Figure 5: The connected part of the propagator $G^{1221}(i-j;\omega_1,\omega_2,\omega_3,\omega_4)$.

Figure 4: Diagramatic representation of the propagator $G^{1221}(i-j;\omega_1,\omega_2,\omega_3,\omega_4)$ in the series expansion in g.

Figure 3: The renormalized function $[\langle \xi_i s_i s_k \xi_k \rangle]_{conn}$.

Figure 2: 1-particle irriducible diagrams that contribute to the renormalization of the coupling constant.

Figure 1: Diagrams 17 for the correlation function $[\langle \xi_i s_i s_k \xi_k \rangle]$ in the series expansion in g.

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$$\frac{1}{k^4+2i\bar{\omega}/\Gamma_0}\left(\frac{1}{k^2+\sqrt{2i\bar{\omega}/\Gamma_0}}\right)F_2 \tag{59}$$

which corrisponds to the diagrams in Fig.[5] with the constant g_r given by (57) and $\bar{\omega}=\omega_2-\omega_1$. The connected term is zero only in the limit of complete time separation. In all the other cases we must calculate the complete correlation function. In this way we have computed the complete $G^{\alpha\beta\gamma\delta}$ propagators for all the values of the indices $\alpha\beta\gamma\delta$. It is easy to see that the long-range limit of the connected part of the expressions (59) coincides with the sum of the terms (28) (29) (30) of the third section.

The expressions that we have derived for the propagators will be used to evaluate the contributions of the diagrams that occur in the loop expansion when the cubic term of the lagrangian (36) is considered non vanishing. After having completed these computations, we will be able to find the correction to the dynamic exponent to the first order in ϵ.

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$$= \frac{1}{k^2 + \sqrt{-2i\bar{\omega}/\Gamma_0}}\, F_1 \left(\frac{k^2}{(i\bar{\omega}/\Gamma_0)^{1/2}} \right) . \tag{56}$$

F_1 is a function of the variable $\left(\frac{k^2}{(i\bar{\omega})^{1/2}} \right)$ that exhibits a constant limit for $\omega \to 0$ and for $k \to 0$. The coupling constant in the low frequency and small moments limit is, according to (55) and to (56),

$$g_r = (k^2 + \sqrt{-2i\bar{\omega}/\Gamma_0}) 1/F_1 \, . \tag{57}$$

To evaluate the total contribution of the loop corrections we must consider the term given by the following diagram:

with renormalized vertices. The value of the loop is:

$$I_2(\bar{\omega}, k) = \int_{-\infty}^{\infty} \left[\frac{d\omega}{2\pi} \frac{1}{k^2 + \sqrt{-2i\omega/\Gamma_0} + \sqrt{-2i(\bar{\omega}-\omega)/\Gamma_0}} \frac{1}{k^2 + \sqrt{2i\omega/\Gamma_0} + \sqrt{2i(\bar{\omega}-\omega)/\Gamma_0}} \times \right.$$

$$\left(\frac{1}{k^2 + \sqrt{-2i\omega/\Gamma_0} + \sqrt{2i(\bar{\omega}-\omega)/\Gamma_0}} + \frac{1}{k^2 + \sqrt{2i\omega/\Gamma_0} + \sqrt{-2i(\bar{\omega}-\omega)/\Gamma_0}} \right) \times$$

$$\left. \frac{8}{\sqrt{-2i\omega/\Gamma_0} + \sqrt{2i\omega/\Gamma_0}} \frac{8}{\sqrt{-2i(\bar{\omega}-\omega)/\Gamma_0} + \sqrt{2i(\bar{\omega}-\omega)/\Gamma_0}} \right] \times$$

$$= \frac{1}{k^4 + 2i\bar{\omega}/\Gamma_0} \left(\frac{1}{k^2 + \sqrt{2i\bar{\omega}/\Gamma_0}} \right) F_2 \left(\frac{k^2}{(i\bar{\omega}/\Gamma_0)^{1/2}} \right), \tag{58}$$

where $F_2(x)$, like $F_1(x)$, has a constant limit for $\omega \to 0$ and $k \to 0$.

The connected part of the free propagator G^{1221} at $T = T_c$ is computed adding up all the diagrams in Fig.[4]. We have:

$$G^{1221}(k;\omega_1,\omega_2,\omega_3,\omega_4)_{conn} = G_0^{1221}(k;\omega_1,\omega_2) G_0^{2122}(k;\omega_3,\omega_4) g_r(k;\bar{\omega}) \frac{1}{F_1} +$$

$$+ G_0^{1222}(k;\omega_1,\omega_2) G_0^{2121}(k;\omega_3,\omega_4) g_r(k;-\bar{\omega}) \frac{1}{F_1} +$$

$$+ G_0^{1221}(k;\omega_1,\omega_2) G_0^{2121}(k;\omega_3,\omega_4) g_r(k;\bar{\omega}) g_r(k;-\bar{\omega}) \frac{1}{F_1^2}$$

$$+\frac{2}{(k^2+\sqrt{2(m^2-i\omega_3/\Gamma_0)}+\sqrt{2(m^2+i\omega_4/\Gamma_0)}}\Bigg]\times$$
$$\frac{2\cdot 4\cdot 2}{(\sqrt{2(m^2-i\omega_2/\Gamma_0)}+\sqrt{2(m^2+i\omega_2/\Gamma_0)})}\frac{2\cdot 4\cdot 2}{(\sqrt{2(m^2-i\omega_3/\Gamma_0)}+\sqrt{2(m^2+i\omega_3/\Gamma_0)})}\times$$
$$\frac{1}{(k^2+\sqrt{2(m^2-i\omega_3/\Gamma_0)}+\sqrt{2(m^2-i\omega_4/\Gamma_0)})}\,, \tag{51}$$

plus another term of the same form proportional to $(\delta(\omega_1+\omega_4)\delta(\omega_2+\omega_3))$. At the first order in g we consider $G = G_0 + G_1$. For G_0 we useœ[A the former solution and we solve the system in the G_1 variables. We apply this procedure in an iterative way and we can show that, defining from a diagramatic point of view

$$G_0^{1221}(i-j;\omega_1,\omega_2,\omega_3,\omega_4) = G_0^{1221}(i-j;\omega_1,\omega_2) = \tag{52}$$

$$G_0^{2221}(i-j;\omega_1,\omega_2,\omega_3,\omega_4) = G_0^{2221}(i-j;\omega_1,\omega_2) = \tag{53}$$

$$G_0^{2222}(i-j;\omega_1,\omega_2,\omega_3,\omega_4) = G_0^{2222}(i-j;\omega_1,\omega_2) = \tag{54}$$

we obtain, for the succeeding order in g, recursive expressions that can be resumed. For istance, the terms that occur for the function $G^{1221}(i-j;\omega_1,\omega_2,\omega_3,\omega_4)$ can be represented with the diagrams in Fig.[4]. To evaluate the complete propagators at $T = T_c$, we can define the renormalized coupling constant

$$g_r = g - g^2 I(\bar{\omega},k) + g^3 I^2(\bar{\omega},k) + ... = \frac{g}{1+g\,I(\bar{\omega},k)}\,, \tag{55}$$

where $I(\bar{\omega},k)$, correspondent to the loop

is given by the integral:

$$I_1(\bar{\omega},k) = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{1}{(k^2+\sqrt{-2i\omega/\Gamma_0}+\sqrt{-2i(\bar{\omega}-\omega)/\Gamma_0}}\times$$
$$\frac{1}{(k^2+\sqrt{2i\omega/\Gamma_0}+\sqrt{-2i(\bar{\omega}-\omega)/\Gamma_0}}\frac{16}{\sqrt{-2i\omega/\Gamma_0}+\sqrt{2i\omega/\Gamma_0}}\times$$

$$G_0^{1221}(k;\omega_1,\omega_2,\omega_3,\omega_4) = \frac{\tilde{K}^{-1}(k)G(-\omega_1)G(\omega_2)\delta(\omega_1+\omega_3)\delta(\omega_2+\omega_4)}{\tilde{K}^{-1}(k)-G(-\omega_1)G(\omega_2)}, \quad (45)$$

$$G_0^{2121}(k;\omega_1,\omega_2,\omega_3,\omega_4) = \frac{\tilde{K}^{-1}(k)G(\omega_1)G(-\omega_2)\delta(\omega_1+\omega_4)\delta(\omega_2+\omega_3)}{\tilde{K}^{-1}(k)-G(\omega_1)G(-\omega_2)}, \quad (46)$$

$$G_0^{2221}(k;\omega_1,\omega_2,\omega_3,\omega_4) = \frac{1}{\tilde{K}^{-1}(k)-G(\omega_1)G(\omega_2)}\Bigg[C(\omega_3)G(-\omega_4)\times$$
$$\frac{\tilde{K}^{-1}G(\omega_3)G(-\omega_4)\left[\delta(\omega_1+\omega_3)\delta(\omega_2+\omega_4)+\delta(\omega_1+\omega_4)\delta(\omega_2+\omega_3)\right]}{\tilde{K}^{-1}(k)-G(\omega_3)G(-\omega_4)}\Bigg]. \quad (47)$$

At low frequency and for $T \to T_c$ we have
$\tilde{K}^{-1}(k) = \frac{1}{\beta^2}K^{-1}(k) = (4+4\Delta T_c+4\tau)(1+ck^2)$, $G(\omega)$ given by the (25) and $C(\omega)$ by the (26) so (45), (46) and (47) become:

$$G_0^{1221}(k;\omega_1,\omega_2,\omega_3,\omega_4) = \frac{4\ \delta(\omega_1+\omega_3)\delta(\omega_2+\omega_4)}{k^2+\sqrt{2(m^2+i\omega_1/\Gamma_0)}+\sqrt{2(m^2-i\omega_2/\Gamma_0)}}\,, \quad (48)$$

$$G_0^{2121}(k;\omega_1,\omega_2,\omega_3,\omega_4) = \frac{4\ \delta(\omega_1+\omega_3)\delta(\omega_2+\omega_4)}{k^2+\sqrt{2(m^2-i\omega_1/\Gamma_0)}+\sqrt{2(m^2+i\omega_2/\Gamma_0)}}\,, \quad (49)$$

$$G_0^{2221}(k;\omega_1,\omega_2,\omega_3,\omega_4) = \frac{\delta(\omega_1+\omega_3)\delta(\omega_2+\omega_4)+\delta(\omega_1+\omega_3)\delta(\omega_2+\omega_4)}{4(k^2+\sqrt{2(m^2-i\omega_1/\Gamma_0)}+\sqrt{2(m^2-i\omega_2/\Gamma_0)})}\times$$
$$\frac{4}{(k^2+\sqrt{2(m^2-i\omega_3/\Gamma_0)}+\sqrt{2(m^2+i\omega_4/\Gamma_0)})}\times$$
$$\frac{4\cdot 2}{(\sqrt{2(m^2-i\omega_3/\Gamma_0)}+\sqrt{2(m^2+i\omega_3/\Gamma_0)})}\,. \quad (50)$$

In the same way, if we calculate the system for $\mu,\nu = 2,2$ we obtain the propagator G^{2222} that for $g=0$ results:

$$G_0^{2222}(k;\omega_1,\omega_2,\omega_3,\omega_4) = \frac{\delta(\omega_1+\omega_3)\delta(\omega_2+\omega_4)}{4(k^2+\sqrt{2(m^2-i\omega_1/\Gamma_0)}+\sqrt{2(m^2-i\omega_2/\Gamma_0)})}\times$$
$$\Bigg[\frac{2}{(k^2+\sqrt{2(m^2+i\omega_3/\Gamma_0)}+\sqrt{2(m^2-i\omega_4/\Gamma_0)})}+$$

$$\sum_{3,4}\Big(-2\,\tilde K^{-1}(k)A^{\alpha\beta\gamma\delta}\delta(1-3)\delta(2-4)+C^{\alpha\beta\gamma\delta}(1,2,3,4)\Big)\times$$
$$\Big(\frac{1}{4}G^{\gamma\delta\mu\nu}(k;3,4,5,6)+\frac{1}{2}\tilde K^{-1}(k)A^{\alpha\beta\gamma\delta}\delta(3-5)\delta(2-6)\Big)=$$
$$=-\delta_{\alpha,\mu}\delta_{\beta,\nu}\tilde K^{-2}(k)\delta(1-5)\delta(2-6)\ , \tag{42}$$

that in the Fourier transform is:

$$\int d\omega_a d\omega_b\,\Big(-2\,\tilde K^{-1}(k)A^{\alpha\beta\gamma\delta}\delta(\omega_1+\omega_a)\delta(\omega_2+\omega_b)+C^{\alpha\beta\gamma\delta}(\omega_1,\omega_2,\omega_a,\omega_b)\Big)\times$$
$$\Big(\frac{1}{4}G^{\gamma\delta\mu\nu}(k;-\omega_a,-\omega_b,\omega_3,\omega_4)+\frac{1}{2}\tilde K^{-1}(k)A^{\alpha\beta\gamma\delta}\delta(-\omega_a+\omega_3)\delta(-\omega_b+\omega_4)\Big)$$
$$=-\delta_{\alpha\mu}\delta_{\beta\nu}\tilde K^{-2}(k)\delta(\omega_1+\omega_3)\delta(\omega_2+\omega_4)\ . \tag{43}$$

For each value of μ and ν (43) is an integral system of four coupled equations in the $G^{11\mu\nu}$, $G^{21\mu\nu}$, $G^{12\mu\nu}$, $G^{22\mu\nu}$ variables. It is obvious that $G^{\alpha\beta\mu\nu}(i,j;t_1,t_2,t_3,t_4)$ is invariant under permutation of the index α,β,μ,ν and of the relatives times t_1,t_2,t_3,t_4. First we solve the system for $\mu,\nu=1,2$. We cannot solve the system (43) exactly, so we consider $G^{\alpha\beta\gamma\delta}$ as a perturbative series in g, and we use a recursive procedure at the different order in g. For $g=0$ the coefficients $C^{\alpha\beta 12}(1,2,3,4)$ are factorized in the time-difference and we obtain:

$$G_0^{1121}(k;\omega_1,\omega_2,\omega_3,\omega_4)=0\ , \tag{44}$$

the 4-spin and 6-spin correlation functions of the MF theory described by the partition function

$$\mathcal{Z} = \int [ds_i][d\hat{s}_i] exp \left[L_0 + \int d1d2 \sum_i \bar{Q}_i^{\alpha\beta}(1,2)\psi_i^\alpha(1)\psi_i^\beta(2) \right], \qquad (37)$$

connected with respect to pairs. For example for $C^{\alpha\beta\gamma\delta}(1,2,3,4)$ we have:

$$C^{\alpha\beta\gamma\delta}(1,2,3,4) = \langle\psi_i^\alpha(1)\psi_i^\beta(2)\psi_i^\gamma(3)\psi_i^\delta(4)\rangle - \langle\psi_i^\alpha(1)\psi_i^\beta(2)\rangle\langle\psi_i^\gamma(3)\psi_i^\delta(4)\rangle \ . \ (38)$$

We have demonstrated that the critical behaviour of these functions is determined only by the zero loop contribution of the correlation functions of the theory descibed by the functional (37). The correlations of the ϕ_i^α fields, which we are interested in, are related to those $\delta Q_i^{\alpha\beta}$ by the relations:

$$[\langle\phi_i^\alpha(1)\phi_i^\beta(2)\rangle]_J = 2\sum_j(\tilde{K}^{-1})_{ij} \ \left(\bar{Q}_j^{\alpha\beta}(1,2) + \langle\delta Q_j^{\alpha\beta}(1,2)\rangle\right), \qquad (39)$$

$$\begin{aligned} [\langle\phi_i^\alpha(1)\phi_i^\beta(2)\phi_k^\gamma(3)\phi_k^\delta(4)\rangle]_J = {} & 4\sum_j(\tilde{K}^{-1})_{ij}\sum_l(\tilde{K}^{-1})_{kl} \\ & \langle\left(\bar{Q}_j^{\alpha\beta}(1,2) + \delta Q_j^{\alpha\beta}(1,2)\right)\left(\bar{Q}_l^{\gamma\delta}(3,4) + \delta Q_l^{\gamma\delta}(3,4)\right)\rangle + \\ & -2(\tilde{K}^{-1})_{ik}A^{\alpha\beta\gamma\delta}\delta(1-3)\delta(2-4). \end{aligned} \qquad (40)$$

In the next section, therefore, we mind to evaluate the correlation functions of the fields $\delta Q_i^{\alpha\beta}(1,2)$.

4.1 The Propagators

Let us consider the expression (36) with the vanishing cubic interaction. The generic propagator

$$\begin{aligned} G^{\alpha\beta\gamma\delta}(i-j;1,2,3,4) &= [\langle\phi_i^\alpha(1)\phi_i^\beta(2)\phi_j^\gamma(3)\phi_j^\delta(4)\rangle]_J - [\langle\phi_i^\alpha(1)\phi_i^\beta(2)\rangle]_J[\langle\phi_j^\gamma(3)\phi_j^\delta(4)\rangle]_J \\ &= 4\sum_l(\tilde{K}^{-1})_{il}\sum_k(\tilde{K}^{-1})_{jk}\langle\delta Q_l^{\alpha\beta}(1,2)\delta Q_k^{\gamma\delta}(3,4)\rangle - 2(\tilde{K}^{-1})_{ij}A^{\alpha\beta\gamma\delta}\delta(1-3)\delta(2-4) \ , \end{aligned} (41)$$

is calculated in free theory and will be used to evaluate the corrections of terms in the loop expansion. In the free theory $G^{\alpha\beta\gamma\delta}(i-j;1,2,3,4)$ is the solution to the following integral system:

where $\tilde{K}_{ij}^{-1} = \frac{z}{\beta^2} K_{ij}^{-1}$ ($K_{ij} = 1$ if i, j are nearest neighbors and zero otherwise) and $A^{\alpha\beta\gamma\delta}$ is such that $A^{1122} = A^{2211} = A^{2112} = A^{1221} = 1$ and 0 otherwise. L_1 is:

$$L_1 = L_0 + \int d1 d2\, Q_i^{\alpha\beta}(12) \psi_i^\alpha(1) \psi_i^\beta(2), \tag{32}$$

where the field ψ_i^α is defined:

$$\psi_i^\alpha = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \phi_i^\alpha = (s_i, i\hat{s}_i) \,, \tag{33}$$

and L_0 is the local part of the theory:

$$L_0 = \int d1 \sum_i \left[i\hat{s}_i(1) \left(-\Gamma_0^{-1} \partial_1 s_i(1) - r_0 s_i(1) - \frac{1}{3!} g s_1^3(1) + \Gamma_0^{-1} i\hat{s}_i(1) \right) \right]. \tag{34}$$

In the previous expressions and in the followings of this section we shall write 1 for t_1, 2 for t_2 and so on.

The solution of the MF theory is well known [3], and it is consistent with the results obtained in the previous section. Let us cosider the fluctuations around the saddle point value $Q_i^{\alpha\beta}(1,2) = \bar{Q}_i^{\alpha\beta}(1,2) + \delta Q_i^{\alpha\beta}(1,2)$. As a result the model can be formulated in terms of the dynamic fluctuation field $\delta Q_i^{\alpha\beta}(1,2)$:

$$\mathcal{Z} = \int \prod_{\alpha,\beta=1,2} D\{\delta Q_i^{\alpha\beta}\} \exp(L(s, \hat{s}, \bar{Q}_i^{\alpha\beta} + \delta Q_i^{\alpha\beta})), \tag{35}$$

where L contains the quadratic and the cubic term of the series expansion around the MF value:

$$\begin{aligned} L &= -\sum_{1,2} \sum_{i,j} \tilde{K}_{i,j}^{-1} \delta Q_i^{\alpha\beta}(1,2) A^{\alpha\beta\gamma\delta} \delta Q_i^{\gamma\delta}(1,2) + \\ &+ \frac{1}{2} \sum_{1,2,3,4} \sum_i \delta Q_i^{\alpha\beta}(1,2) C^{\alpha\beta\gamma\delta}(1,2,3,4) \delta Q_i^{\gamma\delta}(3,4) + \\ &+ \frac{1}{3!} \sum_{1,2,3,4,5,6} \sum_i C^{\alpha\beta\gamma\delta\mu\nu}(1,2,3,4,5,6) \delta Q_i^{\alpha\beta}(1,2) \delta Q_i^{\gamma\delta}(3,4) \delta Q_i^{\mu\nu}(5,6) \,. \end{aligned} \tag{36}$$

The higher order terms can be neglected because we are interested in the behaviour of the Green functions near the critical temperature in the paramagnetic phase. The $C^{\alpha\beta\gamma\delta}(1,2,3,4)$ and $C^{\alpha\beta\gamma\delta\mu\nu}(1,2,3,4,5,6)$ vertices are

3)
$$\frac{1}{2}\left(\frac{1}{2}\sqrt{2}\sqrt{2(2m^2-i\bar{\omega})}\right)\left(\frac{1}{2}\sqrt{2}\sqrt{2(2m^2+i\bar{\omega})}\right)\times$$
$$\frac{2i\sqrt{2}}{\bar{\omega}}\left(\frac{1}{\sqrt{2(m^2-i\bar{\omega})}}-\frac{1}{\sqrt{2(m^2+i\bar{\omega})}}\right)\frac{4}{\sqrt{2(m^2-i\omega_1)}+\sqrt{2(m^2+i\omega_2)}}\times$$
$$\frac{4}{\sqrt{2(m^2-i\omega_3)}+\sqrt{2(m^2+i\omega_4)}}. \tag{30}$$

In the same way we can evaluate all the 4-point functions. The static limit derived from the dynamic functions coincides, order by order in g, with the static results. This is true also for the renormalized functions.

It is not easy to extend this formalism to the short-range model, because in that case the eigenvalue density $\sigma(\lambda)$ is not given by the simple expression (8). Although the results of this section have been useful to us for understanding some properties of the correlation functions in the critical region, we have to leave this approach to go beyond the MF approximation. In the next section we will use the functional integral formalism for a dynamic SG model in dimensions $d = 6 - \epsilon$. The results obtained in this section will be used as comparative terms for the long-range limit of the short-range correlation functions.

4 Fluctuations in short range spin glasses. Functional integral formulation.

To solve a theory with quenched parameters we can use the functional integral formulation for dynamics, introduced by De Dominicis [11]. So we consider an auxiliary field $\hat{s}_i(t)$, [10], and we define a two-component vector field $\phi_i^\alpha = (i\hat{s}_i, s_i)$. After averaging over J_{ij} (in this case we can avoid the replica trick because the functional $\mathcal{Z}$ is normalized), a 4-spins interaction is introduced. It is convenient to decouple this interaction by a gaussian transformation [3]. Then the theory can be defined with a generating functional in the $Q_i^{\alpha\beta}(t,t')$ variables:

$$\mathcal{Z} = \int \prod_{\alpha,\beta=1,2}[DQ_i^{\alpha\beta}(1,2)]\exp\left(-\int d1\ \mathrm{d}2\sum_{i,j}\tilde{K}_{ij}^{-1}Q_i^{\alpha\beta}(1,2)A^{\alpha\beta\gamma\delta}Q_j^{\gamma\delta}(1,2)\right.$$
$$\left.+\ln\int[ds][d\hat{s}]\exp(L_1(s,\hat{s},Q_i^{\alpha\beta})\right), \tag{31}$$

This is an autoconsistent equation that can be solved with an iterative procedure. At finite order in g, Y^n is a polynom in the variables $\xi^n(t)$. We can demonstrate that for the 2-point functions the correction terms do not change their dynamic behaviour for $T = T_c$, because the time dependent part of the self-energy is regular at $T = T_c$. We find for $C(\omega)$ and $G(\omega)$ only a mass renormalization effect

$$G_r(\omega) = (2 + \Delta T_c + \tau) - 2\sqrt{2(m^2 - i\omega)}\,, \tag{25}$$

$$C_r(\omega) = \frac{2 \cdot 4}{\sqrt{2(m^2 - i\omega)} + \sqrt{2(m^2 + i\omega)}}\,. \tag{26}$$

The behaviour of $\chi_{SG}(\omega_1, \omega_2)$ at $T = T_c$ remains unchanged.
As for the generic 4 point function the connected terms occur. For example, for $\langle \xi_i s_i s_k \xi_k \rangle$, we have the diagram of Fig.[1]. The diagrams that we have to sum in order to evaluate the renormalized coupling constant in the dynamic case are drawn in Fig.[2] and in the low frequency limit it results:

$$g_r = \frac{1}{2}\sqrt{2}\sqrt{2(2m^2 - i\omega)}. \tag{27}$$

Therefore the total contribution to $\langle \xi_i s_i s_k \xi_k \rangle$ function, represented in Fig.[3], is the sum of the following terms:

1)
$$-\frac{1}{2}\sqrt{2}\left(\sqrt{2(2m^2 - i\bar{\omega}}\right) \frac{4}{\sqrt{2(m^2 - i\omega_1)} + \sqrt{2(m^2 + i\omega_2)}} \times$$
$$\frac{1}{\sqrt{2(m^2 + i\omega_3)} + \sqrt{2(m^2 + i\omega_4)}} \frac{1}{\sqrt{2(m^2 - i\omega_3)} + \sqrt{2(m^2 + i\omega_4)}} \times$$
$$\frac{16}{\sqrt{2(m^2 - i\omega_3)} + \sqrt{2(m^2 + i\omega_3)}}, \tag{28}$$

2)
$$-\frac{1}{2}\sqrt{2}\left(\sqrt{2(2m^2 + i\bar{\omega}}\right) \frac{4}{\sqrt{2(m^2 - i\omega_3)} + \sqrt{2(m^2 + i\omega_4)}} \times$$
$$\frac{1}{\sqrt{2(m^2 + i\omega_1)} + \sqrt{2(m^2 + i\omega_2)}} \frac{1}{\sqrt{2(m^2 - i\omega_1)} + \sqrt{2(m^2 + i\omega_2)}} \times$$
$$\frac{16}{\sqrt{2(m^2 - i\omega_1)} + \sqrt{2(m^2 + i\omega_1)}}, \tag{29}$$

is compensated by that of the last one

$$-\left((2m^2)^{1/2}\right)\times\left(\frac{2}{(2m^2)^{1/2}}\right)\times\left(\frac{2}{(2m^2)^{1/2}}\right)=-\frac{4}{(2m^2)^{1/2}}. \qquad (17)$$

In the dynamic case we have to solve the Langevin equation for the time-dependent component $Y^n(t)$:

$$\dot{Y}^n=(\beta\lambda^n-1)Y^n-g\sum_{\alpha,\beta,\gamma}Y^\alpha Y^\beta Y^\gamma\sum_i\psi_i^\alpha\psi_i^\beta\psi_i^\gamma\psi_i^n+\xi^n\ , \qquad (18)$$

For $g=0$ the solution is

$$Y^n(t)=\int_0^t e^{-(-\beta\lambda^n+1)(t-t')}\xi^n(t')dt'. \qquad (19)$$

The correlation and the response functions in the ω space (Fourier transform with respect to time) are

$$G(\omega) \;=\; 2\int\frac{d\lambda\,\sigma(\lambda)}{-\beta\lambda+1-i\omega}\propto 2\left(1-\sqrt{\frac{T-T_c}{T_c}-i\omega}\right)\ , \qquad (20)$$

$$C(\omega) \;=\; \int\frac{d\lambda\,\sigma(\lambda)\,2}{(-\beta\lambda+1)^2+\omega^2}\propto\frac{1}{\sqrt{\frac{T-T_c}{T_c}-i\omega}}\ . \qquad (21)$$

and the susceptibility, according to definition (7), results

$$\chi_{S.G.}(\omega_1,\omega_2)=\int\frac{d\lambda\,\sigma(\lambda)}{(-\beta\lambda+1-i\omega_1)(-\beta\lambda+1-i\omega_2)}\ . \qquad (22)$$

From equation (22) it is clear that the susceptibility has a finite limit for $\omega\to 0$ when $T>T_c$ while when $T=T_c$ it shows a critical behaviour such as

$$\chi_{S.G.}\propto\frac{1}{\omega^{1/2}}. \qquad (23)$$

For $g\neq 0$ the solution to the Langevin equation is

$$\begin{aligned}Y^n(t)=&\int_0^t G^n(t-t')\xi^n(t')dt'+\\ -g&\int_0^t dt'G^n(t-t')\sum_{\alpha,\beta,\gamma}Y^\alpha(t')Y^\beta(t')Y^\gamma(t')\sum_i\psi_i^\alpha\psi_i^\beta\psi_i^\gamma\psi_i^n.\end{aligned} \qquad (24)$$

If we write $s_i = \sum_\alpha Y^\alpha \psi_i^\alpha$ in the base of the eigenvectors we obtain:

$$\beta \mathcal{H}_Y = \frac{1}{2}\sum_\alpha (-\beta\lambda^\alpha + 1)(Y^\alpha)^2 + \frac{g}{4!}\sum_{\alpha\beta\gamma\delta} Y^\alpha Y^\beta Y^\gamma Y^\delta \sum_i \psi_i^\alpha \psi_i^\beta \psi_i^\gamma \psi_i^\delta. \tag{11}$$

We can evaluate the Green functions for the component Y^α and, by using (8), go back to the correlation functions of s_i field.

We can start evaluating the correlation functions in the static theory. In the time-independent limit for the Hamiltonian (11) we may consider the non linear interaction as a perturbation and make a series expansion in the coupling constant g. We can show that only the diagrams considered in the Hartree-Fock appressimation give considerable contributions to the free theory, in the thermodynamic limit. In fact the eigenvector index (α for ψ_i^α) plays the same role of the component index in the theories for vector fields, where the Hartree-Fock approximation has been demonstrated valid when the number of the components goes to infinity. We can thus resume all the diagrams at the succeding order in g and find a renormalization of the mass term (the coefficient of the quadratic term in (11)) that for $T \to T_c^+$ results

$$m^2 \propto \left(\frac{T - T_c}{T_c}\right)^2, \tag{12}$$

and a renormalization of the coupling constant g, that in the same region is

$$g_r = (2m^2)^{\frac{1}{2}}. \tag{13}$$

The static susceptibility shows a divergent behaviour for $T \to T_c^+$ with the critic exponent $\gamma = 1$, in agreement with the results obtained with the replica method [1]

$$\begin{aligned} \chi_{S.G.} = \frac{1}{N}\sum_{ik}\langle s_i s_k\rangle^2 &= \int_0^4 \frac{d\lambda\,\sqrt{(4\lambda - \lambda^2)}}{2\pi(\beta\lambda + m^2)^2} \\ &= \frac{2}{(2m^2)^{1/2}} \propto \frac{1}{\tau}. \end{aligned} \tag{14}$$

The generic 4-point function

$$[\langle s_i s_k s_i s_k\rangle]_J = [\langle (s_i)^2\rangle\langle (s_k)^2\rangle]_J + 2[(\langle s_i s_k\rangle)^2)]_J + [\langle s_i s_k s_i s_k\rangle_{conn.}]_J \tag{15}$$

is $O(1/N)$ order and is regular in the critical region. In fact, for $T \to T_c$ the first term is regular and the divergence of the second term

$$2 \times \left(\frac{2}{(2m^2)^{1/2}}\right), \tag{16}$$

where $\alpha, \beta, \gamma, \delta$ can take the values 1 or 2 being $\phi_i^1 = \xi_i$ and $\phi_i^2 = s_i$. We shall see that these quantities represent the propagators of our theory. In this formalism the time-depending spin-glass susceptibility is

$$\chi_{SG}(i-j; t_1-t_4, t_3-t_2) = \frac{1}{N}[\langle s_i(t_1)\xi_j(t_4)\rangle\langle s_j(t_3)\xi_i(t_2)\rangle]_J = G^{1212}(i-j; t_1-t_4, t_3-t_2). \tag{7}$$

In the next sections we will show the beaviour of the general propagator $G^{\alpha\beta\gamma\delta}(i-j; t_1, t_2, t_3, t_4)$, and in particular that of χ_{SG}.

3 Mean Field limit and quadratic fluctuations by diagonalization of the interaction matrix

The theory defined with the Hamiltonian (1) where the J_{ij} are quenched random interactions can be solved by using the general properties of the Random Matrices. It is convenient to apply this approach to an infinite range model ($z = N$ and $N \to \infty$), where the non local propagators $G^{\alpha\beta\gamma\delta}(i, j; t_1, t_2, t_3, t_4)$ represent the corrections at $O(1/N)$ order to the MF correlations. By a proper base change, from the unidimensional spin variables s_i to the eigenvectors ψ^α of the J_{ij} matrix, we manage to decouple the interaction.
So we define ψ^α:

$$\sum_j J_{ij}\psi_j^\alpha = \lambda^\alpha \psi_j^\alpha,$$

where $\alpha = 1......N$ and λ^α is the α-th eigenvalue. The shape of the eigenvalues density $\sigma(\lambda)$, for symmetric matrices with random and statistically indipendent elements, such as J_{ij}, in the limit $N \to \infty$ (N dimension of the matrix), is known [2]:

$$\sigma(\lambda) = \begin{cases} \frac{1}{2\pi}(4-\lambda^2)^{1/2} & |\lambda| < 2 \\ 0 & |\lambda| > 2 \end{cases} . \tag{8}$$

The eigenvectors (that are statistical variables) are found to be orthogonal (in the case of symmetric matrices), and we can choose them to be normalized:

$$\sum_{i=1}^N \psi_i^\alpha \psi_i^\beta = 0\,, \quad \text{se } \alpha \neq \beta \tag{9}$$

$$\sum_{i=1}^N (\psi_i^\alpha)^2 = 1. \tag{10}$$

MF limit (order $O(1/N)$, where N is the number of the sites) using the diagonalization of the intraction random matrix; finally in section 4 we find the general propagator for an effective short range Lagrangian of the field $\delta Q^{\alpha\beta}$ (fluctuations around MF limit).

2 Definition of the model

Let us consider the soft spin version of the Edwards-Anderson model given by the Hamiltonian

$$\beta\mathcal{H} = -\beta\sum_{\langle ij\rangle}\beta J_{ij}s_i s_j + \frac{1}{2}r_0\sum_i s_i^2 + \frac{1}{4!}g\sum_i s_i^4\ , \tag{1}$$

where the sum $\langle ij\rangle$ is over z nearest-neighbor sites and the interactions J_{ij} are quenched random variables with the following distribution:

$$P(J_{ij}) = \frac{1}{(2\pi z)^{1/2}}\exp\left(-z\ J_{ij}^2/2\right)\ . \tag{2}$$

A purely relaxational dynamics can be described by the Langevin equation:

$$\Gamma_0^{-1}\frac{\partial s_i(t)}{\partial t} = -\frac{\partial(\beta\mathcal{H})}{\partial s_i(t)} + \xi_i(t)\ . \tag{3}$$

$\xi_i(t)$ is a Gaussian and white noise with zero mean and variance

$$\langle\xi_i(t)\xi_j(t')\rangle = \frac{2}{\Gamma_0}\delta_{ij}\delta(t-t'),$$

which ensures that locally the fluctuation-dissipation theorem holds.

In the MF limit the physical quantities of interest are the average local correlation function

$$C(t-t') = [\langle s_i(t)s_i(t')\rangle]_J\ , \tag{4}$$

and the average local response function

$$G(t-t') = [\langle s_i(t)\xi_i(t')\rangle]_J\ . \tag{5}$$

Beyond the MF approximation we find the non local fluctuations that are non vanishing

$$G^{\alpha\beta\gamma\delta}(i-j;t_1,t_2,t_3,t_4) = \langle\phi_i^\alpha(t_1)\phi_i^\beta(t_2)\phi_j^\gamma(t_3)\phi_j^\delta(t_4)\rangle - \langle\phi_i^\alpha(t_1)\phi_i^\beta(t_2)\rangle\langle\phi_j^\gamma(t_3)\phi_j^\delta(t_4)\rangle, \tag{6}$$

1 Introduction

The Mean field (MF) predictions of Spin Glasses (SG) theory for $T > T_c$ are generally accepted. This theory has been studied through different approaches all consistent among themselves. Many important results corcerning the equilibrium static properties of SG, have been derived using the replica method [1]. Sompolinsky and A. Zippelius [3], [4], [5], studied a soft spin version of the Edwards-Anderson model with the dynamic formalism, avoiding the replica trick. They defined a Langevin dinamics on the system and analysed the infinite range limit where the MF solution is exact. The static limit derived from dynamics is in agreement with the static prevision obtained with replica. Moreover, dynamic characteristics of the model have been well defined.

Unfortunately the behaviour of short-range SG system is not clear yet. There are different analytic works and simulations about the Ising and Heisemberg SG in finite dimensions, [7], [8], [9], that do not agree with each other. On the other hand, in order to study the corrections to the MF behaviour of the Green functions we can use the Renormalization Group theory and the ϵ-expansion. Chen, Lubensky [14], [15] and Green [16] studied a static Ising model in $d = 6 - \epsilon$ dimensions (6 is the upper critical dimension) with the Replica method and they found the corrections to the second order in ϵ for the critical exponents.

In this work, we have studied the behaviour of the dynamic fluctuations of the short range SG in the critic region $T \to T^+$, in $d = 6 - \epsilon$ dimensions. Our purpose is to extend the analysis of Chen, Lubensky and Green to dynamics and to find the correction to the MF value of the dynamic critical exponent z. Zippelius [6] showed that the Van Hove theory, [12] [13], is correct to the lowest order in ϵ and she found the relation $z = 2(2 - \eta)$. To obtain these results she made some assumptions, which we don't agree with, that lead to a simplified theory and let her derive the correction to z to the first order in ϵ. We have studied the same model with the same general formalism without using these hypothesis. We have been able to derive the exact behaviour of the propagators $G^{\alpha\beta\gamma\delta}(i, j; t_1, t_2, t_3, t_4)$ for any t_1, t_2, t_3, t_4 in the critical region. We have defined the general stucture of the Feynman diagrams that occur at the first order in the loop expansion (they are a lot of more than that considered in the analysis of Zippelius) and we have started to evaluate the contribution of some of these. The definitive computation, which is essential to find the correction to the dynamic exponent z, will be shown in a successive work.

This paper is organized as follows: in section 2 we define the model of the theory; in section 3 we find the form of the quadratic corrections to the

Dynamic fluctuations in a Short-Range Spin Glass model

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Abstract

Dynamic fluctuations of short-range spin glasses are studied in the critical region $T \to T_c^+$. Functional integral formalism is used to define an effective Lagrangian L for the fields $\delta Q_i^{\alpha\beta}(t_1, t_2)$ up to the cubic order in the series expansion around the dynamic Mean-Field value $\bar{Q}^{\alpha\beta}$. The critical behaviour of the non local fluctuations is studied in the quadratic approssimation (free theory). The long-range limit of the correlations, derived in this formalism, are comparated with the correlations of an infinite-range model studied with a different approach.

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