Is there a de Almeida-Thouless line in finite-dimensional spin glasses? (and why you should care)

Peter Young

UNIVERSITY OF CALIFORNIA SANTA CRUZ

Talk at Workshop on "Critical Phenomena in Random and Complex Systems", Capri, September 9-12, 2014

Collaborators:
H. Katzgraber, D. Larson, M.A. Moore

Paper: Phys. Rev. B 87, 024414 (2013), arXiv:1211.7297.

Work supported by the National Science Foundation
WHERE DISCOVERIES BEGIN

Is there a de Almeida-Thouless line in finite-dimensional spin glasses? (and why you should care)

Peter Young

UNIVERSITY OF CALIFORNIA SANTA CRUZ

Talk at Workshop on "Critical Phenomena in Random and Complex Systems", Capri, September 9-12, 2014

Collaborators:
H. Katzgraber, D. Larson, M.A. Moore

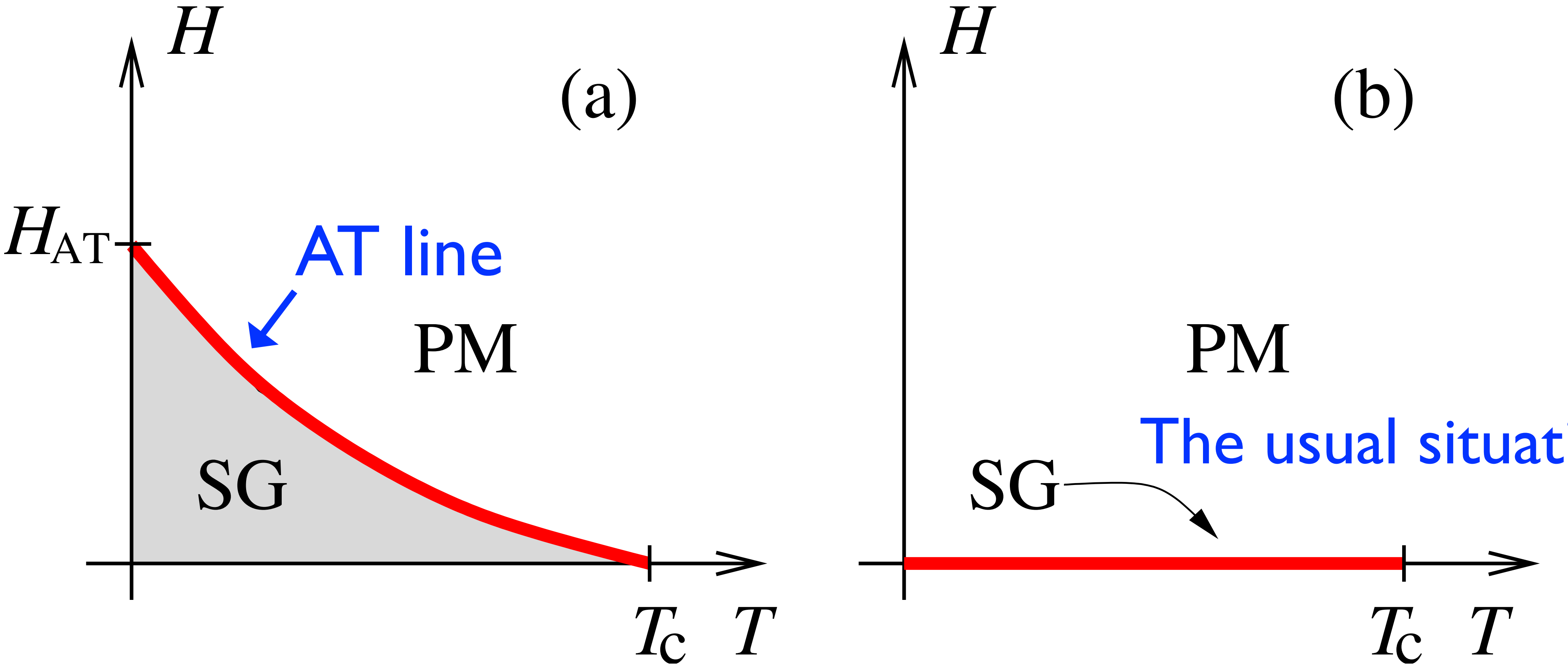
Paper: Phys. Rev. B 87, 024414 (2013), arXiv:1211.7297.

Work supported by the National Science Foundation WHERE DISCOVERIES BEGIN

The Almeida-Thouless line

In MFT (the exact solution of the infinite-range Sherrington-Kirkpatrick model) there's a transition in a field for an Ising spin glass the de Almeida Thouless (AT) line from a spin glass phase (divergent relaxation times, "replica symmetry breaking") to a paramagnetic phase (finite relaxation times, "replica symmetry").

The AT line is a ergodic-non ergodic transition with no change in symmetry



Does an AT line occur in real systems?

The AT line

The AT line

Whether or not the AT line actually occurs in real systems is of interest because:

The AT line

Whether or not the AT line actually occurs in real systems is of interest because:

- It is a transition without symmetry breaking.

The AT line

Whether or not the AT line actually occurs in real systems is of interest because:

- It is a transition without symmetry breaking.
- The two main descriptions of the spin glass state ("replica symmetry breaking" and the "droplet picture"), see later, make different predictions as to whether or not an AT line occurs.

The AT line

Whether or not the AT line actually occurs in real systems is of interest because:

- It is a transition without symmetry breaking.
- The two main descriptions of the spin glass state (“replica symmetry breaking” and the “droplet picture”), see later, make different predictions as to whether or not an AT line occurs.
- The AT line may be related to the putative “ideal glass transition” in structural glasses (see next three slides).

Spin glasses and structural glasses; are they related?

Supercooled liquid

Viscosity: Vogel-Fulcher law $\eta \propto \exp\left[\frac{A}{T - T_0}\right]$

Entropy difference between supercooled liquid and crystal

$$\Delta S \to 0 \text{ for } T \to T_K$$

where T_K is the Kauzmann temperature (the Kauzmann "paradox")

Find $T_K \approx T_0$ (but system drops out of equilibrium at higher T so these are extrapolations.)

Is there an **"ideal glass transition"** at T_K?

Spin glasses and structural glasses

Spin glasses and structural glasses

At mean field (MF) level:

Spin glasses and structural glasses

At mean field (MF) level:

Equations of “mode-coupling theory” of supercooled liquids are the same as the equations for the dynamics of a mean-field p-spin spin glass for $p > 2$.

Spin glasses and structural glasses

At mean field (MF) level:

Equations of “mode-coupling theory” of supercooled liquids are the same as the equations for the dynamics of a mean-field p-spin spin glass for $p > 2$.

Dynamic transition at T_d (which must disappear beyond MF)

Spin glasses and structural glasses

At mean field (MF) level:

Equations of “mode-coupling theory” of supercooled liquids are the same as the equations for the dynamics of a mean-field p-spin spin glass for $p > 2$.

Dynamic transition at T_d (which must disappear beyond MF)

Static transition at a lower temperature T_c.

Spin glasses and structural glasses

At mean field (MF) level:

Equations of "mode-coupling theory" of supercooled liquids are the same as the equations for the dynamics of a mean-field p-spin spin glass for $p > 2$.

Dynamic transition at T_d (which must disappear beyond MF)

Static transition at a lower temperature T_c.
Does T_c correspond to Kauzmann temperature T_K in a glass?

Spin glasses and structural glasses

Spin glasses and structural glasses

Beyond mean field?

Spin glasses and structural glasses

Beyond mean field?

Moore and Yeo (2006) argue that the ideal glass transition corresponds to a spin glass transition in a magnetic field. Why? Argument involved, but perhaps related to the frozen density fluctuations below TK are (a) random (so spin-glass like) and (b) are not symmetric about zero (so like a spin model in a magnetic field).

Spin glasses and structural glasses

Beyond mean field?

Moore and Yeo (2006) argue that the ideal glass transition corresponds to a spin glass transition in a magnetic field. Why? Argument involved, but perhaps related to the frozen density fluctuations below TK are (a) random (so spin-glass like) and (b) are not symmetric about zero (so like a spin model in a magnetic field).

According to Moore and Yeo

Spin glasses and structural glasses

Beyond mean field?

Moore and Yeo (2006) argue that the ideal glass transition corresponds to a spin glass transition in a magnetic field. Why? Argument involved, but perhaps related to the frozen density fluctuations below TK are (a) random (so spin-glass like) and (b) are not symmetric about zero (so like a spin model in a magnetic field).

According to Moore and Yeo

"The question, then, of whether there is a structural glass transition then turns to whether there is an AT line in the spin glass analogue."

Nature of the Spin Glass State?

Numerics and experiments clearly indicate that there is a spin glass transition in three dimensions in zero magnetic field.

But what is the nature of the spin glass state below T_c ?

Two rival descriptions:

- "Replica Symmetry Breaking" (RSB) which is based on Parisi's solution of the spin glass mean field theory (MFT).
- "Droplet theory" of Fisher and Huse, Bray and Moore, McMillan.

Which, if either, is correct?
Here will focus on one aspect for which the two descriptions make opposite predictions and which should be checkable by simulations, namely whether or not there is a line of transitions in a magnetic field (AT line): RSB (YES), droplet theory (NO).

What is known about AT line in finite-d?

- Field theory, Bray and Roberts. d > 6, get Gaussian fixed point. d < 6 no stable perturbative fixed point. Conclusions?
- Real space RG, e.g. Migdal-Kadanoff. Drossel, Bokil & Moore find no transition in a field. Recently Angelini and Biroli find a non-perturbative FP for d greater than about 8. (But does this depend on the details of the method? Not systematic.)
- Experiment. Need to see if relaxation times diverge as T is lowered in a field, or if they just get very large. Careful measurements, Uppsala group (Nordblad et al) find no AT line (but some other groups have come to the opposite conclusion.)
- Simulations: will discuss here (also Ruiz-Lorenzo's talk).

Lower critical dimension (Orbach's talk) in zero field, $d_l \simeq 2.5$.

What is d_l in a field?

How to detect the AT line

In contrast to experiments, in simulations one can compute a static quantity which diverges at the transition in a field. This is $\chi_{SG}(0)$ where

$$\chi_{SG}(\vec{k}) = \frac{1}{N} \sum_{\langle i,j \rangle} \left[\left(\langle S_i S_j \rangle - \langle S_i \rangle \langle S_j \rangle \right)^2 \right]_{av} \exp[i\vec{k} \cdot (\vec{R}_i - \vec{R}_j)]$$

i.e. ∝ square of connected correlation function.

Using "standard" finite-size scaling (FSS), Katzgraber and APY (d=3) and Parisi et al (d=4) do not find an AT line by this approach, although evidence for a AT line in high-d (d > 6?).

To study this question further, it is useful to get information from

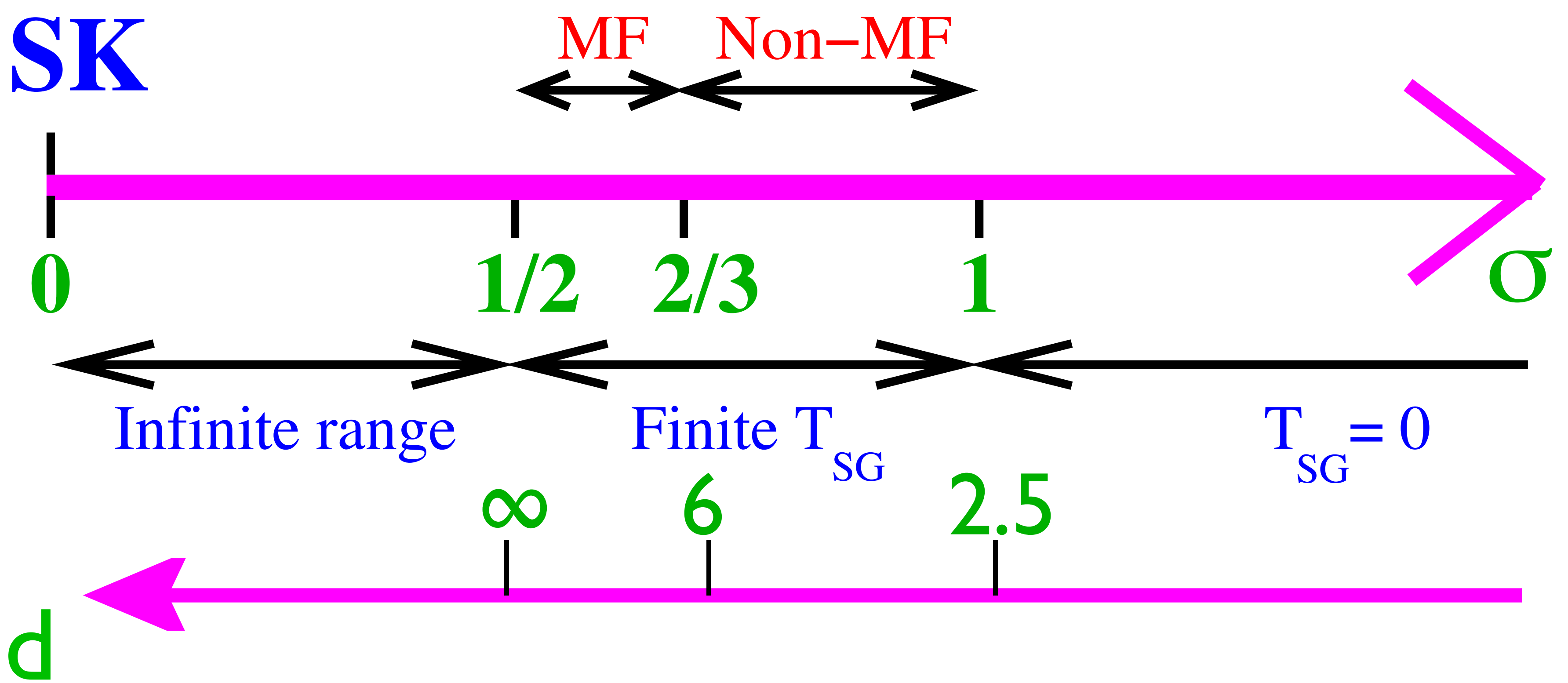
related models in one-dimension with long-range interactions

because these models correspond (closely) to SR models in a range of d, see next slide, so one can study a large range of (linear) sizes (and hence do FSS) for, effectively, a range of dimensions d (**including high-d**).

1-d Models

We take 1-d models where $J_{ij} \sim 1/|r_i - r_j|^\sigma$

Increasing σ is like decreasing d:



For a given d there is a σ(d) for which the LR model in 1-d is a (rough) proxy for a SR model in d dimensions.

Advantages:

- Can study a wide range of d including high-d
- There are many values of L for FSS (and also of k)

The Model

$$\mathcal{H} = -\sum_{\langle i,j \rangle} J_{ij} S_i S_j - \sum_i H_i S_i$$

where

$$[J_{ij}]_{av} = 0, \qquad [J_{ij}^2] \propto 1/|r_i - r_j|^{2\sigma}$$

$$[H_i]_{av} = 0, \qquad [H_i^2] = H^2$$

Recent Results on LR model

To represent d = 3 and d = 4 we take (Baños et. al.)

$\sigma(3) = 0.896$

$\sigma(4) = 0.784$

In **standard finite-size scaling (FSS)** we look for

Intersections of the scale-invariant quantities

ξ_L / L (correlation length, obtained from k=0 and 2π/L)

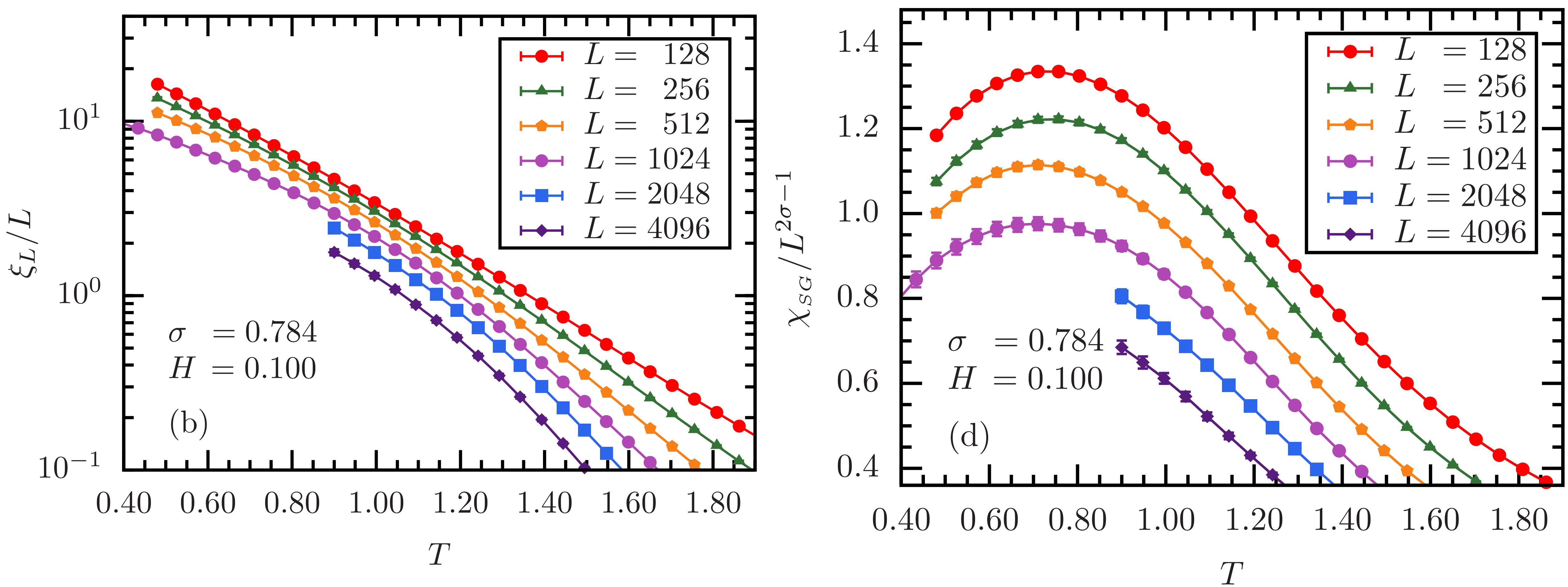
$\chi_{SG}/L^{2-\eta}$ $(2 - \eta = 2\sigma - 1 \text{ here})$ (**η known exactly for LR**)

locate the transition, since, for a scale-invariant quantity X, the finite-size-scaling (FSS) form is

$$X(T,L) = \widetilde{X}\left(L^{1/\nu}(T - T_c) \right)$$

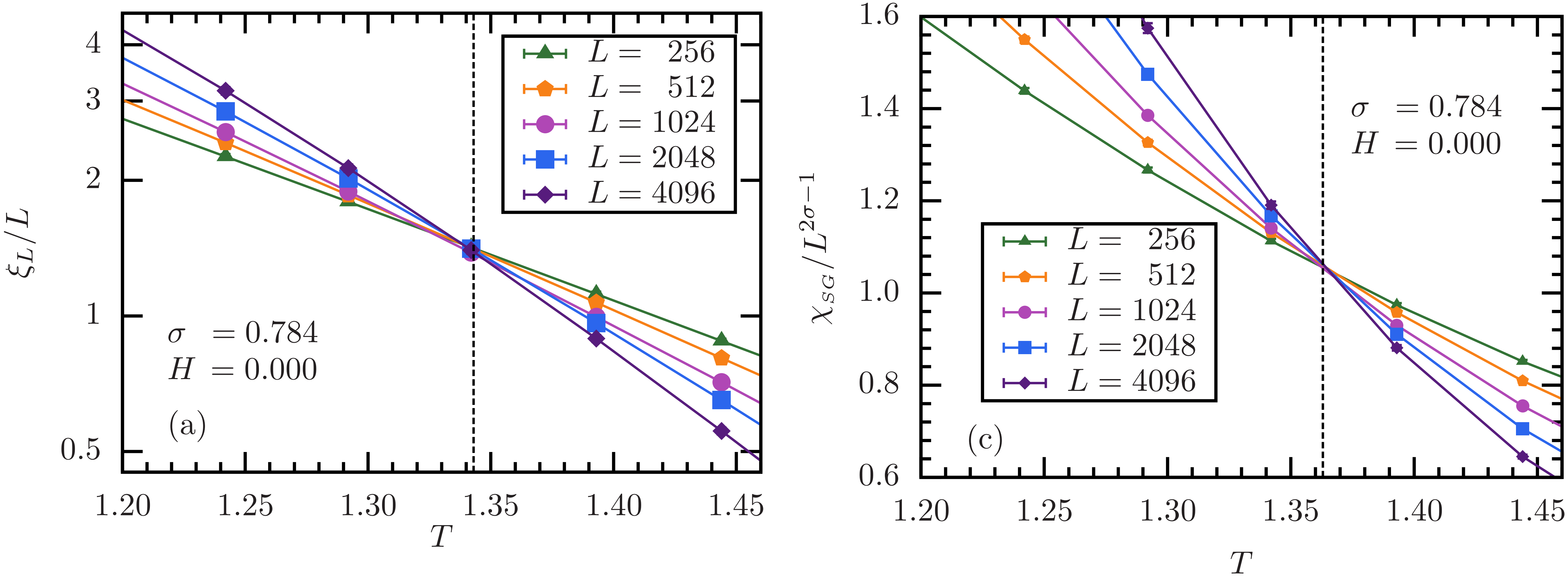
Hence data for scale-invariant quantities for different sizes intersect at T_c.

Standard FSS for σ(4) = 0.784, H=0.1



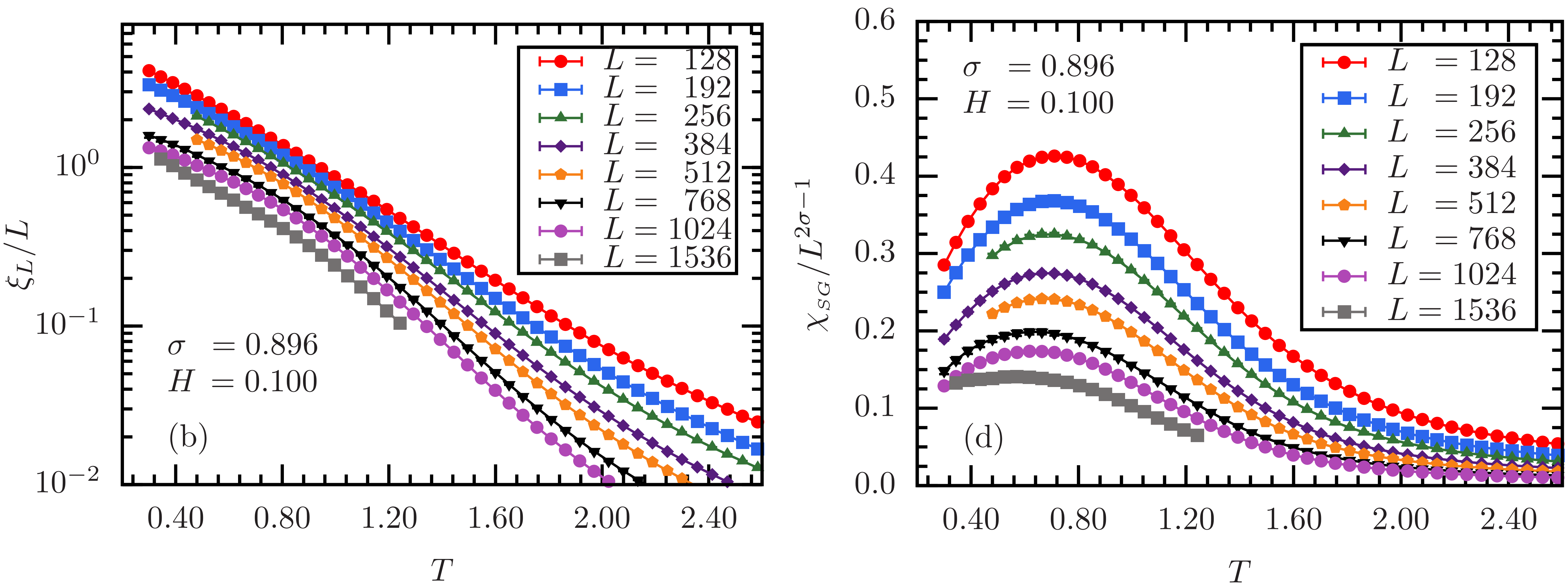
This model is a proxy for d=4.
Data is for H = 0.1.
No sign of intersections, i.e. implies no transition in a field

Standard FSS for σ(4) = 0.784, H=0



This model is a proxy for d=4.
Data here is for H = 0.
Clear intersections, implying there is a transition in zero field

Standard FSS for σ(3) = 0.896, H=0.1



This model is a proxy for d=3.

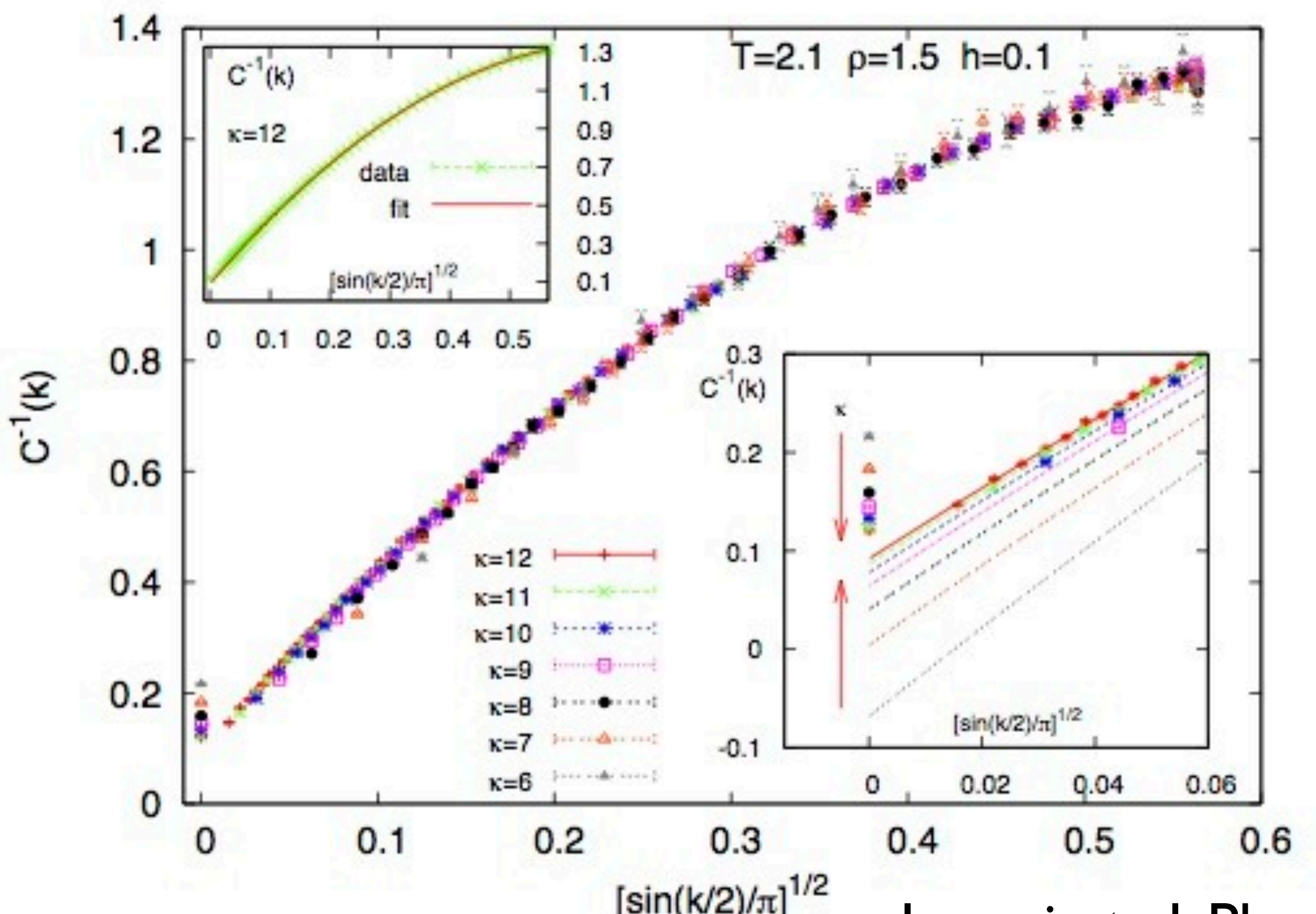
Again no sign of intersections, i.e. implies no transition in a field

“Non-Standard” FSS

Previous analysis used k=0 fluctuations. Leuzzi, Parisi, Ricci-Tersenghi, Ruiz-Lorenzo, PRL, **103**, 267201 (2009) claim, one should avoid k = 0 data because it has large corrections to FSS.

Ornstein-Zernicke form:

$$\chi_{SG}^{-1}(k) = \chi_{SG}^{-1}(0) + Ak^y + \cdots \quad \text{where} \quad \begin{cases} y = 2 & \text{short-range,} \\ y = 2\sigma - 1 & \text{long-range,} \end{cases}$$



We see that

$$\chi_{SG}^{-1}(k \to 0) \neq \chi_{SG}^{-1}(0)$$

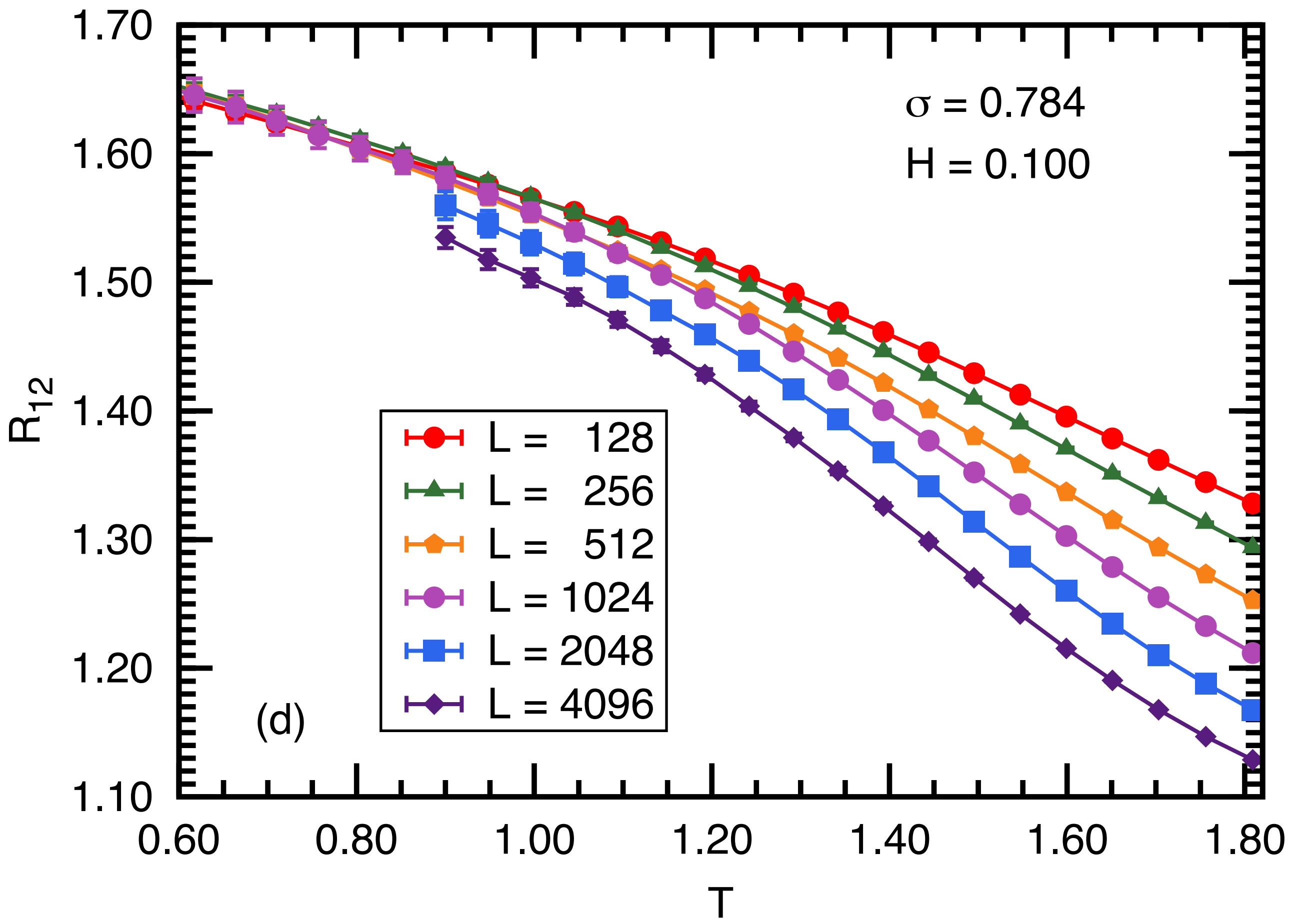
Suggestions:

- look at $\chi_{SG}^{-1}(k \to 0)$
- $R_{12} = \dfrac{\chi_{SG}(k_1)}{\chi_{SG}(k_2)}$ (Ruiz-Lorenzo)

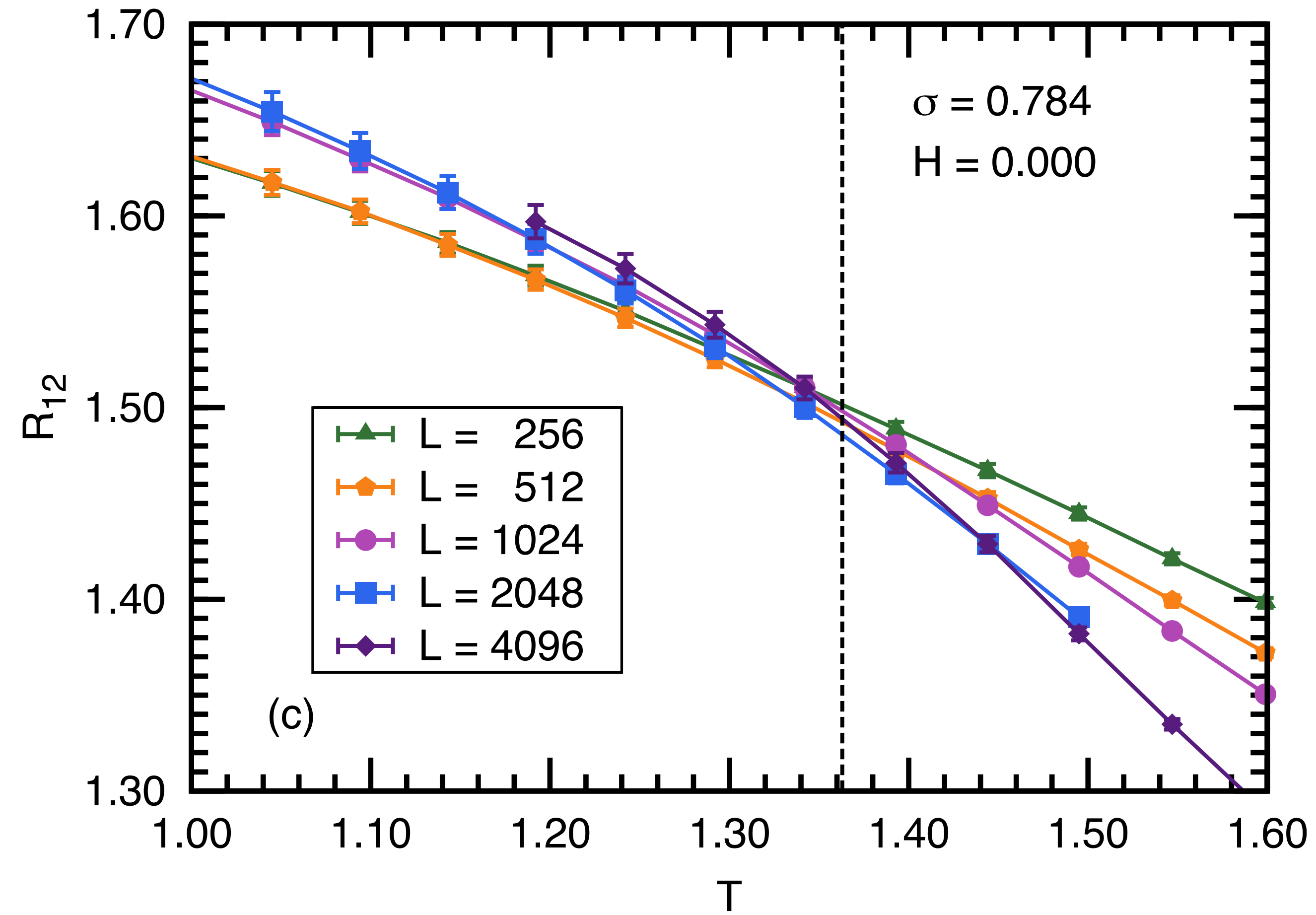
Leuzzi et al, Phys. Rev. Lett. 103, 267201 (2009)

“Non-Standard” FSS for σ(4) , R_{12}

Example of (our) data for σ(4) = 0.784:
This model is a proxy for d=4.



H = 0.1

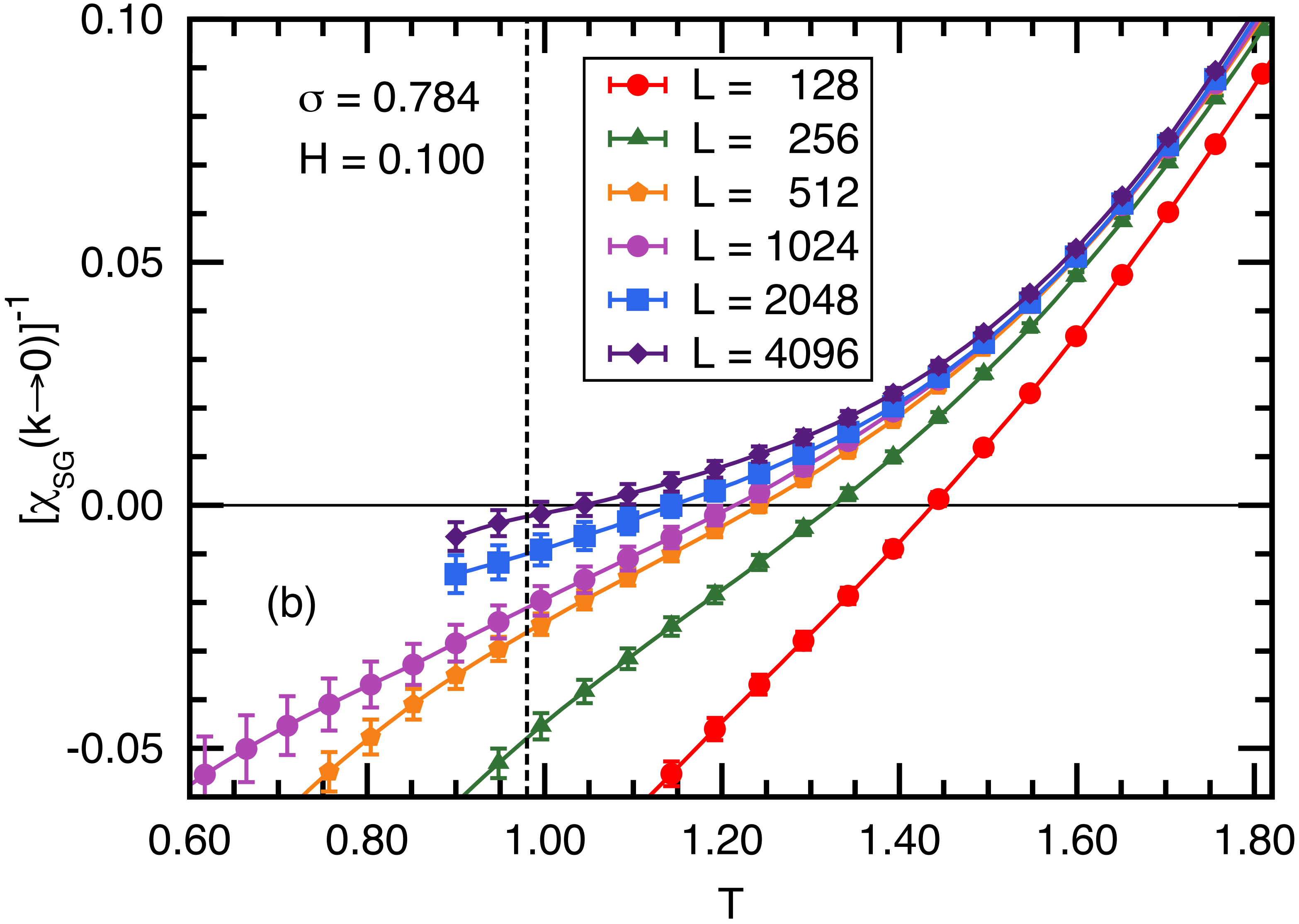


H = 0

For H = 0.1, don’t get intersections, especially for larger sizes.

"Non-Standard" FSS for σ(4), T*(L)

Example of (our) data for σ(4) = 0.784:
This model is a proxy for d=4.



$\chi_{SG}^{-1}(k \to 0)$ vanishes at

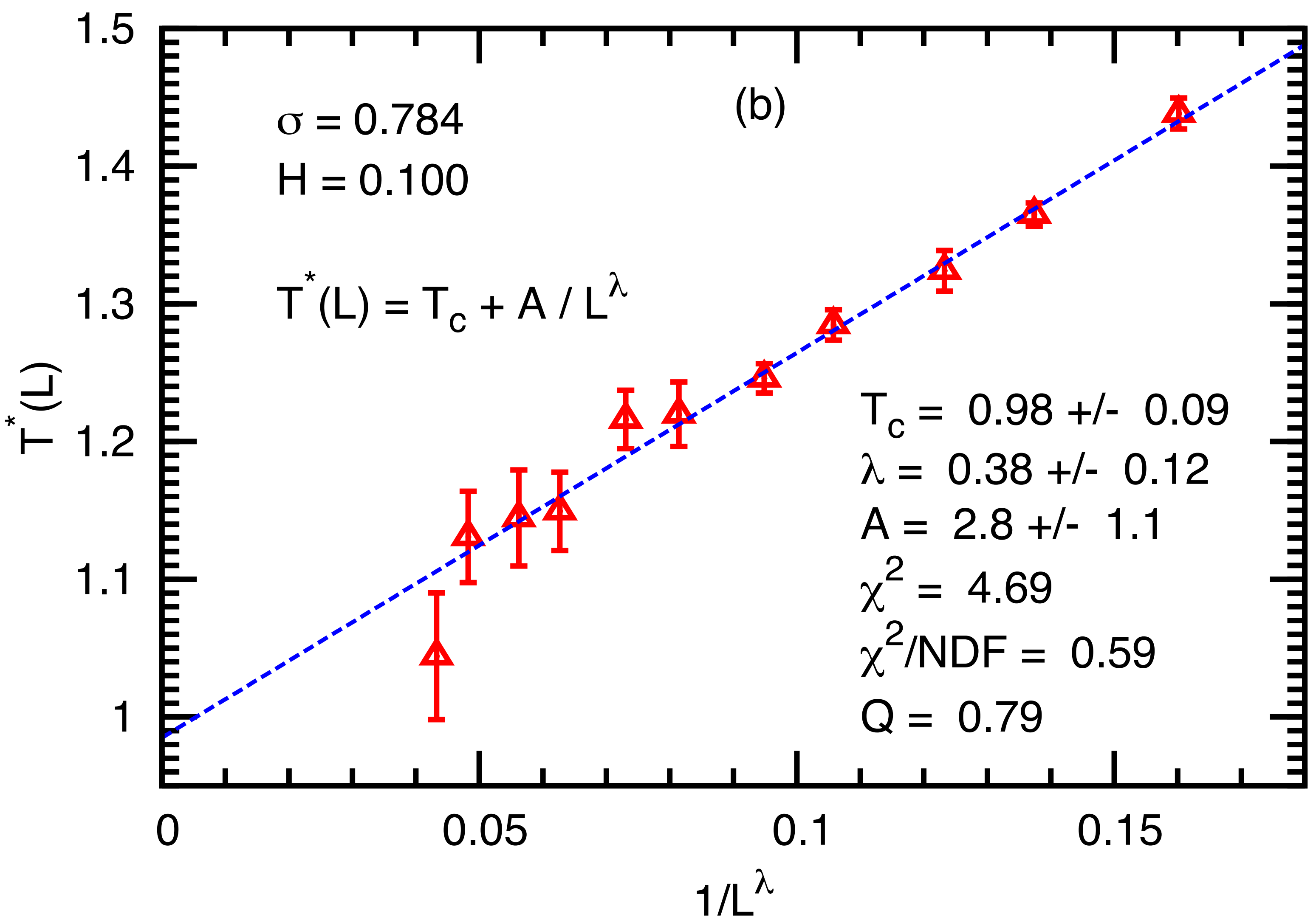
T = T*(L)

Fit T*(L) to

$$T^{\star}(L) = T_c + \frac{A}{L^{\lambda}}$$

“Non-Standard” FSS for σ(4), T*(L)

$T^{*}(L)$ for (our) data for $\sigma(4) = 0.784$:
This model is a proxy for d=4.



$$T^{\star}(L) = T_c + \frac{A}{L^{\lambda}}$$

The fit is good and gives a non-zero T_c. This is also the result of Leuzzi et al.

Summary for σ(4) = 0.784

Is there a transition in a field?

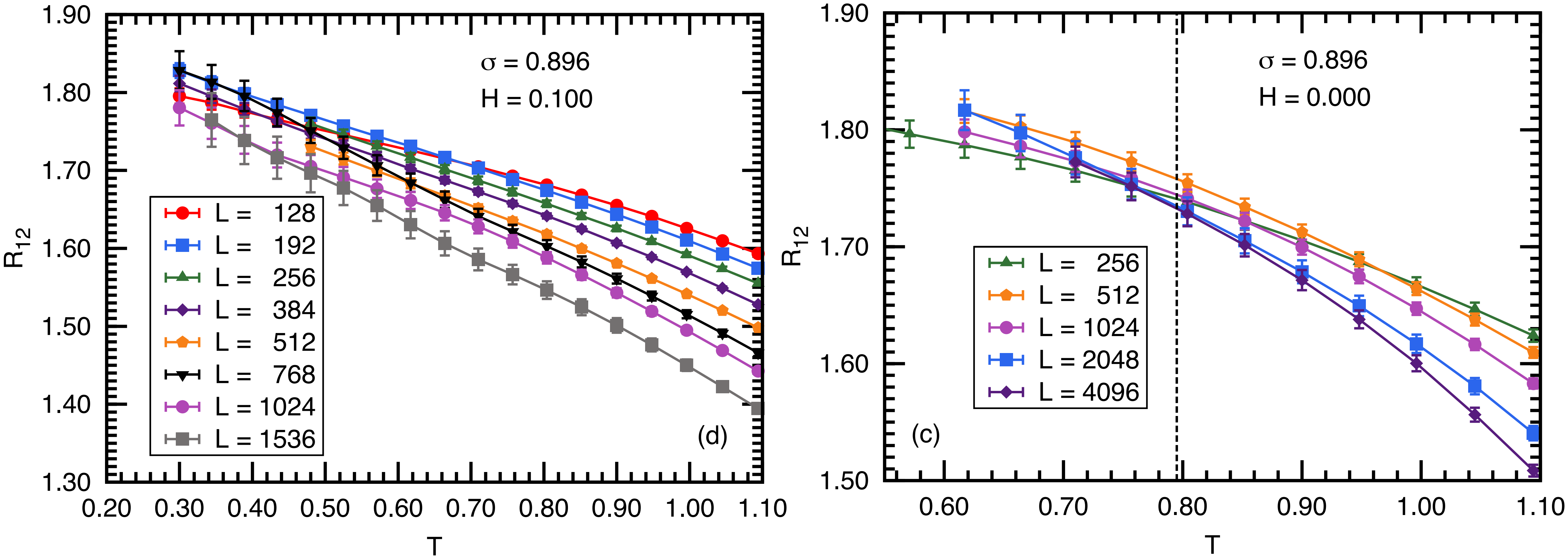
- Standard FSS for ξ_L/L, NO
- Standard FSS for $\chi_{SG}/L^{2\sigma-1}$, NO
- Non-standard FSS for R_{12} , NO
- Non-standard FSS for T*(L), YES

All methods of analysis should give the same result for N →∞.
Which method has the smallest corrections to FSS?

“Non-Standard” FSS for σ(3) , R_{12}

Example of (our) data for σ(3) = 0.896:

This model is a proxy for d=3.



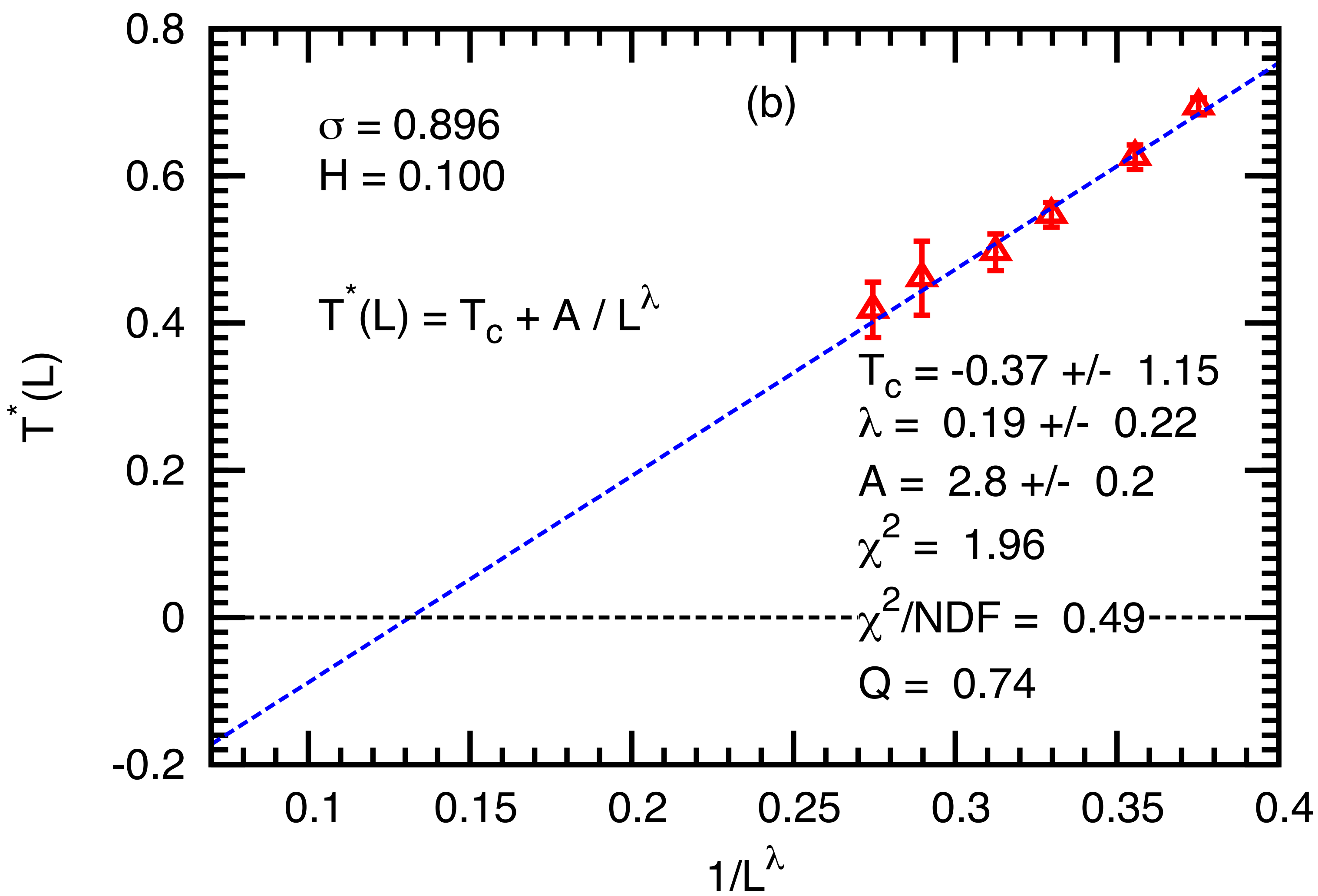
H = 0.1 H = 0

For H = 0.1, data rather ragged, but not clear evidence for a transition. For H = 0, seem to be quite big corrections to FSS.

"Non-Standard" FSS for σ(3), T*(L)

$T^*(L)$ for (our) data for σ(3) = 0.894:
This model is a proxy for d=3.

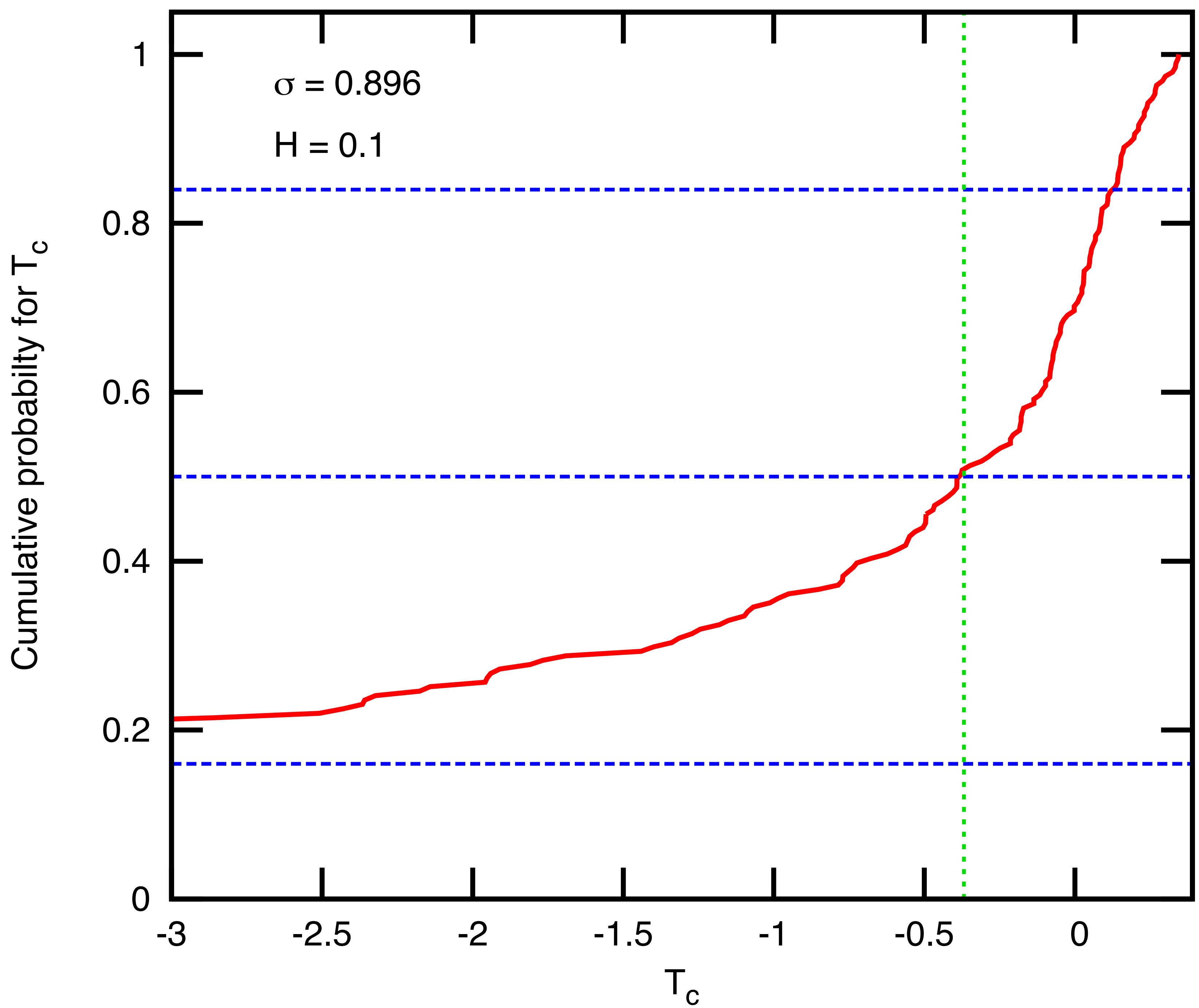
$$T^\star(L) = T_c + \frac{A}{L^\lambda}$$



Seems consistent with Tc = 0, but the given error bars are nonsensically large. This is a non-linear model with parameters rather poorly determined. In these conditions the standard error bars are wrong, so we need a better analysis. Here we use bootstrap.

“Non-Standard” FSS for σ(3), T*(L)

(Our) T_c data for σ(3) = 0.894, analyzed with bootstrap.
This model is a proxy for d=3.



Fitted each bootstrap dataset.
Confidence limit taken: cumulative probability between 16% and 84%.
Gives $T_c < 0.13$.
Only 30% of bootstraps give $T_c > 0$.
Hence, compatible with $T_c = 0$ (the “standard” FSS result).

Note: possibility of Tc → -∞, with correction exponent → 0.
Note too: not symmetric about probability 1/2 and not the integral of a Gaussian.

Summary for $\sigma(3) = 0.896$

Is there a transition in a field?

- Standard FSS for ξ_L / L, NO
- Standard FSS for $\chi_{SG} / L^{2\sigma - 1}$, NO
- Non-standard FSS for R_{12}, ? but NO convincing evidence
- Non-standard FSS for $T^*(L)$, ? but NO convincing evidence

Conclusions

Conclusions

- For σ(4), (proxy for 4d) there is an apparent contradiction:
  3 out of 4 analyses indicate no transition
  1out of 4 analyses indicates a transition
  Which is correct?

Conclusions

- For $\sigma(4)$, (proxy for 4d) there is an apparent contradiction:
  3 out of 4 analyses indicate no transition
  1out of 4 analyses indicates a transition
  Which is correct?

- For $\sigma(3)$ (proxy for 3d):
  "standard" FSS gives no AT line
  "nonstandard" FSS data has a large scatter but is compatible with no AT line.
  Hence probably no AT line, and therefore probably no ideal glass transition.
  But Ruiz Lorenzo's talk gave a different point of view.

Conclusions

- For $\sigma(4)$, (proxy for 4d) there is an apparent contradiction:
  3 out of 4 analyses indicate no transition
  1out of 4 analyses indicates a transition
  Which is correct?

- For $\sigma(3)$ (proxy for 3d):
  "standard" FSS gives no AT line
  "nonstandard" FSS data has a large scatter but is compatible with no AT line.
  Hence probably no AT line, and therefore probably no ideal glass transition.
  But Ruiz Lorenzo's talk gave a different point of view.

Grazie tante