6.0 Static properties of neutron stars

Given the equation of state (EOS), $P = P(\epsilon)$, mass and radius of a neutron star can be obtained from the Tolman-Oppenheimer-Volkov (TOV) equations.

TOV equations, obtained combining hydrostatic equilibrium and Einstein's equations, simply express the requirement that matter in the interior of a neutron star be in equilibrium. They can be written in the form

$$\frac{dP(r)}{dr} = -G \, \frac{\left[\epsilon(r) + P(r)/c^2\right]\left[m(r) + 4\pi r^2 P(r)/c^2\right]}{r^2\left[1 - 2Gm(r)/rc^2\right]} \tag{1}$$

$$m(r) = 4\pi \int_0^r r'^2 dr' \epsilon(r') \ , \tag{2}$$

the initial condition being

$$\epsilon(r = 0) = \epsilon_c \tag{3}$$

Note that in the $c \to \infty$ limit TOV equations reduce to the standard equations of hydrostatic equilibrium, apprp-priate to describe lower density systems, like white dwarfs (see Lecture 1)

For any value of the energy density at the center of the star, ϵ_c, eq.(1), with $m(r)$ given by eq.(2), can be integrated up to the point $r = R$, where the pressure P vanishes. The mass of the star is then given by

$$M = m(R) = 4\pi \int_0^R r^2 dr \epsilon(r) \tag{4}$$

6.1 Stability of the solutions of TOV equations

Equilibrium does not necessarily imply stability. Not all solutions of TOV equations are stable. In general, stability is a very complex issue, that we will not discuss in detail in this notes. However, it can be shown that a necessary condition for stability is

$$\frac{\partial M(\epsilon_c)}{\partial \epsilon_c} > 0 \ . \tag{5}$$

As a consequence, if the curve representing the ϵ_c dependence of the neutron star mass obtained from TOV equations exhibits a maximum at $\epsilon_c = \bar{\epsilon}_c$, then all solutions corresponding to $\epsilon_c < \bar{\epsilon}_c$ are stable, whereas those corresponding to $\epsilon_c > \bar{\epsilon}_c$ are unstable.