Crossovers in the Two Dimensional Ising Spin Glass with ferromagnetic next-nearest-neighbor interactions

Giorgio Parisi[a], Juan J. Ruiz-Lorenzo[a] and Daniel A. Stariolo[b]

[a] Dipartimento di Fisica and INFN, Università di Roma *La Sapienza*
P. A. Moro 2, 00185 Roma (Italy)

[b] Departamento de Fisica, Universidade Federal de Viçosa
36570-000 Viçosa, MG (Brazil)

giorgio.parisi@roma1.infn.it

ruiz@chimera.roma1.infn.it

stariolo@mail.ufv.br

January 21, 1998

Abstract

By means of extensive computer simulations we analyze in detail the two dimensional $\pm J$ Ising spin glass with ferromagnetic next-nearest-neighbor interactions. We found a crossover from ferromagnetic to "spin glass" like order both from numerical simulations and analytical arguments. We also present evidences of a second crossover from the "spin glass" behavior to a paramagnetic phase for the largest volume studied.

1 Introduction

At present it is clear that the lower critical dimension of Edwards-Anderson Ising spin glasses is at some point between $D = 2$ and $D = 3$ [1]. By using different numerical techniques as for instance Monte Carlo simulations, exact ground state calculations and diagonalization of the transfer matrix, it is rather well established that the two dimensional Ising spin glass presents a transition at $T = 0$. Furthermore, an experimental realization of a 2D spin glass also points to a transition only at $T = 0$ [2, 3]. However the values of the critical exponents are still a matter of controversy[1, 4, 5, 6, 7, 3][1].

In a recent paper Lemke and Campbell [9] studied the $2D \pm J$ Ising spin glass with nearest neighbors interactions adding a ferromagnetic interaction between next-nearest-neighbors. Surprisingly, they found numerical evidence for a spin glass transition at finite temperature simulating intermediate lattice sizes.

In order to gain more insight on the nature of the low temperature properties of the model we have computed through extensive simulations a variety of global quantities as well as staggered ones (i.e. observables defined in one of the two sub-lattices in which the system naturally divides), like spin glass and ferromagnetic susceptibilities, specific heat, Binder cumulants, magnetizations and the spin glass order parameter.

In particular we have studied how the originally stable staggered phase is destroyed by the spin glass interaction and finally we have studied if the spin glass phase is stable in the thermodynamic limit.

We will show numerical evidences for two different crossovers: the first one from the staggered ferromagnetic phase to a spin glass phase; and the second one from the spin glass phase to a paramagnetic phase.

The plan of this paper is the following. In the next section we will define the model and describe limiting cases in the parameter space. In the third section we will map the model to the Random Field Ising Model and we will obtain analytical evidences of both crossovers (moreover we will compute the dependence of the first crossover length on the parameters of the model). In sections fourth and fifth we will study numerically the model. Finally we will present our conclusions.

2 The model

The model we are considering is defined by the following Hamiltonian on a square lattice with periodic boundary conditions

$$H = -\sum_{<ij>} J_{ij} S_i S_j - K \sum_{\ll kl \gg} S_k S_l \ , \tag{1}$$

where $< ij >$ denotes sum over all the first nearest neighbors pairs (distance 1) and $\ll kl \gg$ denotes sum over all the second nearest neighbors pairs (distance $\sqrt{2}$).The couplings J_{ij} are quenched variables with $J_{ij} = \pm\lambda$ with probability 1/2. λ and K are positive

[1]Recently has been pointed out by some authors [8] that the two dimensional Ising spin glass shows a phase transition at finite temperature, nonetheless the scaling plots of these works are compatible with a $T = 0$ phase transition in agreement with all the previous cited studies.

constants whose ratio determines the relative strength between spin glass and ferromagnetic interactions. In the rest of the paper we fix $K = 1$. In order to fix the notation we also define the variance of the J_{ij} distribution as $\sigma_J^2 \equiv \overline{J_{ij}^2} = \lambda^2$.

We next describe two limiting cases of the model (for $K = 1$):

- For $\lambda \to \infty$ the system is the $2D$ spin glass for which is known that $T_c = 0$ [1, 4, 5, 6, 7, 3].
- When $\lambda = 0$ the whole lattice decouples into two independent sub-lattices (that we will denote hereafter as sub-lattices 1 and 2, the black and white sub-lattices in a chess-board. Each sub-lattice is itself a two dimensional ferromagnetic Ising model and will have a phase transition (paramagnetic-ferromagnetic) just at the Onsager temperature: $T_c = 2/\log(1+\sqrt{2}) \simeq 2.269$. The order parameter of the phase transition is the so-called staggered magnetization: i.e. the magnetization of one of the two sub-lattices. Obviously, the probability distribution of the total magnetization of the whole lattice for low temperatures will take into account the four possible different magnetizations of the two independent sub-lattices.

So initially we have, when $\lambda = 0$, ferromagnetic order (in both sub-lattices), that can be affected by the introduction of spin glass couplings. The effect of these couplings is to couple (by means of a spin glass interaction) both sub-lattices.

In the next section we will examine analytically the effect of the introduction of random couplings (linking both sub-lattices) in the original stable (staggered) ferromagnetic order.

3 Analytic Results

We will study in this section the analogy, suggested by Lemke and Campbell [9], between the Hamiltonian defined by Eq.(1) and the Random Field Ising Model (RFIM).

We can rewrite the original Hamiltonian Eq.(1) in the following way

$$\mathcal{H} = -\sum_{<i_1 j_1>} \sigma_{i_1}\sigma_{j_1} - \sum_{<i_2 j_2>} \tau_{i_2}\tau_{j_2} - \sum_{i_1}\sum_{j_2(i_1)} J_{i_1 j_2}\sigma_{i_1}\tau_{j_2} \ , \tag{2}$$

where the indices i_1, j_1 (i_2, j_2) run over the sub-lattice 1 (respectively 2), $< i_1 j_1 >$ ($< i_2 j_2 >$) denotes sum to first nearest neighbors pairs in the sub-lattice 1 (respectively 2) and $j_2(i_1)$ denotes the sum over the two nearest neighbors (j_2's) of the site i_1 (in the two positive directions from i_1) in the whole lattice. Moreover we have denoted the variables of the sub-lattice 1 (2) as σ's (respectively τ's).

Now we fix the temperature to a small value and all the spins inside the sub-lattice 1 are fixed in the up state. Next we will examine the cost in energy to do a compact droplet (in the sub-lattice 1) with all its spins flipped down, with the spins of the sub-lattice 2 fixed to an arbitrary configuration.

The Hamiltonian of the model with all the τ spins fixed to an arbitrary configuration is, modulo a constant,

$$\mathcal{H}[\sigma|\tau \text{ fixed}] \quad = \quad -\sum_{<i_1 j_1>} \sigma_{i_1}\sigma_{j_1} - \sum_{i_1}\left[\sum_{j_2(i_1)} J_{i_1 j_2}\tau_{j_2}\right]\sigma_{i_1}$$

$$\equiv \; - \sum_{<i_1 j_1>} \sigma_{i_1}\sigma_{j_1} - \sum_{i_1} h_{i_1}\sigma_{i_1} \; , \tag{3}$$

i.e. a Random Field Ising Model (RFIM) where the magnetic field has zero mean and variance:

$$\overline{h_i h_j} = \begin{cases} 2\sigma_J^2 & \text{if } i = j \; , \\ 0 & \text{elsewhere.} \end{cases} \tag{4}$$

In particular we can think that all the spins in the τ sub-lattice are fixed up. It is easy to reproduce all the steps of the Imry-Ma argument [10, 11] (by turning on the temperature) to show that one can find large regions where it is favorable energetically to flip all the spins inside the region and hence to destroy the long range order of the sub-lattice 1 independently of the configuration of the sub-lattice 2 (and vice-versa).

Thus we have shown that the ferromagnetic order (in either or in both sub-lattices) is unstable against an infinitesimal strength of the spin glass couplings: i.e. initially the sub-lattices 1 and 2 are fixed to up (staggered ferromagnetic order) and we have found that for any configuration τ the sub-lattice 1 disorders, and by redoing the same steps with sub-lattice 1 fixed (and disordered) we can see that the originally ordered sub-lattice 2 disorders too.

Moreover, following Binder [12], we should expect that there exists a crossover length R_c such that for $R < R_c$ the staggered ferromagnetic order is stable but this order is unstable for scales $R > R_c$. The analytical expression for a RFIM with ferromagnetic coupling K and uncorrelated magnetic field with variance σ_h^2 is [12]

$$R_c \propto \exp\left[C\left(\frac{K}{\sigma_h}\right)^2\right] \; , \tag{5}$$

where C is a constant (O(1)), and so for our particular model (where $K = 1$ and $\sigma_h^2 = 2\sigma_J^2$), we finally obtain

$$R_c \simeq \exp\left[\frac{C}{2\sigma_J^2}\right] = \exp\left[\frac{C}{2\lambda^2}\right] \; . \tag{6}$$

Obviously when λ is infinitesimally small (i.e. we put an infinitesimal amount of spin glass disorder in the model) the crossover ratio is exponentially large and ferromagnetic order will only be destabilized in extremely large systems. Nevertheless, in the thermodynamic limit the spin glass disorder is always relevant.

Finally, we can estimate that for $\lambda = 0.5$ (the value that we have used in our numerical simulations presented in this work) $R_c \simeq 7$, assuming that C is just 1.

At this point for $L > R_c$, where L is the linear size of the system, the picture is the following: both sub-lattices are broken in clusters (inside of them all the spins, in average, point in the same direction) of size less than R_c interacting between them. From the Imry-Ma argument it is clear that the "effective" interaction between these clusters is short range (i.e. it could be very large but not infinite). Consequently, we have a two dimensional spin glass with short range interactions. This phase can be thought as a frozen disordered phase (like the spin glass phase) where the clusters play the role of the spin in the usual spin glass phase.

But we know that there exists no spin glass order at finite temperature for short range spin glasses in two dimensions, and so we conclude that there must be a second crossover from the spin glass behavior to paramagnetic behavior as the size of the system increases. We need only a correlation length greater than the range of the interaction between the clusters to return to the usual short range spin glass in two dimensions that has no phase transition. In the next sections we will try to put this fact in more quantitative grounds.

4 Numerical Simulations and Observables

We have simulated, using the Metropolis algorithm, systems with linear sizes ranging from $L = 4$ to $L = 48$ and averaging over 200 to 10000 samples depending on the size. The largest lattice size simulated in reference [9] used in the computation of their Binder cumulant was $L = 12$.

In all the runs we have used an annealing procedure from higher temperatures to the lower ones in order to thermalize the system. In Table 1 we report the statistics we have used. We have performed in the annealing procedure for all the temperatures the same number of thermalization steps (N_T), that we have written in Table 1.

For a given temperature we run N_T steps for thermalization and we measure during $2N_T$ and then we lower the temperature and we repeat the process always with the same N_T. We will return to the issue of the thermalization time at the end of this section.

L	T's	samples	Termalization time
4	[1.5,4.0]	10000	10000
6	[1.5,4.0]	10000	10000
8	[1.5,4.0]	4000	30000
16	[1.5,3.0]	1633	30000
24	[1.5,3.0]	700	60000
32	[1.5,3.0]	1576	90000
48	[1.4,2.5]	426(*)	900000

Table 1: Description of our runs. The step in temperatures was 0.1 in all runs. (*) means that for the $L = 48$ lattice in the measure of the staggered overlap we only have simulated 100 samples.

In the next paragraphs we will describe the observables that we have measured in our numerical simulations.

We have measured the global (m) and staggered (m_s) magnetizations :

$$m \equiv \frac{1}{L^2} \sum_i S_i \ , \ m_s \equiv \frac{2}{L^2} \sum_{i_1} S_{i_1} \ , \tag{7}$$

where the sum runs over all the lattice sites (i) and over a sub-lattice (i_1) respectively.

In order to calculate spin glass quantities we have simulated two replicas α and β in parallel with the same disorder. The overlaps between the replicas, global (q) and staggered (q_s), are:

$$q \equiv \frac{1}{L^2} \sum_i S_i^\alpha S_i^\beta \ , \ q_s \equiv \frac{2}{L^2} \sum_{i_1} S_{i_1}^\alpha S_{i_1}^\beta \ , \tag{8}$$

where, again, the sum runs over all the lattice sites (i) and over one of the two sub-lattices (i_1) respectively.

The magnetic global and staggered susceptibilities (without the β factor) are defined as:

$$\chi \equiv L^2 \left[\overline{\langle m^2 \rangle} - \left(\overline{\langle |m| \rangle} \right)^2 \right] \quad , \quad \chi_s \equiv \frac{L^2}{2} \left[\overline{\langle m_s^2 \rangle} - \left(\overline{\langle |m_s| \rangle} \right)^2 \right] \ . \tag{9}$$

The spin glass or overlap susceptibilities (χ_q, χ_q^s) are defined by:

$$\chi_q \equiv L^2 \left[\overline{\langle q^2 \rangle} - \left(\overline{\langle |q| \rangle} \right)^2 \right] \quad , \quad \chi_q^s \equiv \frac{L^2}{2} \left[\overline{\langle q_s^2 \rangle} - \left(\overline{\langle |q_s| \rangle} \right)^2 \right] \ . \tag{10}$$

We have measured also the Binder cumulants of the magnetization: global g_m and staggered g_m^s,

$$g_m \equiv \frac{1}{2} \left[3 - \frac{\overline{\langle m^4 \rangle}}{\left(\overline{\langle m^2 \rangle} \right)^2} \right] \quad , \quad g_m^s \equiv \frac{1}{2} \left[3 - \frac{\overline{\langle m_s^4 \rangle}}{\left(\overline{\langle m_s^2 \rangle} \right)^2} \right] \tag{11}$$

and Binder parameters of the overlaps: global g_q and staggered g_q^s,

$$g_q \equiv \frac{1}{2} \left[3 - \frac{\overline{\langle q^4 \rangle}}{\left(\overline{\langle q^2 \rangle} \right)^2} \right] \quad , \quad g_q^s \equiv \frac{1}{2} \left[3 - \frac{\overline{\langle q_s^4 \rangle}}{\left(\overline{\langle q_s^2 \rangle} \right)^2} \right] \ . \tag{12}$$

Finally the specific heat is defined by:

$$C_V \equiv \frac{1}{L^2} \left(\overline{\langle \mathcal{H}^2 \rangle} - \overline{\langle \mathcal{H} \rangle}^2 \right) \ . \tag{13}$$

In order to decide a safe thermalization time, N_T, (written in Table 1) we have used the method proposed by Bhatt and Young [5] which consists in running, at the lowest temperature, with an ordered (all spins up) and high temperature initial configurations and monitoring the behavior of the susceptibilities (in our case the non connected overlap susceptibility: $L^2 \overline{\langle q^2 \rangle}$) with the Monte Carlo time. When the two curves reach the same plateau we can say that the system has thermalized. We show in Figures 1 and 2 this procedure for two of our biggest lattices and at the lower temperatures simulated (i.e. $L = 32$ and $T = 1.5$ and $L = 48$ and $T = 1.4$ respectively).

We remark that we have used for *all* temperatures of the annealing procedure the value that we have computed for the lower one (that we have reported in Table 1).

5 Numerical Results

5.1 Crossover Ferromagnetic-Spin Glass

In this sub-section we will show numerical evidences of the first crossover: from a staggered ferromagnetic phase to a "spin glass" phase.

It is natural to study this crossover examining firstly the susceptibility and the Binder cumulant of the staggered magnetization, Figures 3 and 4 respectively. They are the

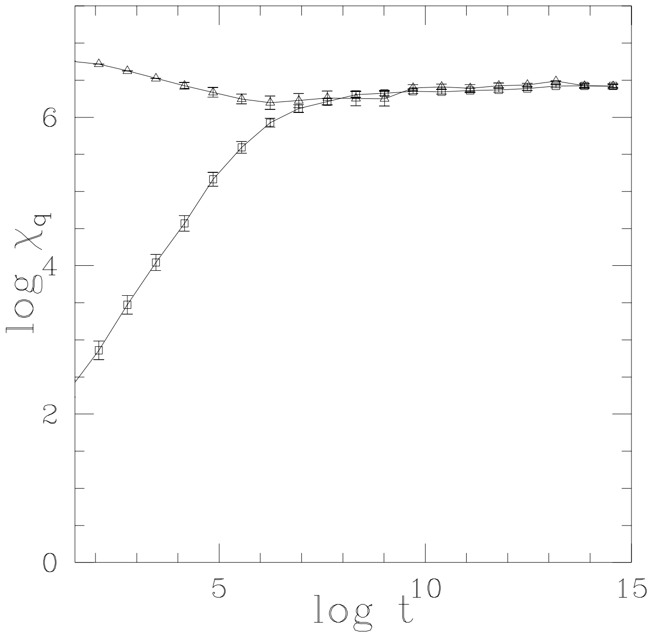

Figure 1: The (non connected) overlap susceptibility against the Monte Carlo time in a double logarithmic scale for one of the lower temperature that we have simulated ($T = 1.5$) and for $L = 32$. The number of samples was 100. The upper curve is from a configuration with all the spins up and the lower curve is with the starting configuration chosen random. One can say that the system has thermalized when there is no difference between the two curves and both stay on a plateau. We have chosen as thermalization time $t = 90000$ (i.e. $\log t = 11.4$) .

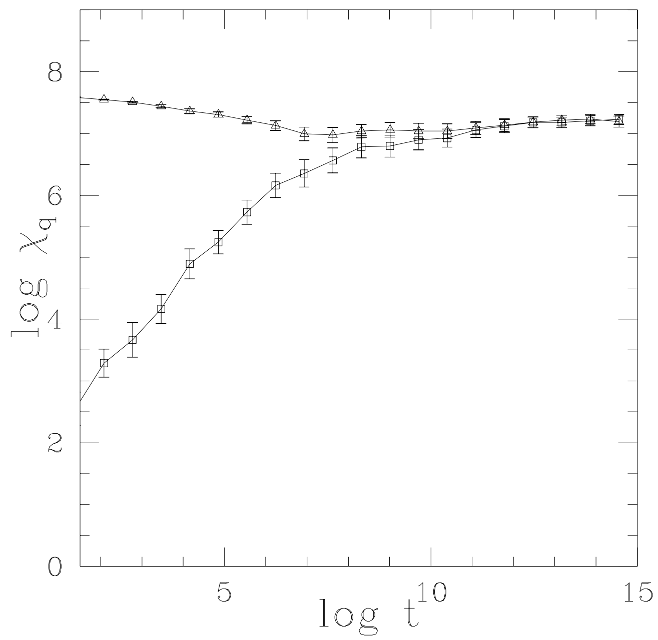

Figure 2: The (non connected) overlap susceptibility against the Monte Carlo time in a double logarithmic scale for the lowest temperature that we have simulated ($T = 1.4$) and for the largest lattice $L = 48$. The number of samples was 25. The upper curve is from a configuration with all the spins up and the lower curve is with the starting configuration chosen random. We have chosen as thermalization time $t = 900000$ (i.e. $\log t = 13.7$) .

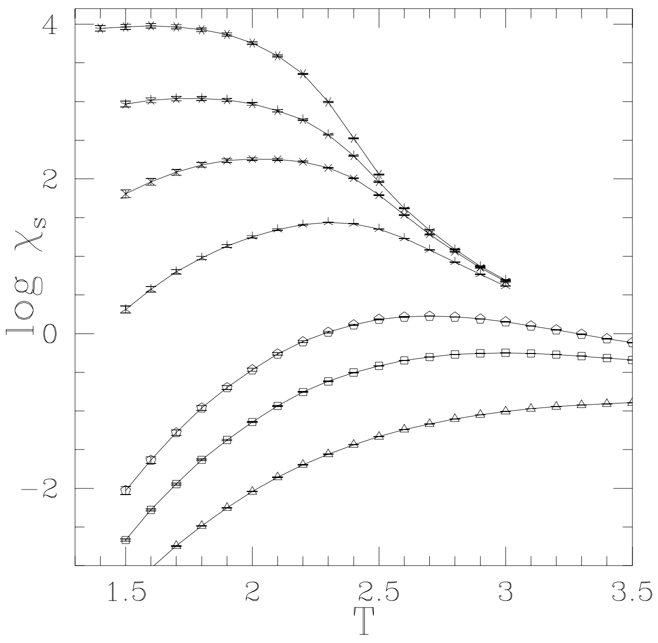

Figure 3: Susceptibility of the staggered magnetization as a function of the temperature. The lattice sizes are (bottom to top): 4, 6, 8, 16, 24, 32 and 48. Here and in the rest of the figures we have used the following symbols for the lattices: $L = 4$ triangle, $L = 6$ square, $L = 8$ pentagon, $L = 16$ three-line star, $L = 24$ four-line star, $L = 32$ five-line star and $L = 48$ six-line star.

L	$T(\chi_s^{\max})$	$\chi_s^{\max}$
8	2.7(1)	1.255(4)
16	2.3(1)	4.21(2)
24	2.0(1)	9.6(2)
32	1.8(2)	19.9(4)
32	1.6(2)	54(2)

Table 2: Maximum of $\chi_s(L,T)$ in temperature and the temperature at which $\chi_s(L,T)$ reaches the maximum.

observables that at $\lambda = 0$ describe the paramagnetic-staggered ferromagnetic phase transition.

In Figure 3 it is possible to see how the point where the staggered susceptibility reaches the maximum drifts quickly to zero with increasing system size. Moreover in Figure 4 there is no crossing of the Binder cumulant: this is a stronger evidence that this parameter (the staggered magnetization) does not show a phase transition at finite temperature, i.e. in the thermodynamic limit $\langle |m_s| \rangle = 0$ for all temperatures different from zero.

More quantitatively, from the data of Table 2 it is clear that the temperatures in which χ_s reaches the maximum, that we denote as $T(\chi_s^{\max})$, goes to zero following a power law:

$$T(\chi_s^{\max}) \propto L^{-0.29(4)}, \tag{14}$$

using only the data of the lattices: 8, 16, 24, 32 and 48. This fit has $\chi^2/\mathrm{DF} = 0.5/3$, where DF means for the number of degrees of freedom in the fit.

This numerical results support our previous analytic result that the staggered ferromagnetic phase is unstable against an infinitesimal perturbation of the kind of a spin glass interaction between the two sub-lattices.

Moreover, we have predicted the presence of a crossover between the staggered ferromagnetic phase and a "spin-glass" like phase for lattices of linear sizes of order 10. To see this crossover we can analyze the specific heat. All the lattice sizes show a maximum near the Onsager temperature ($T_c = 2.26$). We will see that these maxima are the "souvenir" of the staggered phase transition. In particular we can examine the scaling of the maxima, that we denote $C_{\max}(L)$, against the lattice size. The result is given in Figure 5. In this figure we have also plotted the finite size scaling prediction for the pure Ising model ($\lambda = 0$) that is

$$C_{\max}(L) \propto \log L \ . \tag{15}$$

It is clear from Figure 5 that up to $L = 8$ the data follow in good agreement the prediction of the pure model (i.e. up to $L = 8$ the low temperature region is staggered ferromagnetic). But between $L = 8$ and $L = 16$ the system crosses over to a different behavior where there is no divergence of the specific heat, i.e. $\alpha < 0$. This result is in very good agreement with our analytical estimate $R_c \simeq 7$.

Hence, this plot shows us clearly a first crossover between the staggered ferromagnetic phase and a "spin glass" like phase. In the first part of the crossover the specific heat

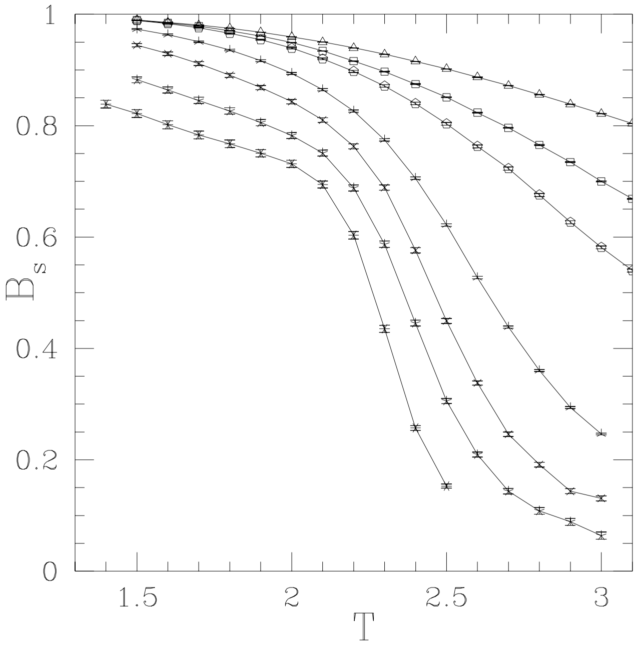

Figure 4: Binder cumulant of the staggered magnetization as a function of the temperature. The lattice sizes are (bottom to top in the right part of the plot): 48, 32, 24, 16, 8, 6 and 4.

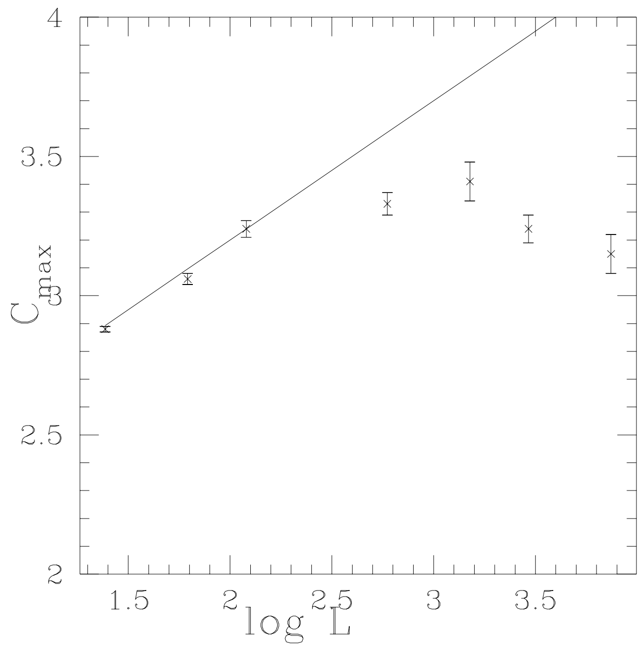

Figure 5: The maximum specific heat as a function of L for $L = 4, 6, 8, 16, 24, 32$ and 48. We have marked the finite size scaling prediction of a logarithmic divergence for the pure model ($\lambda = 0$). The straight line has been obtained from a fit to the formula $A \log L + B$ using $L = 4, 6$ and 8.

diverges logarithmically ($\alpha = 0$) whereas in the second part of the crossover the specific heat does not diverge ($\alpha < 0$).

It is clear that the task that remains is to check that effectively what we have named as a "spin glass" phase has really the properties of a spin glass phase.

To do this we can study the susceptibility and the Binder parameter of the total overlap. In a spin glass phase the magnetization is zero but not the overlap, that becomes the order parameter. If the low temperature phase is spin glass the overlap susceptibility should peak near the transition temperature and the value of the peak should grow with some power of the lattice size (more precisely as $L^{\gamma/\nu}$). Moreover, the analysis of the Binder cumulant should show a clear crossing between curves of different lattice sizes.

In Figures 6 and 7 we show the data for the susceptibility and Binder parameter of the total overlap. We can see again the crossover staggered-spin glass in Figure 7. The curves of the lattice sizes 4, 6 and 8 cross practically at the critical temperature of the pure model ($\lambda = 0$, vertical line). If we examine the crossing point of pairs of lattices for growing sizes, we observe that these crossing points drift to lower temperatures. For instance, the crossing point of the 16 and 32 lattices is near $T = 1.9$. This suggests that the limit of this sequence of crossing points may be zero. This would imply that the spin glass phase is unstable and is crossing over to a paramagnetic phase.

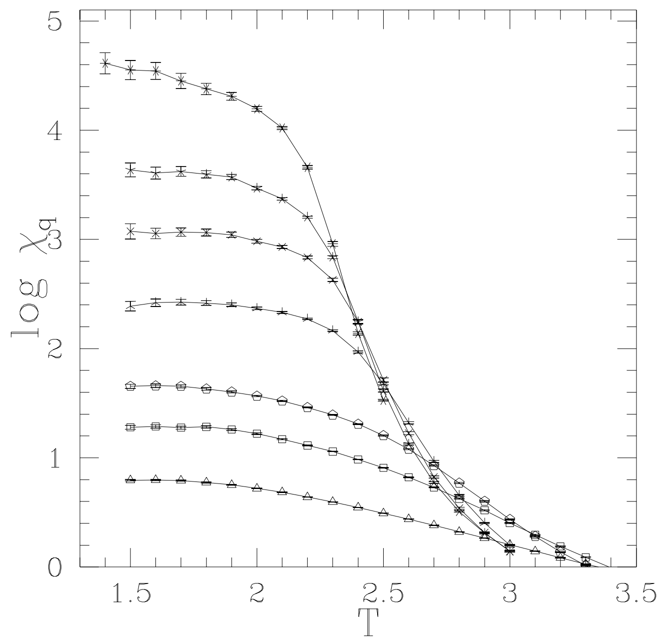

Figure 6: Susceptibility of the total overlap against the temperature. The lattices sizes are (top to bottom): 48, 32, 24, 16, 8, 6 and 4. From this figure it is clear that χ_q for $L = 16, 24, 32$ and 48 does not show a maximum.

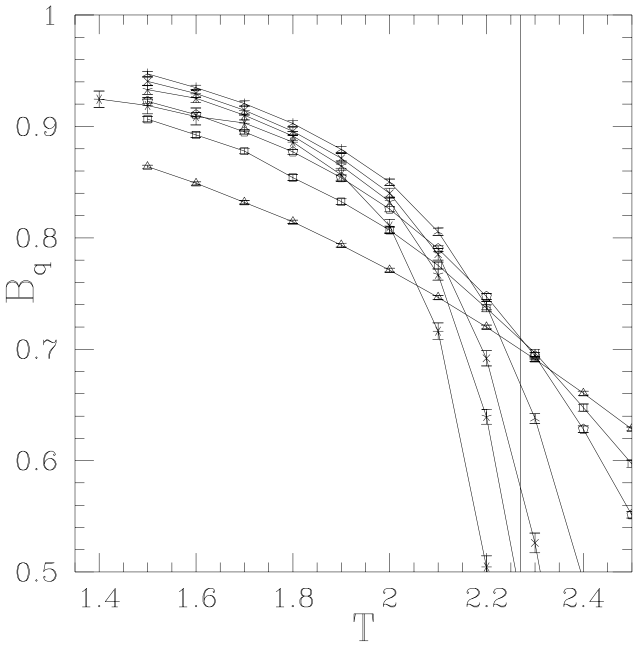

Figure 7: Binder parameter of the total overlap against the temperature. The lattices sizes are (bottom to top in the right part of the plot): 48, 32, 24, 16, 8, 6 and 4. We have marked with a vertical line the Onsager temperature.

L	$T(\chi_{qs}^{\max})$	$\chi_{qs}^{\max}$
8	2.4(1)	2.27(3)
16	2.1(1)	7.2(2)
24	1.9(1)	15.1(3)
32	1.6(2)	28(2)

Table 3: Maximum in temperature of the staggered overlap susceptibility $\chi_q^s(L,T)$, and the temperature at which $\chi_q^s(L,T)$ reaches the maximum. For $L=48$ the error bars do not permit a safe estimate of the maximum.

Moreover, the overlap susceptibility does not present a sharp maximum as a function of the temperature (see the $L=24,32$ and 48 curves). We can extract one conclusions of this fact:

- As long as the spin glass is stable we will expect again that $\chi_q(T)$ should show a sharp maximum as a function of T, or at least that we have independent spin glass order in both sub-lattices (i.e. some sort of staggered spin glass order). From figure 6 it is clear that χ_q does not show a sharp peak in the region that we have simulated (see, for instance, the $L=32$ and $L=48$ data). Moreover the crossing point of the Binder cumulant of the total overlap of two different lattices is drifting to lower values of the temperature, and so we pass to discus the second possible option: spin glass order on both sub-lattices. In this case the staggered overlap susceptibility should have a sharp peak at an intermediate temperature which characterizes a phase transition between a paramagnetic phase and a spin glass one.

To study this issue in more detail, we will examine in the next subsection the overlap defined only in one of the two sub-lattices.

5.2 Crossover Spin Glass-Paramagnetic

In Figures 8 and 9 we show the susceptibility and the Binder cumulant of the staggered overlap (i.e. the overlap computed only in one of the two sub-lattices).

It is clear that these two figures are similar to Figures 3 and 4. For example, there is no crossing in the Binder cumulant. We remark that we have performed small statistic on the $L=48$ lattice and so we obtain large errors. Taking into account only the lattice sizes $L=4,6,8,16,24$ and 32 the effect is clear: the thermodynamical Binder cumulant goes to zero for all the temperatures simulated.

The "apparent" critical temperature (defined as the value where χ_q^s reaches the maximum) goes to zero following the law

$$T(\chi_{qs}^{\max}) \propto L^{-0.23(5)} \tag{16}$$

where we have used L = 8, 16, 24 and 32 in the fit that has $\chi^2/\mathrm{DF} = 0.8/2$. The data used in this fit has been reported in Table 3.

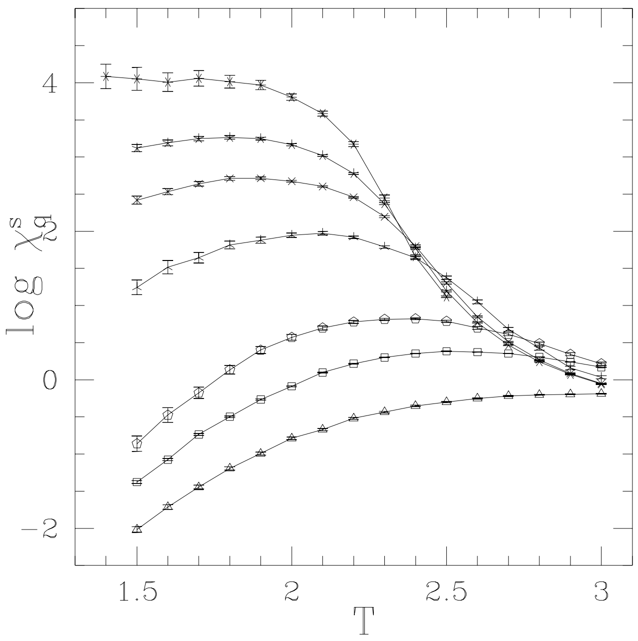

Figure 8: Susceptibility of the staggered overlap against the temperature. The lattices sizes are (bottom to top): 48, 32, 24, 16, 8 and 4.

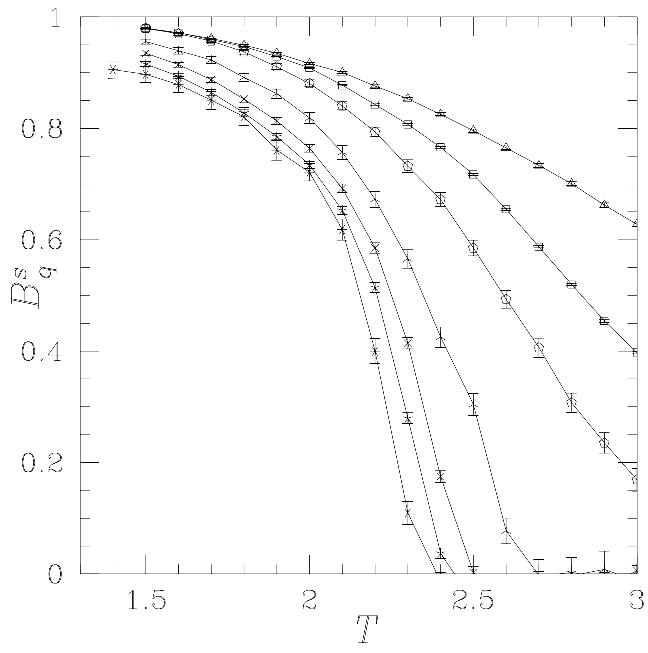

Figure 9: Binder parameter of the staggered overlap against the temperature. The lattice sizes are (bottom to top): 48, 24, 32, 16, 8, and 4.

Within the statistical error the exponent is just the same as in the staggered magnetization. We can compare this figure with that computed for the 2D spin glass[5]: $1/\nu = 0.38(6)$. If we are seeing the pure two dimensional spin glass transition which occurs at $T = 0$, then we will expect a behavior of the apparent critical temperature like $T_{\mathrm{app}} \propto L^{-0.38(6)}$, which is, within the statistical error, the law that we have found for the present model (Eq. (16))[2]: the difference between the two exponents is 0.15(8) (i.e. almost two standard deviations). Obviously our simulations were done in a range of temperatures far away of the critical point ($T = 0$).

Both Figures 8 and 9 suggest that the phase transition is at $T = 0$. In other words, there is not a spin glass phase in the sub-lattices 1 and 2. The whole system seems to be crossing over to a paramagnetic phase.

6 Conclusions

We have studied the two dimensional Ising spin glass model with next nearest neighbor interactions both analytical and numerically.

We have analytically obtained that the system should present two different crossovers as the volume grows: the first one from a staggered ferromagnetic phase to a spin glass

[2]The shift of the apparent critical temperature follows a law: $T_c(L) - T_c \propto L^{-1/\nu}$.

phase for intermediate sizes, and a second crossover between this spin glass phase to a paramagnetic one that will dominate the physics in the thermodynamic limit. Moreover we have obtained the dependence of the first crossover length (staggered-spin glass) on the parameters of the system.

Then we have checked this scenario by performing extensive numerical simulations. We have clearly established the first crossover (staggered-spin glass) and have found strong evidences of the second (spin glass-paramagnetic). In particular, the exponent that governs the shift of the maxima of the spin glass susceptibility is compatible with the known value for the pure two dimensional spin glass.

Acknowledgments

We have run mainly using ALPHA Workstations but the large lattices were run in the RTNN parallel machine[3]. We acknowledge the RTNN project for permitting to us the use of the RTNN computer.

J. J. Ruiz-Lorenzo is supported by an EC HMC (ERBFMBICT950429) grant and thanks to L.A. Fernández for interesting discussions. D. A. Stariolo is partially supported by Conselho Nacional de Desenvolvimento Cientifico e Tecnológico (CNPq), Brazil.

[3]A parallel machine composed by 32 Pentium-Pro at Zaragoza University.

References

[1] E. Marinari, G. Parisi and J. J. Ruiz-Lorenzo, "Numerical Simulations of Spin Glass Systems", in *"Spin Glasses and Random Fields"*, edited by A. P. Young (World Scientific, Singapore, 1997), 130.

[2] C. Dekker, A.F.M. Arts, H.W. de Wijn, A.J. van Duyneveldt and J.A. Mydosh, *Phys.Rev.Lett.* **61**, 1780 (1988);
C. Dekker, A.F.M. Arts and H.W. de Wijn, *Phys.Rev.* **B38**, 8985 (1988);
C. Dekker, A.F.M. Arts, H.W. de Wijn, A.J. van Duyneveldt and J.A. Mydosh, *Phys.Rev.* **B40**, 11243 (1989).

[3] J. A. Mydosh, *Spin Glasses: an Experimental Introduction*, (Taylor & Francis, London 1993).

[4] H. Rieger, *Annual Reviews of Computational Physics II* (World Scientific, Singapore 1995), p. 295.

[5] R.N. Bhatt and A.P. Young, *Phys. Rev.* **B37**, 5606 (1988).

[6] N. Kawashima, N. Hatano and M. Suzuki, *J. Phys.* **A25**, 4985 (1992).

[7] H. Rieger, B. Steckemetz and M. Schreckenberg, *Europhys. Lett.* **27**, 485 (1994);
H. Rieger, L. Santen, U. Blasum, M. Diehl and M. Jünger, *J. Phys.* **A29**, 3939 (1996);
N. Kawashima and H. Rieger, *Europhys. Lett.* **39**, 85 (1997).

[8] T. Shirakura and F. Matsubara, *Phys. Rev. Lett.* **79**, 2755 (1997).

[9] N. Lemke and I. A. Campbell, *Phys. Rev. Lett.* **76**, 4616 (1996).

[10] Y. Imry and S. K. Ma, *Phys. Rev. Lett.* **35**, 1399 (1975).

[11] T. Nattermann, "Theory of the Random Field Ising Model", in *"Spin Glasses and Random Fields"*, edited by A. P. Young (World Scientific, Singapore 1997).

[12] K. Binder, *Z. Phys.* **B50**, 343 (1983).