Twisting and breaking glasses: a replica approach

Hajime Yoshino
Cybermedia Center, Osaka Univ.

HY and F. Zamponi, “The shear modulus of glasses: results from the full replica symmetry breaking solution”, Phys. Rev. E 90, 022302 (2014).

See also the Poster by Corrado Raione

Collaborators

Marc Mézard (ENS Paris)

Replica trick to compute shear-modulus
ex. 3D soft-core system $v(r) \sim r^{-12}$

Francesco Zamponi (ENS, Paris)

Shear-modulus of Hard-sphere glass
in large-D limit

Satoshi Okamura (Osaka Univ.)

Finite temperature MD simulation of
aging in a jamming system

Corrado Raione (ENS, Paris & Univ. Rome)

Pierefrancesco Urbani(CEA, Saclay)

State following approach via
Franz-Parisi potential

Financial Supports

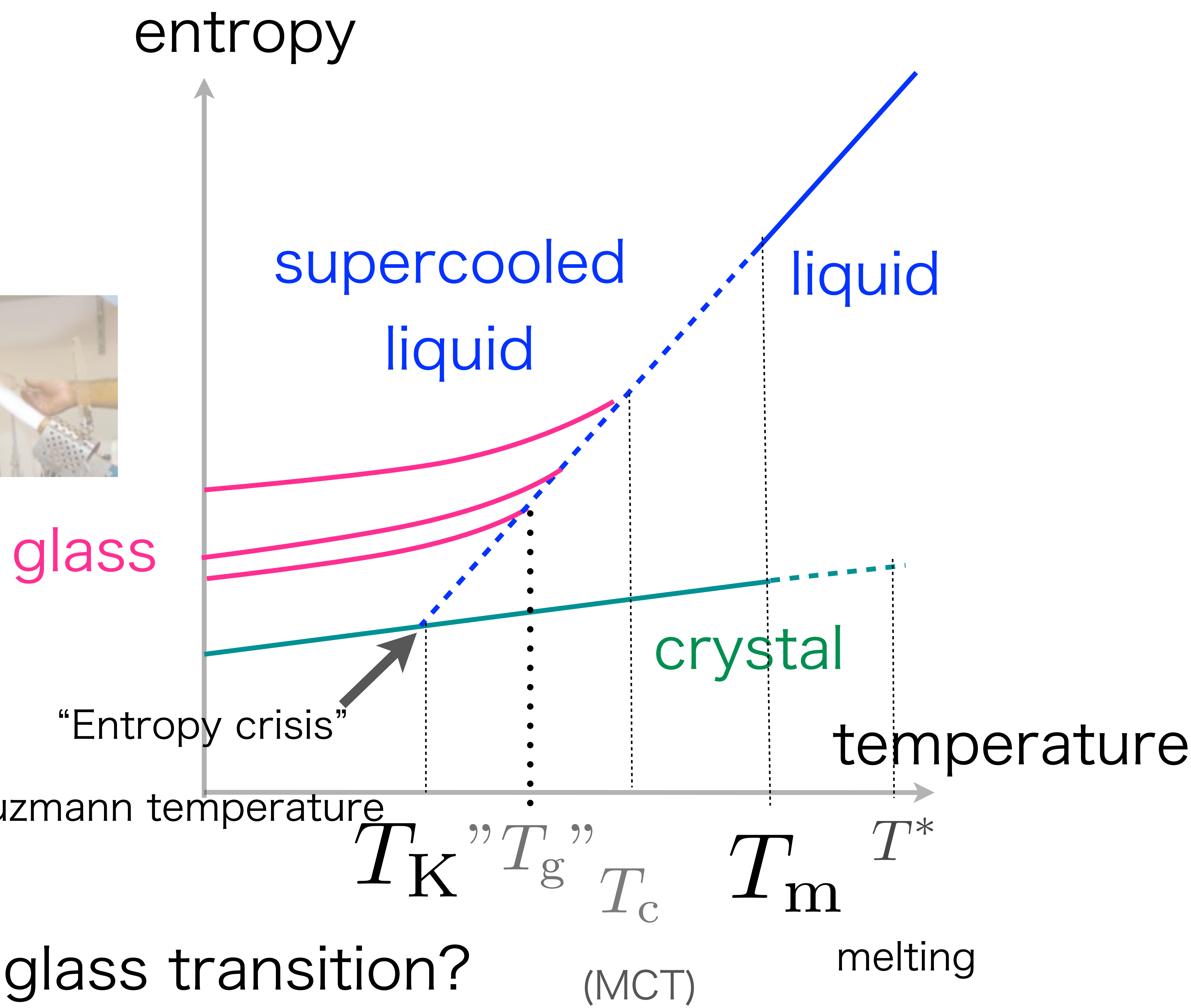
Synergy of Fluctuation and Structure :
Quest for Universal Laws in Non-Equilibrium Systems
2013-2017 Grant-in-Aid for Scientific Research on Innovative Areas, MEXT, Japan

JPS Core-to-Core program 2013-2015 Non-equilibrium dynamics of soft matter and information

Outline

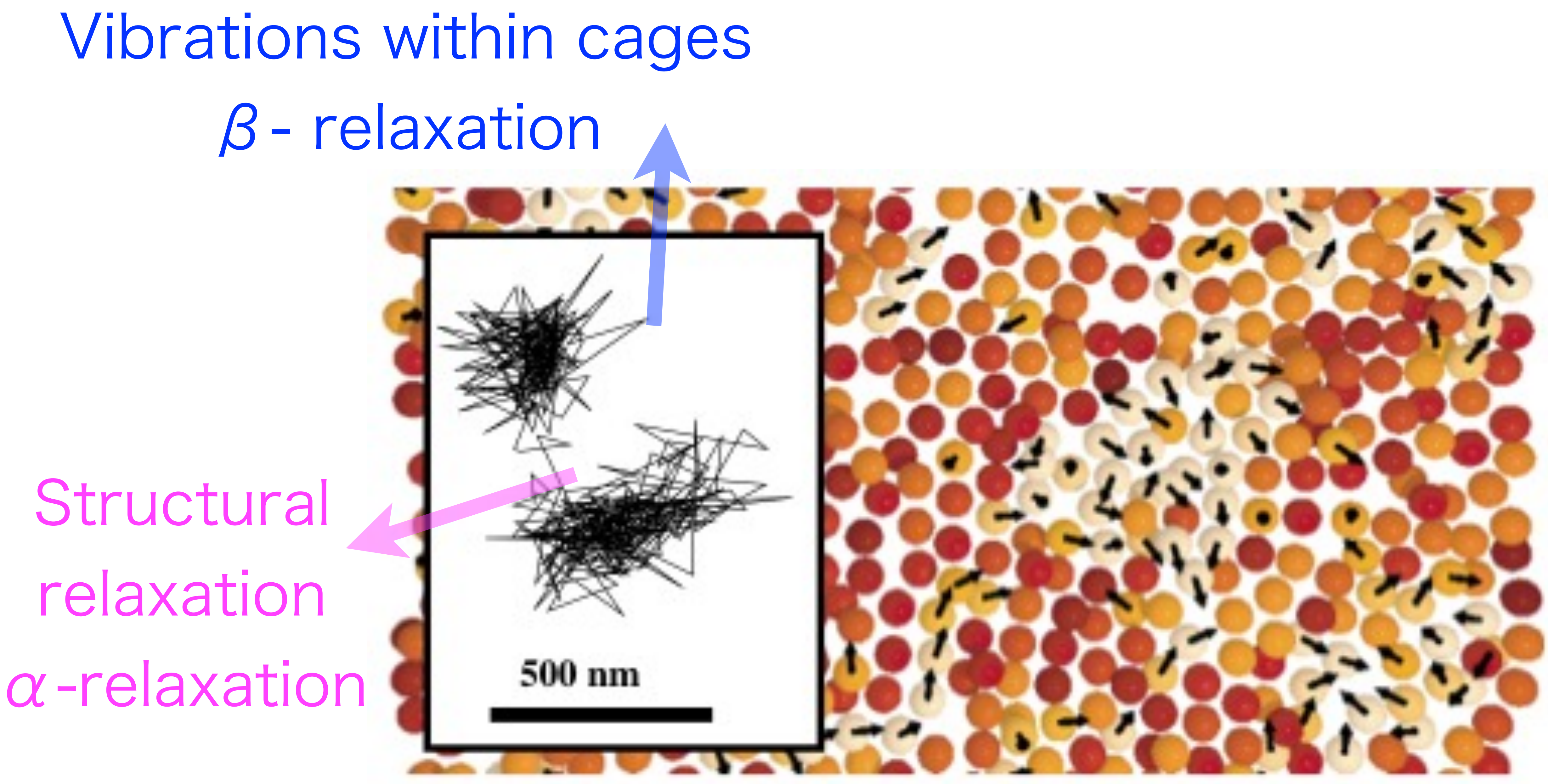
- Introduction:
- Shear on the cloned liquid in the large-d limit (theory)
- Aging around the jamming point (simulation)
- Discussions

Glass transition



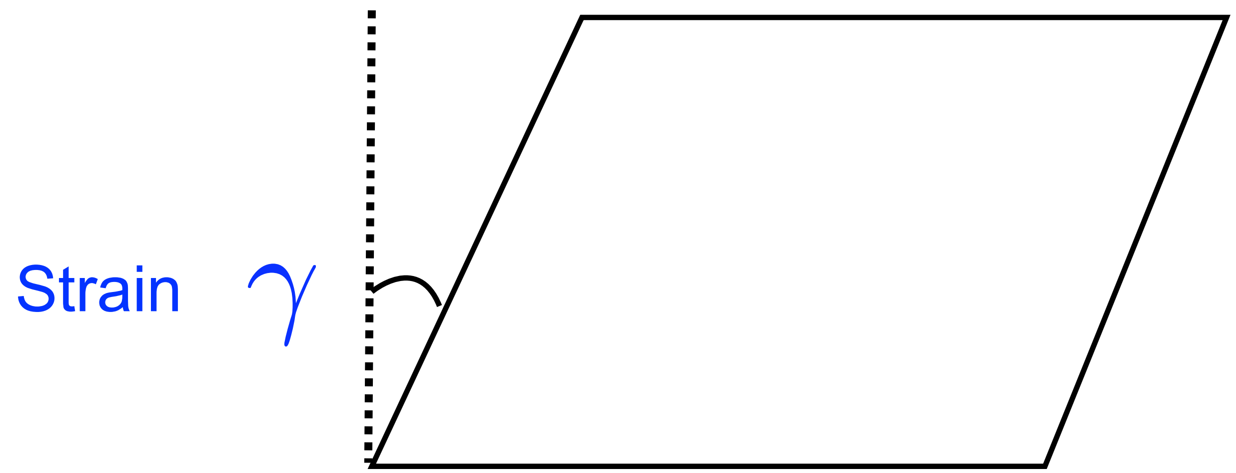
Ideal glass transition?

■ Supercooled liquids



Confocal scope image (E. Weeks and D. Weitz (2002))

Shear modulus: a paradox and a lesson



$$f = F/N$$

$$f(\gamma) = f(0) + \gamma\sigma + \frac{\gamma^2}{2}\mu + \ldots$$

stress

shear modulus or “rigidity”

「水は方円の器にしたがう」水髄方円　筍子

Water conforms to the shape of its container.

liquid	$\mu = 0$
solid	$\mu > 0$

linear response, fluctuation

$\mu = 0$

thermodynamics

Shape of the container should not matter!

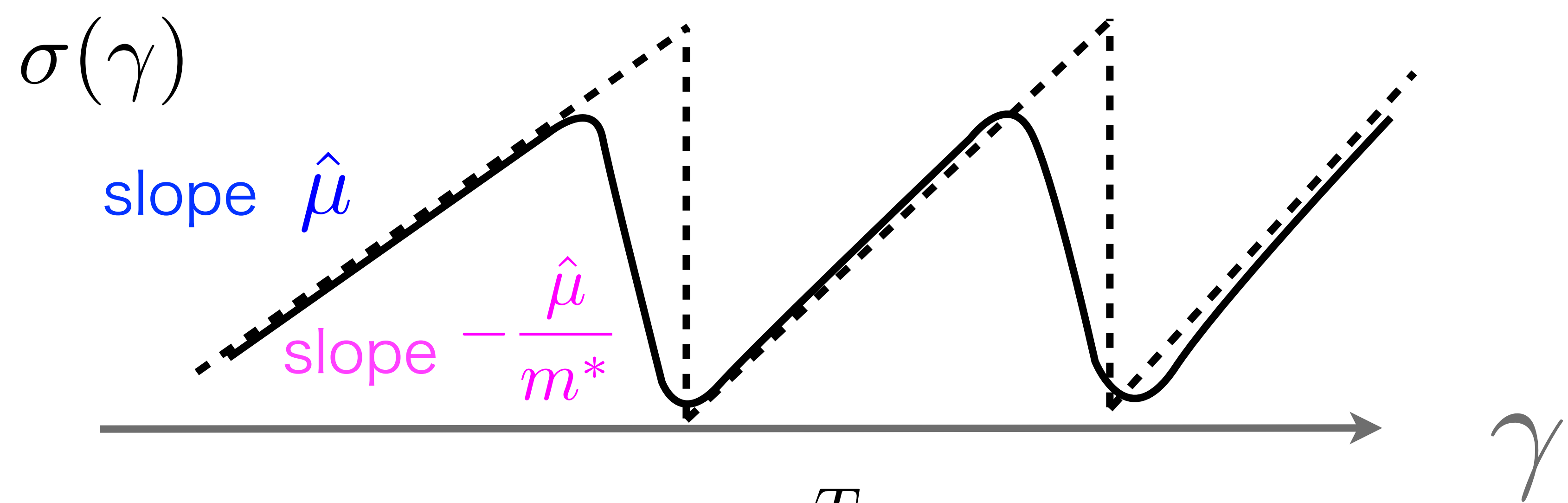
$$\lim_{N\to\infty}\lim_{\gamma\to 0} \neq \lim_{\gamma\to 0}\lim_{N\to\infty}$$

elasticity *must* emerge together with plasticity

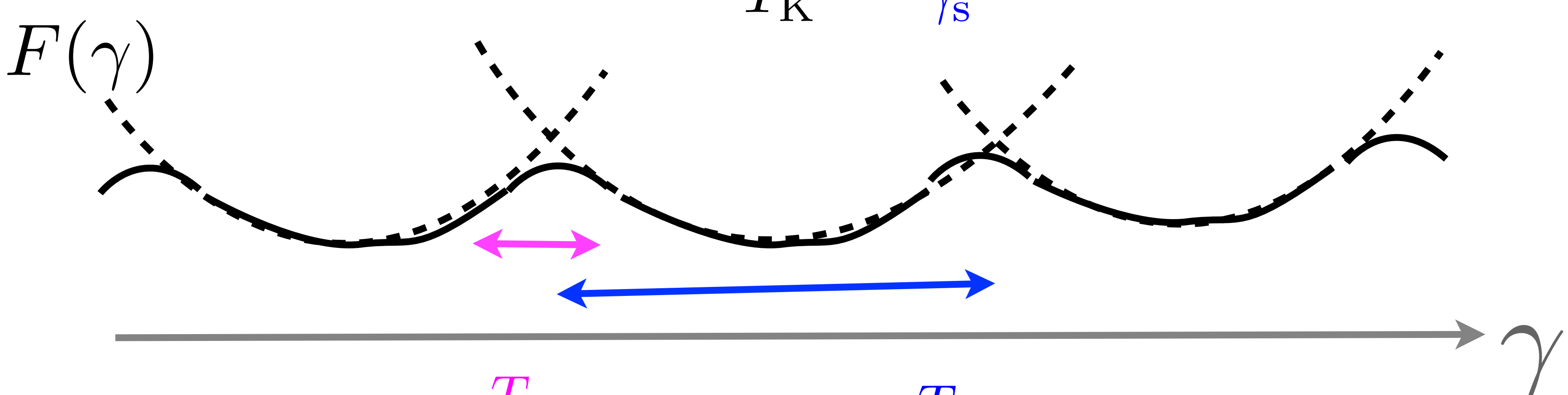
Intra-state and inter-state responses under shear (1RSB)

H. Yoshino and M. Mézard, Phys. Rev. Lett. 105, 015504 (2010).

$$\mu_{ab} = \hat{\mu}\delta_{ab} - \frac{\hat{\mu}}{m^*} \qquad \overline{\mu} = \sum_{b=1}^{m^*} \mu_{ab} = 0$$



$$m^*(T) \simeq \frac{T}{T_{\mathrm{K}}} \simeq \frac{\gamma_{\mathrm{w}}}{\gamma_{\mathrm{s}}}$$



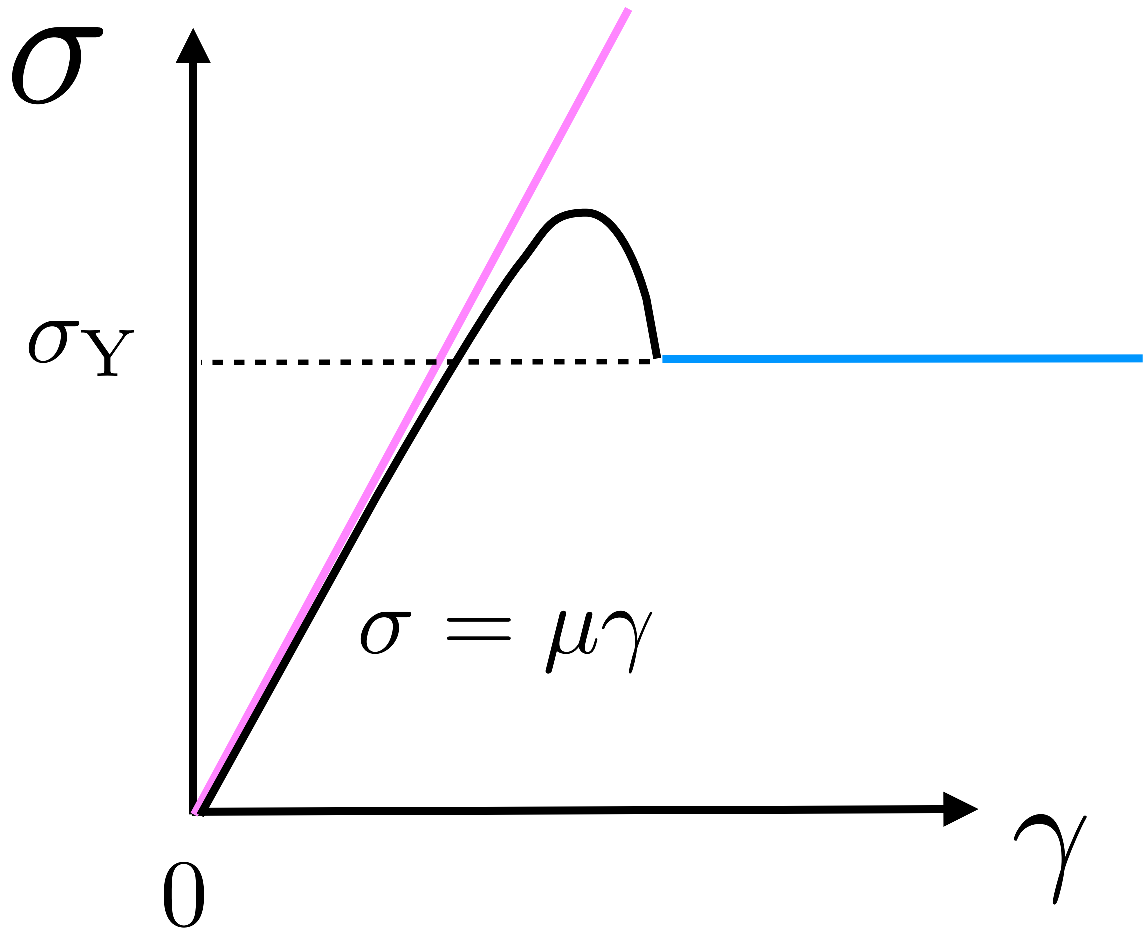
$$\gamma_{\mathrm{w}} = \frac{T}{\Delta\sqrt{N}} \qquad \gamma_{\mathrm{s}} = \frac{T_{\mathrm{K}}}{\Delta\sqrt{N}}$$

C.F. step-wise magnetic response in spin-glasses : H.Y. and T. Rizzo, Phys. Rev. B 77, 104429 (2008).

stress-strain curve

shear-stress (force/area)

yield stress



shear-strain

"state following" :
computation of the Franz-Parisi potential under shear

See the Poster by Corrado Raione

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- Aging around the jamming point (simulation)
- Discussions

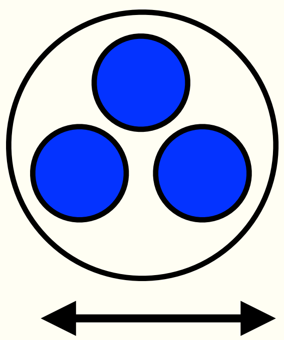
J. Kurchan, G. Parisi and F. Zamponi,.J. Stat. Mech. P10012 (2012).
J. Kurchan, G. Parisi, P. Urbani and F. Zamponi, J. Chem. Phys. B117, 1279 (2013).
P. Charbonneau, J. Kurchan, G. Parisi, P. Urbani, F. Zamponi, arXiv:1310.2549 .
P. Charbonneau, J. Kurchan, G. Parisi, P. Urbani, F. Zamponi, Nature Communications 5, 3725 (2014).

H. Yoshino and F. Zamponi, Phys. Rev. E 90, 022302 (2014).

Basic idea of cloned liquid

m-replicas obeying the same Hamiltonian

$$\mathbf{x}_i^a \quad i = 1, 2, \ldots, N \quad a = 1, 2, \ldots, m$$



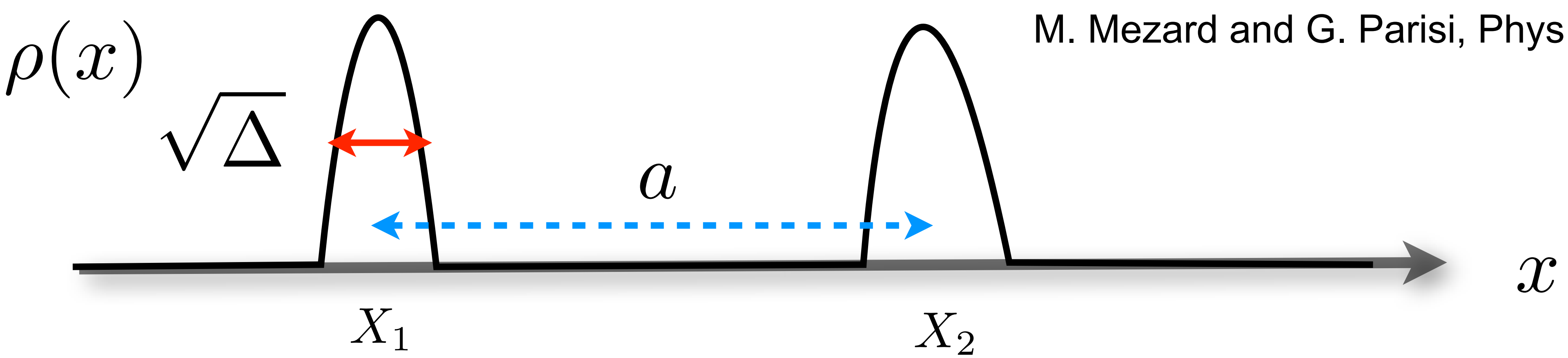
"Cage size"

$$\Delta = \langle (x_i^a - x_i^b)^2 \rangle \qquad H = H_0 - \frac{\epsilon}{4} \sum_i \sum_{a,b} \langle (\mathbf{x}_i^a - \mathbf{x}_i^b)^2 \rangle$$

liquid $\Delta = \infty$ solid $\Delta < \infty$

"Einstein model"

S. Franz and G. Parisi, J. Phys. I France 5 1401 (1995).
R. Monasson, Phys. Rev. Lett. 75 2847 (1995).
M. Mezard and G. Parisi, Phys. Rev. Lett. 82 747 (1999).

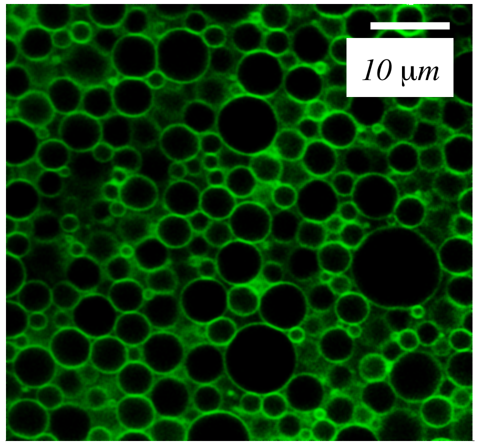


Edwards-Andeson Order Parameter
(non-ergodicity order parameter)

$$q_{\mathrm{EA}} = \lim_{t \to \infty} \frac{1}{N} \sum_{i=1}^{N} \langle s_i(t) s_i(0) \rangle = \frac{1}{N} \sum_{i=1}^{N} \langle s_i \rangle^2$$

Repulsive contact systems

Repulsive colloids, emulsions, granular matter,..

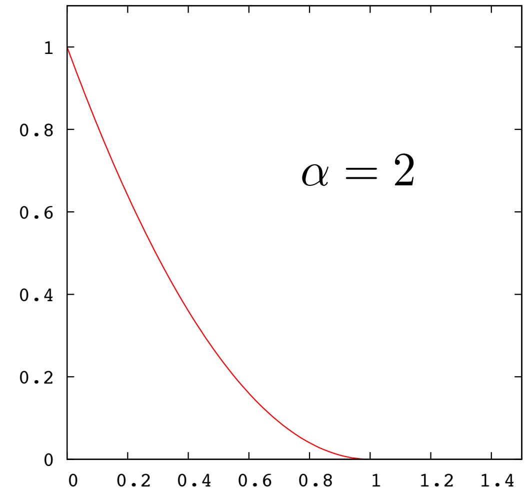


E. R. Weeks,
in "Statistical Physics of Complex Fluids",
Eds. S Maruyama & M Tokuyama
(Tohoku University Press, Sendai, Japan, 2007).

Model potential energy

$$U = \sum_{\langle ij \rangle} v(r_{ij}) \qquad r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$$

$v(r)/\epsilon$



r/D

$$v(r) = \epsilon(1 - r/D)^{\alpha}\theta(1 - r/D)$$

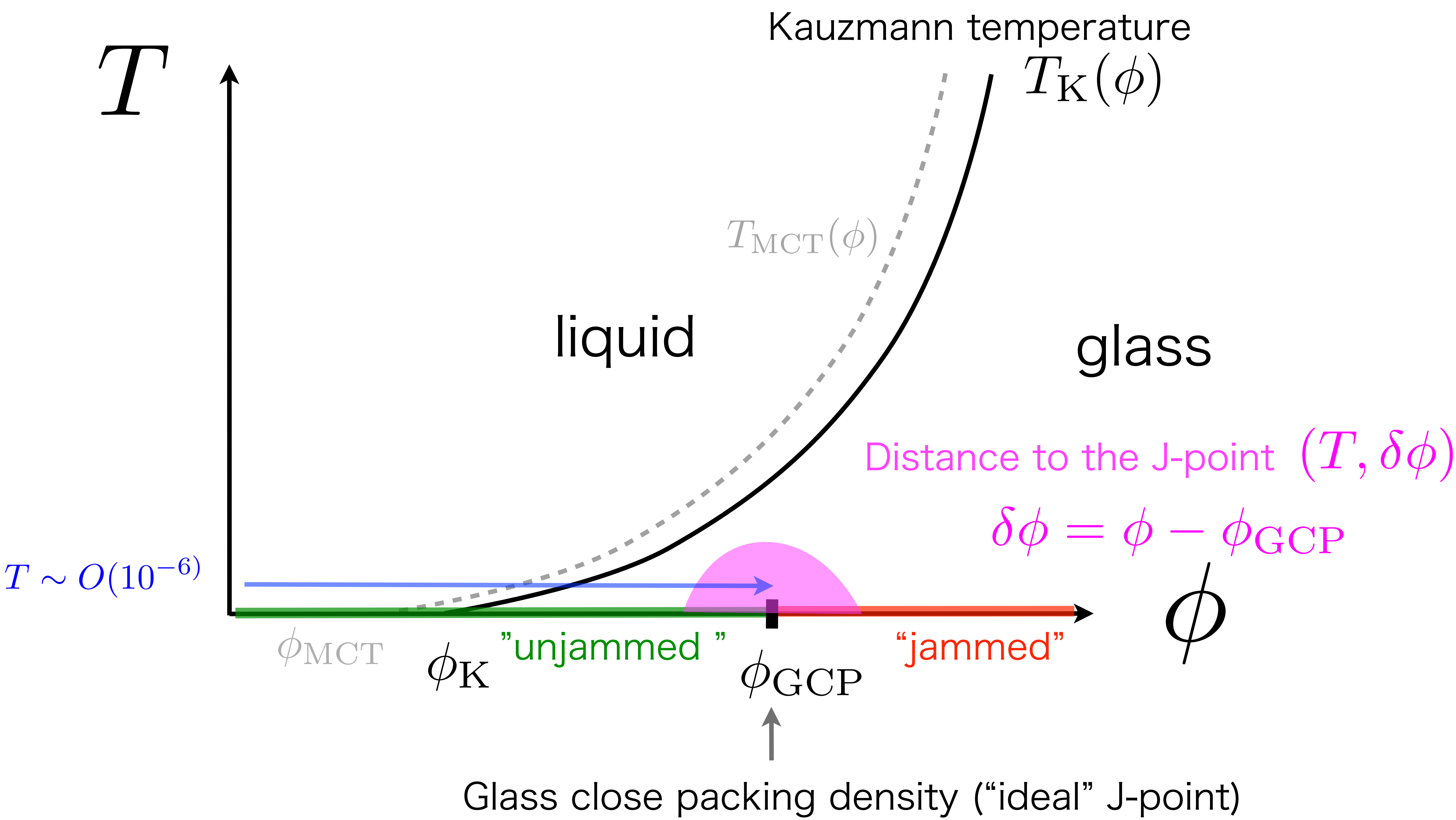
$$\lim_{T \to 0} e^{-v(r)/k_{\mathrm{B}}T} = \theta(r/D - 1)$$

Essentially "hard-spheres" at low temperatures.

Mean-field phase diagram

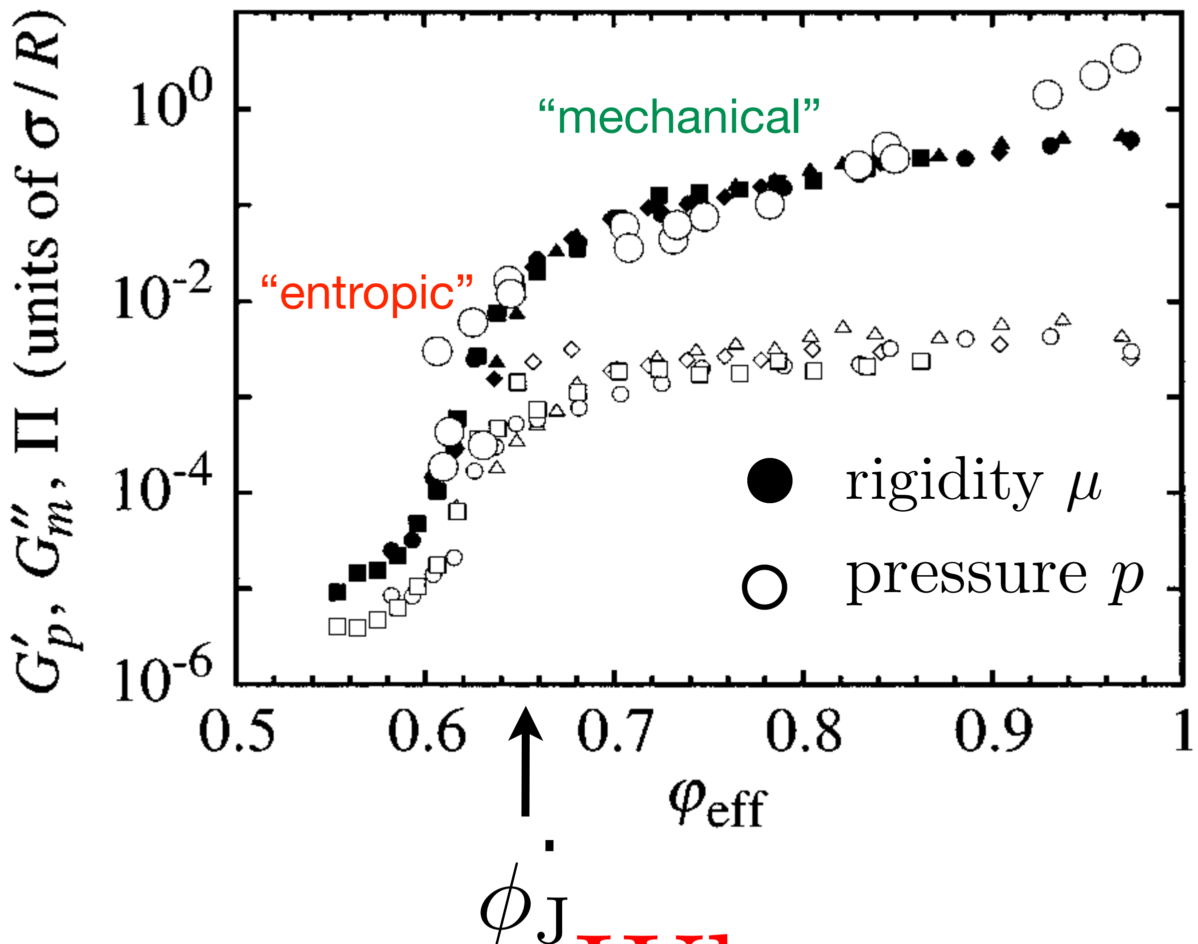
Cloned liquid theory (replica + liquid theory)

G. Parisi and F. Zamponi, Rev. Mod. Phys. 82, 789 (2010)
L. Berthier, H. Jacquin and Z. Zamponi, Phys. Rev. Lett. 106, 135702 (2011) and Phys. Rev. E 84, 051103 (2011).



Experiment: rigidity of emulsions

T. G. Mason, Martin-D Lacasse, Gary Grest, Dov Levine, J Bibette, D Weitz, Physical Review E 56, 3150 (1997)



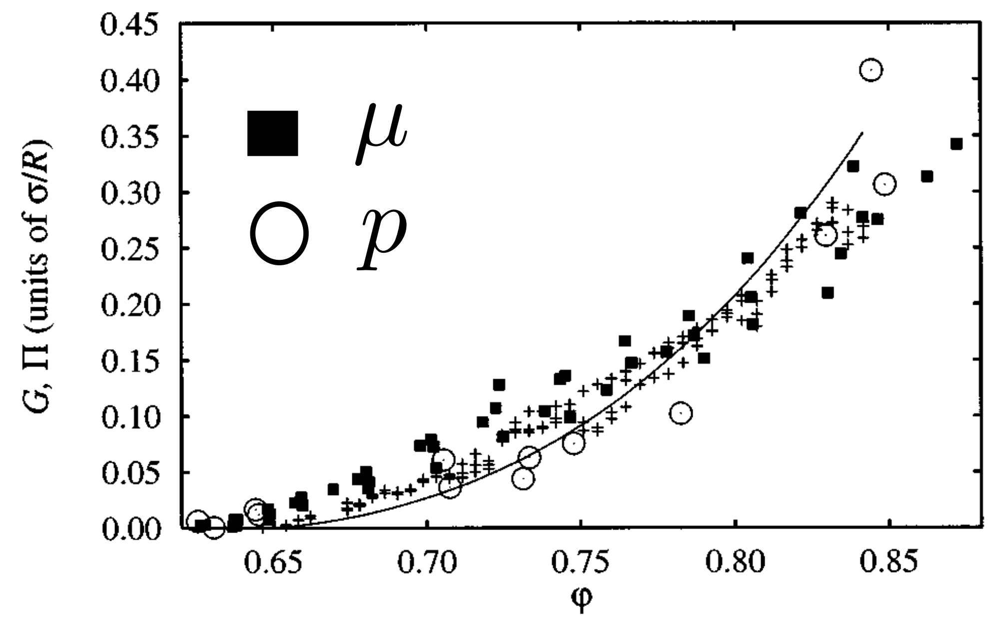


FIG. 1. The scaled shear modulus and osmotic pressure as a function of φ. The computed scaled static shear modulus $G/(\sigma/R)$ (+) and osmotic pressure $\Pi/(\sigma/R)$ (line), as obtained from the model presented in Sec. IV B 2, are compared with the experimental values of $G'_p(\varphi_{\rm eff})$ (■) and $\Pi(\varphi_{\rm eff})$ (○).

Why $\mu \propto \sqrt{\phi - \phi_{\rm J}}$ does not hold?

rigidity (shear-modulus) pressure

$$\mu \sim p$$

measurements at room temperature

$$k_{\rm B}T/\epsilon \sim 10^{-5}$$

Interaction between emulsions: $v(r)/\epsilon = (1 - r/\sigma)^\alpha$

$\alpha > 2?$

M-D. Lacasse, G. S. Grest, D. Levin, T. G. Mason and D. A. Weitz, PRL **76**,3448 (1996)

$\alpha = 2$

I. J. Jorjadze, L-L. Pontani and J. Brujic, PRL **111**, 048302(2013).

1 step RSB approximation

H. Yoshino, AIP Conference Proceedings 1518, 244 (2013)

S. Okamura and H. Yoshino, arXiv:1306.2777 (2013).

$$\lim_{T\to 0} \beta\hat{\mu} = \frac{1}{m^*}\left(\frac{A^*}{m^*}\right)\frac{6\phi}{\pi} y_{\rm liq}^{HS}(\phi_{\rm GCP})^3 \left[c_1 - c_2\sqrt{\frac{A^*}{m^*}} + \ldots\right]$$

ex. Soft-particle case $\alpha = 2$ (3-dim) Berthier-Jacquin-Zamponi (2011)

$\phi_{\rm GCP} = 0.633353..$ $A^*/m^*(\phi_{\rm GCP}) \simeq 9.72187 \times 10^{-5}$ $y_{\rm liq}^{\rm HS}(\phi_{\rm GCP}) \simeq 23.6238$

$\phi < \phi_{\rm GCP}$ $m^* \simeq 20.7487(\phi_{\rm GCP} - \phi)$

Berthier-Jacquin-Zamponi (2011)

$$\lim_{T\to 0} \hat{\mu} \simeq 0.694315T/(\phi_{\rm GCP} - \phi)$$

$$\lim_{T\to 0} p \simeq 1.92178T/(\phi_{\rm GCP} - \phi)$$

$\phi > \phi_{\rm GCP}$ $T/m^* \simeq 0.00835535(\phi - \phi_{\rm GCP})$

Berthier-Jacquin-Zamponi (2011)

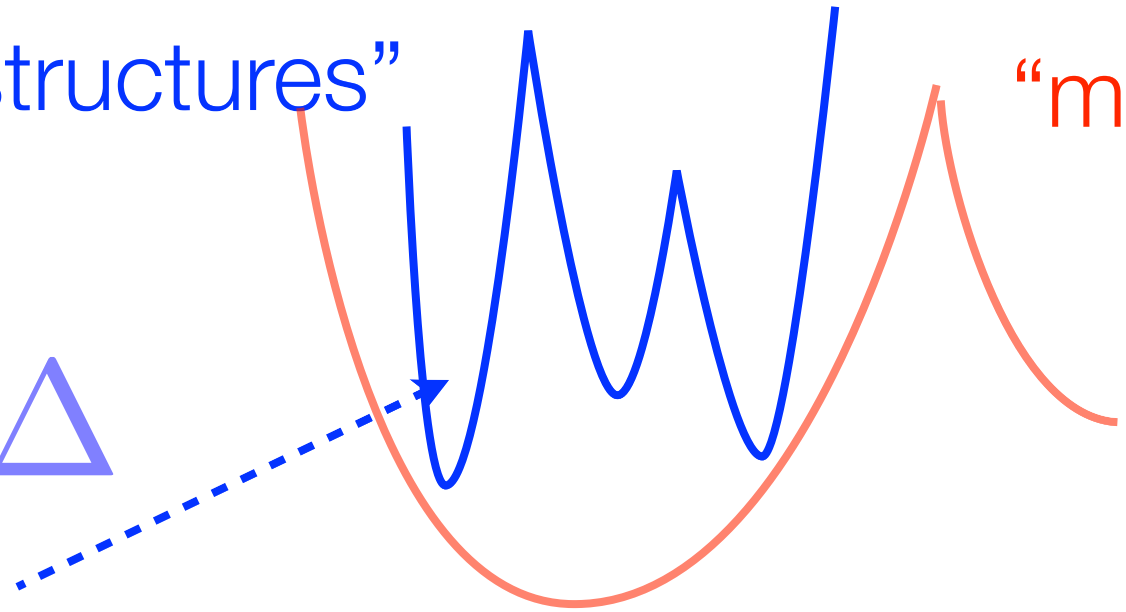
$$\lim_{T\to 0} \hat{\mu} \simeq 0.1239496(\phi - \phi_{\rm GCP})$$

$$\lim_{T\to 0} p \simeq 0.403001(\phi - \phi_{\rm GCP})$$

Harmonic approx. vs 1RSB approx. (2012)

"unjammed" side $\delta\phi < 0$ $\delta\phi = \phi_{\mathrm{J}} - \phi$ $P \propto T/|\delta\phi|$

"inherent structures" "meta-basins"(1RSB)



$\mu \propto T/\Delta$ $\mu \propto T/\Delta$

$\mu_{\mathrm{harmonic}} \propto T/|\delta\phi|^{3/2}$ shear-modulus $\lim_{T\to 0} \mu(T) \propto T/|\delta\phi|$

Brito-Wyart (2006) Yoshino (2012), Okamura-Yoshino(2013)

$\Delta_{\mathrm{harmonic}} \propto |\delta\phi|^{3/2}$ cage size $\lim_{T\to 0} \Delta(T) \propto |\delta\phi|$

Ikeda-Berthier-Birol (2013) Berthier-Jacqin-Zamponi (2011)

Emulsion experiments:
T. G. Mason et al (1997). Guerra-Weitz (2013)

Gardner transition! J. Kurchan, G. Parisi and F. Zamponi,.J. Stat. Mech. P10012 (2012).

Harmonic approx. vs 1RSB approx. (2012)

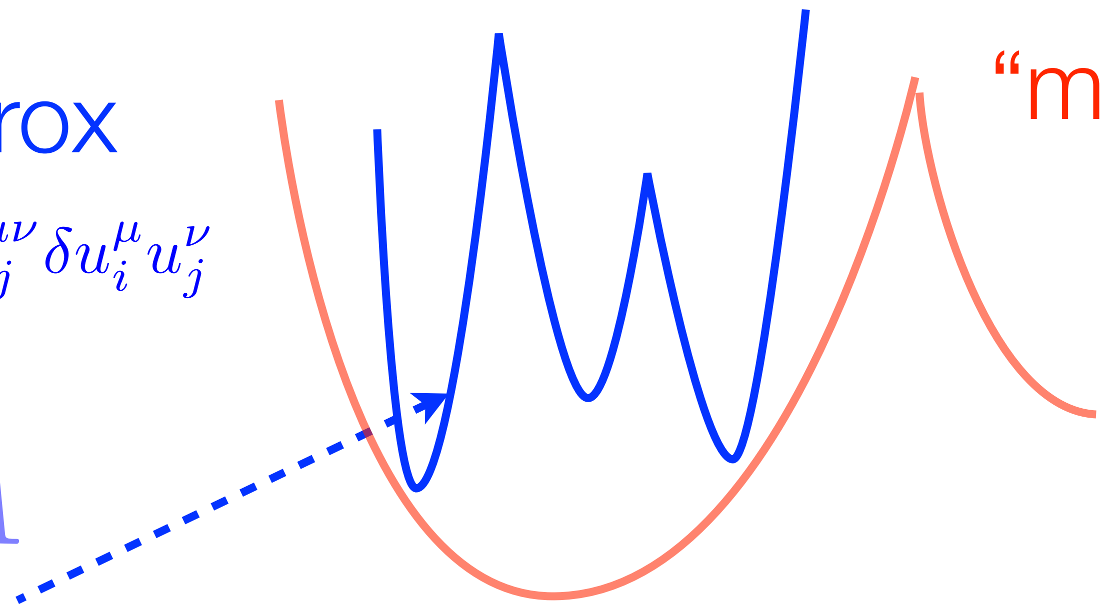
"jammed" side $\delta\phi > 0$ $\delta\phi = \phi_{\rm J} - \phi$ $P \propto |\delta\phi|$

"inherent structures"

"meta-basins" (1RSB)

Harmonic approx

$$H \sim \sum_{i,j} \sum_{\nu\mu} \frac{1}{2} H_{ij}^{\mu\nu} \delta u_i^{\mu} u_j^{\nu}$$



$\mu \propto T/A$ $\mu \propto T/A$

$\mu_{\rm harmonic} \propto \sqrt{\delta\phi}$ shear-modulus $\lim_{T\to 0} \mu({\rm T}) \propto \delta\phi$

Durian (1995), O'Hern et. al. (2003)

Yoshino (2012), Okamura-Yoshino(2013)

$A_{\rm harmonic} \propto T/\sqrt{\delta\phi}$ cage size $\lim_{T\to 0} A(T) \propto T/\delta\phi$

Ikeda-Berthier-Birol (2013)

Berthier-Jacqin-Zamponi (2011)

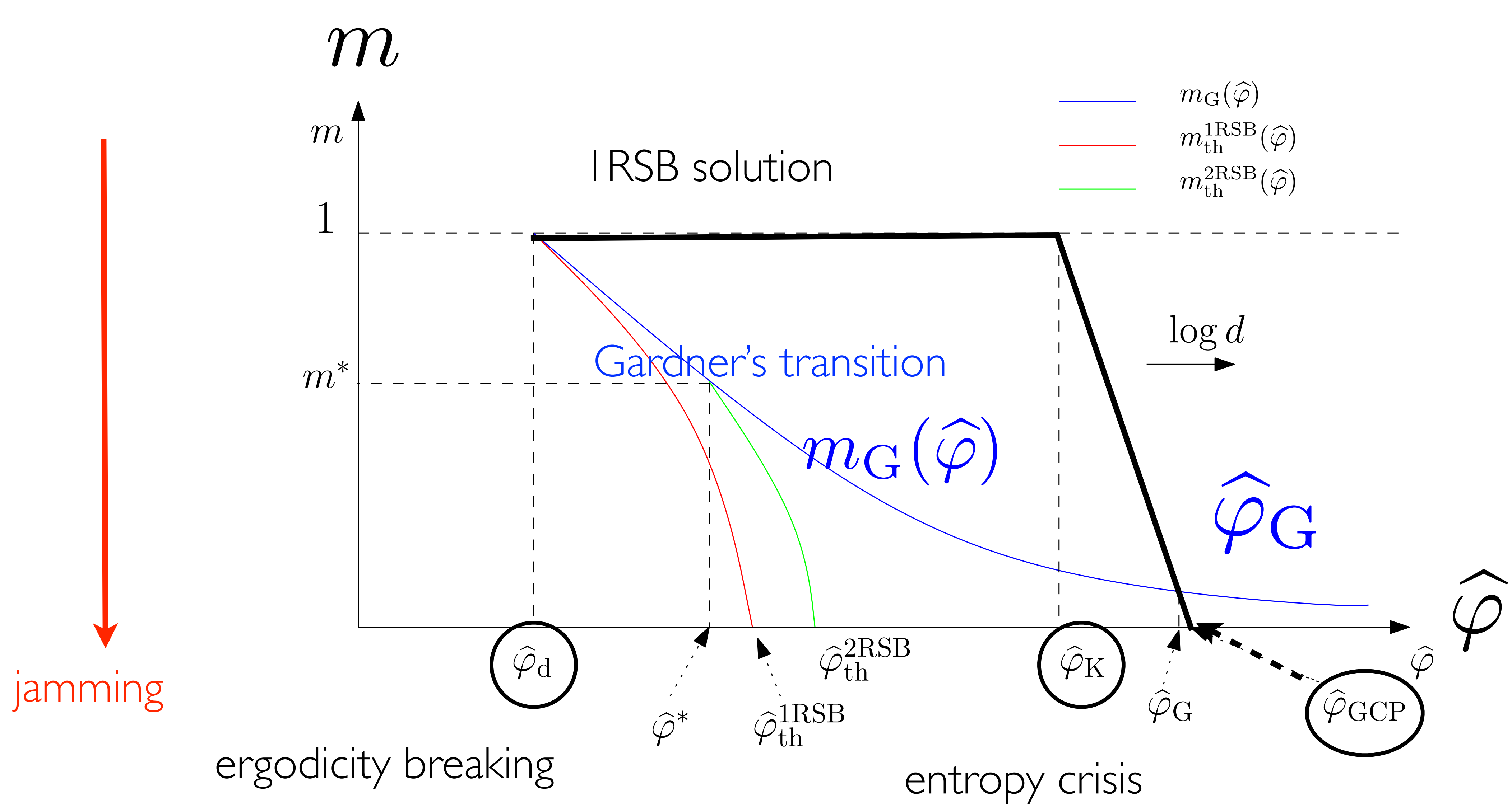
Emulsion experiments:
T. G. Mason et al (1997). Guerra-Weitz (2013)

AT instability - Gardner's transition

JRL De Almeida and D. J. Thouless, Journal of Physics A: Math. Gen. 11, 938 (1978)
E. Gardner, Nucl. Phys. B 257, 747 (1985).

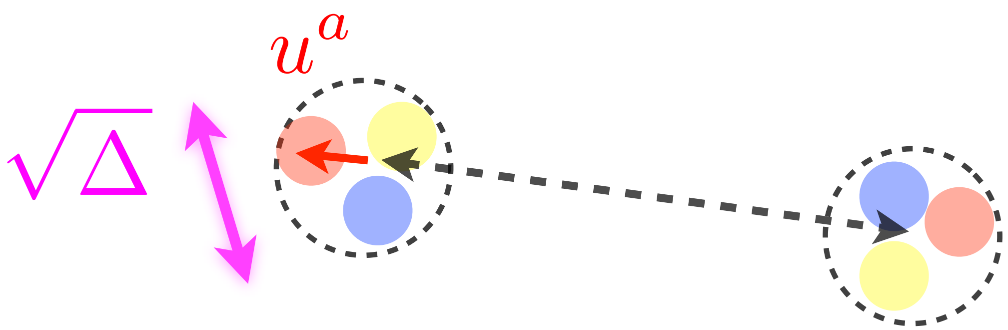
J. Kurchan, G. Parisi, P. Urbani, and F. Zamponi, J. Phys. Chem. B 117, 12979 (2013).

$$\beta p/\rho \propto 1/m$$



Replicated liquid theory of hard spheres $d \to \infty$

molecule made of replicas



J. Kurchan, G. Parisi and F. Zamponi,.J. Stat. Mech. P10012 (2012).

P. Charbonneau, J. Kurchan, G. Parisi, P. Urbani, F. Zamponi, Nature Communications 5, 3725 (2014).

$\overline{x} = \{x_1 \cdots x_m\}$ $\qquad x_a = ((x_a)_1, (x_a)_2, \ldots, (x_a)_m)$

$$-\beta F = \int d\overline{x}\rho(\overline{x})[1 - \log\rho(\overline{x})] + \frac{1}{2}\int d\overline{x}d\overline{y}\rho(\overline{x})\rho(\overline{y})f(\overline{x},\overline{y})$$

Replicated Mayer function

$$f(\overline{x},\overline{y}) = -1 + \prod_{a=1}^{m} e^{-\beta v(|(x_a - y_a)|)}$$

Contact potential

Exact expression of the free-energy as a functional of the glass order parameter

$$-\beta F(\hat{\alpha}, \{\gamma_a\})/N \quad = 1 - \log\rho + d\log m + \tfrac{d}{2}(m-1)\log(2\pi e D^2/d^2) + \tfrac{d}{2}\log\det(\hat{\alpha}^{m,m})$$
$$-\tfrac{d}{2}\widehat{\varphi}\mathcal{F}\left(\Delta_{ab}\right)$$

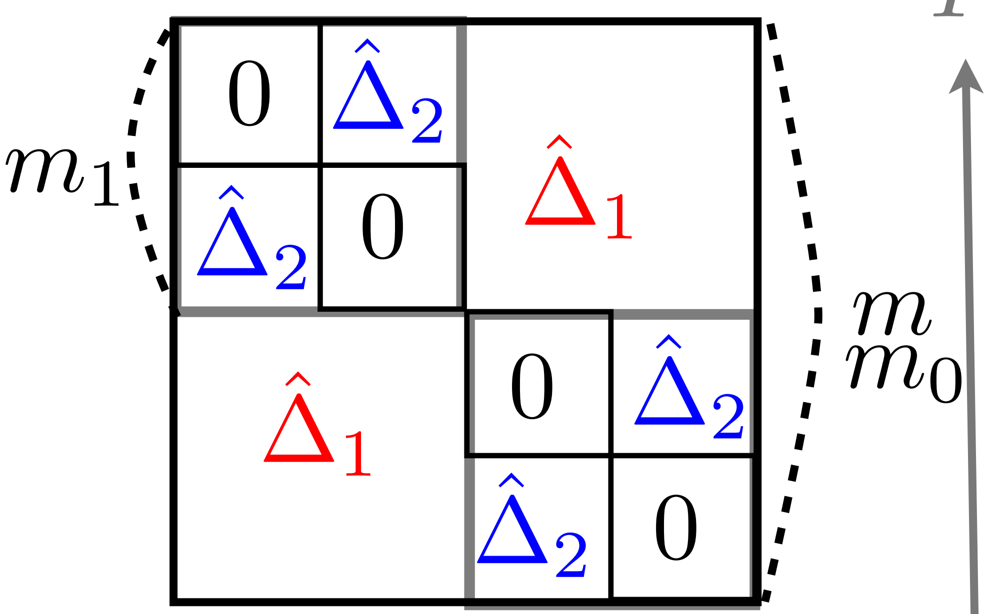
Order parameter $\qquad \Delta_{ab} = \dfrac{d}{D^2}\langle (u_a - u_b)^2 \rangle \qquad \widehat{\varphi} = \dfrac{2^d}{d}\varphi$

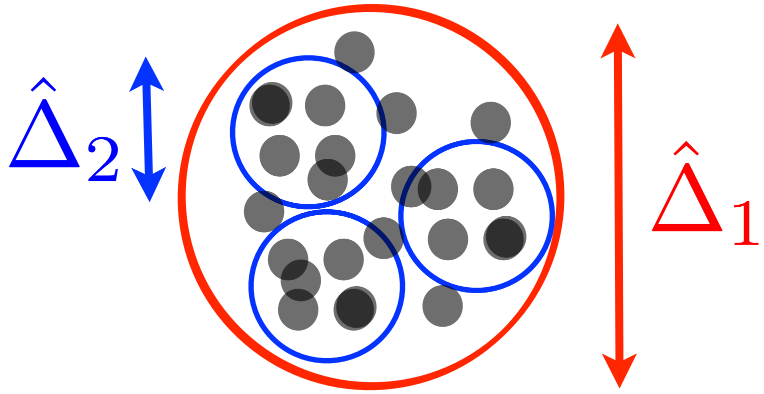
1+continuous Replica Symmetry Breaking

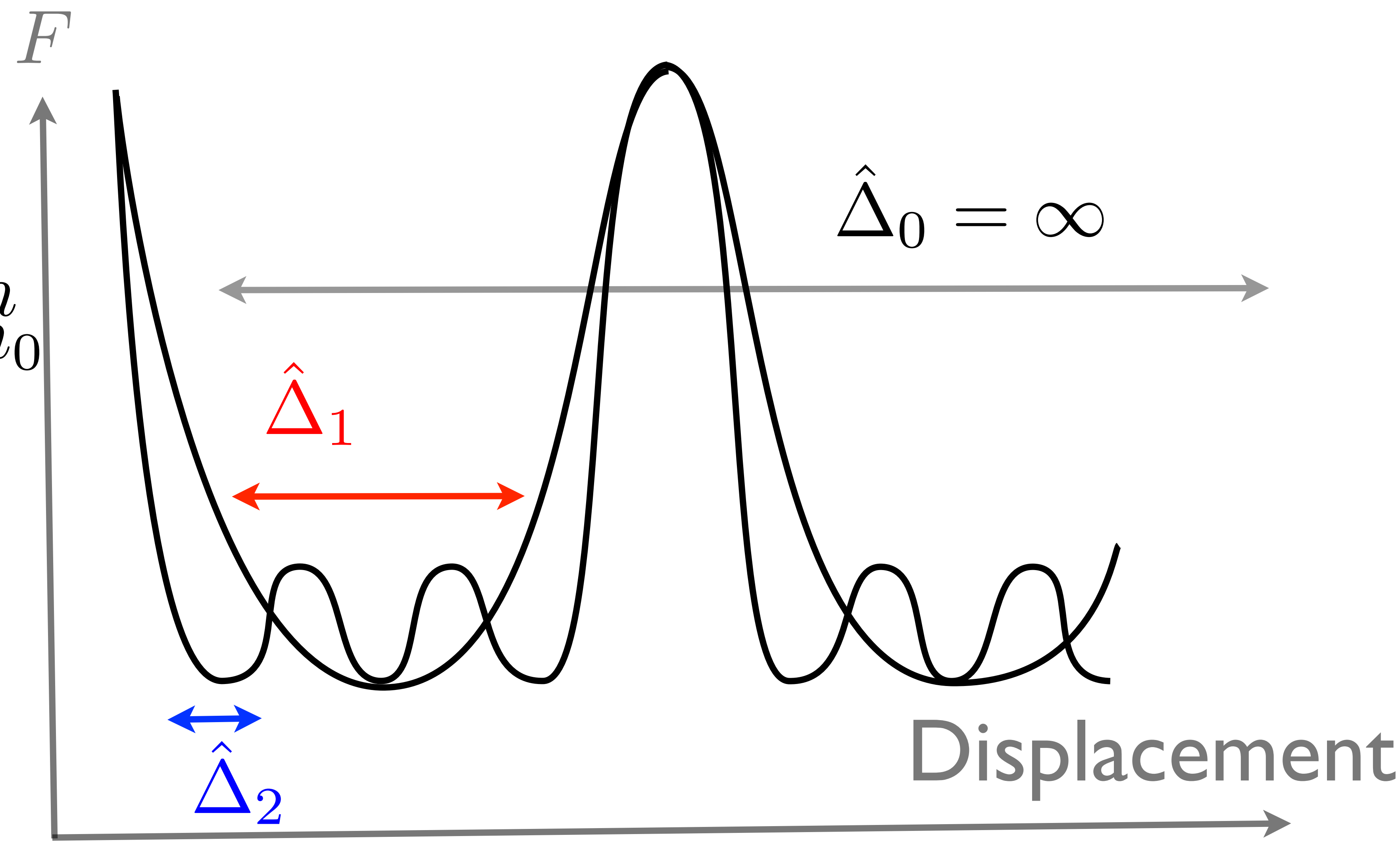
Parisi's matrix (m × m)

previous random first order transition (RFOT) : 1 step RSB

eg. 2 step RSB







$$-\beta F_{\infty\mathrm{RSB}} = -m\int_m^1 \frac{\mathrm{d}x}{x^2}\log\left[\frac{x\Delta(x)}{m}+\int_x^1 \mathrm{d}z\,\frac{\Delta(z)}{m}\right] - \widehat{\varphi}\, e^{-\Delta(m)/2}\int_{-\infty}^{\infty} \mathrm{d}h\, e^h\left[1-e^{mf(m,h)}\right]$$

Parisi's equation

$$\frac{\partial f(x,h)}{\partial x} = \frac{1}{2}\dot{\Delta}(x)\left[\frac{\partial^2 f(x,h)}{\partial h^2} + x\left(\frac{\partial f(x,h)}{\partial h}\right)^2\right],$$

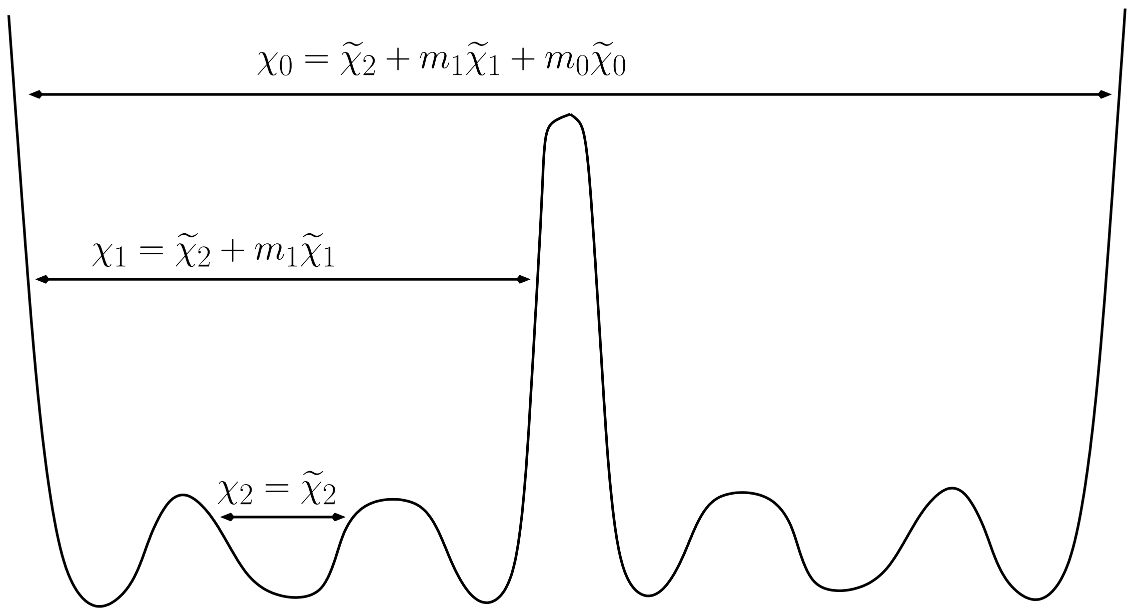
Hard-sphere case

$$f(1,h) = \log\Theta\left[\frac{h}{\sqrt{2\Delta(1)}}\right].$$

(note) Spinglass case : G. Parisi (1980)

massless "replicon" mode, marginal stability

Response in hierarchical energy landscape



$$\chi_i = \sum_{j=i}^{k} m_j \tilde{\chi}_j$$

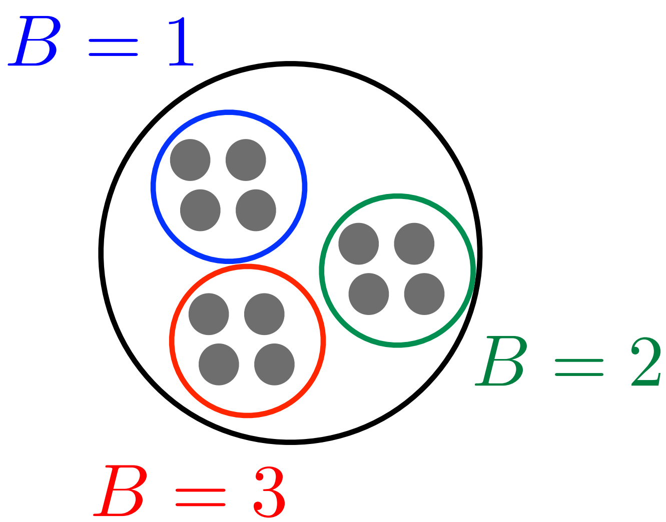
$$\langle O \rangle = -\frac{dF}{dh} = \sum_{\alpha_0} w_{\alpha_0} \sum_{\alpha_1 \in \alpha_0} w_{\alpha_1|\alpha_0} O_{\alpha_1} \qquad O_{\alpha_1} = -\frac{df_{\alpha_1}}{dh}$$

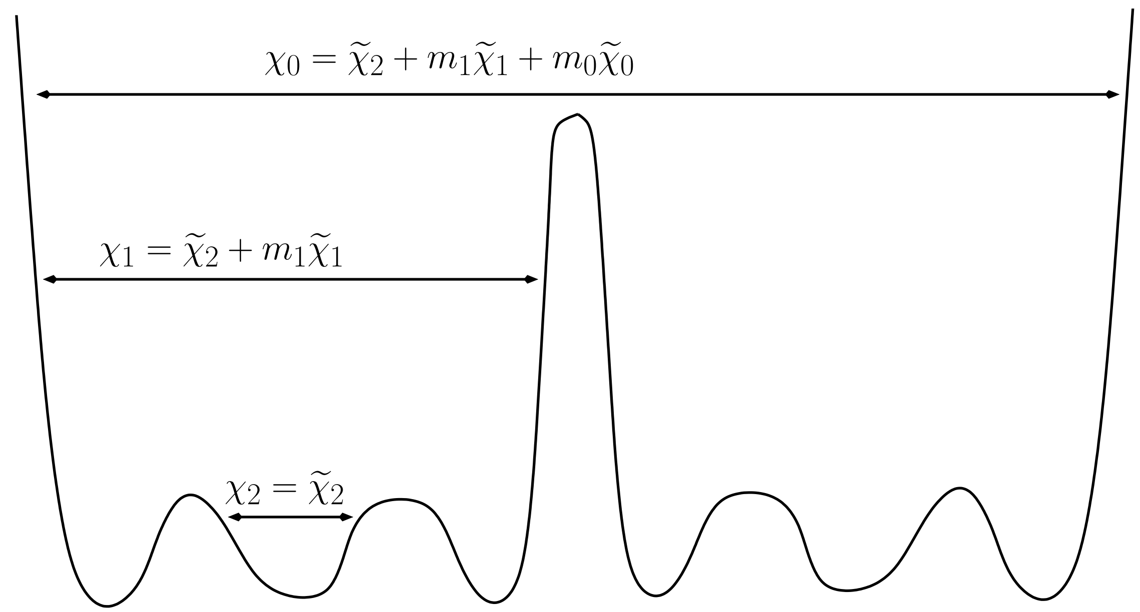
$$\begin{aligned}
\chi &= \frac{d\langle O \rangle}{dh} = -\frac{d^2F}{dh^2} \\
&= \sum_{\alpha_0} w_{\alpha_0} \sum_{\alpha_1 \in \alpha_0} w_{\alpha_1|\alpha_0} \frac{dO_{\alpha_1}}{dh} + \sum_{\alpha_0} w_{\alpha_0} \sum_{\alpha_1 \in \alpha_0} \frac{dw_{\alpha_1|\alpha_0}}{dh} O_{\alpha_1} + \sum_{\alpha_0} \frac{dw_{\alpha_0}}{dh} \sum_{\alpha_1 \in \alpha_0} w_{\alpha_1|\alpha_0} O_{\alpha_1} \\
&= \langle\langle \chi_{\alpha_1} \rangle_1 \rangle_0 + \beta N m_1 \langle\, \langle O^2_{\alpha_1} \rangle_1 - \langle O_{\alpha_1} \rangle_1^2 \,\rangle_0 + \beta N m_0 \left[\langle\langle O_{\alpha_1} \rangle_1^2 \rangle_0 - \langle\langle O_{\alpha_1} \rangle_1 \rangle_0^2 \right] \\
&= \widetilde{\chi}_2 + m_1 \widetilde{\chi}_1 + m_0 \widetilde{\chi}_0
\end{aligned}$$

Response of "cloned system" in hierarchical energy landscape

1RSB case : H. Yoshino and M. Me´zard, Phys. Rev. Lett. 105, 015504 (2010)

$$m_0 = 12$$
$$m_1 = 4$$





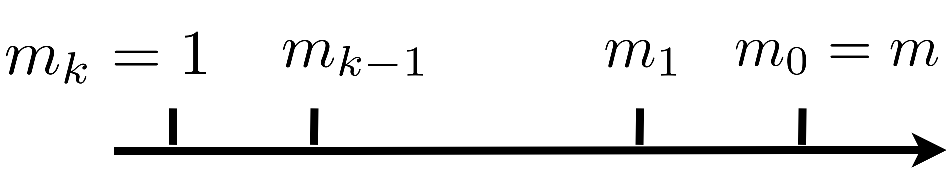
$$\chi_i = \sum_{j=i}^{k} m_j \tilde{\chi}_j$$

$$F_{\rm ml} = -\frac{T}{N}\log Z_{\rm ml} \qquad Z_{\rm ml} = \sum_{\alpha_0} \prod_{B=1}^{m_0/m_1} \left(\sum_{\alpha_1 \in \alpha_0} e^{-\beta N \sum_{a \in B} f_{\alpha_1}(h_a)} \right)$$

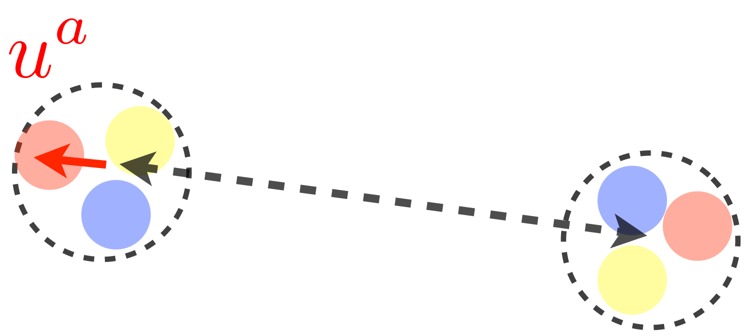
$$\chi_{ab} = \left. \frac{\partial^2 F}{\partial h_a \partial h_b} \right|_{h_a=0} = \tilde{\chi}_2 \delta_{ab} + \tilde{\chi}_1 I_{ab}^{m_1} + \tilde{\chi}_0 I_{ab}^{m_0}$$

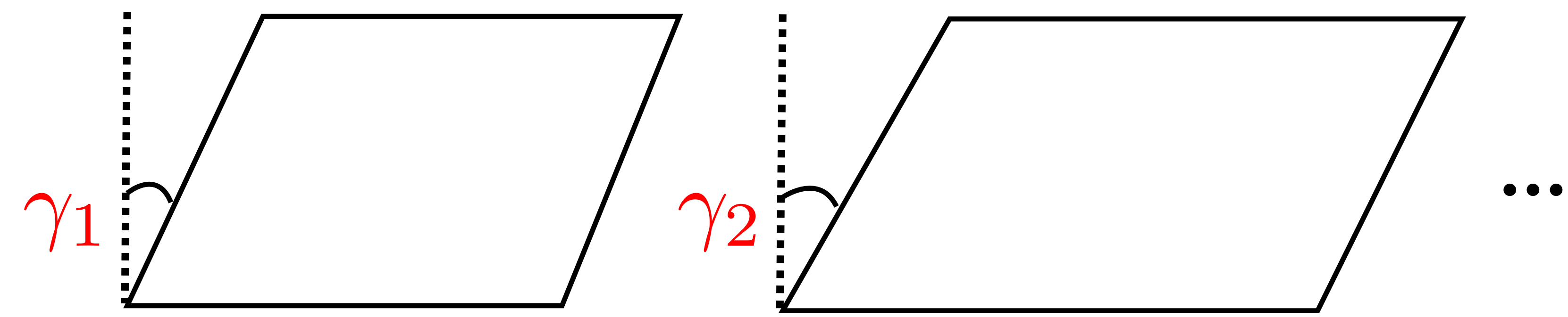
$$I^{m_i} = \overbrace{\begin{array}{|cc|cc|} \hline 1 & 1 & \multicolumn{2}{c|}{\multirow{2}{*}{0}} \\ 1 & 1 & & \\ \hline \multicolumn{2}{|c|}{\multirow{2}{*}{0}} & 1 & 1 \\ & & 1 & 1 \\ \hline \end{array}}^{m_i \text{ (over first block)}}$$

More generally... with k-RSB $\quad \chi_{ab} = \sum_{i=0}^{k} \tilde{\chi}_i I_{ab}^{m_i}$



Twist on the replicated liquid





$$-\beta F(\{\gamma_a\}) = \int d\overline{x}\rho(\overline{x})[1 - \log\rho(\overline{x})] + \frac{1}{2}\int d\overline{x}d\overline{y}\rho(\overline{x})\rho(\overline{y})f_{\{\gamma_a\}}(\overline{x},\overline{y})$$

Replicated Mayer function (under shear)

$$f_{\{\gamma_a\}}(\overline{x},\overline{y}) = -1 + \prod_{a=1}^{m} e^{-\beta v(|S(\gamma_a)(x_a - y_a)|)}$$

$$S(\gamma)_{\mu\nu} = \delta_{\mu\nu} + \gamma\delta_{\nu,1}\delta_{\mu,2}$$

$$(\text{Compression: } C(\gamma)_{\mu\nu} = \delta_{\mu\nu} + \gamma\delta_{\nu,1}\delta_{\mu,1})$$

$$-\beta F(\hat{\alpha},\{\gamma_a\})/N = 1 - \log\rho + d\log m + \frac{d}{2}(m-1)\log(2\pi e D^2/d^2) + \frac{d}{2}\log\det(\hat{\alpha}^{m,m})$$
$$-\frac{d}{2}\widehat{\varphi}\int \frac{d\lambda}{\sqrt{2\pi}}\mathcal{F}\left(\Delta_{ab} + \frac{\lambda^2}{2}(\gamma_a - \gamma_b)^2\right)$$

Small strain expansion

$$F(\{\gamma_a\})/N = F(\{0\})/N + \sum_{a=1}^{m} \sigma_a \gamma_a + \frac{1}{2} \sum_{a,b}^{1,m} \mu_{ab} \gamma_a \gamma_b + \cdots$$

yields shear-modulus matrix

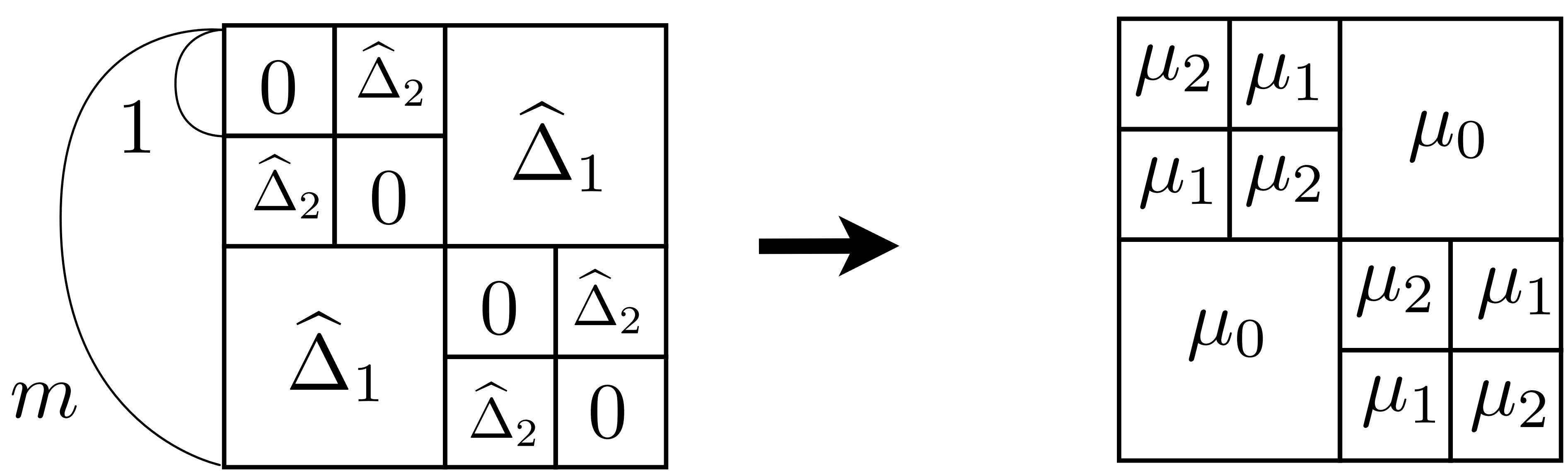
$$\beta \mu_{ab} = \frac{d}{2} \widehat{\varphi} \left[\delta_{ab} \sum_{c(\neq c)} \frac{\partial \mathcal{F}}{\partial \Delta_{ac}} - (1-\delta_{ab}) \frac{\partial \mathcal{F}}{\partial \Delta_{ab}} \right]$$

"sum rule" $\sum_b \mu_{ab} = 0$

$$\beta \hat{\mu}(y) = \frac{1}{m\gamma(y)} \qquad \Delta(y) = \frac{\gamma(y)}{y} - \int_y^{1/m} \frac{dz}{z^2} \gamma(z) \qquad \text{for} \qquad y = x/m$$

Hierarchical RSB

Hierarchical rigidity



$$m < x < 1$$

1RSB case : HY and M. Mezard (2010), HY (2012)

1 step RSB

$$\widehat{\varphi}_{\rm d} < \widehat{\varphi} < \widehat{\varphi}_{\rm Gardner}$$

$$\beta\widehat{\mu}_{ab} = \beta\widehat{\mu}_{\rm EA}\left(\delta_{ab} - \frac{1}{m}\right)$$

H. Yoshino and M. M´ezard, PRL 105, 015504 (2010).

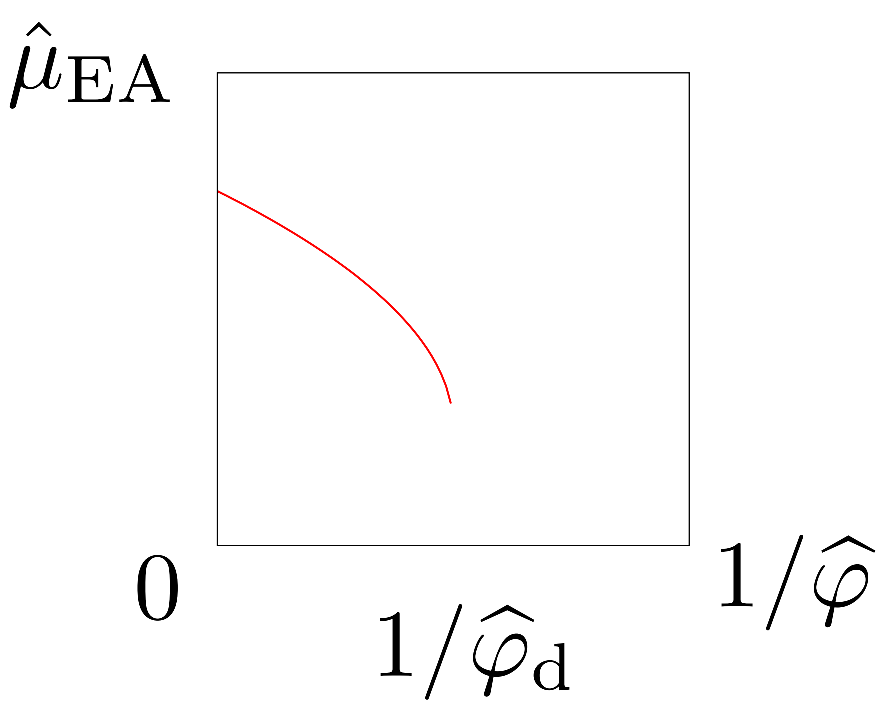
H. Yoshino, The Journal of Chemical Physics 136, 214108 (2012).

$$\beta\hat{\mu}_{\rm EA} = \widehat{\Delta}_{\rm EA}^{-1} \qquad \widehat{\Delta}_{\rm EA} \sim \widehat{\Delta}_d - C(\widehat{\varphi} - \widehat{\varphi}_d)^{1/2}$$

in agreement with MCT

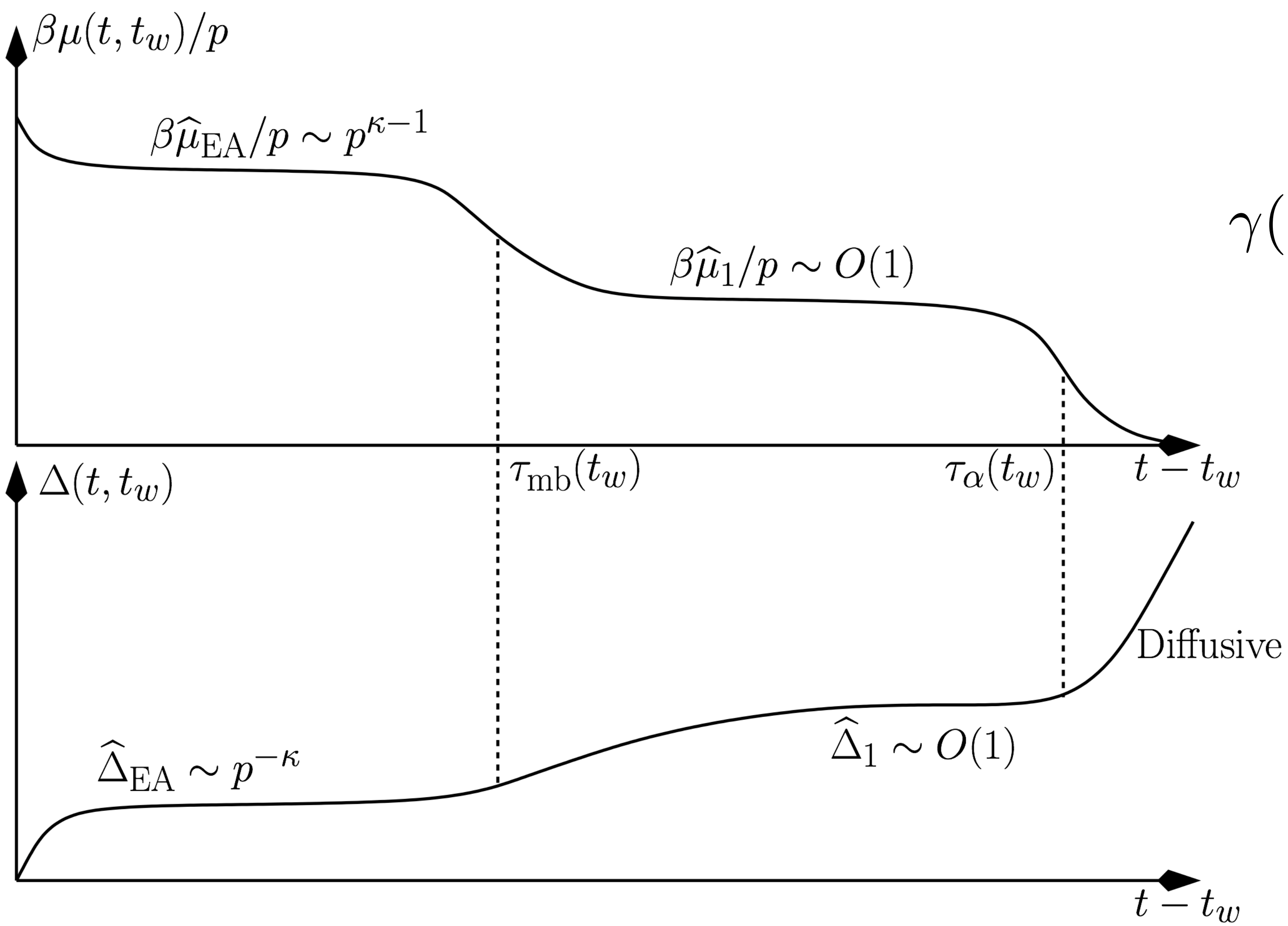
W. Gotze, Complex dynamics of glass-forming liquids: A mode-coupling theory, vol. 143 (Oxford University Press, USA,2009).

G. Szamel and E. Flenner, PRL 107, 105505 (2011).



1+continuous RSB

$$\widehat{\varphi}_{\rm Gardner} < \widehat{\varphi} < \widehat{\varphi}_{\rm GCP}$$



$$\widehat{\varphi} \to \widehat{\varphi}_{\rm GCP}^{-} \qquad p \propto 1/m \to \infty$$

$$\gamma(y) \sim \gamma_\infty y^{-(\kappa-1)} \qquad \kappa = 1.41575..$$

* the plateau modulus

$$\beta\mu_{\rm EA} = 1/\Delta_{\rm EA} \propto m^{-\kappa} \propto p^{\kappa}$$

* the "lowest" plateau modulus

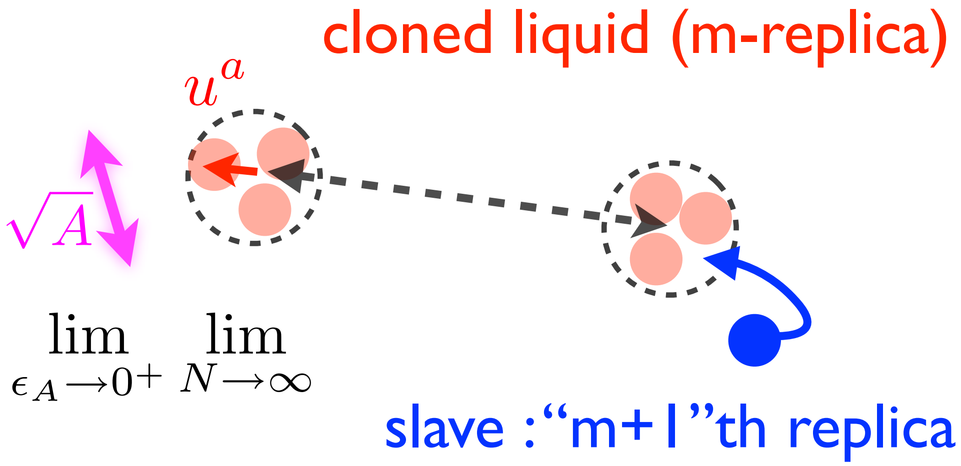
$$\beta\widehat{\mu}(1) = \frac{1}{m\gamma(1)} \propto p$$

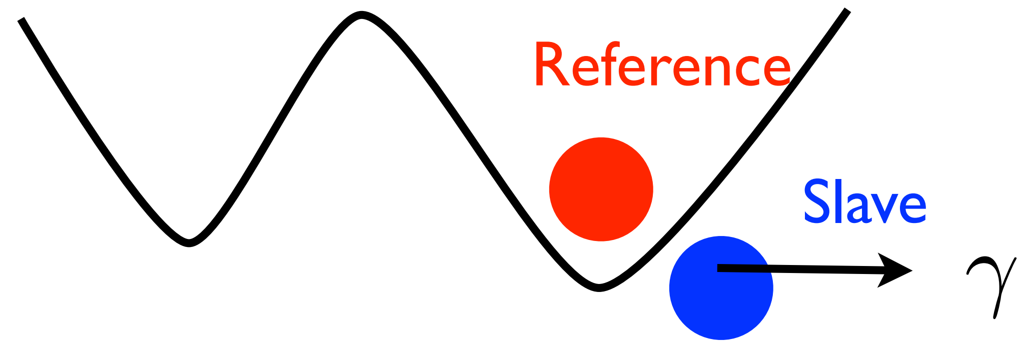
Effective medium approach + numerical simulation

E DeGiuli; E Lerner; C Brito; M Wyart, arXiv:14023834

“Following glassy states” under perturbation

S. Franz and G. Parisi, J. Phys. I France 5 (1995) 1401.
F. Krzakala and L. Zdeborova, Eur. Phys. Lett. , 90 (2010) 66002.





Franz-Parisi potential

$$NV_{\rm FP}(\gamma) = \langle \log Z^e(\gamma) \rangle_m \equiv \frac{1}{Z_m} \int_{\mathbf{r}^1,\dots,\mathbf{r}^{\mathbf{m}}} e^{-\beta \sum_{a=1}^m H[\mathbf{x}^a]} \log Z^e(\gamma)$$

$$Z^e(\gamma) = \log \int_{\mathbf{r}^{m+1}} e^{-\beta H[\{\mathbf{r}_\mathbf{i}^{\mathbf{m+1}}\};\gamma]} \qquad H[\{\mathbf{r}_i\}] = \sum_{i<j} v(\mathbf{r}_i - \mathbf{r}_j)$$

$$-\beta F_{\rm FP}(\gamma) = \log \int_{r^1\dots r^m, r^{m+1}\dots r^{m+s}} e^{-\beta \sum_{a=1}^m H[\{\mathbf{r}^a\}] - \beta \sum_{b=m+1}^{m+s} H[\{\mathbf{r}^\mathbf{b}\},\gamma]}$$

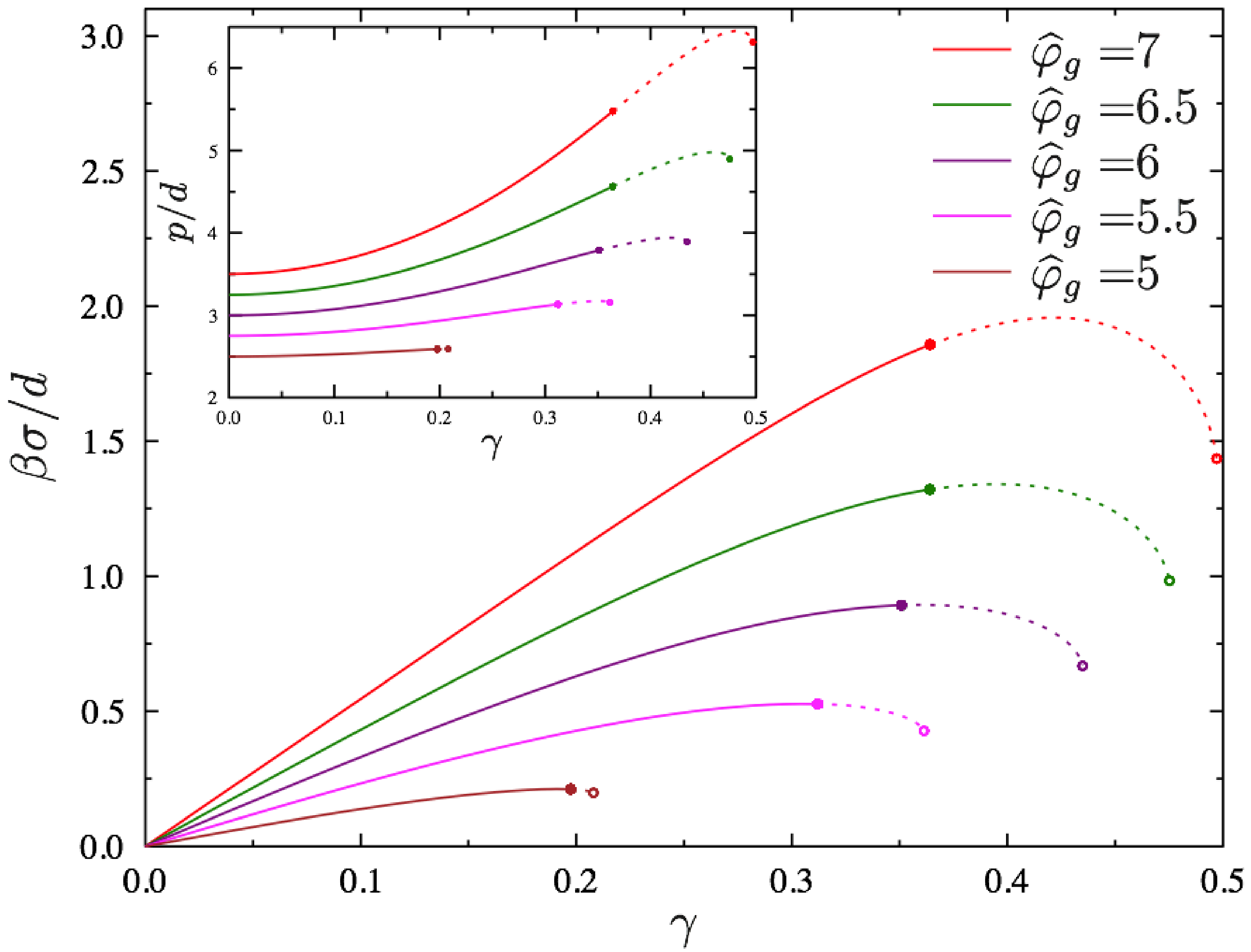
$$= \log \int_{r^1\dots r^m} e^{-\beta \sum_{a=1}^m H[\mathbf{r}^a]} (Z^e(\gamma))^s$$

$$-\beta F_{\rm FP}(\gamma) = \log Z_m + sNV_{\rm FP}(\gamma) + O(s^2).$$

Nonlinear response - yielding

"state following under (de)compression/shear" via Franz-Parisi potential

C. Raione, P. Urbani, H. Yoshino and F. Zamponi, submitted



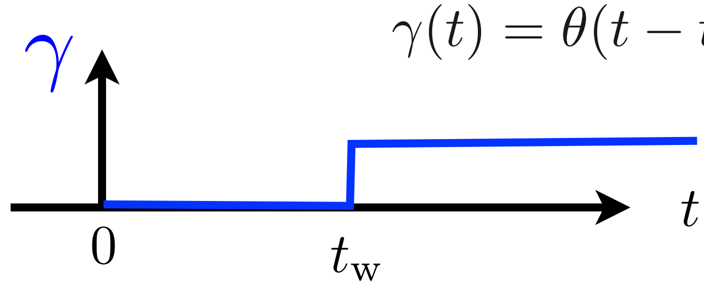
See the Poster by Corrado Raione

MD simulation

shear strain

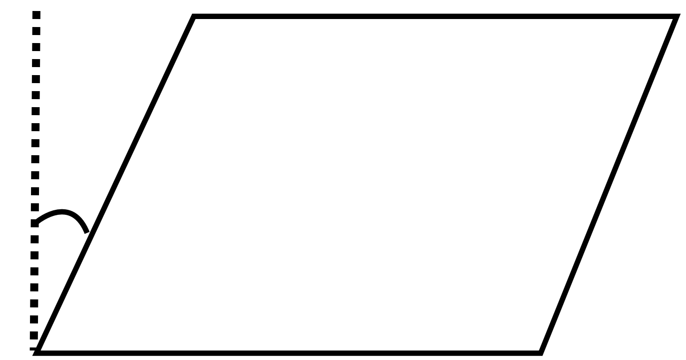
$\gamma(t) = \theta(t - t_{\mathrm{w}})\delta\gamma$

quench $t = 0$



Aging

shear strain γ



Initial configuration

$T/\epsilon = 10^{-3}$

Equilibrium state (liquid)

c.f. McKenna, Narita and Lequeux, J. Rheol. 53, 489 (2009).

response function

$$\mu(t, t_{\mathrm{w}}) = \frac{\delta\langle\sigma(t; t_{\mathrm{w}})\rangle}{\delta\gamma}$$

autocorrelation function

$$C_\sigma(t, t_{\mathrm{w}}) = \langle\sigma(t)\sigma(t_{\mathrm{w}})\rangle$$

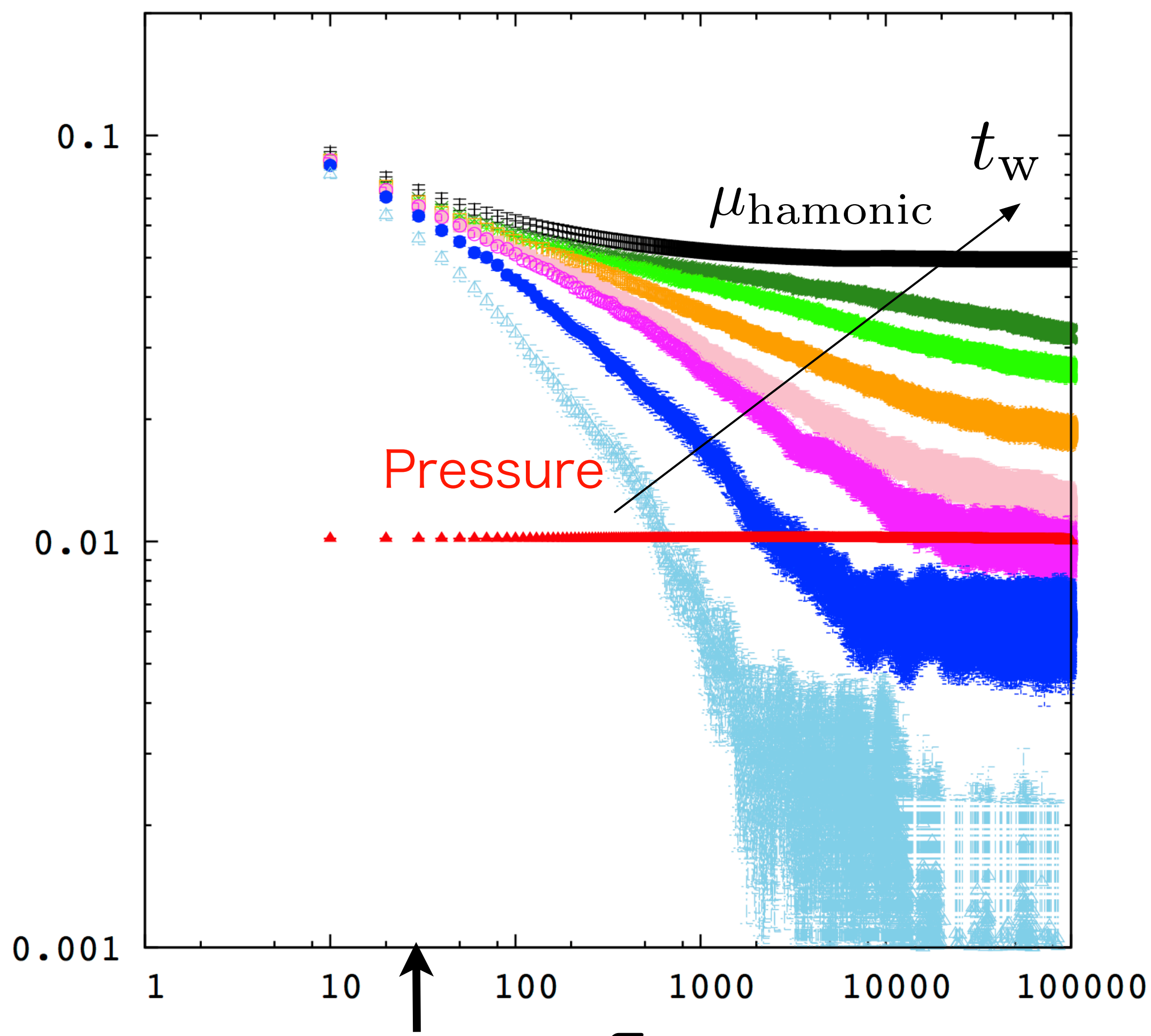
temperature	$k_{\mathrm{B}}T/\epsilon = 10^{-5}$	Langevin simulation
shear-strain	$\gamma = 2.5 \times 10^{-3}$	Lee-Edwards boundary condition
volume fraction	$\varphi = 0.65 - 0.67$	
# of particles	$N = 800, 1600$	time scale $O(t/t_0) = 10^5$
# of sample (initial condition/ Langevin noise)		# of samples : 4096

Outline

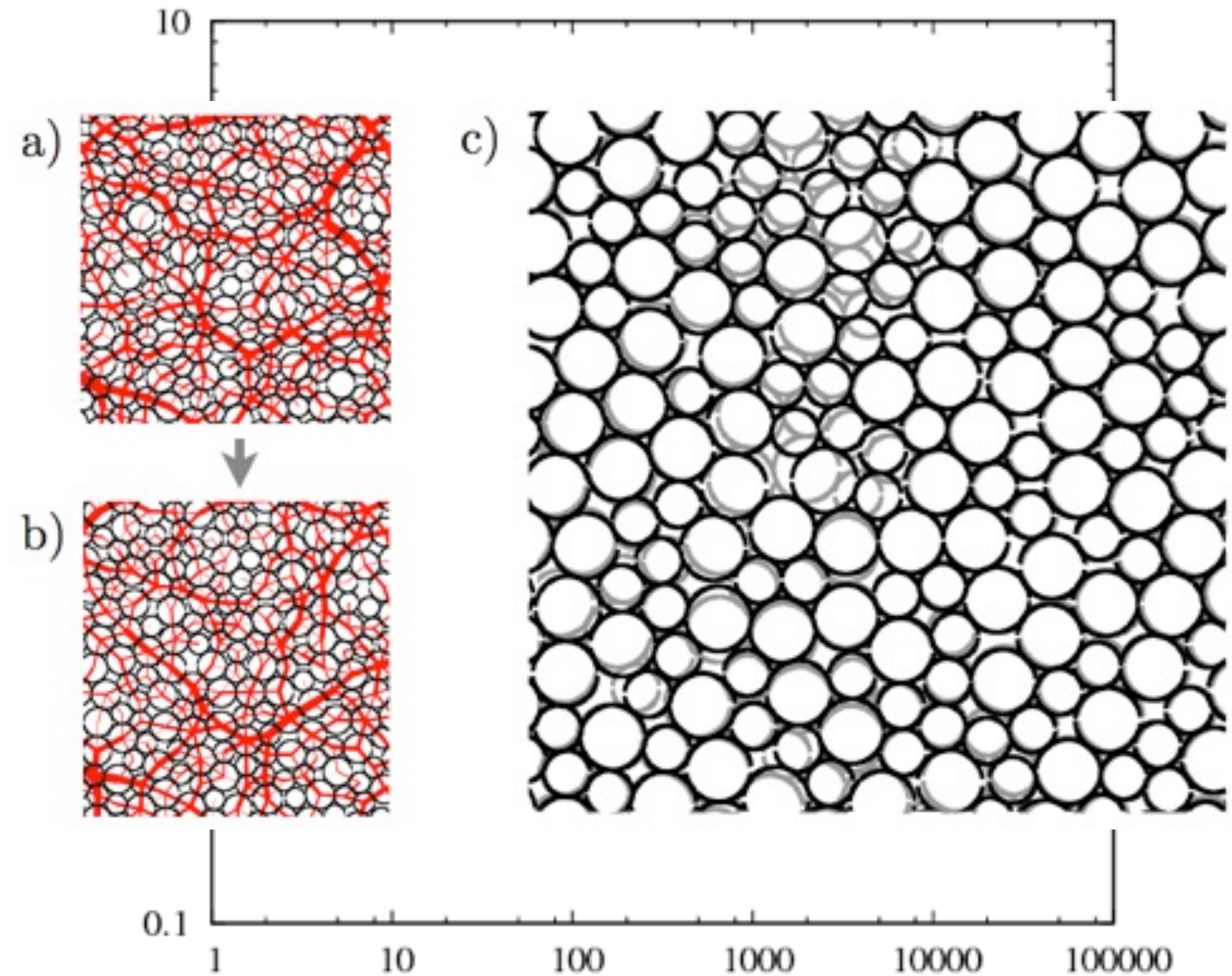
- Introduction:
- Shear on the cloned liquid in the large-d limit (theory)
- Aging around the jamming point (simulation)
- Discussions

$$\varphi = 0.67 \quad k_\mathrm{B}T/\epsilon = 10^{-5}$$

$\sigma(\tau; t_\mathrm{w})/\gamma$



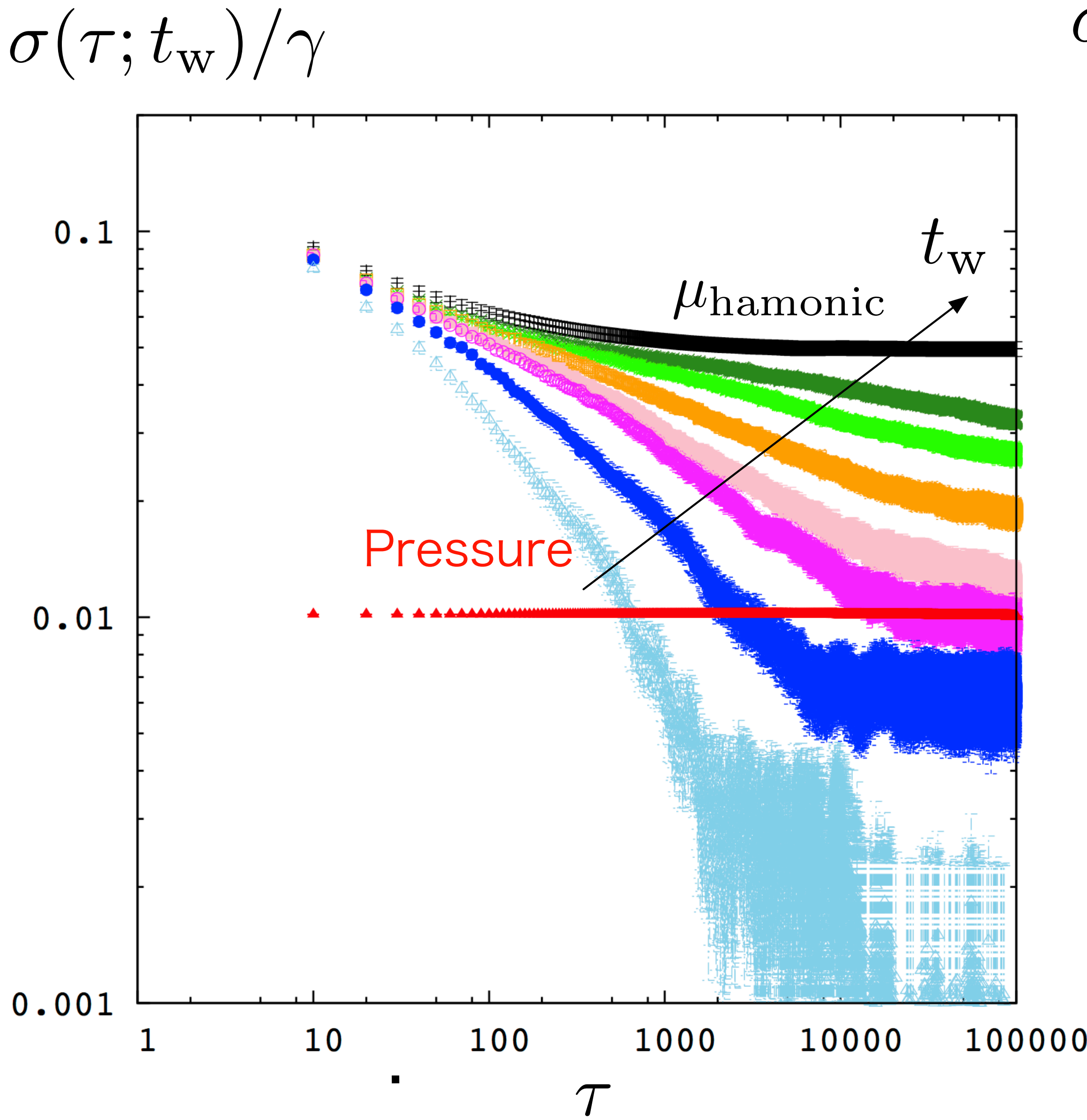
$\beta\langle\sigma(\tau + t_\mathrm{w})\sigma(t_\mathrm{w})\rangle$

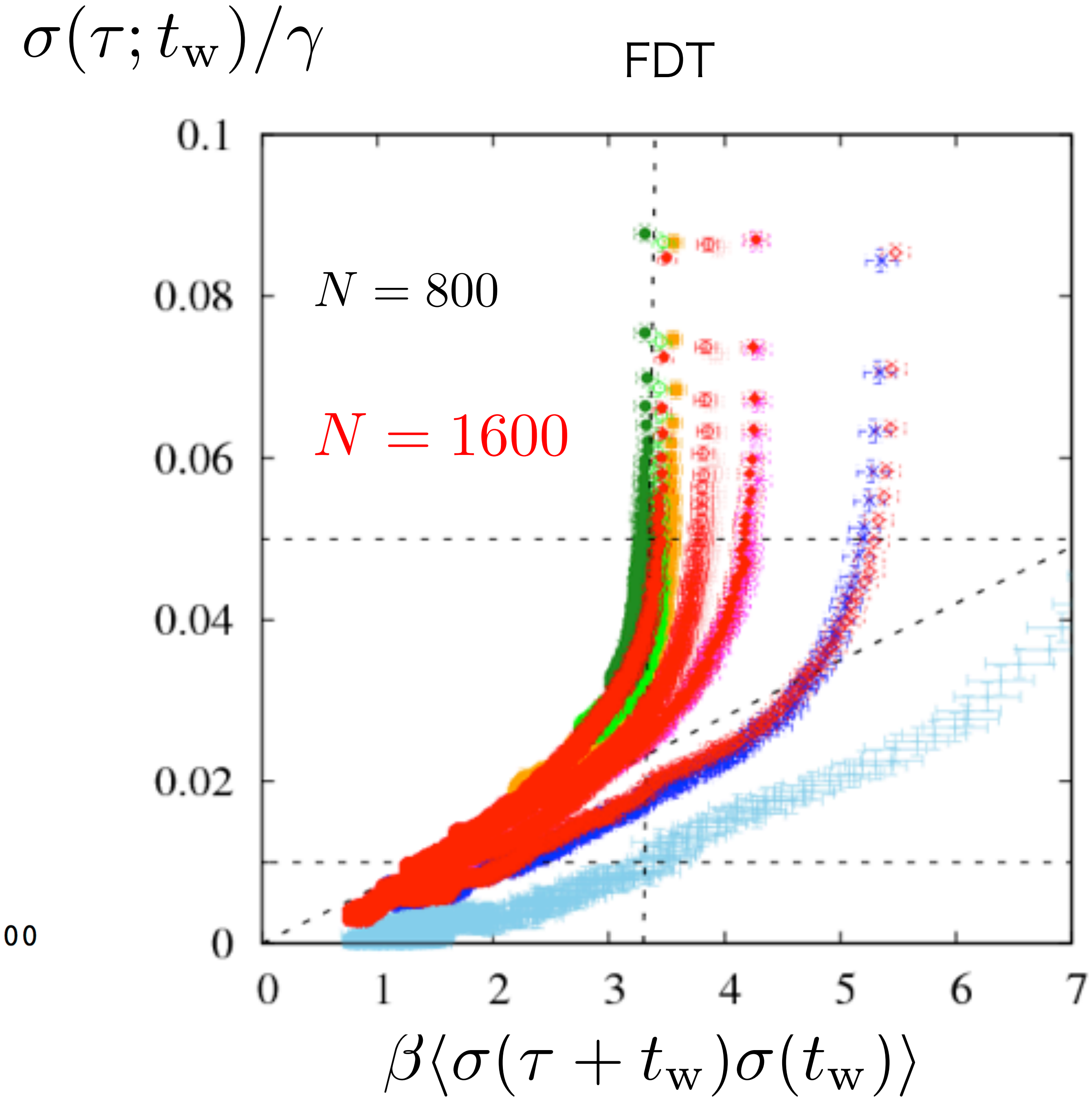


$$t^* = 2\pi/\omega^* = (\varphi = 0.67)$$

$t_\mathrm{w} = 3 \times 10^2, 10^3, 3 \times 10^3, 5 \times 10^3, 10^4, 3 \times 10^4, 10^5$

$$\varphi = 0.67 \quad k_{\mathrm{B}}T/\epsilon = 10^{-5}$$

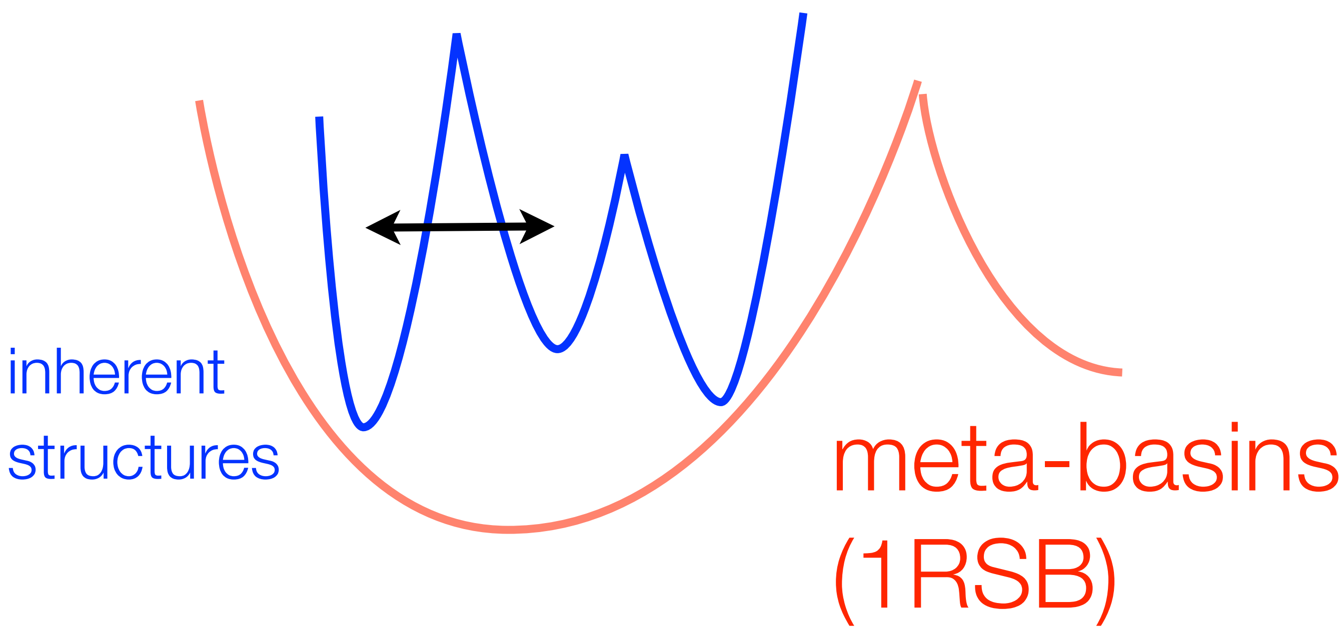




$$t^* = 2\pi/\omega^* = (\varphi = 0.67)$$

$$t_{\mathrm{w}} = 3 \times 10^2, 10^3, 3 \times 10^3, 5 \times 10^3, 10^4, 3 \times 10^4, 10^5$$

Discussion : fluctuation within meta-basin



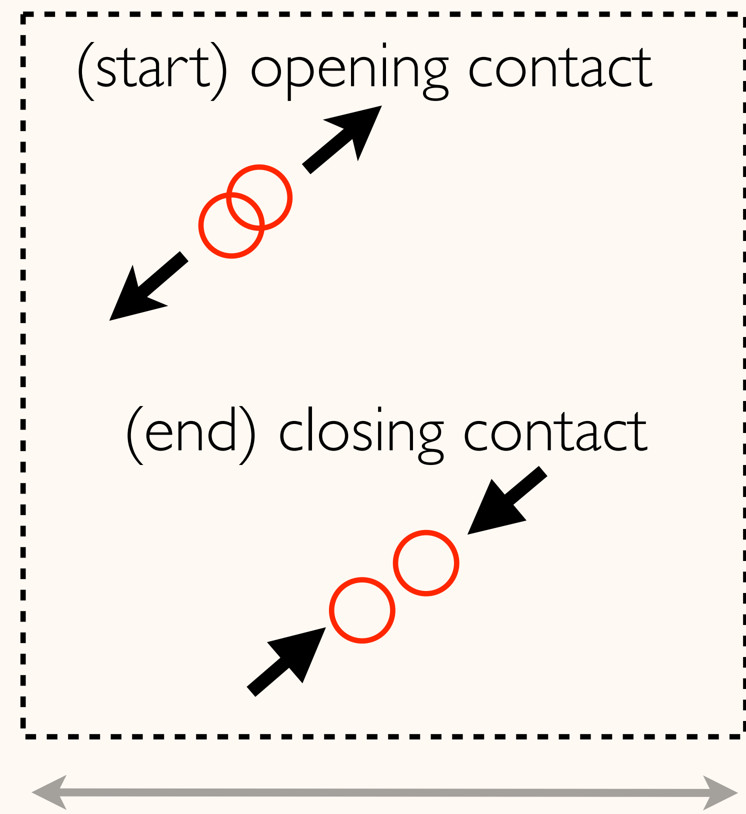
Thermally activated,
localized "floppy modes" ?

NOTE strong relaxation of shear-stress is possible

$$\sigma_{xz} = \frac{1}{V} \sum_{i<j} r_{ij} f(r_{ij}) \frac{x_{ij}}{r_{ij}} \frac{z_{ij}}{r_{ij}}$$ "angular variables

Floppy mode:

$$\delta r_{ij} = |\delta R_i - \delta R_j| = 0$$



$$l_{\rm iso} \sim \delta z^{-1} \sim 1/\sqrt{\phi - \phi_{\rm J}}$$

M.Wyart, Annales de Phys, 30 (3), 1 (2005).

M.Wyart, PRL 109, 125502 (2012).

E. Lerner, G. During and M. Wyart arXiv.1302.3990

■ Summary

Response to shear of a hard-sphere glass in large-dimensional limit

1. Exact free-energy functional under shear

2. Analysis of shear-modulus

   ✳ 1RSB - jump + square-root singularity at Td

   ✳ 1+continuous RSB - (1) hierarchy of rigidities

   (2) anomalous scaling as $\widehat{\varphi} \to \widehat{\varphi}_J$

3. State following under shear - observation of the yield process

Replica can be useful for real life!