Observations of Compact Stars

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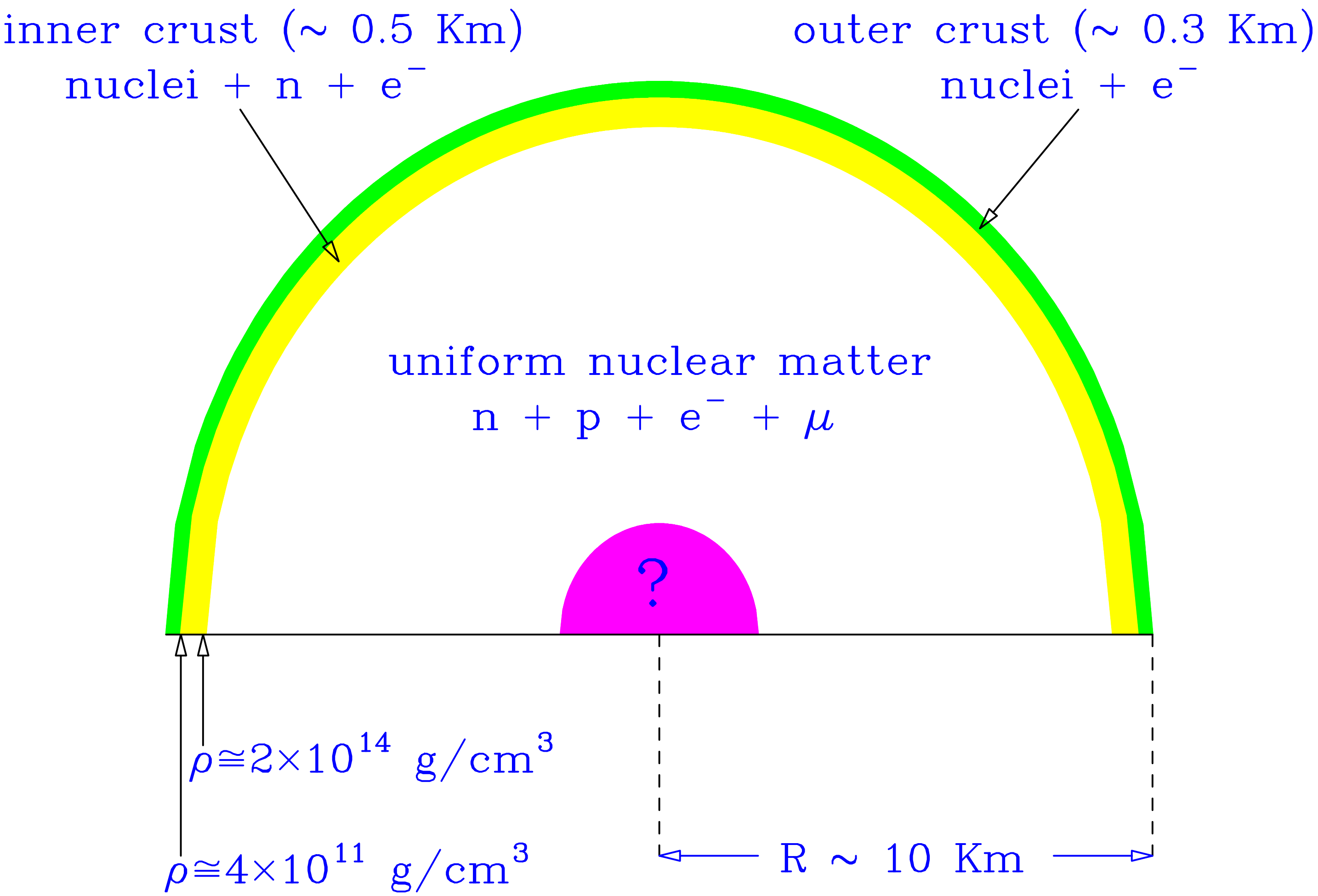
Overview of Neutron Star Structure

- recall: $\rho_0 \approx 0.16\ \mathrm{nucl/fm}^3 = 2.67 \times 10^{14}\ \mathrm{g/cm}^3$



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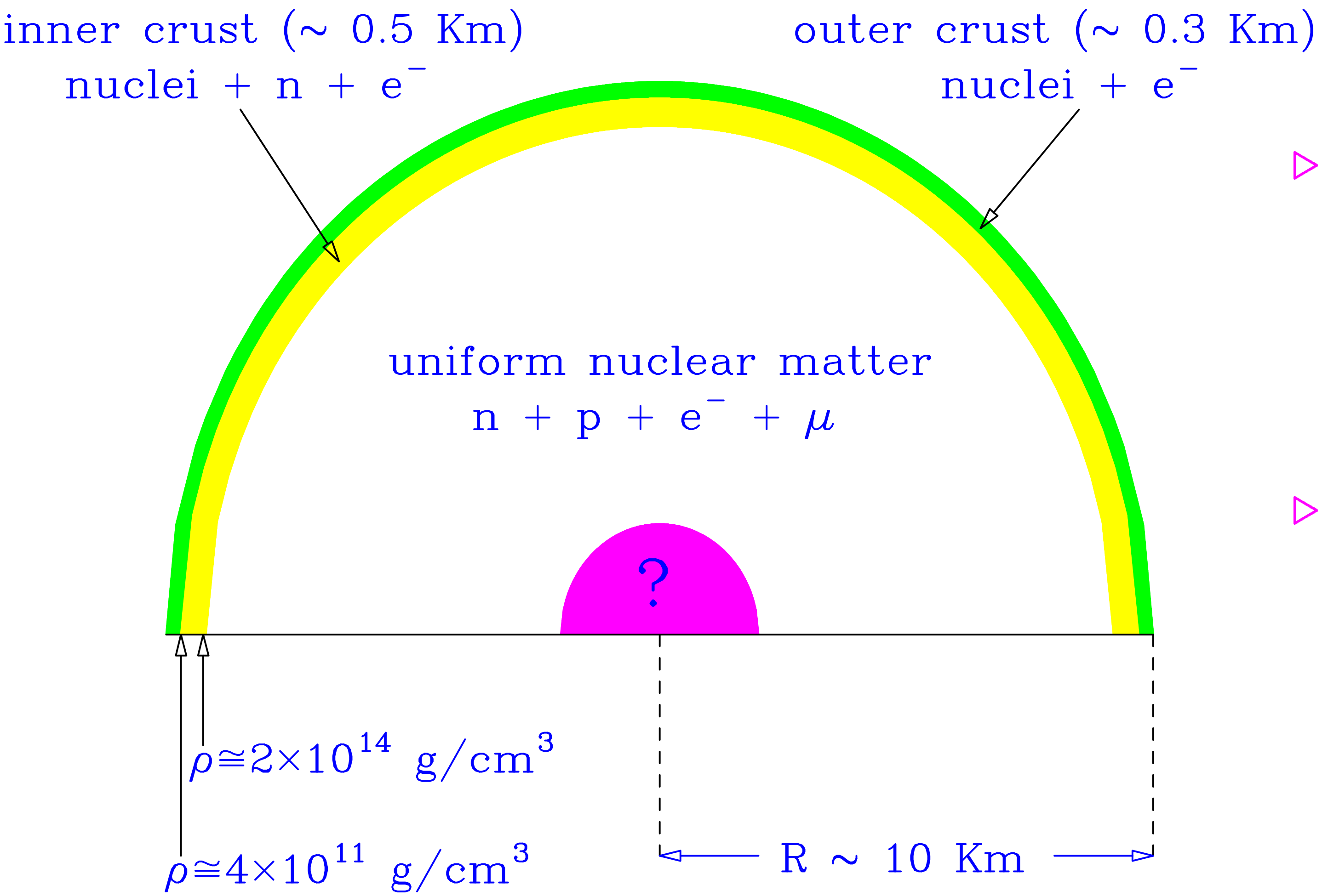
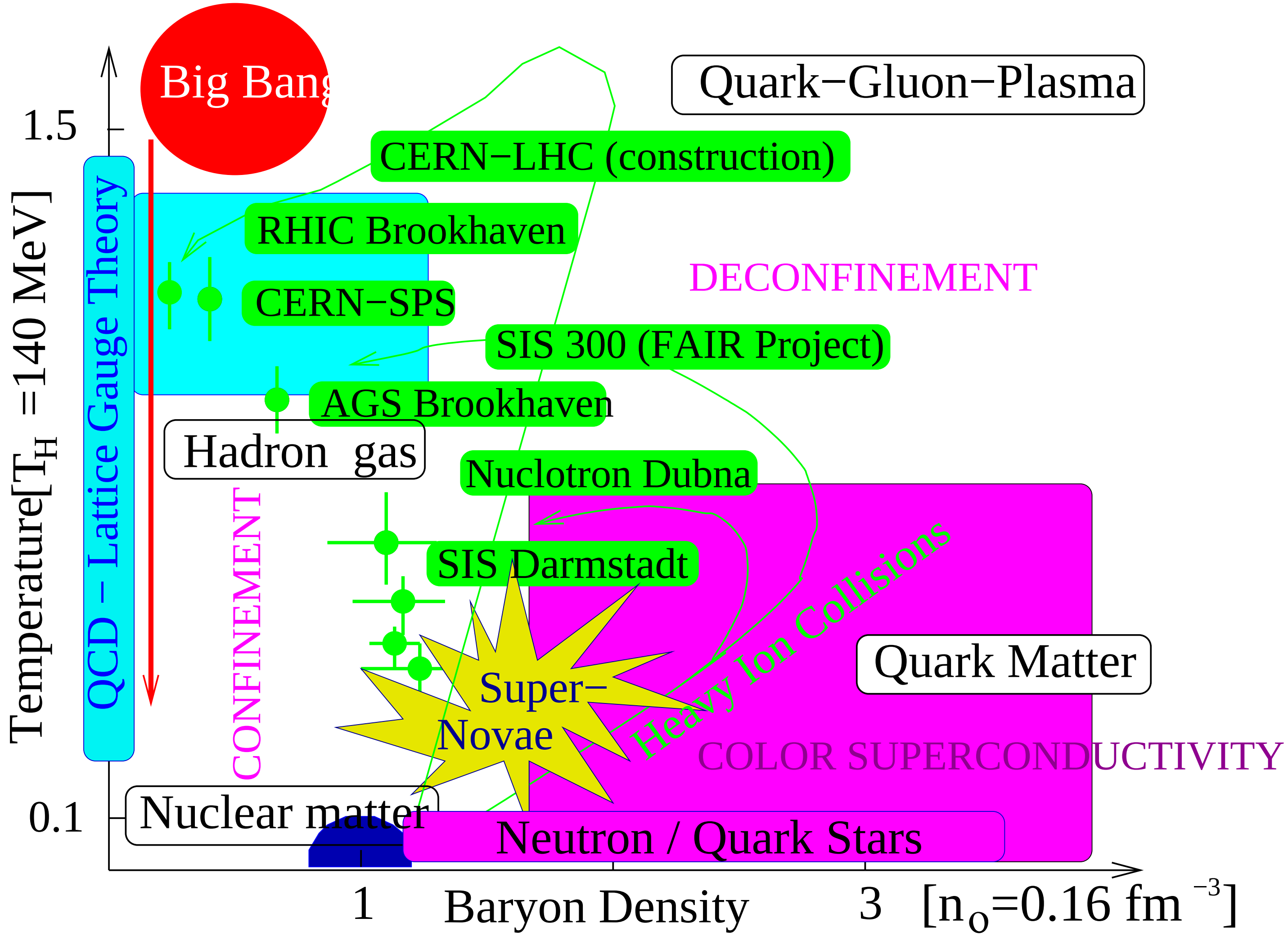


- ? : hyperons, π-condensate, K-condensate, quark matter . . .
- note: most of the neutron star mass is in the region $\rho > \rho_0$

QCD phase diagram



EOS and properties of nonrotating neutron stars

- given the EOS, mass and radius of a nonrotating star can be obtained from the Tolman-Oppenheimer-Volkov (TOV) equations (hydrostatic equilibrium + Einstein eqs)

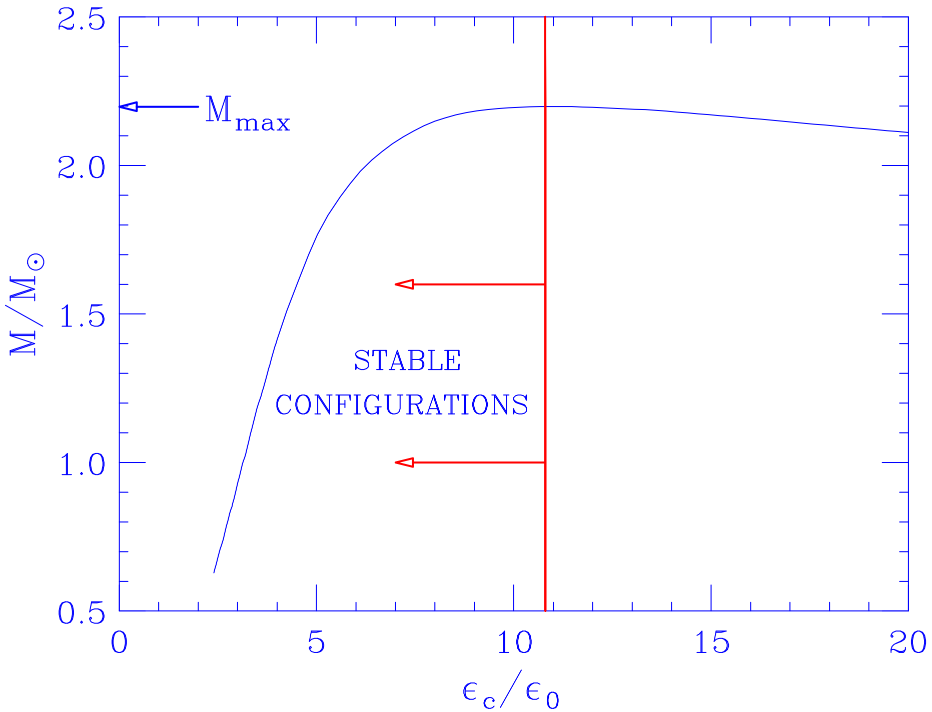
$$\frac{dP(r)}{dr} = -G\,\frac{[\epsilon(r) + P(r)/c^2]\,[M(r) + 4\pi r^2 P(r)/c^2]}{r^2\,[1 - 2GM(r)/rc^2]}$$

$$M(r) = 4\pi \int_0^r r'^2 dr' \epsilon(r') \quad , \quad \epsilon(r=0) = \epsilon_c$$

- solving TOV equations one obtains a set of neutron star configurations, characterized by the radius R, defined through $P(R) = 0$, and the mass $M = M(R)$

Maximum neutron star mass

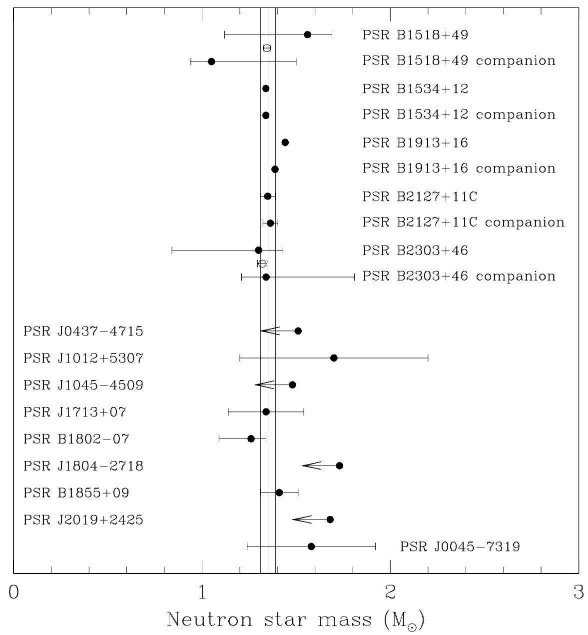
- typical mass-central energy-density curve



- maximum mass given by

$$M_{max} = M(\bar{\epsilon}_c) \quad , \quad \left(\frac{dM}{d\epsilon_c}\right)_{\epsilon_c=\bar{\epsilon}_c} = 0$$

Compilation of measured neutron star masses



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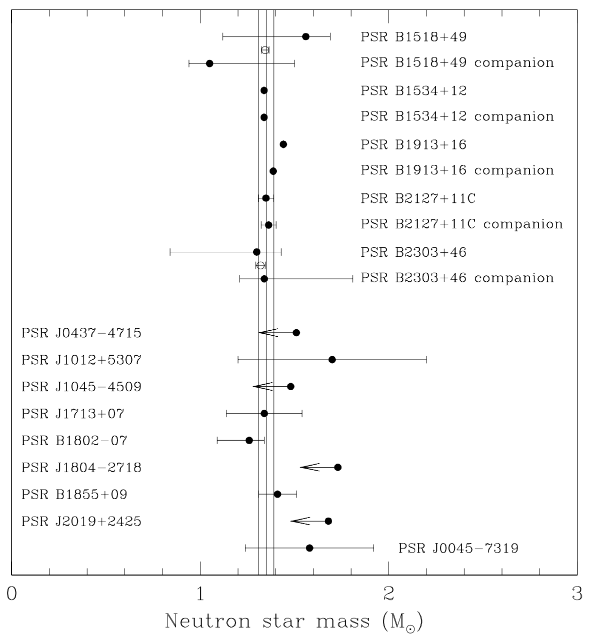

- ▷ Hulse & Taylor: binary pulsar $M = 1.441 \pm .0007 M_\odot$
- ▷ $\sim$ 20 accurate measurements of bynary systems yield $M = 1.35 \pm 0.1 M_\odot$
- ▷ a recent determination of the mass of the X-ray pulsar Vela X-1 yields $M = 1.87^{+0.23}_{-0.17} M_\odot$

Predicted maximum masses vs data

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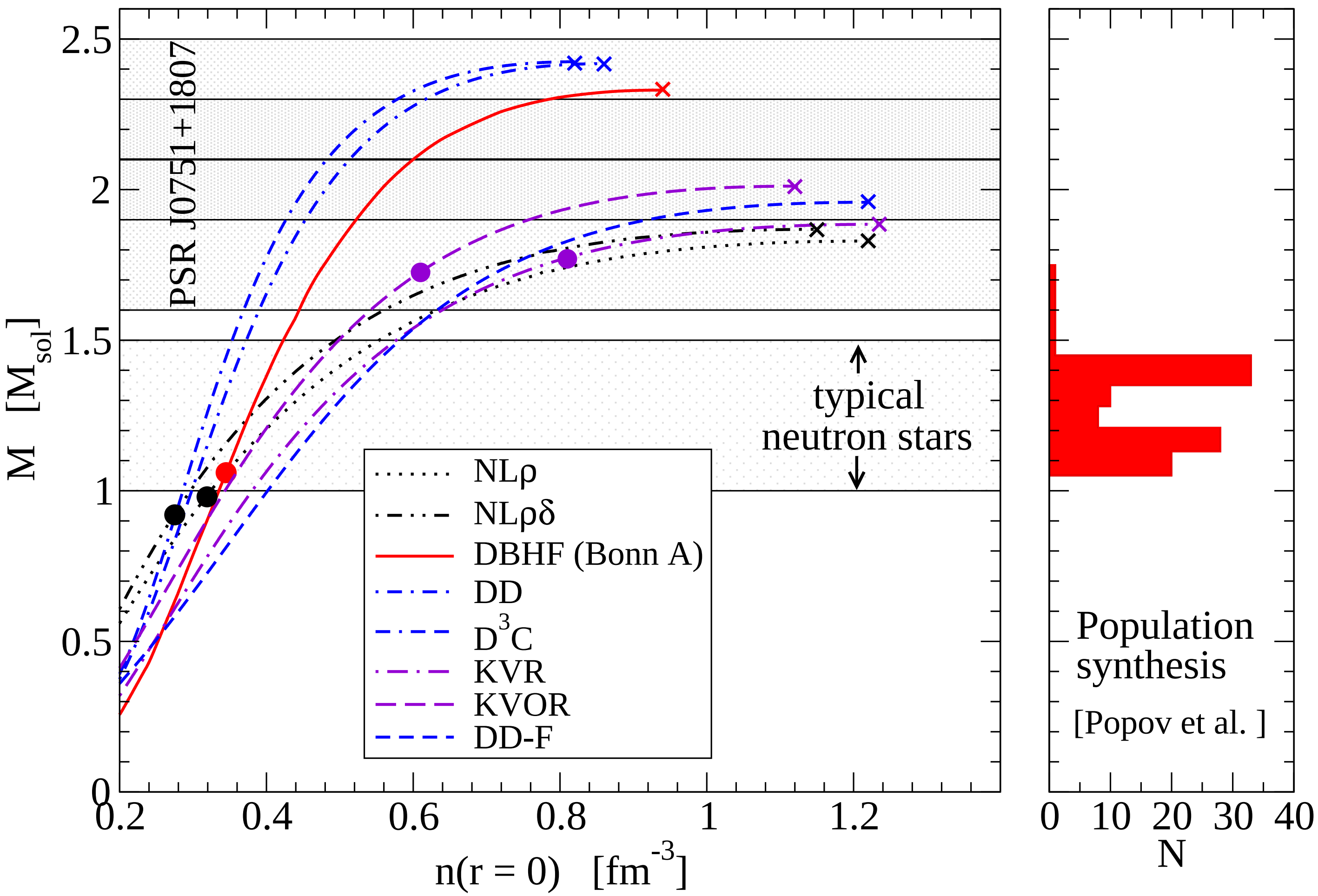
- if confirmed, the measured mass of Vela X-1 will rule out soft EOS, thus leaving little room for the occurrence of "exotic" matter

NS mass vs central density



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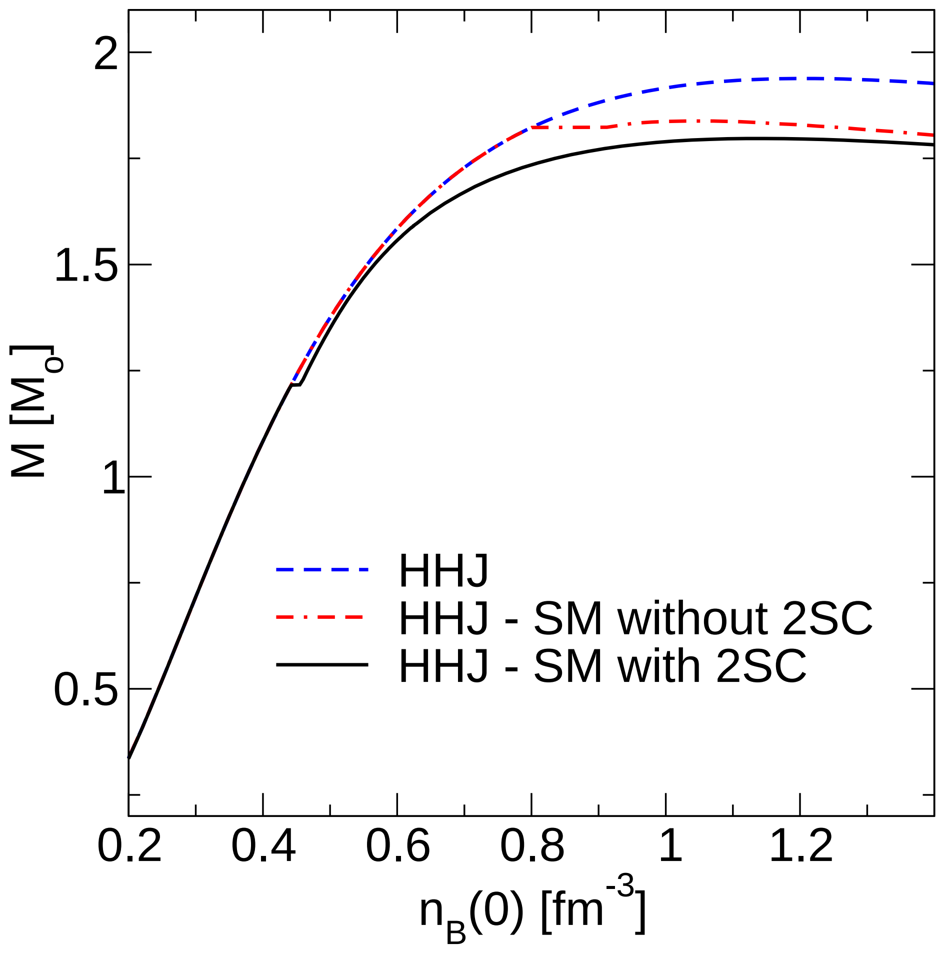
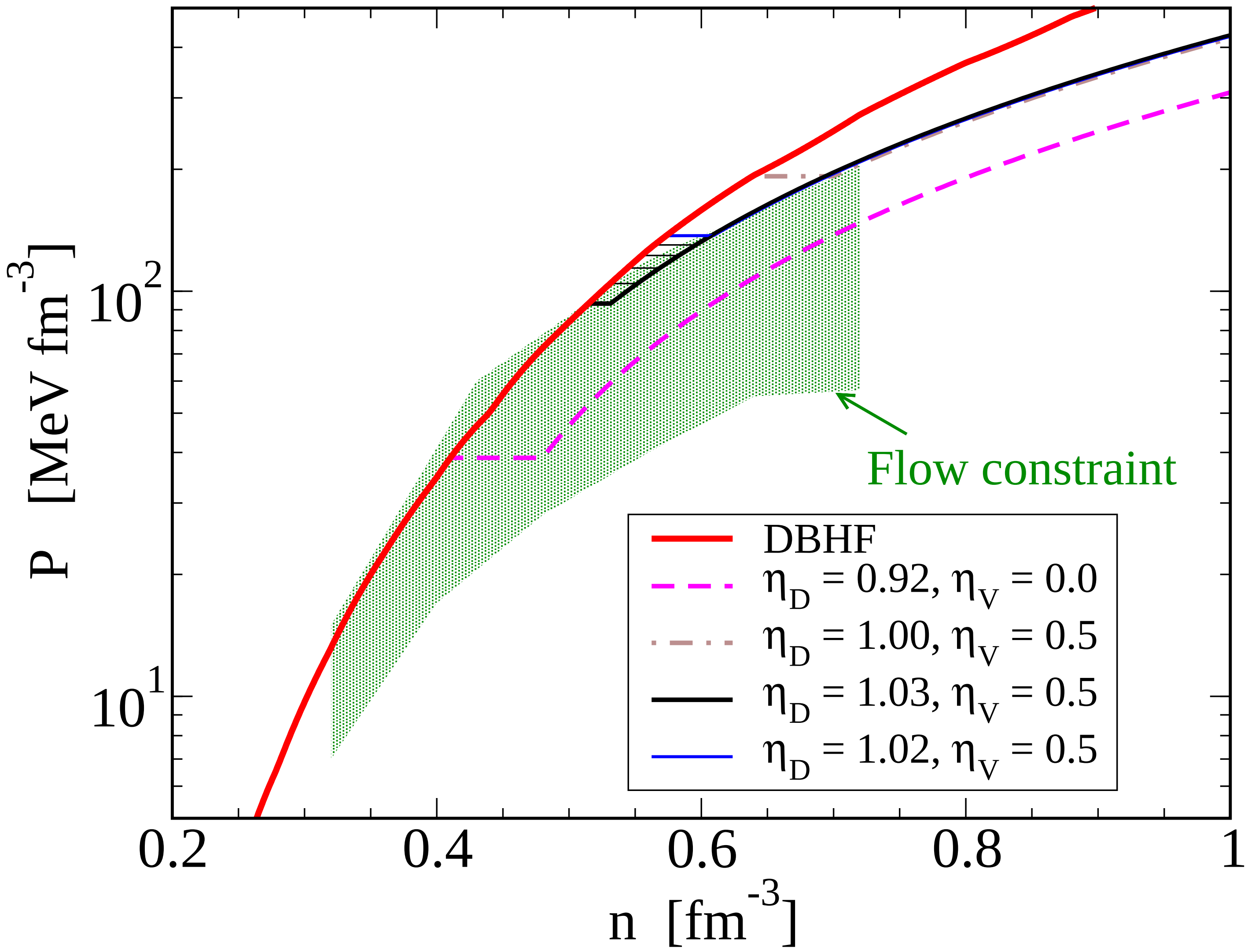


Constraints from heavy-ion collisions



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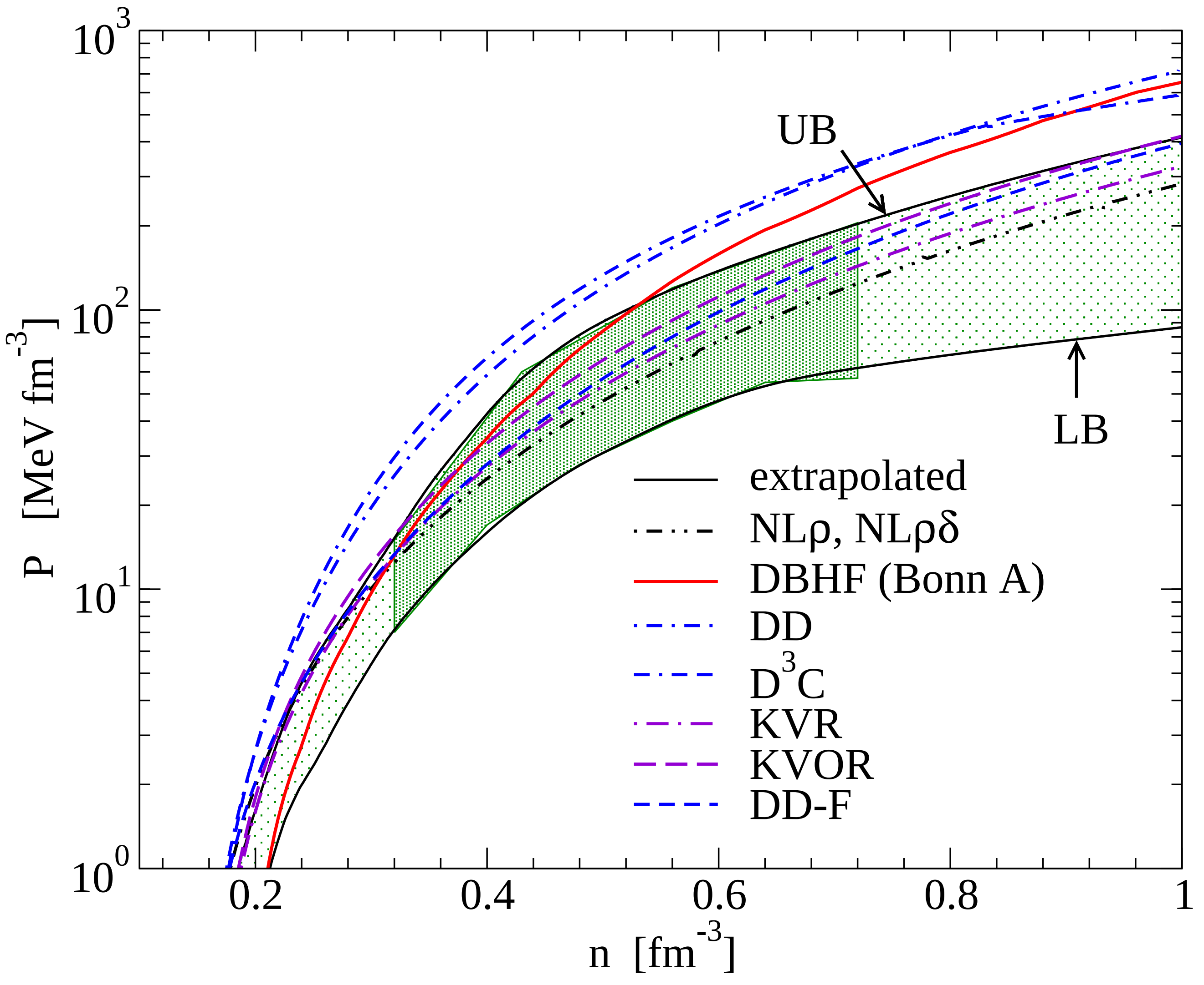

Recent observational developments

- Iron and Oxygen transitions recently observed in the spectra of 28 bursts of the X-ray binary EXO0748-676 correspond to a gravitational redshift $z = 0.35$ (Cottam et al, 2002)

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- Iron and Oxygen transitions recently observed in the spectra of 28 bursts of the X-ray binary EXO0748-676 correspond to a gravitational redshift $z = 0.35$ (Cottam et al, 2002)
- z is related to the mass-radius ratio through

$$R(1+z) = R\left(1 - \frac{2GM}{c^2}\frac{1}{R}\right)^{-1/2}$$

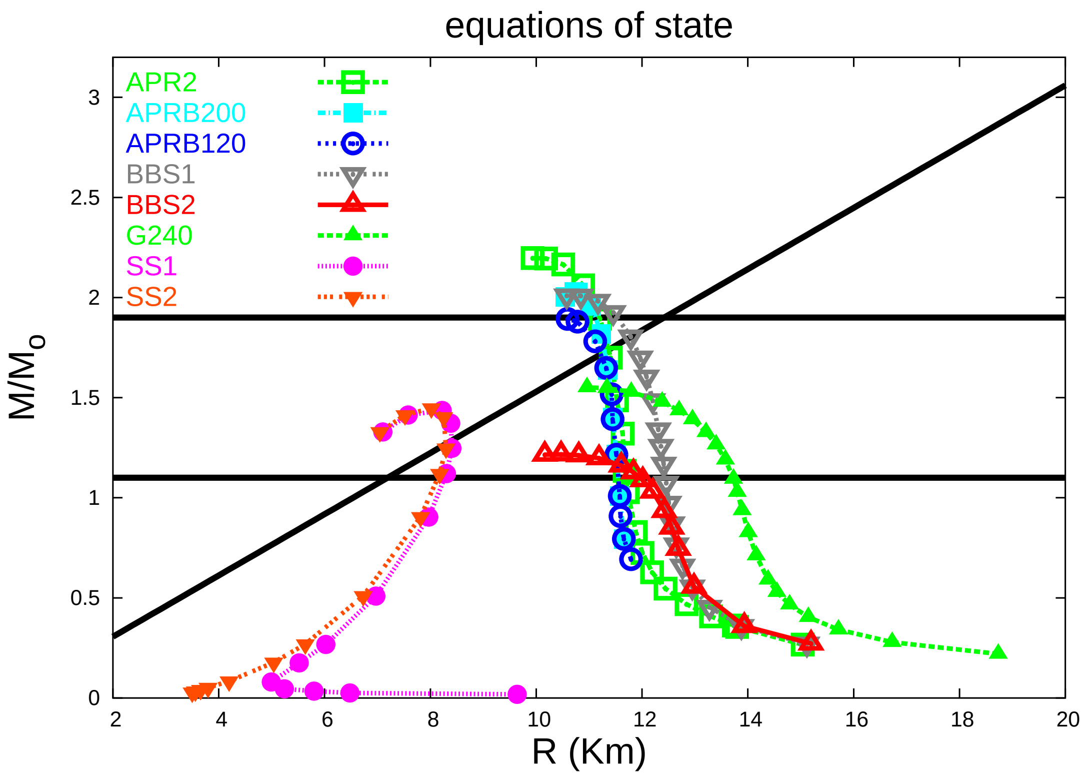
yielding

$$\frac{M}{R} = 0.153\,\frac{M_\odot}{\mathrm{Km}}$$

i.e.

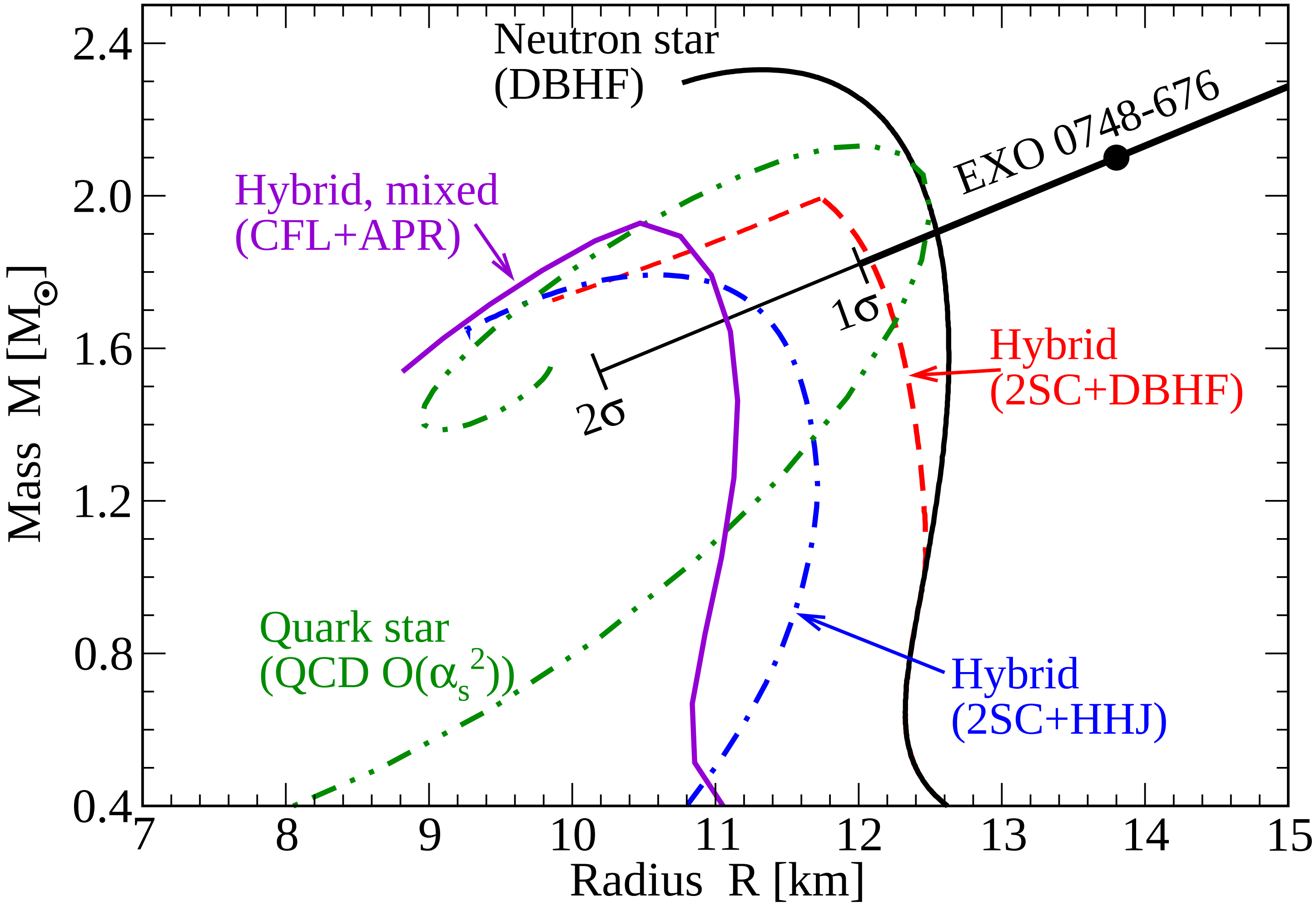
$$1.4 \lesssim M/M_\odot \lesssim 1.8 \iff 9 \lesssim R \lesssim 12\ \mathrm{Km}$$

Predicted M/R ratios vs data

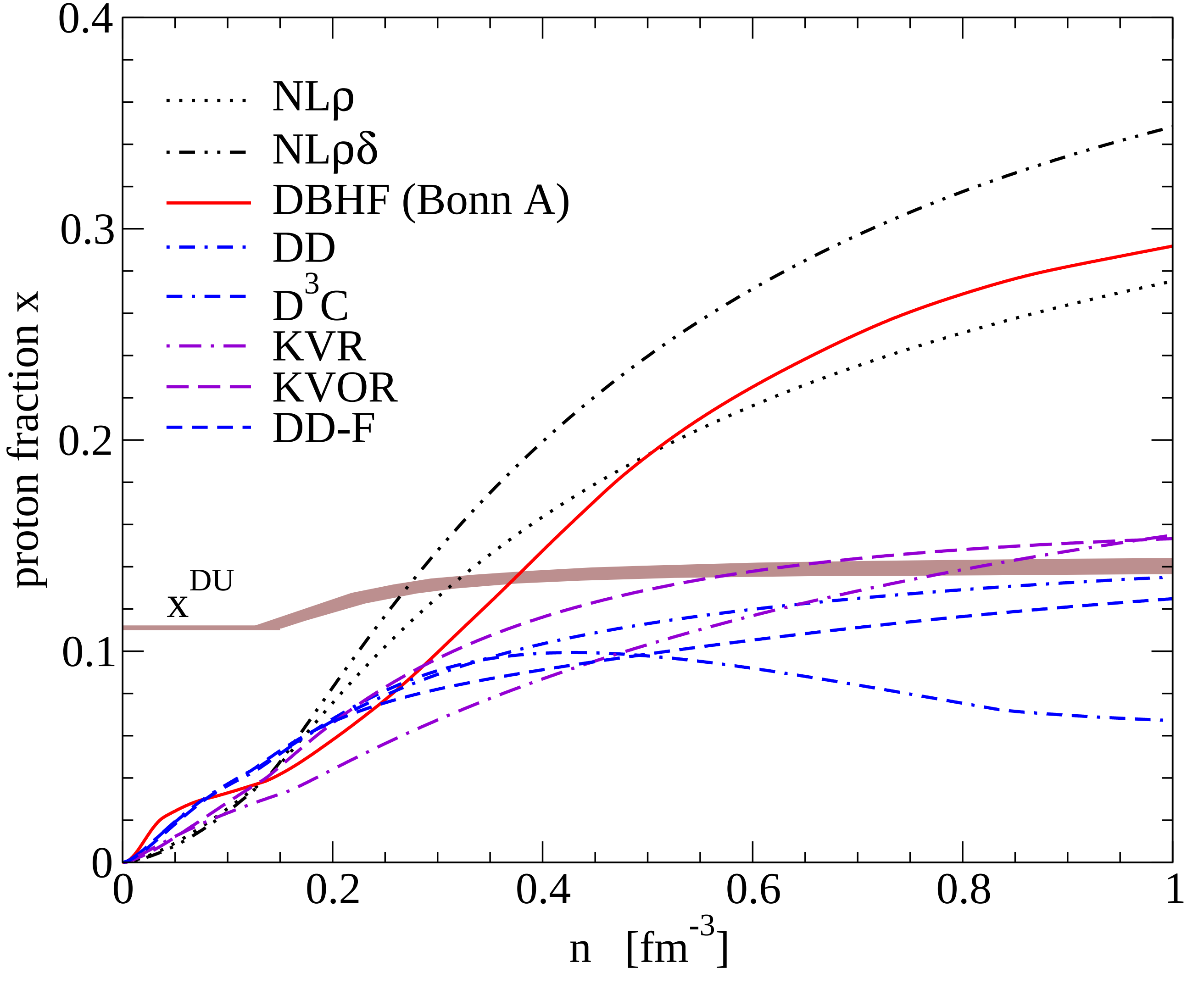


- ▷ APR2, BBS1: nucleons only, nonrelativistic ; APRB120, APR2B200: APR2 + quark matter core
- ▷ BBS2: nucleons + hyperons, nonrelativistic ; G_{240}: nucleons + hyperons, relativistic mean fi eld ; SS1, SS2: strange stars (with and without crust)

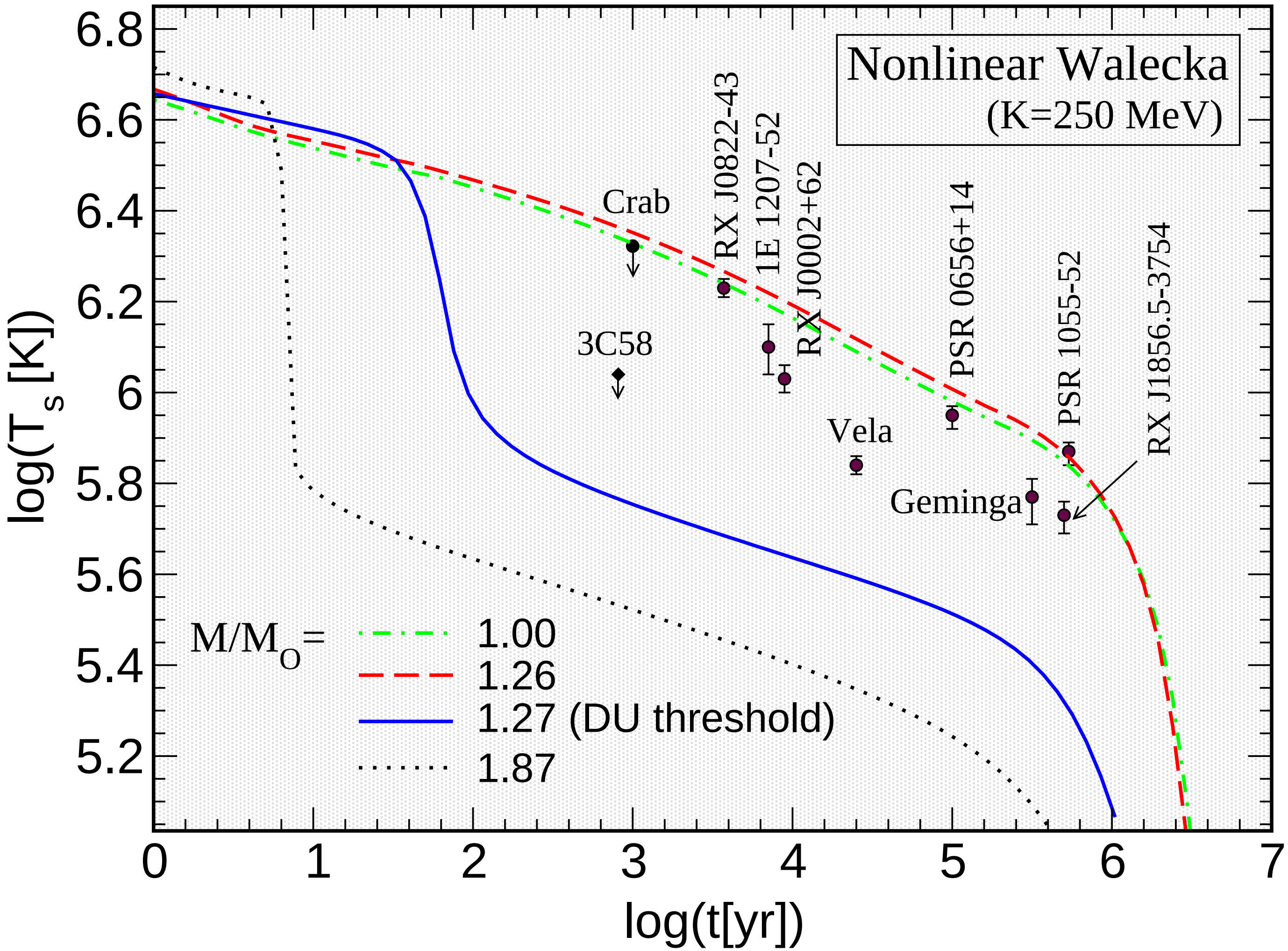
Further constraints on the M/R ratios



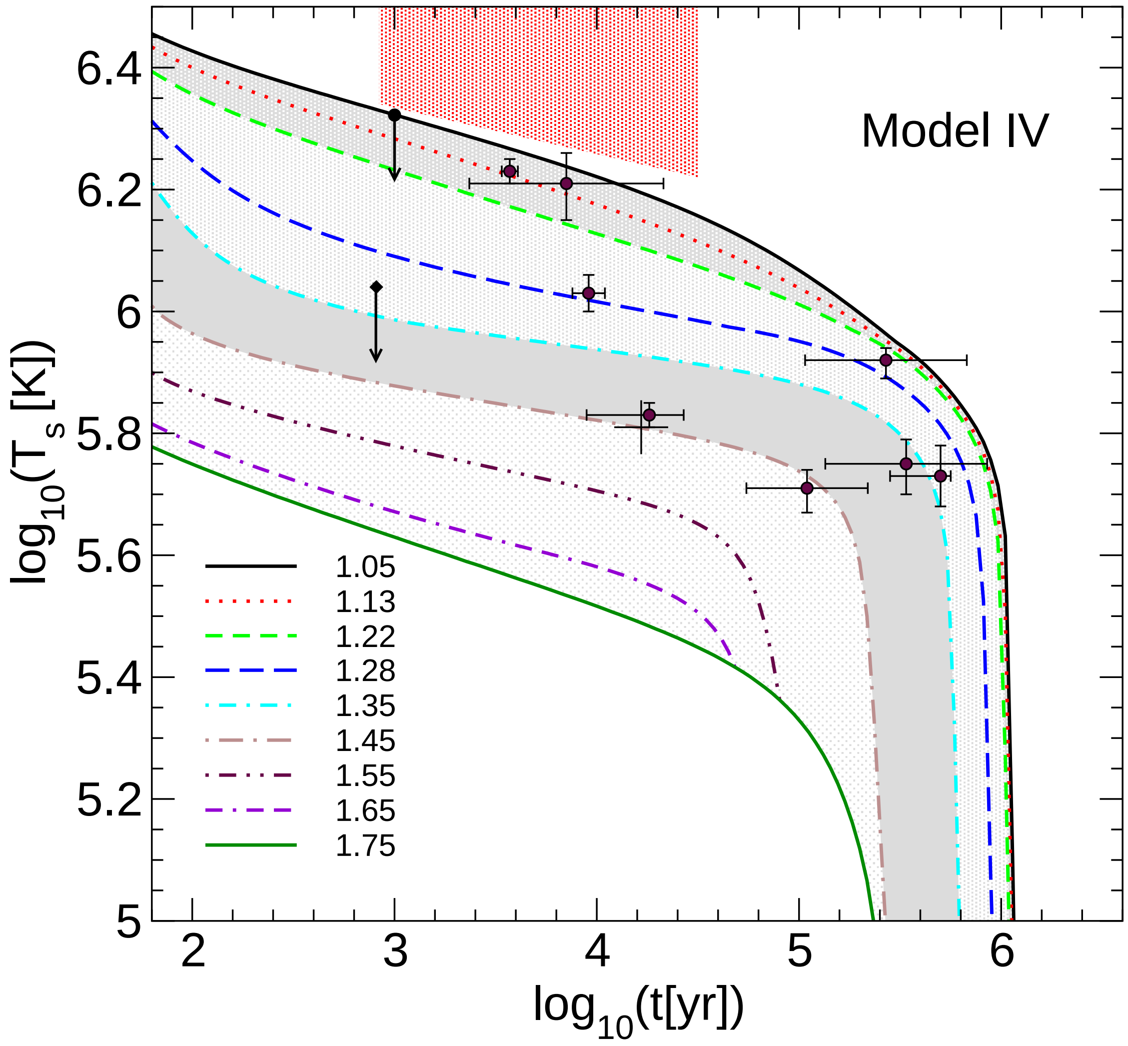
Proton fraction



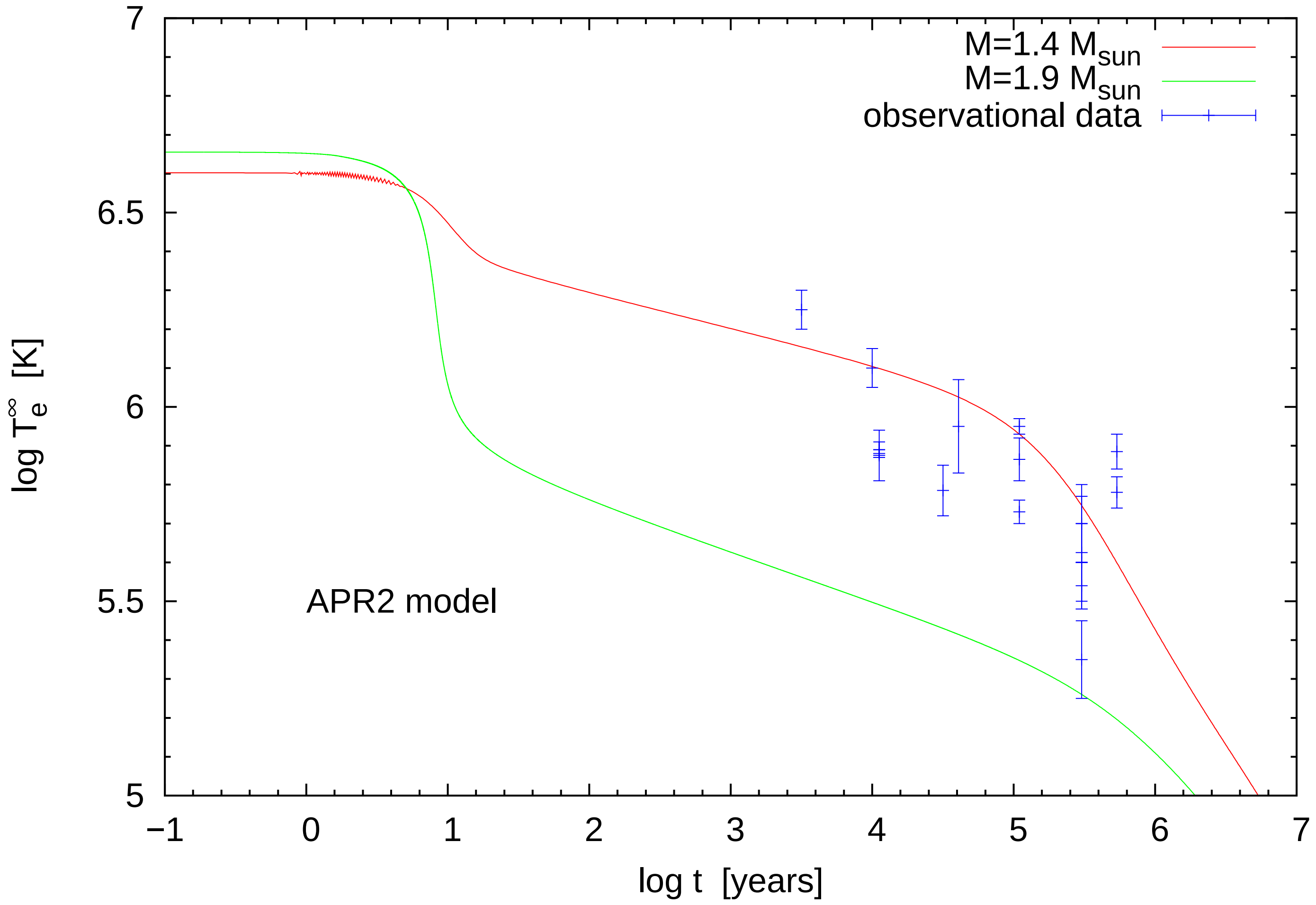
NS cooling: fast vs normal



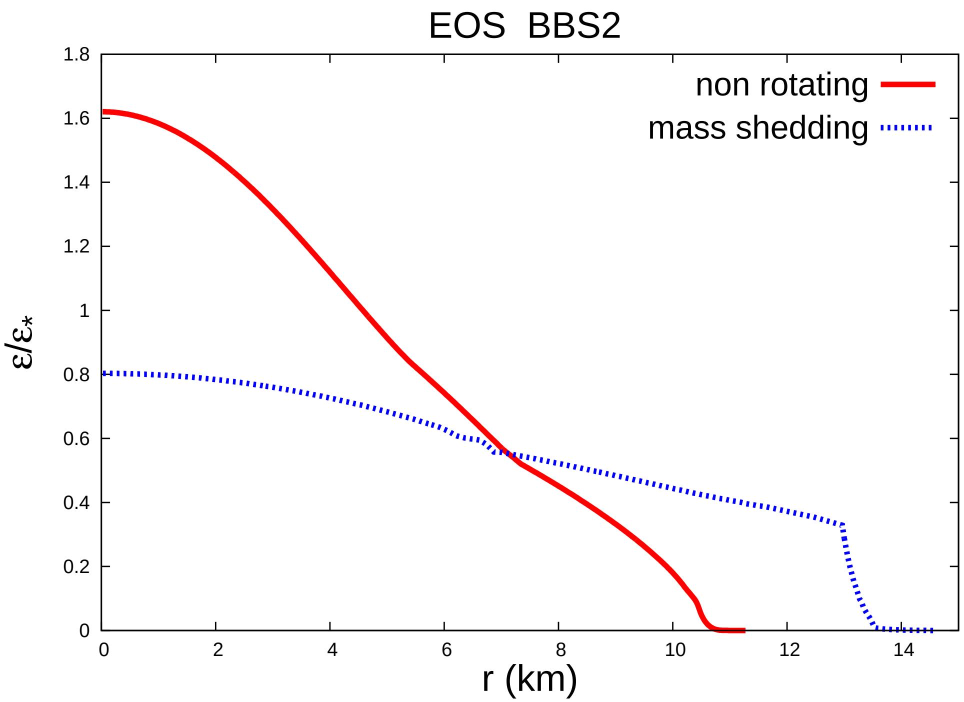
Cooling of hybrid stars

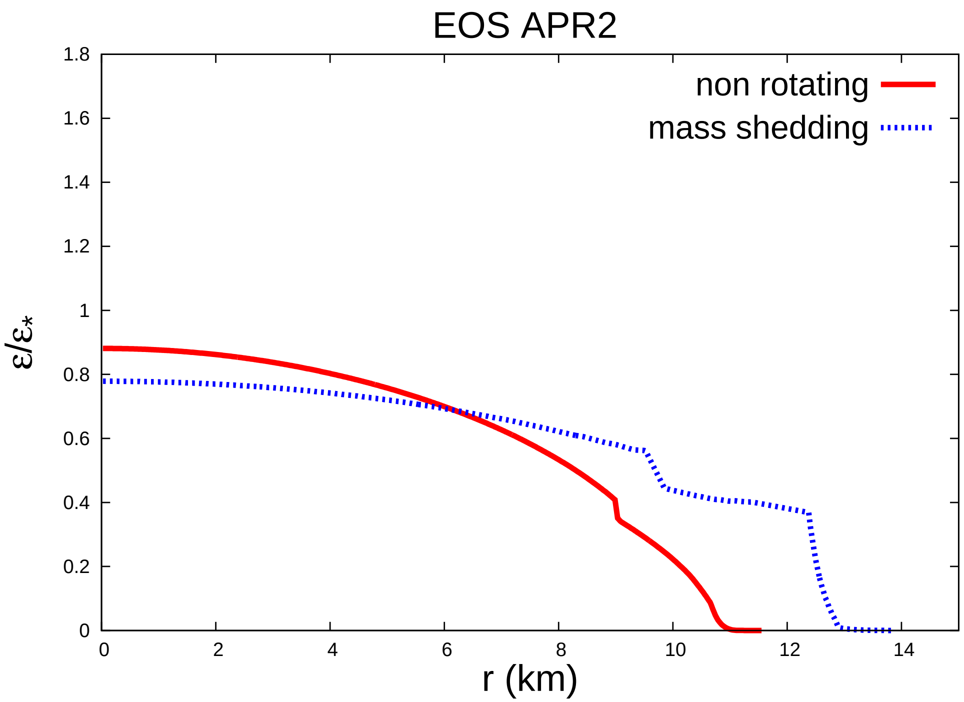


URCA process of hybrid stars

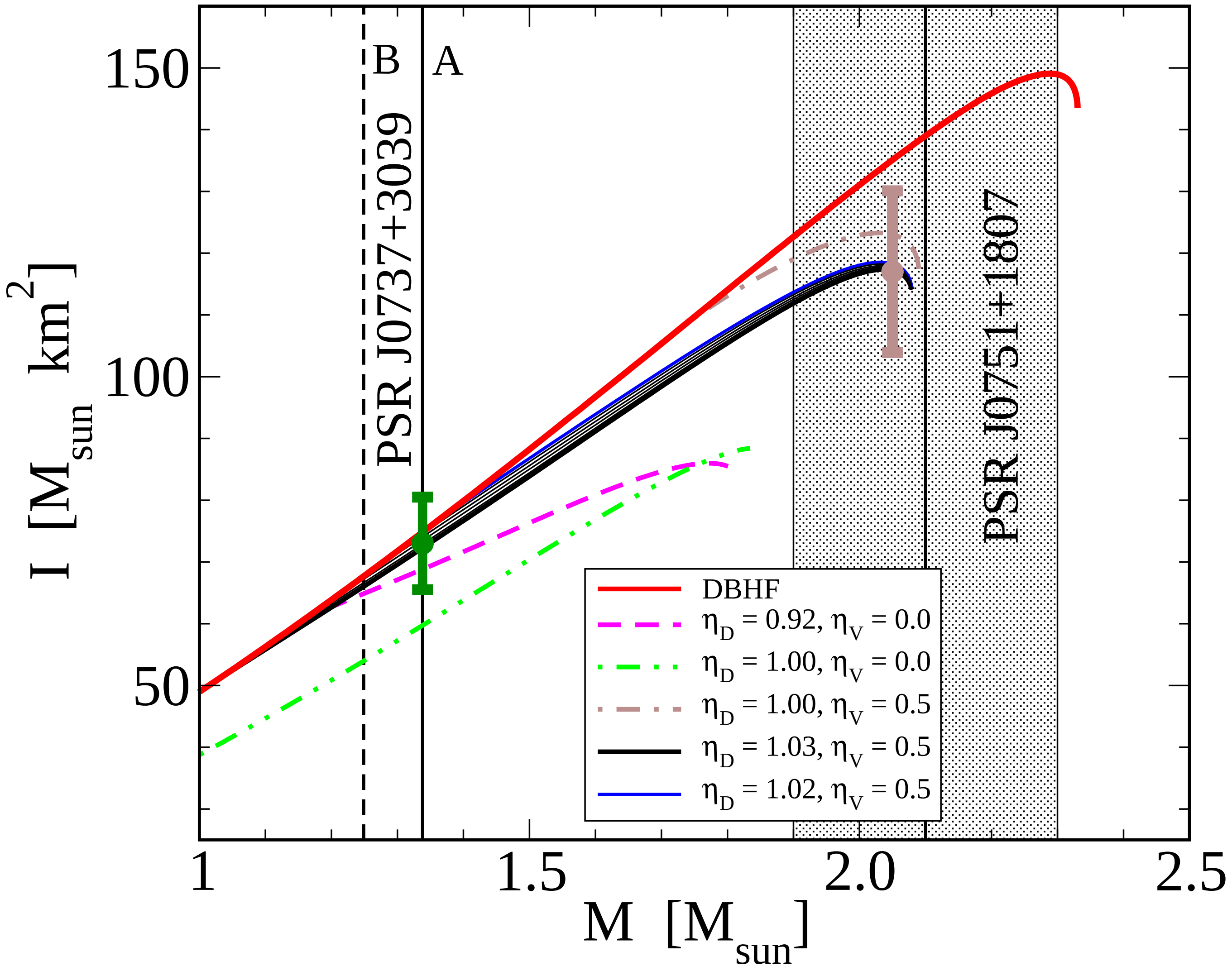


Rotating stars





Moments of inertia



Gravitational waves from neutron stars

- a neutron star emits GW at the (complex) frequencies of its quasi-normal modes
  - g-modes: main restoring force is the buoyancy force
  - p-modes: main restoring force is pressure
  - f-modes: intermediate between g- and p-modes
  - w-modes: pure space-time modes
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- in newtonian theory the frequency of the f-mode is proportional to the average density of the star

GW emission and EOS

- how do neutron star oscillation modes associated with GW emission depend upon the EOS ?

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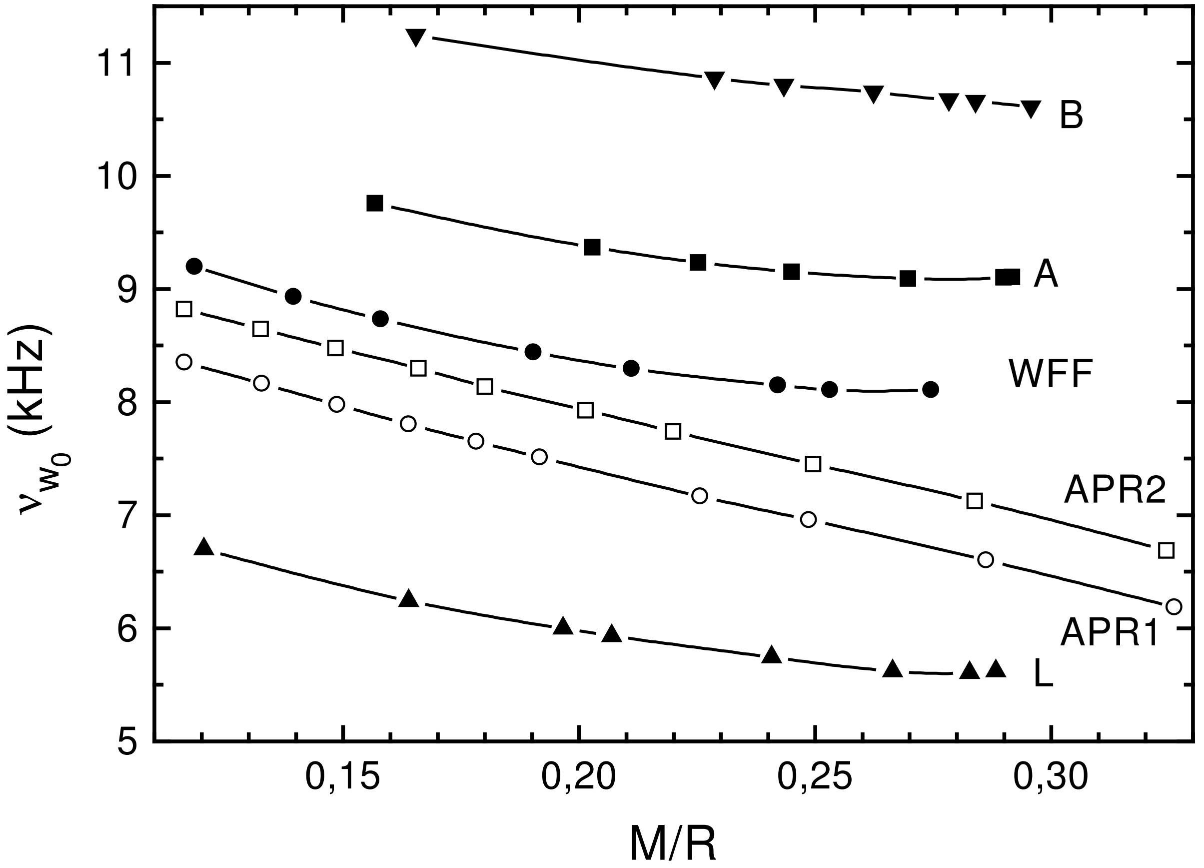
- example: the frequencies of axial (odd parity) w-modes are eigenvalues of a Schrödinger-like equation, whose potential $V_\ell(r)$ explicitly depends upon the EOS

$$V_\ell(r) = \frac{\mathrm{e}^{2\nu(r)}}{r^3} \left\{\ell(\ell+1)r + r^3\left[\epsilon(r) - P(r)\right] - 6M(r)\right\}$$

$$\frac{d\nu}{dr} = -\frac{1}{\left[\epsilon(r) + P(r)\right]}\frac{dP}{dr}$$

GW emission and EOS (continued)



- ▷ frequency of the 1st w-mode vs star compactness (Benhar, Berti & Ferrari, 1999)

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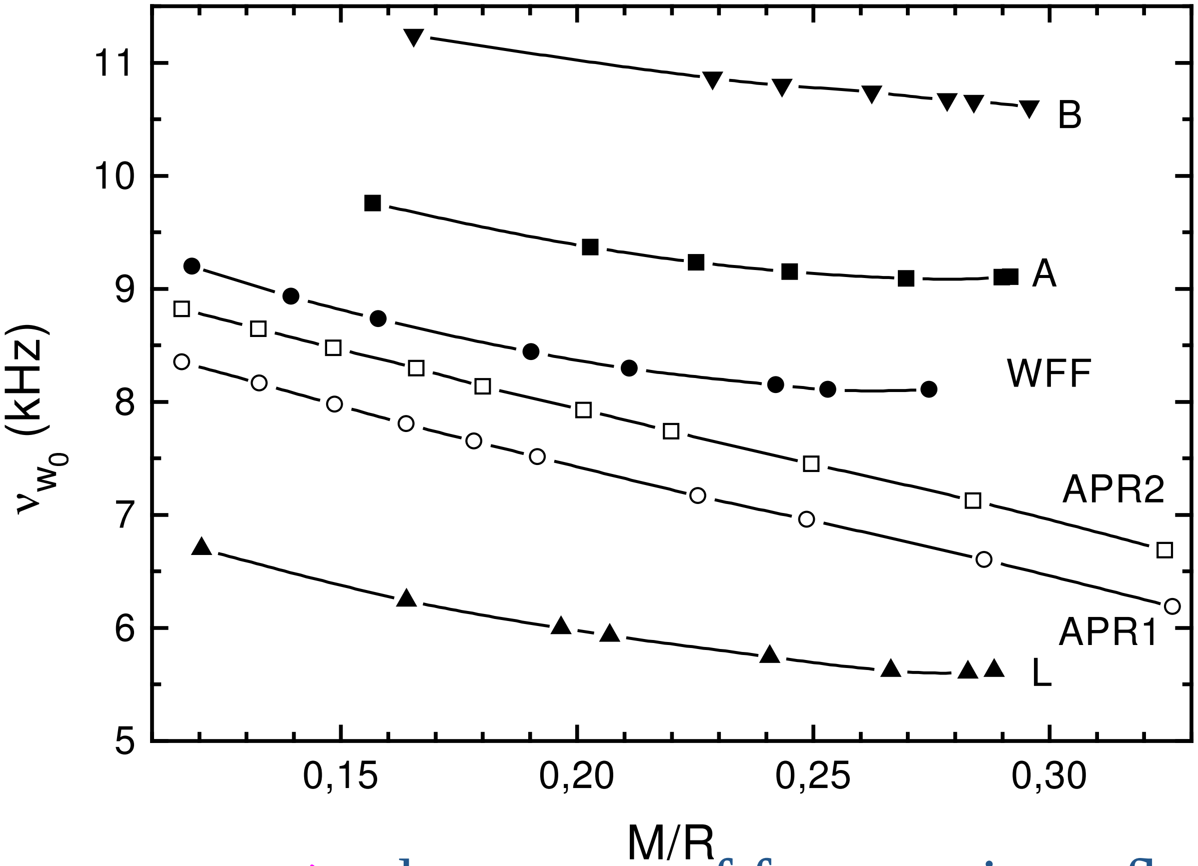
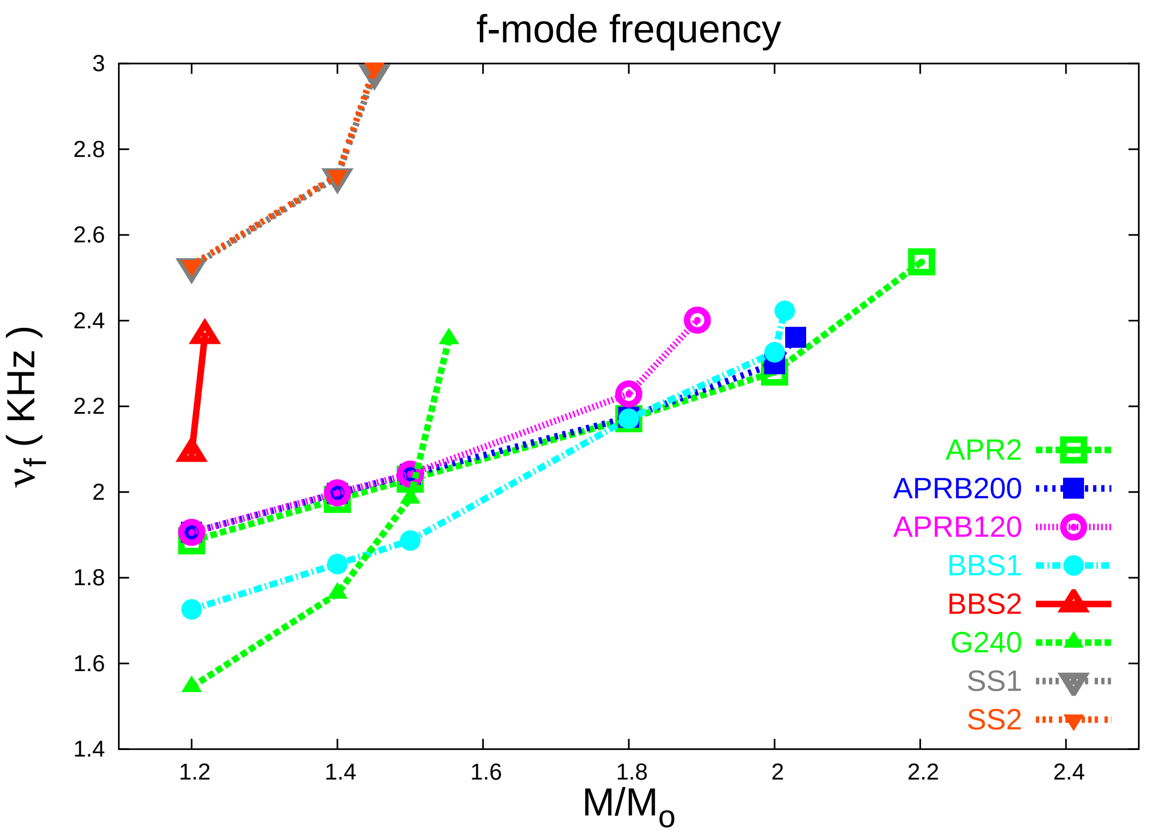


- ▷ frequency of the 1st w-mode vs star compactness (Benhar, Berti & Ferrari, 1999)
- ▷ the pattern of frequencies reflects the stiffness of the EOS. Softer EOS correspond to higher frequencies
- ▷ for a given EOS, the frequency depends weakly upon M/R

GW emission and EOS (continued)

- f-mode frequency as a function of the neutron star mass (Benhar, Ferrari & Gualtieri, 2004)



- stars containing hyperons and strange stars have much higher frequencies

GW emission and EOS (continued)

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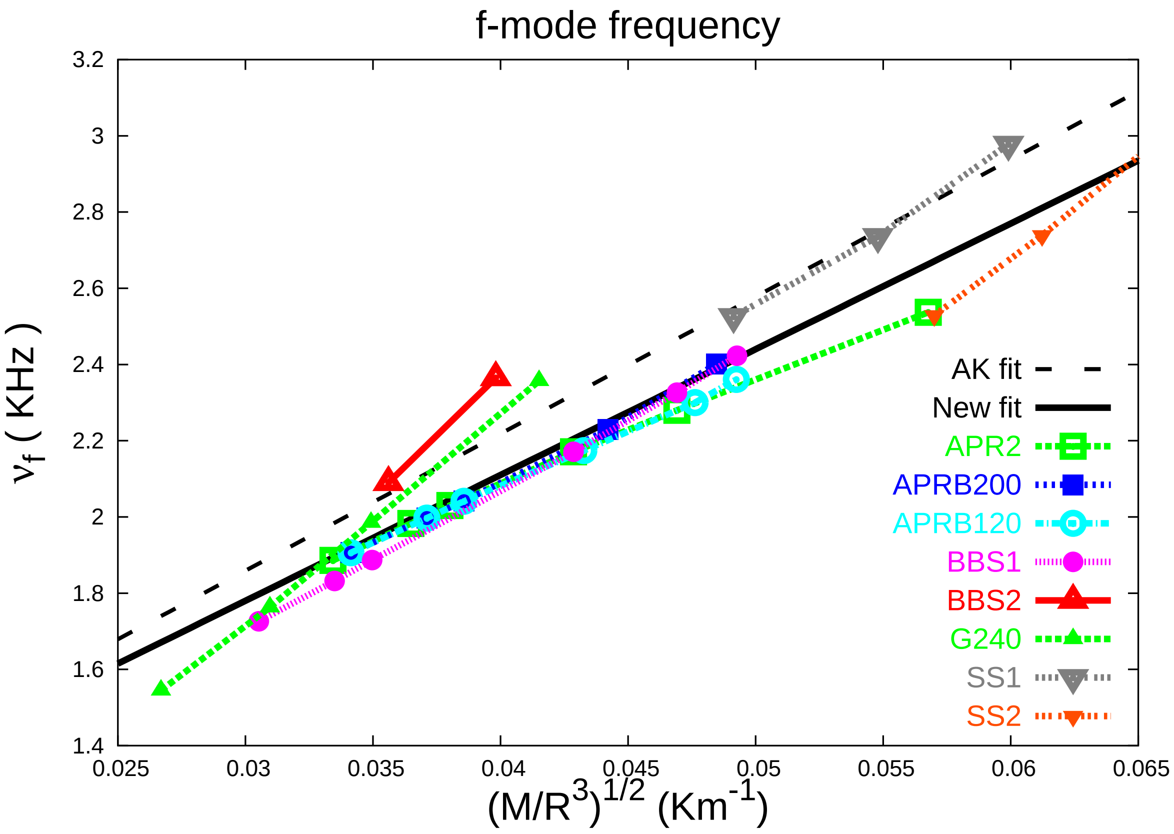


$$\nu_f = a + b\sqrt{\frac{M}{R^3}}$$

$$a = 0.79 \pm 0.09 \text{ kHz}$$

$$b = 33 \pm 2 \text{ km kHz}$$

Extracting M and R from GW frequencies

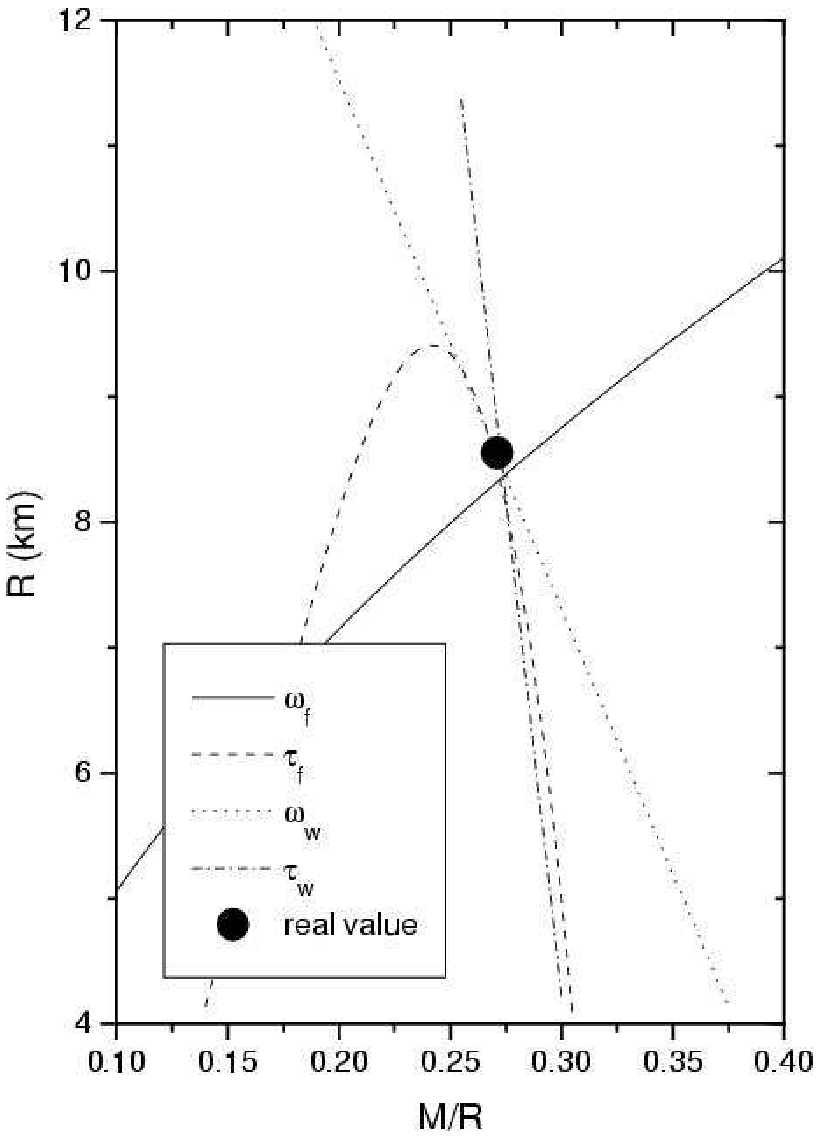
- empirical relations between frequencies and star parameters can also be obtained for the p- and w- modes. For example

$$\nu_w = \frac{1}{K} \left(a + b\frac{M}{R} \right)$$

$$\frac{1}{\tau_w} = 10^{-3}\, M \left[c + d\frac{M}{R} + e\left(\frac{M}{R}\right)^2 \right]$$

- symultaneous detection of GW signals associated with different modes would provide up to five equations for the two unknown R and M

A numerical experiment (Andersson & Kokkotas, 1998)



- select a model polytropic star and compute M and R
- compute frequency and damping time of the f-mode and the 1st w-mode
- plot the four lines corresponding to the empirical relations
- the intersection of the four lines gives the correct M and R with a few percent accuracy

Will GW from neutron stars ever be detected ?

- Assume that the f-mode of a neutron star with $\nu_f = 1.9\ \mathrm{kHz}, \tau_f = 0.184\ \mathrm{s}$ has been excited
- The signal emitted can be modeled as (Ferrari et al, 2003)

$$h(t) = \mathcal{A} e^{(t_{\rm arr} - t)/\tau_f} \sin\left[2\pi\nu_f\left(t - t_{\rm arr}\right)\right] \ ,$$

and the energy stored into the mode is

$$dE_{\rm mode} = \frac{\pi}{2}\nu^2 \,|\, \tilde{h}(\nu)\,|^2 \, dS d\nu$$

- Will the VIRGO interferometer be able to detect this signal ?

Detection of GW from neutron stars (continued)

- VIRGO noise power spectral density ($x = \nu/\nu_0$, $\nu_0 = 500$ Hz)

$$S_n(x) = 10^{-46} \cdot \left\{3.24[(6.23x)^{-5} + 2x^{-1} + 1 + x^2]\right\} \; Hz^{-1},$$

with $x = \nu/\nu_0$ and $\nu_0 = 500$ Hz

- Signal to noise ratio

$$SNR = 2 \left[\int_0^\infty \, d\nu \, \frac{|\tilde{h}(\nu)|^2}{S_n(\nu)} \right]^{1/2}$$

- $SNR = 5$ requires $E_{\rm mode} \sim 6 \times 10^{-7} \; M_\odot$ for a source in our galaxy and $\sim 1.3 \; M_\odot$ for a source in the VIRGO cluster