Recent rigorous results support the predictions of spontaneously broken replica symmetry for realistic spin glasses

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We show that the predictions of spontaneously broken replica symmetry are in perfect agreement with the recent rigorous results obtained by Newman and Stein.

In a very interesting paper Newman and Stein [1] have obtained new exact results for realistic spin glasses. In this note I observe that their results for the quantities that they define are in perfect agreement with the predictions of the spontaneously broken replica symmetry (SBRS). For the reader convenience I recall some predictions of SBRS.

1. We consider a spin glass model at low temperature. The system is in a box of side L, volume $V = L^D$, with fixed boundary condition (e.g. periodic). The Hamiltonian depends on a set of quenched variables J. We define the overlap q as $V^{-1}\sum_i \sigma_i\tau_i$, where σ and τ are two equilibrium configurations. The probability distribution of q is $P_J(q)$. Denoting by an upper bar the average over the quenched disorder J we can define the function $P(q_1) \equiv \overline{P_J(q_1)}$.

In the spin glass phase the funtion is not a simple delta function; it has a more complex structure, which signals the existence of many equilibrium states.

The same results are obtained if before taking the $L \to \infty$ limit and averaging over the quenched disorder we fix to arbitrary values the couplings J in a region of finite size, i.e. we only average over the couplings not belonging to such a finite region.

2. If the function $P(q_1)$ is not a delta function, one finds that the function $P_J(q_1)$ does depend on J and it is not a self-averaging quantity. In other words the quantity $P(q_1,q_2) \equiv \overline{P_J(q_1), P_J(q_2)}$ is not given by the product.$P(q_1)P(q_2)$. (Rigorous arguments in this direction have been recently given by Guerra [2]).

3. If we consider two copies of the same system, which have a mutual overlap q in the $L \to \infty$ limit, the probability of finding a region of size R where the overlap is $p \neq q$ goes to zero as $\exp(-R^\alpha f(p,q))$. The authors of [3] have estimated the exponent α to be equal to $D - \frac{5}{2}$.

4. If the spins σ and τ belong to two systems which differ in the value of the quenched disordered couplings J in a finite (arbitrarily small) portion of the lattice links [4], they turn out to have an overlap which is always equal to the minimum allowed (0 in zero magnetic field).

Spin glasses have a chaotic dependence on the coupling constant.

An apparent contradiction with SBRS is detected in the recent paper [1], in which the authors give two new definitions of a probability distribution of the overlaps q (which we indicate, with abuse of language, again by $P_J(q)$). Such $P_J(q)$ do not depend on J in the large volume limit. However the objects the authors define are different from the ones we usually encounter in the literature. We will show in the following that this J-independence turns out to be in perfect agreement with the SBRS approach.

Let us see which are the predictions of the mean field theory for quantities that are defined in [1]. For simplicity we will consider only two cases.

1. We consider a system of size L and we concentrate our attention on what happens in a box of size R. We define by q_R the overlap of two replicas in this box. We call I the couplings inside the box and E those outside the box. The couplings J are obviously determined by E and I ($J = I \oplus E$) .

Following the first construction of ref. [1] we define

$$P_I^{(1)}(q) \equiv \int d\mu(E) P_J^R(q) \ , \tag{1}$$

where $P_J^R(q)$ is the probability distribution of the overlap q_R, i.e. of the overlap restricted to the region R.

Let us first send $L \to \infty$, or if we prefer, let us consider the case $L >> R$. When R goes to infinity q and q_R are equal and $P_I^{(1)}(q)$ coincides with $\int d\mu(E)P_J(q)$. This last integral does not depend on I, so for large R mean field theory predicts that

$$P_I^{(1)} = P(q) \ , \tag{2}$$

and it is independent from I as rigorously proven in [1].

2. Let us discuss a second definition, $P^{(2)}(q)$, which [5] is inspired by the second construction of ref. [1]. We consider two systems, one with couplings $J_1 = I \oplus E_1$ and the other with couplings $J_2 = I \oplus E_2$. We consider the distribution probability of the overlaps q_R and q among a configuration of the first system and a configuration of the second system, the first overlap (q_R) being restricted to the region of size R (where the couplings are I for both systems). We introduce the corresponding probability distributions which obviously depend on the couplings I, E_1 and E_2.

We define

$$P_I^{(2)}(q_R) \equiv \int d\mu(E_1) d\mu(E_2) P^R_{I,E_1,E_2}(q_R) \ ,$$
$$P_I(q) \equiv \int d\mu(E_1) d\mu(E_2) P_{I,E_1,E_2}(q) \ . \tag{3}$$

Also in this case $P_I(q)$ and $P_I^{(2)}(q)$ coincide in the large R limit. $P_I^{(2)}(q) = \delta(q)$, due to the chaotic nature of spin glasses. $P_I^{(2)}$ is independent from I, as proven in [1] and it is different from $P_I^{(1)}$.

The SBRS theory is sophisticated enough to give different answers to different questions, spurring from different definitions of the overlap distribution. In all cases it gives the correct answer, i.e. it declares self-averaging objects that one can rigorously prove to be self-averaging (even for realistic spin glasses) and non-self-averaging quantities that are rigorously shown to be non-self-averaging.

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