Fluctuations in glassy systems and the Random FIeld Ising Model

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Overview

Overview of the presentation

1. Fluctuations in Glassy Systems
2. Glasses under constraints
3. Dynamical line and critical points
4. Replica Field Theory
5. From RFT to RFIM.

Work in collaboration with:
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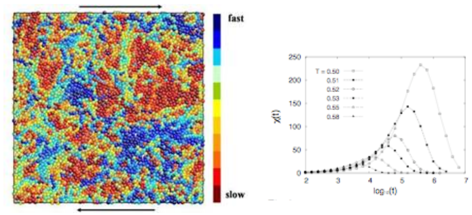
Critical Fluctuations in Structural Glassy systems

Dynamical heterogeneities, growth of correlations - compared to critical fluctuations.
Persistence of high and low mobility regions over long (τ_α) times
typical size grows moderately



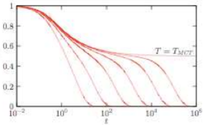
MCT & Replicas: describe the growth as a dynamical and geometrical critical phenomenon of ergodicity breaking. Formation of long-living metastable states.

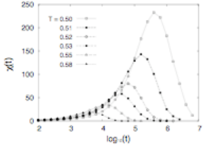
Dynamical heterogeneities II

Corrections to MF:

- Critical Fluctuations below D_{UCD}
- Activation Neglect !

Equilibrium method to study off-equilibrium and/or dynamical fluctuations





time scale separation:

If $C(t) \approx q_{EA}$ the system is sampling according to restricted Gibbs.

$$\mu(S|S_0) = \frac{1}{Z[S_0]} e^{-\beta H(S)} \prod_x \delta\left(Q_x(S, S_0) - q_{EA} \pm \eta\right)$$

Glassy critical points

Universal effective theory around the dynamical transition.
Problem Critical properties of an avoided transition.
Is there a regime where the scaling regime is observable ?

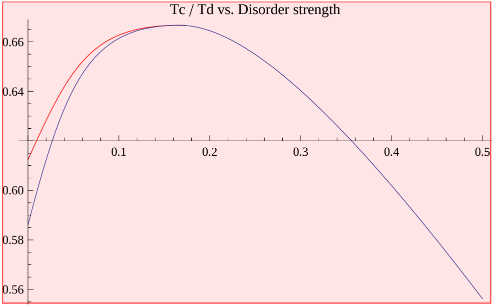
Higher order transition singularities
Points of **dynamic** and **thermodynamic** singularity.
Critical growth of correlations and No-Activation

- Disorder: liquids in porous media
- glass-glass transitions / Gardner-Gross Kanter Sompolinsky transition

- Glasses under constraints
  - Coupling between "clones" : annealed vs. quenched
  - Pinning of particles
  - Coupling induced transitions and critical points

Liquids in disordered (porous) media

HNC & MCT.



Depending on the disorder strength either discontinuous or continuous transition.

Glasses under constraints

Epsilon coupling

Annealed coupling

$$\mu(S_1, S_2) = \frac{1}{Z_2} e^{-\beta[H(S_1)+H(S_2)]-N\epsilon Q(S_1,S_2)}$$

$n = 2$ replicas: Universality of Ising

Quenched Coupling: Choose S_0 as an equilibrium configuration

$$\mu(S|S_0) = \frac{1}{Z[S_0]} e^{-\beta H(S)-N\epsilon Q(S,S_0)}$$

S_0 Self-generated disorder - RFIM universality

Glasses under constraints II

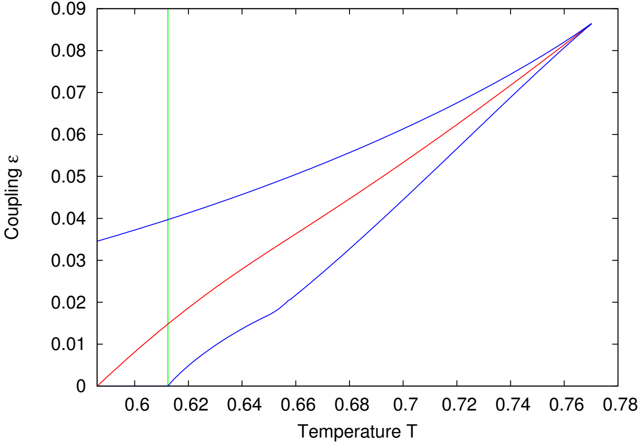
Particle pinning

$$\mu(S|S_0) = \frac{1}{Z[S_0]} e^{-\beta H(S)} \delta(Q_A(S, S_0) - 1)$$

$Q_x(S, S_0) = 1$ for $x \in A$
S_0 Self-generated disorder - RFIM universality

Phase diagram

Mean Field predictions (+ numerics)



- Epsilon coupling : 1st order transition line
- Particle pinning : Glass transition (1RSB) line.

Field theoretical formulation

Study of local overlap fluctuations two equivalent formulations:

$$e^{-W[q(x)|S_0]} = \frac{1}{Z}\sum_S e^{-\beta H(S)}\delta\left(Q_x(S,S_0) - q(x)\right)$$

$$e^{-\Gamma[\epsilon(x)|S_0]} = \frac{1}{Z}\sum_S e^{-\beta H(S)+\int dx\ \epsilon(x)(Q_x(S,S_0)-q(x))}$$

$n \to 1$ replica field theory.

Depending on the problem different replica symmetry
Epsilon coupling $\epsilon(x) = const.$; S_0 privileged $\to S_{n-1}$
Particle pinning: $Q_x(S_a, S_0) = 1$ for $x \in A \to S_n$.
Dynamical heterogeneities and glassy critical points $\epsilon(x) = 0 \to S_n$.

Sources of fluctuations

Decompose χ_4 or G_4:

- Isoconfigurational fluctuations: different thermal histories for fixed S_0.

$$G_{th}(x-y) = [[\langle Q_x(S,S_0) Q_y(S,S_0)\rangle - \langle Q_x(S,S_0)\rangle\langle Q_y(S,S_0)\rangle]]$$

- Initial condition fluctuations: effect of heterogeneity

$$G_{het}(x-y) = [[\langle Q_x(S,S_0)\rangle\langle Q_y(S,S_0)\rangle]] - [[\langle Q_x(S,S_0)\rangle]]^2$$

- Disorder (if present).

Sources of fluctuations

$$G_4(x-y) = [[\langle Q_x(S,S_0)Q_y(S,S_0)\rangle - [[\langle Q_x(S,S_0)\rangle]]^2$$
$$= G_{th}(x-y) + G_{het}(x-y)$$
$$\chi_* = \int dx\ G_*(x)$$

For all critical points we have analyzed S_0 acts as a random field on the overlap fluctuations : "Self-generated disorder".

the whole disribution of $W(q(x)|S_0)$ is important Moments of W (1st and 2nd) associated to overlap correlation functions

- Fluctuations around the Plateau value of correlation in the Beta regime close to the MCT (putative) transition are in the same universality class of the spinodal transition of the Random Field Ising Model, when expressed in a reparametrization invariant way.

$$\Gamma[\phi] = \int dx \, \frac{1}{2}\phi(x)[m_1 + \nabla^2]\phi(x) + h(x)\phi(x) + g_3\phi(x)^3$$
$$m_1 \propto \sqrt{(}T_c - T)$$

- At points of continuous transition $g_3 \to 0$ the effective theory becomes

$$\Gamma[\phi] = \int dx \, \frac{1}{2}\phi(x)[m_1 + \nabla^2]\phi(x) + h(x)\phi(x) + g_4\phi(x)^4$$

Effective action formalism $W[\rho_{ab}(x,y)]$

Effective action and its fluctuations: the replica formalism

$$\rho_{ab}(x,y) = \frac{1}{V^2} \sum_{i,j}^{1,N} \langle \delta(x_i^a - x)\delta(x_j^b - y) \rangle$$

$$q_{ab}(x) = \int dy \; w(x-y)\rho_{ab}(x,y)$$

glass transition: non trivial value of off-diagonal ($a \neq b$) values of $\rho_{ab}(x,y)$ ($a \neq b$) for $n \to 1$.

Effective action formalism $W[\rho_{ab}(x,y)]$

Mean-Field level

Suppose of having an approximate scheme to compute $W[\rho_{ab}(x,y)]$ that gives a critical glass transition. (e.g. HNC)

Field theory of fluctuations

- Quadratic fluctuations
  Study $M_{ab;cd}(x,y;z,w) = \frac{\delta^2 W}{\delta\rho_{ab}(x,y)\delta\rho_{cd}(z,w)}$ and identify the zero modes
- Interaction
  Project fluctuations on the zero modes
  $\delta\rho_{ab}(x,y) = \phi_{ab}(\frac{x+y}{2})k_0(x-y)$.
- Study the interaction between the soft modes
- Gradient expansion
  Keep terms of lowest scaling dimension
- Ginzburg criterion
- RG computations

S_n Symmetric Effective action

MCt transition + continuous glass transitions.

$$
\begin{aligned}
S = \quad & \int dx\, \frac{1}{2} \sum_{a,b} \phi_{ab}(x)[m_1 - \nabla^2]\phi_{ab}(x) + \\
& \frac{1}{2} m_2 \sum_a \left(\sum_b \phi_{ab} \right)^2 + \frac{1}{2} m_3 \left(\sum_{a,b} \phi_{ab} \right)^2 + \\
& \frac{1}{6}[w_1 \mathrm{tr}\, \phi^3 + w_2 \sum_{a,b} \phi_{ab}^3] \\
& + \ \textit{subdominant cubic terms} \\
& + [u_1 \mathrm{tr}\, \phi^4 + u_2 \sum_{a,b} \phi_{ab}^4 + u_3 \sum_{a,b,c} \phi_{ab}^2 \phi_{ac} \phi_{bc}] \\
& + \ \textit{other quartic terms}
\end{aligned}
$$

$m_1 \propto \sqrt{(T_c - T)} \to 0$

$m_2, \ m_3$ associated to fluctuations.

Bare parameters can be computed

Analysis of Mass matrix

Matrix of quadratic fluctuations:

$$M_{ab;cd} = m_1 \frac{\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}}{2} + m_2 \frac{\delta_{ac} + \delta_{bd} + \delta_{ad} + \delta_{bc}}{2} + m_3$$

de Almeida - thouless

$$\lambda_L = m_1 + (n-1)\eta$$
$$\lambda_A = \text{finite}$$
$$\lambda_R = m_1$$

Degeneracy of eigenvalues typical of Random Field systems:

$$G_{th}(k) \sim \frac{1}{m_1 + k^2}$$
$$G_{het}(k) \sim \frac{1}{(m_1 + k^2)^2}$$

within gaussian approximation.

Dynamical transition

Zero modes: Longitudinal + Replicons.

ϕ_{ab} mixes different representations of S_n : different scaling dimensions.

Cardy change of basis (1-to-1)

$$\phi_{ab} = \phi + \frac{\omega}{2} + \omega U_{ab} + \chi_{ab}$$
$$U_{ab} = \delta_{b,a-(-1)^a} \quad \sum_{ab} U_{ab}\chi_{ab} = 0 \quad \sum_b \chi_{ab} = 0$$

ϕ, ω χ_{ab} well defined dimension

$$S = \int dx\, \omega(x) \left(-\Delta\phi(x) + m_1\phi(x) + 3g\, \phi(x)^2 - (m_2 + m_3)\omega(x)\right) + \frac{1}{2}\sum_{ab} \chi_{ab}(x)^2 \left(-\Delta + m_1 + 6g\, \phi(x)\right)$$

Most singular terms : Parisi-Sourlas action of ϕ^3-RFIM

Effective description

$\phi(x) = \phi_{0a}(x)$ local overlap fluctuation, leading order

$$\Gamma[\phi] = \int dx \; \frac{1}{2}\phi(x)[m_1 + \nabla^2]\phi(x) + h(x)\phi(x) + g_3\phi(x)^3$$

$$m_1 \propto \sqrt(T_c - T) \;\; g_3 = w_1 - w_2$$

$$\overline{h(x)h(y)} = |m_2 + m_3|\delta(x-y)$$

$h(x)$ parameterizes the effect of the reference configuration S_0.

$D_{UC} = 8$; Scaling laws; finite size effects in MF models.

Unstable field theory (as it should)
Is there a regime where ϕ^3-RFIM fluctuations can be observed ?
high D ?

Numerics

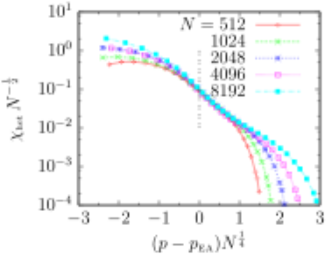
Fredricksen-Andersen model on random regular graph.

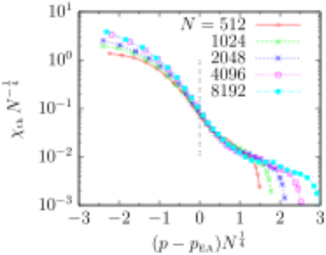
trivial thermodynamics

Numerical evidence for an ideal MCt dynamical transition

Persistence : number of spins that have not flipped up to time t

Finite size scaling of χ_{th} and χ_{het}





Glassy critical points

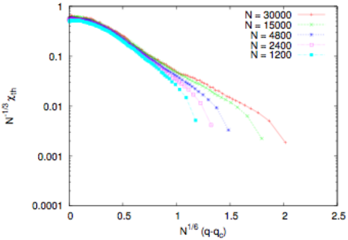
At critical points $w_1 = w_2$; $g_3 = 0$
Control of subdominant cubic terms and quartic terms in the RFT.
Cubic terms superficially as singular as quartic ones.
Cubic terms vanish at the one loop level.

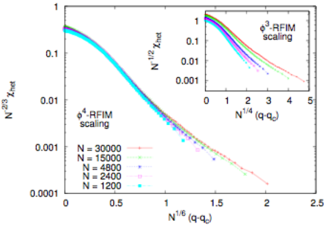
$$\Gamma[\phi] = \int dx \, \frac{1}{2}\phi(x)[m_1 + \nabla^2]\phi(x) + h(x)\phi(x) + g_4\phi(x)^4$$
$$m_1 \propto T - T_c$$
$$\overline{h(x)h(y)} = |m_2 + m_3|\delta(x-y)$$
$$g_4 = u_1 + u_2 - u_3$$

N.B. dynamic and equilibrium transitions coincide
Critical properties should be observable and coincide with the ones of ϕ^4-RFIM.

Numerics

Particle pinning : 3-XOR-SAt on Random Regular Graph $z = 8$
Finite N correction to MF and fluctuations.
Critical point can be identified.





Epsilon coupling

Annealed case: $n = 2$: no S_0, scalar order parameter ϕ Universality class of Ising

Quenched case: a much simpler analysis: Longitudinal and Anomalous eigenvalues degenerate at the transition.
Critical field $\psi_a = \phi_{0a} + \frac{\gamma}{n-2} \sum_{\substack{b=1 \\ b\neq a}}^{n-1} \phi_{ab}$.
Analysis of S_{n-1} invarians gives ϕ^4-RFIM to all orders in perturbation theory.

Prediction:

Fluctuations of different overlaps should be strongly correlated. Identities relating correlation functions that can be tested in numerical simulations. e.g.

$$g_{01;23}(x) = \langle q_{01}(x) q_{23}(0) \rangle = \gamma g_{01;02}(x) = \gamma \langle q_{01}(x) q_{02}(0) \rangle$$

Sketch of derivation

Longitudinal eigenvalue $\lambda_L \to 0$: Correlated fluctuations of $\delta p = \frac{1}{n-1}\sum_a \phi_{0,a}$ and $\delta q = \frac{1}{(n-1)(n-2)}\sum_{a,b}\phi_{ab}$.

$$L \propto \delta p + \gamma\, \delta q$$

Anomalous eigenvalue $\lambda_A = \lambda_L + (n-1)\eta$

$$A^a \propto (\phi_{0,a} - \delta p) + \gamma\, \frac{1}{n-2}\sum_b (\phi_{ab} - \delta q)$$

Critical field

$$\psi_a(x) = \phi_{0,a}(x) + \gamma\, \frac{1}{n-2}\sum_b \phi_{ab}(x)$$

Dimensional analysis of one index replica invariants. RFIM at critical points all orders in perturbation theory.

Summary

- Critical glassy fluctuations are subject to an effective random field Self-generated disorder.
- Discontinuous Dynamical transitions ϕ^3-RFIM. Is there a regime where critical behavior is observable ?
- Continuous transitions + quenched ϵ-coupling critical points ϕ^4-RFIM
- Continuous criticality is observable
- the field parameterizes the effect of initial condition/reference configuration