[2] H. Sompolinsky, *Phys. Rev. Lett.*, **47**, 935 (1981).
[3] L. Cugliandolo and J. Kurchan, *Phys. Rev. Lett.*, **71**, 173 (1993); *Phil. Mag.***71** 501 (1995).
[4] L. Cugliandolo and J.Kurchan, *J. Phys.* , **A27**, 5749 (1994).
[5] S. Franz and M. Mezard, *Physica*, **A209**, 1 (1994).
[6] L. Cugliandolo and P. Le Doussal, *Phys. Rev.*, **E53**, 1525 (1996).
[7] P. Ranieri *J. Phys. I (France)* **6**, 807 (1996).
[8] L. F. Cugliandolo and D.S. Dean, *J. Phys.* , **28**, 4213 (1995).
[9] S. Ciuchi and F. De Pasquale, *Nucl. Phys.*, **B300** [FS22], 31 (1988).
[10] J.M. Kosterlitz, D.J. Thouless and R.C. Jones, *Phys. Rev. Lett.*, **36**, 1217 (1976).
[11] M. L. Metha, *Random matrices and the statistical theory of energy level Academic, New York, 1967.*
[12] M. Campellone, G. Parisi and P. Ranieri *work in progress.*

$Time Regime$	ω	$\frac{\partial\omega}{\partial t_4}$	$\sum\lambda$	λ	$\frac{\partial\lambda}{\partial t_3}$	γ
$(t_1t_2t_3t_4)$	$t_1^{-5/2}$	$T\sum\lambda$	$O(1)$	$O(1)$	$T\gamma$	$O(1)$
$(t_1t_2)(t_3t_4)$	$t_1^{-5/2}$	$t_1^{-5/2}$	t_1^{-3}	$O(1)$	t_1^{-1}	$t_1^{-3/2}$
$t_1t_2(t_3t_4)$	$t_1^{-1/2}$	$t_1^{-1/2}$	t_1^{-1}	$O(1)$	t_1^{-1}	$t_1^{-3/2}$
$t_1(t_2t_3)t_4$	$t_1^{3/2}$	$t_1^{1/2}$	$O(1)$	$O(1)$	t_1^{-1}	$t_1^{-3/2}$
$(t_1t_2)t_3t_4$	$t_1^{-1/2}$	$t_1^{-3/2}$	t_1^{-2}	$O(1)$	t_1^{-1}	$t_1^{-3/2}$
$(t_1t_2t_3)t_4$	$t_1^{-1/2}$	$t_1^{-3/2}$	t_1^{-2}	$O(1)$	t_1^{-1}	$t_1^{-3/2}$
$t_1(t_2t_3t_4)$	$t_1^{-1/2}$	$t_1^{-1/2}$	t_1^{-1}	$O(1)$	t_1^{-1}	$t_1^{-3/2}$
$t_1t_2t_3t_4$	$t_1^{3/2}$	$t_1^{1/2}$	$O(1)$	$O(1)$	t_1^{-1}	$t_1^{-3/2}$

(35)

To see if equations (21) and (22) hold one has to look at the third and sixth column. One sees that the FDT relations for the fluctuations hold only in the first regime, where all the times are at a finite distance from each other. In the other regimes they still can be generalized introducing an $X_{t_1}(C) \propto t_1^{-1/2}$ as in the case of the two-times functions. Let us also remark that $\omega(t_1,t_2,t_3,t_4)$ diverges every time t_1-t_2 and t_3-t_4 are order $O(t_1)$. In this cases, the function $\omega(t_1,t_2,t_3,t_4)$, (19), has not a well defined limit. Moreover, in the FDT regime the functions $\gamma(t_1,t_2,t_3,t_4)$, $\lambda(t_1,t_2,t_3,t_4)$ and $\omega(t_1,t_2,t_3,t_4)$ are asymptotically dependent on the times only through $\tau = t_1+t_2-t_3-t_4$, while in the aging regimes an explicit dependence from the times that are at distance $O(t_1)$ remains. Finally we would like to point out that τ is a characteristic time scale that determines whether we are in an FDT regime ($\tau \sim O(1)$) or not ($\tau \sim O(t_1)$), assuming the same role as $t-t'$ in the case of the two times functions.

V. CONCLUSIONS

In this work we calculated explicitly the four-times functions for the spherical 2-spin glass model in the low temperature phase. We calculated the asymptotic behaviour of these functions in the possible time regimes that can be selected with four ordered times. We found out that there is a time regime in which one can relate the functions through a relation similar to the fluctuation-dissipation theorem for the equilibrium dynamics of the two-times functions. In this work we provide the first explicit expression for the dynamical four-point functions for a spin glass model in the cold phase. It is our intention to use the results obtained here to analyze less trivial spin glass models describing short range interactions of $p>2$ spins. In these models there is lot to be understood about the dynamical propagators and about their static limit [12]. Unfortunately the equations for the propagators are integro-differential equations that cannot be solved without the use of an ansatz on their asymptotic form. We think that the knowledge of the dynamical structure of the propagators for the spherical $p=2$ spin glass model can help us in view of these further developments.

[1] H. Sompolinsky and A. Zippelius, *Phys. Rev. B*, **25**, 6860 (1982).

We shall study the fluctuations around the mean field solutions and in particular we shall be interested in understanding if equations (21) and (22) are verified.

In the large-time limit form the functions (29)(30) (31) read

$$\tilde{\gamma}(t_1,t_2,t_3,t_4) = \left[\frac{t_1 t_2}{t_3 t_4}\right]^{\frac{3}{4}} e^{-2[t_1+t_2-t_3-t_4]} \frac{I_1[2(t_1+t_2-t_3-t_4)]}{(t_1+t_2-t_3-t_4)} \tag{32}$$

$$\begin{aligned}\tilde{\lambda}(t_1,t_2,t_3,t_4) = & \left[\frac{4(t_1 t_2 t_3)}{t_4(t_1+t_2+t_3-t_4)^2}\right]^{\frac{3}{4}} \\ & \left[1 - T\int_0^{2(t_1+t_2-t_3-t_4)} dt' \frac{e^{-t'} I_1[t']}{t'(1-\frac{t'}{2(t_1+t_2+t_3-t_4)})^{(\frac{3}{2})}}\right]\end{aligned} \tag{33}$$

$$\begin{aligned}\tilde{\omega}(t_1,t_2,t_3,t_4) = & \, q_{EA}^2\sqrt{4\pi}\left[\frac{16(t_1 t_2 t_3 t_4)}{(t_1+t_2+t_3+t_4)^2}\right]^{\frac{3}{4}} \left[2(1-q_{EA}^2) - 2T + 2T \times \right. \\ \int_0^{t_3} dt'' \frac{\Gamma(t'') e^{-4t''}}{(1-\frac{2t''}{t1+t2+t3+t4})^{3/2}} & \left.\left[1 - q_{EA}^2 - T\int_0^{2(t_1+t_2+t_3-t_4-2t'')} dt' \frac{e^{-t'} I_1[t']}{[t'(1-\frac{t'}{2(t_1+t_3+t_2+t_4-2t'')})^{3/2}]}\right]\right]\end{aligned} \tag{34}$$

Note that the function $\tilde{\omega}$ is divergent as $t_1^{3/2}$ in all the time regimes. This is due to the fact that $\tilde{\omega}(t_1,t_2,t_3,t_4)$ diverges in the static limit.

However, in the definition (19) of the $\omega(t_1,t_2,t_3,t_4)$ we have subtract this overall behaviour on the equilibrium time scale, and in this regime we can consider only the dynamics on the time differences.

The structure of the possible time regimes is in principle similar to that for the two point correlation functions: the times are either far or close to each other.

In a generic four-time function we can have some times which are close and some which are far away from each other and this complicates the separation in time sectors of the time space because the function may be in local equilibrium with respect to some times and aging with respect to others. We will now study the functions in all the possible times regimes obtainable by four ordered times. We will calculate their asymptotic behaviour for each regime. The result of our analysis is that relations (21) and (22) hold if and only if all the four times are close to each other. That means if there are no aging time scales. Otherwise the dependence upon the aging times (*i.e.* they are far from each other) will dominate the functions.

The results that we obtain are summarized in the table below where with $\sum\lambda$ we indicated in a compact form the r.h.s. of the equation (21).On the first column we indicated the time regime we were considering, using curly brackets to group times whose difference remains finite in the limit of $t_1 \to \infty$.

$$\frac{\partial}{\partial t_4}\omega(t_1,t_2,t_3,t_4) = 2T\Big(2\lambda(t_1,t_2,t_3,t_4) - \lambda(t_1,t_1,t_3,t_4) - \lambda(t_2,t_2,t_3,t_4)$$
$$-2\lambda(t_1,t_2,t_4,t_4) + \lambda(t_1,t_1,t_4,t_4) + \lambda(t_2,t_2,t_4,t_4)\Big), \qquad (21)$$
$$\frac{\partial}{\partial t_3}\lambda(t_1,t_2,t_3,t_4) = T\gamma(t_1,t_2,t_3,t_4). \qquad (22)$$

Let us remark that in equation (17) (18) (19) we have by hand subtracted the part of the term that is of order $O(1)$, and the quantities defined are all of order $O(1/N)$.

In the functional formalism the above functions are the propagators of the fields $Q^{\alpha\beta}(t_1,t_2)$ introduced in [1] evaluated at zero momentum. This simple model will provide us with the first explicit calculation of the dynamical propagators for a spin glass in the low temperature phase.

Let us now set the time order $t_4 < t_3 < t_2 < t_1$. Using the fact that the model is quadratic we can write

$$\gamma(t_1,t_2,t_3,t_4) = \tilde{\gamma}(t_1,t_2,t_3,t_4) + \tilde{\gamma}(t_1,t_2,t_4,t_3) \qquad (23)$$
$$\lambda(t_1,t_2,t_3,t_4) = \tilde{\lambda}(t_1,t_2,t_3,t_4) + \tilde{\lambda}(t_2,t_1,t_3,t_4) \qquad (24)$$
$$\omega(t_1,t_2,t_3,t_4) = 4\tilde{\omega}(t_1,t_2,t_3,t_4) + 4\tilde{\omega}(t_1,t_2,t_4,t_3) - 4\tilde{\omega}(t_1,t_1,t_3,t_4) -$$
$$4\tilde{\omega}(t_2,t_2,t_3,t_4) - 4\tilde{\omega}(t_1,t_2,t_3,t_3) - 4\tilde{\omega}(t_1,t_2,t_4,t_4) +$$
$$2\tilde{\omega}(t_1,t_1,t_3,t_3) + 2\tilde{\omega}(t_2,t_2,t_3,t_3) + 2\tilde{\omega}(t_1,t_1,t_4,t_4) + 2\tilde{\omega}(t_2,t_2,t_4,t_4) \qquad (25)$$

where we have defined

$$\tilde{\gamma}(t_1,t_2,t_3,t_4) \doteq \overline{\langle s_i(t_1)\xi_j(t_3)\rangle\langle s_i(t_2)\xi_j(t_4)\rangle} \qquad (26)$$
$$\tilde{\lambda}(t_1,t_2,t_3,t_4) \doteq \overline{\langle s_i(t_1)s_j(t_3)\rangle\langle s_i(t_2)\xi_j(t_4)\rangle} \qquad (27)$$
$$\tilde{\omega}(t_1,t_2,t_3,t_4) \doteq \overline{\langle s_i(t_1)s_j(t_3)\rangle\langle s_i(t_2)s_j(t_4)\rangle}. \qquad (28)$$

One gets

$$\tilde{\gamma}(t_1,t_2,t_3,t_4) = \frac{1}{N}\theta(t_1-t_3)\theta(t_2-t_4)\sqrt{\frac{\Gamma(t_3)\Gamma(t_4)}{\Gamma(t_1)\Gamma(t_2)}}\frac{I_1[2(t_1+t_2-t_3-t_4)]}{(t_1+t_2-t_3-t_4)} \qquad (29)$$
$$\tilde{\lambda}(t_1,t_2,t_3,t_4) = \frac{1}{N}\theta(t_2-t_4)\frac{\sqrt{\Gamma(t_4)}}{\sqrt{\Gamma(t_1)\Gamma(t_2)\Gamma(t_3)}}\left[\frac{I_1[2(t_1+t_2+t_3-t_4)]}{(t_1+t_2+t_3-t_4)}\right.$$
$$\left.+2T\int_0^{t_3} dt'\Gamma(t')\frac{I_1[2(t_1+t_2+t_3-t_4-2t')]}{(t_1+t_2+t_3-t_4-2t')}\right] \qquad (30)$$
$$\tilde{\omega}(t_1,t_2,t_3,t_4) = \frac{1}{N}\frac{1}{\sqrt{\Gamma(t_1)\Gamma(t_2)\Gamma(t_3)\Gamma(t_4)}}\times$$
$$\left[2T\int_0^{t_3}dt'\Gamma(t')\frac{I_1[2(t_1+t_2+t_3+t_4-2t')]}{(t_1+t_2+t_3+t_4-2t')}+\right.$$
$$2T\int_0^{t_4}dt'\Gamma(t')\frac{I_1[2(t_1+t_2+t_3+t_4-2t')]}{(t_1+t_2+t_3+t_4-2t')}+$$
$$\left.4T^2\int_0^{t_3}dt'\int_0^{t_4}dt''\Gamma(t')\Gamma(t'')\frac{I_1[2(t_1+t_2+t_3+t_4-2t'-2t'')]}{(t_1+t_2+t_3+t_4-2t'-2t'')}\right]. \qquad (31)$$

• For $\lambda \simeq O(1) < 1$ and for $T < T_c$, the system is in the so called *aging* regime: the correlation functions depend on both time variables (in this case through λ) and not only on time differences. Moreover the FDT relation is not valid although it can be generalized introducing a function $X_{t_1}(C)$ [3] [4] by the relation

$$G(t_1,t_2) = \frac{X_{t_1}(C)}{T}\frac{\partial C(t_1,t_2)}{\partial t_2}\theta(t_1-t_2). \tag{16}$$

For large times one has that $X_{t_1}(C) \to X(C)$. The function $X(C)$ generally characterizes the type of aging dynamics of the model. In this model the slowness of the dynamics is due to the flatness of the energy landscape and is therefore qualitatively similar to ordinary domain coarsening. This kind of dynamics is in general associated with a $X(C) = 0$ which is indeed the case for this model in which it has been found that, in the aging regime, $X_{t_1}(C)$ scales as $t_1^{-1/2}$ for large t_1, [8]. We shall see that this will be the case also for the four-times functions.

IV. DYNAMICAL FLUCTUATIONS

The problem of the dynamical fluctuations around the mean field solution for spin glass models, for $T < T_c$ has not been faced yet. We shall therefore study the four-point functions in this model in which it is possible to determine the explicit temporal behaviour of these functions and we will describe the different large-time asymptotic regimes.

Let us now introduce the four-time correlation functions

$$\gamma(t_1,t_2,t_3,t_4) \doteq \frac{1}{N^2}\sum_{i,j}\left(\overline{\langle s_i(t_1)s_i(t_2)\xi_j(t_3)\xi_j(t_4)\rangle} - \overline{\langle s_i(t_1)s_i(t_2)\rangle}\,\overline{\langle \xi_j(t_3)\xi_j(t_4)\rangle}\right) \tag{17}$$

$$\lambda(t_1,t_2,t_3,t_4) \doteq \frac{1}{N^2}\sum_{i,j}\left(\overline{\langle s_i(t_1)s_i(t_2)s_j(t_3)\xi_j(t_4)\rangle} - \overline{\langle s_i(t_1)s_i(t_2)\rangle}\,\overline{\langle s_j(t_3)\xi_j(t_4)\rangle}\right) \tag{18}$$

$$\omega(t_1,t_2,t_3,t_4) \doteq \frac{1}{N^2}\sum_{i,j}\Big(\overline{\langle (s_i(t_1)-s_i(t_2))^2(s_j(t_3)-s_j(t_4))^2\rangle} - \overline{\langle (s_i(t_1)-s_i(t_2))^2\rangle}\,\overline{\langle (s_j(t_3)-s_j(t_4))^2\rangle}\Big). \tag{19}$$

The functions γ, λ, ω are related to the fluctuations around the saddle point of $Q_i^{\alpha\beta}(t_1,t_2)$, [1] [2] that gives, as mean field solutions, the two-point correlations and response functions. Note that $\gamma(t_1,t_2,t_3,t_4)$ is related to the dynamical χ_{SG}:

$$\chi_{SG} = \frac{1}{N}\sum_{ij}\overline{\frac{\partial\langle s_i(t_1)\rangle}{\partial h_j(t_3)}\frac{\partial\langle s_i(t_2)\rangle}{\partial h_j(t_4)}}\,, \tag{20}$$

while $\omega(t_1,t_2,t_3,t_4)$ is the dynamical four-points correlation function defined in such way to stay finite in the asymptotic limit of $t_1 \to \infty$ with $t_1 \sim t_2 \sim t_3 \sim t_4$, i.e. on the equilibrium time scale.

In this regime one expects to verify some FDT-like relations

$$s_\alpha(t) = s_{\alpha(t_0)} e^{\alpha(t-t_0)-\int_{t_0}^{t} z(\tau)d\tau} + \int_{t_0}^{t} e^{\alpha(t-t'')-\int_{t_0}^{t} z(\tau')d\tau'} (h_\alpha(t'') + \xi_\alpha(t''))dt'', \tag{9}$$

where again α labels the eigenvectors of J_{ij}, and the initial time is $t = 0$. We introduce the correlation and response function which read:

$$C(t_1, t_2) = \frac{1}{N}\sum_i^N \langle s_i(t_1) s_i(t_2)\rangle, \tag{10}$$

$$G(t_1, t_2) = \frac{1}{N}\sum_i^N \frac{\partial\langle s_i(t_1)\rangle}{\partial h_i(t_2)} = \frac{1}{N}\sum_i^N \frac{\partial\langle s_i(t_1)\rangle}{\partial \xi_i(t_2)} = \frac{1}{N}\sum_i^N \frac{1}{2T}\langle s_i(t_1)\xi_i(t_2)\rangle, \tag{11}$$

where the last equality is valid because the noise has a Gaussian distribution probability. In the $N \to \infty$ limit and in absence of the external field the above functions have the form [8]

$$C(t_1, t_2) = \frac{1}{\sqrt{\Gamma(t_1)\Gamma(t_3)}} \left[\frac{I_1[2(t_1+t_2)]}{(t_1+t_2)} + 2T\int_0^{t_2} dt'\, \Gamma(t') \frac{I_1(t_1+t_2-2t')}{(t_1+t_2-2t')}\right] \tag{12}$$

$$G(t_1, t_2) = \sqrt{\frac{\Gamma(t_1)}{\Gamma(t_2)}} \frac{I_1(t_1-t_2)}{(t_1-t_2)}, \tag{13}$$

where $I_1[x]$ is the modified Bessel function and the function

$$\Gamma(t) \doteq e^{2\int_{t_0}^{t} z(\tau)d\tau} \tag{14}$$

is fixed by implementing the spherical condition $C(t,t) = 1$ in equation (12).

The simplicity of this model allows one to obtain the mean field solutions (12) (13) directly, simply averaging over the distribution of the eigenvalues of the matrix J_{ij}.

For other models of spin glasses we do not manage to obtain the explicit form of $C(t_1, t_2)$ and $G(t_1, t_2)$.

A quite general procedure, introduced in [1], allows to obtain closed equations for $C(t_1, t_2)$ and $G(t_1, t_2)$ as the saddle point solutions of a dynamical generating functional. In particular, in the case the off-equilibrium dynamics of the p-spin model, with $p > 2$, and the SK model, we are not able to solve these equations and therefore to determine the explicit time dependence of the correlation and response functions but we can only predict the structure of their asymptotic behaviour. This model provides us with an explicit solution, and allows us to control the assumptions and the ansatz we used to obtain analytical results for the dynamics of more complicated systems.

For the correlation functions $C(t_1, t_2)$ and $G(t_1, t_2)$ it has been shown that there are basically two asymptotic time scales and that they can be distinguished by the variable $\lambda = t_2/t_1$ [8].

• For $\lambda \simeq 1$ the system is in an equilibrium regime, in which the functions depend only on the difference of the two arguments (time translational invariance) and the FDT relation

$$G(t_1 - t_2) = \frac{1}{T}\frac{\partial C(t_1 - t_2)}{\partial t_2}\theta(t_1 - t_2), \tag{15}$$

holds at all temperatures.

where we have set $h_i = 0$ for all i and z is a Lagrange multiplier introduced to enforce the constraint. For $N \to \infty$ the distribution of the eigenvalues of the matrix J_{ij} follows the Wigner semi-circle law [11]:

$$\rho(\lambda) = \frac{1}{2\pi}\sqrt{4-\lambda^2}\,, \qquad \lambda \in [-2,2]. \tag{4}$$

In this limit one can formally write the saddle point equations on Z which reproduce a softer version of the constraint

$$1 = \frac{1}{N}\sum_{\alpha}^{N}\langle s_\alpha^2\rangle = \int_{-2}^{2} d\lambda \rho(\lambda)\frac{1}{2z-\beta\lambda}. \tag{5}$$

In equation (5) $s_\alpha = \sum_{i=1}^{N}\phi_i^\alpha s_i$ is the projection of the spin variables on the α-th eigenvector of the matrix of the couplings J_{ij}.

It can be seen that below a critical temperature $T_c = 1$ the second of the equalities in (5) does not hold because a spontaneous magnetization arises along the eigenvector with eigenvalue 2 (or -2). For $T < T_c$ the value of the Lagrangian multiplier remains fixed at the branch point $z = \beta$ and a magnetization $\langle s_2\rangle \propto N^{1/2}$ appears.

The spin glass susceptibility is defined as follows:

$$\chi_{SG} = \frac{1}{N}\sum_{ij}\overline{\left(\langle s_i s_j\rangle - \langle s_i\rangle\langle s_j\rangle\right)^2} = \frac{1}{N}\sum_{\alpha}\frac{1}{(2z-\beta\lambda^\alpha)^2}, \tag{6}$$

In the large-N limit χ_{SG} diverges as $1/(T-T_c)$ for $T \to T_c^+$ and it remains infinite in the whole frozen phase. This model is critical for all temperatures below T_c.

The computation of the average free energy can also be done using the replica method, [9], where a replica symmetric ansatz solve the model exactly. One can reproduce the results for χ_{SG} using the following identity:

$$\chi_{SG} = \lim_{n\to 0}\left[\langle\delta q_{\alpha\beta}^2\rangle - 2\langle\delta q_{\alpha\beta}\delta q_{\alpha\gamma}\rangle + \langle\delta q_{\alpha\beta}\delta q_{\gamma\delta}\rangle\right], \tag{7}$$

where the $\delta q_{\alpha\beta}$ are the fluctuations around the saddle point of the replicated partition function. It can be seen that each of the terms of equation (7) diverges below T_c.

III. THE MEAN FIELD DYNAMICS

We shall mainly deal with the dynamical behaviour of the model. A Langevin dynamics for the Hamiltonian (1) joined with the spherical constraint gives the following equation of motion

$$\frac{\partial s_i(t)}{\partial t} = \sum_j J_{ij}s_j(t) - z(t)s_i(t) + h_i(t) + \xi_i(t) \tag{8}$$

where $z(t)$ is a time-dependent Lagrange multiplier, and $\xi_i(t)$ is a Gaussian noise with zero mean and variance $\langle\xi_i(t)\xi_j(t')\rangle = 2T\delta_{ij}\delta(t-t')$. In this model the solution of (8) for the component s_α can be explicitly written

Unfortunately it is not clear how to approach the equations of motion below T_c unless one knows which are the large-time asymptotic regimes for these four-time functions at low temperatures.

Furthermore one would like to understand if, and in which time regimes, a kind of fluctuation-dissipation relation can be written for these four-time dependent quantities.

These questions can by answered in the case of a simple spin-glass model which can be solved explicitly. It describes soft spins interacting through quenched random long range couplings and forced to satisfy a global spherical constraint. Statically, this so called *spherical $p = 2$ spin glass* does not present the peculiar features typical of other spin glasses and reveals itself to be a sort of disguised ferromagnet. Below a critical temperature T_c the system freezes in one of two possible states spontaneously breaking the symmetry under the parity operator that connects them.

Nevertheless, for temperatures below T_c and if the dynamics of the system begins in a random initial configuration, the model presents a non trivial out of equilibrium dynamical regime exhibiting the so called aging phenomenon and a violation of the fluctuation-dissipation theorem.

The simplicity of this model allowed for an explicit calculation of the two-point correlation and response functions both at equilibrium and in the aging regime. The violations of the FDT were calculated in the aging regime and have been related to a function X_t which asymptotically vanishes as $t^{-1/2}$ [8].

In this work we shall calculate the explicit form of the four-point correlation functions. Hopefully, this simple case will provide us some hints on how to approach more complicated models which cannot be solved without the use of the functional methods and therefore without an ansatz on the asymptotic behaviour of the solution.

II. THE STATICS

The spin glass spherical model is described by the Hamiltonian

$$H = -\frac{1}{2}\sum_{i \neq j} J_{ij} s_i s_j - \sum_i h_i s_i, \tag{1}$$

where $h_i(t)$ is an external magnetic field, and the spin variables s_i are forced to satisfy the spherical constraint

$$\sum_i^N s_i^2 = N. \tag{2}$$

The couplings J_{ij} are symmetric quenched random variables extracted from a Gaussian probability distribution with $\overline{J_{ij}} = 0$ and $\overline{J_{ij}^2} = 1/N$. The mean field solution of the statics is rather simple for this model [10]. Note that, using conventional notations, we have indicated by $\langle . \rangle$ the averages over the Boltzmann distribution, and by $\bar{\cdot}$ the averages over the different realizations of the quenched disorder J_{ij}.

The partition function is

$$Z = \int_{-\infty}^{\infty} \frac{dz}{2\pi i} \prod_i^N ds_i \exp\left(\frac{\beta}{2} \sum_{i\neq j} J_{ij} s_i s_j - z \sum_i s_i^2 \right), \tag{3}$$

Dynamical fluctuations in an exactly solvable model of spin glasses.

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In this work we calculate the dynamical fluctuations at O(1/N) in the low temperature phase of the $p = 2$ spherical spin glass model. We study the large-times asymptotic regimes and we find, in a short time-differences regime, a fluctuation dissipation relation for the four-point correlation functions. This relation can be extended to the out of equilibrium regimes introducing a function X_t which, for large time t, as $t^{-1/2}$ as in the case of the two-point functions.

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I. INTRODUCTION

The mean field Langevin dynamics for spin glasses has been quite extensively studied in recent years [1], [2], [3], [4], [5], [6], [8]. The main result has been that for low enough temperatures there is an off-equilibrium regime where the dynamics of the system depends on its whole history up to the beginning of its observation and often this feature is accompanied by a loss of the validity of the fluctuation dissipation theorem (FDT). Though it is in general very difficult to calculate the explicit time dependence of the correlation functions, except for some simple models, [8], their behaviour in all the different asymptotic regimes has been well understood [3] [4].

The main question we are left with is how to extend the mean field picture for finite dimensional systems. From the analytical point of view this amounts to take into account the corrections to the mean field limit.

In all generality one can determine closed equations for the two-point correlation functions as saddle points of an appropriate functional of a two time dependent field $Q_i^{\alpha\beta}(t_1, t_2)$, [1]. In this formalism, to consider the dynamical fluctuations around this mean field limit, one has to solve the equations of motion for the propagators, which are some four-times correlation functions and are related to the dynamical spin glass susceptibility.

At the dynamical critical temperature, where dynamical scaling is supposed to hold and the off-equilibrium features are not relevant, these equations have been solved for some models [7].