High-dimensional surprises near the glass and the jamming transitions: connecting spin glasses with reality

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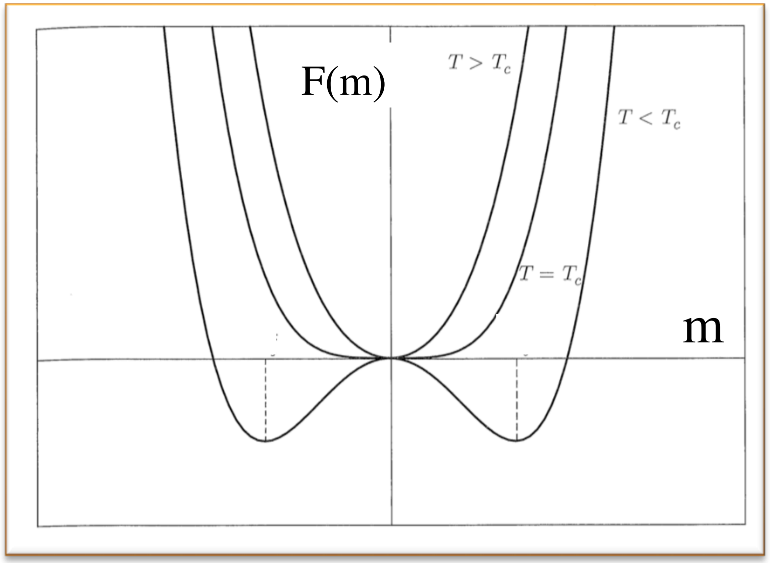
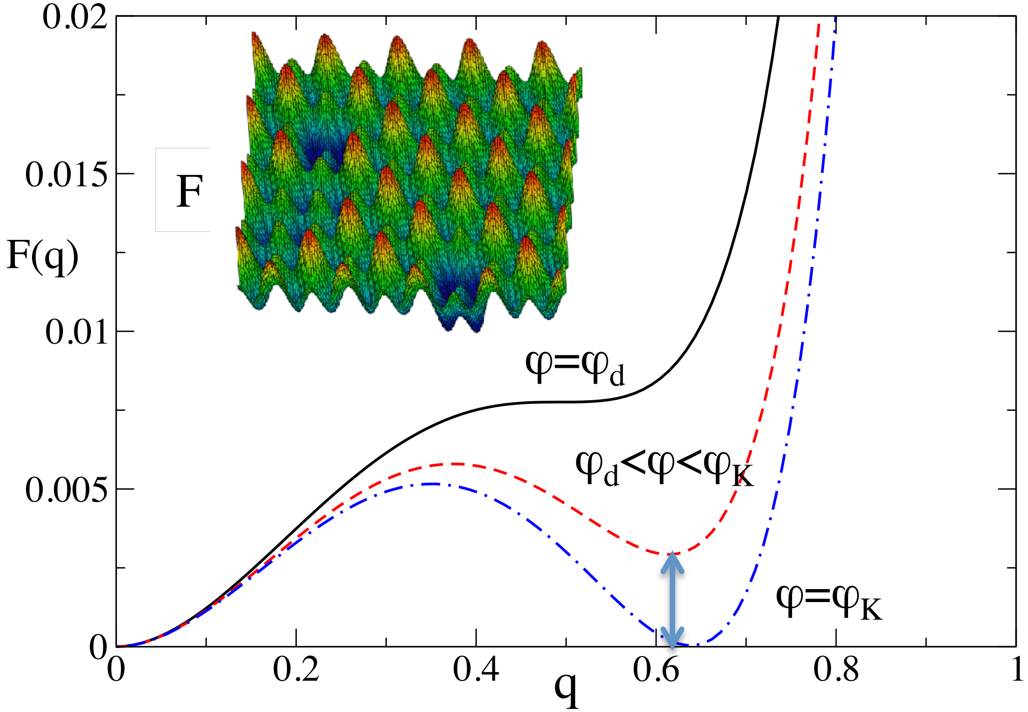
Spin-based MF picture for glasses (RFOT)



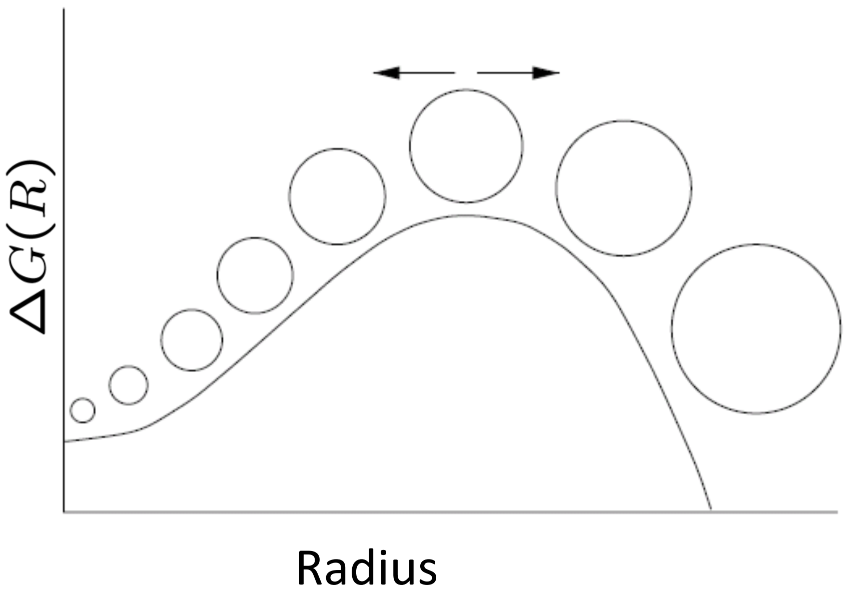


Exact in $d=\infty$.

The dynamics gets infinitely sluggish at the onset of the breakdown in states φ_d.

Kirkpatrick, Thirumalai, Wolynes (1987-1989); Parisi *et al.*
Debenedetti and Stillinger, Nature (2001)
Franz, Parisi (1997); KPZ (2012)

Problem 0: Crystal nucleation



The crystal state is completely orthogonal to this description and it thought to interfere with its observation.

Typical solution: complex alloys

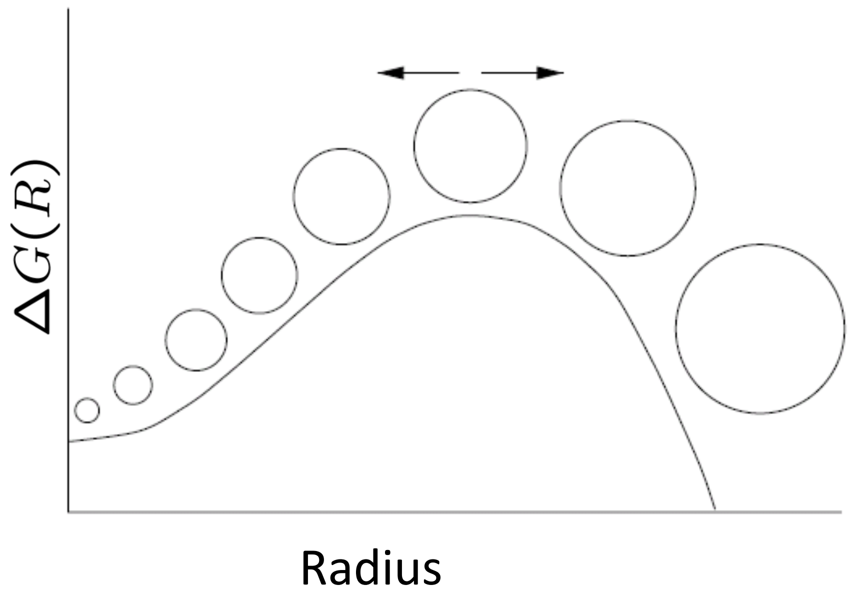
Other possible solution: increase spatial dimension

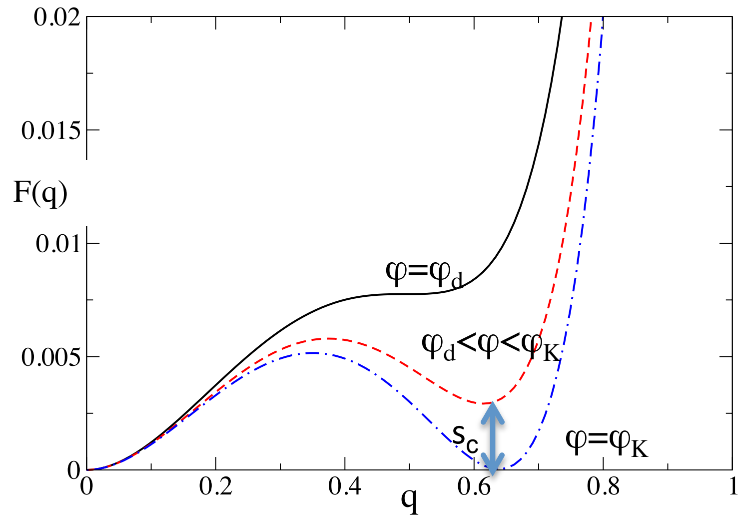
$$\Delta G(R) = \gamma S_{d-1} R^{d-1} - \Delta\mu \rho_s V_d R^d$$

$$\Delta G^{\dagger}(R^*) \sim \frac{(2d\pi)^{d/2}\gamma^d}{\Delta\mu^{d-1}}$$

van Meel, Fortini, Charbonneau, Charbonneau, PRE (2009-2010)

Problem 1: Glass-glass nucleation and (facilitated) hopping





$$\Delta G(R) = \gamma S_{d-1} R^{d-1} - s_c \rho V_d R^d$$

$$\Delta G^{\dagger}(R^*) \sim \frac{(2d\pi)^{d/2}\gamma^d}{s_c^{d-1}}$$

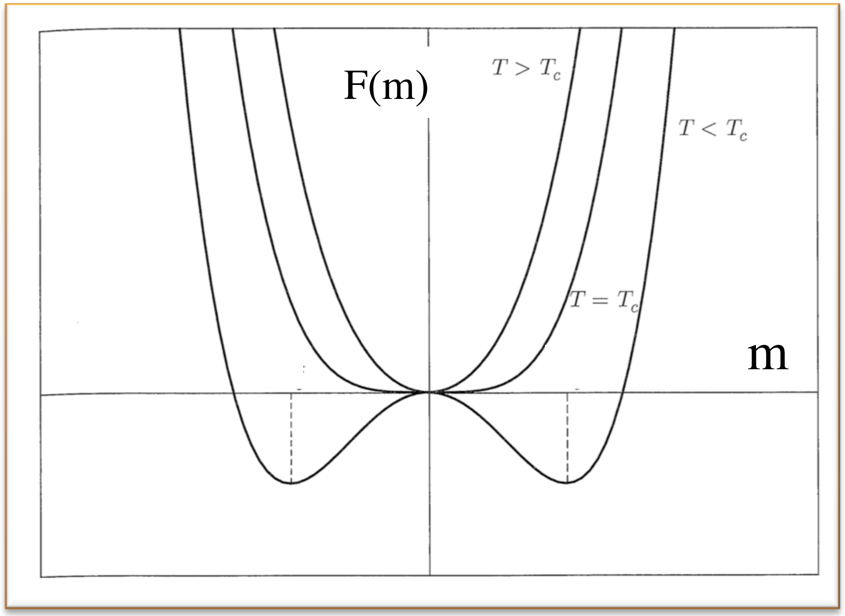
Mean-field hopping: see Yuliang Jin's poster

Once replica symmetry is broken, nucleation from one state to another is still possible.

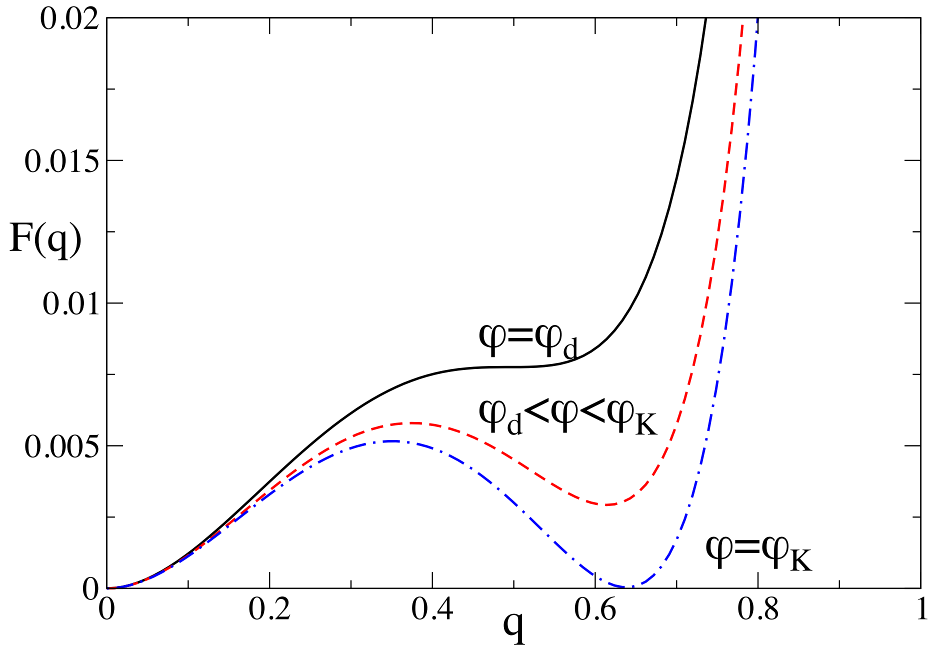
Typical solution: patch theory, look in crossover area

Other possible solution: increase spatial dimension

Problem 2: Below d_u fluctuations renormalize, or worse



For an Ising-type field theory d_u=4

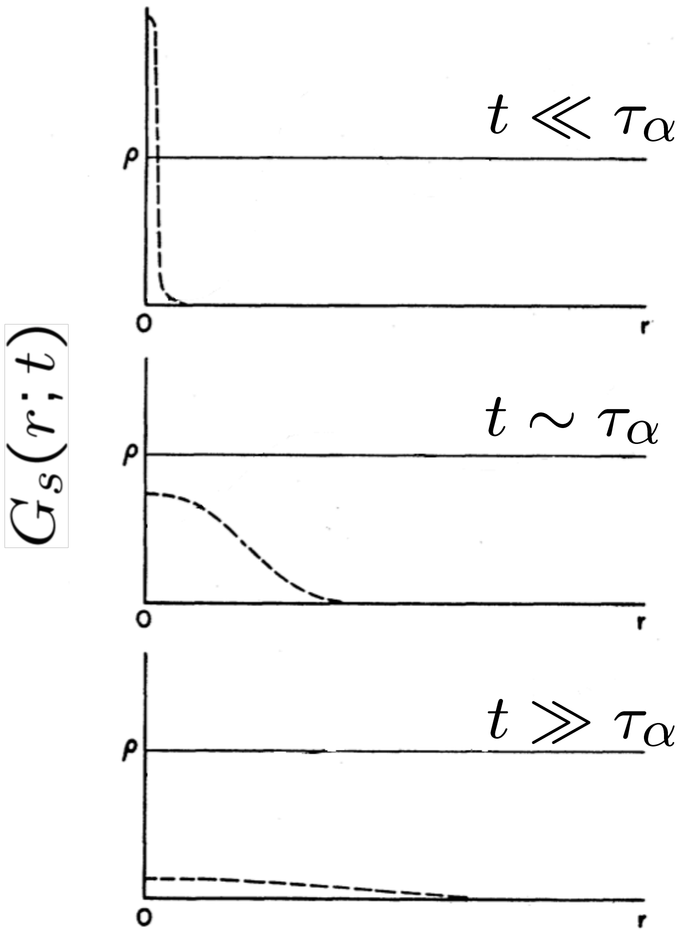


For the RFOT dynamical transition d_u=8

Typical solutions: wave hands or throw everything away
Other possible solution: increase spatial dimension

Biroli, Bouchaud (2007); Franz *et al.* (2011)

Problem 3: Caging order parameter is non-trivial to describe



van Hove (1954)

$$G_s(r;t) \propto \frac{1}{N}\sum_{i=1}^{N}\langle \delta(|\mathbf{r}_i(t) - \mathbf{r}_i(0)| + r)\rangle$$

Various treatments make different approximations => quantitatively different results at finite *d*.

RT: static (free energy) description, assumes Gaussian cage
MCT: (non-linear) dynamical description is not Gaussian

But in a (simple) mean-field limit they should correspond.

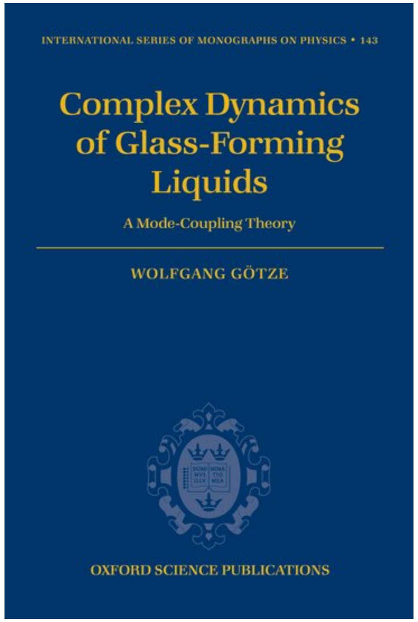
Typical solutions: hope that all is well in the mean-field limit
Other possible solution: increase spatial dimension to check

Example 1: MF view of MCT

The Mode-Coupling Theory of supercooled liquids

Does it wear any clothes?

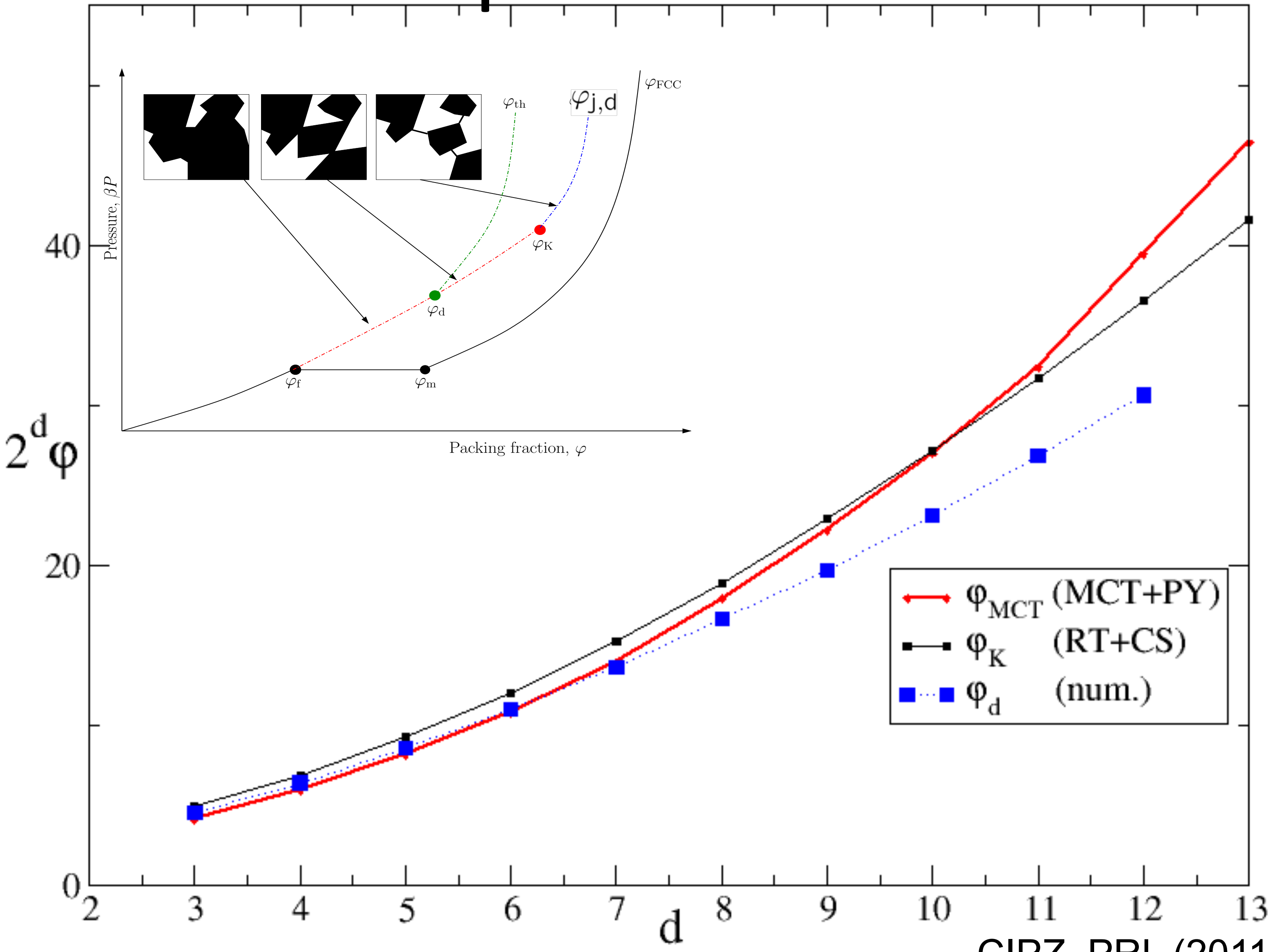
$$\varphi_{\mathrm{RT}} \sim 4.8d2^{-d}$$
$$\varphi_{\mathrm{MCT}} \sim 0.22d^2 2^{-d}$$



Glass transition of hard spheres in high dimensions,
Authors: Bernhard Schmid, Rolf Schilling
arXiv:1003.4559.

Mode-Coupling Theory as a Mean-Field Description of the Glass Transition
Authors: Atsushi Ikeda, Kunimasa Miyazaki
arXiv:1003.5472

MF problem with MCT



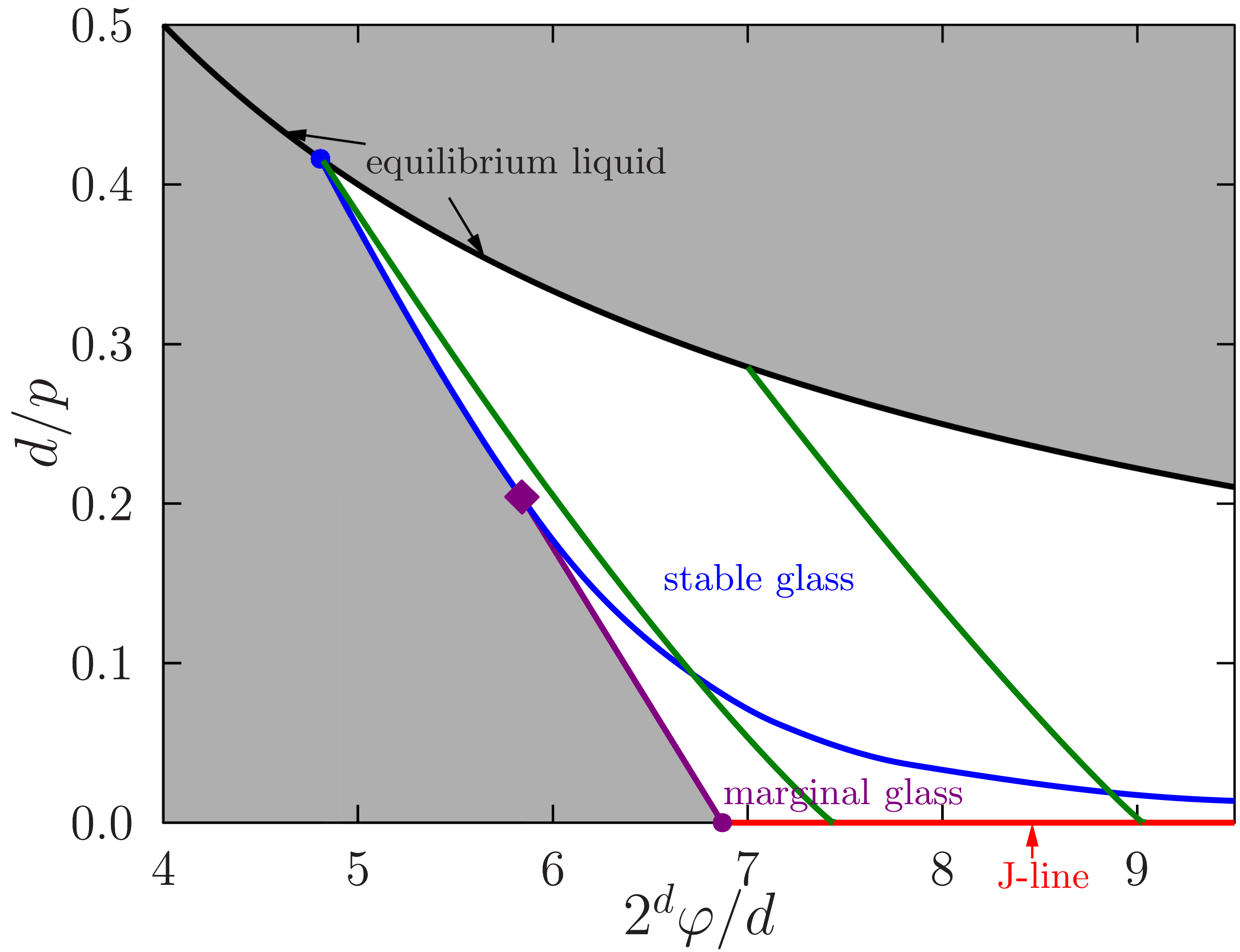
CIPZ, PRL (2011); PNAS (2012)

Example 2: Jamming

Out-of-equilibrium critical transition, hence describing it requires a good microscopic glass theory.

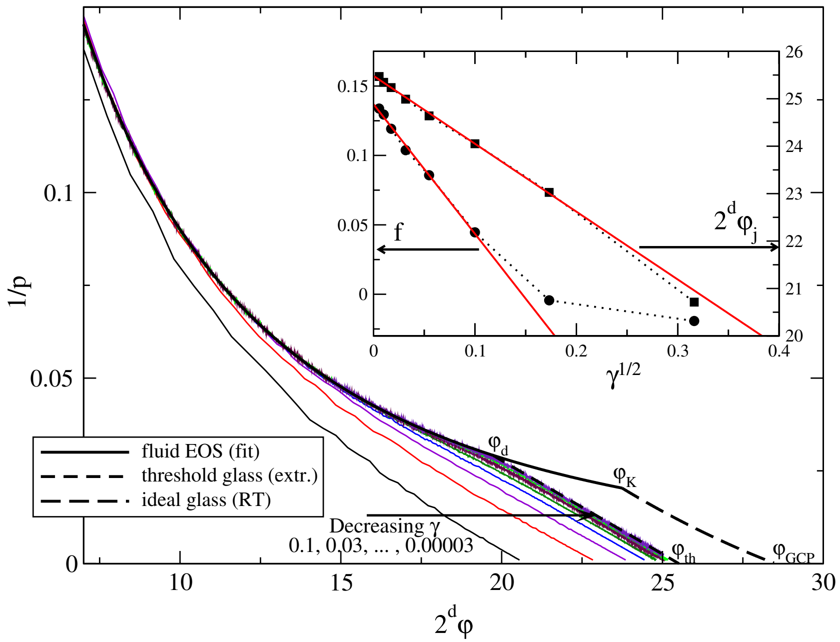
->Stringent test of glass theories.

HS phase diagram (GP's talk tomorrow)

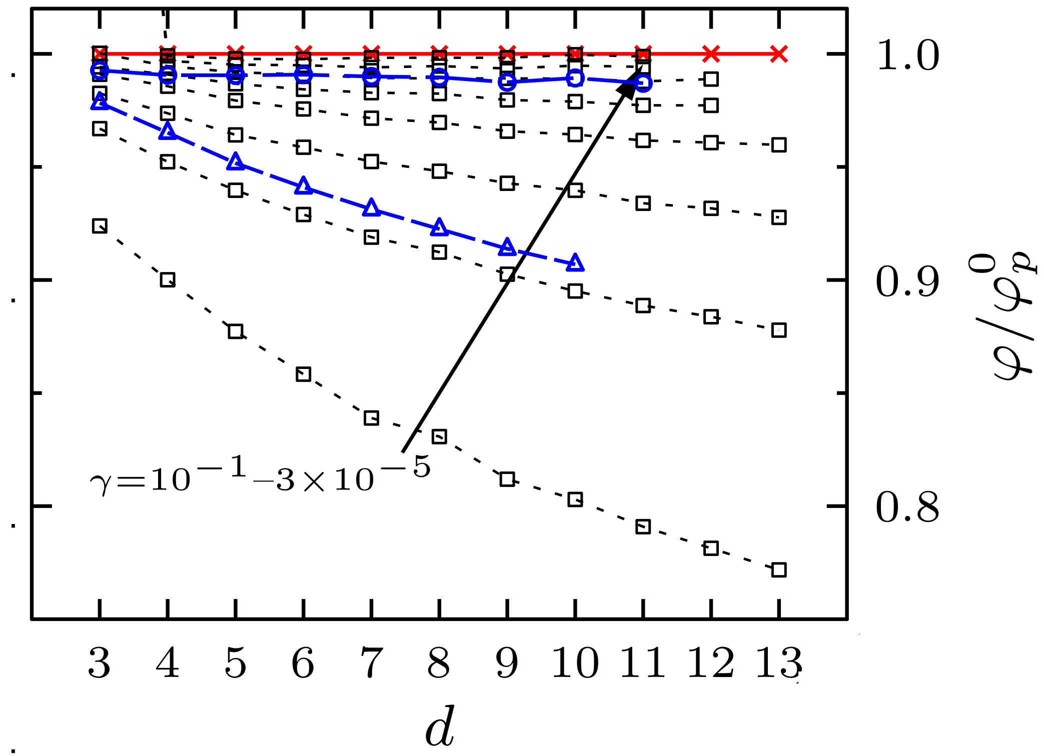


CKUPZ Nat. Comm. (2014) & other papers PZ, KPZ, KPUZ, CKPUZ (2010-2014)

Evolving Density

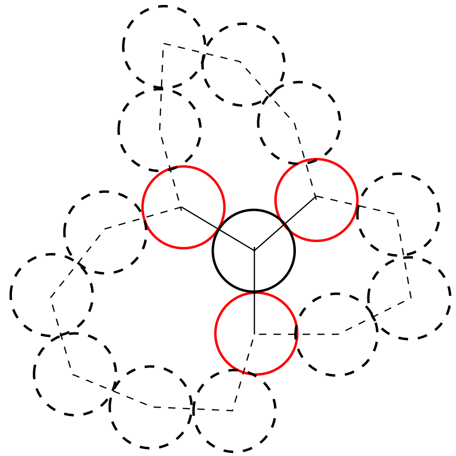


Consistent with *d*=3 results
Chaudhuri, Berthier, Sastry PRL (2010)



CIPZ PRL (2011); CCPZ PRL (2012)

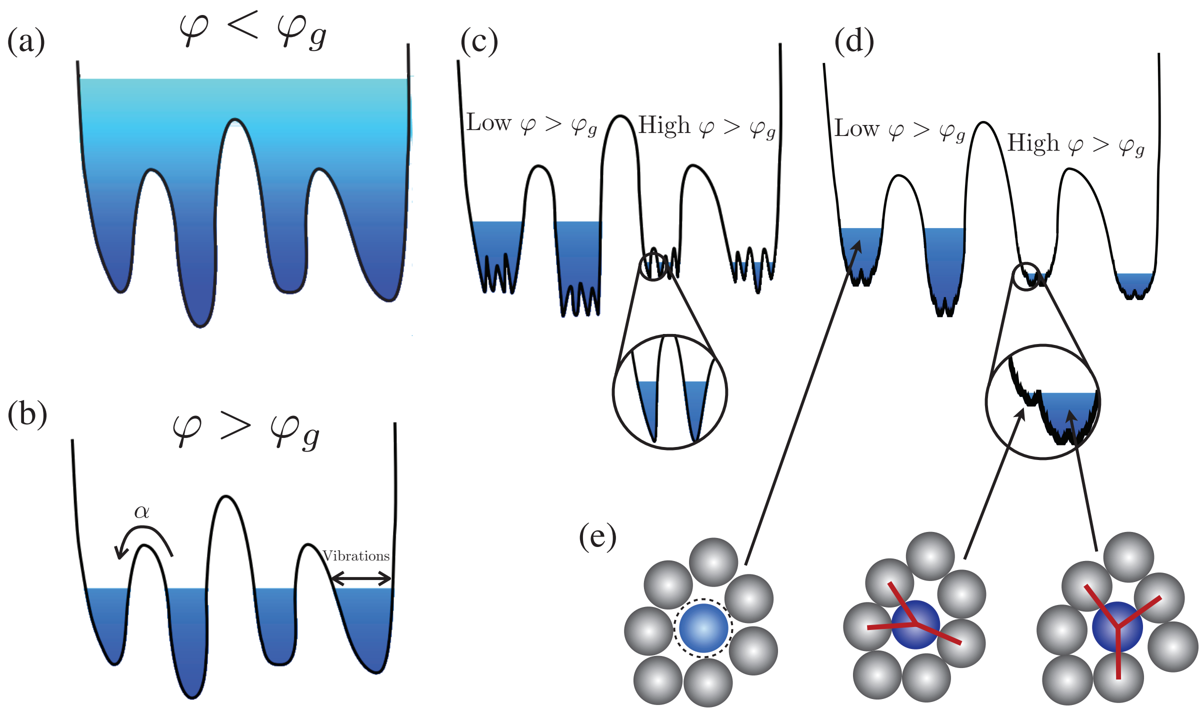
Cage Collapse



Between the liquid and jamming, something must happen, because

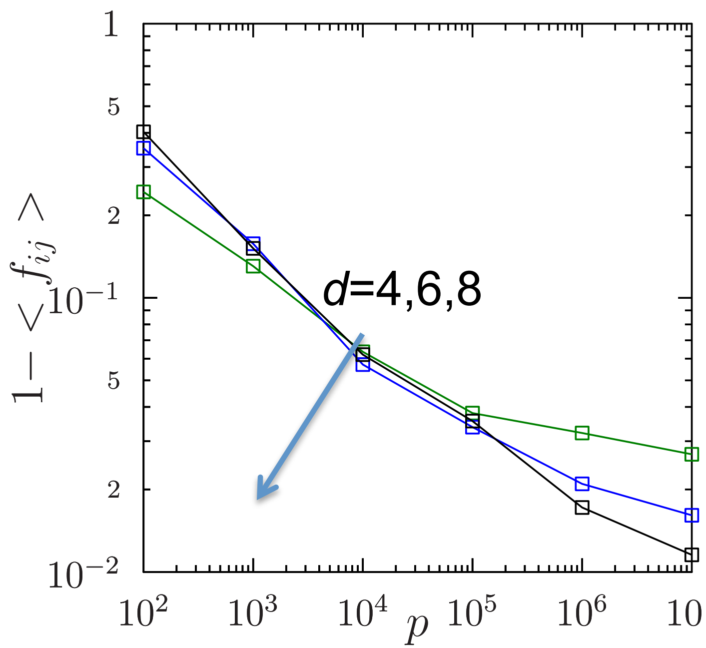
$$\bar{z} = 2d \text{ (isostaticity)}$$

$$\bar{z} \sim e^{d} \text{ (liquid shell)}$$



CKUPZ Nature Comm. (2014)

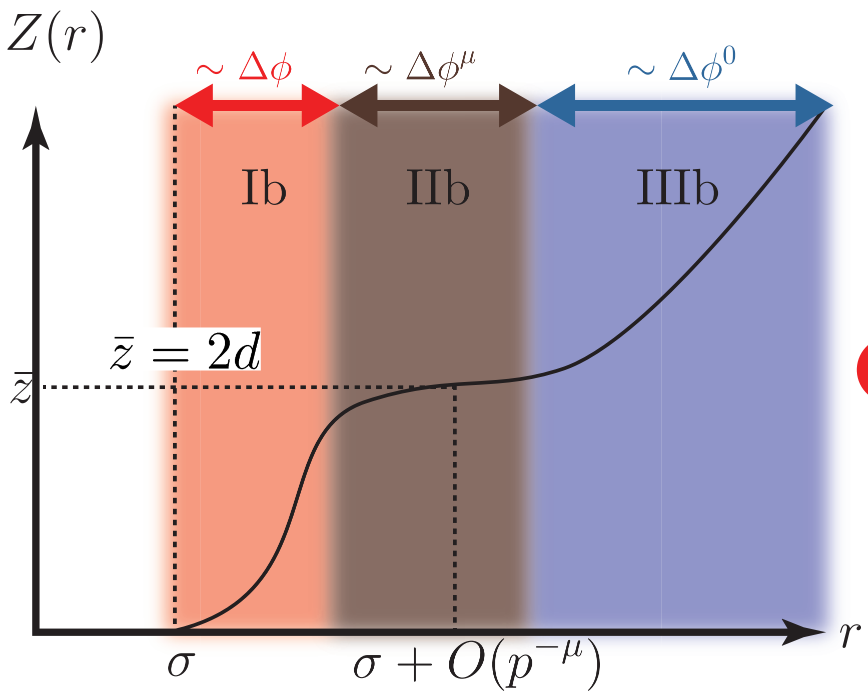
How determined are the force contacts (and the rattlers)?

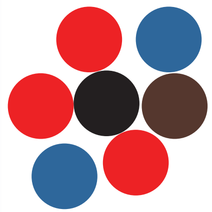


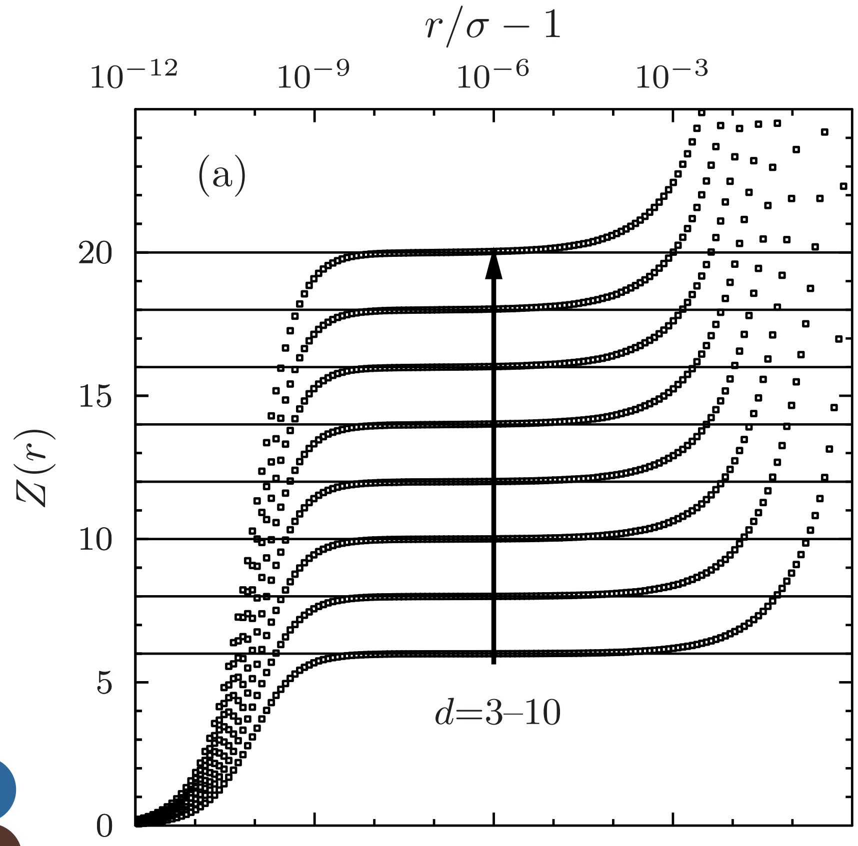
Jamming (marginally stable) cage

As proposed by Wyart PRL (2012), two critical regimes follow from marginality.

1RSB solution does not: power-law exponents are 0.

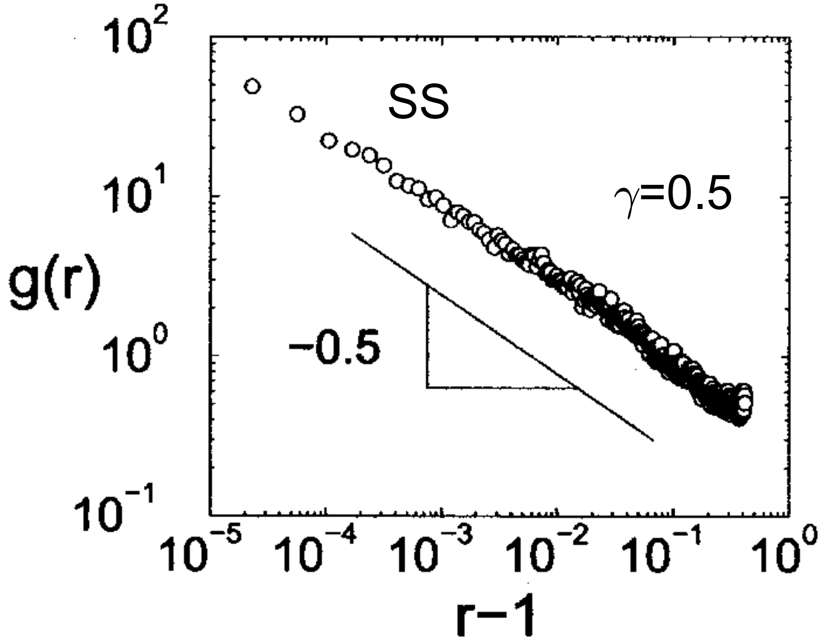




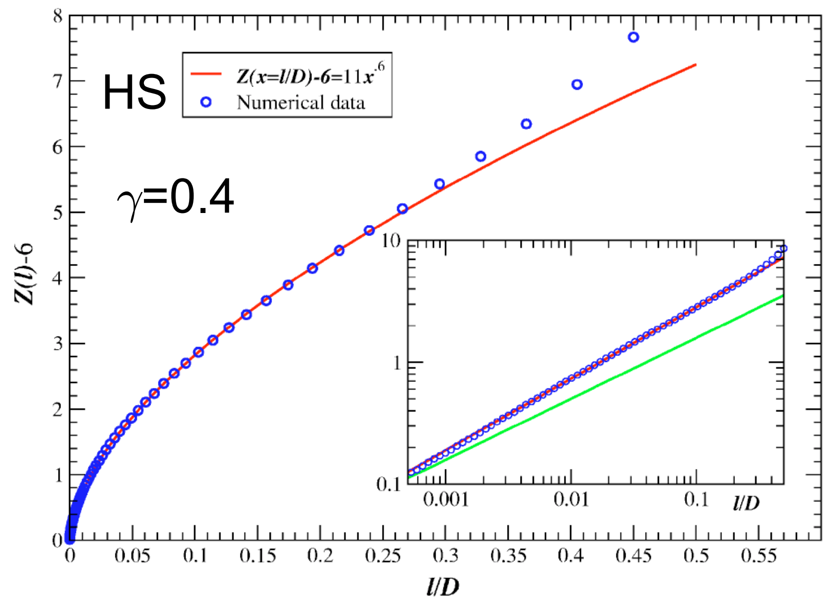


CCPZ PRL (2012)

1^{st} power law: near contacts

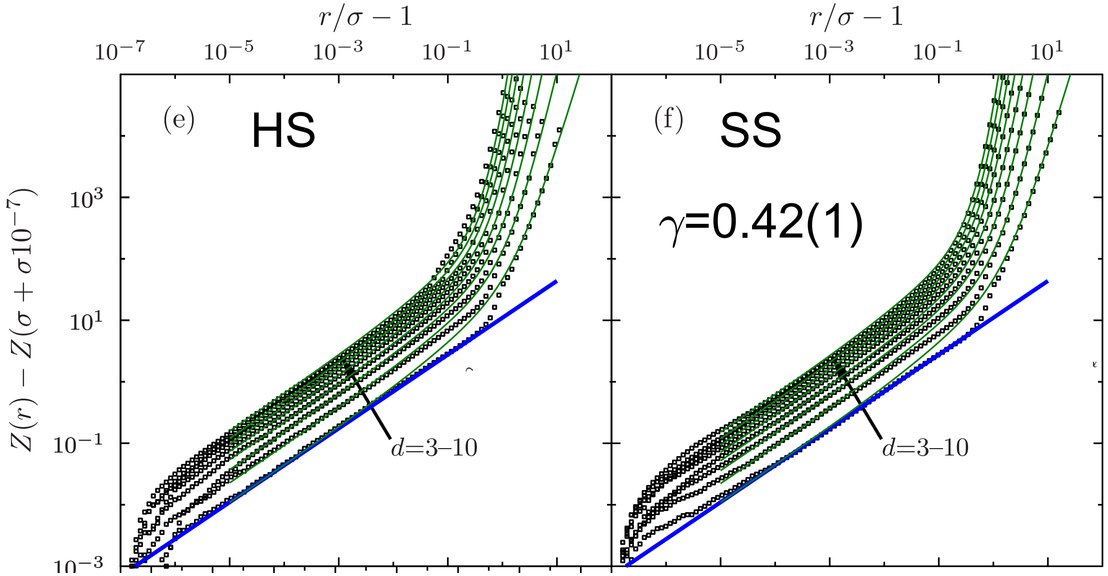


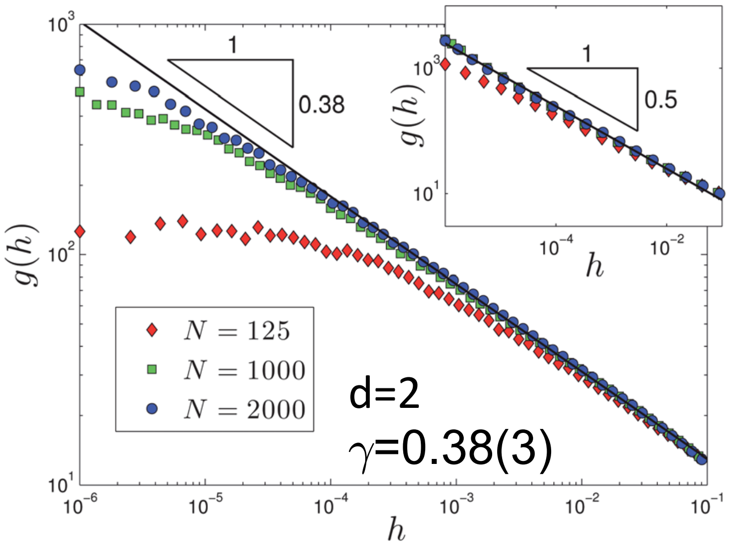
Silbert, Liu, Nagel, PRE (2005)

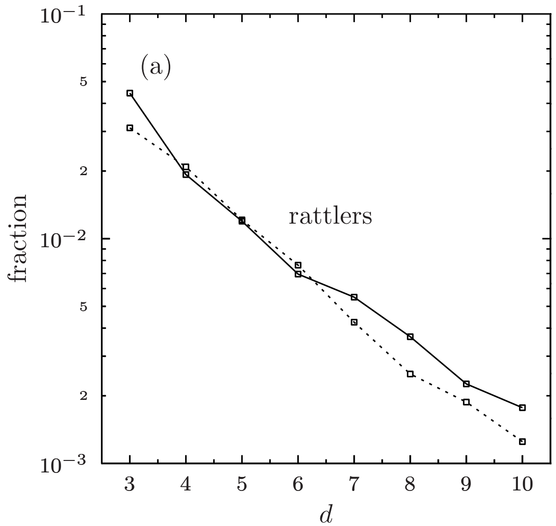


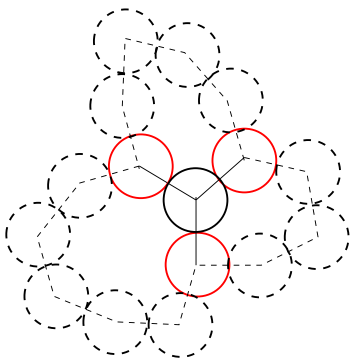
Donev, Stillinger, Torquato, PRE (2005);
Skoge *et al*. PRE (2006)

Near contacts: remove rattlers









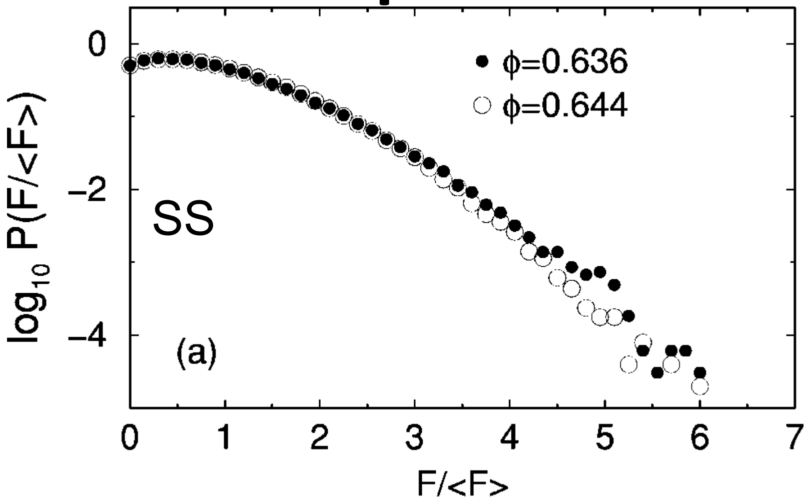
fullRSB γ=0.41269

Suggest d_u=d_l=2 (?)

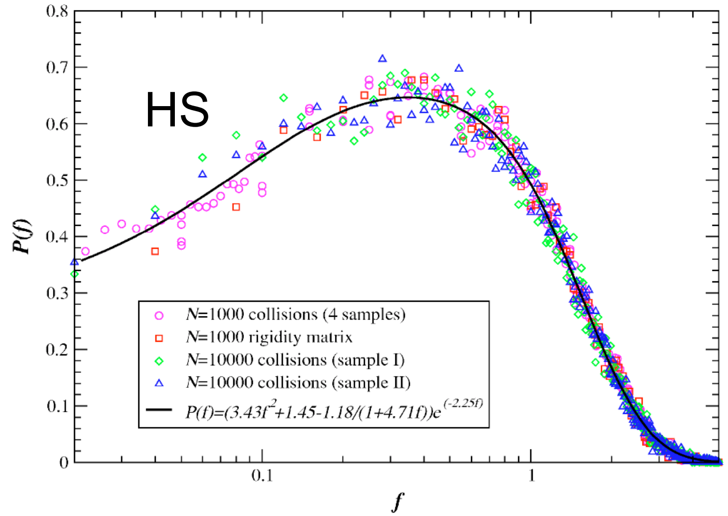
Agrees with finite-size scaling arguments of Goodrich and Liu PRL (2012).

CCPZ PRL (2012); Lerner, Duering, Wyart Soft Matter (2013)

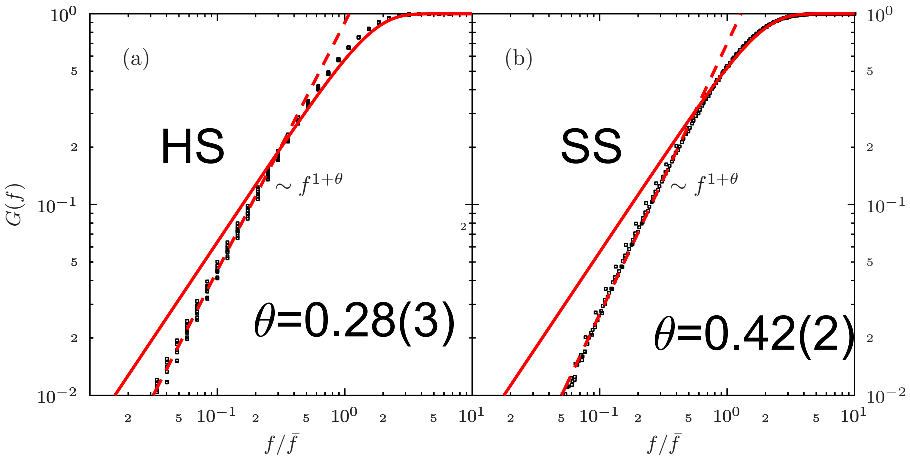
2^{nd} power law: force contacts



O'Hern et al. PRE (2003)

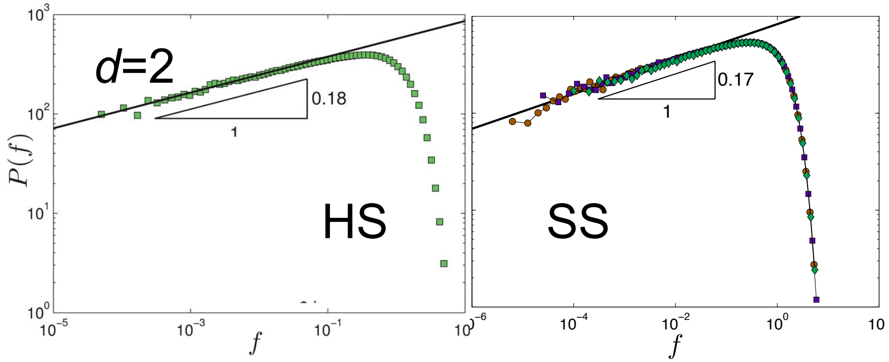


Donev, Stillinger, Torquato, PRE (2005)



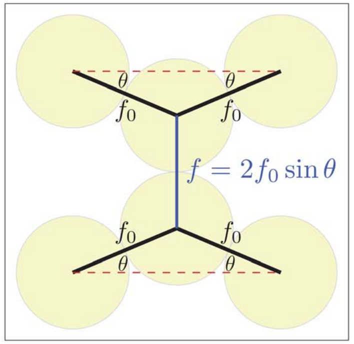
CCPZ PRL (2012)

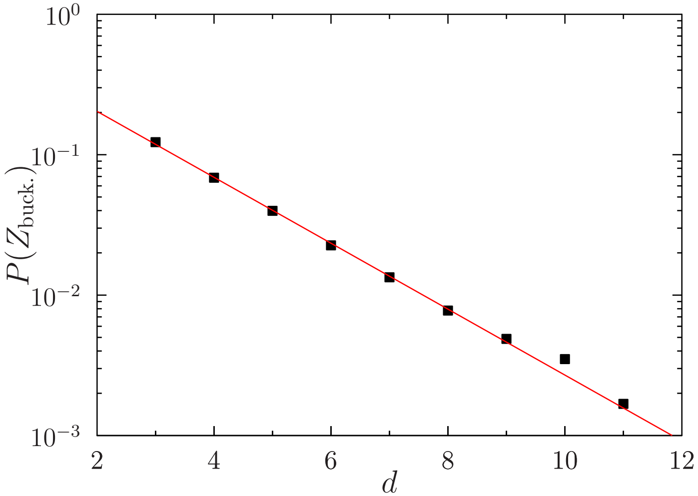
Embarrassingly different…

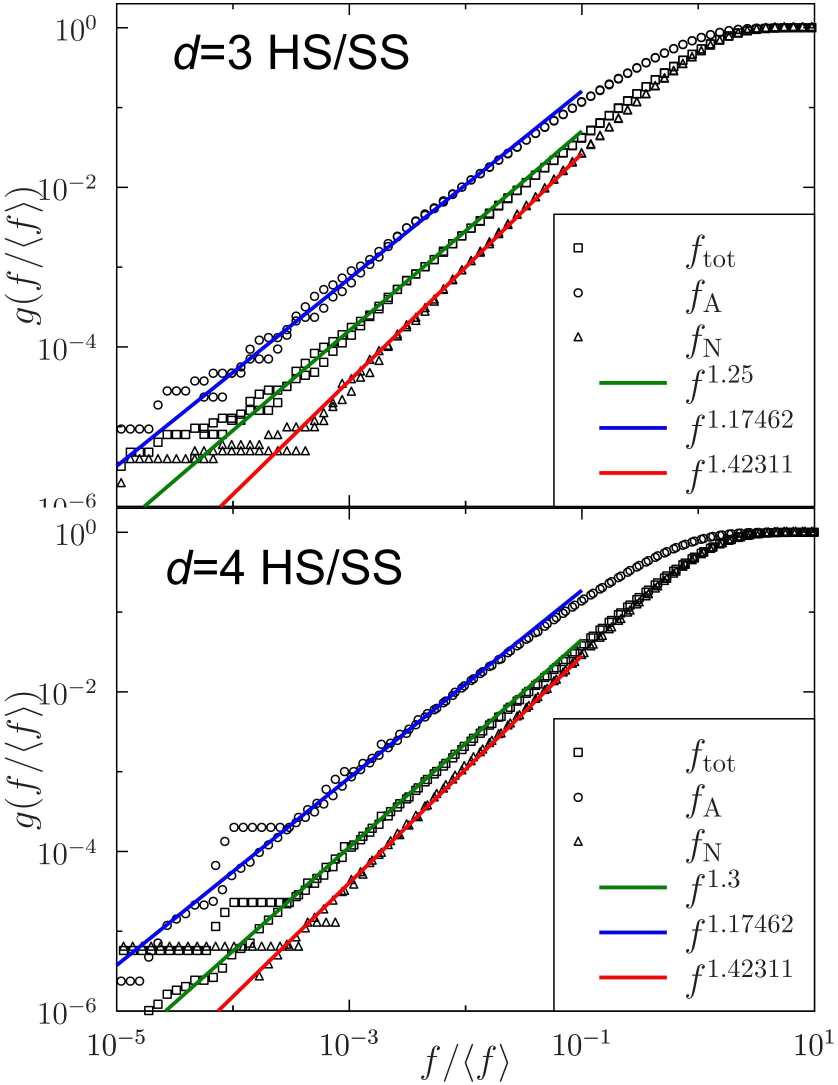


Lerner, Duering, Wyart Soft Matter (2013); DeGiuli et al. arXiv:1402.3834

Force contacts: remove bucklers

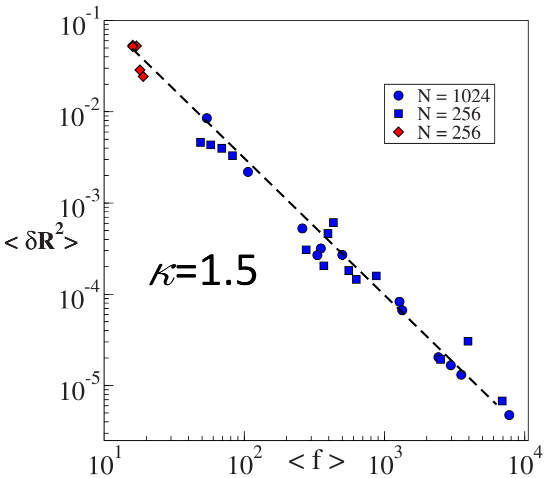






CCPZ unpublished (2014)

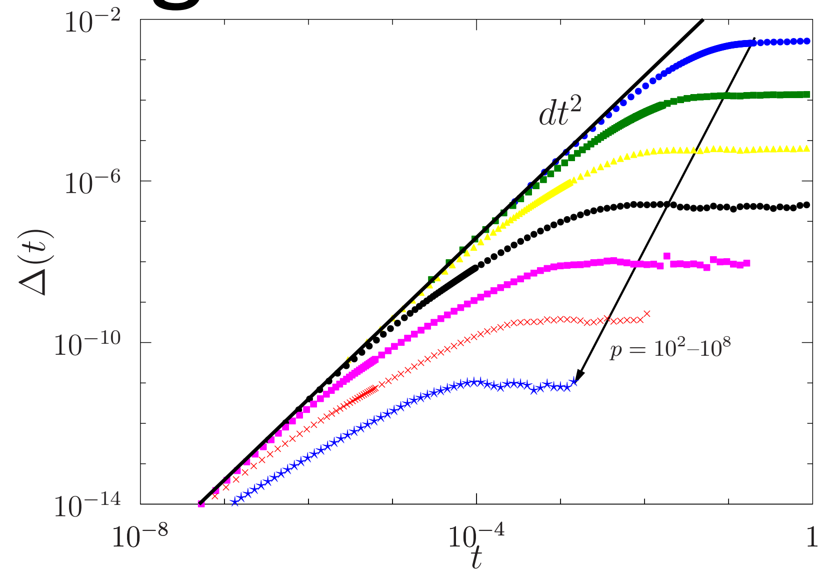
3^{rd} power law: cage evolution

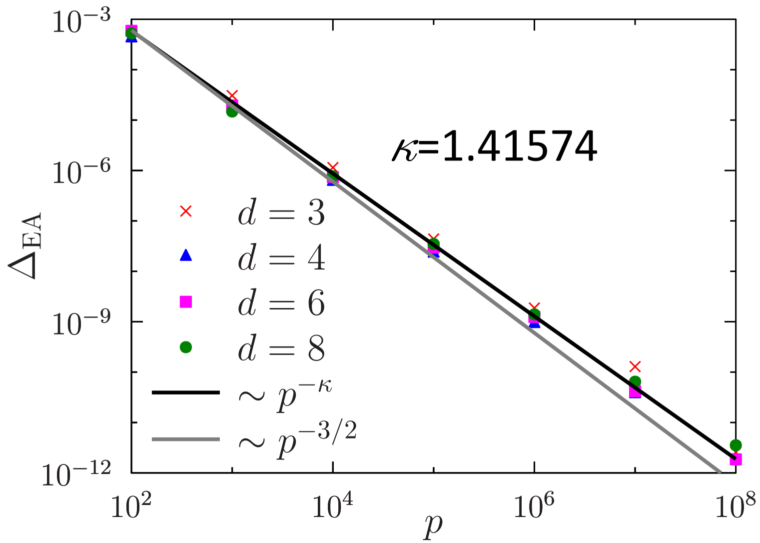


Brito and Wyart J. Chem. Phys. (2009)

1RSB solution gives 0.

High pressure numerical challenges:
-eliminate rattlers
-network and thus rattlers reorganize with time





CKUPZ Nat. Comm. (2014)

Problem 4: Can only approach dynamical transition from one side

Typical solutions: get out of equilibrium and pray

Other possible solution: High dimensions can't help much, so… plant a glass instead!

Example 3: Mari-Kurchan (MK) model

HS interacting via shifted potential

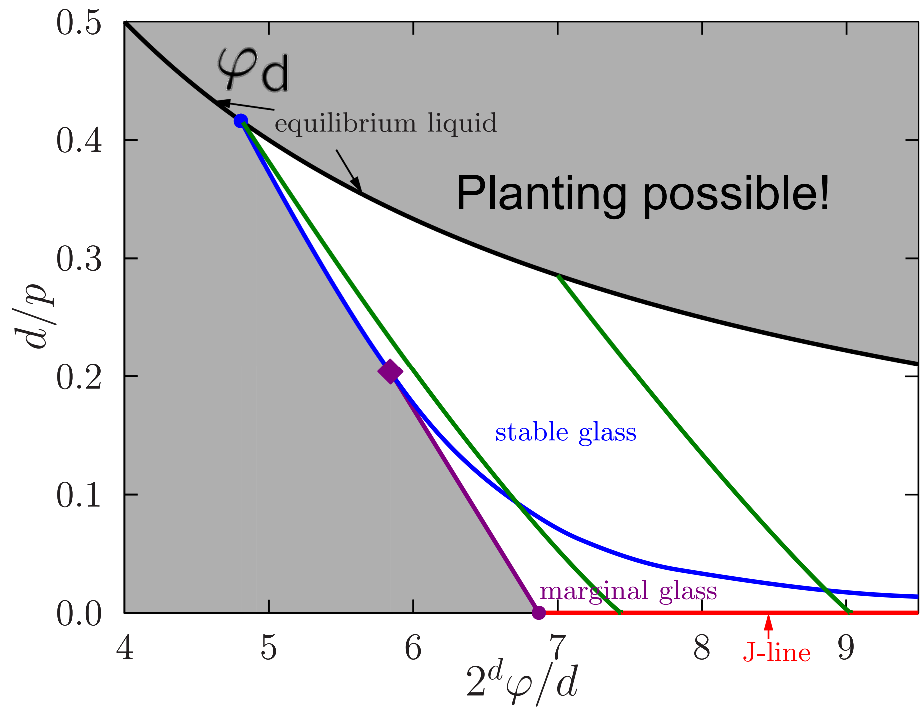
$$H_\Lambda = \sum_{j>i=1,N} V(|x_i - x_j + \Lambda_{ij}|)$$

$\Lambda_{ij} \in [0,L]^d$ with a flat distribution. If A is near B and B is near C, A is not necessarily near C. In the liquid phase

$$g(x) = \exp(-\beta V(x))$$

i.e., $p = \beta P/\rho = 1 + \beta B_2 \rho$

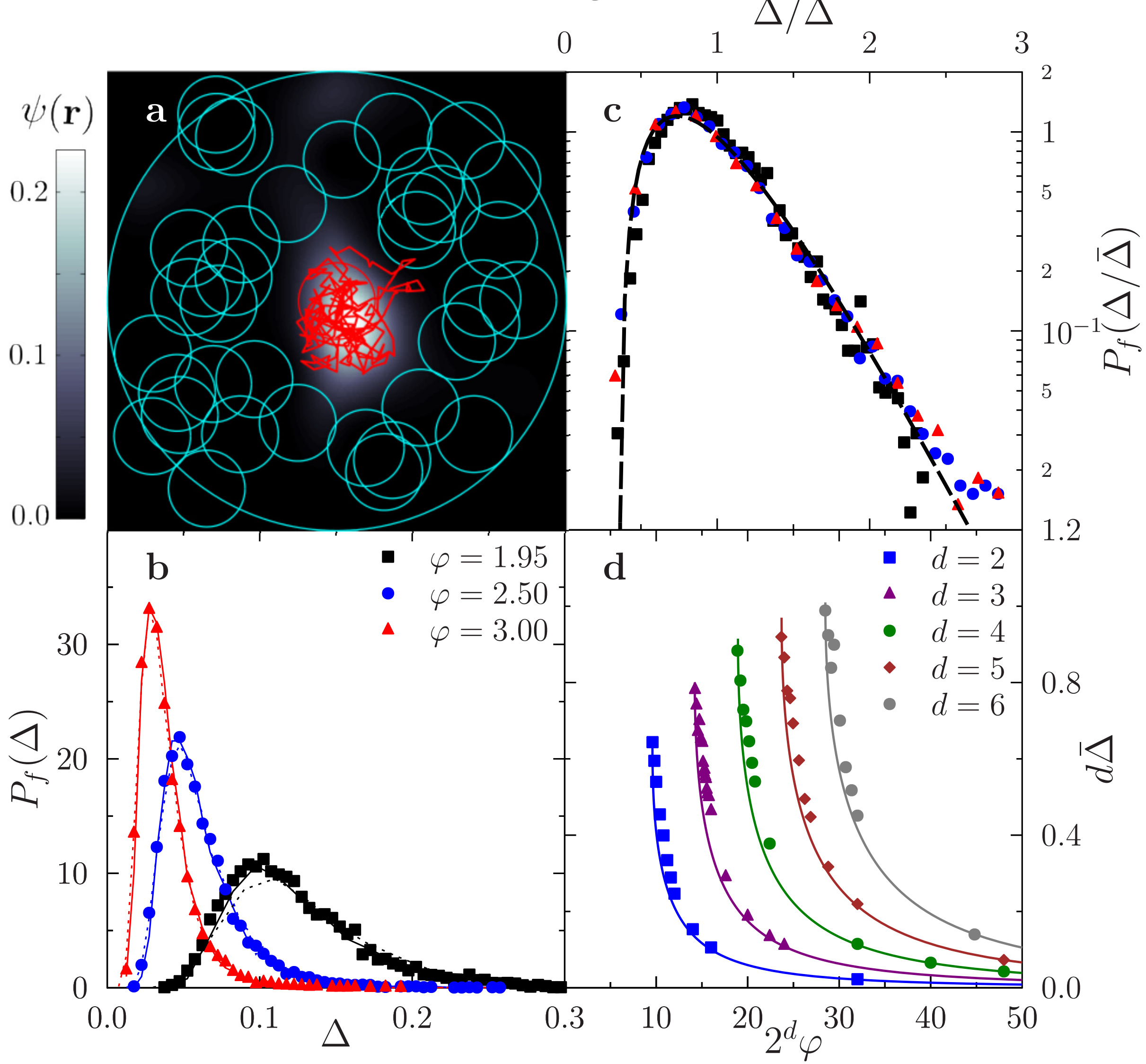
- Same high *d* partition function as HS.
- Mean-field critical behavior.
- Nucleation suppressed.
- Critical fluctuations don't couple with displacements.
- No spatial notion -> limited facilitation.
- Ideal off-lattice finite *d* MF model!

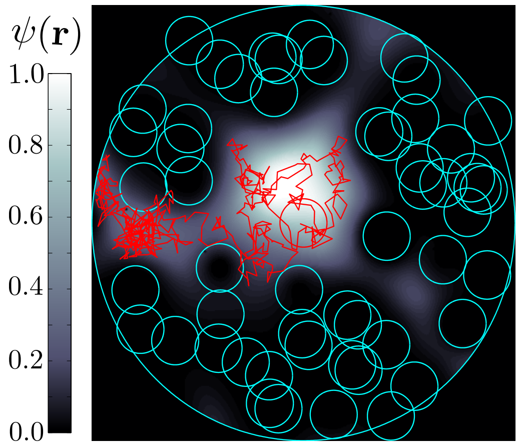


MK, PRL (2009); JCP (2011)

Mézard, Parisi, Tarzia, Zamponi, J. Stat. Phys. (2011)

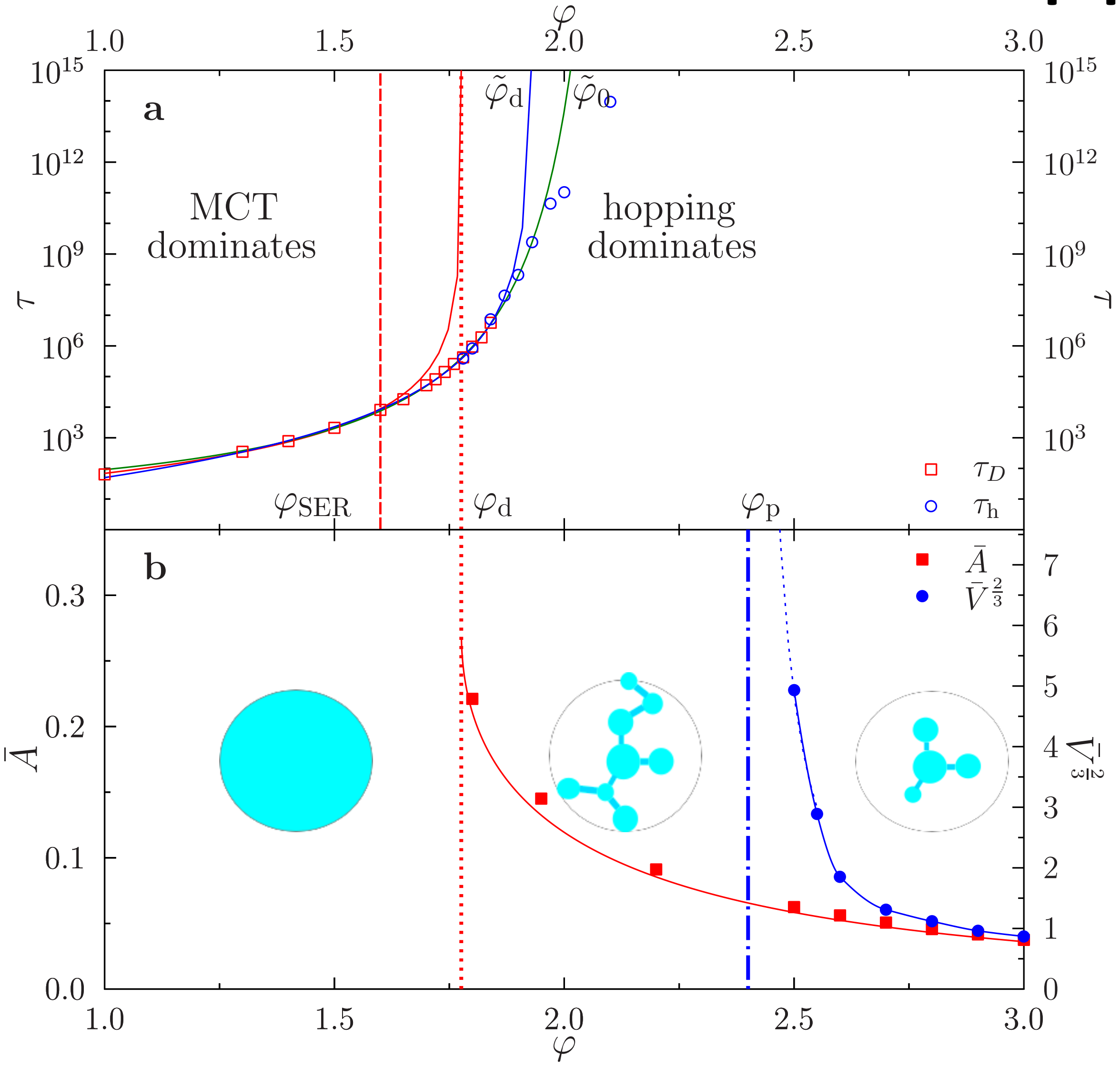
Cavity reconstruction





CJPZ, PNAS (in press), arXiv:1407.5677

Finite *d* MF -> RFOT + hopping



Conclusions

- Some qualitative *and quantitative* features of the mean-field fullRSB solution persist all the way down to *d*=2. How ubiquitous is fullRSB?
- Refinement of critical exponents and of deviations is ongoing (robust?).
- Gardner transition may be experimentally testable (ask Yuliang or Bea).
- Much to do about the dynamical transition and connection with void percolation (Jin and PC, arXiv:1409.0688)