LECTURE 6

6.0 Static properties of neutron stars

Given the equation of state (EOS), $P = P(\epsilon)$, mass and radius of a neutron star can be obtained from the Tolman-Oppenheimer-Volkov (TOV) equations.

TOV equations, obtained combining hydrostatic equilibrium and Einstein's equations, simply express the requirement that matter in the interior of a neutron star be in equilibrium. They can be written in the form

$$\frac{dP(r)}{dr} = -G\,\frac{\left[\epsilon(r) + P(r)/c^2\right]\left[m(r) + 4\pi r^2 P(r)/c^2\right]}{r^2\left[1 - 2Gm(r)/rc^2\right]} \tag{1}$$

$$m(r) = 4\pi \int_0^r {r'}^2 dr' \epsilon(r') \ , \tag{2}$$

the initial condition being

$$\epsilon(r = 0) = \epsilon_c \tag{3}$$

Note that in the $c \to \infty$ limit TOV equations reduce to the standard equations of hydrostatic equilibrium, apprpriate to describe lower density systems, like white dwarfs (see Lecture 1)

For any value of the energy density at the center of the star, ϵ_c, eq.(1), with $m(r)$ given by eq.(2), can be integrated up to the point $r = R$, where the pressure P vanishes. The mass of the star is then given by

$$M = m(R) = 4\pi \int_0^R r^2 dr \epsilon(r) \tag{4}$$

6.1 Stability of the solutions of TOV equations

Equilibrium does not necessarily imply stability. Not all solutions of TOV equations are stable. In general, stability is a very complex issue, that we will not discuss in detail in this notes. However, it can be shown that a necessary condition for stability is

$$\frac{\partial M(\epsilon_c)}{\partial \epsilon_c} > 0 \ . \tag{5}$$

As a consequence, if the curve representing the ϵ_c dependence of the neutron star mass obtained from TOV equations exhibits a maximum at $\epsilon_c = \overline{\epsilon}_c$, then all solutions corresponding to $\epsilon_c < \overline{\epsilon}_c$ are stable, whereas those corresponding to $\epsilon_c > \overline{\epsilon}_c$ are unstable.