A Monte Carlo approach to the study of medium-light hypernuclei properties

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December 16, 2011

Outline

1. The method
   - DMC
   - DMC for nuclei: AFDMC

2. Hypernuclei
   - hyperon-nucleon interaction
   - AFDMC for hypernuclei

3. Preliminary results

4. Conclusions and perspectives

Diffusion Monte Carlo

DMC = projection method

$$\tau = \frac{it}{\hbar} \quad \Rightarrow \quad -\frac{\partial}{\partial \tau}\psi(\boldsymbol{R},\tau) = \mathcal{H}\psi(\boldsymbol{R},\tau)$$

$$\psi(\boldsymbol{R},\tau) = e^{-(\mathcal{H}-E_0)\tau}\psi(\boldsymbol{R},0) \qquad \psi(\boldsymbol{R},0) = \sum_{n=0}^{\infty} c_n \varphi_n(\boldsymbol{R})$$

$$= \sum_{n=0}^{\infty} c_n e^{-(E_n-E_0)\tau}\varphi_n(\boldsymbol{R})$$

$$\Rightarrow \quad \lim_{\tau\to\infty} \psi(\boldsymbol{R},\tau) = c_0\varphi_0(\boldsymbol{R})$$

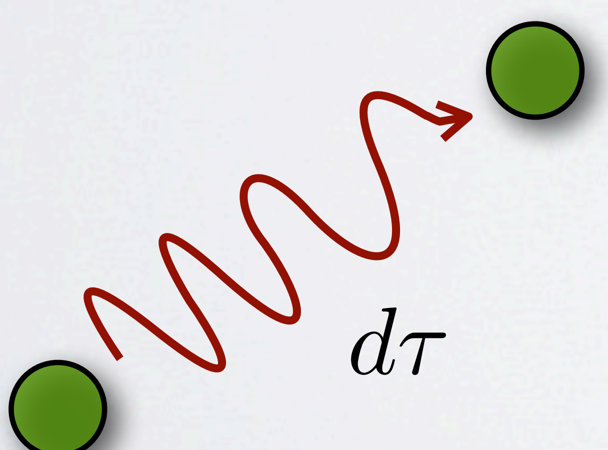
Diffusion Monte Carlo

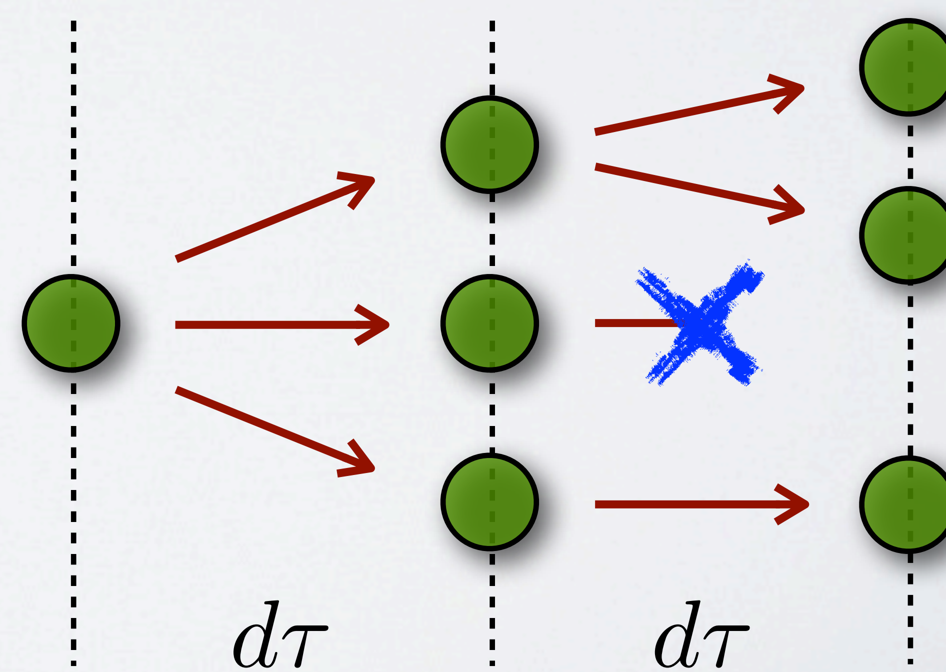
$$\psi(\boldsymbol{R}, \tau + d\tau) = \langle \boldsymbol{R} | \psi(\tau + d\tau) \rangle$$
$$= \int \underbrace{\langle \boldsymbol{R} | \, e^{-(\mathcal{H} - E_0) d\tau} \, | \boldsymbol{R}' \rangle}_{G(\boldsymbol{R}, \boldsymbol{R}', d\tau)} \langle \boldsymbol{R}' | \psi(\tau) \rangle \, d\boldsymbol{R}'$$

walkers

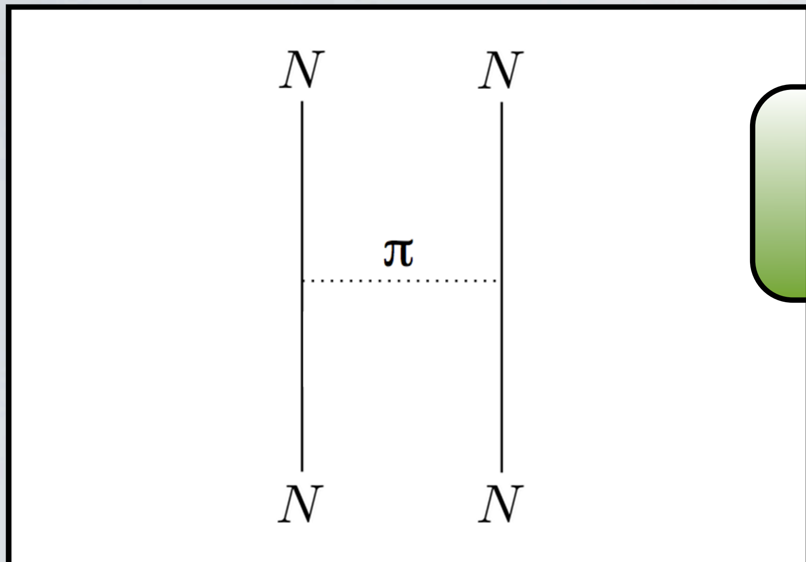
kinetic term

potential term



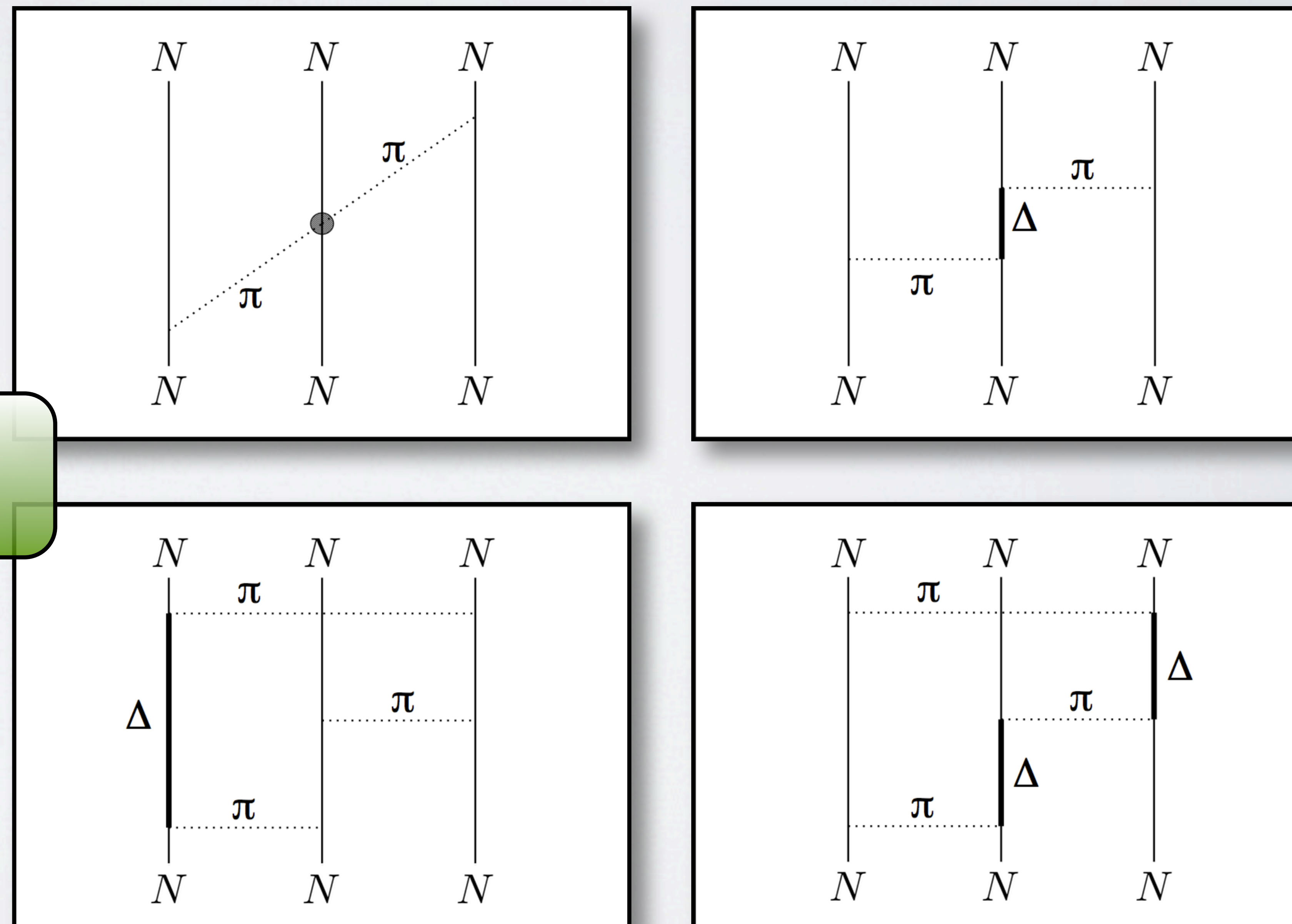


Nucleon-nucleon interaction



2 body

3 body



Nucleon-nucleon interaction

2 body

$$V_{ij} = \sum_{p=1}^{n} v_p(r) \mathcal{O}_{ij}^{p}$$

$$\mathcal{O}_{ij}^{p=1,8} = \{1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}, \boldsymbol{L}_{ij} \cdot \boldsymbol{S}_{ij}\} \otimes \{1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\}$$

$$\mathcal{O}_{ij}^{p=9,14} = \left\{L_{ij}^2, L_{ij}^2 \left(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\right), \left(\boldsymbol{L}_{ij} \cdot \boldsymbol{S}_{ij}\right)^2\right\} \otimes \{1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\}$$

$$\mathcal{O}_{ij}^{p=15,18} = \{T_{ij}, (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)\, T_{ij}, S_{ij}\, T_{ij}, \tau_i^z + \tau_j^z\}$$

3 body

$$V_{ijk} = C_{2\pi}^{SW} \mathcal{O}_{ijk}^{2\pi,SW} + C_{2\pi}^{PW} \mathcal{O}_{ijk}^{2\pi,PW} + C_{3\pi}^{\Delta R} \mathcal{O}_{ijk}^{3\pi,\Delta R} + C^{R} \mathcal{O}_{ijk}^{R}$$

Auxiliary Field Diffusion Monte Carlo

Problem:

$$\mathcal{P} \sim \mathrm{e}^{-\frac{1}{2}\lambda d\tau \mathcal{O}^2} \quad \Rightarrow \quad \psi \neq \mathcal{A}\left(\bigotimes_i c_i \cdot \varphi_i\right)$$

$$\sum \frac{A!}{Z!(A-Z)!} 4^A \text{ components}$$

Idea: Hubbard-Stratonovich transformation

$$\mathrm{e}^{-\frac{1}{2}\lambda d\tau \mathcal{O}^2} = \frac{1}{\sqrt{2\pi}} \int dx \ \mathrm{e}^{-\frac{x^2}{2}+\sqrt{-\lambda d\tau} x \mathcal{O}}$$

MC calculation

auxiliary field

Auxiliary Field Diffusion Monte Carlo

$$\text{AV6: } \mathcal{O}_{ij}^{p=1,6} = \underbrace{\{1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}\} \otimes \{1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\}}_{V_{SD} + V_{SI}}$$

$$V_{SD} = \frac{1}{2}\sum_{n=1}^{A}\sum_{\alpha=1}^{3}\lambda_n^{(\tau)}\mathcal{O}_{n\alpha}^{(\tau)2} + \frac{1}{2}\sum_{n=1}^{3A}\lambda_n^{(\sigma)}\mathcal{O}_n^{(\sigma)2} + \frac{1}{2}\sum_{n=1}^{3A}\sum_{\alpha=1}^{3}\lambda_n^{(\sigma\tau)}\mathcal{O}_{n\alpha}^{(\sigma\tau)2}$$

$\mathcal{A}_{i,j}^{(\tau)} : \quad A \times A$

$\mathcal{A}_{i\alpha,j\beta}^{(\sigma)} : \quad 3A \times 3A$

$\mathcal{A}_{i\alpha,j\beta}^{(\sigma\tau)} : \quad 3A \times 3A$

good for Hubbard-Stratonovich $\longrightarrow$ $15A$ auxiliary fields

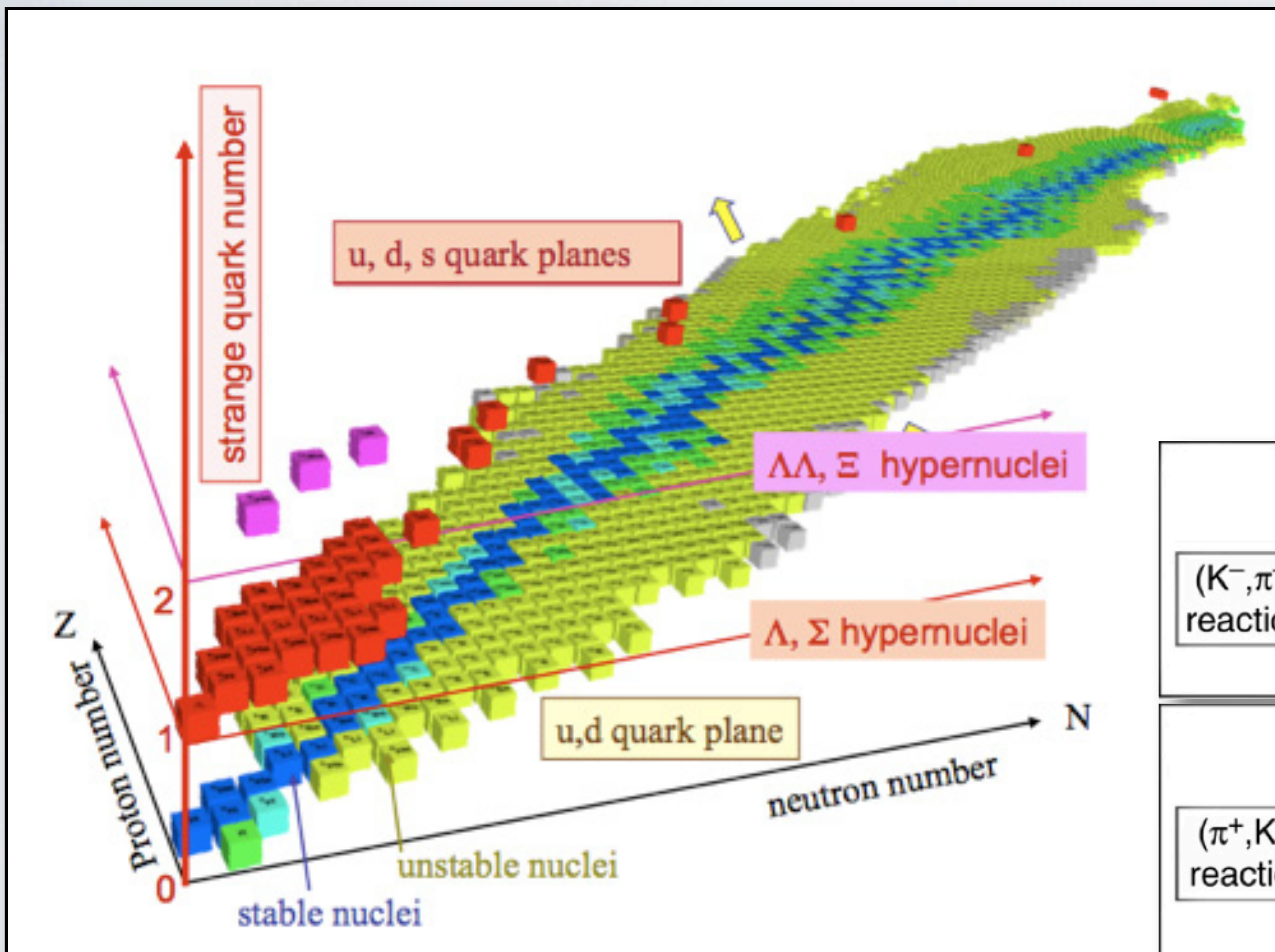
⇒ rotation of spin-isospin states

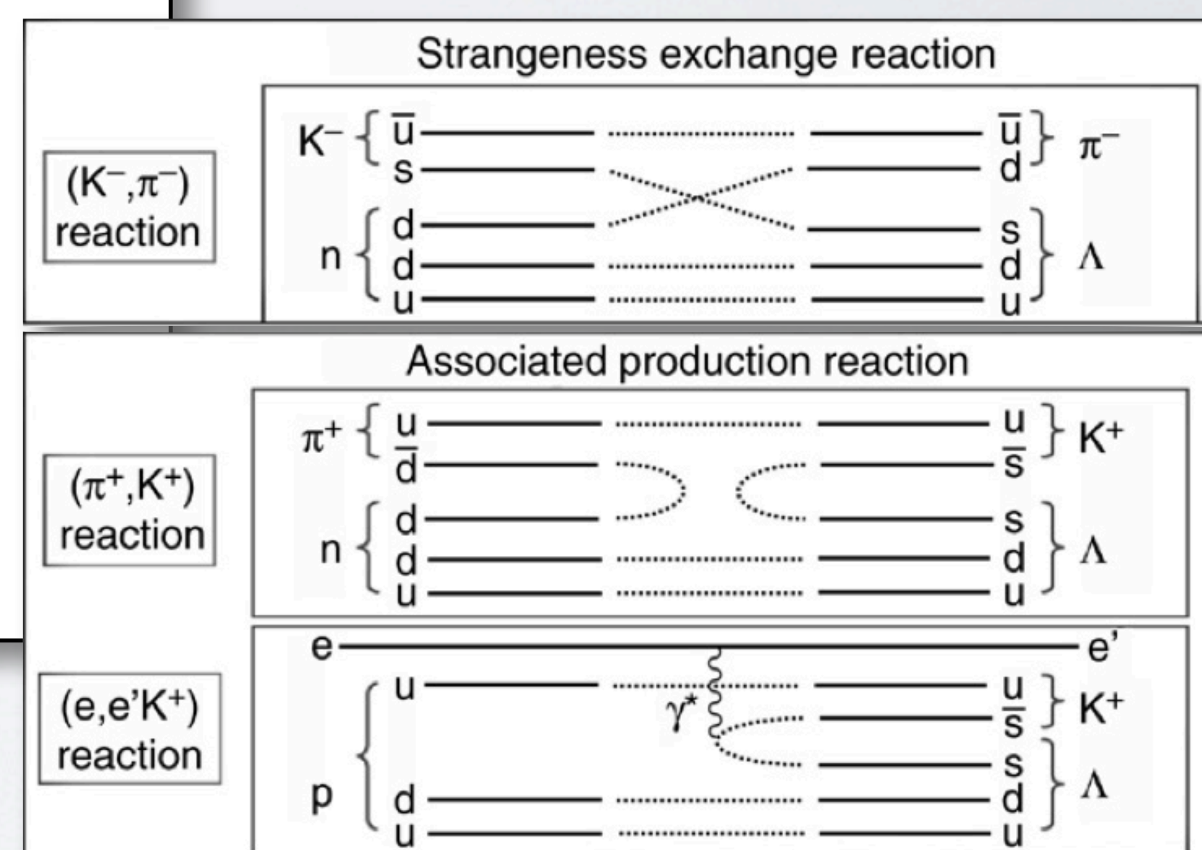
Hypernuclei

$$\Lambda = u + d + s$$

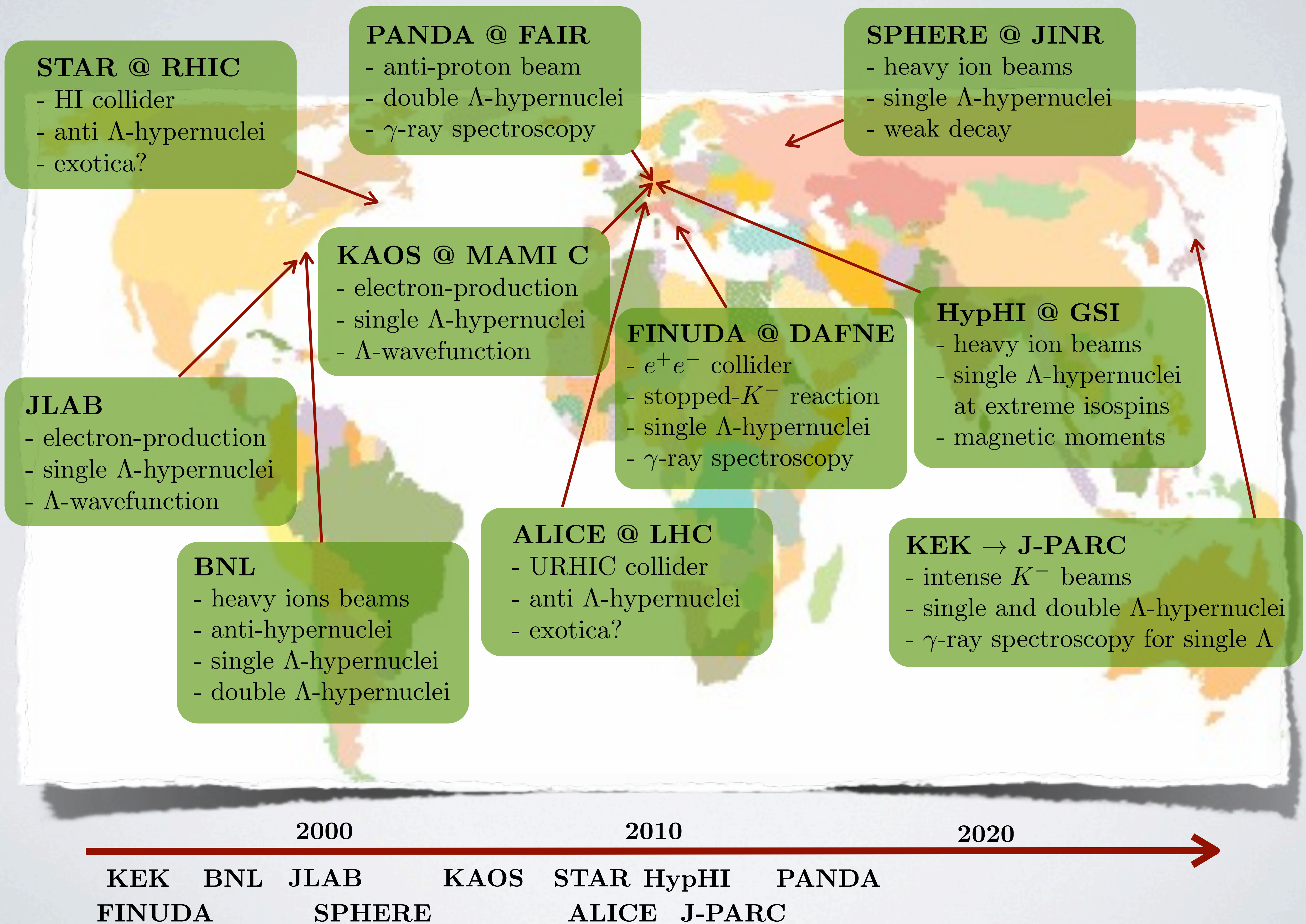
single Λ hypernucleus

double Λ hypernucleus

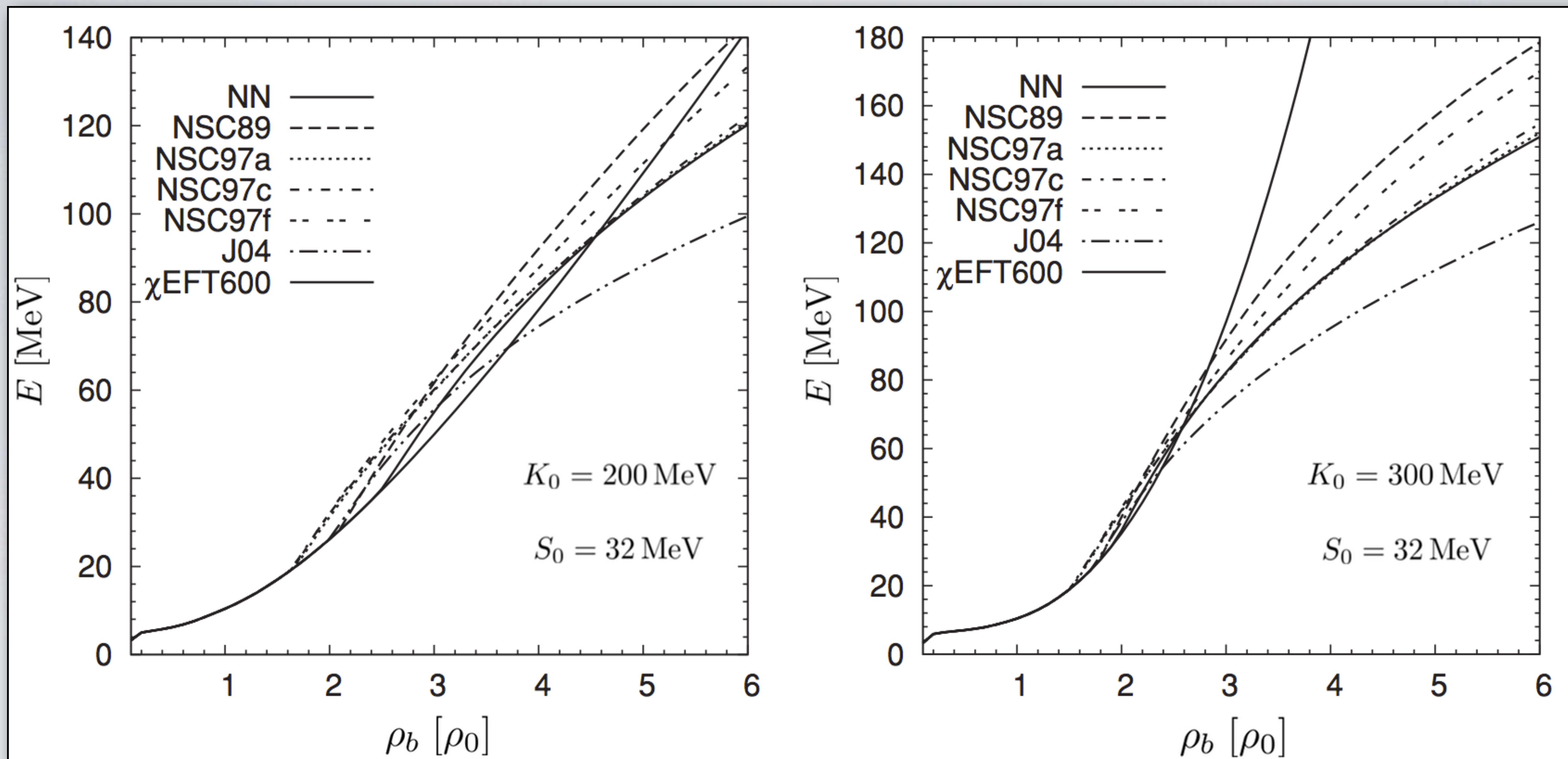




Hypernuclei

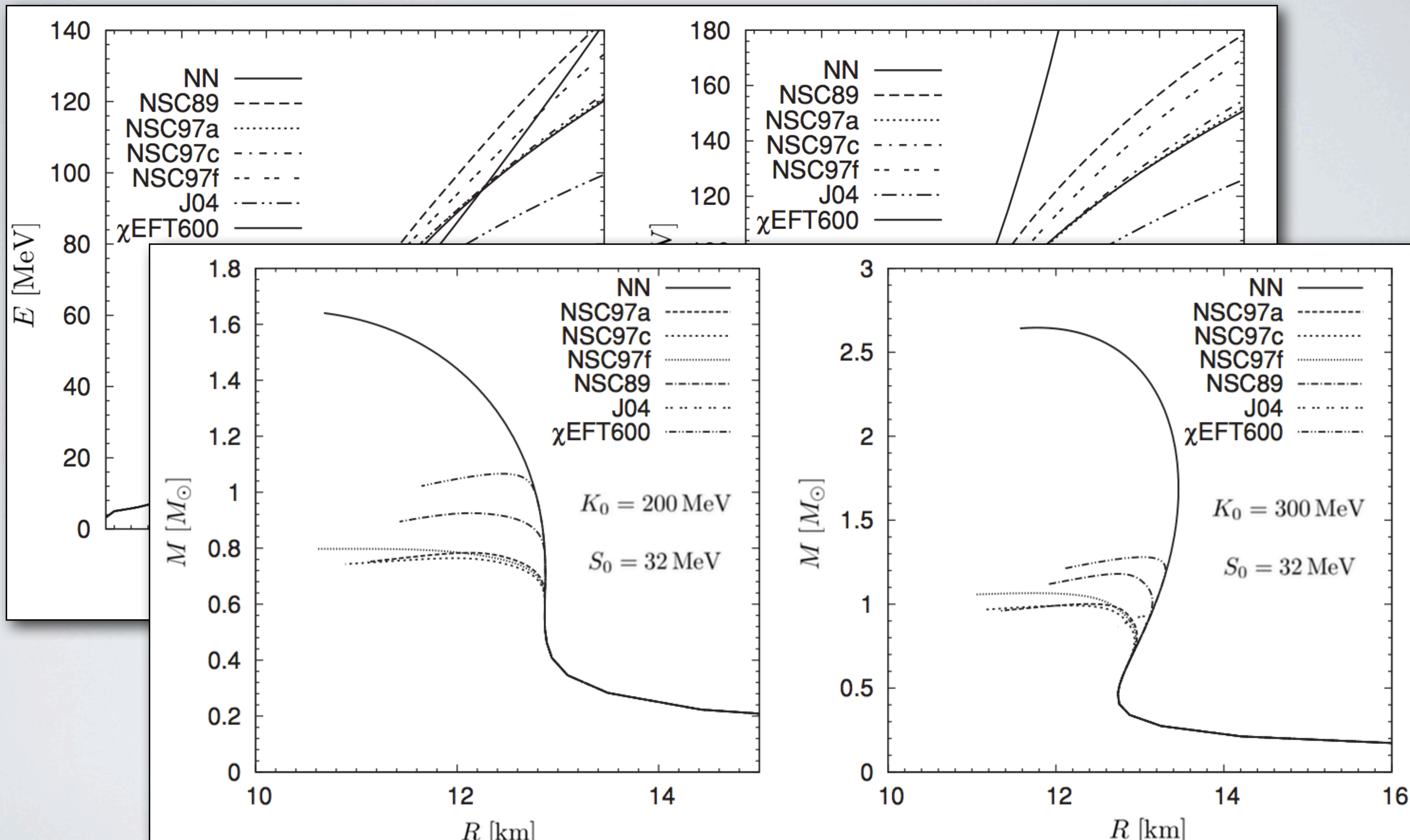


Hypernuclei



H. Đapo, B.-J. Schaefer, and J. Wambach. Appearance of hyperons in neutron stars. Phys. Rev. C, 81(3):035803, Mar 2010

Hypernuclei



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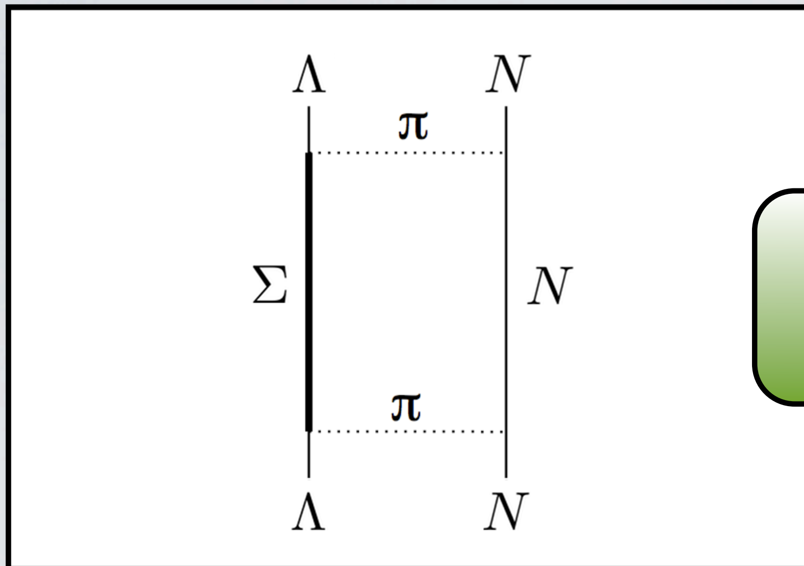
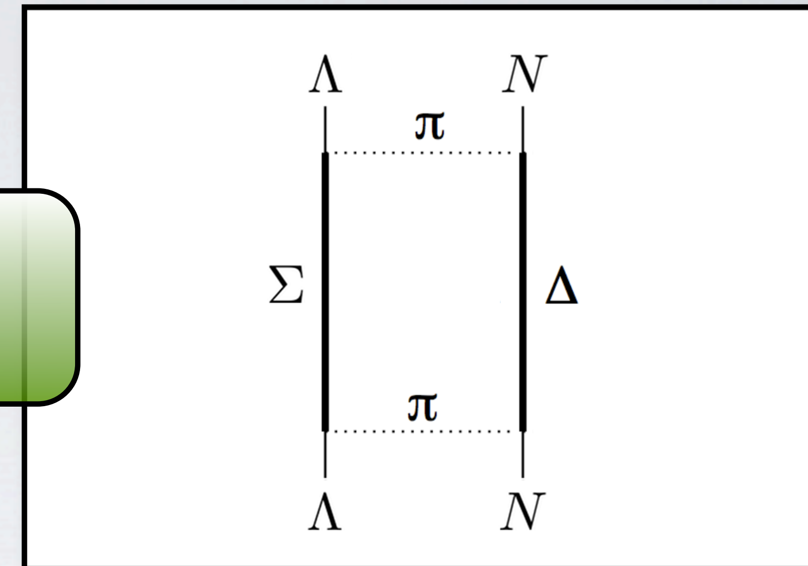
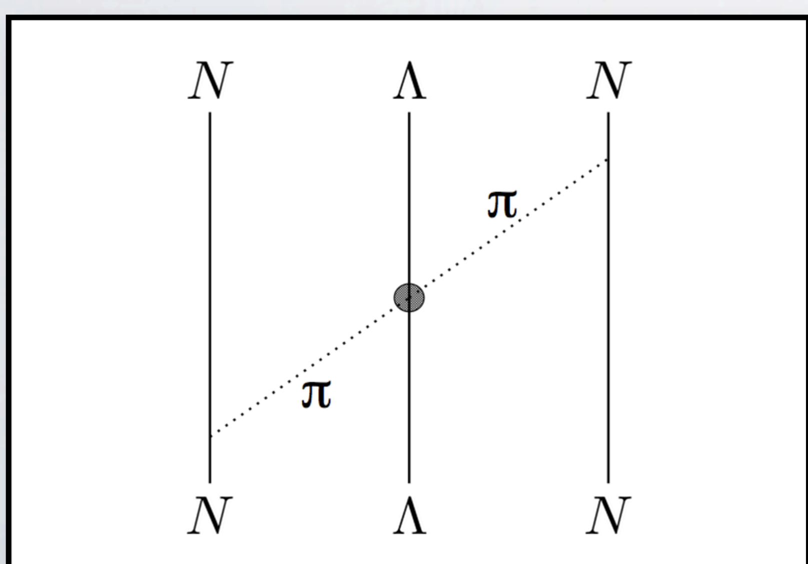
Hyperon-nucleon interaction[1]

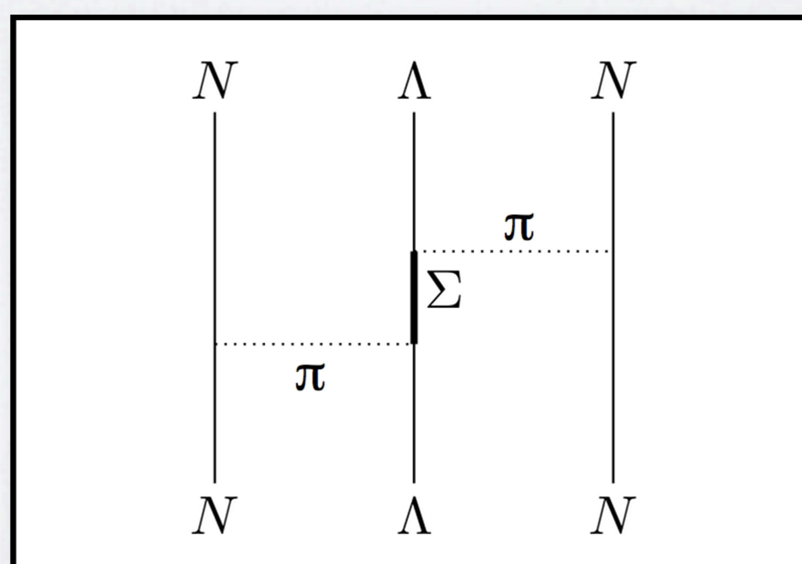


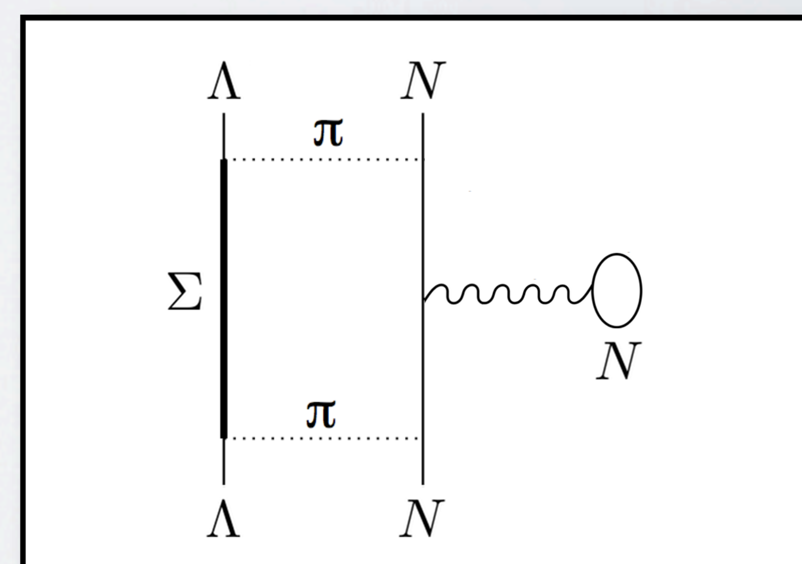
2 body



3 body







Hyperon-nucleon interaction[1]

2 body

$$V_{\Lambda i}(r) = v_0(r) + v_0(r)\varepsilon(P_x - 1) + \frac{1}{4}v_\sigma T_\pi^2(m_\pi r)\boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_i$$

$$v_0(r) = v_c(r) - v_{2\pi}(r)$$

$$v_c(r) = W_c\left[1 + \mathrm{e}^{\frac{r-\bar{r}}{a}}\right]^{-1}$$

$$v_{2\pi}(r) = \bar{v}\, T_\pi^2(m_\pi r)$$

HS ok!

3 body

only 2-body operators → HS ok!

$$V_{\Lambda ij} = C_{2\pi}^{SW}\,\mathcal{O}_{\Lambda ij}^{2\pi,SW} + C_{2\pi}^{PW}\,\mathcal{O}_{\Lambda ij}^{2\pi,PW} + C^D\mathcal{O}_{\Lambda ij}^D$$

The idea

nucleus $\psi_{nuc} = Det_N \, J_{NN}$

hypernucleus $\psi_{hyp} = Det_N \, J_{NN} \, Det_\Lambda \, J_{N\Lambda}$

propagator via HS $\longrightarrow$ $15A + 3A \cdot A_\Lambda$ auxiliary fields

$$B_\Lambda = \frac{\langle \psi_{nuc} | \mathcal{H}_N | \psi_{nuc} \rangle}{\langle \psi_{nuc} | \psi_{nuc} \rangle} - \frac{\langle \psi_{hyp} | \mathcal{H}_N + \mathcal{H}_\Lambda | \psi_{hyp} \rangle}{\langle \psi_{hyp} | \psi_{hyp} \rangle}$$

Hyp: nuclear effects cancel

information about the hyperon-nucleon interaction

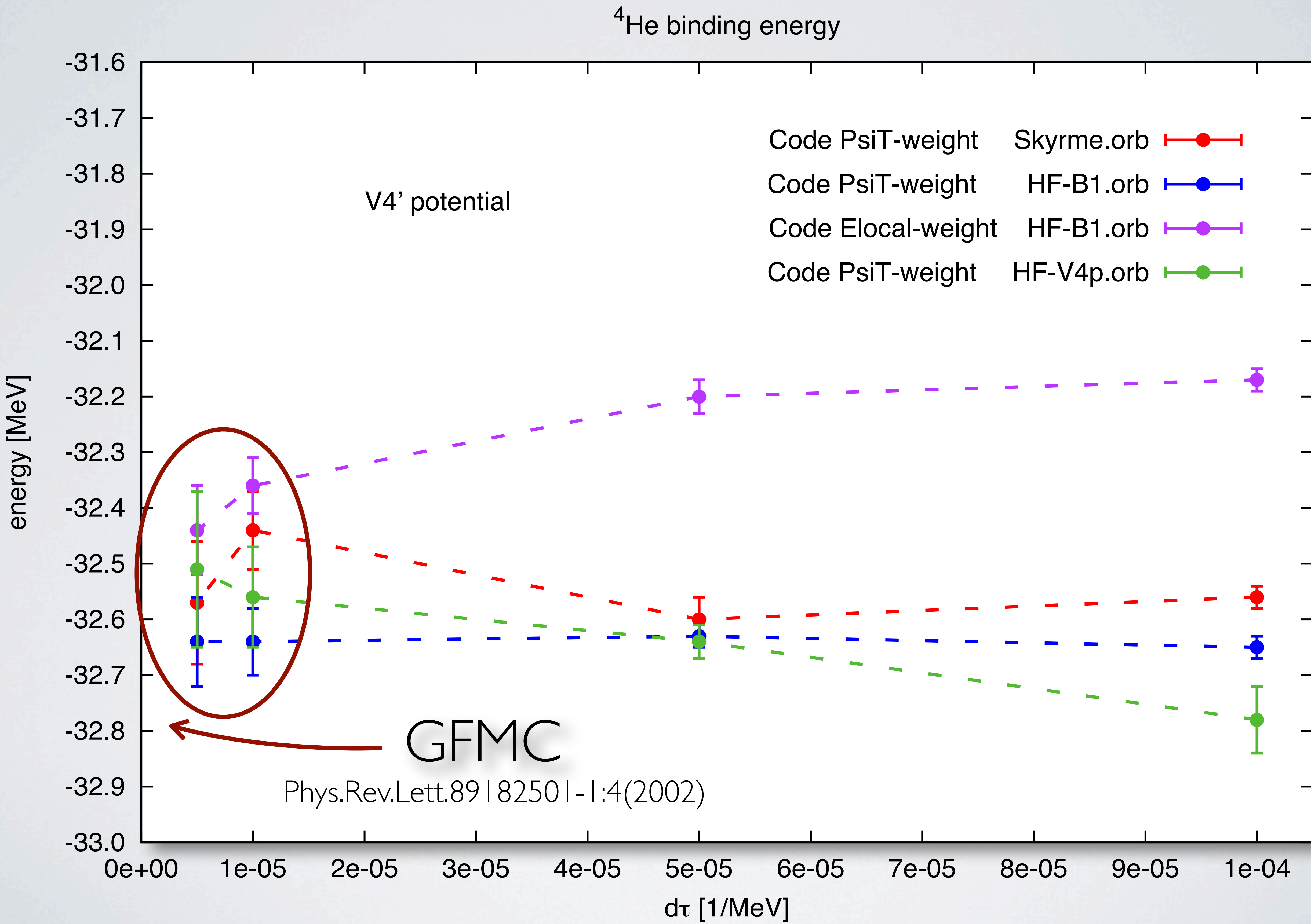
The idea: behind the scene

Starting point: robust method for nuclei + good wave function

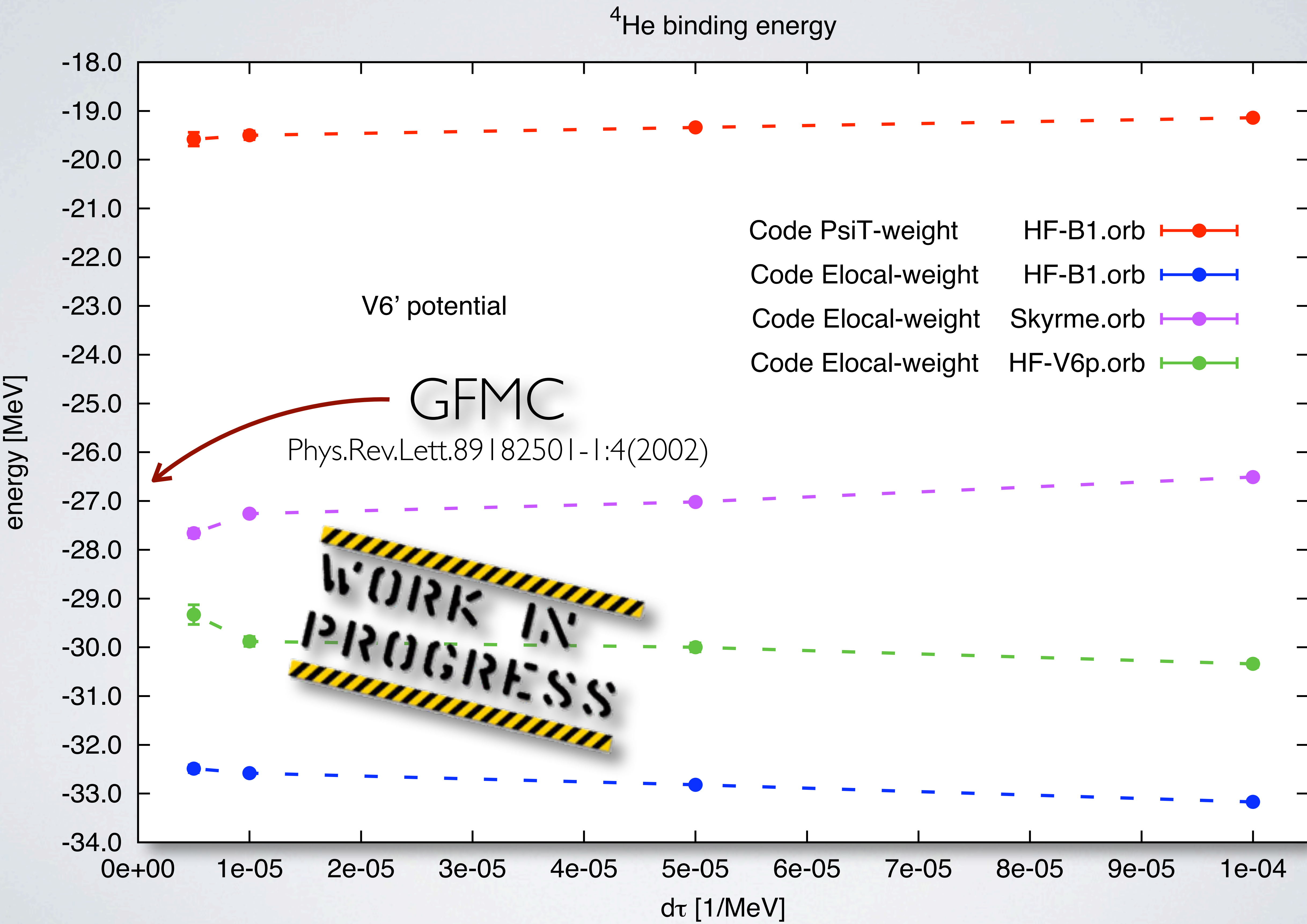
different AFDMC approaches → work in progress

different single particle orbitals → present and future collaborations

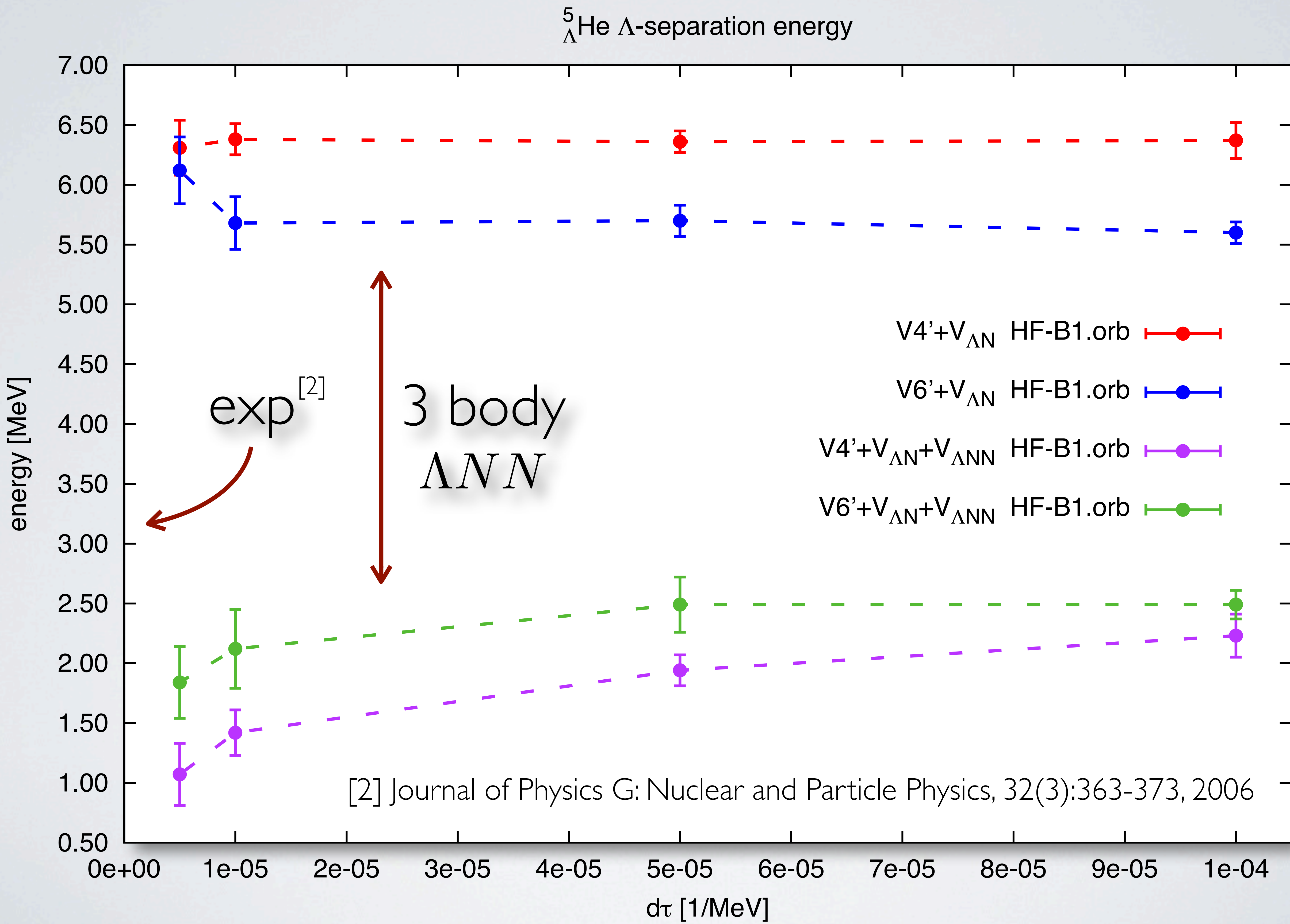
The idea: behind the scene



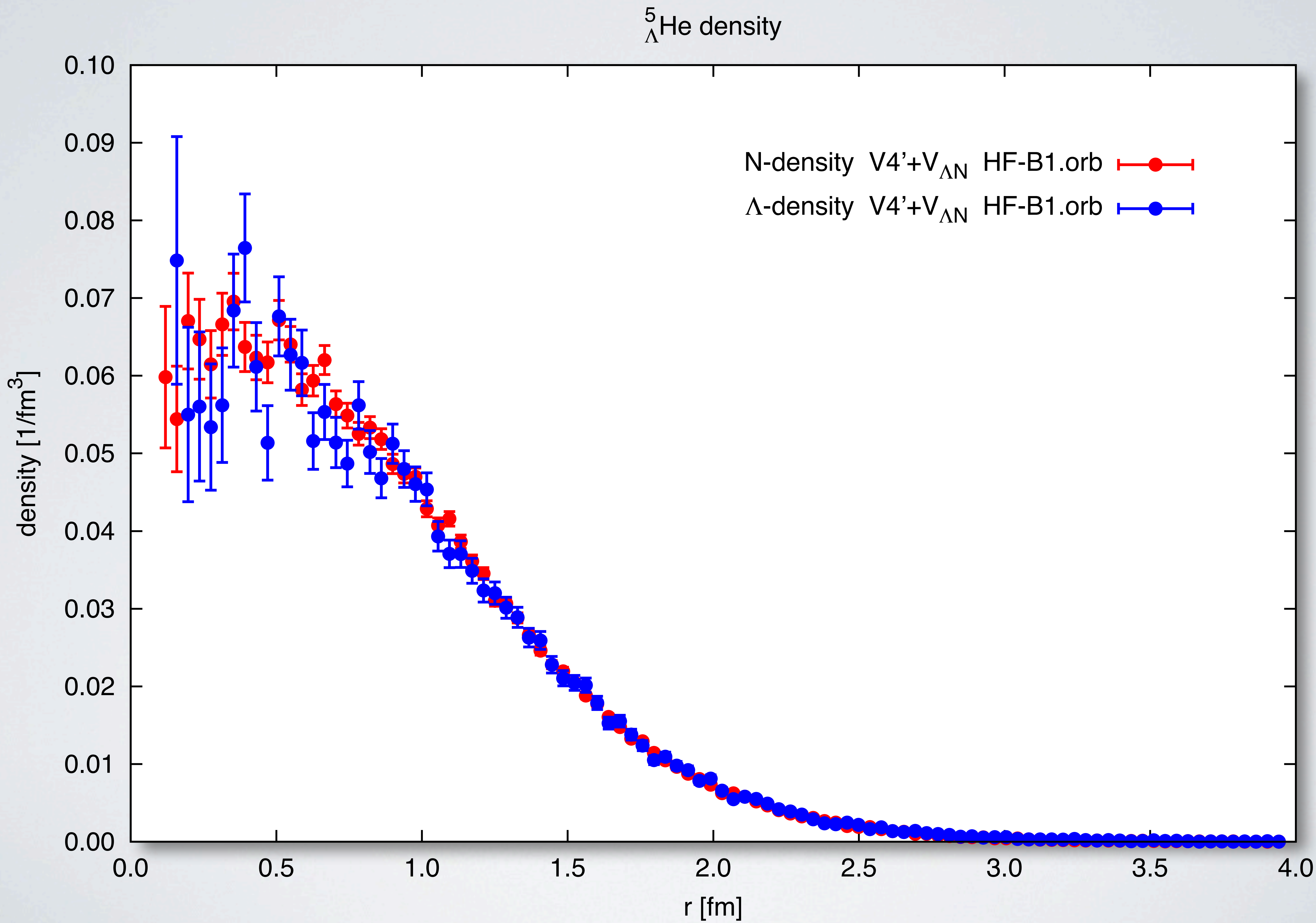
The idea: behind the scene



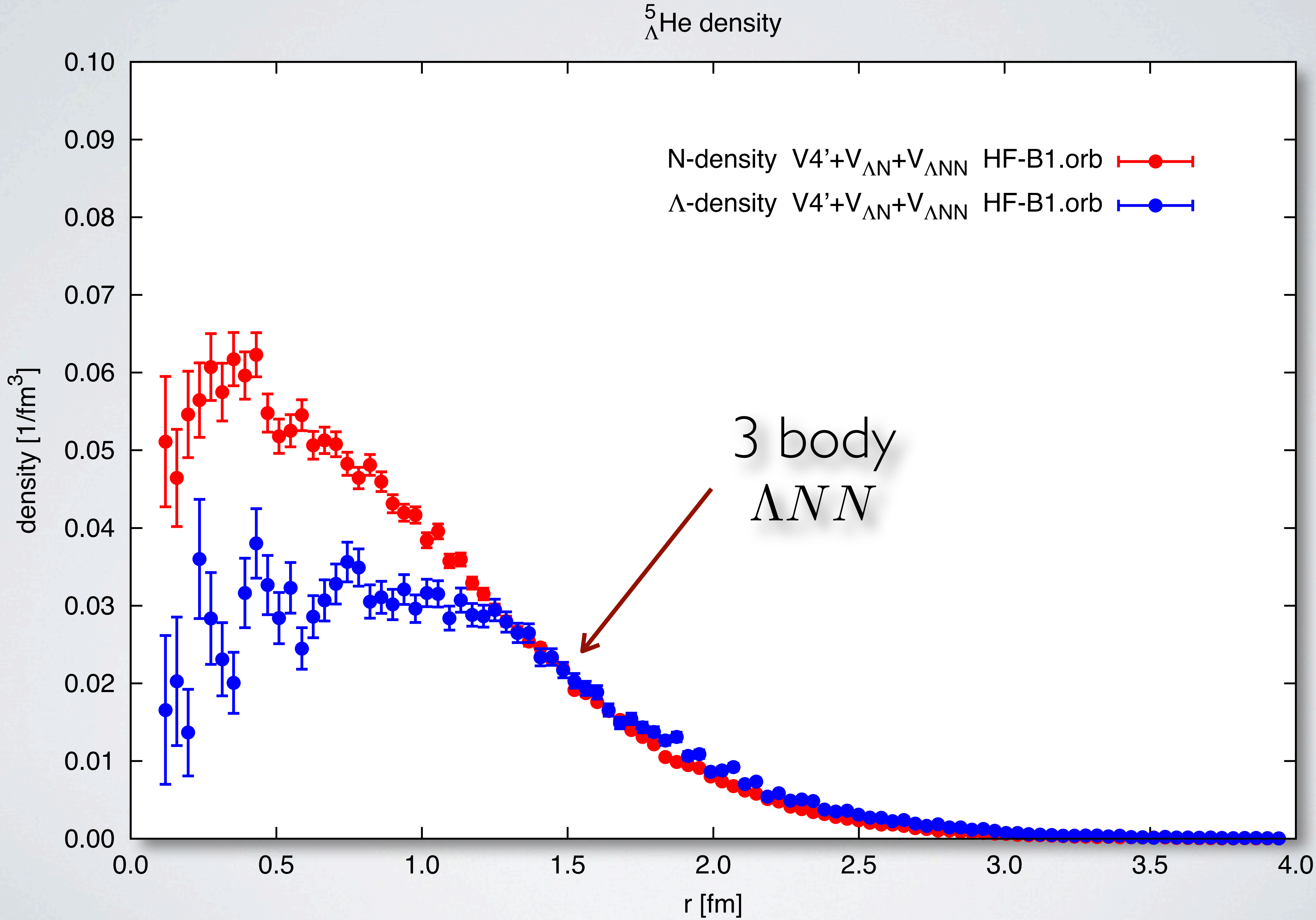
Hypernuclei preliminary results



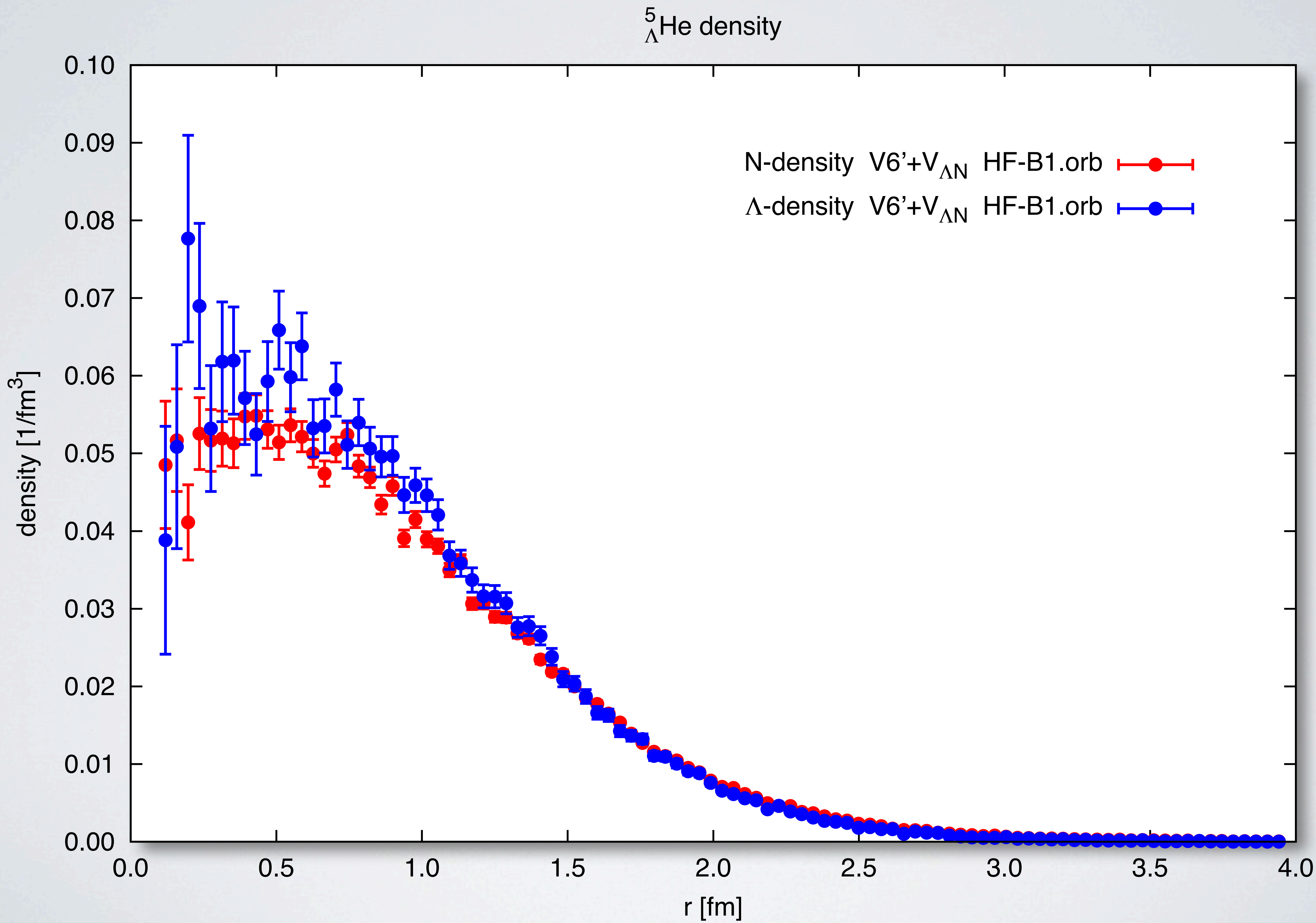
Hypernuclei preliminary results



Hypernuclei preliminary results



Hypernuclei preliminary results



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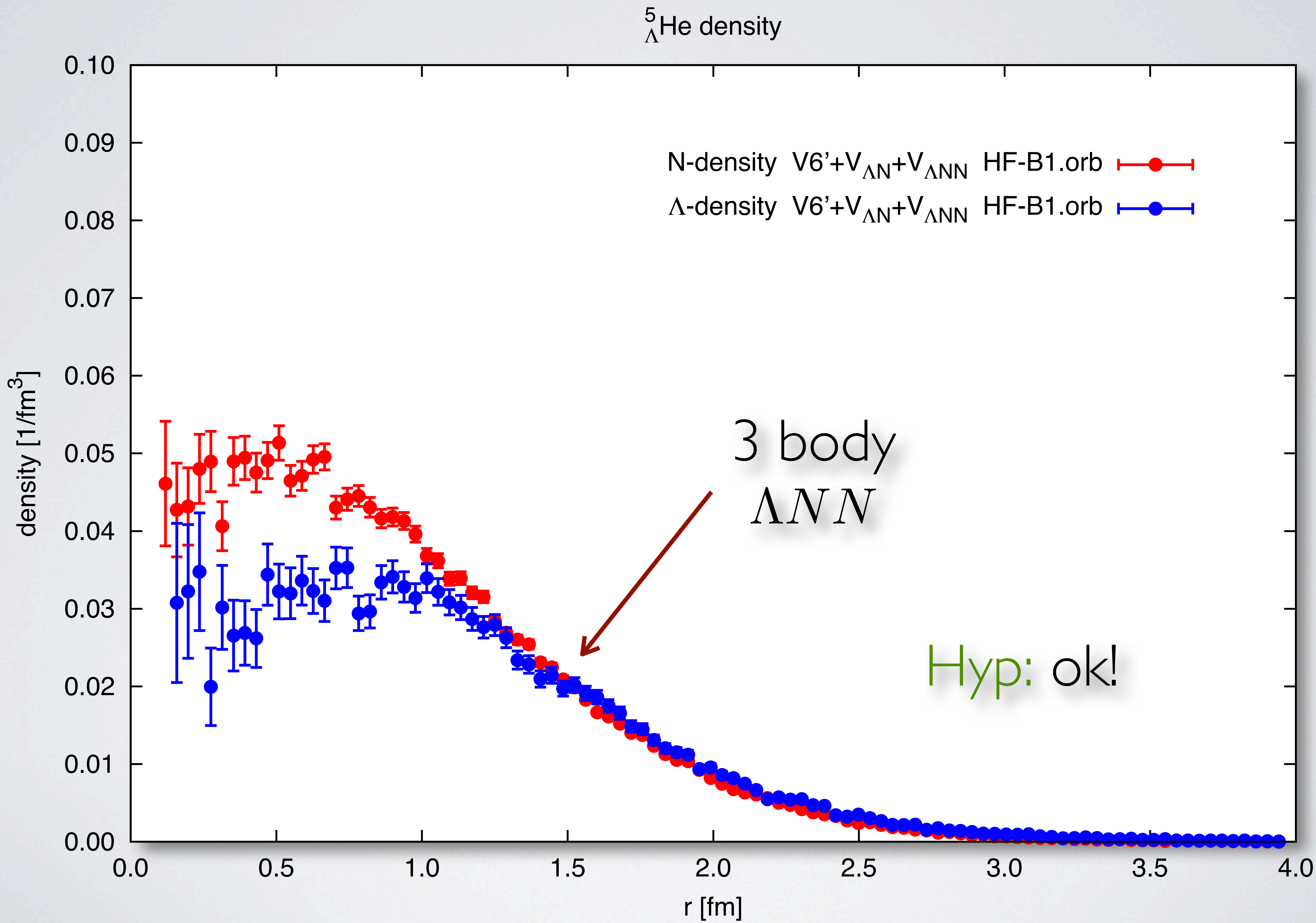

Conclusions and perspectives

AFDMC algorithm can be easily extended to the study of hypernuclear systems

- study of heavier Λ-hypernuclei: ${}^{7}_{\Lambda}\mathrm{He}\,,\,{}^{17}_{\Lambda}\mathrm{O}$

  ⇨ development of an accurate hyperon-nucleon potential

- study of $\Lambda\Lambda$-hypernuclei: ${}^{\;\,6}_{\Lambda\Lambda}\mathrm{He}$

  ⇨ development of an accurate hyperon-hyperon potential

Conclusions and perspectives

To do
(high priority)

- → deeper investigation in the AFDMC algorithm
- → better trial wave function
  - → single particle orbitals
  - → correlation functions

Thank you

[1] Hypernuclear potentials:

A. A. Usmani, Steven C. Pieper, and Q. N. Usmani. Variational calculations of the Λ–separation energy of the $^{17}_{\Lambda}\mathrm{O}$ hypernucleus. Phys. Rev. C, 51(5):2347–2355, May 1995.

A. A. Usmani and S. Murtaza. Variational Monte Carlo calculations of $^{5}_{\Lambda}\mathrm{He}$ hypernucleus. Phys. Rev. C, 68(2):024001, Aug 2003.

A. A. Usmani. ΛN space–exchange correlation effects in the $^{5}_{\Lambda}\mathrm{He}$ hypernucleus. Phys. Rev. C, 73(1):011302, Jan 2006.