1-loop contribution to the dynamical exponents in spin glasses

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Abstract

We evaluate the corrections to the mean field values of the x and z exponents at the first order in the ϵ-expansion, for $T = T_c$. We find that both x and z are decreasing when the space dimension decreases.

We want to investigate the purely relaxational dynamics of a short-range spin glass model for $T \to T_c^+$, in the framework of the ϵ-expansion. The dynamical properties of the model in the mean field theory are very different from those of the models whose dynamical properties are usually investigated in the literature. These new feature make the computation of the dynamical critical exponents much more involved than that of the usual.

In a previous work, [6], we evaluated the Gaussian dynamical fluctuations of the order parameter around the MF limit. The aim of this letter is to pursue this analysis, by considering the 1-loop correction to the mean field (MF) theory in a renormalization group calculation. At the first order in ϵ, unlike Zippelius [5], we find results that disagree with the conventional Van Hove theory, because we obtain a correction to the kinetic coefficient already to the lowest order in the loop expansion. In this work, first we evaluate the correction to MF value of the critical exponents x, that describes the critical slowing down of the dynamical order parameter at the critical point, then we evaluate the correction to the z exponent that describes the critical slowing down of the dynamical spin glass susceptibility χ_{SG}. Finally, we check that the scaling low, which connect these two exponents, is verified.

We study the soft-spin version of the EA model given by the Hamiltonian:

$$\beta\mathcal{H} = -\beta \sum_{\langle ij \rangle} J_{ij} s_i s_j + \frac{1}{2} r_o \sum_i s_i^2 + \frac{1}{4!} g \sum_i s_i^4 \ , \tag{1}$$

where J_{ij} are random Gaussian interactions between the nearest-neighbors sites, with zero average and mean square fluctuations $[(J_{ij})^2] = j^2/n$ (n is the coordination number). Purely relaxation dynamics is introduced by the Langevin equation:

$$\Gamma_0^{-1} \frac{\partial s_i(t)}{\partial t} = -\frac{\partial(\beta\mathcal{H})}{\partial s_i(t)} + \eta_i(t), \tag{2}$$

where η is the usual Gaussian noise with zero average and variance $\langle \eta_i(t)\eta_j(t')\rangle = \frac{2}{\Gamma_0}\delta_{ij}\delta(t-t')$. The interesting physical quantities in MF theory are the averaged response and correlation functions, defined respectively:

$$\overline{G}(t-t') = \left[\frac{\partial \langle s_i(t)\rangle_\eta}{\partial h_i(t')}\right]_J \tag{3}$$

$$\overline{C}(t-t') = [\langle s_i(t) s_i(t')\rangle_\eta]_J \ , \tag{4}$$

where the angular brackets $\langle ..\rangle_\eta$ refer to averages over the noise and the square brackets $[..]_J$ over quenched disorder.

Moreover, considering the Gaussian fluctuation, we can define the dynamical spin glass susceptibility as follow:

$$\chi_{SG}(i-j; t_3 - t_1, t_2 - t_4) = \left[\frac{\partial \langle s_i(t_3)\rangle_\eta}{\partial h_j(t_1)} \frac{\partial \langle s_i(t_2)\rangle_\eta}{\partial h_j(t_4)}\right]_J . \tag{5}$$

The dynamical scaling implies that the decay of $C(t)$ is governed by a characteristic time τ, which diverges at T_c, as, for long t, we can write:

$$C(t) = \frac{1}{t^x} \tilde{q}^+(t/\tau), \tag{6}$$

where $\tilde{q}^+$ is the universal scaling function in the region $T \to T_c^+$. The relaxation time divergence, at the critical point, is connected to the correlation length divergence through the dynamical exponent z:

$$\tau \propto \xi^z . \tag{7}$$

According to the scaling hypothesis, this exponent is related to the slowing down of the spin glass susceptibility, and we expect to have:

$$\chi_{SG}(k,\omega)=\omega^{\frac{2-\eta}{z}}\tilde{f}(k^z/\omega) \tag{8}$$

The MF behaviour of this model ($n=N$, number of spins, long-range limit), in the critical region, is well known [1], [2], [3]. In the low-frequency limit the response and correlation functions are respectively:

$$\overline{G}(\omega)=\left(1-\sqrt{-i\omega}\right), \tag{9}$$

$$\overline{C}(\omega)=\frac{2}{\sqrt{i\omega}+\sqrt{-i\omega}}, \tag{10}$$

while, in the Gaussian approximation, the spin glass susceptibility is

$$\chi_{SG}(k,\omega_1,\omega_2)=\frac{1}{k^2+\sqrt{-i\omega_1}+\sqrt{-i\omega_2}}. \tag{11}$$

The MF value of the dynamical critical exponents x and z, as known, are $1/2$ and 4 respectively.

To deal with Langevin disordered dynamic theory as usual we use the dynamic functional integral method [7]. In this formalism, it is conventional to introduce an auxiliary field $\hat{s}_i(t)$ and to define an effective Lagrangian of an Hubbard-Stratanovich field $Q_i^{\alpha\beta}(t,t')$, [2], [5], such as:

$$2\langle Q^{\alpha\beta}_{k=0}(t_1,t_2)\rangle_{L(Q_{\alpha\beta})}=[\langle\phi_i^\alpha(t_1)\phi_i^\beta(t_2)\rangle_\eta]_J \tag{12}$$

where the two component vector field is defined:

$$\phi_i^\alpha=(i\hat{s}_i,s_i). \tag{13}$$

In considering the correction to MF approximation we derive the following Lagrangian, as series expansion around the order parameter saddle-point value $\overline{Q^{\alpha\beta}}(t,t')$, [3], [5], [6]:

$$\begin{aligned}L(\delta Q^{\alpha\beta})=&-\sum_{t_1,t_2}\sum_{i,j}\tilde{K}^{-1}_{i,j}\delta Q_i^{\alpha\beta}(t_1,t_2)A^{\alpha\beta\gamma\delta}\delta Q_j^{\gamma\delta}(t_1,t_2)+\\&\frac{1}{2}\sum_{t_1,t_2,t_3,t_4}\sum_i\delta Q_i^{\alpha\beta}(t_1,t_2)C^{\alpha\beta\gamma\delta}(t_1,t_2,t_3,t_4)\delta Q_i^{\gamma\delta}(t_3,t_4)+\\&\frac{1}{3!}\sum_{t_1,t_2,t_3,t_4,t_5,t_6}\sum_i C^{\alpha\beta\gamma\delta\mu\nu}(t_1,t_2,t_3,t_4,t_5,t_6)\delta Q_i^{\alpha\beta}(t_1,t_2)\delta Q_i^{\gamma\delta}(t_3,t_4)\delta Q_i^{\mu\nu}(t_5,t_6)\,.\end{aligned} \tag{14}$$

For the structure and the meaning of each term of (14), the reader is referred to [6]. The non local connected propagators of the theory are defined as follow:

$$\begin{aligned}
G^{\alpha\beta\gamma\delta}(i-j;t_1,t_2,t_3,t_4) &= [\langle\phi_i^\alpha(t_1)\phi_i^\beta(t_2)\phi_j^\gamma(t_3)\phi_j^\delta(t_4)\rangle_\eta]_J - \\
&\quad [\langle\phi_i^\alpha(t_1)\phi_i^\beta(t_2)\rangle_\eta]_J[\langle\phi_j^\gamma(t_3)\phi_j^\delta(t_4)\rangle_\eta]_J \\
&= 4\sum_l(\tilde{K}^{-1})_{il}\sum_k(\tilde{K}^{-1})_{jk}\langle\delta Q_l^{\alpha\beta}(t_1,t_2)\delta Q_k^{\gamma\delta}(t_3,t_4)\rangle_{L(Q_{\alpha\beta})} \\
&\quad -2(\tilde{K}^{-1})_{ij}A^{\alpha\beta\gamma\delta}\delta(1-3)\delta(2-4)\ . \qquad (15)
\end{aligned}$$

In [6] we evaluated the critical behaviour of these propagators for each combination of the indices $\alpha,\beta,\gamma,\delta$ and in any time interval, when the cubic interactions is vanishing. We write down the general structure of $G^{2211}(\omega_1,\omega_2,\omega_3,\omega_4)$ which is present in several of the following 1-loop functions:

$$\begin{aligned}
G^{2211}(k;\omega_1,\omega_2,\omega_3,\omega_4) &= [\delta(\omega_1+\omega_3)\delta(\omega_2+\omega_4)+\delta(\omega_2+\omega_3)\delta(\omega_1+\omega_4)] \\
&\quad \tilde{G}^{2211}(k;\omega_1\omega_2)+\delta(\omega_1+\omega_2+\omega_3+\omega_4)\tilde{\tilde{G}}^{2211}(k;\omega_1,\omega_2,\omega_3,\omega_4) \qquad (16)
\end{aligned}$$

where:

$$\tilde{G}^{2211}(k;\omega_1,\omega_2) = \frac{1}{ck^2+\sqrt{-i\omega_1}+\sqrt{-i\omega_2}} \qquad (17)$$

$$\begin{aligned}
\tilde{\tilde{G}}^{2211}(k;\omega_1,\omega_2,\omega_3,\omega_4) &= \left(\overline{C}(\omega_1)\tilde{G}^{2211}(k;-\omega_1,\omega_2)+\overline{C}(\omega_2)\tilde{G}^{2211}(k;\omega_1,-\omega_2)\right)\times \\
&\quad \tilde{G}^{2211}(k;\omega_1,\omega_2)\tilde{G}^{2211}(k;-\omega_3,-\omega_4)g_r(k;\omega_1+\omega_2) \qquad (18)
\end{aligned}$$

$$g_r(k;\omega_1+\omega_2) = \left(ck^2+\sqrt{-i(\omega_1+\omega_2)}\right)F_1\left(\frac{k^2}{(\omega_1+\omega_2)^{1/2}}\right); \qquad (19)$$

where $F_1(x)$ is an homogenous function of the $\frac{k^2}{\sqrt{\omega}}$. We recall that in this formalism the time-dependent spin glass susceptibility (5) is:

$$\chi_{SG}(i-j;t_3-t_1,t_2-t_4) = [\langle s_i(t_3)\eta_j(t_1)\rangle_\eta\langle s_j(t_2)\eta_i(t_4)\rangle_\eta]_j = \tilde{G}^{1221}(i-j;t_3-t_1,t_2-t_4). \qquad (20)$$

Let us consider the 1-loop correction to the "free" theory. We intend to use the propagators derived in [6] to evaluate the contribution of the 1-loop Feynman diagrams to the mean value $\overline{Q^{\alpha\beta}}$, to the bare propagators $G^{\alpha\beta\gamma\delta}$, and to the bare cubic vertices.

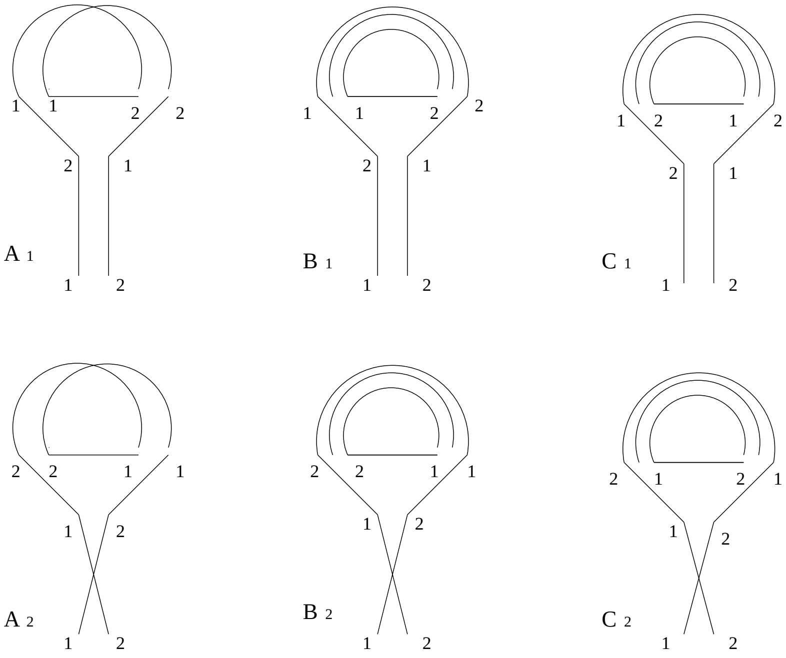

Figure 1:

We consider the 1-loop Feynman diagrams as a g-series expansion, by using the correspondent propagators. In analogy with the static, we can guess that the g-dependent part of the 1-loop corrections is not singular at the critical point, as soon as $g \neq 0$. The physical quantities have to be not affected by the value of g, provided that it is not zero. The behaviour at $g = 0$ is rather different and the ϵ-expansion starts from $D = 8$, [12].

Concerning the 1-point function, let us consider the response function, that as a consequence of eq. (12) is:

$$G(\omega) = 2\langle Q^{21}_{k=0}(\omega)\rangle \tag{21}$$

The diagrams that occur in 1-loop correction to the MF value of the x exponent are shown in Fig.[1]: two continuous lines represent a bare factorized in time propagator

$$\tilde{G}^{\alpha\beta\gamma\delta}(k; w_1, w_2)\,[\delta(\omega_1+\omega_3)\delta(\omega_2+\omega_4) + \delta(\omega_1+\omega_4)\delta(\omega_2+\omega_3)]\,,$$

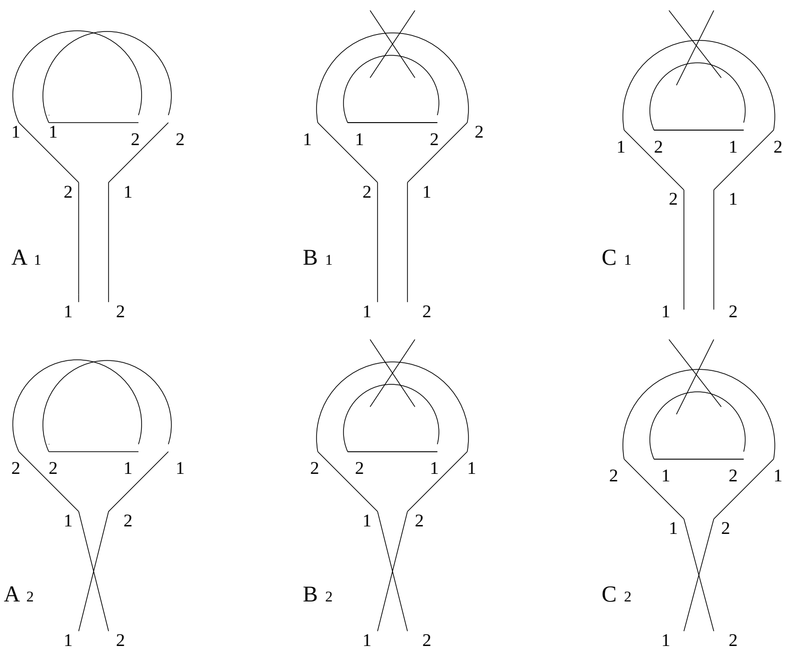

Figure 2:

three lines the bare connected in time propagator

$$\tilde{\tilde{G}}^{\alpha\beta\gamma\delta}(k; w_1, w_2, \omega_3, \omega_4)\delta(\omega_1 + \omega_2 + \omega_3 + \omega_4)$$

and, finally, the triangle in the center of the diagrams is a factorized in time cubic vertex. We succeed in evaluating the singular behaviour of the diagrams A_1 and A_2, while we have to use a trick to take into account the contribution from the others. By using the series expansion in g of the propagators in the loop, see [6], we obtain a correspondent g series of the two 1P.I. diagrams, but the zero term is missing. Let us add and subtract the term we need. As in the static case (where we deal with just the momentum variable k, and we easily manage to resum the g-series), we can suppose that the resumation of the series give a non singular behaviour at the critical point (the presence of g remove the pole at zero momentum for $T = T_c$). We left with the diagrams A_1, A_2 and with the two zero-order in g diagrams that we need for the series resumation (with negative sign), (see Fig.[2]):

$$\left(\frac{u}{2}\right)\ 2\left[\int\frac{d^dk}{(2\pi)^d}\frac{1}{ck^2+2\sqrt{i\omega}}-2\int\frac{d^dk}{(2\pi)^d}\frac{1}{ck^2+\sqrt{i\omega}}\right] \tag{22}$$

The propagators involved in these "new" diagrams are represented, in Fig.[2], with two crossed lines. By evaluating the previous integrals for $d=6$, we find that, at the first order in the loop expansion, the response function is

$$\begin{aligned} G(\omega) &= 1-(-i\omega)^x = \overline{G}(\omega)+2\ u\ \langle\delta Q^{21}(\omega)\rangle \\ &= 1-\sqrt{-i\omega}-\frac{k_6}{(2\pi)^6}u^2\left[I(\omega=0)+\frac{1}{4}\sqrt{-i\omega}\,ln(\omega)\right] \end{aligned} \tag{23}$$

As for the statics u has been introduced as expansion parameter in the cubic vertices, to apply the renormalization group method for critical phenomena. The factor u^2 in the equation (23) is due to the fact that the saddle point response function (9) is of order $(\frac{1}{u})$.

In the same way, we derive the flux equation of the 1P.I. vertex functions which allow us to determine the fixed point below 6 dimension. As for the conventional Langevin dynamical theories, the IR stable fixed point below 6 dimension is the same that the static one (i.e., we find the same relevant diagrams):

$$(u^*)^2=\frac{(2\pi)^6}{k_6}\frac{\epsilon}{2}. \tag{24}$$

The series expansion in ϵ for the static exponents is evaluated up the 3th order in Ref. [10] and Ref. [11].

Let us determine the first order correction for the dynamical ones. Substituting the fixed point value (24) into (23) we obtain for x:

$$x=\frac{1}{2}\left(1-\frac{\epsilon}{4}\right). \tag{25}$$

In the same way, we find that the relevant 1-loop 1P.I. diagrams for the Self-Energy $\Sigma(\omega_1,\omega_2)$, shown in Fig [3], give the following contribution to the spin glass susceptibility (20):

$$A:\frac{u^2}{2}\int\frac{d^dk}{(2\pi)^d}\left(\frac{1}{c(p-k)^2+2\sqrt{i\omega_1}}\frac{1}{ck^2+\sqrt{i\omega_1}+\sqrt{-i\omega_2}}\right) \tag{26}$$

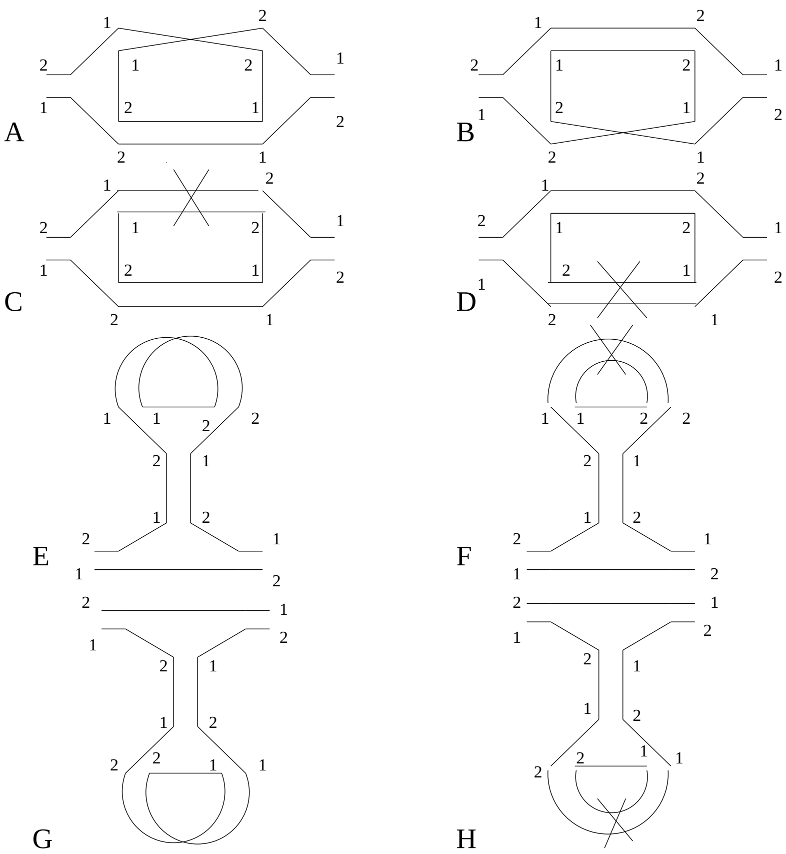

Figure 3:

$$B : \frac{u^2}{2} \int \frac{d^d k}{(2\pi)^d} \left(\frac{1}{c(p-k)^2 + 2\sqrt{-i\omega_2}} \frac{1}{ck^2 + \sqrt{i\omega_1} + \sqrt{-i\omega_2}} \right) \quad (27)$$

$$C : -2\frac{u^2}{2} \int \frac{d^d k}{(2\pi)^d} \left(\frac{1}{c(p-k)^2 + \sqrt{i\omega_1}} \frac{1}{ck^2 + \sqrt{-i\omega_2}} \right) \quad (28)$$

$$D : -2\frac{u^2}{2} \int \frac{d^d k}{(2\pi)^d} \left(\frac{1}{c(p-k)^2 + \sqrt{-i\omega_2}} \frac{1}{ck^2 + \sqrt{i\omega_1}} \right) \quad (29)$$

$$E : \frac{u^2}{2}\, 2\, \frac{1}{cp^2 + \, 2\sqrt{i\omega_1}} \int \frac{d^d k}{(2\pi)^d} \frac{1}{ck^2 + 2\sqrt{i\omega_1}} \quad (30)$$

$$F : -2\frac{u^2}{2}\, 2\, \frac{1}{cp^2 + \, 2\sqrt{i\omega_1}} \int \frac{d^d k}{(2\pi)^d} \frac{1}{ck^2 + \sqrt{i\omega_1}} \quad (31)$$

$$G : \frac{u^2}{2}\, 2\, \frac{1}{cp^2 + \, 2\sqrt{-i\omega_2}} \int \frac{d^d k}{(2\pi)^d} \frac{1}{ck^2 + 2\sqrt{-i\omega_2}} \quad (32)$$

$$H : -2\frac{u^2}{2}\, 2\, \frac{1}{cp^2 + \, 2\sqrt{-i\omega_2}} \int \frac{d^d k}{(2\pi)^d} \frac{1}{ck^2 + \sqrt{-i\omega_2}} \quad (33)$$

At the first order in ϵ, the sum of the contributions to z exponent of the diagrams A, B, C, D is vanishing. On the other hand, from the sum of the diagrams E, F, G and H we obtain the following contribution to the 1P.I. 2-point function:

$$\chi_{SG}^{-1}(p; -\omega_1, \omega_2)\Big|_{p=0} = \overline{\chi_{SG}^{-1}}(-\omega_1, \omega_2) + \Sigma(\omega_1, \omega_2) =$$
$$\sqrt{i\omega_1} + \sqrt{-i\omega_2} + \frac{u^2}{4} \frac{k_6}{(2\pi)^6} \left(\sqrt{i\omega_1}\; ln(\omega_1) + \sqrt{-i\omega_2}\; ln(\omega_2) \right) \quad (34)$$

At the fixed point u^* given from (24), we obtain the following correction to z exponent:

$$z = \frac{(2-\eta)}{\frac{1}{2}\left(1 + \frac{\epsilon}{4}\right)} = 4\left(1 - \frac{\epsilon}{12}\right) \quad (35)$$

The scaling relation between the exponents x and z, that we recall to be

$$x = \frac{d - 2 + \eta}{2z}, \quad (36)$$

is verified, at the first order in ϵ.

Numerical simulations for the exponents x and z can be found in the literature, the values are $z \approx 7$ and $x \approx .06$ in dimensions $D = 3$, [13], and

$z \approx 5$ and $x \approx .15$ in dimensions $D = 4$, [14]. Our prediction states that both the values of x and z are decreasing when the dimension decreases. This is true for x but not for z. The apparent discrepancy that we have with the behaviour of z should not worry us. In fact, also in the static case, the critical exponents for spin glasses have a badly convergent ϵ-expansion and the prediction of this expansion can be hardly applied in 3 or 4 dimensions.

A numerical study of what happens in 5 dimensions is necessary. Moreover it should be noticed that usual arguments imply that our computation predicts, without ambiguities, that the logarithmic corrections in 6 dimensions are such to decrease the effective value of the exponents.

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CAPTIONS FOR ILLUSTRATIONS

Fig.1 1-loop diagrams that contribute to the x exponent.

Fig.2 g-series resumation of the 1-loop diagram contribution to the x exponent.

Fig.3 g-series resumation of the 1-loop diagram contribution to the Self-Energy.