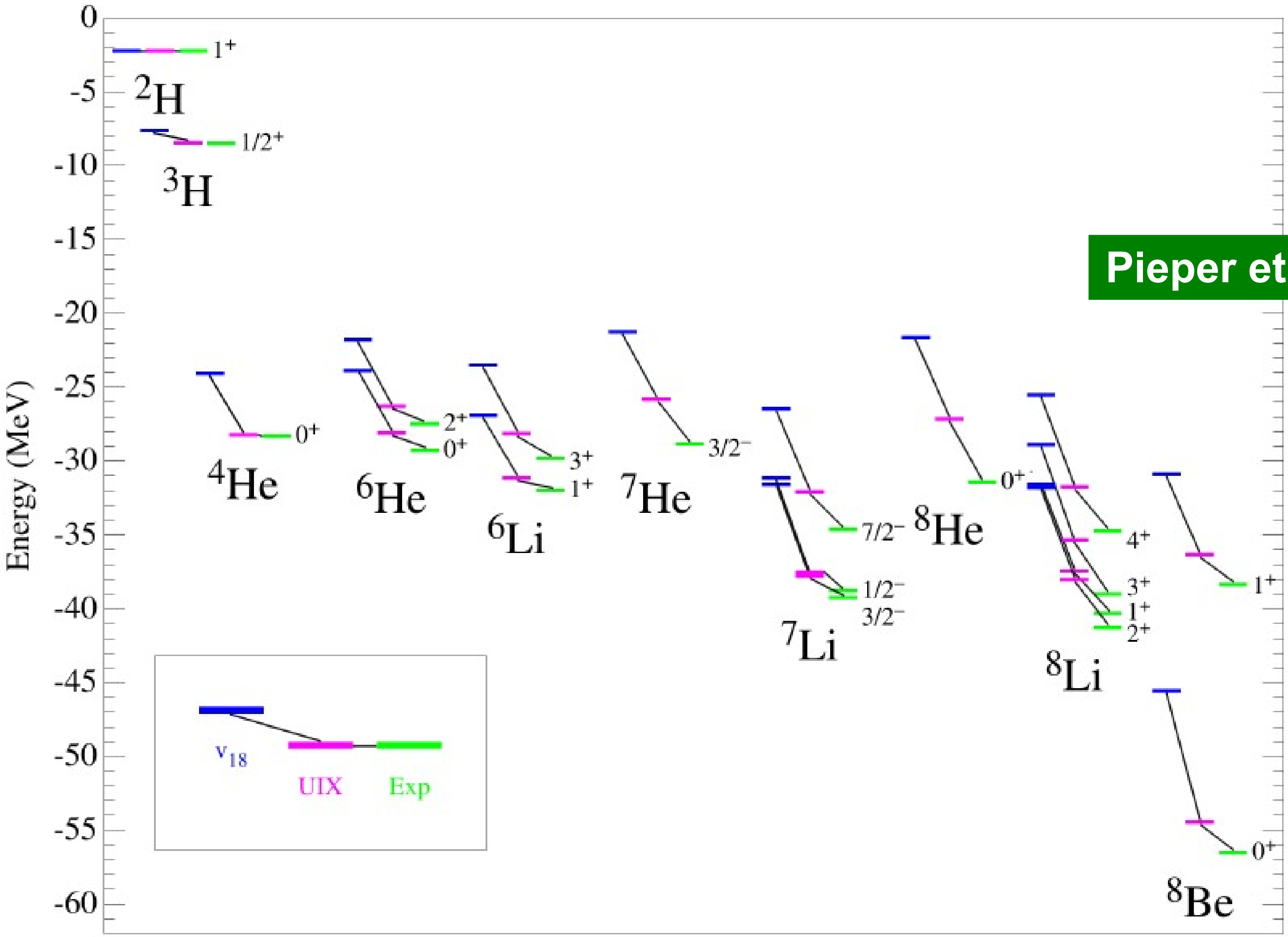
For $V^{2\pi}$ there are no a priori reasons to stop at the first order in the perturbative expansion in the coupling constant **g_0~10**.

Adjusting U_0 to reproduce the correct value of ρ_0, calculated within the FHNC/SOC framework, makes the potential affected by the uncertanties of the many-body technique.

Are calculations with UIX really “ab initio”?

Urbana IX three body potential

Improved description of energy levels of light nuclei



Pieper et al 2001

Urbana IX three body potential

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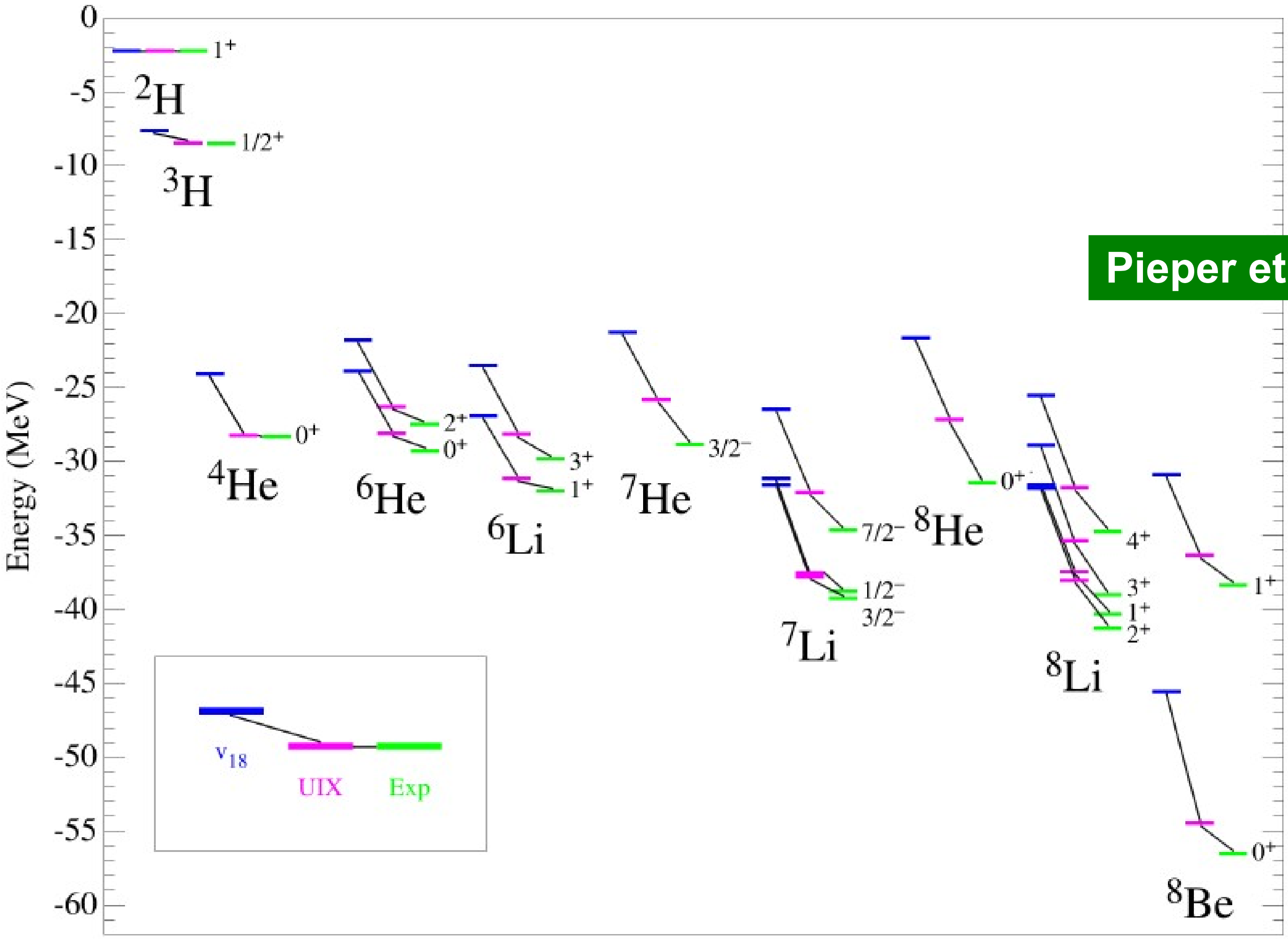


Pieper et al 2001

Still some discrepancies with experimental data!

Urbana IX three body potential

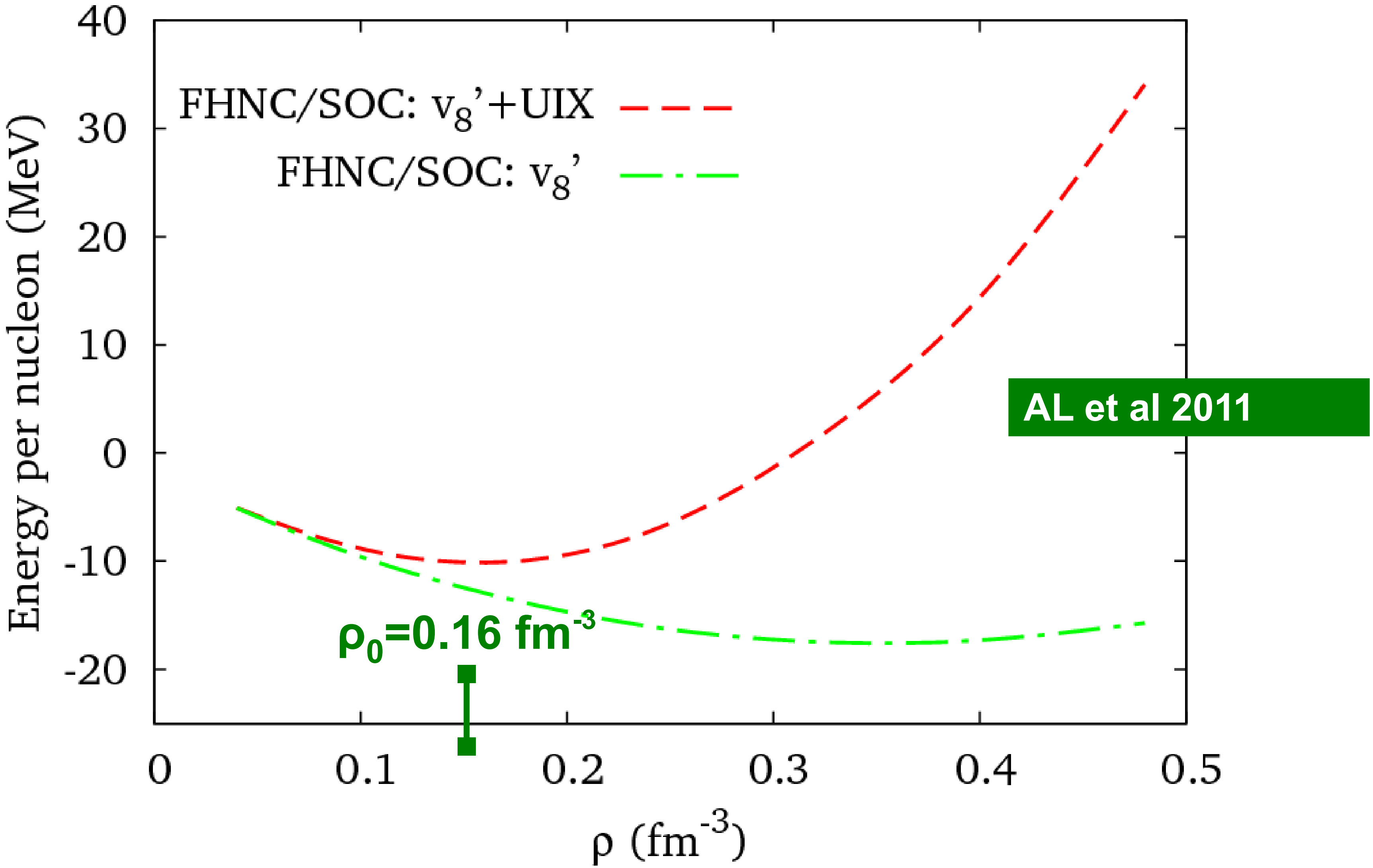
When two body potential only is considered:

- The *nd* scattering length still is not compatible with the experimental value.

$$
{}^2a_{nd} = \begin{cases} 1.258\,\text{fm} & \rightarrow v_{18} \\ 0.578\,\text{fm} & \rightarrow v_{18} + \text{UIX} \\ 0.645 \pm 0.003 \pm 0.007\,\text{fm} & \rightarrow \text{Experiment} \end{cases}
$$

Urbana IX three body potential

SNM saturation density is well reproduced



Urbana IX three body potential

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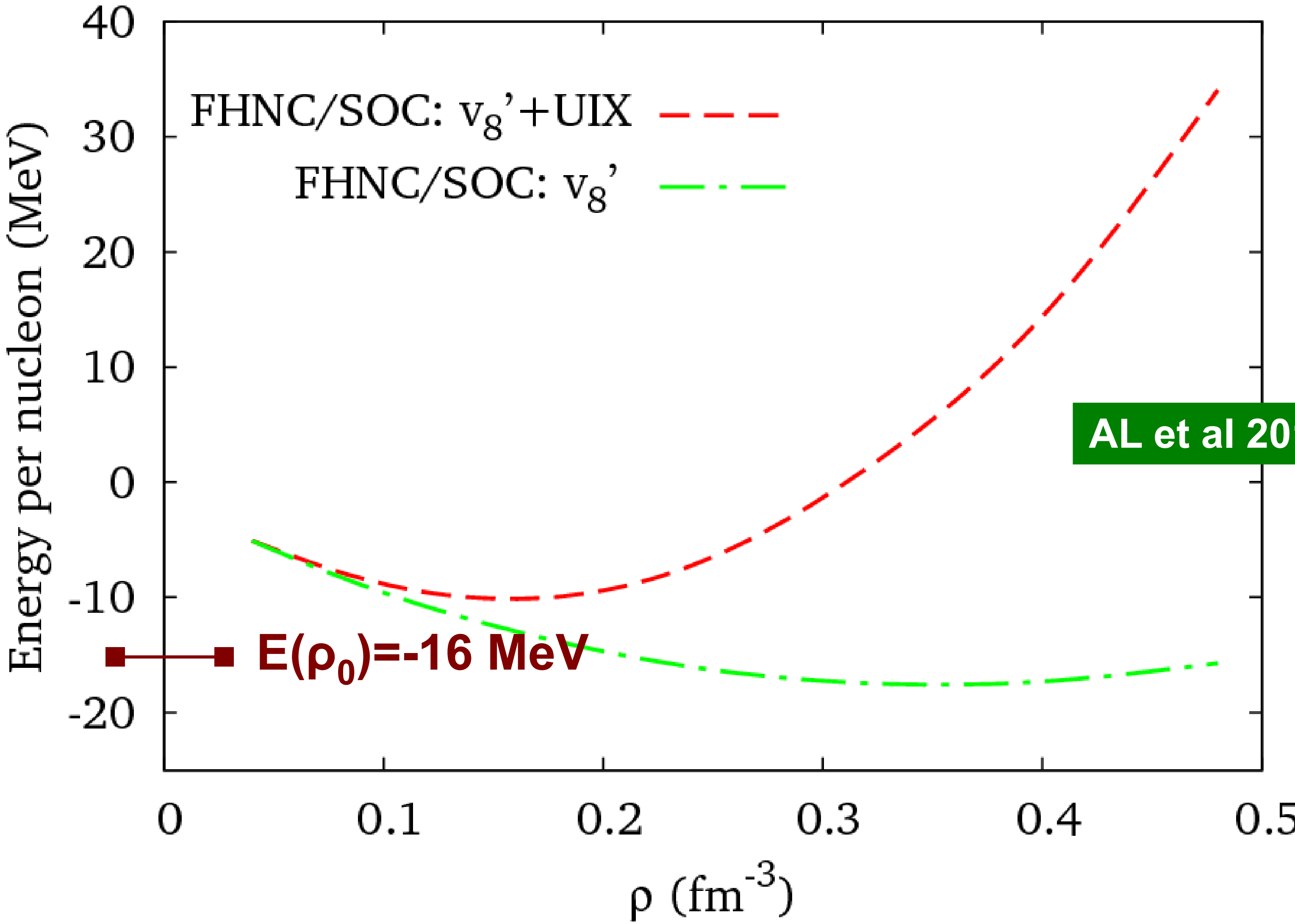

AL et al 2011

SNM is underbound : $E(\rho_0)$= -11 MeV instead of -16 MeV!

Urbana IX three body potential

Akmal et al. ascribed the underbing of SNM to deficiencies of the variational wave function.

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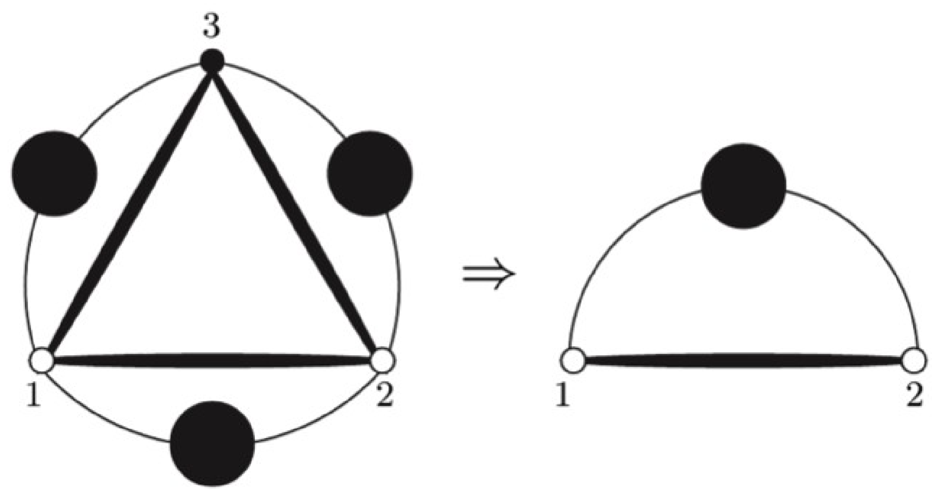
It may be wrong!

We have included UIX three body interactions in the AFDMC computational scheme in an effective way.

Density dependent two body potential

Tenet: UIX three body potential can be replaced by an effective two-nucleon potential, obtained through an average over the degrees of freedom of the third particle

UIX: V_{123}

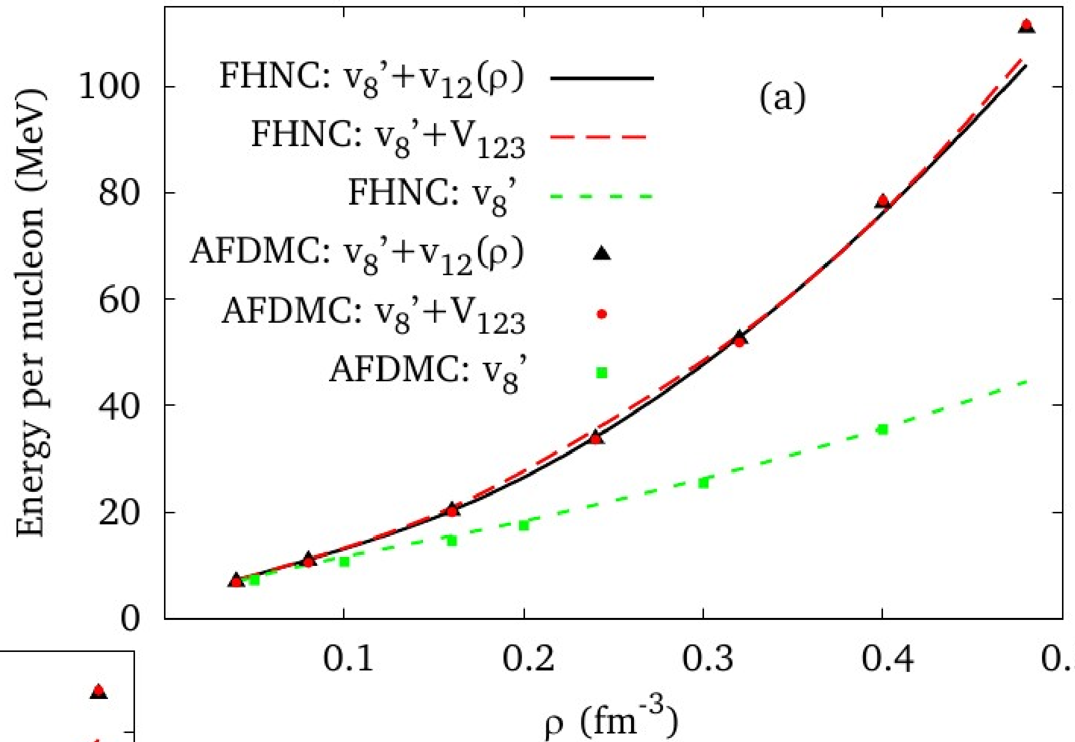


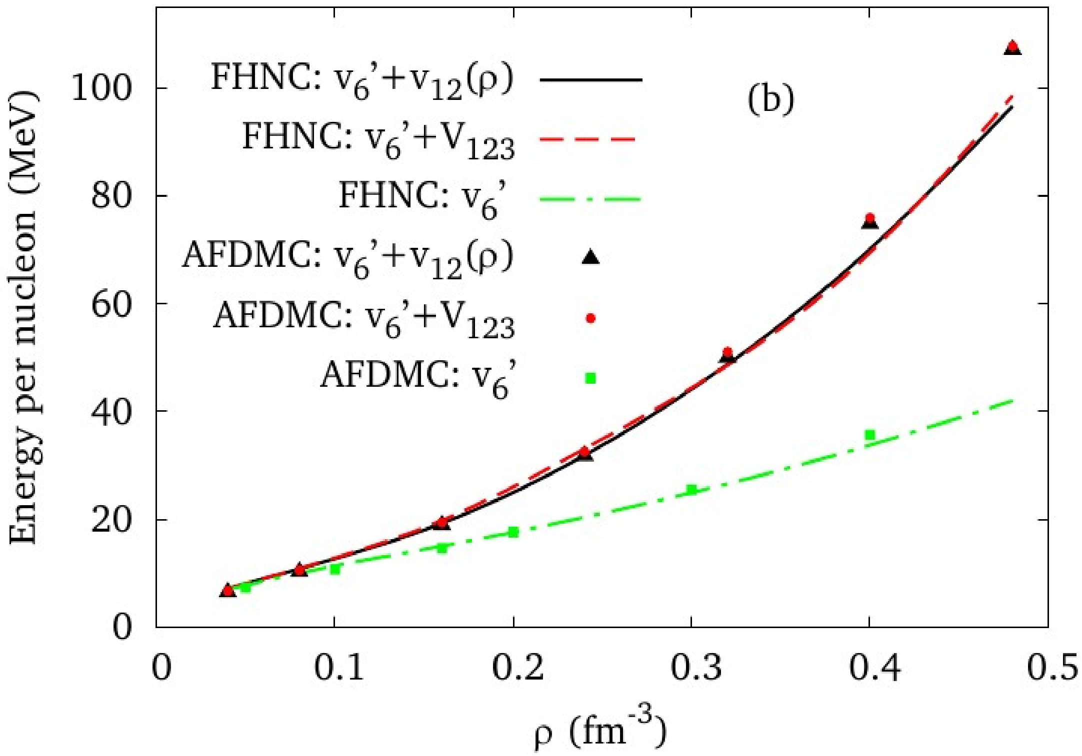
ddp: $v_{12}(\rho)$

- Obtained from a microscopic model of the three-nucleon force providing an accurate description of the properties of light nuclei
- Could be used to include the effects of three nucleon interactions in the calculation of the nucleon-nucleon scattering cross section in the nuclear medium.
- Has been implemented in AFDMC

Results for pure neutron matter

Argonne v_8' as two-body potential

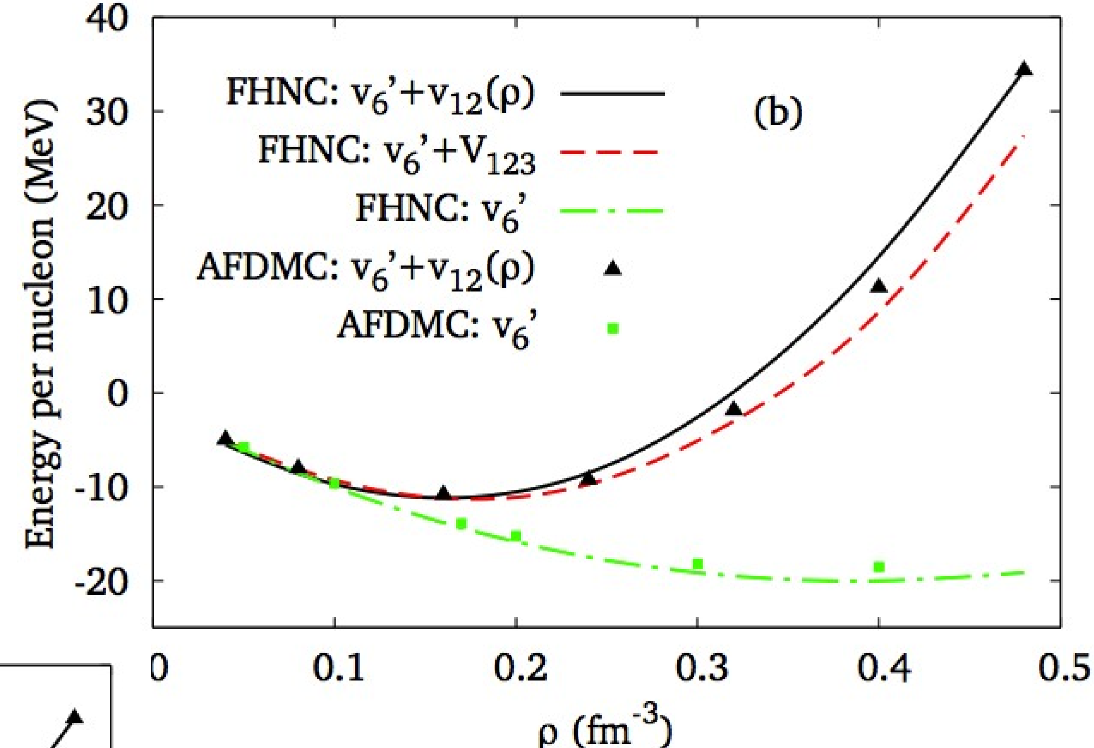


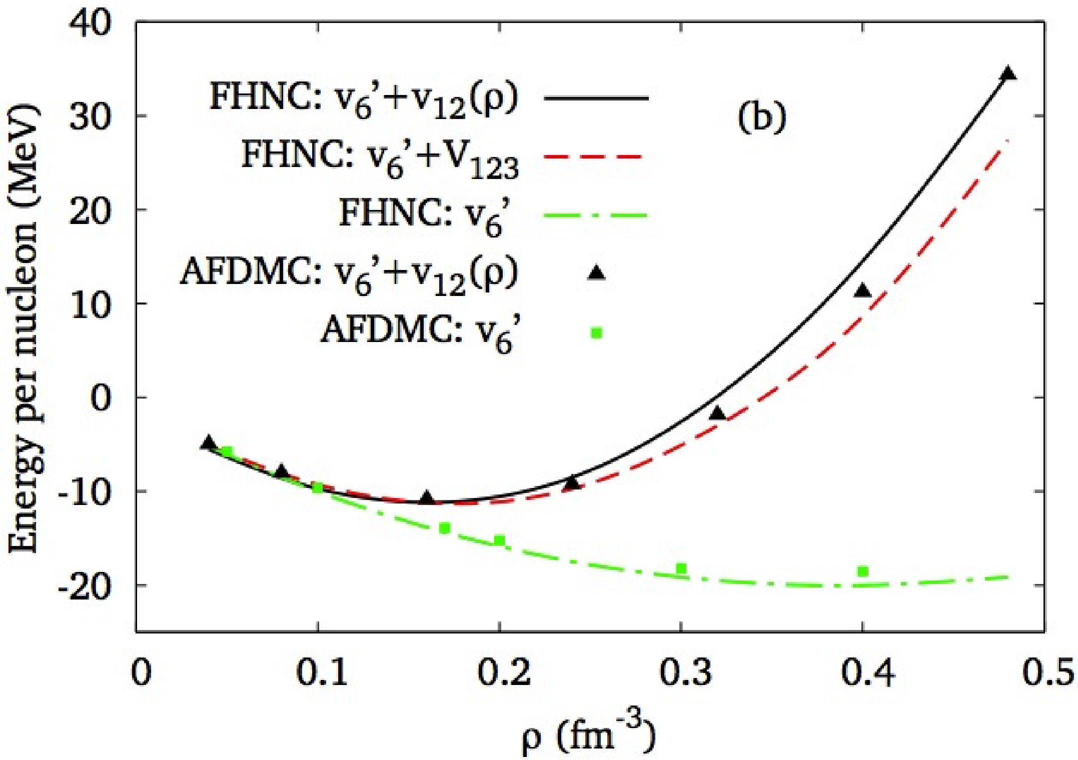


Argonne v_6' as two-body potential

Results for symmetric nuclear matter

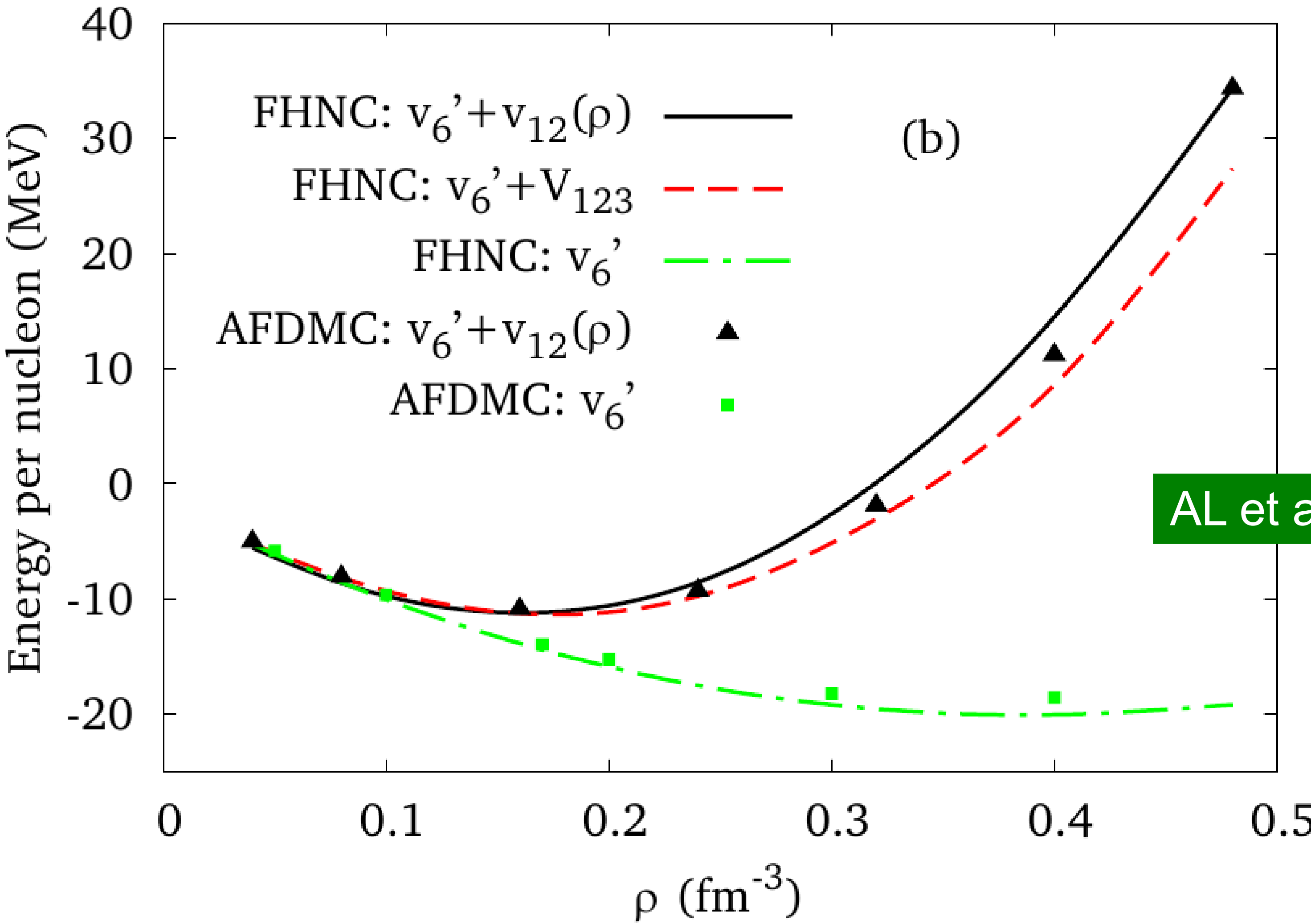
Argonne v_8' as two-body potential





Argonne v_6' as two-body potential

UIX in symmetric nuclear matter



AL et al 2011

Auxiliary Field Diffusion Monte Carlo do not show a lowering of the variational FHNC/SOC result for $E(\rho_0)$.

We need to go beyond the UIX potential!

How to go beyond Urbana IX ?

Same strategy used for NN potential?

Decomposition of NN potential in spin-isospin structures respecting the symmetry of the interaction

Argonne v_{18}, or v_8'

$$\hat{V}_{12} = \sum_{p=1}^{n} v^p(r_{12}) \hat{O}^p_{12}$$

Fitting the **huge amount** of nn and pn data ⇒ Shape of the radial functions $v^p(r)$

Following the same strategy adopted for the NN potential seems not to be feasible without an additional theoretical guidance.

- Large variety of different possible structures in the three-nucleon force
- Diffuculties in extracting information of three nucleon force from NNN data

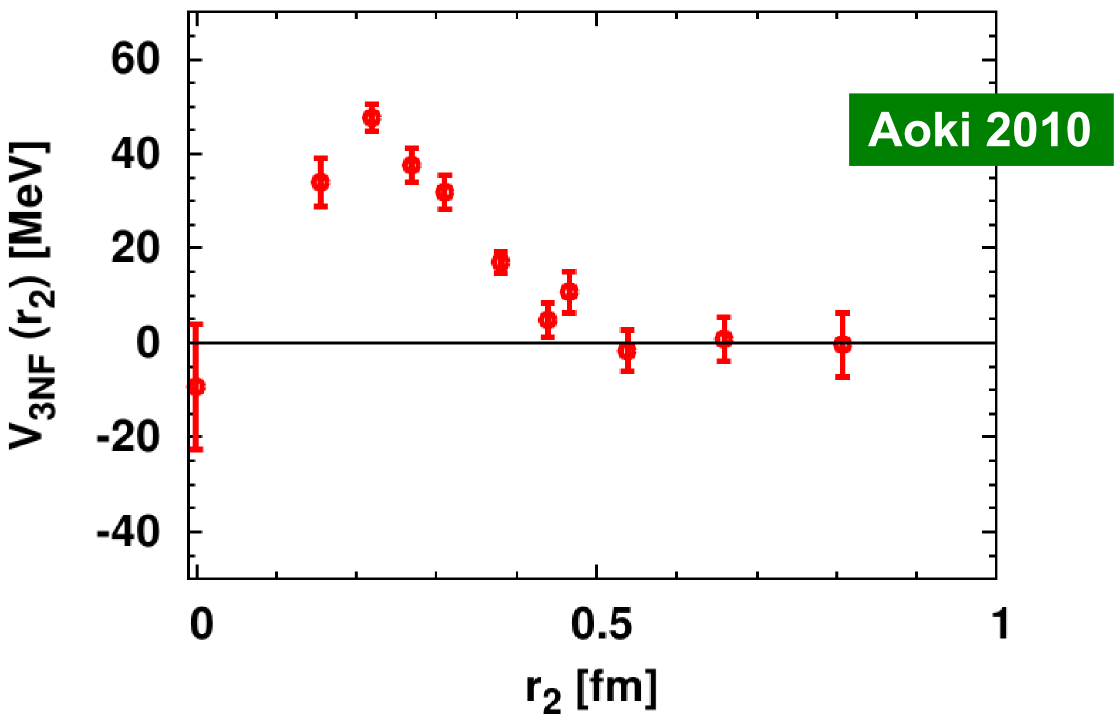
Describing interacting nucleons

THE SYSTEM

- Pions with external momentum of the order of m_π
- Non-relativistic nucleons with spatial-momentum of the order of m_π.

QCD, the underlying theory, IS NOT PERTURBATIVE AT THIS SCALE!

Lattice QCD



Effective theories describing phenomena which occur at a chosen length (or energy) range.

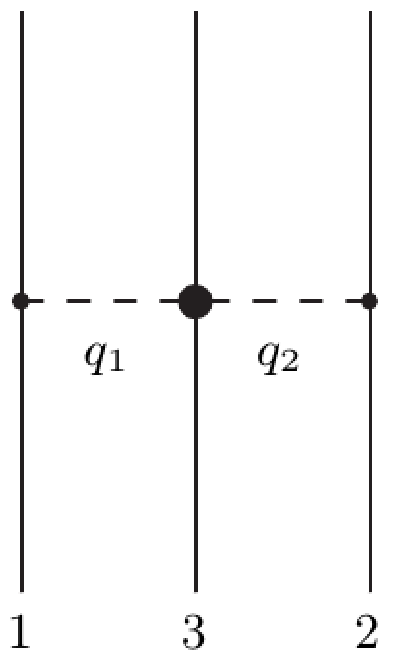
ChPT

Chiral 3-body potential at NNLO

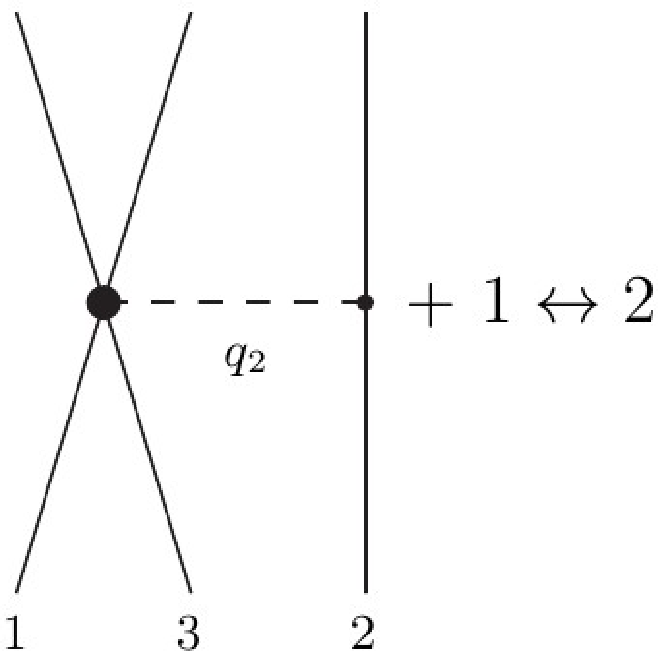
In a theory without explicit Δ degrees of freedom, the first contribution to the chiral 3NF appears at N^2LO in the Weinberg counting scheme.

The interaction is described by three different physical mechanisms

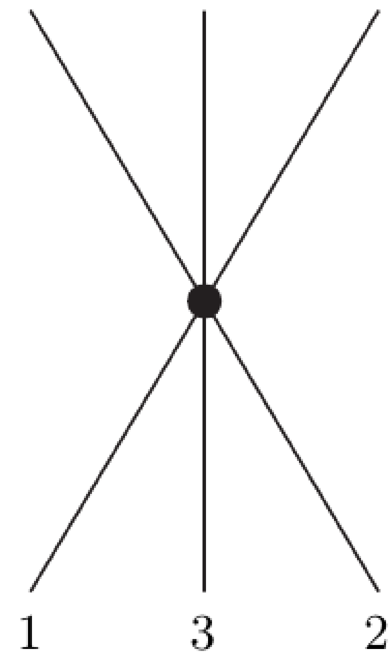
Two-pion exchange (TPE)



One-pion exchange (OPE)



Contact term



Cyclic sum

$$V^{\chi}(1,2,3) = V^{\chi}(1:2,3) + V^{\chi}(2:1,3) + V^{\chi}(3:1,2)$$

Chiral 3-body NNLOL potential

Fourier transforming the Chiral NNLO 3-body potential, originally derived in momentum space, yields a local expression in coordinate space:

NNLOL potential

$$V^{\chi}(3:12) = \int \frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\tilde{V}^{\chi}(3:12)F_\Lambda(q_1^2)F_\Lambda(q_2^2)\mathrm{e}^{i\mathbf{q}_1\cdot\mathbf{r}_{13}}\mathrm{e}^{i\mathbf{q}_2\cdot\mathbf{r}_{23}}$$

$$F_\Lambda(q_i^2) = \exp\Big(-\frac{q_i^4}{\Lambda^4}\Big)$$

- Depend on transferred momenta
- Generate powers of q/Λ beyond NNLO

Radial functions appearing in the coordinate space version of the chiral NNLO potential:

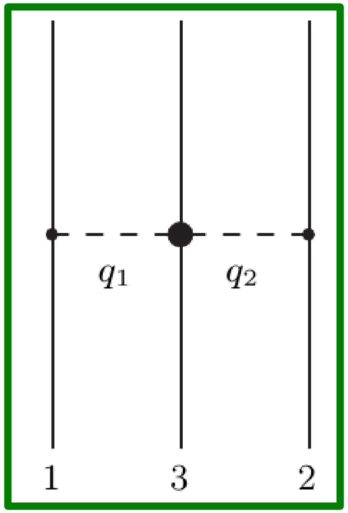
$$y(r) = \frac{z_1'(r)}{r}$$

$$t(r) = \frac{1}{r^2}\Big(z_1''(r) - \frac{z_1'(r)}{r}\Big) = \frac{1}{r}y'(r)$$

$$z_n(r) = \frac{4\pi}{m_\pi^3}\int \frac{d^3q}{(2\pi)^3}\frac{F_\Lambda(q^2)}{(q^2+m_\pi^2)^n}e^{i\mathbf{q}\cdot\mathbf{r}}$$

$$= \frac{2}{\pi m_\pi^3}\int dq q^2 \frac{F_\Lambda(q^2)}{(q^2+m_\pi^2)^n}j_0(qr)$$

TPE term



$$= c_1 V_1(3:12) + c_3 V_3(3:12) + c_4 V_4(3:12)$$

The coordinate space expressions are

$$
\begin{aligned}
V_1(3:12) &= W_0\, \tau_{12}(\boldsymbol{\sigma}_1 \cdot \vec{r}_{13})(\boldsymbol{\sigma}_2 \cdot \vec{r}_{23}) y(r_{13}) y(r_{23}) \\
V_3(3:12) &= W_0\, \tau_{12}[\sigma_{12} y(r_{13}) y(r_{23}) \\
&+ (\boldsymbol{\sigma}_1 \cdot \vec{r}_{23})(\boldsymbol{\sigma}_2 \cdot \vec{r}_{23}) t(r_{23}) y(r_{13}) \\
&+ (\sigma_1 \cdot \vec{r}_{13})(\sigma_2 \cdot \vec{r}_{13}) t(r_{13}) y(r_{23}) \\
&+ (\vec{r}_{13} \cdot \vec{r}_{23})(\sigma_1 \cdot \vec{r}_{13})(\sigma_2 \cdot \vec{r}_{23}) t(r_{13}) t(r_{23})] \\
V_4(3:12) &= W_0\, (\boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)[(\boldsymbol{\sigma}_3 \cdot \boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_1) y(r_{13}) y(r_{23}) \\
&+ (\boldsymbol{\sigma}_3 \cdot \vec{r}_{23} \times \boldsymbol{\sigma}_1)(\boldsymbol{\sigma}_2 \cdot \vec{r}_{23}) t(r_{23}) y(r_{13}) \\
&+ (\boldsymbol{\sigma}_2 \cdot \vec{r}_{13} \times \boldsymbol{\sigma}_3)(\boldsymbol{\sigma}_1 \cdot \vec{r}_{13}) t(r_{13}) y(r_{23}) \\
&+ (\boldsymbol{\sigma}_3 \cdot \vec{r}_{23} \times \vec{r}_{13})(\sigma_1 \cdot \vec{r}_{13})(\sigma_2 \cdot \vec{r}_{23}) t(r_{13}) t(r_{23})]
\end{aligned}
$$

$$W_0 = \left(\frac{m_\pi}{F_\pi}\right)^4 \frac{g_A^2 m_\pi^2}{(4\pi)^2}$$

TPE term & TM potential

Tucson Melbourne potential has **the same spin-isospin structure** of the chiral NNLOL potential TPE term.

It corresponds to the following choice of the constants

$$W_0 = \left(\frac{g m_\pi}{8\pi m_N}\right)^2 m_\pi^4 \qquad c_1 = \frac{a}{m_\pi^2} \quad , \quad c_3 = 2b \quad , \quad c_4 = -4d$$

The cutoff function is not an exponential, but

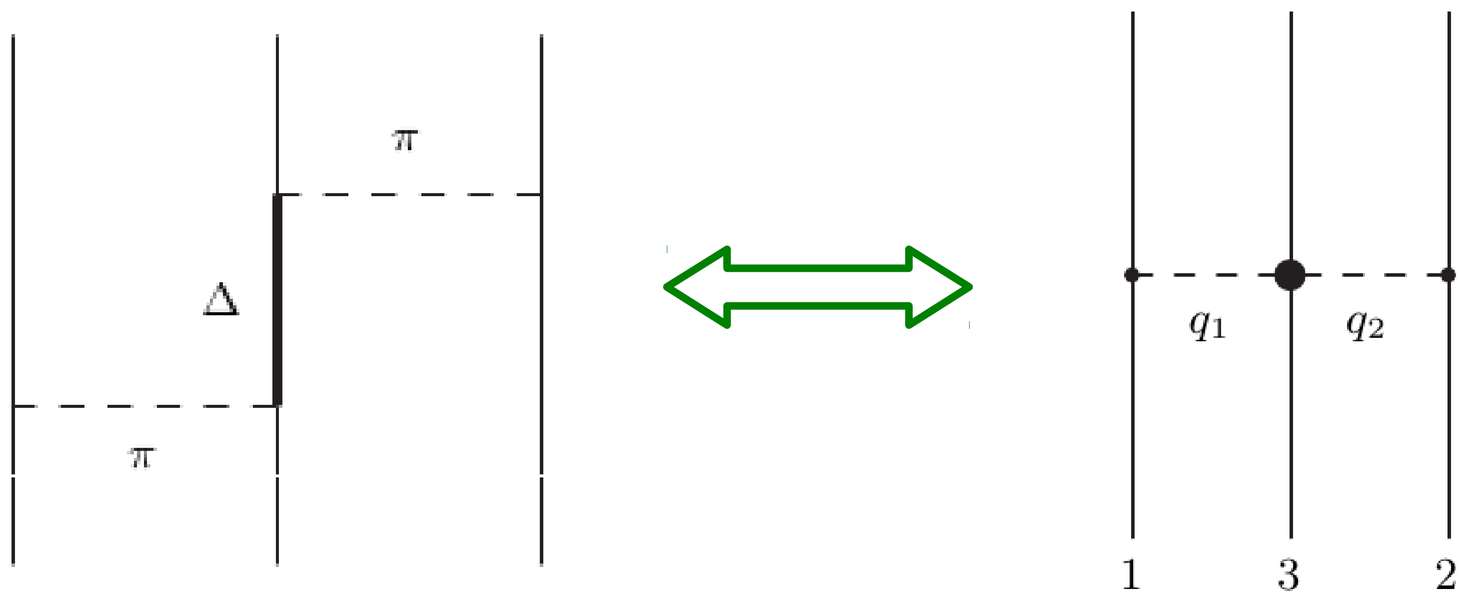
$$F_\Lambda(q^2) = \left(\frac{\Lambda^2 - m_\pi^2}{\Lambda^2 + q^2}\right)^2$$

The analytic expression of the radial functions is in this case known

$$\begin{cases} y(r) = \dfrac{e^{-r\Lambda}}{2m_\pi^3 r^3}\Big[2 - m_\pi^2 r^2 - 2(1 + m_\pi r)e^{r(\Lambda - m_\pi)} \\ \qquad + r\Lambda(2 + r\Lambda)\Big] \\ t(r) = \dfrac{e^{-r\Lambda}}{2m_\pi^3 r^5}\Big[-6 + 2(3 + 3m_\pi r + m_\pi^2 r^2)e^{r(\Lambda - m_\pi)} \\ \qquad + m_\pi^2 r^2(1 + r\Lambda) - r\Lambda[6 + r\Lambda(3 + r\Lambda)]\Big]\,. \end{cases}$$

TPE term and UIX

V_3 and V_4 correspond to the anticommutator and to the commutator term present in $V^{2\pi}$ of UIX potential



Given the following relations between constants and radial functions

$$\begin{cases} bW_0 = 4A_{2\pi} \\ dW_0 = 4C_{2\pi} \end{cases} \qquad \begin{cases} Y(r) = y(r) + \frac{r^2}{3}t(r) \\ T(r) = \frac{r^2}{3}t(r) \end{cases}$$

V_a term **is not present** in UIX.

OPE term

From the chiral Lagrangian, the OPE original momentum space version is

$$V^{OPE}(3:12) = -c_D V_0^D \frac{(\boldsymbol{\sigma}_2 \cdot \mathbf{q}_2)}{(q_2^2 + m_\pi^2)} \Big[\alpha_1 (\boldsymbol{\sigma}_1 \cdot \mathbf{q}_2) \tau_{12} + \alpha_2 (\boldsymbol{\sigma}_1 \cdot \mathbf{q}_2) \tau_{13} + \alpha_3 \mathbf{q}_2 (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_3) \vec{\tau}_2 \cdot (\vec{\tau}_1 \times \vec{\tau}_3) + 1 \leftrightarrow 2 \Big]$$

$$V_0^D = \frac{g_A}{8 F_\pi^4 \Lambda_\chi}$$

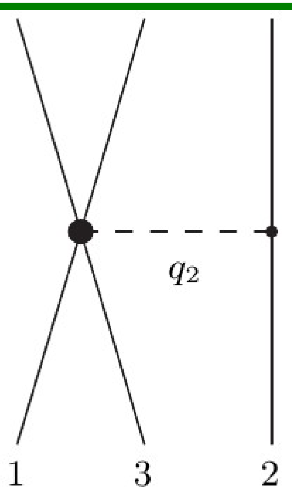
When applied to an antisymmetric wavefunction

$$V^{\mathrm{OPE}}(3:12)\mathcal{A}_{12}|\Psi\rangle \qquad \mathcal{A}_{12} = 1 - \frac{(1+\sigma_{12})}{2}\frac{(1+\tau_{12})}{2}$$

all three different contact structures lead to the same expression

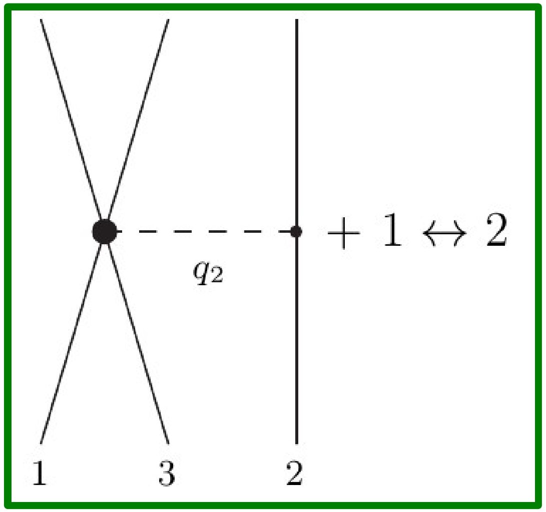
Epelbaum et al. (2002)

It is convenient to consider only one



$$+ 1 \leftrightarrow 2 = -c_D V_0^D \frac{(\boldsymbol{\sigma}_2 \cdot \mathbf{q}_2)}{(q_2^2 + m_\pi^2)} \Big[\alpha_1 (\boldsymbol{\sigma}_1 \cdot \mathbf{q}_2) \tau_{12} + 1 \leftrightarrow 2 \Big]$$

OPE term



Coordinate space

$$= c_D W_0^D \tau_{12}[\sigma_{12} y(r_{23}) z_0(r_{13}) + (\boldsymbol{\sigma}_1 \cdot \vec{r}_{23})(\boldsymbol{\sigma}_2 \cdot \vec{r}_{23}) t(r_{23}) z_0(r_{13}) + 1 \leftrightarrow 2]$$

$$W_0^D = \frac{m_\pi^6}{(4\pi)^2} V_0^D$$

This is the Fourier transform of just one of the equivalent contact terms

Because of the regulator dependence on the momentum transferred $\mathbf{q}_2$ $\Rightarrow$

$$\delta(r_{13}) \longrightarrow \frac{m_\pi^3}{4\pi} z_0(r_{13})$$

No more contact terms!

The antisymmetrization operator also exchange **the position of nucleons**

In principle all the different terms need to be considered.

Contact term

From the chiral Lagrangian, the momentum space version of contact term is

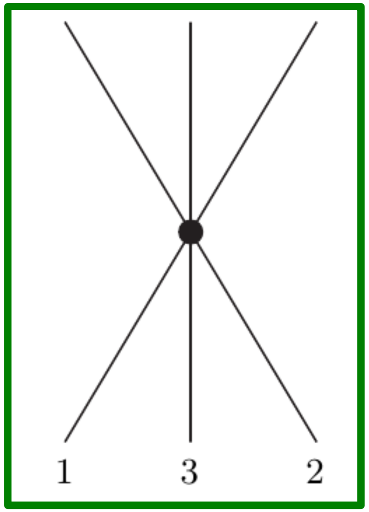
$$V^{cont}(3:12) = c_E(\beta_1 + \beta_2\sigma_{12} + \beta_3\tau_{12} + \beta_4\sigma_{12}\tau_{12} + \beta_5\sigma_{12}\tau_{23} + \beta_6(\vec{\sigma}_1 \times \vec{\sigma}_2)\cdot\vec{\sigma}_3 + (\vec{\tau}_1 \times \vec{\tau}_2)\cdot\vec{\tau}_3 + 1 \leftrightarrow 2)$$

Once multiplied by the antisimmetrization operator, all the terms give the same contribution: it is possible to choice just one of those

$$V^{cont}(3:12) = c_E W_0^E \tau_{12}$$

$$W_0^E = \frac{m_\pi^6}{(4\pi)^2}\frac{1}{F_\pi^4 \Lambda_\chi}$$

Coordinate space



$$= \quad c_E W_0^E \tau_{12} z_0(r_{13}) z_0(r_{23})$$

In principle all the different terms need to be considered.

Contact term in TM' and UIX

- TM' potential has a repulsive three-nucleon contact term

$$V_E(3:12) = c_E W_0^E z_0(r_{13}) z_0(r_{23})$$

NO τ_{12}!

$$W_0^E = \left(\frac{g m_\pi}{8\pi m_N}\right)^2 \frac{9 m_\pi^2}{\Lambda_\chi}$$

$$z_0(r) = \frac{e^{-r\Lambda}}{8\pi\Lambda}(m_\pi^2 - \Lambda^2)^2$$

- V^R of UIX, although not a contact term, has the same structure of the contact term of NNLOL

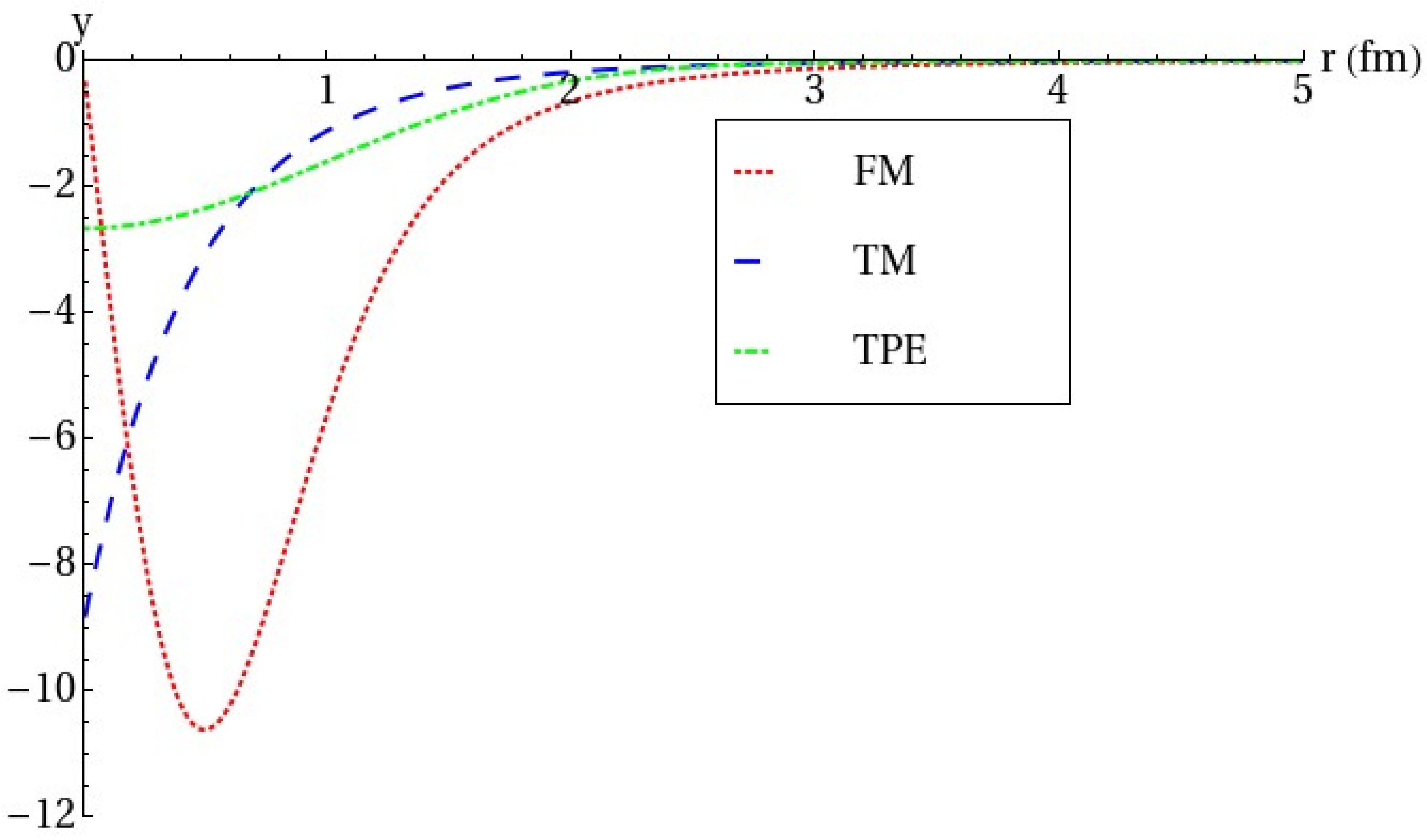
$$\begin{cases} T^2(m_\pi r) \longleftrightarrow z_0(r) \\ U_0 \longleftrightarrow c_E W_0^E \end{cases}$$

NO τ_{12}!

Chiral NNLOL, TM' and UIX

Some of the contributions of NNLOL potential also appear in TM' and UIX.

The differences among the potentials reside in the radial functions and in the constants.



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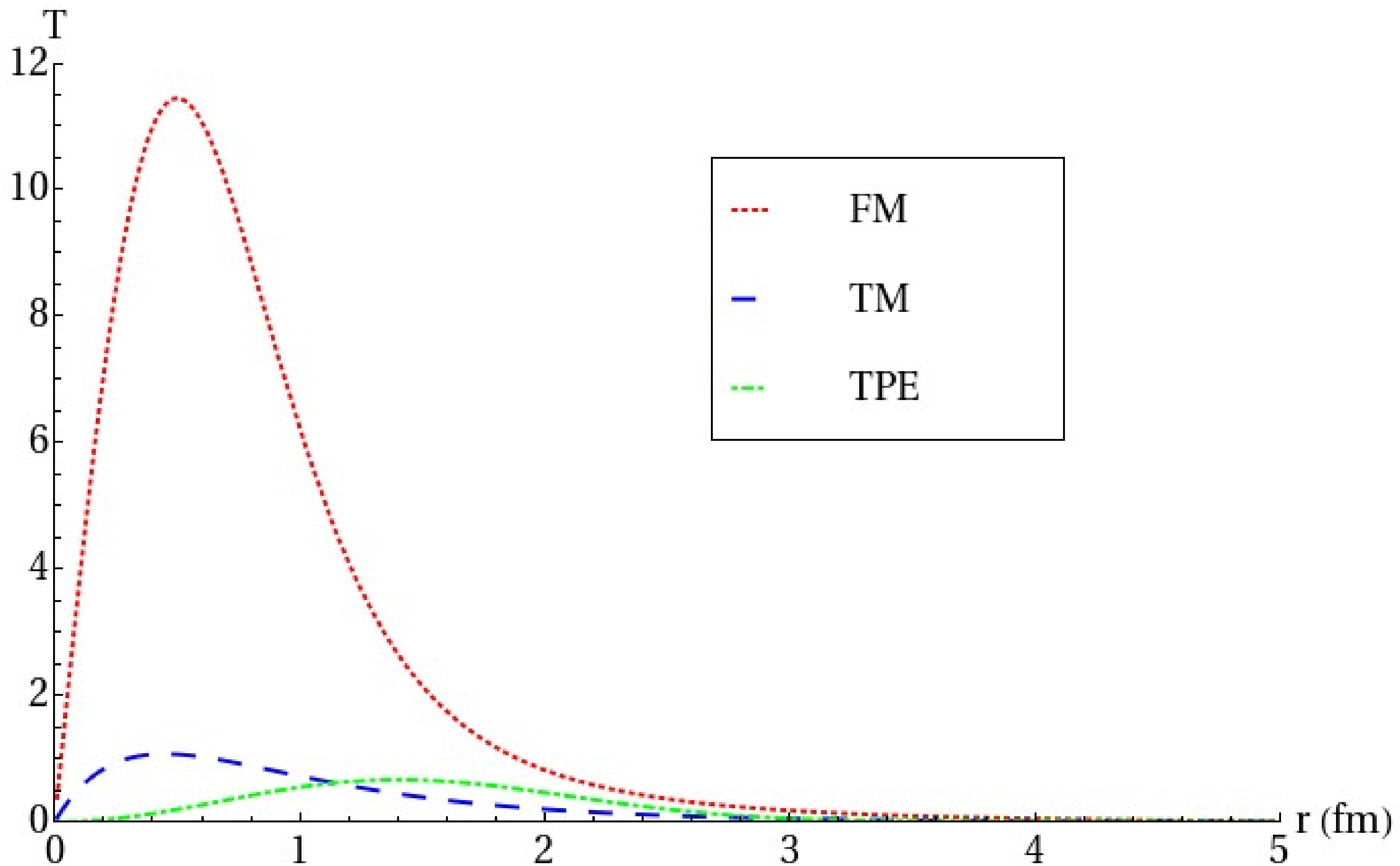

3-body potential analysys

Kievsky et al. in 2010 have found the best-fit values for the TM' and NNOL 3-body potentials to simultaneously reproduce

$$\begin{cases} B(^3H) = -8.482 \text{ MeV} \\ B(^4He) = -28.30 \text{ MeV} \\ {}^2a_{nd} = 0.645 \pm 0.003 \pm 0.007 \end{cases}$$

3-body potential analysys

Kievsky et al. in 2010 have found the best-fit values for the TM' and NNOL 3-body potentials to simultaneously reproduce

$$\begin{cases} B(^3\mathrm{H}) = -8.482 \ \mathrm{MeV} \\ B(^4\mathrm{He}) = -28.30 \ \mathrm{MeV} \\ ^2a_{nd} = 0.645 \pm 0.003 \pm 0.007 \end{cases}$$

Chiral NNLOL potential

Potential	c_3 (MeV^{-1})	c_4 (MeV^{-1})	c_D	c_E
NNLOL$_1$	-0.00448	-0.001963	-0.5	0.100
NNLOL$_2$	-0.00448	-0.002044	-1.0	0.000
NNLOL$_3$	-0.00480	-0.002017	-1.0	-0.030
NNLOL$_4$	-0.00544	-0.004860	-2.0	-0.500

$$\begin{cases} c_1 = 0.00081 \, \mathrm{MeV}^{-1} \\ \Lambda_\chi = 700 \, \mathrm{MeV} \\ \Lambda = 500 \, \mathrm{MeV} \end{cases}$$

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TM' potential

Potential	$b(m_\pi^{-3})$	$d(m_\pi^{-3})$	c_E	$\Lambda(m_\pi)$
TM'_1	-8.256	-4.690	1.0	4.0
TM'_2	-3.870	-3.375	1.6	4.8
TM'_3	-2.064	-2.279	2.0	5.6

$$a = -0.87\, m_\pi^{-1}$$

NNLOL contact term issue in nuclear matter

Full momentum space expression of NNLO contact term

$$V^{cont}(3:12) = c_E(\beta_1 + \beta_2\sigma_{12} + \beta_3\tau_{12} + \beta_4\sigma_{12}\tau_{12} + \beta_5\sigma_{12}\tau_{23}$$
$$+ \beta_6(\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \vec{\sigma}_3 + (\vec{\tau}_1 \times \vec{\tau}_2) \cdot \vec{\tau}_3 + 1 \leftrightarrow 2)$$

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Fourier transform of the isospin and scalar term yields

$$V_E^{\tau}(3:12) = c_E^{\tau} W_0^E \tau_{12} z_0(r_{13}) z_0(r_{23})$$

$$V_E^{I}(3:12) = c_E^{I} W_0^E z_0(r_{13}) z_0(r_{23})$$

NNLOL contact term issue in nuclear matter

Full momentum space expression of NNLO contact term

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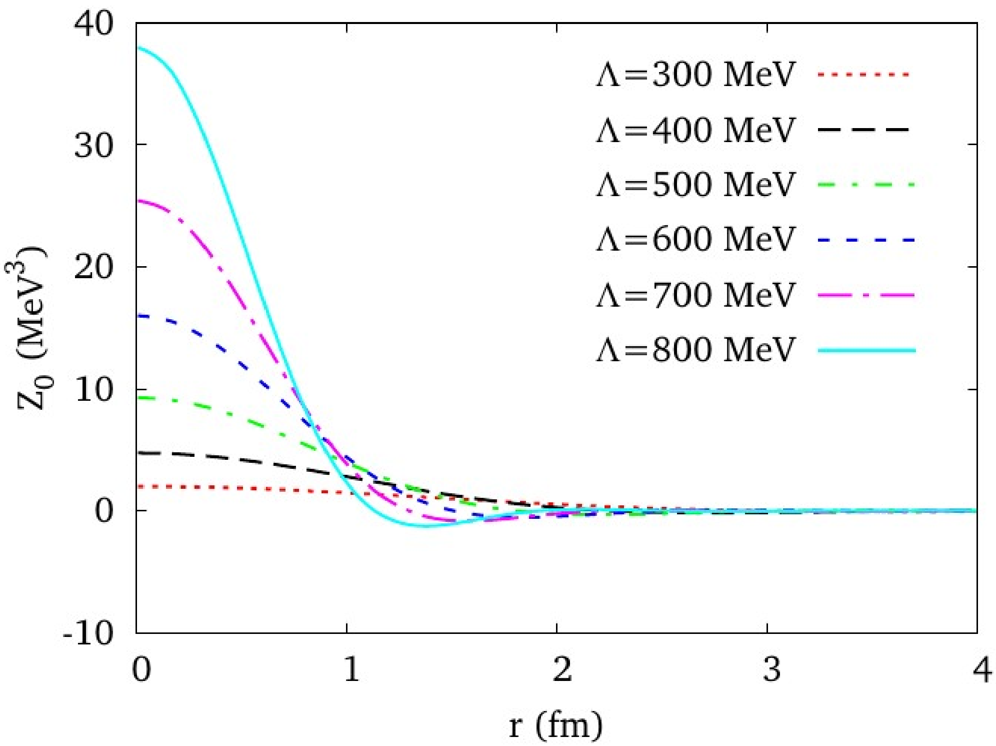
$$V_E^\tau(3:12) = c_E^\tau W_0^E \tau_{12} z_0(r_{13}) z_0(r_{23})$$

$$V_E^I(3:12) = c_E^I W_0^E z_0(r_{13}) z_0(r_{23})$$

Convenient normalization for the radial function

$$Z_0(r) = \frac{(4\pi)}{m_\pi^3} z_0(r)$$

$$\lim_{\Lambda\to\infty} Z_0(r) = \delta(r)$$



NNLOL contact term issue in nuclear matter

In the limit of infinite cutoff

$$\sum_{cycl} V_0^E \delta(\mathbf{r}_{13})\delta(\mathbf{r}_{23})\tau_{12}\mathcal{A}_{123} = -\sum_{cycl} V_0^E \delta(\mathbf{r}_{13})\delta(\mathbf{r}_{23})\mathcal{A}_{123}$$

The terms are equivalent!

Crucial for the above equivalence

The exchange of particles lying in the same position is the identity!

$$e^{i\mathbf{k}_{ij}\cdot\mathbf{r}_{ij}}\delta(\mathbf{r}_{ij}) = 1$$

At finite values of the cutoff this is not true anymore.

Not considering all the terms lead to ambiguities in both PNM and SNM

NNLOL contact term issue in nuclear matter

Assume that reproducing the binding energies of light nuclei and $^2a_{nd}$ and requires a repulsive V_E $\Rightarrow$ $\begin{cases} c_E^\tau < 0 \\ c_E^I > 0 \end{cases}$

Pure Neutron Matter (PNM)

For Pauli principle, in PNM the expectation value of a three-body contact term is zero.

WHILE...

$$\langle V_E^{I,\tau} \rangle_{PNM} = \mathcal{O}\left(\frac{q^4}{\Lambda^4}\right)$$

Λ (MeV)	$\langle V_E^{I,\tau_{12}} \rangle_{PNM}^{FG} / A$ (MeV)
300	9.15
400	5.95
500	3.60
600	2.15
700	1.30
800	0.81
∞	0

Furthermore

$$\langle \tau_{12} \rangle_{PNM} = 1 \Rightarrow \begin{cases} \langle V_E^I \rangle_{PNM} > 0 \\ \langle V_E^{\tau_{12}} \rangle_{PNM} < 0 \end{cases}$$

Fixing only one of the contact terms of NNLOL on low energy observables leads to ambiguity in PNM.

NNLOL contact term issue in nuclear matter

Assume that reproducing the binding energies of light nuclei and $^2a_{nd}$ and requires a repulsive V_E ⟹ $\begin{cases} c_E^\tau < 0 \\ c_E^I > 0 \end{cases}$

Symmetric Nuclear Matter (SNM)

Infinite cutoff

$$\frac{\langle V_E^{\tau_{12}} \rangle_{SNM}^{FG}}{A} = -\frac{3}{16}\rho^2 V_0^E$$

$$\frac{\langle V_E^{I} \rangle_{SNM}^{FG}}{A} = \frac{3}{16}\rho^2 V_0^E$$

Finite cutoff

Λ (MeV)	$\langle V_E^{\tau_{12}} \rangle_{SNM}^{FG}/A$ (MeV)	$\langle V_E^{I} \rangle_{SNM}^{FG}/A$ (MeV)
300	-2.61	10.21
400	-3.61	8.15
500	-4.37	6.93
600	-4.87	6.30
700	-5.15	5.98
800	-5.30	5.81
∞	-5.55	5.55

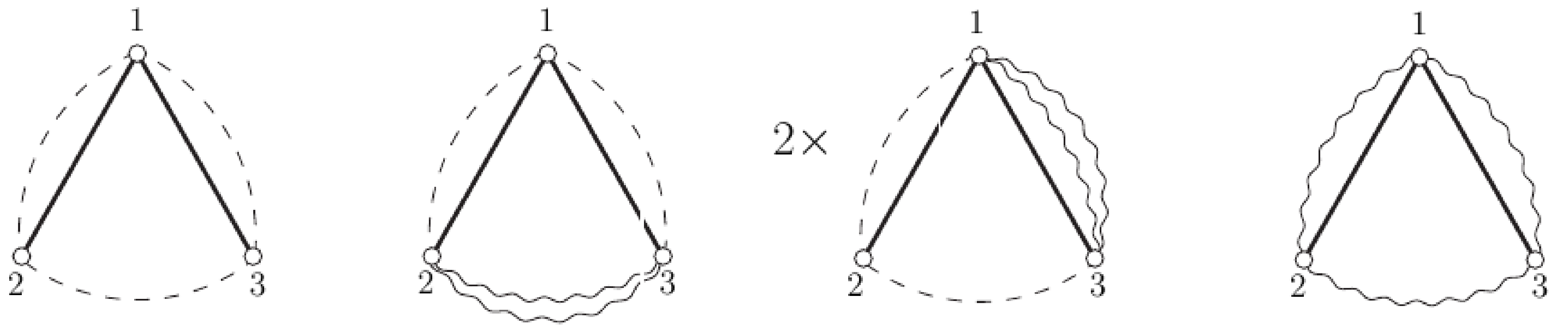
The two contact term are equivalent in the limit of infinite cutoff only

3-body potentials in CBF

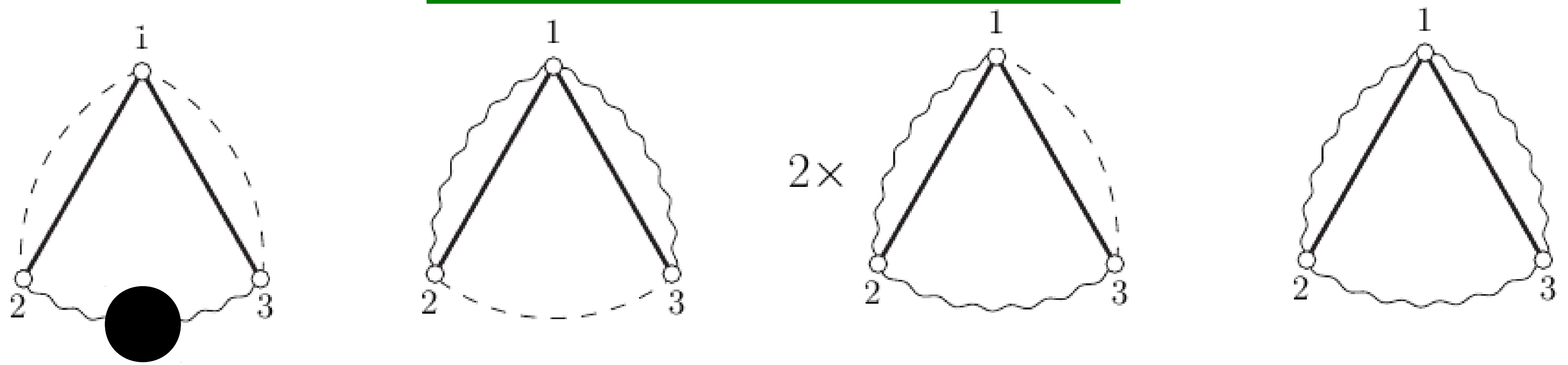
Expectation value of the three body potential

$$\langle V \rangle = \frac{\int dx_1 \dots dx_A \Phi_0^*(x_1 \dots x_A) F^\dagger V_{123} F \Phi_0(x_1 \dots x_A)}{\int dx_1 \dots dx_A \Phi_0^*(x_1 \dots x_A) F^\dagger F \Phi_0(x_1 \dots x_A)}$$

Diagrams involved for the contact term V_E



Diagrams for TPE and OPE



3-body potentials in CBF: simulated annealing

Variational energy depends on a set of parameters

$$E_V = E_V(d_c, d_t, \beta_p, \alpha_p)$$

For the minimization of E_V ⇒ **Simulated annealing procedure**

3-body potentials in CBF: simulated annealing

Variational energy depends on a set of parameters

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3-body potentials in CBF: simulated annealing

Variational energy depends on a set of parameters

$$E_V = E_V(d_c, d_t, \beta_p, \alpha_p)$$

For the minimization of E_V ⟹ **Simulated annealing procedure**

It consists in a Metropolis algorithm

$$s = \{d_c, d_t, \beta_p, \alpha_p\} \qquad s' = \{d'_c, d'_t, \beta'_p, \alpha'_p\}$$

with $P_{s,s'} = \exp\left[-\frac{E(s') - E(s)}{T}\right]$

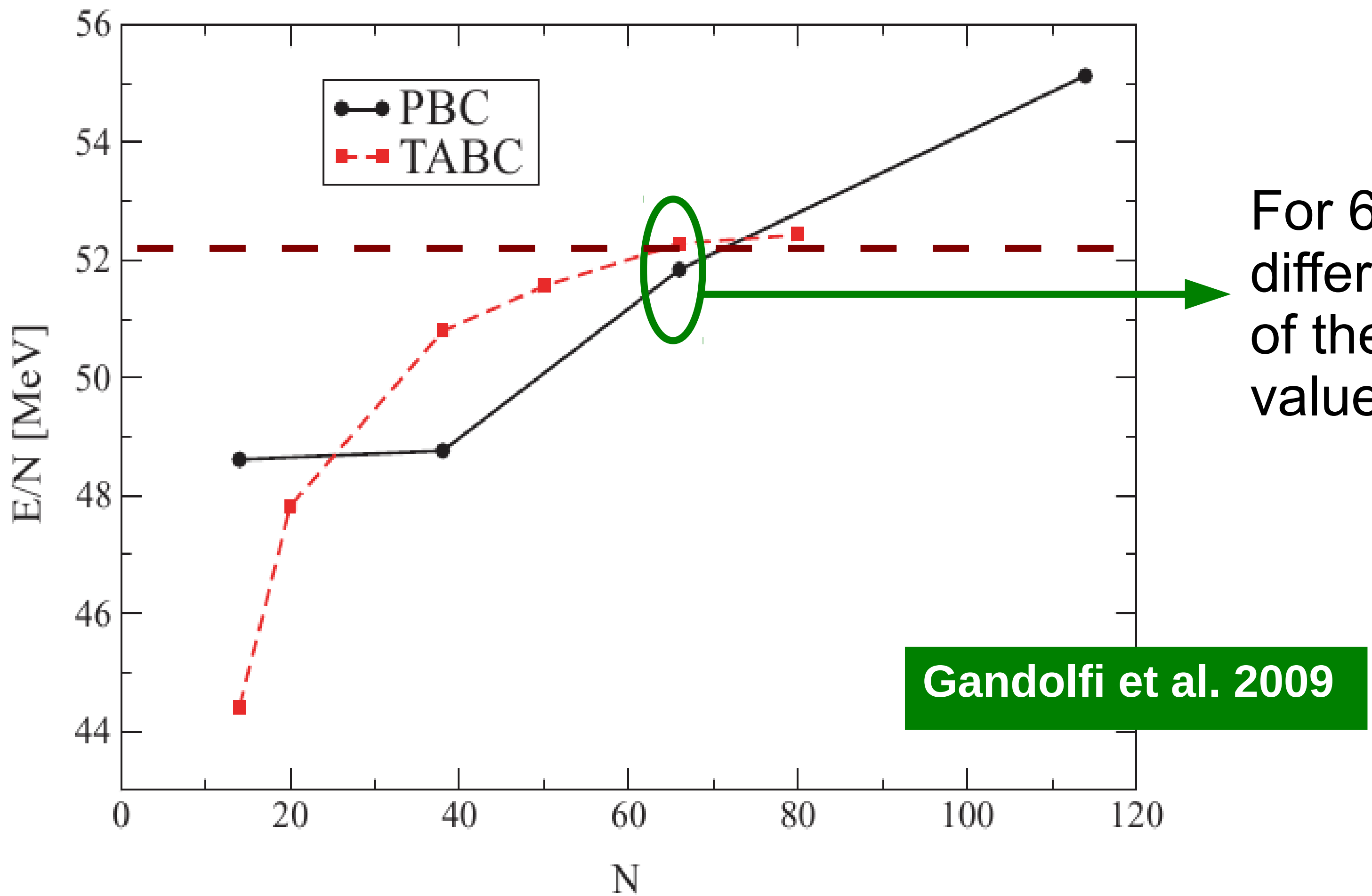
As T is lowered, the parameters stay closer to the minimum of E_V

To keep the violations under control ⟹ Constrained optimization

$$\begin{cases} |E_{PB} - E_{JF}| < 10\%\, T_F \\ \left|\rho \int d\vec{r}_{12}(g^c(r_{12}) - 1) + 1\right| < 0.03 \\ \left|\frac{\rho}{3} \int d\vec{r}_{12} g^\tau(r_{12}) + 1\right| < 0.03 \end{cases}$$

3-body potentials AFDMC

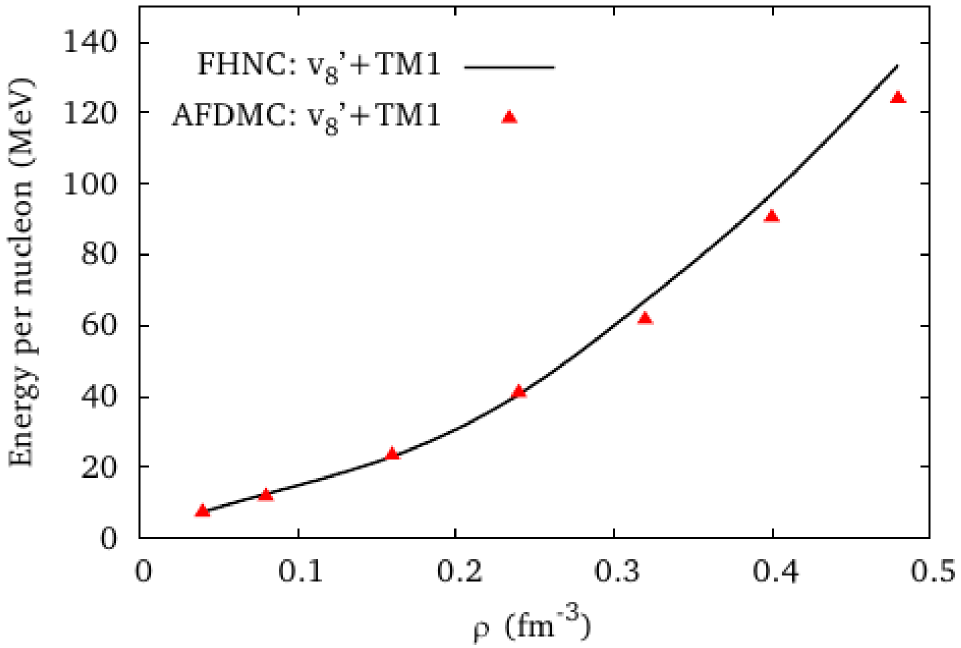
AFDMC simulations for PNM with **66 neutrons in periodic box system.**

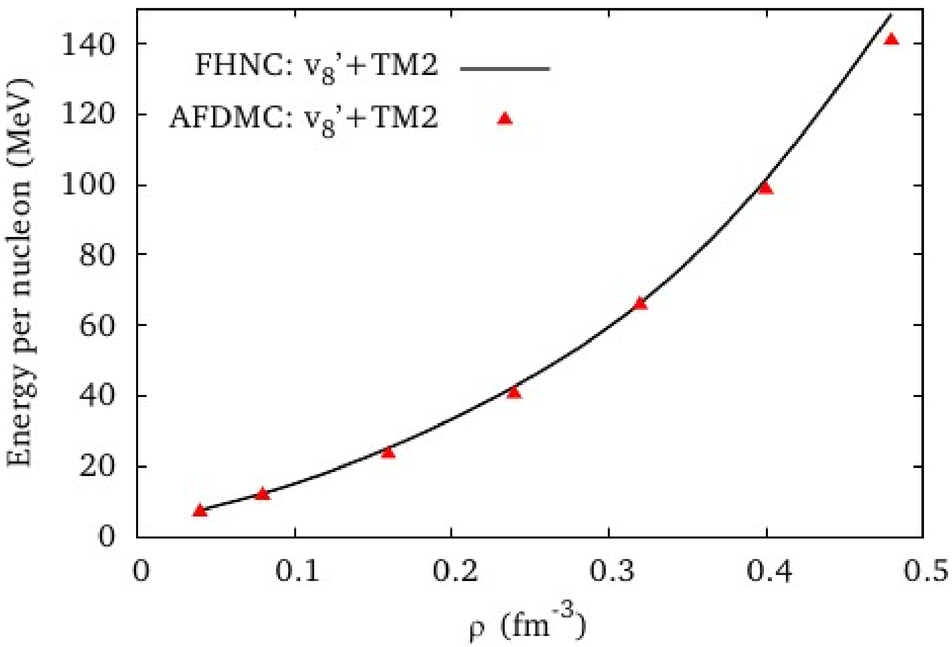


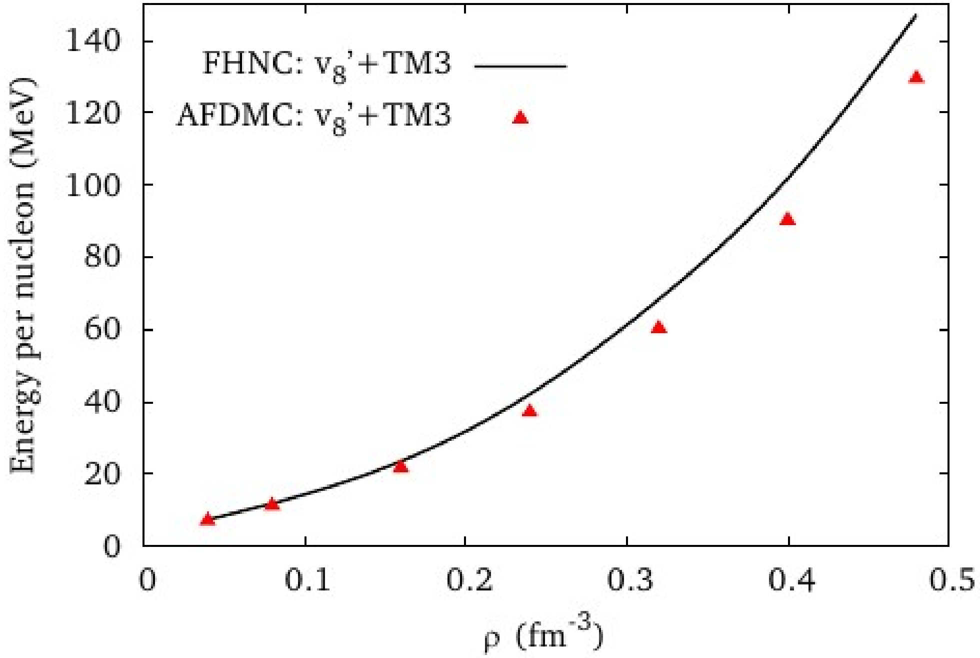
For 66 fermions difference around 2% of the asymptotic value of TABC

	$A=14$	$A=38$	$A=66$	$A=114$	∞
$E/A(\mathrm{MeV})$	56.51	53.50	55.43	56.58	55.71

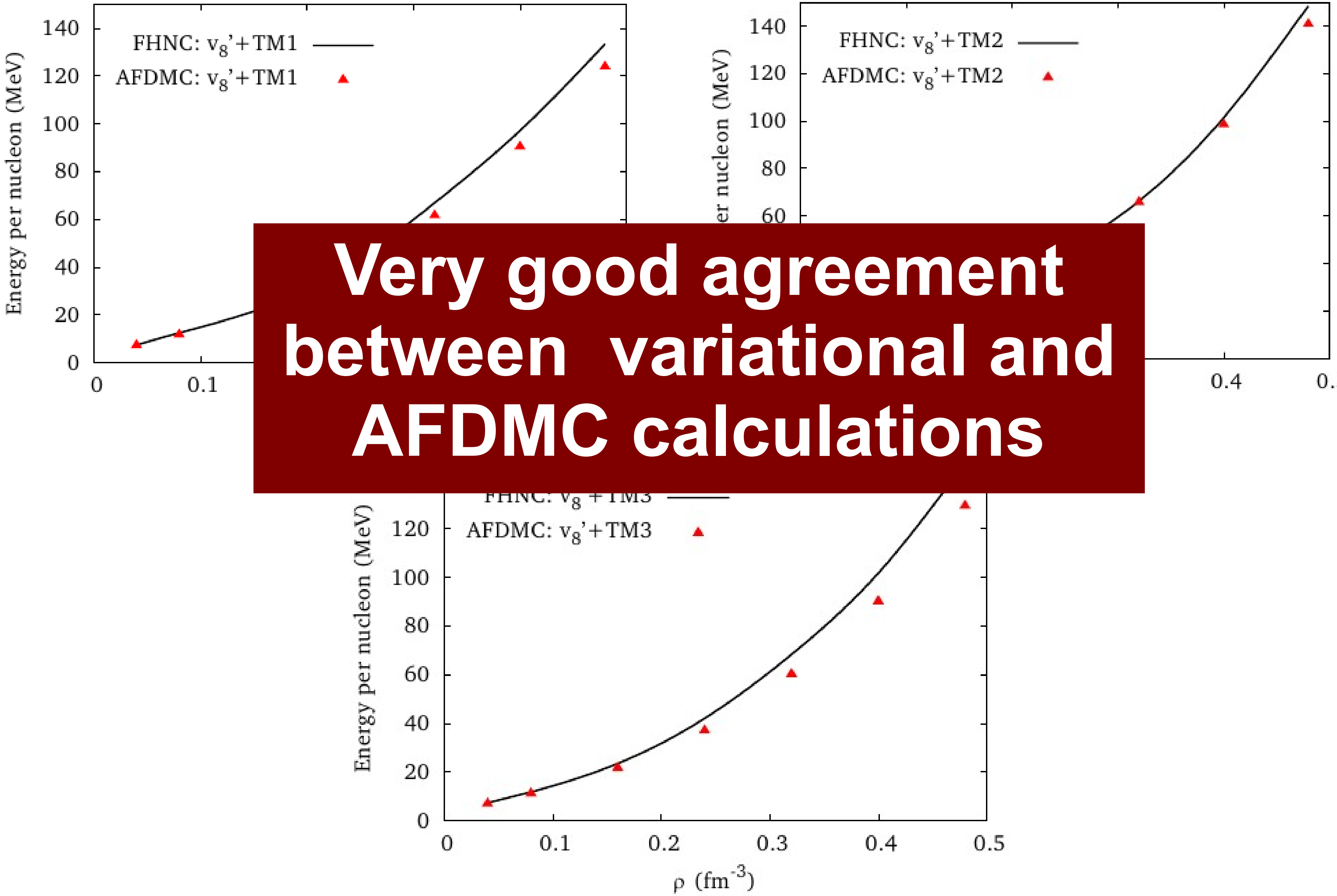
TM' results for PNM





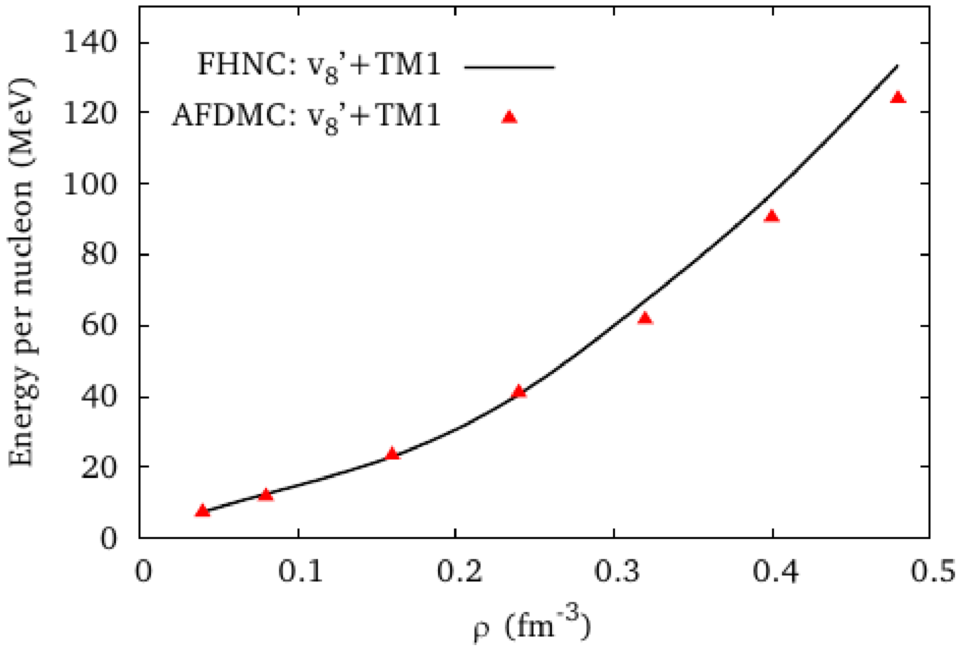


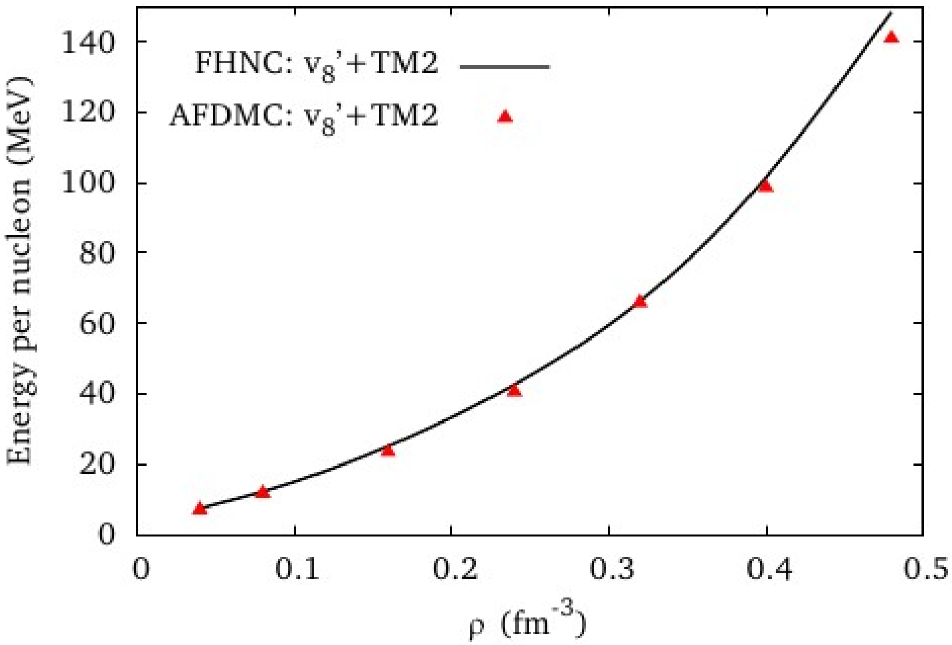
TM' results for PNM

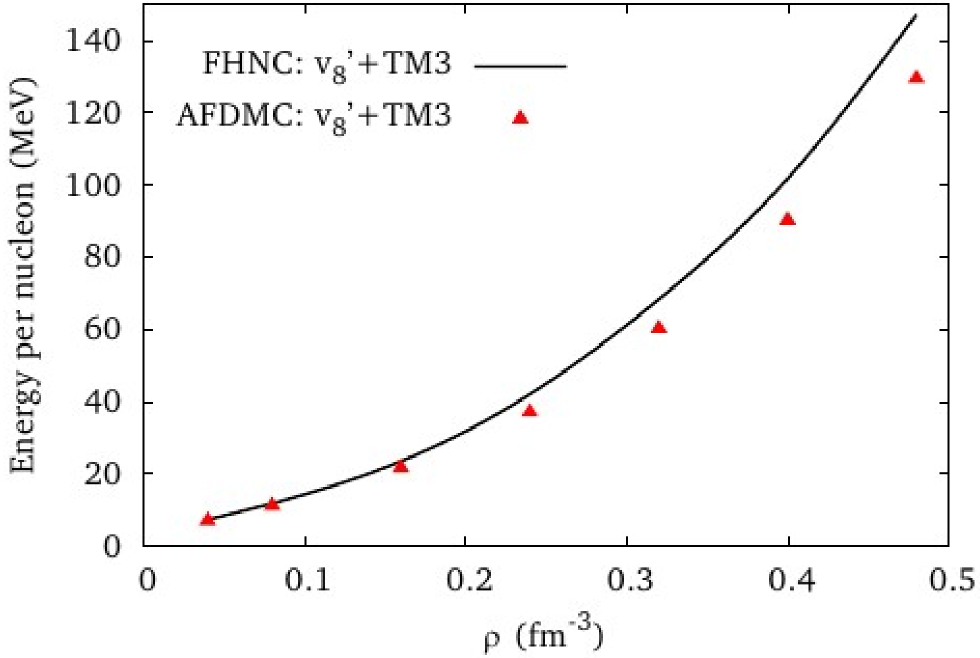


Very good agreement between variational and AFDMC calculations

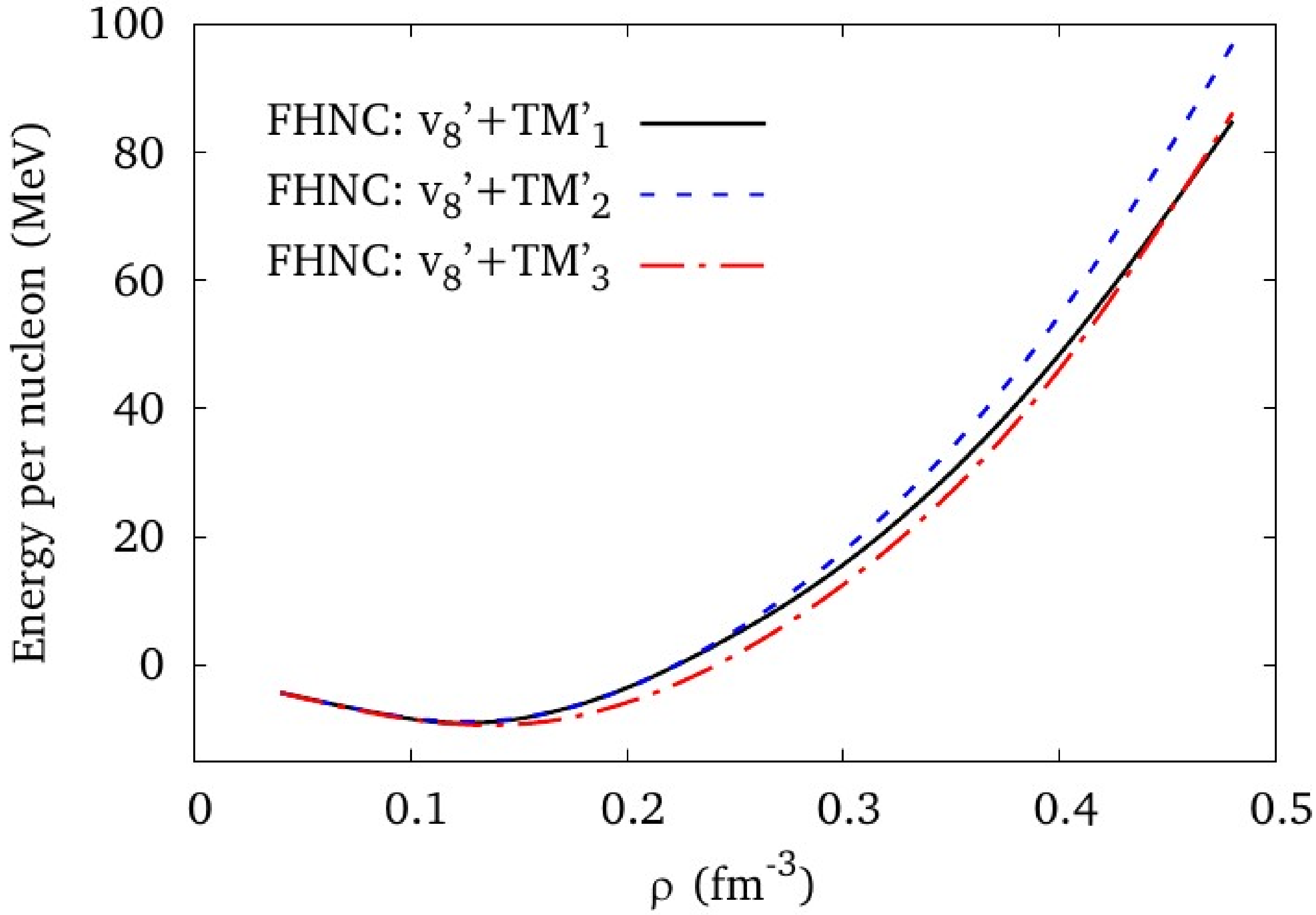
TM' results for PNM







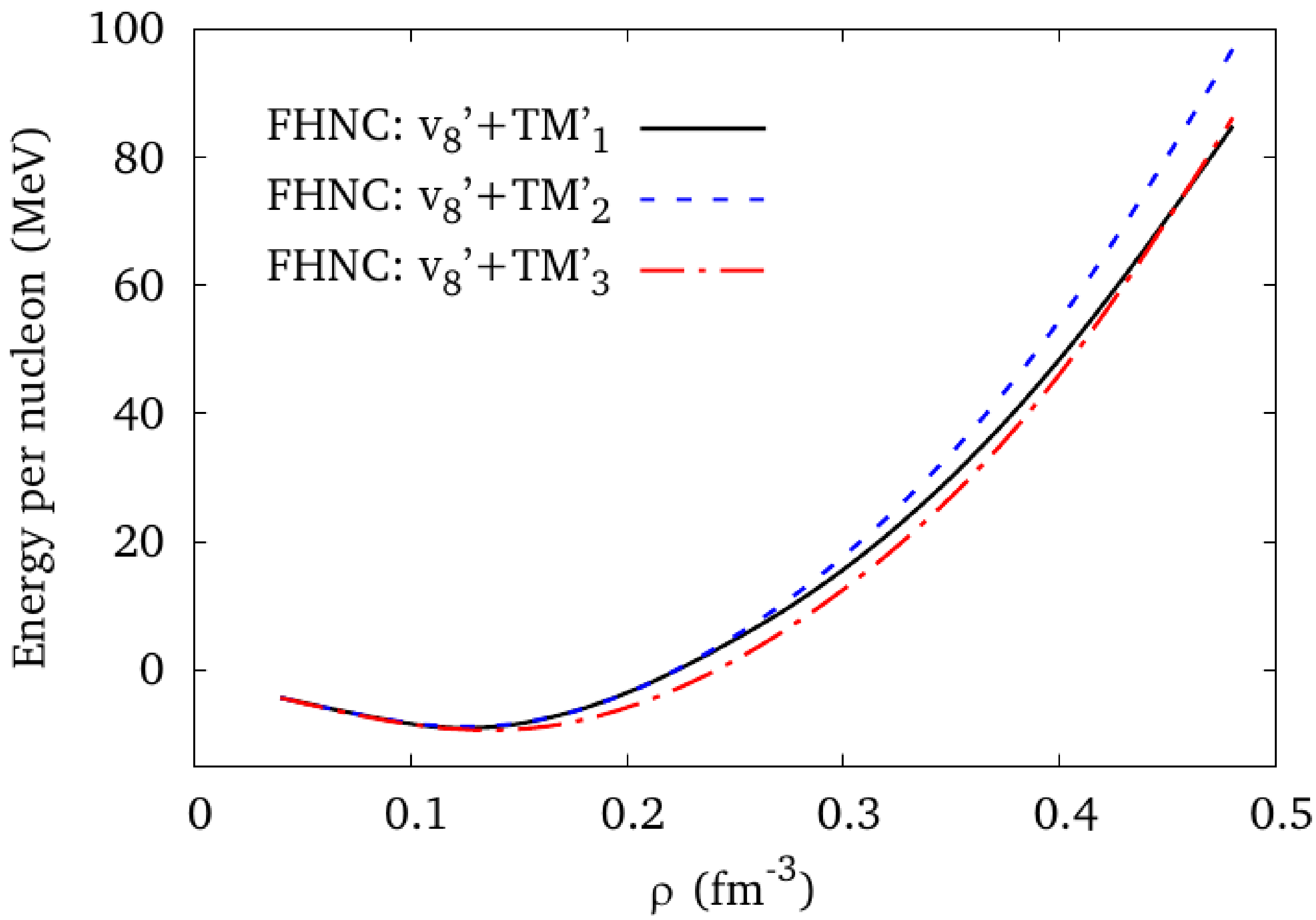
TM' results for SNM



The three Equations of State are very close to each other

Is this happening beacuse three body force are designed to reproduce $^2a_{nd}$ also?

TM' results for SNM

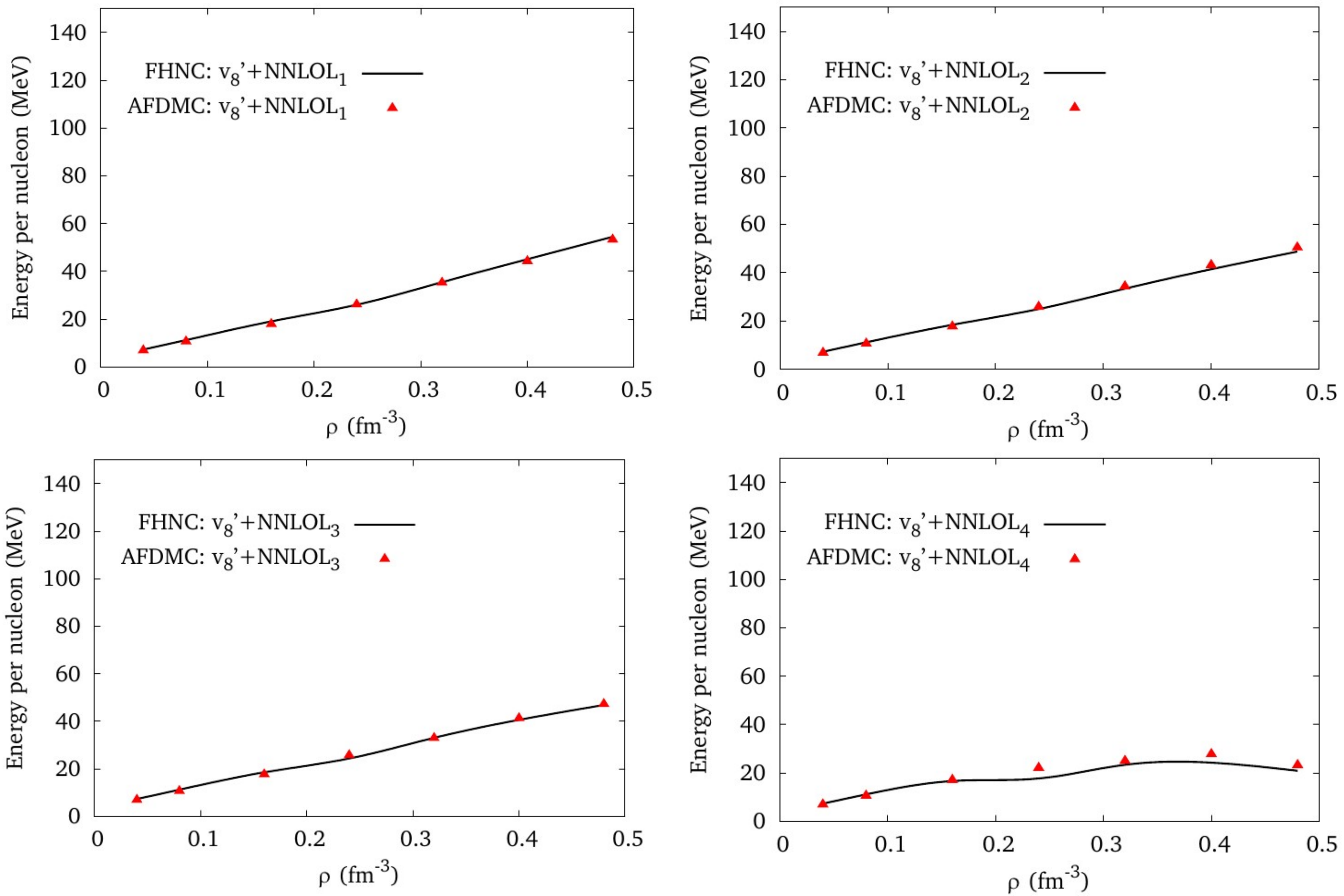


	TM'_1	TM'_2	TM'_3
ρ_0 (fm^{-3})	0.12	0.13	0.14
E_0 (MeV)	-9.0	-8.8	-9.4
K (MeV)	266	243	249

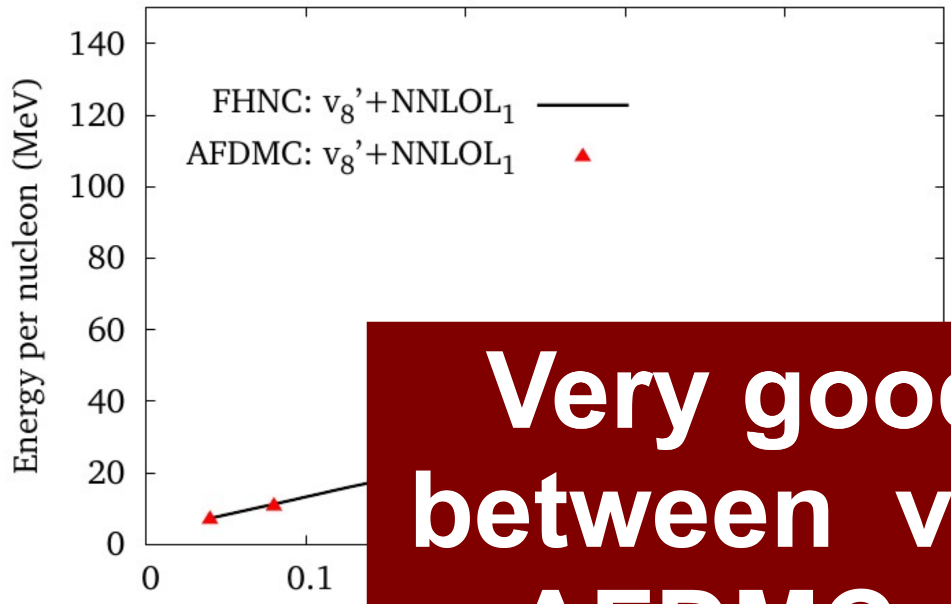
Experimental values

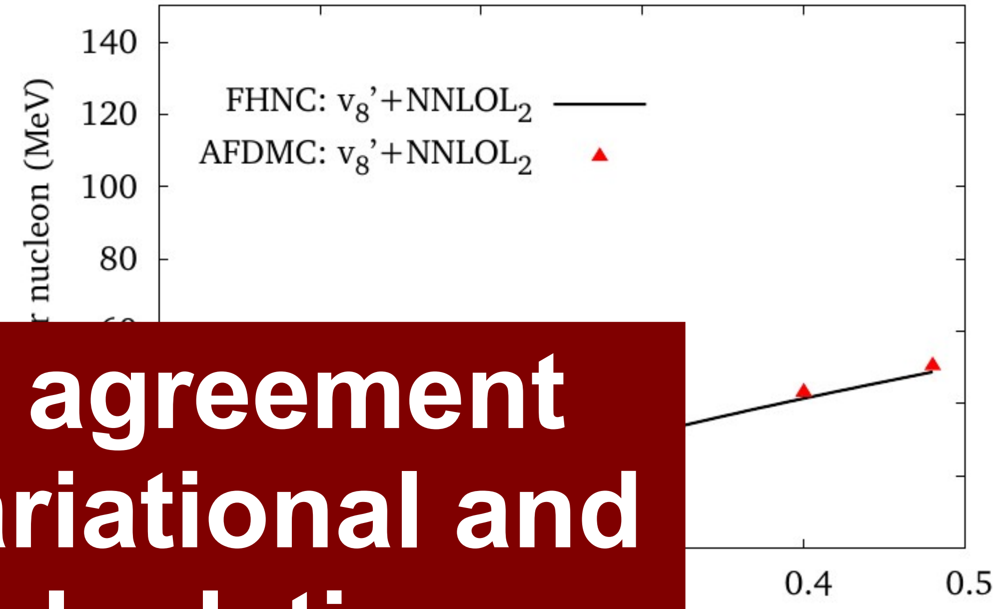
ρ_0 (fm^{-3})	0.16
E_0 (MeV)	-16.0
K (MeV)	240

Chiral NNLOL results for PNM

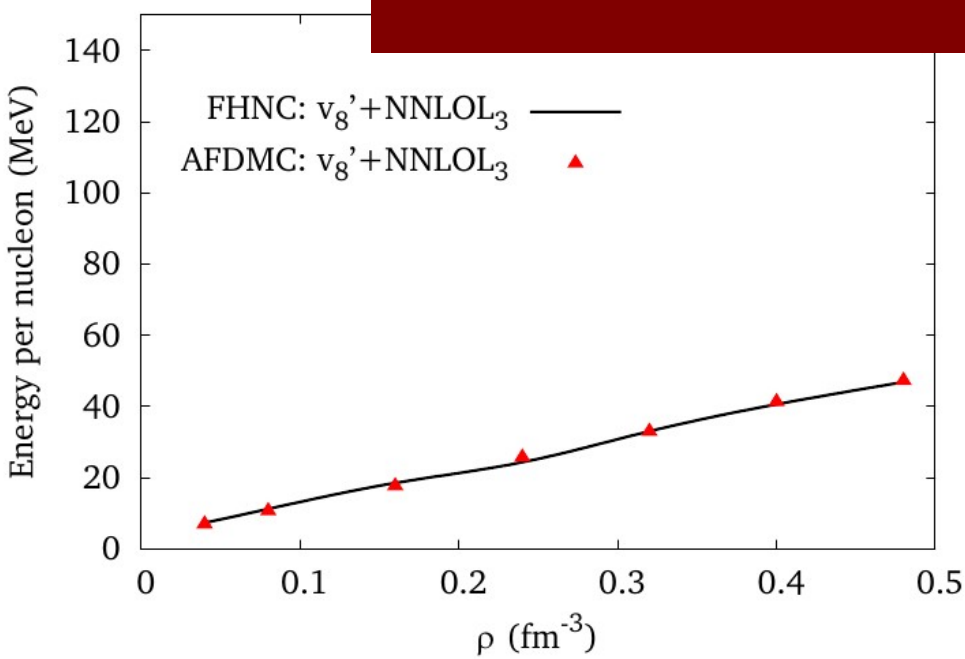


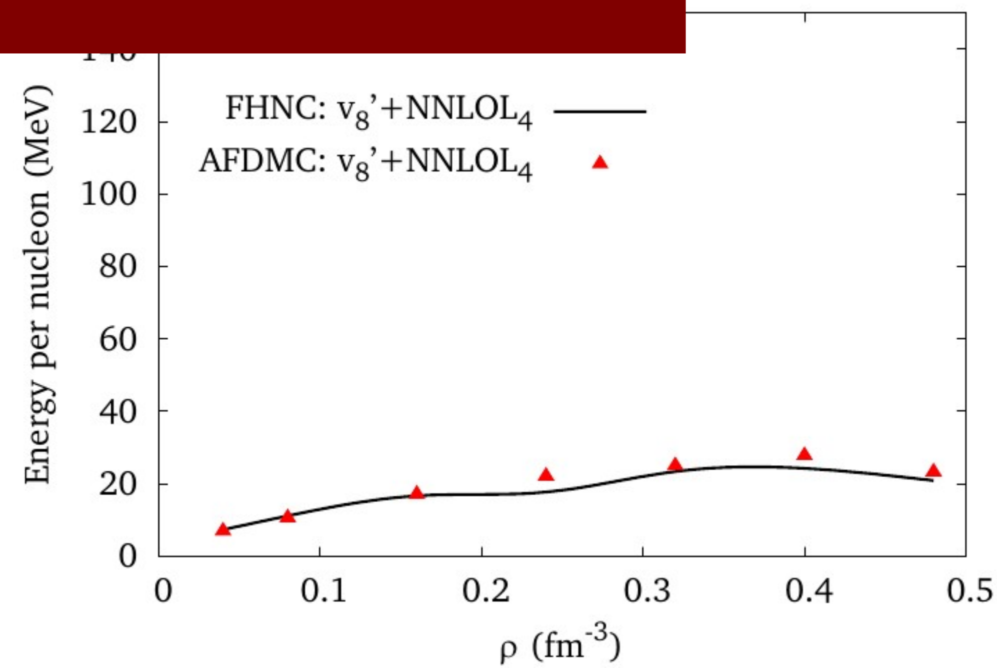
Chiral NNLOL results for PNM



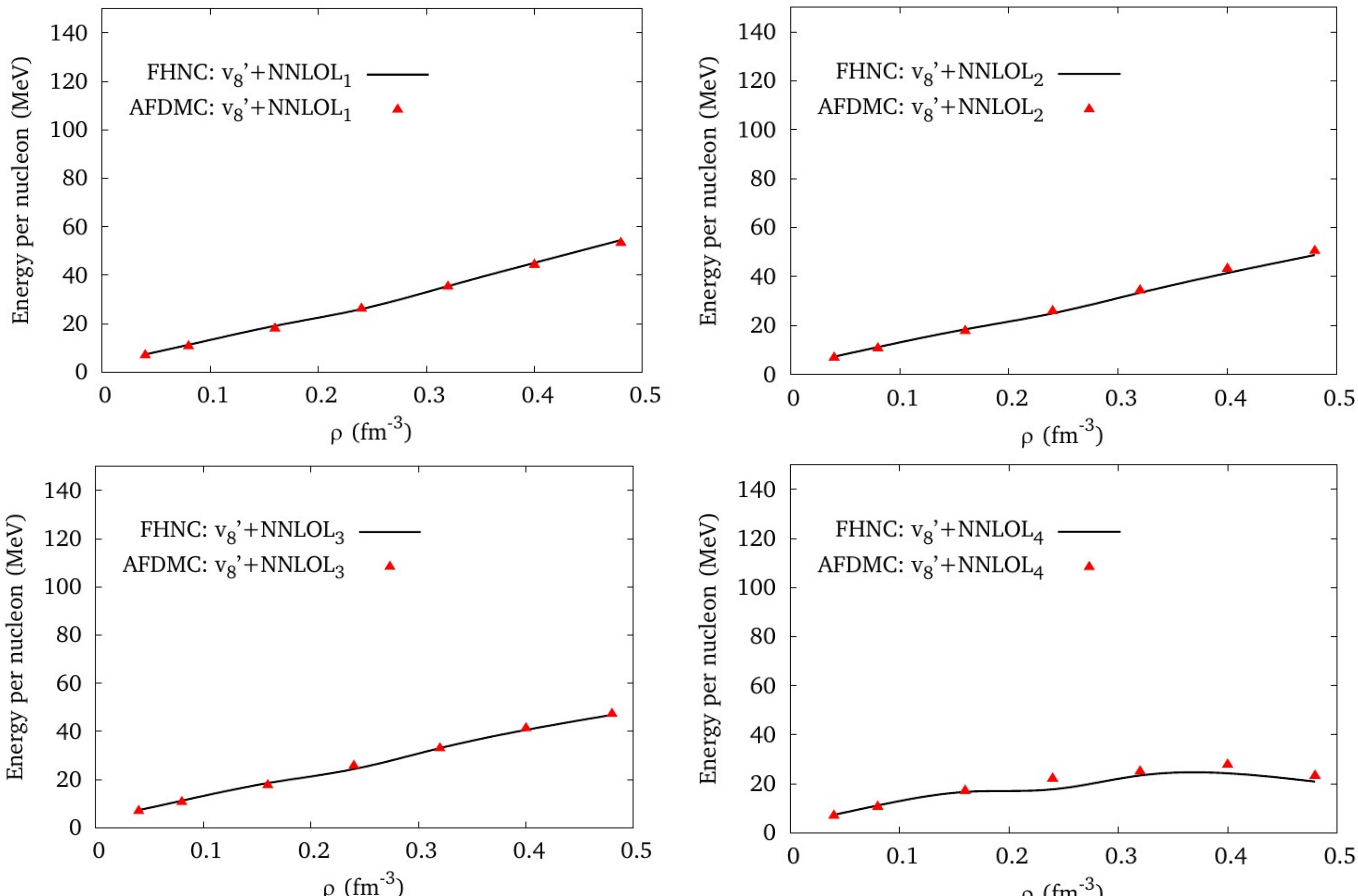


Very good agreement between variational and AFDMC calculations

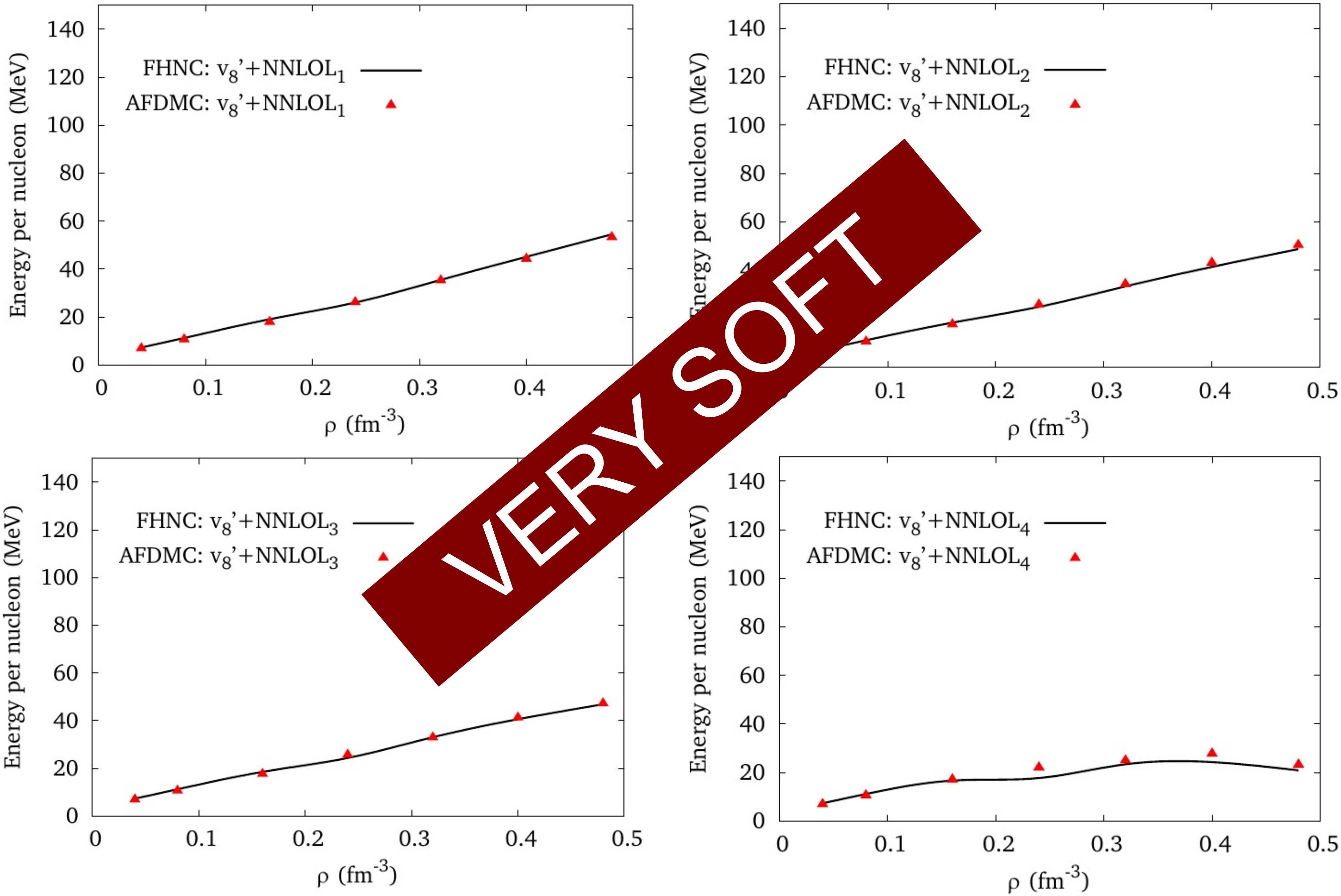




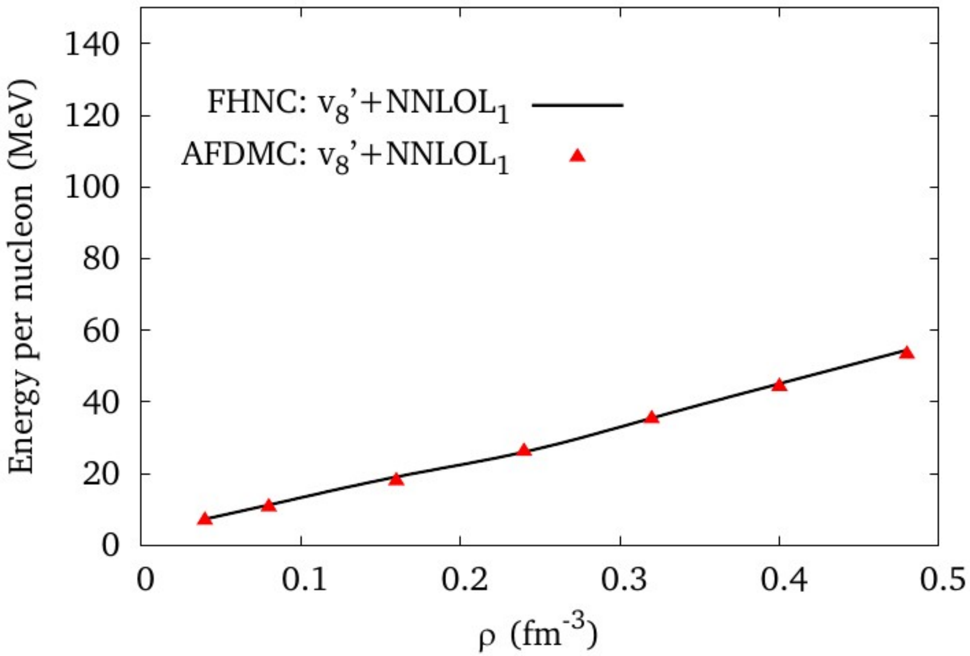
Chiral NNLOL results for PNM

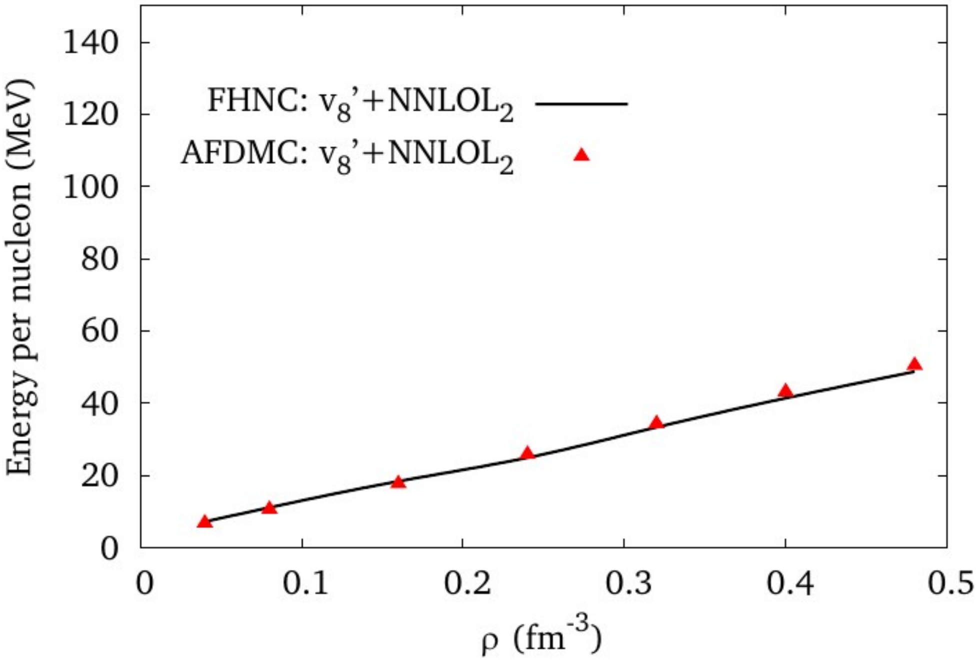


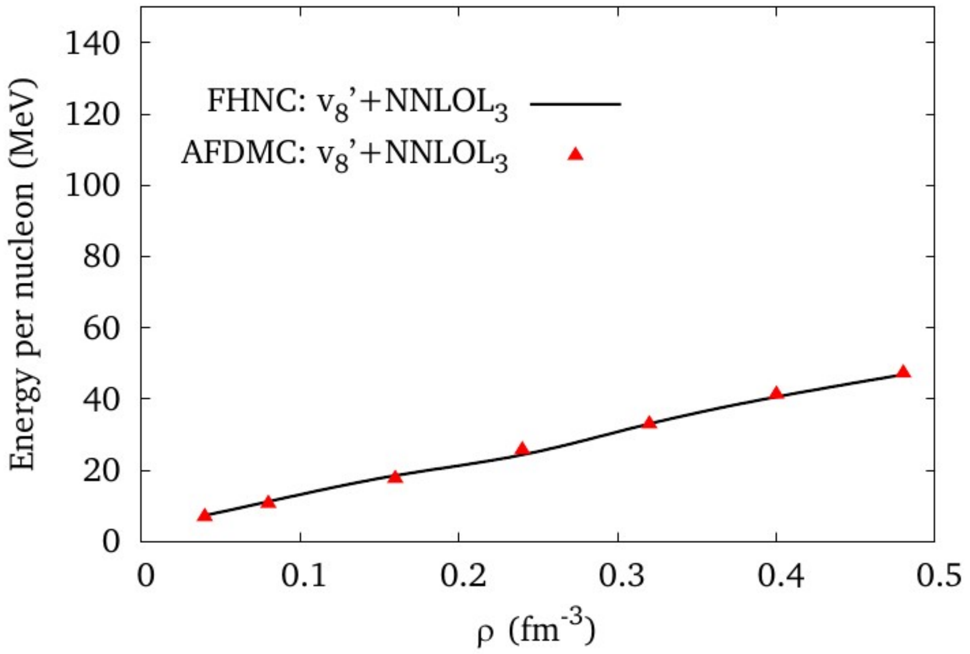
Chiral NNLOL results for PNM

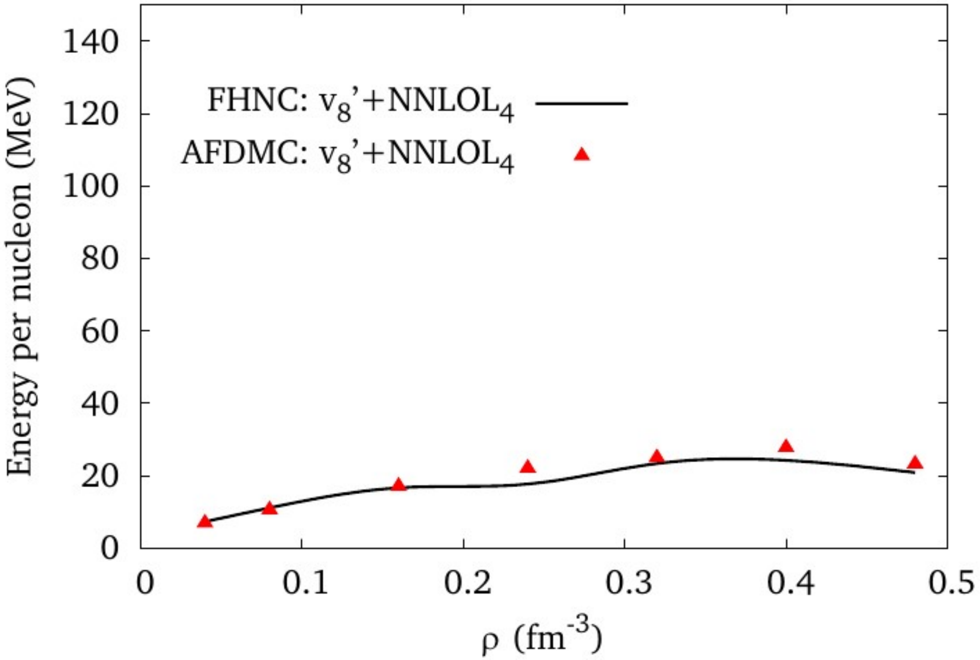


Chiral NNLOL results for PNM

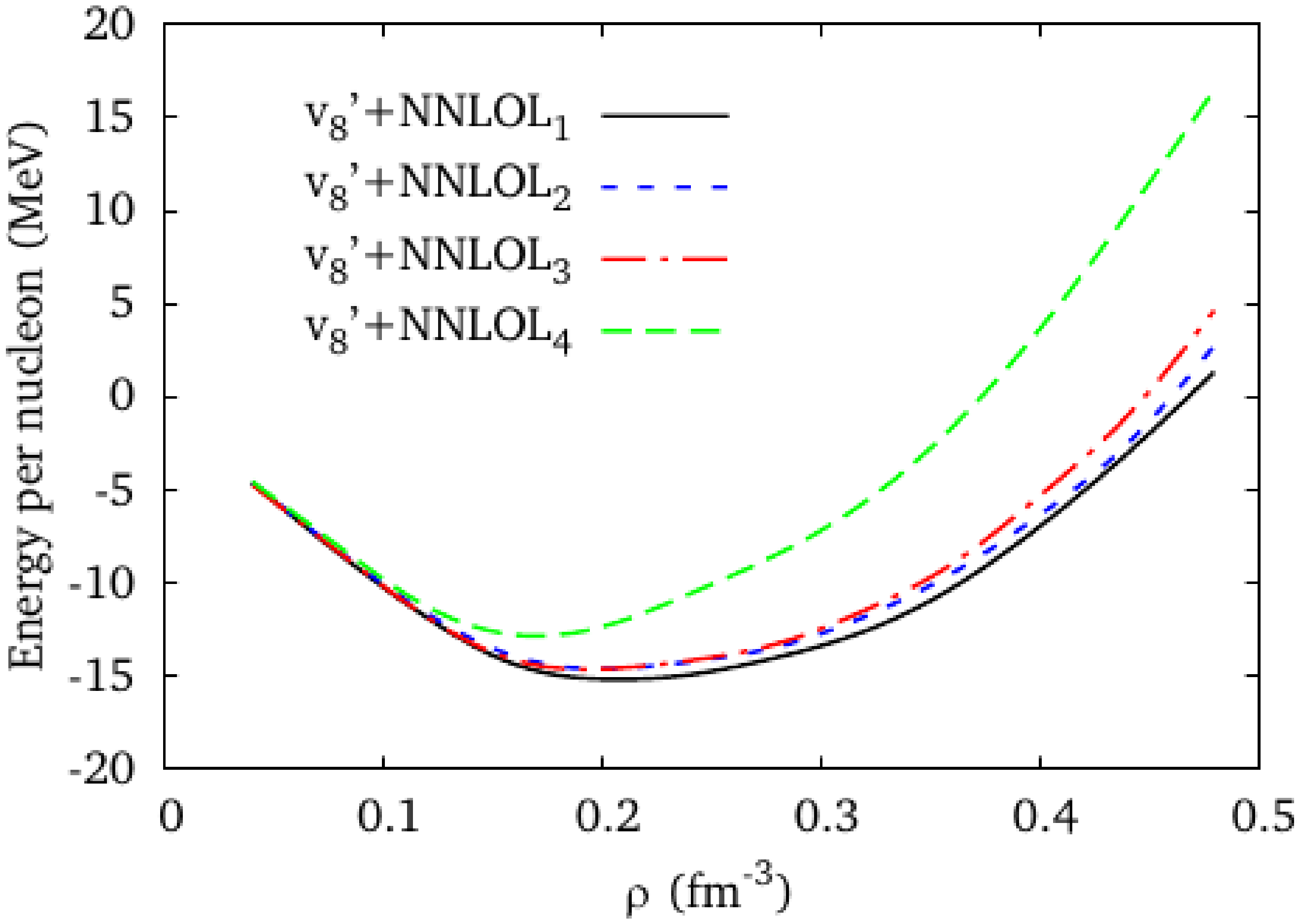








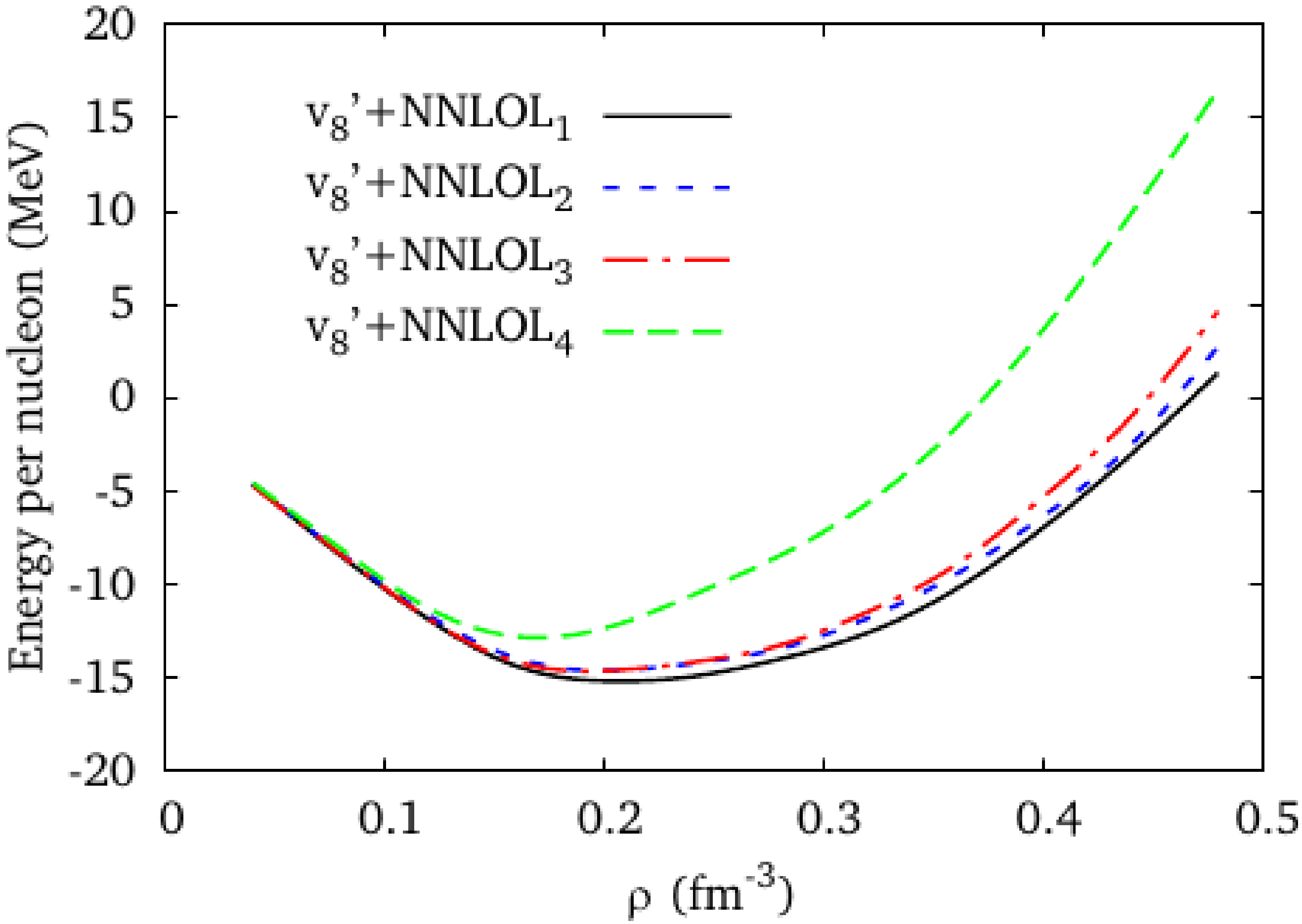
Chiral NNLOL results for SNM



NNLOL$_4$ has a large negative c_E

- stiffest SNM EoS
- softest PNM EoS

Chiral NNLOL results for SNM



	NNLOL_1	NNLOL_2	NNLOL_3	NNLOL_4
ρ_0 (fm^{-3})	0.21	0.20	0.19	0.17
E_0 (MeV)	-15.2	-14.6	-14.6	-12.9
K (MeV)	198	252	220	310

⟺

Experimental values

ρ_0 (fm^{-3})	0.16
E_0 (MeV)	-16.0
K (MeV)	240

Conclusions

- UIX potential fails to reproduce the binding energy of symmetric nuclear matter.

- Contact term of NNLOL potential suffers of cutoff dependence. Its contribution in nuclear matter can not be evaluated fitting low energy observables.

- No one of the potential considered simultaneoulsy explains the binding energy and the saturation density of SNM.

- $NNLOL_4$ and TM'_3 potentials provide reasonable value for the saturation density of SNM although **not involving any parameters adjusted to reproduce it.**

- Deriving potential from chiral perturbation theory is still a promising approach.

  NNNLO potential is now available as well as NNNNLO contact terms!

  More low energy observables are needed