On Rational Expectations in a Model of Inflation

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Abstract

In economic models expectations about future parameters influence the behaviour of the agents. To have a well-defined model one has to write down a closed system of equations that relates expected and real values. Infinite rationality with perfect foresight provides a simple and appealing recipe. Other possibilities include adaptive expectations and/or evolving, adaptive agents. In this paper we examine critically some of these ideas in a simple model for inflation.

1 Introduction

In this note we want to make some observations on a model originally proposed by Sargent and Wallace [1] as a possible explanation of hyperinflation and later challenged and modified by Marcet and Sargent [2]. In both versions the dynamical equations have two fixed points at which inflation remains stationary. In the Sargent-Wallace version the agent uses rational expectations to form an estimate of the future rate of inflation whereas in the Marcet-Sargent version he forms forecasts using a least squares regression of price on the once lagged price.

The particular feature that interests us is that the stability of the fixed points changes from one version to the other. While Sargent and Wallace found that the higher inflation rate is stable against the lower rate fixed point, Marcet and Sargent found almost exactly the contrary. This is therefore an interesting laboratory to compare different hypothesis on how agents shape their expectations.

We will first discuss the possibility of rational expectations leading to perfect foresight. We will show that prediction of the near feature is controlled by a boundary condition on the far future and therefore there is no rational calculation that can lead to perfect foresight. Sargent and Wallace derive difference evolution equations by equating real values of inflation and estimated ones. We show that these equations have a natural interpretation as deterministic **backward** in time equations. The only behaviour consistent with a forward in time interpretation corresponds to agents forming their estimates at time t so that prices at t confirm the expectations of the previous generation.

We then discuss the Marcet-Sargent version of the model. Now the agent is supposed to extrapolate from past series of inflation data the level of next period prices. However by construction the agent does not use today's price in his extrapolation. As a consequence, in the more interesting range of parameters the lower fixed point is stable and the higher one is unstable.

The situation changes drastically when the agent is allowed to take into consideration today's prices when forming an estimate of tomorrow's inflation. This requires a certain degree of recursive reasoning but no anticipation on the future. The agent has to solve a second degree equation to determine his prediction of the rate of inflation. The two solutions of such equation determine two different future paths for the prices and inflation rates. In a large range of parameters the fixed points now fall into the two different branches and are both stable.

2 The model and its fixed points

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To focus on the issues we discuss the simplest version of the Sargent Wallace model. It consists of a model of overlapping generations with constant population and an exogenously specified endowment stream of a single type of consumption good. At each period of time a constant number of *young* people are born that live for 2 periods. Each of them is endowed with w_1 units of the consumption good when young and w_2 when old. All agents act identically and therefore their actions are represented by a single agent. His decision to consume follows from the maximisation of a **utility** function $\log(c_y(t)c_o(t+1))$ constrained by the overall budget:

$$c_y(t)p(t) + c_o(t+1)p(t+1) \leq w_1 p(t) + w_2 p(t+1) \tag{1}$$

The utility function is intertemporal and therefore the young agent has to form an estimate of tomorrow's rate of inflation $\pi^*(t) = p^*(t+1)/p(t)$ (p are the prices, p^* are the corresponding estimates). Solving

$$\begin{aligned} max(\log(c_y(t)c_o(t+1))) \\ \text{subject to} \\ c_y(t)p(t) + c_o(t+1)p^*(t+1) \leq w_1 p(t) + w_2 p(t+1), \end{aligned} \tag{2}$$

the consumption when young becomes:

$$c_y(t) = \frac{w_1 + w_2\pi^*(t)}{2} \tag{3}$$

while of course $c_o(t+1)$ is determined from the budget constraint which due to the concavity and monotonicity of the utility function is always saturated.

This kind of model is often used to discuss a scenario where the young borrows to increase his consumption. Instead in this case $w_1 > w_2$ so that the young agent prefers to exchange part of his endowment for currency to spend later. In this situation the monetary authority can issue an unbacked currency to finance a deficit G. The budget constraint reads:

$$h_t - h_{t-1} = p(t)G \tag{4}$$

where h_t is the total currency (per young person) at time t. We may assume that the market clears so that:

$$(w_1 - c_y(t))p(t) = h_t \tag{5}$$

Equations (4) and (5) do not determine the future course of the economical system because $c_y(t)$ depends on expected future inflation. However the possible stationary inflation rates can be determined without ambiguity. In this case:

$$\begin{aligned} \frac{w_1 - w_2\pi}{2} &= \frac{h_{t-1}}{p(t)} + G \\ &= \frac{w_1 - w_2\pi}{2\ \pi} + G \end{aligned} \tag{6}$$

This equation can be solved to obtain either two possible stationary values for the inflation rate or none . The right hand side of eq 6 is the supply from the young while the left hand side has two terms that represent the old agent's demand and the government deficit. The condition for existence of the fixed points is:

$$G \leq \frac{w_2 + w_1}{2} - \sqrt{w_1 w_2} \tag{7}$$

The interpretation of the model requires in addition that:

$$w_1 > w_2;\ G > 0;\ \pi > 1 \tag{8}$$

Otherwise one can analytically continue the model to a scenario of positive interest rates.

Both fixed points are meaningful. For instance, in the limit $w_2 \to 0$ with w_1 and G constant,the higher fixed point diverges. On the smaller fixed point the three terms in the equation are of the same order while on the larger one the old agent's demand goes to zero. If the young agent believes that inflation will be very large, he will reduce drastically his supply to the market. The government will devaluate currency so as to reduce to zero the acquistal power of the old generation and get hold of all the supply. This confirms the young agent's estimate and therefore explains the stationarity of the higher rate of inflation.

To check the stability of the fixed points, in the next section we will reintroduce the time dependence and discuss different ways to close the equations depending on which mechanism is used to predict inflation.

3 Dynamics: Rational Expectations vs. Least Squares Regression

In this model the equilibrium between supply and demand determines the price in a rather peculiar way because supply depends on the expected rate of inflation rather than on price. We can substitute 4 on equation 5 to rewrite it as:

$$\frac{w_1 - w_2\pi^*(t)}{2} = \frac{w_1 - w_2\pi^*(t-1)}{2\pi(t-1)} + G \tag{9}$$

Two time dependent functions must be determined.

At this point Sargent and Wallace invoke rational expectations and perfect foresight to impose $\pi^*(t) = \pi(t)$. The resulting equation:

$$\frac{w_1 - w_2\pi(t)}{2} = \frac{w_1 - w_2\pi(t-1)}{2\pi(t-1)} + G \tag{10}$$

was then interpreted by the authors as a deterministic forward in time equation that allows the agent to "guess" the right value for the rate of inflation based on past values. A little reflection shows that this interpretation can not be correct because there is simply no way to calculate from past data the **correct** value of tomorrow's inflation. The real fact is that estimates today are determined by a future final time condition rather than by an initial condition.

The difference evolution equation derived by equating estimated values and real values of inflation can be read backwards by an agent to predict inflation on any date if

- he knows the estimate of inflation that a future generation will make on a future date
- all generations in between have the same information
- all generations in between try to play the same rational game

On the other hand if the agent uses eq.10 in the forward direction in time, i.e. if he chooses $\pi(t)$ from 10 what he is doing (compare 9) is to adjust his estimate so that the price at time t *fulfils an earlier prophecy by a different agent, whatever that was.* In this way he neither maximises his utility nor the previous agent's utility.

It is well known that differential equations stability properties depend on whether one reads them forward or backward. In fact when read backward the **lower** rate of inflation fixed point becomes the stable attractor while when read forward the stable fixed point is the higher one.

In the second paper Marcet and Sargent introduced an explicit mechanism of formation of estimates of next period's price level using a least squares regression of price on the once lagged price:

$$\pi^*(t) = \frac{\sum_{i=0}^{t-2} p(i)^2 \pi(i)}{\sum_{i=0}^{t-2} p(i)^2} \tag{11}$$

and therefore a new dynamical equation:

$$\pi^*(t) = (1 - g_{t-1})\pi^*(t-1) + g_{t-1}\pi(t-2) \tag{12}$$

where

$$g_{t-1} = \frac{p(t-2)^2}{\sum_{i=0}^{t-2} p(i)^2} \tag{13}$$

is a positive, less than one, slowly varying function of the past prices up to but excluding $p(t-1)$. It was proven in [2] that the difference equations system 9 and 12 has a lower stable fixed point if $g < \pi_2 - 1$ where π_2 is the larger fixed point.

It is remarkable that the estimate of the rate of inflation at time t is expressed as a weighted average of past rates of inflation **up to** time $t-2$. That means that the agent does not use today's prices to infer tomorrow's inflation. This is a strong (unrealistic?) behavioural hypothesis that as we will show in next section is at the basis of Marcet and Sargent conclusion that the higher inflation fixed point is unstable.

4 Bounded Rationality

We will now explore what happens if we include today's price in equation 12. The simplest generalisation would be to replace $\pi(t-2)$ by $\pi(t-1)$:

$$\pi^*(t) = (1 - g)\ \pi^*(t-1) + g\ \pi(t-1) \tag{14}$$

Again substituting eq 14 into 9 we derive:

$$\frac{w_1 - w_2\pi^*(t)}{2} = \frac{g\ (w_1 - w_2\pi^*(t-1))}{2(\pi^*(t) - (1-g)\pi^*(t-1))} + G \tag{15}$$

This equation again has either two solutions for $\pi^*(t)$ as a function of $\pi^*(t-1)$ or none. In the latter case demand is larger than supply for all prices. Otherwise there are two possible equilibria (supply=demand) characterised by two different price levels. In our case supply decreases with price (whereas in Marcet Sargent it was constant). A higher price today is interpreted as signal for a higher rate of inflation tomorrow. The demand also decreases but eventually at high prices levels off because G, the per capita deficit, is by construction independent of price. In the lower price equilibrium the supply curve is steeper than the demand curve and therefore this point is stable under tâtonnement dynamics. The contrary occurs at the higher point. The higher price is a consequence of a lower level of supply consequential to a pessimistic estimate of tomorrow's inflation. Figure 1 shows a typical case with parameters such that the two inflation fixed points are $\pi_1 = 1.5$, $\pi_2 = 4$ and $g = .5$. We do not see any reason, on the context of this model, to disregard one of the two equilibria. On the hypothesis that the economic agents must forecast inflation it is quite reasonable that they find two solutions. Many models in economy (and in physics) show similar multiple equilibria. Adding noise sometimes help to discriminate which is the more robust. It would be interesting to embed this two player game into a larger model to check these questions.

The two stationary inflation fixed points are now in the two different equilibrium branches and they are both attractors if the system moves along one equilibrium branch. Figure 2 shows a typical case with the same values of the parameters as in Fig.1. One may speculate on the following scenario: in the absence of any noise, and the parameters of the economy fixed, inflation will stabilise and stay at one of the fixed points. On an adiabatic change of the parameters the system will move towards the new fixed point along the same branch. On the other hand if an abrupt change in the parameters and/or the level of noise is large with respect to the difference between the two branches there may be a sudden jump from one equilibrium to the other. It may then be difficult to restore the previous situation.

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References

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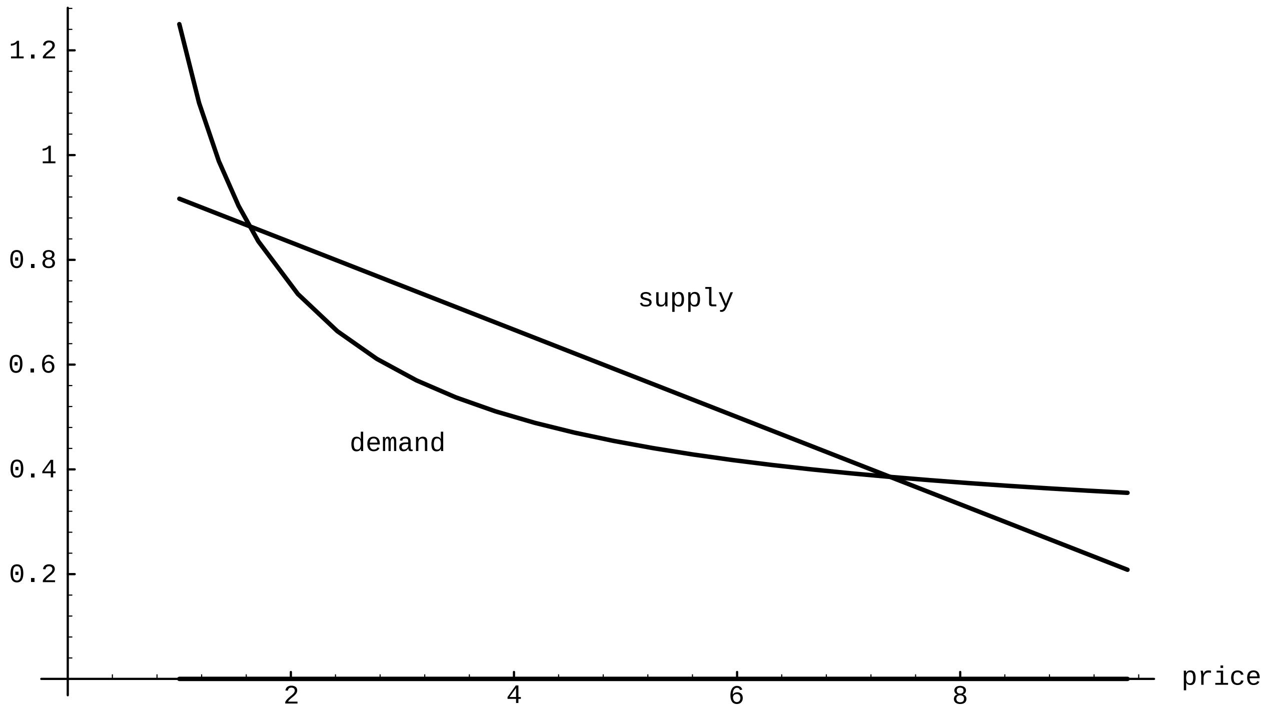

Figure 1: Supply and demand curves as functions of relative price ($= p(t)/p(t-1)$) at time t for the following parameter values: $\pi_1 = 1.5$; $\pi_2 = 4$; $h_{t-1}/p(t-1) = 1$; $g = .5$. For these values: $w_1/w_2 = 6$ and $G = 0.25$

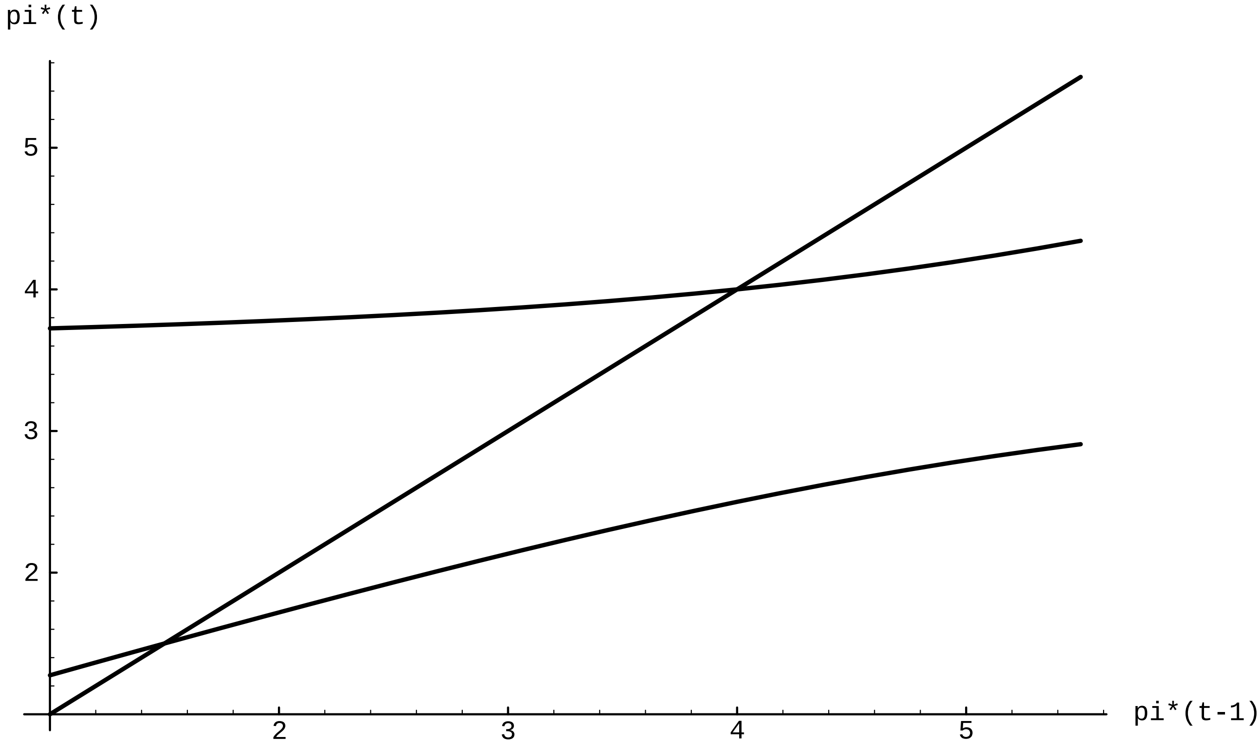

Figure 2: The two possible curves for the time dependence of the inflation's estimates. The crossings represent the two possible fixed points. The derivative of $\pi^*(t)$ as a function of $\pi^*(t-1)$ is less than one at both fixed points therefore they are both stable.