Appendix

1. Dirac spinors

We will use a normalization suitable to describe messless particles.
A plane wave solution of the Dirac equation will be written:

$$\psi(x) = \frac{1}{\sqrt{2\omega_k}}\, u^{(s)}(k)\, \frac{e^{-ikx}}{\sqrt{\Omega}} \qquad (A.1)$$

where $\Omega = L^3$ is the volume of the normalization box and $\omega_k = +\sqrt{|\vec{k}|^2 + m^2}$, m being the particle mess.

The $u^{(s)}$ corresponding to positive energy solutions $(s=1,2)$ read

$$u^{(s)}(k) = N \begin{pmatrix} \chi_s \\ \dfrac{\vec{\sigma}\cdot\vec{k}}{\omega_k+m}\chi_s \end{pmatrix}, \qquad (A.2)$$

while for negative energy we have

$$u^{(s+2)}(k) = N \begin{pmatrix} -\dfrac{\vec{\sigma}\cdot\vec{k}}{\omega_k+m}\chi_s \\ \chi_s \end{pmatrix}. \qquad (A.3)$$

In eqs. (A.2) and (A.3) χ_s are the Pauli spinors

$$\chi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{A.4}$$

and $\vec{\sigma} \equiv (\sigma_x, \sigma_y, \sigma_z)$ with

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The normalization of $\psi(x)$

$$\int d^3x \, \psi^\dagger(x) \psi(x) = 1 \tag{A.5}$$

requires

$$u^{(s)\dagger}(k) \, u^{(s)}(k) = 2\omega_k , \tag{A.6}$$

implying in turn

$$N = \sqrt{\omega_k + m} . \tag{A.7}$$

Defining the adjoint spinor $\bar{u}^{(s)}(k) = u^{(s)\dagger}(k)\gamma_0$ we find

$$\bar{u}^{(r)}(k) \, u^{(s)}(k) = \delta_{rs} \, 2m$$

and

$$\sum_s u_\alpha^{(s)}(k)\,\bar{u}_\beta^{(s)}(k) = (\gamma_\mu k^\mu + m)_{\alpha\beta} = (\not{k} + m)_{\alpha\beta}\,. \qquad (A.8)$$

#

Proof: Using (A.2) and

$$\bar{u}^{(s)}(k) = N \begin{pmatrix} \chi_s^\dagger \\ -\frac{\vec{\sigma}\cdot\vec{k}}{\omega_k + m}\chi_s^\dagger \end{pmatrix}$$

the adjoint of $\vec{\sigma}\chi$ is $\chi^\dagger\vec{\sigma}$

The matrix of elements $u_\alpha^{(s)}(k)\,\bar{u}_\beta^{(s)}(k)$ can be easily constructed, with the result:

$$N^2 \begin{pmatrix} I & -\frac{\vec{\sigma}\cdot\vec{k}}{\omega_k+m} \\ \frac{\vec{\sigma}\cdot\vec{k}}{\omega_k+m} & -\left(\frac{\vec{\sigma}\cdot\vec{k}}{\omega_k+m}\right)^2 \end{pmatrix} = \begin{pmatrix} \omega_k + m & -\vec{\sigma}\cdot\vec{k} \\ \vec{\sigma}\cdot\vec{k} & m-\omega_k \end{pmatrix}$$

$$= \gamma_0\omega_k - \vec{\gamma}\cdot\vec{k} + Im = \gamma_\mu k^\mu + m = \not{k} + m$$

#

According to Dirac's theory, the negative energy solutions (A.3) can be associated with antiparticles carrying positive energy and momentum $-\vec{k}$. Hence, antiparticle can be described by the spinors

$$v^{(1)}(k) = u^{(4)}(-k)$$
$$v^{(2)}(k) = u^{(3)}(-k) \quad , \qquad (A.9)$$

yielding

$$v^{(s)\dagger}(k)\, v^{(s)}(k) = 2\omega_k \qquad (A.10)$$
$$\bar{v}^{(r)}(k)\, v^{(s)}(k) = -\,\delta_{rs}\, 2m \quad , \qquad (A.11)$$

and

$$\sum_s v_\alpha^{(s)}(k)\, \bar{v}_\beta^{(s)}(k) = (\not{k} - m)_{\alpha\beta} \quad . \qquad (A.12)$$

In the limit of nonrelativistic particles, having $(k/m) \ll 1$, eqs. (A.2) and (A.9) reduce to

$$u^{(s)}(k) \to \sqrt{2m} \begin{pmatrix} \chi_s \\ 0 \end{pmatrix} \qquad (A.13)$$

$$v^{(s)}(k) \to \sqrt{2m} \begin{pmatrix} 0 \\ \chi_s \end{pmatrix} . \qquad (A.14)$$

A massless particle has $\omega_k = |\vec{k}|$ and travels at the speed of light. Its velocity cannot be changed by a Lorentz transformation. As a consequence, for massless particles helicity (given by $\vec{\sigma}\cdot\vec{k}/2|\vec{k}|$) is an intrinsic property, independent of the reference frame.

The zero mass limit of the positive and negative energy spinors read

$$u^{(1)} = \sqrt{\omega_k}\begin{pmatrix} \chi_1 \\ \chi_1 \end{pmatrix} \qquad u^{(2)} = \sqrt{\omega_k}\begin{pmatrix} \chi_2 \\ -\chi_2 \end{pmatrix} \tag{A.15}$$

$$u^{(3)} = \sqrt{\omega_k}\begin{pmatrix} -\chi_1 \\ \chi_1 \end{pmatrix} \qquad u^{(4)} = \sqrt{\omega_k}\begin{pmatrix} \chi_2 \\ \chi_2 \end{pmatrix}$$

Particles with positive and negative helicity are called right handed and left handed, respectively. The states $u^{(1)}$ and $u^{(4)}$ describe right handed particles, while $u^{(2)}$ and $u^{(3)}$ describe left handed particles.

Using the Pauli-Dirac representation of the γ matrices

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \qquad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

The matrix

$$\gamma_5 = i\,\gamma_0\,\gamma_1\,\gamma_2\,\gamma_3 \tag{A.16}$$

can be written

$$\gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} . \tag{A.17}$$

It follows that

$$\frac{1}{2}(1-\gamma_5)u^{(2)} = u^{(2)} \qquad \frac{1}{2}(1-\gamma_5)u^{(3)} = u^{(3)} \tag{A.18}$$

and

$$\frac{1}{2}(1-\gamma_5)u^{(1)} = \frac{1}{2}(1-\gamma_5)u^{(4)} = 0 \tag{A.19}$$

Right and left handed neutrinos are described by $u^{(1)}$ and $u^{(2)}$, respectively, while $u^{(3)}$ and $u^{(4)}$ describe right and left handed antineutrinos. From eqs. (A.18) and (A.19) it follows that a current of the V−A form

$$\bar{\psi}\,\gamma_\mu\,(1-\gamma_5)\,\psi \tag{A.20}$$

couples to right handed antineutrinos or left handed neutrinos only.

2. Traces of combinations of γ matrices

We will make use of the properties of γ matrices

$$\{\gamma_\mu, \gamma_\nu\} = 2 g_{\mu\nu}$$

$$\gamma_0^2 = I \qquad \gamma_\alpha^2 = -I \quad (\alpha = 1,2,3)$$

and, defining $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$

$$\{\gamma_5, \gamma_\mu\} = 0$$

$$\gamma_5^2 = I$$

Moreover, remember that

$$T(AB) = T(BA)$$

T1 $\qquad \mathrm{Tr}(\gamma_\mu\gamma_\nu) = 4 g_{\mu\nu}$

Proof:

$$\mathrm{Tr}(\gamma_\mu\gamma_\nu) = \frac{1}{2}\left[\mathrm{Tr}(\gamma_\mu\gamma_\nu) + \mathrm{Tr}(\gamma_\nu\gamma_\mu)\right]$$

$$= \frac{1}{2}\mathrm{Tr}(\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu) = \frac{1}{2}\mathrm{Tr}(2 g_{\mu\nu})$$

$$= g_{\mu\nu}\,\mathrm{Tr}(I) = 4 g_{\mu\nu}$$

\# T2 The trace of the product of any odd number of γ matrices vanishes

Proof:

$$\mathrm{Tr}(\gamma_{\mu_1}\cdots\gamma_{\mu_n}) = \mathrm{Tr}(\gamma_{\mu_1}\cdots\gamma_{\mu_n}\gamma_5\gamma_5)$$

Use $\{\gamma_\mu,\gamma_5\}=0$ to move γ_5 to the left in n steps. It follows that

$$\mathrm{Tr}(\gamma_{\mu_1}\cdots\gamma_{\mu_n}) = (-)^n\,\mathrm{Tr}(\gamma_5\gamma_{\mu_1}\cdots\gamma_{\mu_n}\gamma_5).$$

Use now $\mathrm{Tr}(AB)=\mathrm{Tr}(BA)$ to obtain

$$\mathrm{Tr}(\gamma_{\mu_1}\cdots\gamma_{\mu_n}) = (-)^n\,\mathrm{Tr}(\gamma_{\mu_1}\cdots\gamma_{\mu_n})$$

which implies, for n odd, $\mathrm{Tr}(\gamma_{\mu_1}\cdots\gamma_{\mu_n})=0$.

\# T3 $\mathrm{Tr}(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma)=4(g_{\mu\nu}g_{\rho\sigma}-g_{\mu\rho}g_{\nu\sigma}+g_{\mu\sigma}g_{\nu\rho})$

Proof:

$$\mathrm{Tr}(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) = 2g_{\rho\sigma}\,\mathrm{Tr}(\gamma_\mu\gamma_\nu)-\mathrm{Tr}(\gamma_\mu\gamma_\nu\gamma_\sigma\gamma_\rho)$$

$$= 8g_{\rho\sigma}g_{\mu\nu}-2g_{\sigma\nu}\,\mathrm{Tr}(\gamma_\mu\gamma_\rho)+\mathrm{Tr}(\gamma_\mu\gamma_\sigma\gamma_\nu\gamma_\rho)$$

$$= 8g_{\rho\sigma}g_{\mu\nu}-8g_{\sigma\nu}g_{\mu\rho}+2g_{\sigma\mu}\,\mathrm{Tr}(\gamma_\nu\gamma_\rho)-\mathrm{Tr}(\gamma_\sigma\gamma_\mu\gamma_\nu\gamma_\rho)$$

over →

$$= 8\left(g_{\mu\nu} g_{\rho\sigma} - g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho}\right) - \mathrm{Tr}\left(\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma\right)$$

Note that the result T3 implies that the trace of the product of four γ matrices vanishes unless the product involves two pairs of γ matrices with the same index.

\# T4 $\mathrm{Tr}\left(\gamma_5 \gamma_{\mu_1} \cdots \gamma_{\mu_n}\right) = 0$ for any odd n

This result simply follows from T2 and the definition of γ_5 (it involves four γ matrices).

\# T5 $\mathrm{Tr}\left(\gamma_5 \gamma_\mu \gamma_\nu\right) = 0$

Proof:

$$\mathrm{Tr}\left(\gamma_5 \gamma_\mu \gamma_\nu\right) = i\, \mathrm{Tr}\left(\gamma_0 \gamma_1 \gamma_2 \gamma_3 \gamma_\mu \gamma_\nu\right)$$

$$= 2i\, g_{\mu\nu}\, \mathrm{Tr}\left(\gamma_0 \gamma_1 \gamma_2 \gamma_3\right) - i\, \mathrm{Tr}\left(\gamma_0 \gamma_1 \gamma_2 \gamma_3 \gamma_\nu \gamma_\mu\right)$$

The first term in the second line vanishes due to T3. The second can be rewritten

$$-i\, \mathrm{Tr}\left(\gamma_0 \gamma_1 \gamma_2 \gamma_3 \gamma_\nu \gamma_\mu\right) = -2i\, g_{\nu 3}\, \mathrm{Tr}\left(\gamma_0 \gamma_1 \gamma_2 \gamma_\mu\right) + i\, \mathrm{Tr}\left(\gamma_0 \gamma_1 \gamma_2 \gamma_\nu \gamma_3 \gamma_\mu\right)$$

and again, due to T3, $\mathrm{Tr}(\gamma_0\gamma_1\gamma_2\gamma_\mu)=0$.
This procedure can be repeated over and over, with the final result

$$\mathrm{Tr}(\gamma_5\gamma_\mu\gamma_\nu) = -\mathrm{Tr}(\gamma_\nu\gamma_5\gamma_\mu) = -\mathrm{Tr}(\gamma_5\gamma_\mu\gamma_\nu)$$

T6 $\mathrm{Tr}(\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) = -4i\,\varepsilon_{\mu\nu\rho\sigma}$

where

$$\varepsilon_{\mu\nu\rho\sigma} = \begin{cases} +1 & \text{if } (\mu\nu\rho\sigma) \text{ is an even permutation of } (0123) \\ -1 & \text{if it is an odd permutation} \\ 0 & \text{otherwise} \end{cases}$$

Proof:

If any two indices $\mu\nu\rho\sigma$ are the same

$$\mathrm{Tr}(\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma)=0$$

Take $\mu=\rho$, $\mu\neq\nu$, $\mu\neq\sigma$

$$\mathrm{Tr}(\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) = -\mathrm{Tr}(\gamma_5(\gamma_\mu)^2\gamma_\nu\gamma_\sigma)$$

$$= \pm\,\mathrm{Tr}(\gamma_5\gamma_\nu\gamma_\sigma) = 0$$

Use $(\gamma_0)^2 = I$, $(\gamma_\alpha)^2 = -I$ $(\alpha=1,2,3)$ and T5

Obviously the trace $Tr(\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma)$ also vanishes if three or four indices among μ,ν,ρ and σ are the same, since $Tr(\gamma_5\gamma_\mu) = Tr(\gamma_5) = 0$.

In order to have a nonzero trace the indices $\mu\nu\rho\sigma$ must be any permutation of 0123.

Consider the identical permutation:

$$\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma = i\,\gamma_0\gamma_1\gamma_2\gamma_3\gamma_0\gamma_1\gamma_2\gamma_3$$

$$= -i\,\gamma_0\gamma_1\gamma_2(\gamma_3)^2\gamma_0\gamma_1\gamma_2 = -i\,\gamma_0\gamma_1(\gamma_2)^2(\gamma_3^2)\gamma_0\gamma_1$$

$$= i\,\gamma_0(\gamma_1)^2(\gamma_2)^2(\gamma_3)^2\gamma_0 = i(\gamma_0)^2(\gamma_1)^2(\gamma_2)^2(\gamma_3)^2$$

$$= -iI$$

Hence

$$Tr(\gamma_5\gamma_0\gamma_1\gamma_2\gamma_3) = -i\,Tr(I) = -4i$$

Using the anticommutation rules of the γ matrices it can be easily seen that any even permutation of the indices leads to the same result, while for odd permutations one gets $4i$.

T7 $\quad \mathrm{Tr}(\gamma_\mu \not{p} \gamma_\nu \not{k}) = 4[p_\mu k_\nu + p_\nu k_\mu - g_{\mu\nu}(pk)]$

Proof:

$$\mathrm{Tr}(\gamma_\mu \not{p} \gamma_\nu \not{k}) = p^\rho k^\sigma \, \mathrm{Tr}(\gamma_\mu \gamma_\rho \gamma_\nu \gamma_\sigma)$$

$$= 4 p^\rho k^\sigma (g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho} - g_{\mu\nu} g_{\rho\sigma})$$

T8 $\quad \mathrm{Tr}[\gamma_\mu (1-\gamma_5) \not{p} \gamma_\nu (1-\gamma_5) \not{k}]$

$$= 2\,\mathrm{Tr}(\gamma_\mu \not{p} \gamma_\nu \not{k}) + 8i\,\varepsilon_{\mu\rho\nu\sigma} p^\rho k^\sigma$$

Proof:

$$\gamma_\mu (1-\gamma_5) \not{p} \gamma_\nu (1-\gamma_5) \not{k}$$

$$= \gamma_\mu \not{p} \gamma_\nu \not{k} - \gamma_\mu \gamma_5 \not{p} \gamma_\nu \not{k} - \gamma_\mu \not{p} \gamma_\nu \gamma_5 \not{k}$$

$$+ \gamma_\mu \gamma_5 \not{p} \gamma_\nu \gamma_5 \not{k}$$

$$= \gamma_\mu \not{p} \gamma_\nu \not{k} + \gamma_\mu (\gamma_5)^2 \not{p} \gamma_\nu \not{k}$$

$$- \gamma_\mu \gamma_5 \not{p} \gamma_\nu \not{k} - \gamma_\mu \gamma_5 \not{p} \gamma_\nu \not{k}$$

$$= 2(\gamma_\mu \not{p} \gamma_\nu \not{k}) + 2(\gamma_5 \gamma_\mu \not{p} \gamma_\nu \not{k})$$

Use T6 to get the result

T9 $\quad \mathrm{Tr}(\gamma_\mu \not{p} \gamma_\nu \not{k})\, \mathrm{Tr}(\gamma^\mu \not{k}' \gamma^\nu \not{p}')$

$$= 32\left[(pk')(kp') + (pp')(kk')\right]$$

Proof:

$$\mathrm{Tr}(\gamma_\mu \not{p} \gamma_\nu \not{k})\, \mathrm{Tr}(\gamma^\mu \not{k}' \gamma^\nu \not{p}')$$

$$= 16\left[p_\mu k_\nu + p_\nu k_\mu - g_{\mu\nu}(pk)\right]\left[k'^\mu p'^\nu + k'^\nu p'^\mu - g^{\mu\nu}(k'p')\right]$$

$$= 16\Big[(pk')(kp') + (pp')(kk') - (kp)(k'p') + (pp')(kk')$$

$$+ (pk')(kp') - (pk)(p'k') - (k'p')(pk) - (k'p')(pk)$$

$$+ 4(pk)(k'p')\Big]$$

$$= 32\left[(pk')(kp') + (pp')(kk')\right]$$

T10 $\quad \mathrm{Tr}(\gamma_\mu \not{p} \gamma_\nu \gamma_5 \not{k})\, \mathrm{Tr}(\gamma^\mu \not{k}' \gamma^\nu \gamma_5 \not{p}')$

$$= 32\left[(pk')(kp') - (pp')(kk')\right]$$

Proof: Using T8 we can write

$$\mathrm{Tr}(\gamma_\mu \not{p} \gamma_\nu \gamma_5 \not{k})\, \mathrm{Tr}(\gamma^\mu \not{k}' \gamma^\nu \gamma_5 \not{p}')$$

$$= \mathrm{Tr}(\gamma_5 \gamma_\mu \not{p} \gamma_\nu \not{k})\, \mathrm{Tr}(\gamma_5 \gamma^\mu \not{k}' \gamma^\nu \not{p}')$$

$$= p^\rho k^\sigma \, \mathrm{Tr}(\gamma_5 \gamma_\mu \gamma_\rho \gamma_\nu \gamma_\sigma)\, k'_{\rho'} p'_{\sigma'} \, \mathrm{Tr}(\gamma_5 \gamma^\mu \gamma^{\rho'} \gamma^\nu \gamma^{\sigma'})$$

$$= p^\rho k^\sigma k'_{\rho'} p'_{\sigma'} \, (-i4\varepsilon_{\mu\rho\nu\sigma})(-i4\varepsilon^{\mu\rho'\nu\sigma'})$$

Use now the contraction property of $\varepsilon_{\mu\rho\nu\sigma}$

$$\varepsilon_{\mu\nu\rho\sigma}\, \varepsilon^{\mu\nu\rho'\sigma'} = -2\left(g^\rho_{\rho'} g^\sigma_{\sigma'} - g^\rho_{\sigma'} g^\sigma_{\rho'}\right)$$

we can rewrite

$$\mathrm{Tr}(\gamma_\mu \not{p} \gamma_\nu \gamma_5 \not{k}) \, \mathrm{Tr}(\gamma^\mu \not{k}' \gamma^\nu \gamma_5 \not{p}')$$

$$= -16\, p^\rho k^\sigma k'_{\rho'} p'_{\sigma'} \, \varepsilon_{\mu\rho\nu\sigma} \varepsilon^{\mu\rho'\nu\sigma'}$$

$$= -16\, p^\rho k^\sigma k'_{\rho'} p'_{\sigma'} \left[-2\left(g^\rho_{\rho'} g^\sigma_{\sigma'} - g^\rho_{\sigma'} g^\sigma_{\rho'}\right)\right]$$

$$= 32\left[(pk')(kp') - (pp')(kk')\right]$$

T11 $\mathrm{Tr}\left[\gamma^\mu (1-\gamma_5)\not{p}\gamma^\nu(1-\gamma_5)\not{k}\right]$

$$\times \mathrm{Tr}\left[\gamma_\mu(1-\gamma_5)\not{k}'\gamma_\nu(1-\gamma_5)\not{p}'\right] = 256\,(pk')(kp')$$

Proof: use T8 to obtain

$$\mathrm{Tr}\left[\gamma^\mu(1-\gamma_5)\not{p}\gamma^\nu(1-\gamma_5)\not{k}\right]\mathrm{Tr}\left[\gamma_\mu(1-\gamma_5)\not{k}'\gamma_\nu(1-\gamma_5)\not{p}'\right]$$

$$= \left[2\,\mathrm{Tr}(\gamma^\mu \not{p} \gamma^\nu \not{k}) + 8i\,\varepsilon^{\mu\rho\nu\sigma} p_\rho k_\sigma\right]$$

$$\times\left[2\,\mathrm{Tr}(\gamma_\mu \not{k}' \gamma_\nu \not{p}') + 8i\,\varepsilon_{\mu\rho'\nu\sigma'}(k')^{\rho'}(p')^{\sigma'}\right]$$

$$= 4\,\mathrm{Tr}\left(\gamma^\mu \not{p} \gamma^\nu \not{k}\right) \mathrm{Tr}\left(\gamma_\mu \not{k}' \gamma_\nu \not{p}'\right)$$

$$+ 16 i\, \mathrm{Tr}\left(\gamma^\mu \not{p} \gamma^\nu \not{k}\right) \varepsilon_{\mu\rho'\nu\sigma'} (k')^{\rho'} (p')^{\sigma'}$$

$$+ 16 i\, \mathrm{Tr}\left(\gamma_\mu \not{k}' \gamma_\nu \not{p}'\right) \varepsilon^{\mu\rho\nu\sigma} p_\rho k_\sigma$$

$$- 64\, \varepsilon_{\mu\rho'\nu\sigma'} \varepsilon^{\mu\rho\nu\sigma} p_\rho k_\sigma (k')^{\rho'} (p')^{\sigma'}$$

Using T7, T8 and the contraction property of $\varepsilon_{\mu\rho\nu\sigma}$ exploited to obtain T10 we can rewrite the above expression

$$64\left[p_\mu k_\nu + p_\nu k_\mu - g_{\mu\nu}(pk)\right]\left[(k')^\mu (p')^\nu + (k')^\nu (p')^\mu - g^{\mu\nu}(k'p')\right]$$

$$+ 64 i \left[p^\mu k^\nu + p^\nu k^\mu - g^{\mu\nu}(pk)\right] \varepsilon_{\mu\rho'\nu\sigma'} (k')^{\rho'} (p')^{\sigma'}$$

$$+ 64 i \left[k'_\mu p'_\nu + k'_\nu p'_\mu - g_{\mu\nu}(p'k')\right] \varepsilon^{\mu\rho\nu\sigma} p_\rho k_\sigma$$

$$+ 128\left[(pk')(kp') - (pp')(kk')\right]$$

The second and third line do not contribute, since

$$\varepsilon_{\mu\rho\nu\sigma} p^\mu k^\nu p^\rho k^\sigma = 0$$

The remaining contributions yield

$$128\left[(pk')(kp') + (pp')(kk')\right]$$

$$+ 128\left[(pk')(kp') - (pp')(kk')\right] = 256\,(pk')(kp')$$