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NEUTRINO EMISSIVITIES OF NEUTRON STARS[1]

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ABSTRACT

Neutrino emissivities from the modified Urca process, considered previously by Bahcall and Wolf, and neutrino pair bremsstrahlung from nucleon-nucleon scattering, considered previously by Flowers *et al.*, are recalculated using a nucleon-nucleon interaction that consists of a long-range, one-pion-exchange tensor part and a short-range part parametrized with nuclear Fermi liquid (Landau) parameters. Effects of short-range correlations on the tensor part are approximated through insertion of a cutoff in the one-pion exchange potential. The resulting emissivities differ dramatically from previous calculations: the emissivities from the Urca process and the neutron-neutron neutrino pair bremsstrahlung exceed the results of Bahcall and Wolf and Flowers *et al.*, respectively, by an order of magnitude. It is argued that these results are due to the one-pion-exchange tensor force in the nucleon-nucleon interaction used here. The neutrino mean free path arising from a reaction related to the Urca process is also calculated, and the influence of finite neutrino mean free paths and nucleon superfluidity on neutrino emissivities is briefly discussed.

Subject headings: dense matter — neutrinos — nuclear reactions — stars: neutron

I. INTRODUCTION

With the recent establishment of an upper limit on the Crab pulsar blackbody flux (Wolff *et al.* 1975; Toor and Seward 1977), the subject of neutron star cooling via neutrino emission has become of renewed interest. Among the important processes contributing to this cooling are the modified Urca process,

$$n + n \rightarrow n + p + e^- + \bar{\nu}_e\,, \tag{1}$$

and its inverse, and, if pion condensates exist, quasi-particle β-decay,

$$u \rightarrow u + e^- + \bar{\nu}_e\,. \tag{2}$$

Here the u quasi-particles are linear combinations of proton and neutron states. Ordinary neutron decay,

$$n \rightarrow p + e^- + \bar{\nu}_e\,, \tag{3}$$

is strongly suppressed in neutron stars in thermal equilibrium by a large energy-momentum mismatch.

Process (1) and a simplified version of (2),

$$n + \pi^- \rightarrow n + e^- + \bar{\nu}_e\,, \tag{4}$$

were first treated in connection with neutron star cooling by Bahcall and Wolf (1965), who found that the neutrino emissivity from the pion decay process exceeds that from the Urca process by several orders of magnitude at interior temperatures near 10^9 K. In obtaining the pion decay emissivity, these authors assumed a zero-momentum, noninteracting condensate. When interactions are included, a condensate with finite momentum is preferred. However, this does not substantially alter the result, as was demonstrated by Maxwell *et al.* (1977).

The discovery of weak neutral currents (Hasert *et al.* 1973; Benvenuti *et al.* 1974; Barish *et al.* 1974) suggested two other processes that might be important: neutrino pair bremsstrahlung from neutron-neutron and neutron-proton scattering,

$$n + n \rightarrow n + n + \nu + \bar{\nu}\,, \tag{5}$$

$$n + p \rightarrow n + p + \nu + \bar{\nu}\,, \tag{6}$$

which we hereafter refer to as the $nn\nu\bar{\nu}$ and $np\nu\bar{\nu}$ processes, respectively. Both processes may involve either electron-type neutrinos or muon-type neutrinos. They were first treated in connection with neutron star cooling by Flowers, Sutherland, and Bond (1975), who obtained for the $np\nu\bar{\nu}$ emissivity a result comparable with the Urca process and for the $nn\nu\bar{\nu}$ emissivity a result roughly two orders of magnitude smaller.

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Other processes not involving nucleons, such as photoneutrino production, plasmon neutrino production, and neutrino pair production through e^-e^+ annihilation, have been studied elsewhere (Dicus 1972) and found to be unimportant at typical neutron star densities and temperatures.

The large discrepancy in neutrino emissivities calculated in the absence and presence of pion condensates suggests that cooling observations may provide information concerning the existence of pion condensates in neutron star interiors. In particular, if the emissivities calculated in the absence of a condensate are insufficient to account for the upper limit on the Crab pulsar temperature, as deduced from the upper limit on the blackbody flux (Wolff *et al.* 1975), then the existence of pion condensates in neutron star interiors would be strongly indicated.

The three noncondensate processes described above—processes (1), (5), and (6)—all involve a strong nucleon-nucleon (NN) interaction as well as the weak interaction responsible for the neutrino production. To date this strong interaction has been treated either by computing the overlap integrals associated with the initial and final nucleon wave functions (Bahcall and Wolf 1965) or through the use of a Fermi liquid parametrization (Flowers, Sutherland, and Bond 1975). In neither case was the best known part of the NN interaction—the one-pion-exchange (OPE) piece—treated explicitly. However, for the purpose of obtaining neutrino emissivities in neutron star interiors, this may be the most important part. In particular, the average interparticle separation in neutron matter at nuclear matter density is 2.2 fm, which is large enough that the longest range part of the NN interaction (i.e., the OPE part) should dominate. Moreover, the tensor piece of the OPE interaction can conspire with the axial part of the weak hadronic current to give much larger matrix elements than obtained with a central NN interaction. Such is the case for the $nn\nu\bar{\nu}$ process, as will be shortly demonstrated.

Motivated by these considerations, we have recomputed the Urca and neutral current bremsstrahlung emissivities with the OPE term included explicitly in the NN interaction. In the nonrelativistic approximation this term consists of both tensor and spin-spin parts and depends upon the magnitude of the momentum transfer between the interacting nucleons (k-dependence). To describe the short-range part of the interaction, which cannot be treated in perturbation theory, we employ Fermi liquid parameters (Migdal 1967). Such a parametrization of the effective interaction is especially suitable for calculating neutrino emissivities since the participating nucleons must all lie near their Fermi surfaces.

To obtain expressions for the various neutrino emissivities with this form for the NN interaction, we impose a number of simplifying approximations. First, the nucleons are treated nonrelativistically in the NN interaction and the hadronic part of the weak interaction. Second, we expand the nucleon propagator in powers of the inverse nucleon mass and discard all but the lowest order term. This is equivalent to the neglect of nuclear recoil terms. In the emissivities such terms are of order $(p_F/m^*)^2$, where p_F and m^* are the Fermi momentum and effective mass, respectively. Thus, for neutrons at nuclear matter density they are at most 20% of the lowest order term with the values used here for $p_F(n)$ and m_n^*. The proton recoil terms are even less important.

Performance of the phase space integrals is facilitated by imposing three further approximations. In particular, since the nucleons and electrons participating in neutrino-producing processes must all lie near their Fermi surfaces, we can equate their momentum magnitudes with the corresponding Fermi momenta in all angular integrals. Then the angular and energy parts of the phase space integrals separate. We can also neglect the neutrino momenta in comparison with the electron and nucleon Fermi momenta since the neutrinos are thermal. This simplifies the phase space integrals by eliminating the neutrino momenta from the momentum conserving delta functions. Finally, in the Urca process we adopt a triangle approximation, in which only the neutron momenta are retained in the momentum conserving delta function. Thus, we keep only the lowest order term in an expansion of the Urca emissivity in powers of $p_F(e)/p_F(n)$. Since the terms of odd order in this expansion vanish, the error in this approximation is at most of order $[p_F(e)/p_F(n)]^2$ times the leading term, and can hence be neglected.

Within these approximations and using the uncorrelated form of the OPE interaction, analytical expressions can be obtained for the Urca and neutral-current bremsstrahlung emissivities if only the direct Born terms are retained in the corresponding matrix elements. Such a calculation is outlined in some detail in § IV following a brief summary of physical conditions in neutron star interiors in § II and a more explicit description of the NN interaction in § III. The following section (V) contains numerical results and also treats the effects of short-range correlations in the OPE interaction. In § VI the OPE exchange terms arising from the antisymmetry requirement on the nucleon wave functions are considered (note that the Landau part of the NN interaction is antisymmetric by construction). The influence of ρ exchange, which also contains a tensor piece not included in the Landau interaction, is the subject of § VII. Finally, in the concluding section we compare our results with those obtained previously and briefly treat the effects of nucleon superfluidity and finite mean free paths on our emissivities.

II. NEUTRON STAR INTERIORS

Neutron stars probably originate in the aftermath of supernova explosions with densities a few times nuclear matter density (0.17 fm^{-3}) and interior temperatures near or somewhat above 10^{11} K ($\sim$10 MeV). Very shortly after formation the temperature drops to within the 10^9–10^{10} K range, the interior matter occupies its ground state, and the star as a whole is in thermal, β, and charge equilibrium (Tsuruta and Cameron 1965, 1966; Baym and Pethick 1975). Because of the extremely high densities and relatively low temperatures involved (in comparison with the relevant Fermi energies), the electrons, protons, and neutrons in the interior are all degenerate. This has the consequence that the interior is approximately isothermal (Tsuruta and Cameron 1966). The electrons are also extremely relativistic, while the nucleons are approximately nonrelativistic.

These degeneracy and relativity characteristics, together with the equilibrium conditions, permit explicit expressions to be obtained for the electron and nucleon Fermi momenta, which are required to derive numerical results for the neutrino emissivities. In particular, the neutron Fermi momentum is given by ($\hbar = c = 1$)

$$p_F(n) = (3\pi^2\rho)^{1/3} \approx 340(\rho/\rho_0)^{1/3}\ \mathrm{MeV}/c\,, \tag{7}$$

where ρ_0 is nuclear matter density. Charge equilibrium requires that the electron and proton Fermi momenta be equal. Combining this with the β-equilibrium condition,

$$\mu_n = \mu_p + \mu_e\,, \tag{8}$$

approximating the chemical potentials here by the corresponding Fermi energies, and neglecting the proton Fermi energy in comparison with the electron Fermi energy, we find for the electron Fermi momentum

$$p_F(e) \approx p_F^2(n)/2m_n \approx 62(\rho/\rho_0)^{2/3}\ \mathrm{MeV}/c\,. \tag{9a}$$

This result assumes that the neutron and proton potential energies are equal in neutron star interiors. In actuality, because the $T = 0$ interaction is more attractive than the $T = 1$ interaction, the proton is more bound than the neutron. To take this into account we choose a somewhat larger value for $p_F(e)$

$$p_F(e) \approx 85(\rho/\rho_0)^{2/3}\ \mathrm{MeV}/c\,, \tag{9b}$$

which we will employ in what follows.

In addition to the neutron and electron Fermi momenta, numerical results for the neutrino emissivities require values for the neutron and proton effective masses in high-density neutron matter. A number of studies of these effective masses have been carried out, but to date the results are not conclusive. Using a Fermi liquid theory approach, Bäckman, Källman, and Sjöberg (1973) obtained the value 0.8 for the effective mass m_n^*/m_n in neutron matter at densities slightly above ρ_0. A similar calculation by Sjöberg (1976) in neutron star matter (nuclear matter in β-equilibrium) indicates that the proton effective mass m_p^*/m_p might be lower than the neutron one. However, Sjöberg's calculation does not include the induced interaction, which might alter the effective masses appreciably at high densities. In view of these uncertainties we simply set both m_n^*/m_n and m_p^*/m_p equal to 0.8.

Before concluding this section, it should be mentioned that neutron stars probably possess rather large magnetic fields (Ruderman 1972) and that both neutron and proton superfluidity may occur over extensive regions in the interior (Migdal 1959). Magnetic fields can affect neutron star cooling by altering the structure of the crust material, but are probably insufficient to affect the emissivities themselves (Pethick 1978). On the other hand, nucleon superfluidity will affect the emissivities qualitatively by restricting the phase space available to the participating nucleons. We have not included such effects in the present calculations, but rather have assumed throughout that the interior matter is normal. In the conclusion we will briefly treat the effects of superfluidity qualitatively.

III. THE NUCLEON–NUCLEON INTERACTION

As discussed in the Introduction, we employ an NN interaction consisting of an OPE part and a Fermi liquid, or Landau, part. Nonrelativistically, the momentum-space representation of the OPE interaction in the absence of correlations is

$$V_{\mathrm{OPE}} = \left(\frac{f}{m_\pi}\right)^2 \boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{k}\left(\frac{-1}{k^2 + m_\pi^2}\right)\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{k}(\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)})\,, \tag{10}$$

where the σ's and τ's are respectively the Pauli spin matrices and isospin matrices associated with the two nucleon lines, k is the momentum transfer, and f is the p-wave πN coupling constant. Due to vertex renormalization mechanisms which spread the πNN vertex over a finite region of space, the quantity f is actually not constant but contains a momentum dependence typically parametrized by a monopole form factor. We have not included such a form factor in the present calculations but have simply fixed f equal to its value at the pion pole:

$$f^2 = 4\pi \times 0.08 \approx 1\,. \tag{11}$$

At typical neutron star densities, the renormalized coupling constant is not very far from this value.

Equation (10) does not include short-range correlations induced by the hard core of the NN interaction, which prevent nucleons from approaching each other too closely. To take account of such correlations, it is necessary to multiply the position space representation of V_{OPE} by a squared correlation function, $f^2(r)$. Fourier transforming back to momentum space then gives

$$\tilde{V}(k) = \int d^3r \exp(-i\boldsymbol{k}\cdot\boldsymbol{r}) f^2(r) V_{\mathrm{OPE}}(r)\,. \tag{12}$$

Various choices can be made for $f(r)$, which result in different forms for the correlated potential. A particularly simple choice is the unit step function

$$f(r) = \theta(r - d)\,, \tag{13}$$

which cuts off the uncorrelated potential within a distance d and leaves it unaltered outside of d. With this choice for $f(r)$, equation (12) yields (Dahlblom *et al.* 1964)

$$\tilde{V}(k) = \frac{f^2 \exp(-m_\pi d)}{3k}\left[\frac{k \cos kd + m_\pi \sin kd}{k^2 + m_\pi^2}(S_{12}(\hat{k}) + \boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}) - 3\,\frac{1 + m_\pi d}{m_\pi^2 d} j_1(kd) S_{12}(\hat{k})\right]\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}\,, \tag{14}$$

where j_1 is the $l = 1$ spherical Bessel function, and the tensor operator S_{12} is given by

$$S_{12}(\hat{k}) = 3\boldsymbol{\sigma}^{(1)}\cdot\hat{k}\boldsymbol{\sigma}^{(2)}\cdot\hat{k} - \boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}\,. \tag{15}$$

An alternative choice for $f(r)$ is

$$f^2(r) = 1 - j_0(q_c r)\,, \tag{16}$$

where j_0 is the $l = 0$ spherical Bessel function, and q_c is a cutoff momentum typically chosen equal to the mass of the ω meson, the exchange of which is supposed to be the origin of the hard core of the NN interaction. This form for $f(r)$ leads to a correlated potential of the form

$$\tilde{V}(k) = -\left(\frac{f}{m_\pi}\right)^2\left[\left(\frac{1}{k^2 + m_\pi^2} - \frac{1}{k^2 + q_c^2 + m_\pi^2}\right)\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{k}\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{k} - \frac{1}{3}\frac{q_c^2}{k^2 + q_c^2 + m_\pi^2}\,\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}\right]\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)} \tag{17}$$

with the zero momentum transfer value employed for the πN coupling constant f.

The OPE interaction describes only the long-range part of the NN interaction. The short-range part arises primarily from exchange of heavy mesons and is approximately constant for momentum transfers small compared with the meson masses. We use the Landau Fermi liquid parameters to describe this part of the interaction.

In symmetric nuclear matter the particle-hole interaction can be expressed in the form (Migdal 1967)

$$F(\boldsymbol{k}_1, \boldsymbol{k}_2) = f + f'\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)} + g\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)} + g'\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}\,. \tag{18}$$

For small momentum transfers we need consider only momenta $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ close to the Fermi surface. Then the functions on the right-hand side of (18) depend only on the angle between $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$. Hence, they can be expanded in Legendre polynomials:

$$f = \sum_l f_l P_l(\cos\theta)\,, \tag{19}$$

with analogous expansions for f', g, and g'. Since this expansion is expected to converge rapidly, one usually retains only the first few terms of the series. Theoretical values for the parameters are in rough agreement with empirical ones deduced from the properties of heavy nuclei.

In neutron star matter—i.e., neutron matter with a small fraction of protons—the isospin dependence of the particle-hole interaction is not as simple as in (18). The relation between the scattering amplitude and the particle-hole interaction, which is straightforward in symmetric nuclear matter (Bäckman 1969), is also nontrivial in neutron star matter. In addition there are no Landau parameters for neutron star matter available at present. Therefore, as a first approximation we use the parameters for symmetric nuclear matter and neglect the difference between the scattering amplitude and the particle-hole interaction. We also drop the higher ($l > 0$) parameters and assume that the short-range part of the interaction is independent of momentum transfer. This part of the interaction is then local and of zero range in configuration space.

Explicit values for the parameters are (Sjöberg 1973; Anastasio and Brown 1977)

$$F_0' = 0.7\,, \qquad G_0 = G_0' = 1.1\,, \tag{20}$$

where

$$F_0' = N(0) f_0' \tag{21}$$

with analogous expressions for the other parameters. Here

$$N(0) = 2m^* p_F(n)/\pi^2\hbar^3 \tag{22}$$

is the density of states at the Fermi surface in symmetric nuclear matter, with $p_F = 1.36\ \mathrm{fm}^{-1}$ and $m^* = m$. A value for F_0 has not been given, since the spin-summed matrix elements are independent of this parameter, as will shortly be demonstrated.

IV. NEUTRINO EMISSIVITIES IN THE BORN APPROXIMATION

a) *Matrix Elements*

The direct diagrams contributing to the $nn\nu\bar{\nu}$ process, $np\nu\bar{\nu}$ process, and Urca process matrix elements in the Born approximation are illustrated in Figures 1, 2, and 3. In each of these figures, the 4-momentum imparted to the lepton pair is denoted q, and the momentum transferred between the nucleons either $\boldsymbol{k}$ or $\boldsymbol{k}'$. Momentum conservation requires that

$$\boldsymbol{k}' = \boldsymbol{k} + \boldsymbol{q}\,, \tag{23}$$

so that $\boldsymbol{k}$ and $\boldsymbol{k}'$ are not exactly equal. However, for all three processes $|\boldsymbol{k}|$ and $|\boldsymbol{k}'|$ are large compared with $|\boldsymbol{q}|$ over most of the allowed phase space; thus $\boldsymbol{k}$ and $\boldsymbol{k}'$ can be equated without introducing significant errors into the emissivities.

In Figure 2, the first four diagrams involve a different final state from the last four diagrams so that the matrix element contributions from the two groups of diagrams must be added incoherently. In order to distinguish the two matrix element contributions, we will hereafter denote the first set of diagrams (A–D) group I diagrams and the second set (E–H) group II diagrams. Also, there are three diagrams in addition to those in Figure 3 which contribute to the Urca matrix element. These additional diagrams are the analogs of the Figure 3 diagrams with the neutron and proton labels on the outgoing nucleon lines interchanged (and with the location of the weak interaction shifted accordingly). Since they involve a different final state from the diagrams in Figure 3, they must be added incoherently with those diagrams. Moreover, after the phase space integrals are performed, they give a contribution to the Urca emissivity that is identical to that of the Figure 3 diagrams. Hence, they may be included in the calculation by merely doubling the emissivity obtained with the Figure 3 diagrams alone.

To describe the weak interaction, we employ the Weinberg-Salam model (Weinberg 1972), which yields the usual nonrelativistic expression for the charged current interaction

$$\mathscr{L}_c = \frac{G}{\sqrt{2}}\chi_p{}^+(\delta_{\mu 0} - g_A\delta_{\mu i}\sigma_i)\chi_n l_\mu\,, \tag{24}$$

where χ_n and $\chi_p{}^+$ are Pauli spinors representing the incoming neutron and outgoing proton respectively, $G = 8.74 \times 10^{-5}$ MeV fm^3 is the weak Fermi coupling constant, $g_A = 1.26$ is the axial vector renormalization, and l_μ, the lepton current, is given by

$$l_\mu = \bar{u}(q_1)\gamma_\mu(1 - \gamma_5)u(q_2)\,, \tag{25}$$

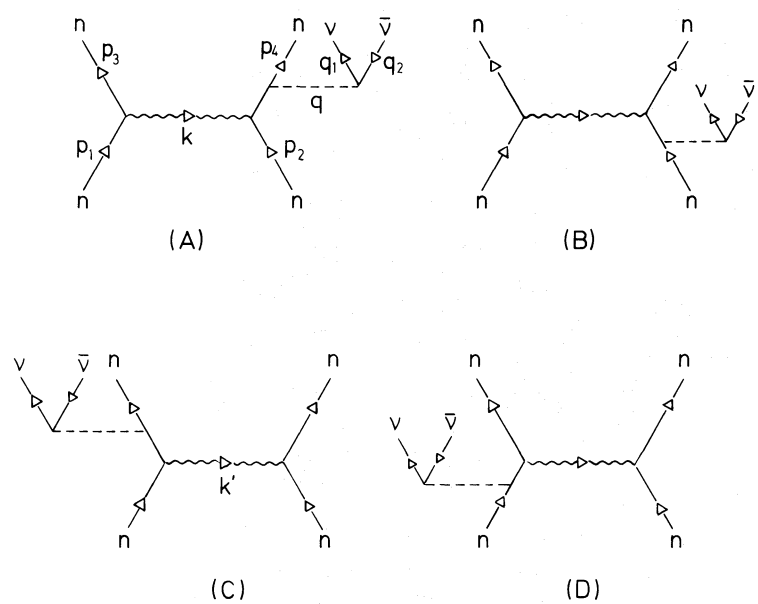

FIG. 1.—Diagrams for the $nn\nu\bar{\nu}$ process (5). The directed solid lines indicate neutrons or neutrinos as indicated, the directed wavy line indicates the nucleon-nucleon interaction involving exchange of momentum k or k', and the dashed line indicates the weak interaction involving exchange of momentum q.

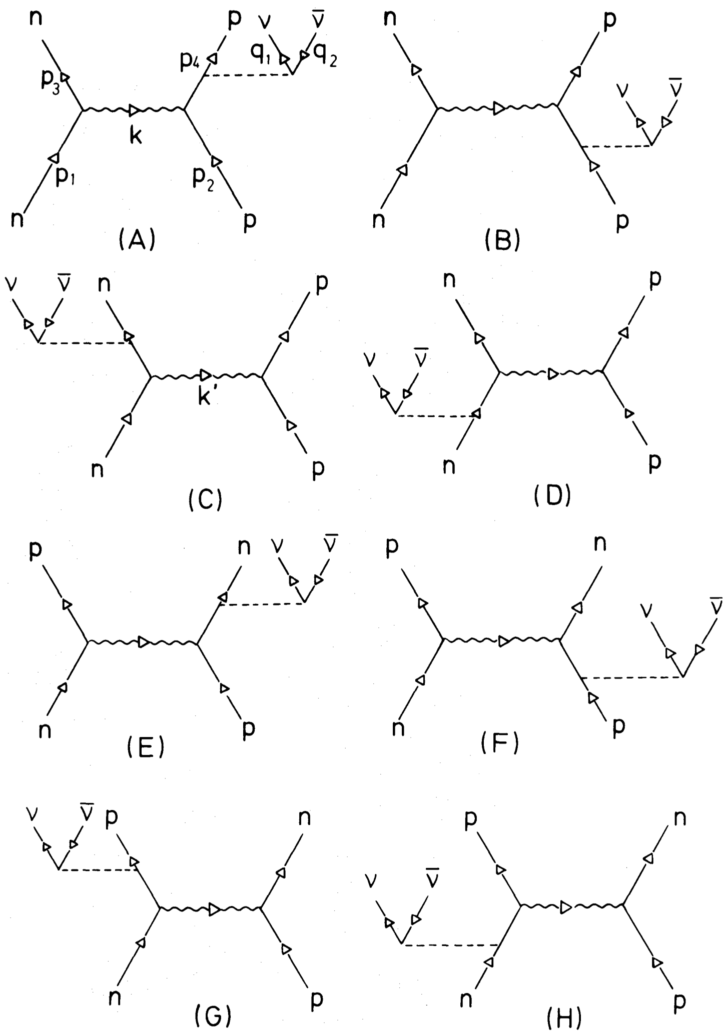

FIG. 2.—Diagrams for the $np\nu\bar{\nu}$ process (6). Labeling is as in Fig. 1.

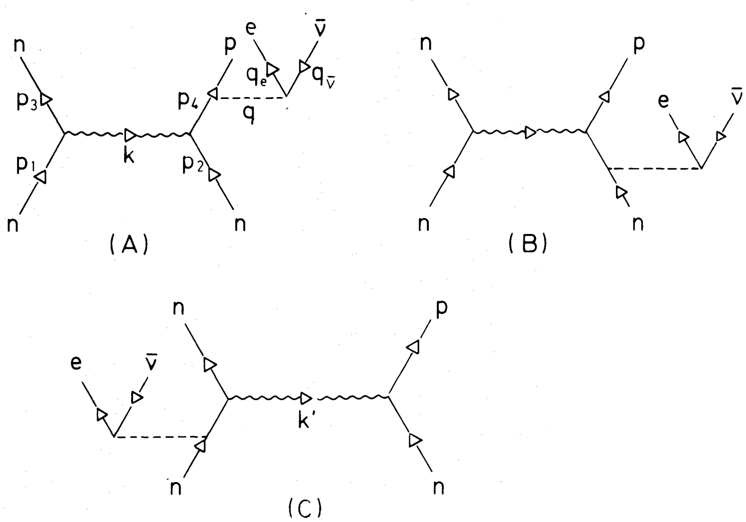

FIG. 3.—Diagrams for the Urca process (1). Labeling as in Fig. 1.

with q_1 and q_2 equal to the lepton 4-momenta. The corresponding neutral current interactions are

$$\mathscr{L}_n = -\frac{G}{2\sqrt{2}}\chi_1^+(\delta_{\mu 0} - g_A\delta_{\mu i}\sigma_i)\chi_2 l_\mu \tag{26a}$$

for neutrons and

$$\mathscr{L}_p = \frac{G}{2\sqrt{2}}\chi_1^+(c_v\delta_{\mu 0} - g_A\delta_{\mu i}\sigma_i)\chi_2 l_\mu \tag{26b}$$

for protons, where c_v is related to the Weinberg angle, θ_W, by

$$c_v = 1 - 4\sin^2\theta_W\,. \tag{27}$$

For the nucleon propagator we have, nonrelativistically,

$$iG(p \pm q, E_p \pm \omega) = i/(E_p \pm \omega - E_{p\pm q})\,, \tag{28a}$$

where E_p is the energy associated with the external nucleon line, $E_{p\pm q}$ is the energy associated with the internal nucleon line, and ω is the total lepton energy. The sign is chosen positive if the weak interaction is attached to an outgoing nucleon line; otherwise, negative. Expanding this propagator in powers of the inverse nucleon mass and keeping only the lowest order term, as discussed in the Introduction yields

$$iG(p \pm q, E_p \pm \omega) = \pm i\omega^{-1}\,, \tag{28b}$$

with the sign as before.

If we now combine equations (26a) and (28b) with equation (10) for the uncorrelated OPE interaction, we obtain for the OPE contribution of diagram A of Figure 1 to the $nn\nu\bar{\nu}$ matrix element

$$M^{(A)}{}_{nn,\mathrm{OPE}} = \frac{G}{2\sqrt{2}}\left(\frac{f}{m_\pi}\right)^2 \omega^{-1} l_\mu[\chi_4^+(\delta_{\mu 0} - g_A\delta_{\mu i}\sigma_i)\boldsymbol{\sigma}\cdot\boldsymbol{k}\chi_2(k^2 + m_\pi^2)^{-1}\chi_3^+\boldsymbol{\sigma}\cdot\boldsymbol{k}\chi_1] \tag{29}$$

where the numerical subscripts on the χ's denote the corresponding nucleon lines, and the isospin part of the matrix element has been evaluated. The corresponding contribution from diagram B of Figure 1 differs from the above only in the order of the interactions and the sign of the nucleon propagator. Consequently, when the two diagrams are summed, the vector part of the weak interaction vanishes. A similar cancellation occurs between diagrams C and D of Figure 1. Note that the axial part of the weak interaction does not cancel, because the Pauli spin matrices do not commute. Summing the four diagrams in Figure 1, we obtain for the OPE contribution to the $nn\nu\bar{\nu}$ matrix element

$$M_{nn,\mathrm{OPE}} = \frac{G}{2\sqrt{2}}\left(\frac{f}{m_\pi}\right)^2 g_A\omega^{-1}(k^2 + m_\pi^2)^{-1} l_i 2i\varepsilon_{mik}k_m \times [\chi_4^+\boldsymbol{\sigma}\cdot\boldsymbol{k}\chi_2\chi_3^+\sigma_k\chi_1 + \chi_3^+\boldsymbol{\sigma}\cdot\boldsymbol{k}\chi_1\chi_4^+\sigma_k\chi_2]\,, \tag{30}$$

where repeated indices are summed over the three spatial coordinates only.

The vanishing of the vector part of the weak interaction is a property not only of the OPE contribution to the $nn\nu\bar{\nu}$ matrix element, but of the OPE contributions to the $np\nu\bar{\nu}$ and Urca matrix elements as well. It is also a property of all the Landau contributions to the matrix elements.

Summing diagrams A–D of Figure 2, we obtain for the OPE contribution of group I diagrams to the $np\nu\bar{\nu}$ matrix element

$$M^{(\mathrm{I})}{}_{np,\mathrm{OPE}} = \frac{G}{2\sqrt{2}}\left(\frac{f}{m_\pi}\right)^2 g_A\omega^{-1}(k^2 + m_\pi)^{-1} l_i 2i\varepsilon_{mik}k_m \times [\chi_3^+\boldsymbol{\sigma}\cdot\boldsymbol{k}\chi_1\chi_4^+\sigma_k\chi_2 - \chi_4^+\boldsymbol{\sigma}\cdot\boldsymbol{k}\chi_2\chi_3^+\sigma_k\chi_1]\,. \tag{31a}$$

The corresponding contribution from group II diagrams is

$$M^{(\mathrm{II})}{}_{np,\mathrm{OPE}} = \frac{G}{\sqrt{2}}\left(\frac{f}{m_\pi}\right)^2 g_A\omega^{-1}(k^2 + m_\pi^2)^{-1} l_i 2k_i[\chi_4^+\boldsymbol{\sigma}\cdot\boldsymbol{k}\chi_2\chi_3^+\chi_1 - \chi_3^+\boldsymbol{\sigma}\cdot\boldsymbol{k}\chi_1\chi_4^+\chi_2]\,. \tag{31b}$$

For the OPE contribution to the Urca matrix element, we have

$$M_{\mathrm{URCA,OPE}} = \frac{G}{\sqrt{2}}\left(\frac{f}{m_\pi}\right)^2 g_A\omega^{-1}(k^2 + m_\pi^2)^{-1} l_i\{2k_i\chi_3^+\boldsymbol{\sigma}\cdot\boldsymbol{k}\chi_1\chi_4^+\chi_2 + 2k_m[\chi_4\boldsymbol{\sigma}\cdot\boldsymbol{k}\chi_2\chi_3^+(i\varepsilon_{imk}\sigma_k - \delta_{im})\chi_1]\}\,. \tag{32}$$

Concerning the Landau contributions to the various matrix elements, we first observe that the matrix element of the unit isospin operator vanishes for all diagrams involving charge exchange so that contributions from these diagrams can involve only the parameters f' and g' (hereafter, the subscript 0 on the Landau parameters is to be understood). Furthermore, due to cancellations of the same type that eliminate the vector part of the weak interaction, matrix element contributions from the parameters f and f' vanish among the various diagrams not involving charge exchange. Hence, contributions from these diagrams can involve only g and g'.

Among the four diagrams contributing to the $nn\nu\bar{\nu}$ matrix element, there are further cancellations. In this case, not only do the f and f' contributions cancel pairwise between diagrams A and B and between C and D, but the g and g' contributions cancel as well between the sum of A and B and the sum of C and D. Hence, the $nn\nu\bar{\nu}$ matrix element has no contribution in lowest order in the nuclear propagator from the Landau part of the NN interaction. Note that this result does not depend upon the particular functional form of the Landau parameters, but is a property solely of the particular configuration of spin operators associated with the Landau interaction. In other words, a more complicated interaction, such as a tensor interaction, is mandatory to obtain a lowest order contribution to the $nn\nu\bar{\nu}$ matrix element.

For the other processes, the Landau contributions to the matrix elements are

$$M^{(\mathrm{I})}{}_{np,\mathrm{L}} = \frac{G}{2\sqrt{2}}(g - g')(g_A\omega^{-1})l_i 4i\varepsilon_{imk}[\chi_3{}^+\sigma_k\chi_1\chi_4{}^+\sigma_m\chi_2]\,, \tag{33a}$$

$$M^{(\mathrm{II})}{}_{np,\mathrm{L}} = \frac{G}{\sqrt{2}}(f' - g')(g_A\omega^{-1})2l_i[\chi_4{}^+\sigma_i\chi_2\chi_3{}^+\chi_1 - \chi_3\sigma_i\chi_1\chi_4{}^+\chi_2]\,, \tag{33b}$$

and

$$M_{\mathrm{URCA,L}} = \frac{G}{\sqrt{2}}(g_A\omega^{-1})l_i\{2(f' - g')[\chi_3{}^+\sigma_i\chi_1\chi_4\chi_2 - \chi_4{}^+\sigma_i\chi_2\chi_3{}^+\chi_1] + 2i\varepsilon_{imk}(g - g')[\chi_3{}^+\sigma_m\chi_1\chi_4{}^+\sigma_k\chi_2]\}\,. \tag{34}$$

To obtain the spin-summed matrix elements, we simply sum the OPE and Landau contributions, square, and evaluate the traces. After contraction with the lepton trace, given by

$$\mathrm{Tr}\,(l_j{}^+l_i) = 8(q_{1j}q_{2i} + q_{1i}q_{2j} + q_1q_2g_{ij} + i\varepsilon_{i\alpha j\beta}q_1{}^\alpha q_2{}^\beta)\,, \tag{35}$$

where g_{ij} is the metric tensor, and $\epsilon_{i\alpha j\beta}$ is the completely antisymmetric tensor of rank 4, this yields for the $nn\nu\bar{\nu}$ matrix element

$$\sum_{\mathrm{spins}} |M_{nn}|^2 = 64G^2\left(\frac{f}{m_\pi}\right)^4 g_A{}^2\left(\frac{k^2}{k^2 + m_\pi{}^2}\right)^2\omega^{-2}(\omega_1\omega_2 - \boldsymbol{q}_1\cdot\hat{k}\boldsymbol{q}_2\cdot\hat{k})\,, \tag{36}$$

where ω_1 and ω_2 are the neutrino energies corresponding to the momenta $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$.

In the approximation that the neutrino momenta are neglected compared with the nucleon momenta—in particular, in the momentum-conserving delta function—the second term in (36) involving $(\boldsymbol{q}_1\cdot\hat{k})(\boldsymbol{q}_2\cdot\hat{k})$ vanishes in the phase space integration. Hence, it may be omitted, leaving an expression that involves only the lepton energies and the magnitude of the nucleon momentum transfer, k:

$$\sum_{\mathrm{spins}} |M_{nn}|^2 = 64G^2\left(\frac{f}{m_\pi}\right)^4 g_A{}^2\left(\frac{k^2}{k^2 + m_\pi{}^2}\right)^2\frac{\omega_1\omega_2}{\omega^2}\,. \tag{37}$$

The corresponding expressions for the $np\nu\bar{\nu}$ and Urca processes are

$$\sum_{\mathrm{spins}} |M_{np}{}^{(\mathrm{I})}|^2 = 64G^2g_A{}^2\left[\left(\frac{f}{m_\pi}\right)^4\left(\frac{k^2}{k^2 + m_\pi{}^2}\right)^2 + 4(g - g')\left(\frac{f}{m_\pi}\right)^2\frac{k^2}{k^2 + m_\pi{}^2} + 6(g - g')^2\right]\frac{\omega_1\omega_2}{\omega^2}\,, \tag{38a}$$

$$\sum_{\mathrm{spins}} |M_{np}{}^{(\mathrm{II})}|^2 = 128G^2g_A{}^2\left[\left(\frac{f}{m_\pi}\right)^4\left(\frac{k^2}{k^2 + m_\pi{}^2}\right)^2 + 2(f' - g')\left(\frac{f}{m_\pi}\right)^2\frac{k^2}{k^2 + m_\pi{}^2} + 3(f' - g')^2\right]\frac{\omega_1\omega_2}{\omega^2}\,, \tag{38b}$$

and

$$\sum_{\mathrm{spins}} |M_{\mathrm{URCA}}|^2 = 256G^2g_A{}^2\left\{2\left(\frac{f}{m_\pi}\right)^4\left(\frac{k^2}{k^2 + m_\pi{}^2}\right)^2 + 2[(g - g') + (f' - g')]\left(\frac{f}{m_\pi}\right)^2\frac{k^2}{k^2 + m_\pi{}^2} + 3[(g - g')^2 + (f' - g')^2]\right\}\frac{\omega_1\omega_2}{\omega^2}\,, \tag{39}$$

where the Urca result has been multiplied by 2 to include diagrams not shown in Figure 3.

b) *Phase Space Integrals*

The emissivity or luminosity per unit volume for a particular neutrino process is the integral over the total phase space available to the participating particles of the product of the total neutrino energy and the spin-summed matrix element. Explicitly ($\hbar c = 1$)

$$\varepsilon_\nu = \frac{2\pi}{\hbar}\int\left[\prod_{i=1}^{4}\frac{d^3p_i}{(2\pi)^3}\right]\frac{d^3q_1}{2\omega_1(2\pi)^3}\frac{d^3q_2}{2\omega_2(2\pi)^3}\,\delta(E_f - E_{in})(2\pi)^3\delta^{(3)}(\boldsymbol{P}_f - \boldsymbol{P}_{in})\frac{1}{s}\left(\sum_{\text{spins}}|M|^2\right)\omega_\nu\mathscr{S}\,, \tag{40}$$

where the p_i are the nucleon momenta, the q_i and ω_i are the lepton momenta and energies, $\delta(E_f - E_{in})$ is the energy conserving delta function, $\delta^{(3)}(\boldsymbol{P}_f - \boldsymbol{P}_{in})$ is the momentum-conserving delta function, s is a symmetry factor, ω_ν is the total neutrino energy, and $\mathscr{S}$ is the appropriate product of Fermi-Dirac distribution functions.

To evaluate this expression for a particular process, we first perform the integrals over the neutrino phase space. This can be easily accomplished if the neutrino momenta are dropped from the momentum-conserving delta function. Next, we effect a separation of the angular and energy parts of the remaining phase space by performing the angular integrals with the momenta of the degenerate fermions approximated by the corresponding Fermi momenta. In particular, we make the replacement

$$d^3p_i = d^3p_i\int dE_i\delta(E_i - E_{p_i}) \to d^3p_i\delta[E_i - E_F(i)]\int dE_i\,, \tag{41}$$

where $E_F(i)$ is the Fermi energy of particle i, and use either the nonrelativistic relation

$$\delta[E_i - E_F(i)] = [m_i^*/p_F(i)]\delta[p_i - p_F(i)] \tag{42a}$$

or the relativistic one,

$$\delta[E_i - E_F(i)] = \delta[p_i - p_F(i)]\,, \tag{42b}$$

according to whether i is a nucleon or an electron. Such a procedure is justified by the smallness of kT compared with the Fermi energies.

We now introduce an integral over the momentum transfer by inserting

$$1 = \int d^3k\delta^{(3)}(\boldsymbol{k} - \boldsymbol{p}_1 + \boldsymbol{p}_3)\,. \tag{43}$$

With this insertion it is found that the angular part of the phase space collapses to a single integral over the magnitude of $\boldsymbol{k}$, the limits of which are fixed by the delta functions in (41). Evaluating this integral yields an emissivity expression that involves the product of the energy integral and some function of the Fermi momenta. For the $nn\nu\bar{\nu}$ process, this expression is

$$\epsilon_{nn} = \frac{64}{15}\frac{G^2g_A{}^2m_n{}^{*4}}{(2\pi)^9\hbar}\left(\frac{f}{m_\pi}\right)^4 p_F(n)F\left(\frac{m_\pi}{2p_F(n)}\right)I_{\nu\bar{\nu}}\,, \tag{44}$$

where $F(x)$ is defined by

$$F(x) = 1 - \frac{3}{2}x\tan^{-1}\left(\frac{1}{x}\right) + \frac{1}{2}\left(\frac{x^2}{1+x^2}\right) \tag{45}$$

and the energy integral, $I_{\nu\bar{\nu}}$, can be expressed in dimensionless form, to within errors of at most $\exp[-E_F(n)/kT]$, as

$$I_{\nu\bar{\nu}} = (kT)^8\int_0^\infty dy y^4\int_{-\infty}^{\infty}\left[\prod_{i=1}^{4}\frac{dx_i}{e^{x_i}+1}\right]\delta\left(\sum_{i=1}^{4}x_i - y\right) = \frac{1}{6}(kT)^8\int_0^\infty dy\frac{y^5(y^2+4\pi^2)}{e^y - 1} \tag{46}$$

with $y = \omega_\nu/kT$. To evaluate the integrals over the x_i's in the above, we employed a technique described by Baym and Pethick (1978).

If the second relation for $I_{\nu\bar{\nu}}$ in (46) is inserted into (44), one obtains

$$\epsilon_{nn} = \frac{32}{45}\frac{G^2g_A{}^2m_n{}^{*4}}{(2\pi)^9\hbar}\left(\frac{f}{m_\pi}\right)^4 p_F(n)F\left(\frac{m_\pi}{2p_F(n)}\right)(kT)^8\int_0^\infty dy\frac{y^5(y^2+4\pi^2)}{e^y - 1}\,. \tag{47}$$

This is a particularly useful form for ϵ_{nn} in that it yields an expression for the $nn\nu\bar{\nu}$ neutrino distribution function $f_{\nu\bar{\nu}}(y)$, defined by

$$\epsilon_{nn} \equiv \int d^3q_1d^3q_2f_{\nu\bar{\nu}}(y)\omega_\nu\,. \tag{48}$$

In particular, performing the integrals in (48) after inserting

$$1 = \int d\omega_\nu\delta(\omega_\nu - \omega_1 - \omega_2) \tag{49}$$

and comparing the result with (47), one finds

$$f_{\nu\bar{\nu}}(y) = \frac{16}{3}\frac{Gg_A{}^2 m_n{}^{*4}}{(2\pi)^{11}\hbar}\left(\frac{f}{m_\pi}\right)^4 p_{\rm F}(n) F\left[\frac{m_\pi}{2p_{\rm F}(n)}\right]\left(\frac{kT}{y}\right)\frac{y^2+4\pi^2}{e^y-1}\,. \tag{50}$$

In terms of this distribution function the average energy imparted to the neutrinos in the $nn\nu\bar{\nu}$ process is

$$\bar{\omega}_\nu = \frac{\int d^3q_1 d^3q_2 f_{\nu\bar{\nu}}(y)\omega_\nu}{\int d^3q_1 dq_2 f_{\nu\bar{\nu}}(y)} \approx 5.8kT\,. \tag{51}$$

Returning to the emissivity, evaluation of the integral in (47) yields

$$\epsilon_{nn} = \frac{41}{14175}\frac{G^2 g_A{}^2 m_n{}^{*4}}{2\pi\hbar}\left(\frac{f}{m_\pi}\right)^4 p_{\rm F}(n) F\left[\frac{m_\pi}{2p_{\rm F}(n)}\right](kT)^8\,. \tag{52}$$

The corresponding expressions for the group I and group II contributions to the $np\nu\bar{\nu}$ emissivity are

$$\epsilon_{np}{}^{(\rm I)} = \frac{82}{14175}\frac{G^2 g_A{}^2 m_n{}^{*2} m_p{}^{*2}}{2\pi\hbar}\left(\frac{f}{m_\pi}\right)^4 p_{\rm F}(e)\alpha_{\rm I}(kT)^8 \tag{53a}$$

and

$$\epsilon_{np}{}^{(\rm II)} = \frac{82}{14175}\frac{G^2 g_A{}^2 m_n{}^{*2} m_p{}^{*2}}{2\pi\hbar}\left(\frac{f}{m_\pi}\right)^4 p_{\rm F}(e)\alpha_{\rm II}(kT)^8\,, \tag{53b}$$

where the functions $\alpha_{\rm I}$ and $\alpha_{\rm II}$ are defined by

$$\alpha_{\rm I} \equiv F\left[\frac{m_\pi}{2p_{\rm F}(e)}\right] + 4\left(\frac{f}{m_\pi}\right)^{-2}(g-g')\left\{1-\frac{m_\pi}{2p_{\rm F}(e)}\tan^{-1}\left[\frac{2p_{\rm F}(e)}{m_\pi}\right]\right\} + 6\left(\frac{f}{m_\pi}\right)^{-4}(g-g')^2 \tag{54a}$$

and

$$\alpha_{\rm II} \equiv 2\left[\frac{p_{\rm F}{}^2(n)}{p_{\rm F}{}^2(n)+m_\pi{}^2}\right]^2 + 4\left(\frac{f}{m_\pi}\right)^{-2}(f'-g')\frac{p_{\rm F}{}^2(n)}{p_{\rm F}{}^2(n)+m_\pi{}^2} + 6\left(\frac{f}{m_\pi}\right)^{-4}(f'-g')^2\,, \tag{54b}$$

with F given by (45).

In $\alpha_{\rm II}$ we have retained only the zeroth order term in $p_{\rm F}(e)/p_{\rm F}(n)$. The total $np\nu\bar{\nu}$ emissivity is just the sum of (53a) and (53b). Note that since the energy integral associated with the $np\nu\bar{\nu}$ process is identical to that for the $nn\nu\bar{\nu}$ process, the average energies imparted to the neutrinos in the two processes will be the same.

For the Urca process, the emissivity calculation is greatly simplified if the triangle approximation discussed in the introduction is adopted. Within this approximation, only the neutron momenta appear in the momentum conserving delta function, so that after the angular and energy parts of the phase space have been separated, the electron and proton angular integrals can be performed immediately. Introducing the momentum transfer and evaluating the remaining angular integrals then yields

$$\epsilon_{\rm URCA} = 512\frac{G^2 g_A{}^2 m_n{}^{*3} m_p{}^*}{(2\pi)^9\hbar}\left(\frac{f}{m_\pi}\right)^4 p_{\rm F}(e)\alpha_{\rm URCA} I_{\rm URCA}\,, \tag{55}$$

where

$$\alpha_{\rm URCA} \equiv 2\left(\frac{p_{\rm F}{}^2(n)}{p_{\rm F}{}^2(n)+m_\pi{}^2}\right)^2 + 2\left(\frac{f}{m_\pi}\right)^{-2}[(g-g')+(f'-g')]\frac{p_{\rm F}{}^2(n)}{p_{\rm F}{}^2(n)+m_\pi{}^2} + 3\left(\frac{f}{m_\pi}\right)^{-4}[(g-g')^2+(f'-g')^2]\,; \tag{56}$$

and in dimensionless form, the Urca energy integral is

$$I_{\rm URCA} = (kT)^8\int_0^\infty dy y^3\int_{-\infty}^{\infty}\left[\prod_{i=1}^{5}\frac{dx_i}{e^{x_i}+1}\right]\delta\left(\sum_{i=1}^{5}x_i - y\right) = \frac{1}{24}(kT)^8\int_0^\infty dy\frac{y^3(y^4+10\pi^2y^2+9\pi^4)}{e^y+1}\,, \tag{57}$$

with, again, $y = \omega_\nu/kT$.

In analogy with $f_{\nu\bar{\nu}}$, a neutrino distribution function can be defined for the Urca process:

$$\epsilon_{\rm URCA} \equiv \int d^3q_\nu f_{\rm URCA}(y)\omega_\nu \,. \tag{58}$$

Using (57) in (55) and comparing the result with (58), we obtain for this function

$$f_{\rm URCA}(y) = \frac{32}{3}\frac{G^2 g_A{}^2 m_n{}^{*3} m_p{}^*}{(2\pi)^{10}\hbar}\left(\frac{f}{m_\pi}\right)^4 \alpha_{\rm URCA} p_{\rm F}(e)(kT)^4 \frac{y^4 + 10\pi^2 y^2 + 9\pi^4}{e^y + 1}\,; \tag{59}$$

and for the average energy imparted to the antineutrino in the Urca process

$$\bar{\omega}_\nu = \frac{\int d^3q_\nu f_{\rm URCA}(y)\omega_\nu}{\int d^3q_\nu f_{\rm URCA}(y)} \approx 4.7kT\,. \tag{60}$$

Finally, evaluating the integral in (57) yields

$$\epsilon_{\rm URCA} = \frac{11513}{60480}\frac{G^2 g_A{}^2 m_n{}^{*3} m_p{}^*}{2\pi\hbar}\left(\frac{f}{m_\pi}\right)^4 p_{\rm F}(e)\alpha_{\rm URCA}(kT)^8\,. \tag{61}$$

V. NUMERICAL RESULTS WITH AND WITHOUT CORRELATIONS

To obtain numerical results for the various neutrino emissivities, we first evaluate the auxiliary functions defined in the previous section. Using the results of §§ II and III (eqs. [7], [9b], and [20]) and expanding the quantity $p_{\rm F}{}^2(n)/(p_{\rm F}{}^2(n) + m_\pi{}^2)$ and its square in $(m_\pi/p_{\rm F}(n))^2$ and keeping only the lowest order terms reduces those functions to

$$\begin{aligned} F\left[\frac{m_\pi}{2p_{\rm F}(n)}\right] &\approx F[0.21(\rho/\rho_0)^{1/3}]\,,\\ \alpha_{\rm I} = F\left[\frac{m_\pi}{2p_{\rm F}(e)}\right] &\approx F[0.82(\rho/\rho_0)^{2/3}]\,,\\ \alpha_{\rm II} &\approx 1.52 - 0.57(\rho_0/\rho)^{2/3}\,,\\ \alpha_{\rm URCA} &\approx 1.76 - 0.63(\rho_0/\rho)^{2/3}\,. \end{aligned} \tag{62}$$

The density dependence exhibited in these expressions is quite weak: at $\rho = \rho_0$, we obtain

$$F\left(\frac{m_\pi}{2p_{\rm F}(n)}\right) \approx 0.59\,, \quad \alpha_{\rm I} \approx 0.11\,, \quad \alpha_{\rm II} \approx 0.95\,, \quad \alpha_{\rm URCA} \approx 1.13\,; \tag{63a}$$

while at $\rho = 5\rho_0$,

$$F\left(\frac{m_\pi}{2p_{\rm F}(n)}\right) \approx 0.74\,, \quad \alpha_{\rm I} \approx 0.49\,, \quad \alpha_{\rm II} \approx 1.33\,, \quad \alpha_{\rm URCA} \approx 1.54\,. \tag{63b}$$

Hence, within the expected range of neutron star densities, it is not an unreasonable approximation to simply use the values at nuclear matter density. Then the emissivities become, in cgs units:

$$\epsilon_{nn} = (7.8 \times 10^{19})\left(\frac{m_n{}^*}{m_n}\right)^4\left(\frac{\rho}{\rho_0}\right)^{1/3} T_9{}^8 \text{ ergs cm}^{-3}\text{ s}^{-1}\,, \tag{64a}$$

$$\epsilon_{np} = \epsilon_{np}{}^{\rm (I)} + \epsilon_{np}{}^{\rm (II)} = (7.5 \times 10^{19})\left(\frac{m_n{}^*}{m_n}\right)^2\left(\frac{m_p{}^*}{m_p}\right)^2\left(\frac{\rho}{\rho_0}\right)^{2/3} T_9{}^8 \text{ ergs cm}^{-3}\text{ s}^{-1}\,, \tag{64b}$$

$$\epsilon_{\rm URCA} = (2.7 \times 10^{21})\left(\frac{m_n{}^*}{m_n}\right)^3\left(\frac{m_p{}^*}{m_p}\right)\left(\frac{\rho}{\rho_0}\right)^{2/3} T_9{}^8 \text{ ergs cm}^{-3}\text{ s}^{-1}\,, \tag{64c}$$

where factors of 2 have been included in ϵ_{nn} and ϵ_{np} to account for μ-neutrino production and in $\epsilon_{\rm URCA}$ to account for the inverse Urca process, $T_9 = T/10^9$ K, and equations (7) and (9b) for the neutron and electron Fermi momenta have been used.

These results do not include short-range correlations in the OPE interaction. To study the influence of such correlations on the emissivities, we first adopted the form (14) for $\tilde{V}(k)$, which is based on a simple position-space cutoff for the correlation function. The major disadvantage of this form is that it necessitates numerical evaluation of the phase space integrals.

TABLE 1

RATIOS OF EMISSIVITIES COMPUTED WITH THE CORRELATED AND UNCORRELATED OPE POTENTIAL AT NUCLEAR MATTER DENSITY FOR THE $nn\nu\bar{\nu}$, $np\nu\bar{\nu}$, AND URCA PROCESSES AS A FUNCTION OF d

d(fm)	R_{nn}	R_{np}	$R_{\rm Urca}$
0.55	0.68	0.56	0.78
0.7	0.56	0.66	0.68
0.85	0.44	0.77	0.58

Our results with this form are summarized in Table 1. Here the ratio of the emissivities computed with the correlated and uncorrelated OPE interactions at nuclear matter density is shown for each neutrino process as a function of the cutoff distance d. Note that the ratios do not depend strongly on the cutoff within a reasonable range of values. Also, of the three emissivities, that of the $nn\nu\bar{\nu}$ process is influenced most by correlations. This is just what is expected since the $nn\nu\bar{\nu}$ process involves the largest momentum transfers and hence smallest inter-nucleon distances.

In order to check the results in Table 1 and to determine whether the effects of correlations depend on the particular form of the correlated potential, we have recomputed the $nn\nu\bar{\nu}$ emissivity using the form (17) for $\tilde{V}(k)$. With this form the phase space integrals can be evaluated analytically in the same manner as for the uncorrelated interaction. We find that the resulting emissivity is a factor of 0.56 less than that without correlations at nuclear matter density and for $q_c = m_\omega \approx 780$ MeV. At $\rho = 5\rho_0$, the reduction factor is 0.36. These results indicate that correlation effects are not strongly dependent on the density. Moreover, as comparison with Table 1 reveals, they are relatively insensitive to the form of the correlated interaction.

Using the results of Table 1 for $d = 0.7$ fm, which is typically the range of correlation functions, we obtain for the neutrino emissivities, including correlations:

$$\epsilon_{nn} = (4.4 \times 10^{19})\left(\frac{m_n^*}{m_n}\right)^4\left(\frac{\rho}{\rho_0}\right)^{1/3} T_9{}^8 \text{ ergs cm}^{-3}\text{ s}^{-1}\,, \tag{65a}$$

$$\epsilon_{np} = (5.0 \times 10^{19})\left(\frac{m_n^*}{m_n}\right)^2\left(\frac{m_p^*}{m_n}\right)^2\left(\frac{\rho}{\rho_0}\right)^{2/3} T_9{}^8 \text{ ergs cm}^{-3}\text{ s}^{-1}\,, \tag{65b}$$

$$\epsilon_{\rm URCA} = (1.8 \times 10^{21})\left(\frac{m_n^*}{m_n}\right)^3 \frac{m_p^*}{m_p}\left(\frac{\rho}{\rho_0}\right)^{2/3} T_9{}^8 \text{ ergs cm}^{-3}\text{ s}^{-1}\,. \tag{65c}$$

Comparison of the different contributions to the α-functions, given by equations (54) and (56), indicates that these emissivities arise almost entirely from the OPE contributions to the matrix elements. The Landau contributions merely reduce the emissivities somewhat through the interference terms in the squared matrix elements.

If, somewhat arbitrarily, we adopt the Bäckman, Källman, and Sjöberg (1973) value 0.8 for m_n^*/m_n and the same value for m_p^*/m_p, the emissivities become

$$\epsilon_{nn} = (1.8 \times 10^{19})(\rho/\rho_0)^{1/3} T_9{}^8 \text{ ergs cm}^{-3}\text{ s}^{-1}\,, \tag{66a}$$

$$\epsilon_{np} = (2.0 \times 10^{19})(\rho/\rho_0)^{2/3} T_9{}^8 \text{ ergs cm}^{-3}\text{ s}^{-1}\,, \tag{66b}$$

$$\epsilon_{\rm URCA} = (7.4 \times 10^{20})(\rho/\rho_0)^{2/3} T_9{}^8 \text{ ergs cm}^{-3}\text{ s}^{-1}\,, \tag{66c}$$

We believe these to be reasonable estimates of the Urca and neutral-current bremsstrahlung emissivities within the present approximations and in the absence of better effective mass values. It should be emphasized, however, that the effective mass dependences exhibited in equations (65) are quite strong.

For a neutron star of 10 km radius with constant density equal to ρ_0, equations (66) yield the following luminosities:

$$L_{nn}{}^{(\nu)} = (7.5 \times 10^{37}) T_9{}^8 \text{ ergs s}^{-1}\,, \tag{67a}$$

$$L_{np}{}^{(\nu)} = (8.4 \times 10^{37}) T_9{}^8 \text{ ergs s}^{-1}\,, \tag{67b}$$

$$L_{\rm URCA}{}^{(\nu)} = (3.1 \times 10^{39}) T_9{}^8 \text{ ergs s}^{-1}\,. \tag{67c}$$

VI. EXCHANGE TERMS

Thus far, we have ignored the antisymmetrization requirement on the nucleon wave functions in the OPE matrix elements. To satisfy this requirement, one must consider, in addition to the diagrams illustrated in Figures

1–3, a set of diagrams analogous to the direct diagrams but with the outgoing nucleon lines (or, alternatively, the incoming nucleon lines) interchanged. In the matrix elements these exchange diagrams must be summed coherently with the direct diagrams and hence can significantly affect the resulting emissivities.

To ascertain the importance of exchange terms, we have recomputed the neutrino emissivities for all three processes including both the direct and exchange OPE contributions to the matrix elements but omitting the Landau contributions. As discussed in the previous section, the Landau contributions do not affect the emissivities in a major way. Hence, their omission is not serious.

For the $nn\nu\bar{\nu}$ process, the direct and exchange contributions to the matrix element differ only in the arrangement of the spinors, the value of the nucleon momentum transfer, and the overall sign. In particular, the spinors χ_1 and χ_4 are joined in the exchange terms instead of χ_1 and χ_3 as in the direct terms. The two momentum transfers differ by

$$\boldsymbol{k}'' - \boldsymbol{k} = \boldsymbol{p}_3 - \boldsymbol{p}_4\,, \tag{68}$$

where $\boldsymbol{k}''$ is the exchange term momentum transfer, and the neutrino momentum has been neglected. Because of the close relationship between the direct and exchange terms, the exchange contribution to the matrix element can be obtained immediately from the direct contribution. Adding the two contributions and summing over spins yields

$$\sum_{\text{spins}} |M_{nn}|^2 = 64G^2 g_A{}^2 \left(\frac{f}{m_\pi}\right)^4 \left[\left(\frac{k^2}{k^2 + m_\pi{}^2}\right)^2 + \left(\frac{k''^2}{k''^2 + m_\pi{}^2}\right)^2 + \frac{k^2 k''^2}{(k^2 + m_\pi{}^2)(k''^2 + m_\pi{}^2)}\right]\left(\frac{\omega_1\omega_2}{\omega^2}\right), \tag{69}$$

where we have used the fact that in the $nn\nu\bar{\nu}$ process $\boldsymbol{k}$ and $\boldsymbol{k}''$ are orthogonal when the neutrino momenta are neglected. Note in this expression that the direct and exchange contributions interfere *constructively* in the spin-summed matrix element.

The relationship between the direct and exchange terms is not so simple for the $np\nu\bar{\nu}$ and Urca processes because of the different isospin projections of the outgoing nucleons. Nevertheless, it is still a simple matter to derive the exchange contributions to the matrix elements from the direct contributions. For the total spin-summed matrix elements, we obtain

$$\sum_{\text{spins}} |M_{np}{}^{(\mathrm{I})}|^2 = 64G^2 g_A{}^2 \left(\frac{f}{m_\pi}\right)^4 \left[\left(\frac{k^2}{k^2 + m_\pi{}^2}\right)^2 + 2\left(\frac{k''^2}{k''^2 + m_\pi{}^2}\right)^2 - \frac{k^2 k''^2}{(k^2 + m_\pi{}^2)(k''^2 + m_\pi{}^2)}\right]\frac{\omega_1\omega_2}{\omega^2} \tag{70}$$

and

$$\sum_{\text{spins}} |M_{\mathrm{URCA}}|^2 = 256G^2 g_A{}^2 \left(\frac{f}{m_\pi}\right)^4 \left[\left(\frac{k^2}{k^2 + m_\pi{}^2}\right)^2 + \left(\frac{k''^2}{k''^2 + m_\pi{}^2}\right)^2 - \frac{k^2 k''^2 - \frac{1}{2}(\boldsymbol{k}\cdot\boldsymbol{k}'')^2 - \frac{1}{4}(\boldsymbol{k}\times\boldsymbol{k}'')^2}{(k^2 + m_\pi{}^2)(k''^2 + m_\pi{}^2)}\right]\frac{\omega_e\omega_\nu}{\omega^2}\,, \tag{71}$$

where we have omitted terms involving the neutrino momenta and in the $np\nu\bar{\nu}$ matrix element, we have again $\boldsymbol{k}\cdot\boldsymbol{k}'' = 0$. The group II contribution to the $np\nu\bar{\nu}$ matrix element is just (70) with k and k'' interchanged.

Evaluation of the phase space integrals is straightforward for the Urca process if the triangle approximation is adopted. Within this approximation, momentum conservation requires

$$\boldsymbol{k} = \boldsymbol{p}_1 - \boldsymbol{p}_3 = -\boldsymbol{p}_2 \tag{72a}$$

and

$$\boldsymbol{k}'' = \boldsymbol{k} + \boldsymbol{p}_3 = \boldsymbol{p}_1\,. \tag{72b}$$

Hence,

$$k^2 = k''^2 = p_{\mathrm{F}}{}^2(n)\,, \tag{73a}$$

$$\boldsymbol{k}\cdot\boldsymbol{k}'' = \tfrac{1}{2}p_{\mathrm{F}}{}^2(n)\,, \tag{73b}$$

and the matrix element given by (71) reduces to

$$\sum_{\text{spins}} |M_{\mathrm{URCA}}|^2 = 256G^2 g_A{}^2 \left(\frac{f}{m_\pi}\right)^4 \frac{21}{16}\left[\frac{p_{\mathrm{F}}{}^2(n)}{p_{\mathrm{F}}{}^2(n) + m_\pi{}^2}\right]^2 \frac{\omega_e\omega_\nu}{\omega^2}\,. \tag{74}$$

Integrating this matrix element over the allowed phase space, we find for the ratio of the Urca emissivities with and without exchange terms (and without the Landau contributions)

$$\epsilon_{\mathrm{URCA}}{}^{(\mathrm{exch})}/\epsilon_{\mathrm{URCA}}{}^{(\mathrm{no\ exch})} = \tfrac{21}{16} \approx 1.3\,. \tag{75}$$

Thus, inclusion of exchange terms in the Urca matrix element increases the resulting emissivity, but only slightly.

For the $nn\nu\bar{\nu}$ process the phase space integrals can be done analytically without further approximations. For the ratio of the $nn\nu\bar{\nu}$ emissivities with and without exchange terms we find

$$\epsilon_{nn}^{(\text{exch})}/\epsilon_{nn}^{(\text{no exch})} = \left\{F_{\text{ex}}\left[\frac{m_\pi}{2p_{\text{F}}(n)}\right]\right\}\Big/\left\{F\left[\frac{m_\pi}{2p_{\text{F}}(n)}\right]\right\} \approx 2.8\,, \tag{76}$$

where $F_{\text{ex}}(x)$ is given by

$$F_{\text{ex}}(x) = 3 - 5 \times \tan^{-1}\left(\frac{1}{x}\right) + \frac{x^2}{1+x^2} + \frac{x^2}{(1+2x^2)^{1/2}}\tan^{-1}\left[\frac{(1+2x^2)^{1/2}}{x^2}\right], \tag{77}$$

and $F(x)$ by (45).

To perform the phase space integrals for the $np\nu\bar{\nu}$ process we find it convenient to simplify the integrand. Since k and k'' in (70) are constrained by the inequalities

$$0 \le k \le 2p_{\text{F}}(e) \tag{78}$$

and

$$p_{\text{F}}(n) - p_{\text{F}}(e) \le k'' \le p_{\text{F}}(n) + p_{\text{F}}(e)\,, \tag{79}$$

respectively, and $p_{\text{F}}(n)$ is much larger than $p_{\text{F}}(e)$, we can replace k'' by $p_{\text{F}}(n)$ in (70) and expand the matrix element in $m_\pi^2/[m_\pi^2 + p_{\text{F}}^2(n)]$ (≈ 0.14 at $\rho = \rho_0$). If the terms above first order in this expansion are dropped, we obtain

$$\sum_{\text{spins}} |M_{np}^{(\text{I})}|^2 = 64G^2 g_A^2\left(\frac{f}{m_\pi}\right)^4\left[1 - 2\frac{m_\pi^2}{m_\pi^2 + p_{\text{F}}^2(n)}\left(1 + \frac{m_\pi^2}{m_\pi^2+k^2}\right) + \left(\frac{m_\pi^2}{m_\pi^2+k^2}\right)^2\right]\frac{\omega_1\omega_2}{\omega^2}\,. \tag{80}$$

The phase space integrals can now be readily evaluated. Since the group II diagrams yield an identical contribution to the emissivity, they are included by just doubling the group I result. Our result for the $np\nu\bar{\nu}$ process is then (excluding Landau contributions)

$$\epsilon_{np}^{(\text{exch})}/\epsilon_{np}^{(\text{no exch})} = 2\frac{\alpha_{\text{ex}}}{\alpha_{\text{I}}' + \alpha_{\text{II}}'} \approx 1.3\,, \tag{81}$$

where α_{I}' and α_{II}' are given by equations (54) with the Landau parameters set to zero and α_{ex} is given by

$$\alpha_{\text{ex}} = 1 - 2\frac{m_\pi^2}{m_\pi^2 + p_{\text{F}}^2(n)} + \left[\frac{1}{2} - 2\frac{m_\pi^2}{m_\pi^2 + p_{\text{F}}^2(n)}\right]\frac{m_\pi}{2p_{\text{F}}(n)}\tan^{-1}\left[\frac{2p_{\text{F}}(e)}{m_\pi}\right] + \frac{m_\pi^2}{2[4p_{\text{F}}^2(e) + m_\pi^2]}\,. \tag{82}$$

The numbers on the right hand sides of (76) and (81) were obtained at nuclear matter density, but neither ratio is strongly density dependent.

Our results reveal that both the $nn\nu\bar{\nu}$ and $np\nu\bar{\nu}$ emissivities are increased by the inclusion of exchange terms in the matrix elements. For the $np\nu\bar{\nu}$ process, the increase is moderate, as for the Urca process. By contrast, the increase in the $nn\nu\bar{\nu}$ emissivity is quite sizable. We believe that this is primarily due to the different isospin structures of these processes. In particular the $nn\nu\bar{\nu}$ process clearly has only a $T = 1$ contribution, which is pure tensor, while the $np\nu\bar{\nu}$ and Urca processes have both $T = 1$ and $T = 0$ tensor contributions (the weak axial current flips the nucleon isospin state from $T = 1$ to $T = 0$) and contributions from the spin-spin interaction. The tensor interaction is greatly enhanced by exchange in the $T = 1$ channel but relatively unchanged by exchange in the $T = 0$ channel. On the other hand, the contribution from the spin-spin interaction is reduced by the inclusion of exchange. Hence, for the $np\nu\bar{\nu}$ and Urca emissivities there is a competition between the increase in the tensor contribution and the decrease in the spin-spin one so that these emissivities should be only slightly affected by the inclusion of exchange. The $nn\nu\bar{\nu}$ emissivity, however, which arises solely from the tensor interaction and is, moreover, pure $T = 1$, should be increased substantially. Our results confirm these expectations.

VII. ρ-EXCHANGE

The Landau parameters discussed in § III describe only part of the short-range contributions to the NN-interaction—in particular, those terms associated with the central, $\boldsymbol{\tau}\cdot\boldsymbol{\tau}$, $\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}$, and $(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma})(\boldsymbol{\tau}\cdot\boldsymbol{\tau})$ spin-isospin operators. In addition to these terms, there is a short-range contribution of tensor character arising from the exchange of ρ-mesons. This short-range tensor interaction has the opposite sign of the OPE tensor interaction and hence could significantly decrease the neutrino emissivities obtained with the OPE tensor alone.

To check this possibility, we calculated the $nn\nu\bar{\nu}$ emissivity with both π- and ρ-exchange included in the NN-interaction. The $nn\nu\bar{\nu}$ emissivity is, as we saw in § V, the process which is most sensitive to the short-range behavior of the interaction. Hence, we expect that the effects of ρ-exchange should be largest for this emissivity.

Using the identity

$$(\boldsymbol{\sigma}^{(1)} \times \boldsymbol{k})\cdot(\boldsymbol{\sigma}^{(2)} \times \boldsymbol{k}) \equiv k^2\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)} - \boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{k}\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{k}\,, \tag{83}$$

the ρ-exchange interaction can be cast in the form

$$V_\rho = \left(\frac{f_\rho}{m_\rho}\right)^2\left(\frac{-1}{k^2 + m_\rho^2}\right)[k^2\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)} - \boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{k}\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{k}]\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}\,, \tag{84}$$

where $m_\rho \approx 770$ MeV is the ρ mass (treating the ρ as a sharp resonance), and f_ρ, the p-wave ρN coupling constant, is given by

$$f_\rho^2 = (m_\rho/2m_N)^2 g_\rho^2(1 + \kappa)^2 \approx 38 \tag{85}$$

in the absence of vertex renormalization mechanisms. To obtain the numerical value for f_ρ^2, we used $g_\rho^2/4\pi = 0.5$ and the value, $\kappa = 5$.

The $\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}$ term in (84) does not contribute to the $nn\nu\bar{\nu}$ matrix element as explained in § IV, and can hence be dropped. The remaining term, containing the tensor, has the same spin-isospin structure as the uncorrelated OPE interaction. Thus, inclusion of ρ-exchange in the NN interaction alters the $nn\nu\bar{\nu}$ matrix element only in the k-dependence. After summing over spins, we obtain

$$\sum_{\text{spins}} |M_{nn}|^2 = 64G^2 g_A^2\left(\frac{f}{m_\pi}\right)^4\left[\left(\frac{k^2}{k^2 + m_\pi^2}\right)^2 + C_\rho^2\left(\frac{k^2}{k^2 + m_\rho^2}\right)^2 - 2C_\rho\frac{k^4}{(k^2 + m_\pi^2)(k^2 + m_\rho^2)}\right]\frac{\omega_1\omega_2}{\omega^2}\,, \tag{86}$$

where

$$C_\rho \equiv \frac{(f_\rho/m_\rho)^2}{(f/m_\pi)^2} \approx 1.25\,. \tag{87}$$

This yields for the ratio of emissivities with and without ρ-exchange

$$\frac{\epsilon_{nn}^{(\pi+\rho)}}{\epsilon_{nn}^{(\pi)}} = 1 + C_\rho^2\frac{F[m_\rho/2p_F(n)]}{F[m_\pi/2p_F(n)]} - 2C_\rho\frac{F[m_\pi/2p_F(n), m_\rho/2p_F(n)]}{F(m_\pi/2p_F(n))}\,, \tag{88}$$

where $F(x)$ is given by (45) and $F(x_1, x_2)$ by

$$F(x_1, x_2) \equiv 1 + \frac{x_1^3}{x_2^2 - x_1^2}\tan^{-1}\left(\frac{1}{x_1}\right) + \frac{x_2^3}{x_1^2 - x_2^2}\tan^{-1}\left(\frac{1}{x_2}\right)\,. \tag{89}$$

At $\rho = \rho_0$, (88) reduces to

$$\epsilon_{nn}^{(\pi+\rho)}/\epsilon_{nn}^{(\pi)} = 0.46\,; \tag{90a}$$

and at $\rho = 5\rho_0$ to

$$\epsilon_{nn}^{(\pi+\rho)}/\epsilon_{nn}^{(\pi)} = 0.25\,. \tag{90b}$$

Inclusion of correlations of the form (16) in both the OPE and ρ-exchange interactions alters these results only slightly: instead of (90), we get

$$\epsilon_{nn}^{(\pi+\rho)}/\epsilon_{nn}^{(\pi)} = 0.39 \tag{91a}$$

at $\rho = \rho_0$ and

$$\epsilon_{nn}^{(\pi+\rho)}/\epsilon_{nn}^{(\pi)} = 0.21 \tag{91b}$$

at $\rho = 5\rho_0$. Thus, for the $nn\nu\bar{\nu}$ emissivity correlation effects on the OPE and ρ-exchange tensor contributions approximately cancel.

Equations (90), or alternatively (91), reveal that ρ-exchange affects the $nn\nu\bar{\nu}$ emissivity rather substantially and in a manner that is somewhat density dependent. It is interesting that the influence of exchange terms on this emissivity, as given by equation (76), and that of ρ-exchange largely offset each other. This indicates that the $nn\nu\bar{\nu}$ emissivity is well represented by the results of § V with neither exchange terms nor ρ-exchange included in the matrix element. For the other neutrino processes, we found in the previous section that exchange terms influence the emissivities quite moderately. We expect, furthermore, that the influence of ρ-exchange is quite moderate for these processes, as discussed above. Hence, both the $np\nu\bar{\nu}$ and Urca emissivities should also be well represented by the results of § V.

VIII. DISCUSSION AND CONCLUSIONS

Equations (66c) for the Urca emissivity and (66a) for the $nn\nu\bar{\nu}$ emissivity exceed the previous results, obtained by Bahcall and Wolf (1965) and Flowers, Sutherland, and Bond (1975), respectively, by nearly an order of magnitude at nuclear matter density. We believe this large difference to be due to the long-range and tensor character of the

OPE term included in our NN interaction. In partial support of this belief, we note, with regard to the $nn\nu\bar{\nu}$ emissivity, that the Flowers *et al.* result exhibits no contribution from lowest order terms in the nucleon propagator.

In contrast to the $nn\nu\bar{\nu}$ and Urca results, equation (66b) for the $np\nu\bar{\nu}$ emissivity is less than the Flowers *et al.* result. This is rather surprising, particularly since the Flowers *et al.* result does not contain contributions from group II diagrams (as evidenced by the collection of T-matrices in their eq. [51]), which provide the largest contribution to ϵ_{np} in our calculation. However, Flowers *et al.* employ neutron-neutron Landau parameters to describe the neutron-proton Landau interaction. In our notation the appropriate combination of parameters for the latter is $g - g'$, whereas $g + g'$ is the neutron-neutron combination. Since g and g' are nearly equal, use of the neutron-neutron combination to describe the neutron-proton interaction will clearly result in a large overestimate of the $np\nu\bar{\nu}$ emissivity.

Note that the Born terms considered here, for a given effective NN-interaction, include all contributions of order T^8 to the neutral current processes. The lowest order corrections are of higher order in the temperature, and hence small compared to the leading terms. For the Urca process the situation is different, since for this process the correction terms are of the same order in the temperature as the Born term. In particular the rescattering term, where the nucleons interact both before and after the weak interaction, could be important. This is essentially the process considered by Bahcall and Wolf and recently by Sawyer and Soni (1978). However, since their results are roughly an order of magnitude smaller than ours, it seems that the Urca emissivity is also dominated by the OPE interaction. Hence, we believe that inclusion of the rescattering terms would not change our results significantly.

In the absence of nucleon superfluidity, the present result for ϵ_{URCA} exceeds ϵ_{nn} and ϵ_{np} by a factor of 40. Hence, the Urca process will dominate the production of neutrinos in normal fluid neutron stars without π^- condensates. The situation is somewhat different if the interior nucleons are superfluid. In that case, limitations on the phase space available to the participating nucleons severely hinder the neutrino emission processes. Qualitatively, superfluidity reduces ϵ_{np} and ϵ_{URCA} by a factor $\exp[-(\Delta_n(0) + \Delta_p(0))/kT]$ [for $\Delta_p(0) > \Delta_n(0)$], where $\Delta_n(0)$ and $\Delta_p(0)$ are the zero-temperature gaps in the neutron and proton spectra at the Fermi surface. For ϵ_{nn}, the reduction factor is $\exp[-2\Delta_n(0)/kT]$. Clearly, in regions where protons but not neutrons are superfluid, ϵ_{np} and ϵ_{URCA} will both be suppressed relative to ϵ_{nn}, so that the latter quantity will dominate the total neutrino emissivity.

A number of authors have studied the influence of nucleon superfluidity on neutrino emissivities in considerable detail. These include Wolf (1966), Itoh and Tsuneto (1972), and Malone (1974).

Up to now it has been assumed that neutrinos, once produced, escape from neutron stars without interacting further with the neutron star matter. Thus, the luminosities given by equations (67) were obtained by simply multiplying the emissivities evaluated at an average value of the density by the stellar volume. At moderate temperatures—a few times 10^9 K and lower—this procedure is probably valid. At higher temperatures, however, neutrino absorption mechanisms may be strong enough to seriously hinder the exodus of neutrinos from the star, as Sawyer and Soni (1977) first pointed out. Such absorption would significantly lower neutrino luminosities by restricting the volume contributing to the luminosities to a thin spherical shell lying just below the surface.

To estimate the magnitude of absorption effects, one can use the further observation of Sawyer and Soni (1977) that neutrino absorption and emission mechanisms are closely related. This relationship is particularly apparent within our formalism. Consider, for example, the process.

$$\nu + n + n \rightarrow n + p + e^- \,, \tag{92}$$

which is just the absorption analog of the Urca process and, as such, is one of the primary absorption mechanisms in neutron star matter. The contribution to the neutrino mean free path from this process is given by

$$\lambda_\nu^{-1} = 2\pi \int \left[\prod_{i=1}^{4} \frac{d^3p_i}{(2\pi)^3}\right] \frac{d^3q_e}{(2\pi)^3 2\omega_e}\, \delta(E_f - E_{in})(2\pi)^3 \delta^{(3)}(\boldsymbol{p}_f - \boldsymbol{p}_{in}) \left(\frac{1}{2}\right) \frac{1}{2\omega_\nu} \left(\sum_{\text{spins}} |M|^2\right) \mathscr{S}_\lambda \,, \tag{93}$$

where the factor of $\frac{1}{2}$ is a symmetry factor, and $\mathscr{S}_\lambda$ is the appropriate combination of Fermi-Dirac functions. (Note that $\mathscr{S}_\lambda$ has two terms: the usual term and a blocking term containing contributions to λ_ν^{-1} from Pauli inhibition of the inverse process [see Baym and Pethick 1978].)

If we ignore the neutrino energy in the nucleon propagator, the matrix element in the above is just the Urca matrix element. Using that observation, (93) can be reduced to the simple form

$$\lambda_\nu^{-1} = (2\pi)^3 c^{-1}(e^{-y} + 1) f_{\text{URCA}}(-y) = (2\pi)^3 c^{-1}(e^{y} + 1) f_{\text{URCA}}(y) \,, \tag{94}$$

which, upon substitution of (59) for f_{URCA}, becomes

$$\lambda_\nu(y) = (8.2 \times 10^{14})(\rho_0/\rho)^{2/3} T_9^{-4}(y^4 + 10\pi^2 y^2 + 9\pi^4)^{-1} \text{ cm} \tag{95}$$

with $y = \omega_\nu/kT$. For a neutron star of 10 km radius with constant density equal to nuclear matter density, this yields

$$\lambda_\nu(y)/R = (8.2 \times 10^{8}) T_9^{-4}(y^4 + 10\pi^2 y^2 + 9\pi^4)^{-1} \,. \tag{96}$$

Finally, choosing $y = 4.7$, the average Urca process neutrino energy, we obtain

$$\lambda_\nu/R = (2.3 \times 10^5)T_9^{-4}\,. \tag{97}$$

Equation (97) reveals that a neutrino of average energy produced in the Urca process will have a mean free path less than the stellar radius if T_9 exceeds 22. Thus, for temperatures of order 10^{10} K and above, absorption mechanisms will significantly affect neutrino luminosities. This actually represents an upper limit since only one contribution to the mean free path has been considered.

Let us now briefly consider the neutrino luminosity from a star for which $\lambda_\nu \ll R$, keeping only the Urca contribution. For a neutron star with constant density and $\lambda_\nu > R$ the luminosity can be expressed as

$$L_{\rm URCA} = \tfrac{4}{3}\pi R^3 \epsilon_{\rm URCA} = \tfrac{4}{3}\pi R^3 \int d^3q_\nu 2 f_{\rm URCA}(y)\omega_\nu \tag{98}$$

where the extra factor of 2 takes account of the inverse Urca process. For $\lambda_\nu \ll R$, only a surface layer of thickness λ_ν contributes to the luminosity. Thus, we must replace (98) by

$$L_{\rm URCA} = \tfrac{1}{4}(8\pi R^2)\int d^3q_\nu f_{\rm URCA}(y)\omega_\nu\lambda_\nu(y)\,, \tag{99}$$

which, upon substitution of (94) for λ_ν, yields

$$L_{\rm URCA} = \frac{1}{4}(8\pi R^2 c)\int \frac{d^3q_\nu}{(2\pi)^3}\frac{\omega_\nu}{\exp(\omega_\nu/kT)+1}\,. \tag{100}$$

The integrand of (100) is just a Fermi-Dirac energy distribution function for particles with zero chemical potential. Hence, the Pauli principle is automatically satisfied in this expression.

Equation (100) can also be expressed in the form

$$L_{\rm URCA} = \frac{1}{4}(8\pi R^2 c)\int \frac{dy}{2\pi^2}\frac{y^3}{e^y+1}(kT_s)^4\,, \tag{101}$$

which is a blackbody expression for neutrinos with an effective surface temperature T_s. This is exactly what one expects for $\lambda_\nu \ll R$, since in that case the neutron star is completely saturated with neutrinos. A more complete study of neutrino transport at high temperatures has recently been carried out by Sawyer and Soni (1978).

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