~~Diluted one-dimensional spin glasses with long-range interactions undergo a phase transition in presence of an external magnetic field~~

Spin glasses in a field

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in honor of E. Brezin, G. Jona-Lasinio and G. Parisi

Giorgio, me and the computers

- 20 years of intense interactions
- Mainly mentoring, thanks Giorgio!
- Computer programming is not writing a code...
  - optimization
  - search for new models and tools of analysis
  - always with a clear physical picture in mind
- Uncovering new physics with a computer is an "art"...
  ...and Giorgio is an artist ;-)
- Building most powerful computers:
  APE100, APEmille, Janus and Janus II
  - essential for studying non-perturbative effects

Spin glasses in a field

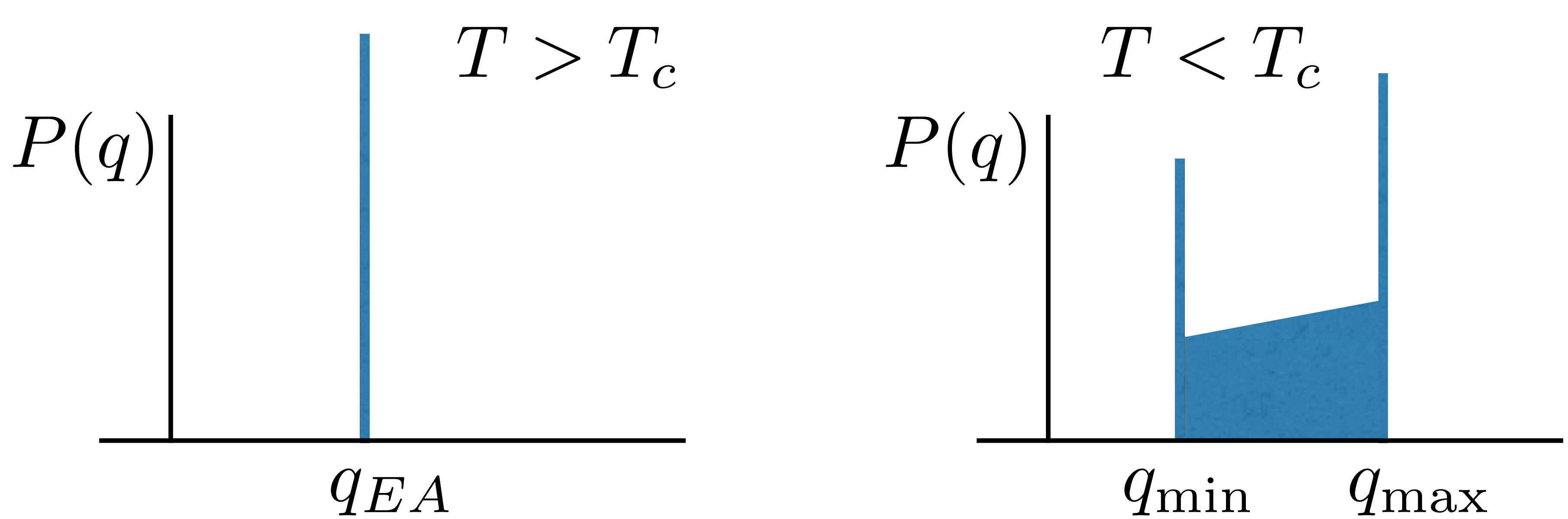
- N Ising variables $s_i = \pm 1$

$$\mathcal{H} = -\sum_{(ij)\in E} J_{ij} s_i s_j - \sum_i h_i s_i$$

- Interaction network/graph: edge set E
- Random couplings $J_{ij} = \pm 1$
- External field:
  - uniform $h_i = h$
  - Gaussian $h_i \sim N(0, h^2)$
- Model justification (not really needed in this context…)
  - many applications, mainly inference problems

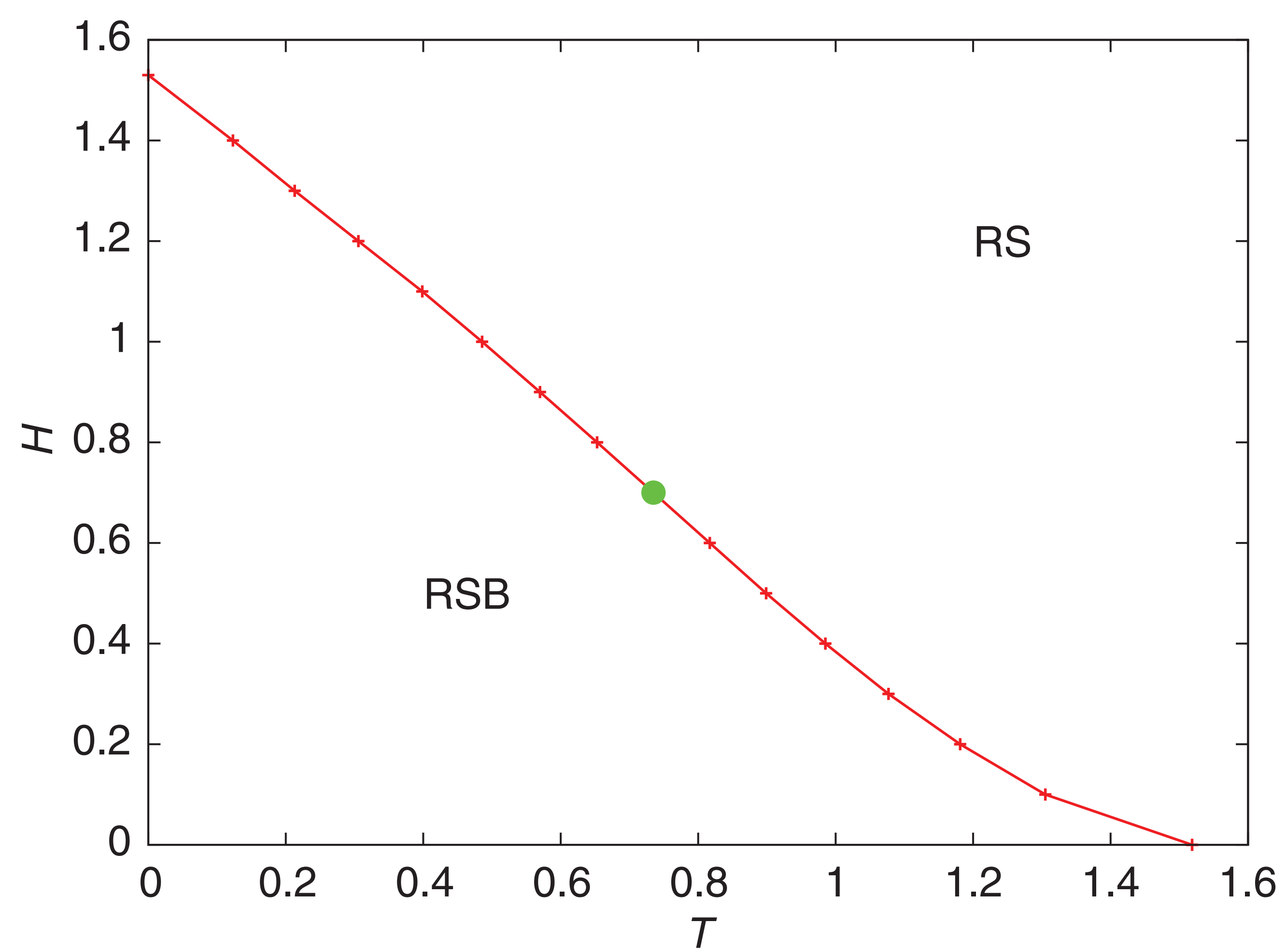
Fully connected, SK model

- E is the complete graph
- rescale J by $1/\sqrt{N}$
- Parisi solution: full replica symmetry breaking (FRSB)
- Overlap between 2 configurations <u>s</u> and <u>t</u> is $q \equiv N^{-1} \sum_i s_i t_i$
- many "states" and broad P(q) below <u>dAT line</u> Tc(h)



Sparse random graphs

- Erdos-Renyi (ER), random regular graph (RRG)
- finite degree -> local fluctuations
  (more similar to low-dimensional lattices)
- mean-field approx. not valid
  -> Bethe-Peierls approx.
  (Mézard & Parisi, 2001)
- analytically known:
  - RS & 1RSB solutions
  - critical properties
  - non-diverging dAT line
    $h_c(T=0) < \infty$
- FRSB is a challenging...

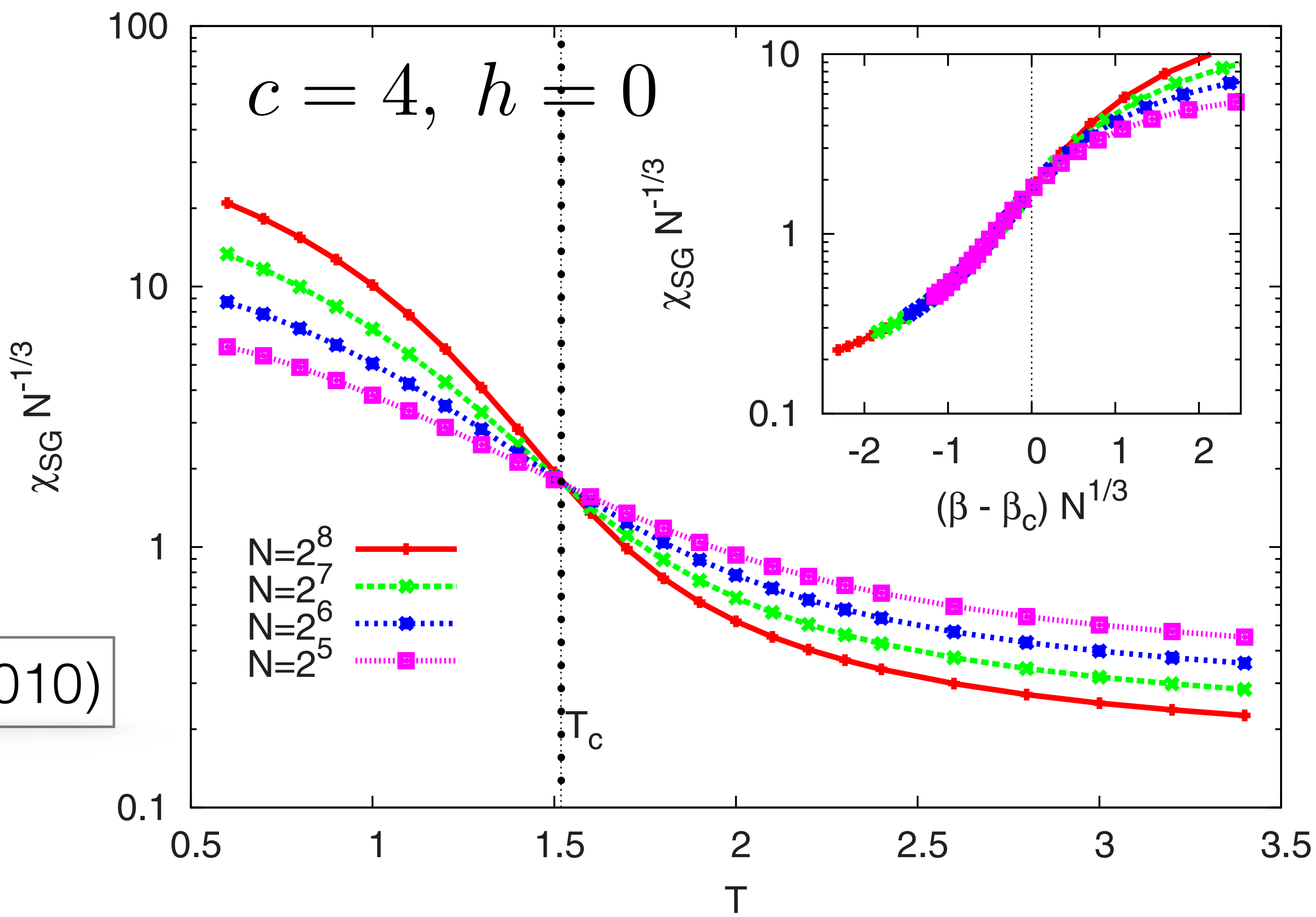


Spin glasses on RRG

- Ideal for comparing analytics and numerics
  - Monte Carlo running time O(N)
  - estimate of the critical temperature via the crossing of the scaled susceptibility $N^{-\frac{1}{3}}\chi_{\mathrm{SG}}$

$$\chi_{\mathrm{SG}} = \int dr\, G(r)$$

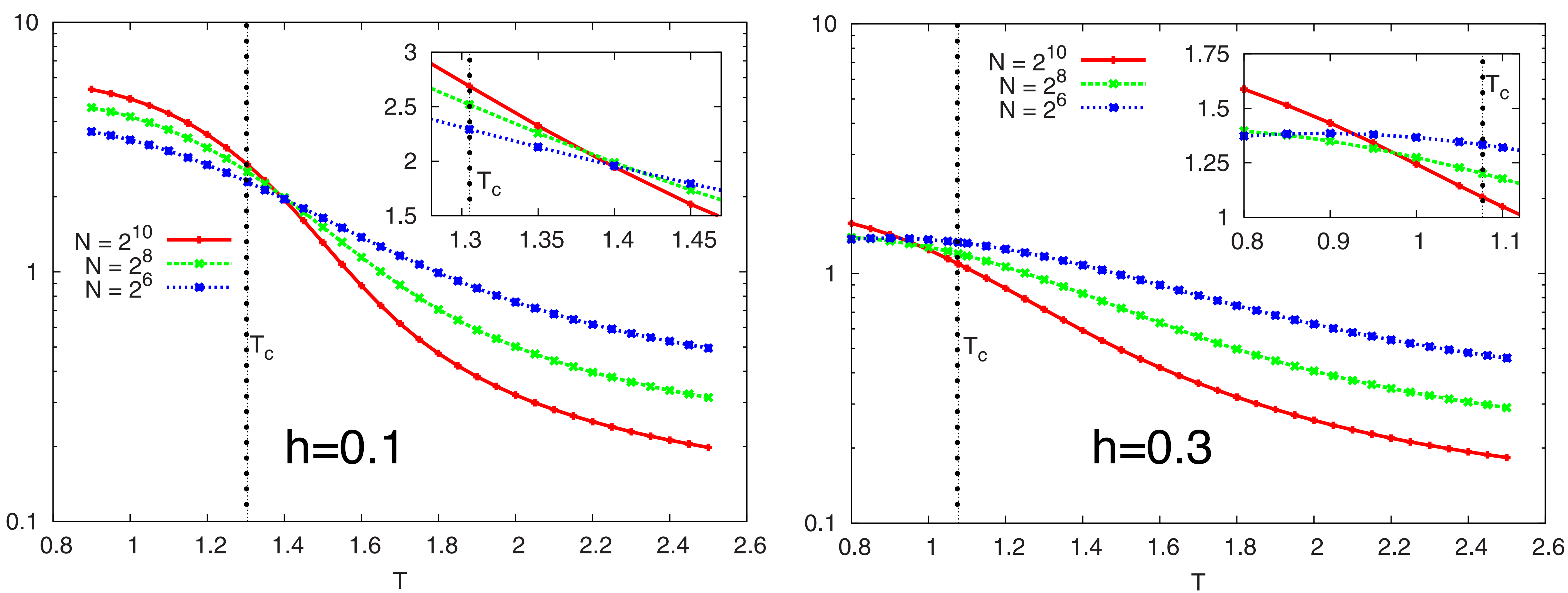
$$G(r) = \overline{\langle s_0 s_r \rangle_c^2}$$



Takahashi, FRT, Kabashima, PRB (2010)

Spin glasses in field on RRG

- Strong finite size corrections
  - crossing points of $N^{-\frac{1}{3}}\chi_{\rm SG}$ are far from analytical predictions



Takahashi, FRT, Kabashima, PRB (2010)

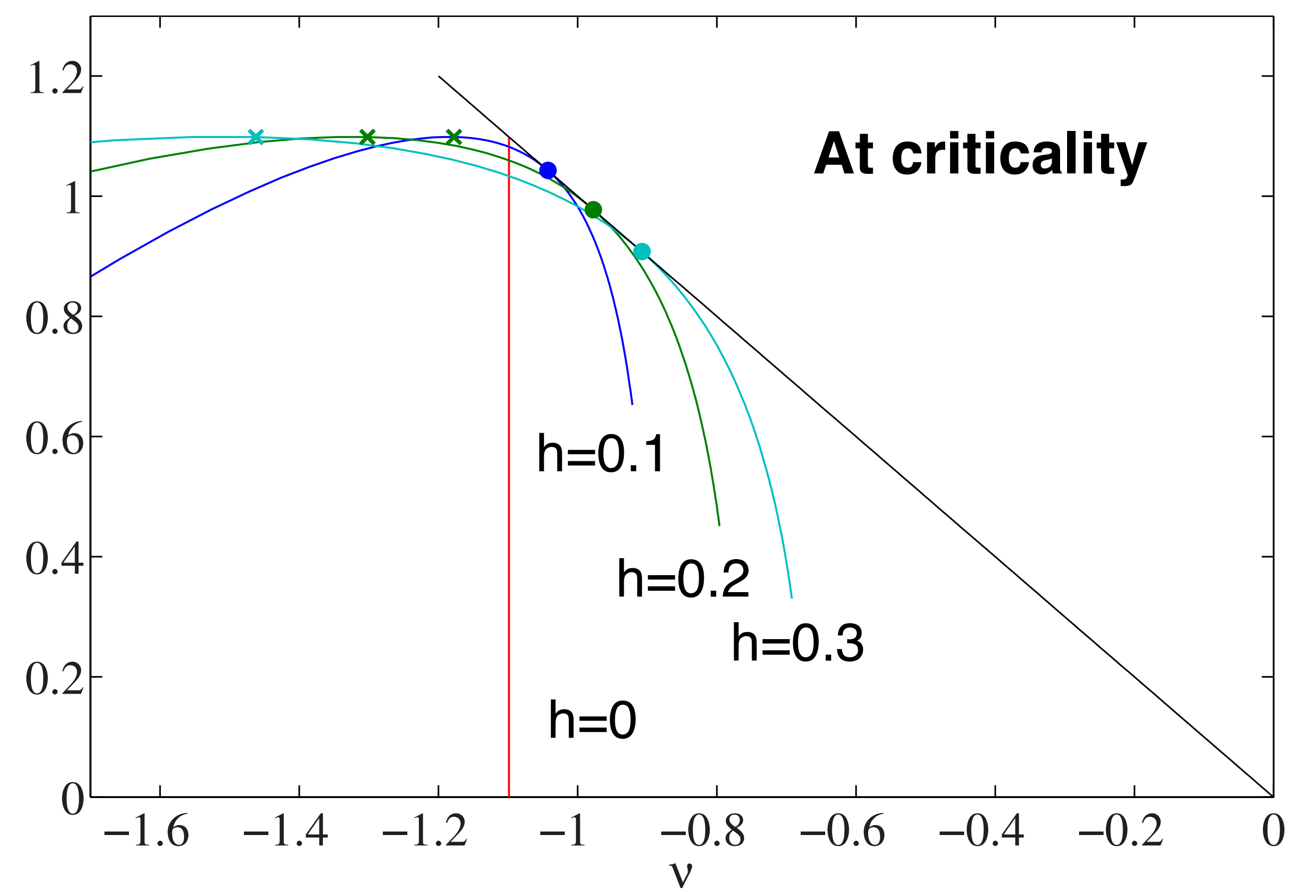
Spin glasses in field on RRG

- Strong finite size corrections are related to local heterogeneities

$$h = 0 \implies G(r) = \tanh(\beta)^r$$

$$h > 0 \implies G(r) \sim \exp(\nu\, r)$$

ν has a broad probability distribution for h > 0

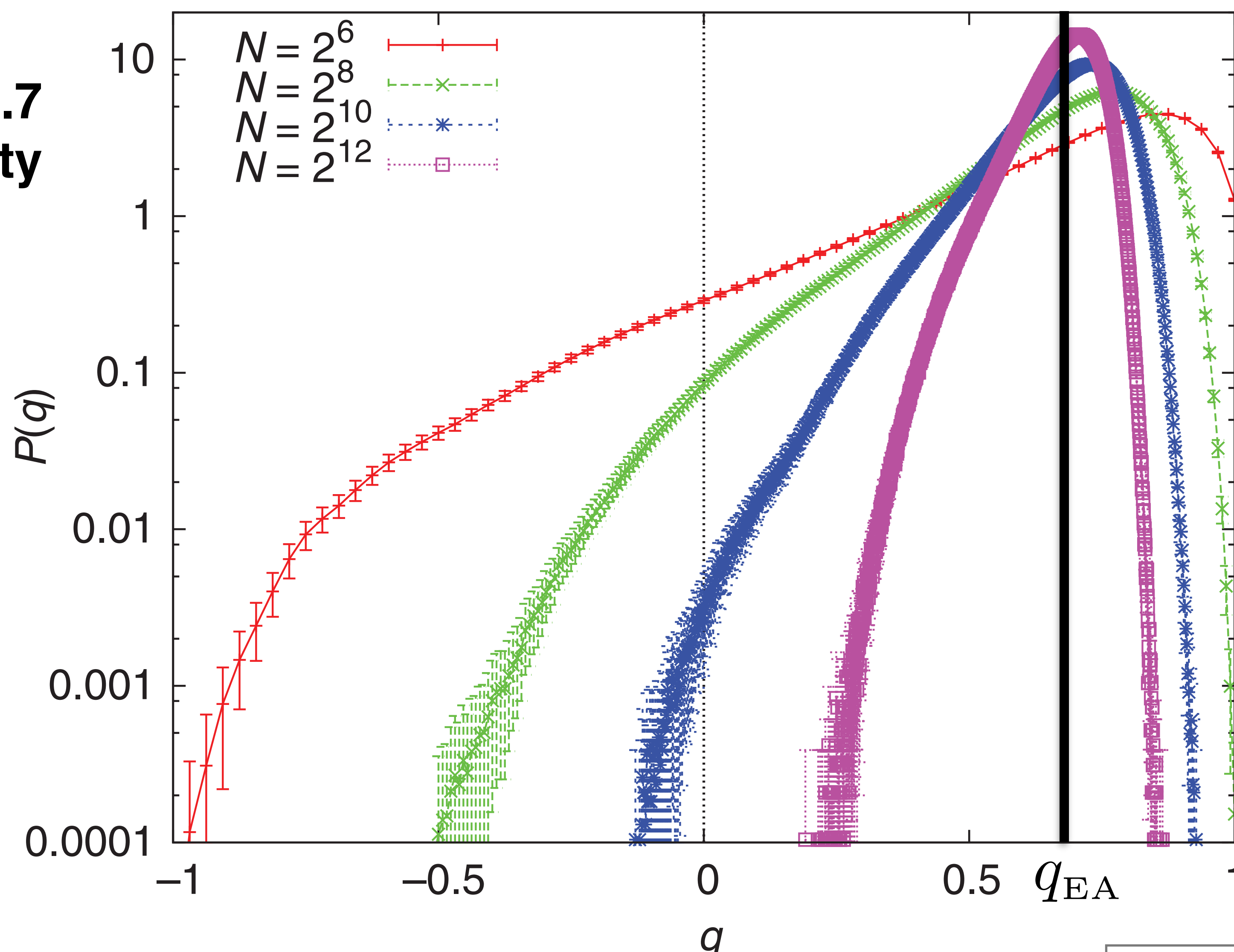


Takahashi, FRT, Kabashima, PRB (2010)

Spin glasses in field on RRG

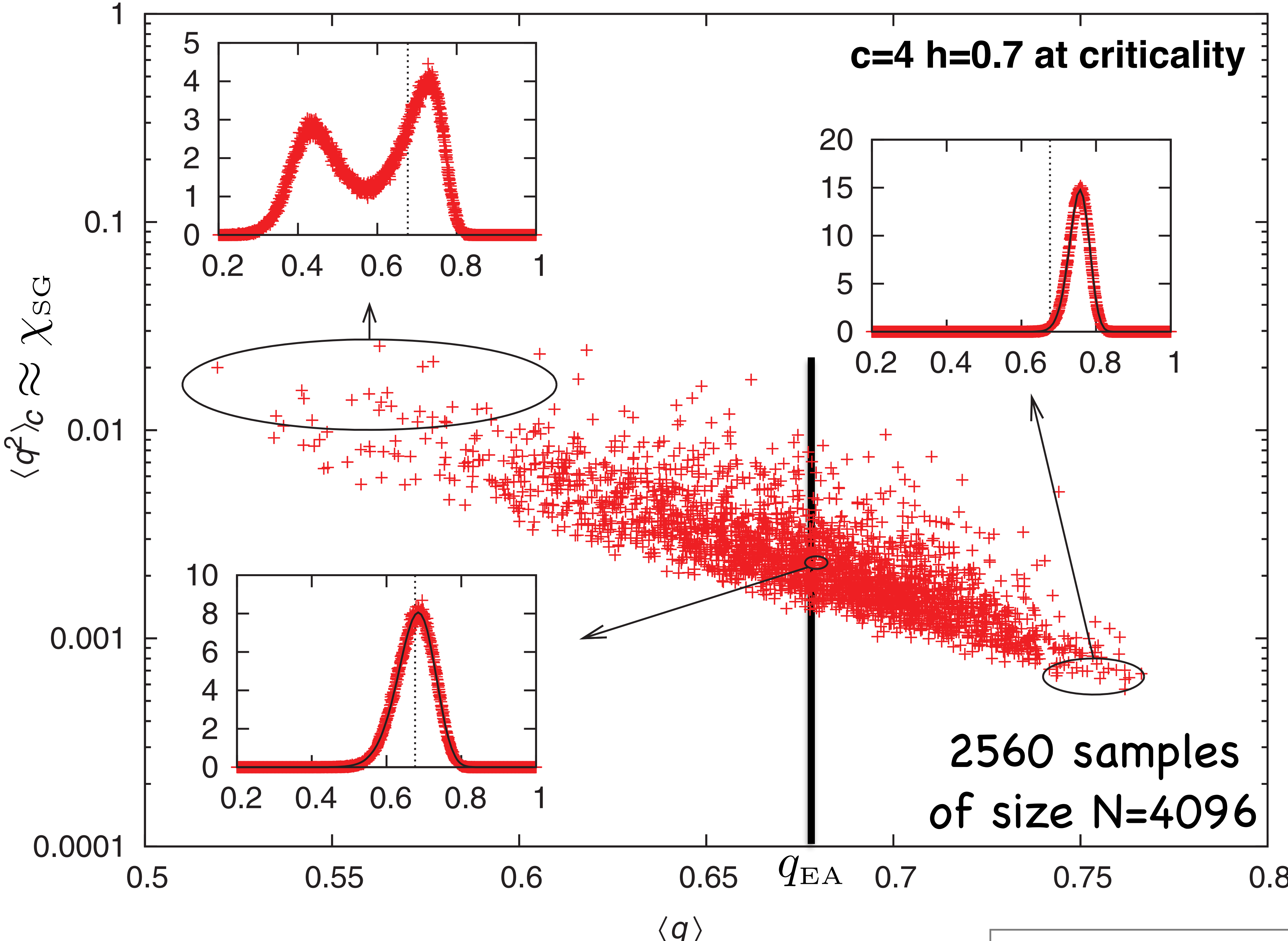
- Strong finite size corrections also in global quantities

c=4 h=0.7
at criticality



Parisi, FRT, PhilMag (2012)

Spin glasses in field on RRG

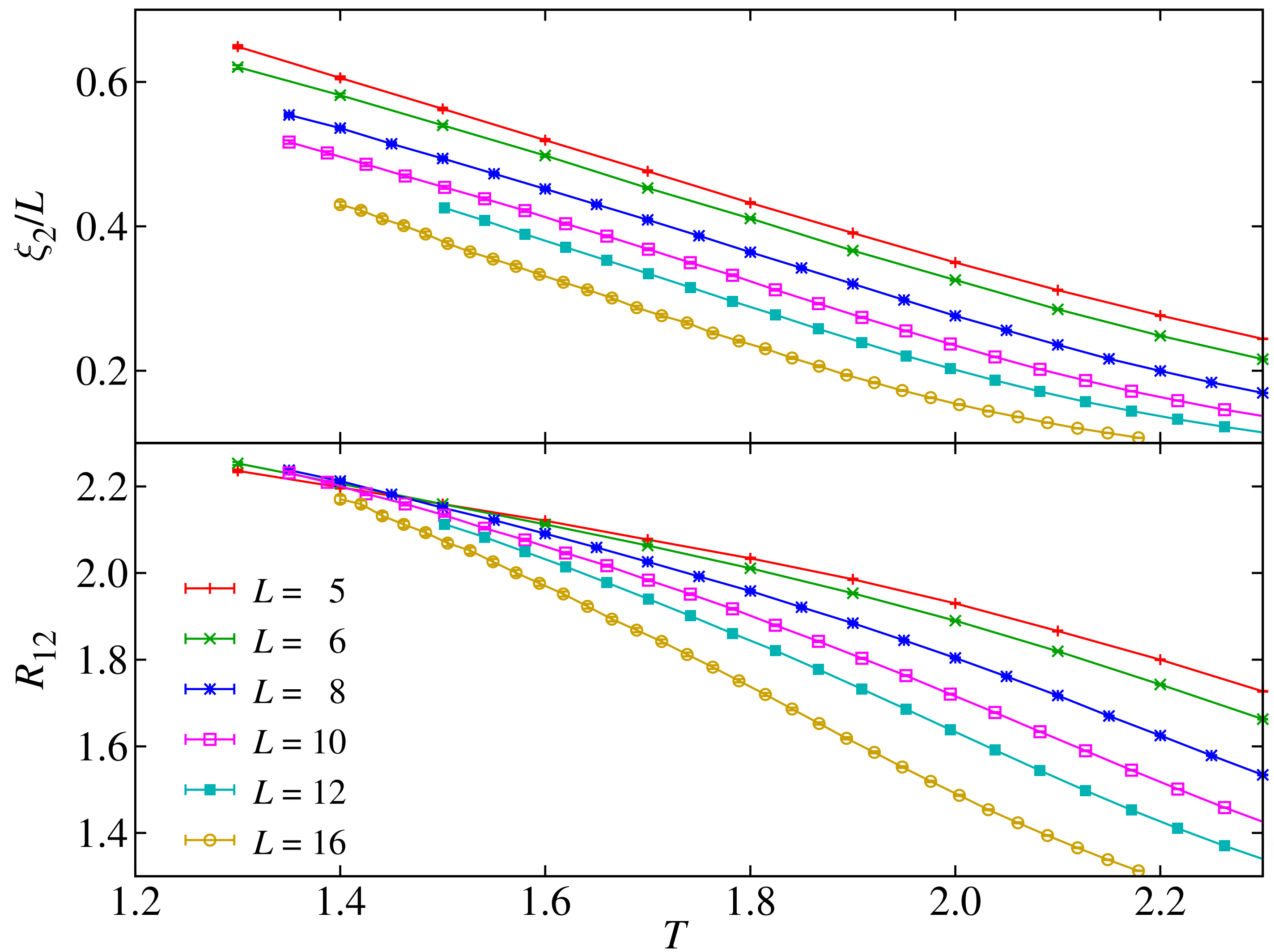


Parisi, FRT, PhilMag (2012)

D=4 spin glasses in uniform field

- $$\xi_2 = \frac{1}{2\sin(\pi/L)}\left(\frac{\hat{G}(0)}{\hat{G}(\boldsymbol{k}_1)} - 1\right)^{1/2}$$

$$R_{12} = \frac{\hat{G}(\boldsymbol{k}_1)}{\hat{G}(\boldsymbol{k}_2)} \qquad \boldsymbol{k}_1 = (2\pi/L, 0, 0, 0), \boldsymbol{k}_2 = (2\pi/L, 2\pi/L, 0, 0)$$



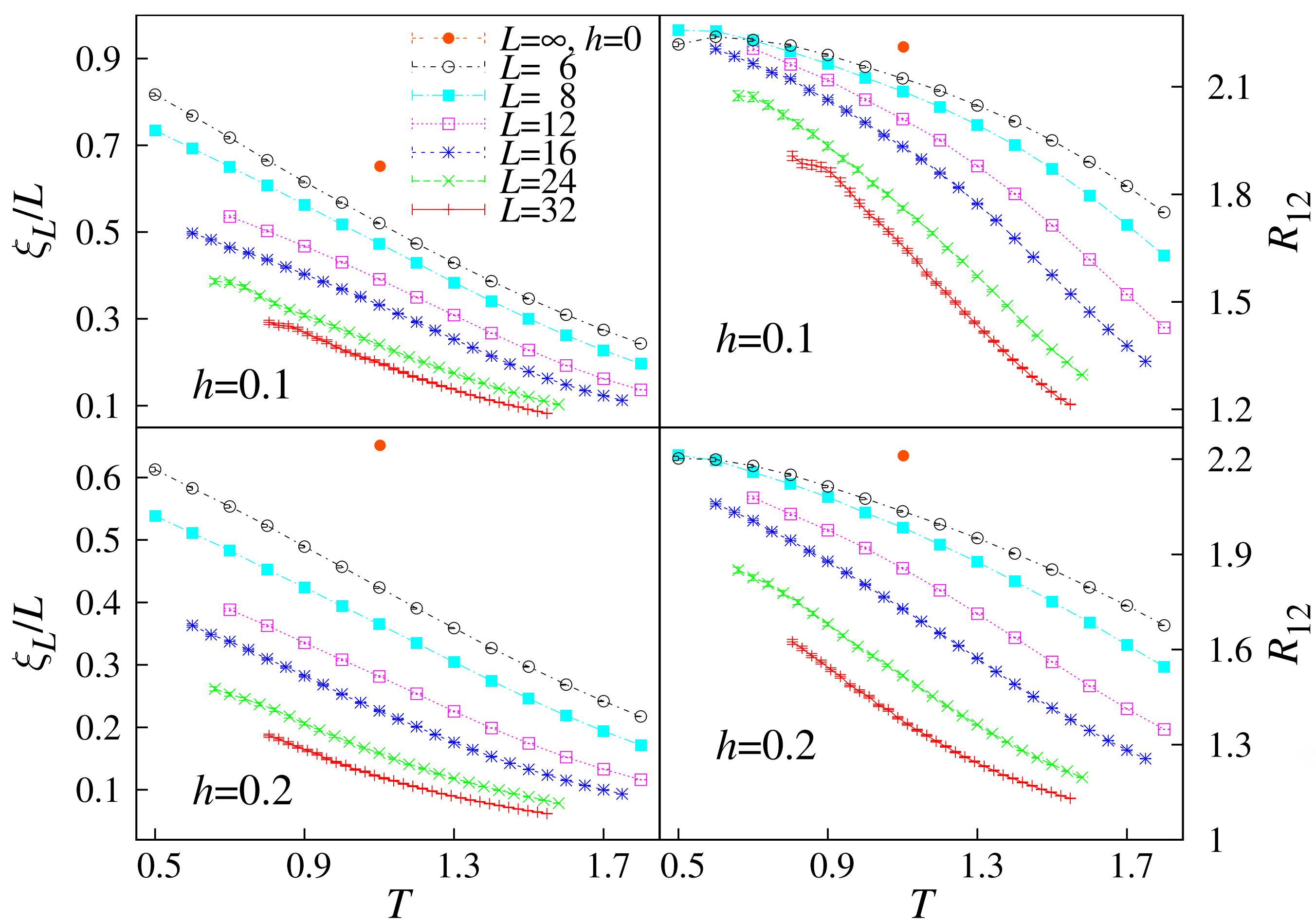
$h = 0.15$

$T_c(h = 0) = 2.002(10)$

Janus collaboration, PNAS (2012)

D=3 spin glasses in uniform field

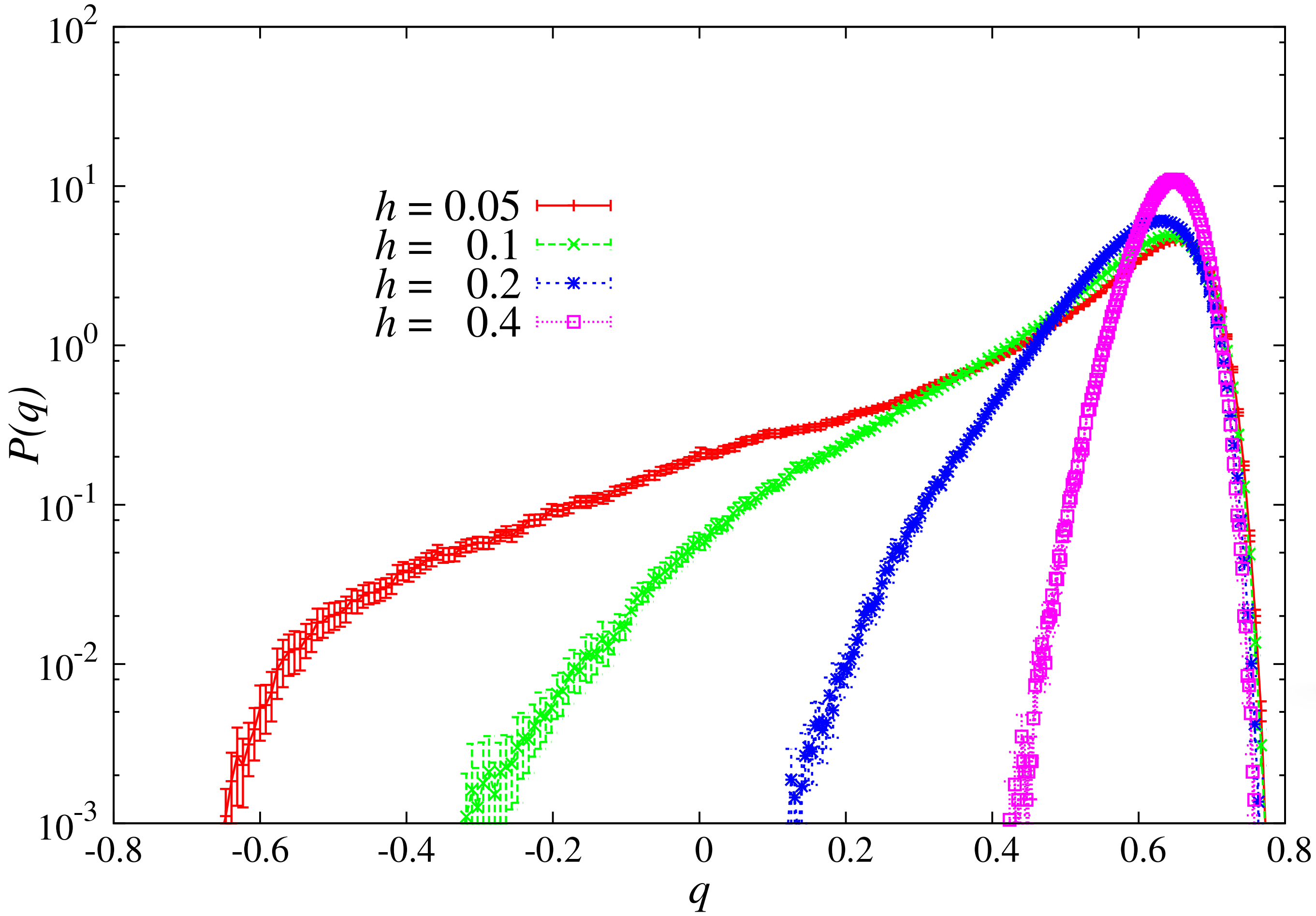
- Data from Janus supercomputer:
  - best data currently available
  - approaching the experimental timescales...
  - ...and still strong finite size corrections!
- Standard methods of analysis -> no phase transition in field



Janus collaboration, JSTAT (2014)

D=3 spin glasses in uniform field

- The largest size L=32 at the lowest temperature T=0.805128 still shows strong fluctuations (as in RRG)

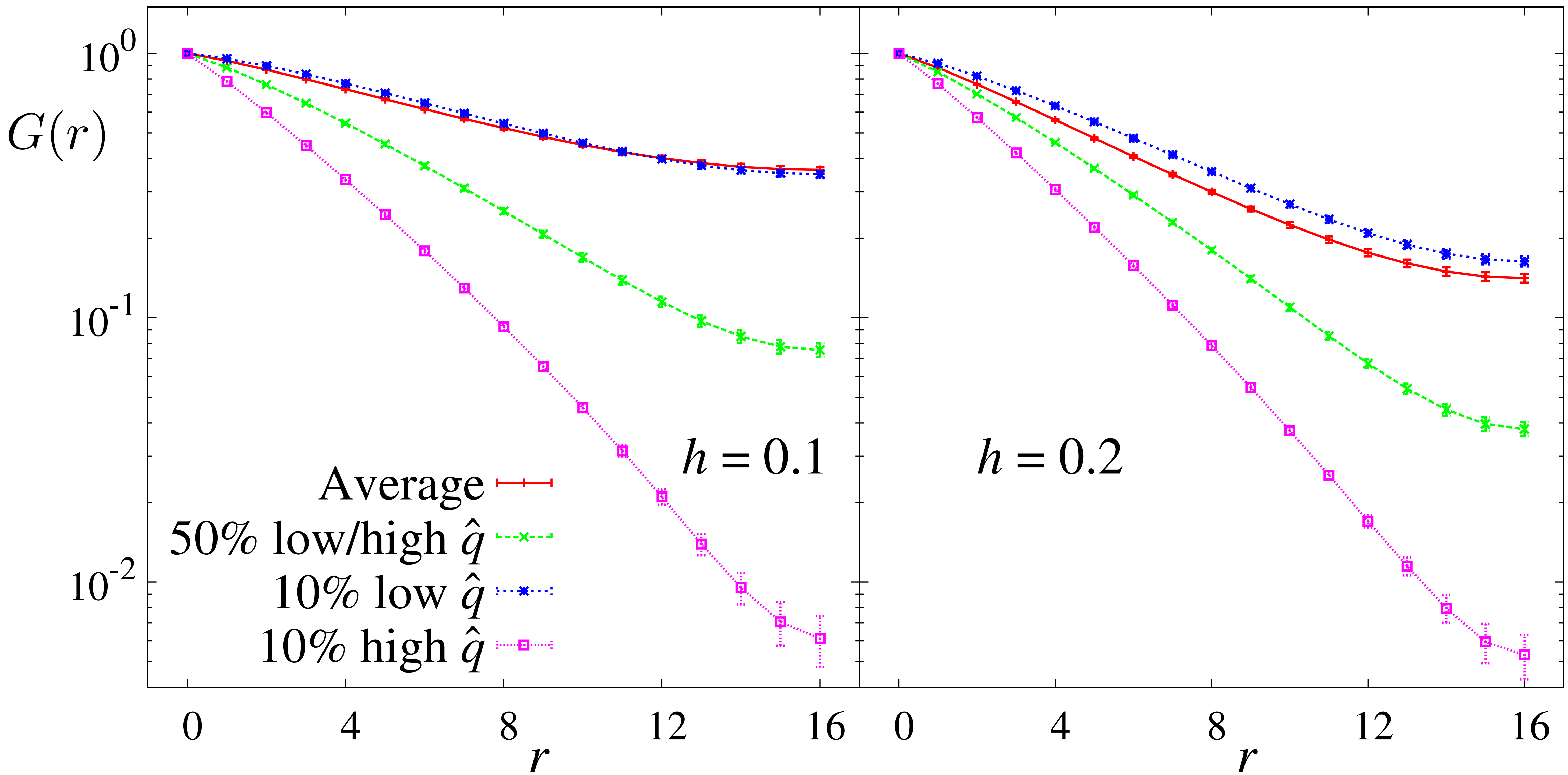


Janus collaboration, JSTAT (2014)

- P(q) should be a delta function in the paramagnetic phase!

D=3 spin glasses in uniform field

- Conditioning on the value of the overlap we get surprises!



Janus collaboration, JSTAT (2014)

D=3 spin glasses in uniform field

- All measurements from different samples together $\{(O_i, \hat{q}_i)\}$
- Conditional expectation

conditioning variate

$$\mathbb{E}[O|q] = \frac{\mathbb{E}[O\ \delta_{\hat{q},q}]}{\mathbb{E}[\delta_{\hat{q},q}]} \qquad \mathbb{E}[O] = \int dP(\hat{q})\, \mathbb{E}[O|\hat{q}]$$

$$\mathbb{V}[O] = \int dP(\hat{q})\, \mathbb{V}[O|\hat{q}] + \int dP(\hat{q})\, \big(\mathbb{E}[O|\hat{q}] - \mathbb{E}[O]\big)^2$$

Janus collaboration, JSTAT (2014)

D=3 spin glasses in uniform field

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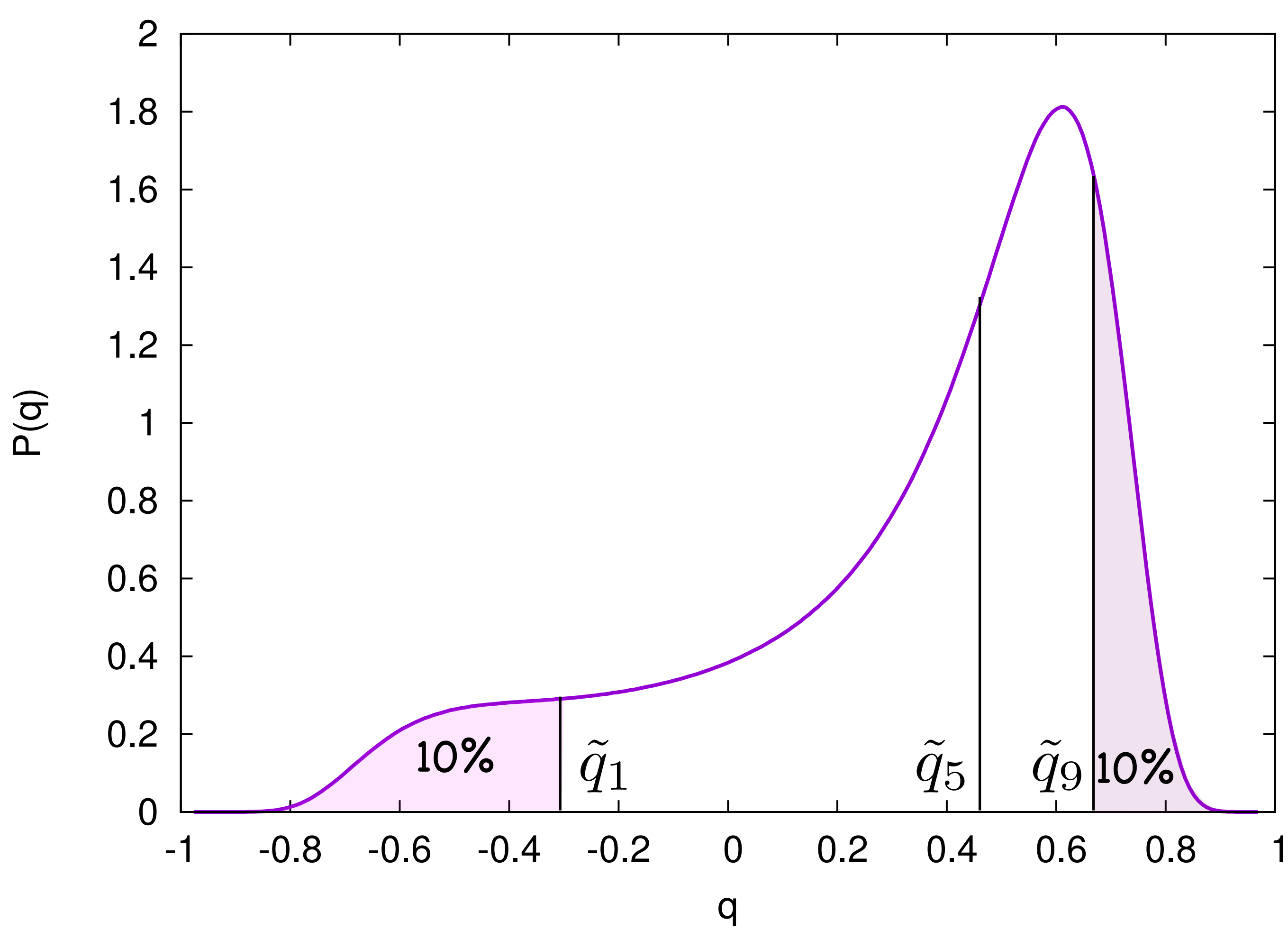
$$\mathbb{V}[O] = \underbrace{\int dP(\hat{q})\, \mathbb{V}[O|\hat{q}]}_{\text{as small as possible (and hopefully self-averaging)}} + \underbrace{\int dP(\hat{q})\, \left(\mathbb{E}[O|\hat{q}] - \mathbb{E}[O]\right)^2}_{\text{as large as possible}}$$

Janus collaboration, JSTAT (2014)

D=3 spin glasses in uniform field

- Use of deciles:
  - pros -> constant statistics
  - cons -> values of decile separators $\tilde{q}$ change with size



Janus collaboration, JSTAT (2014)

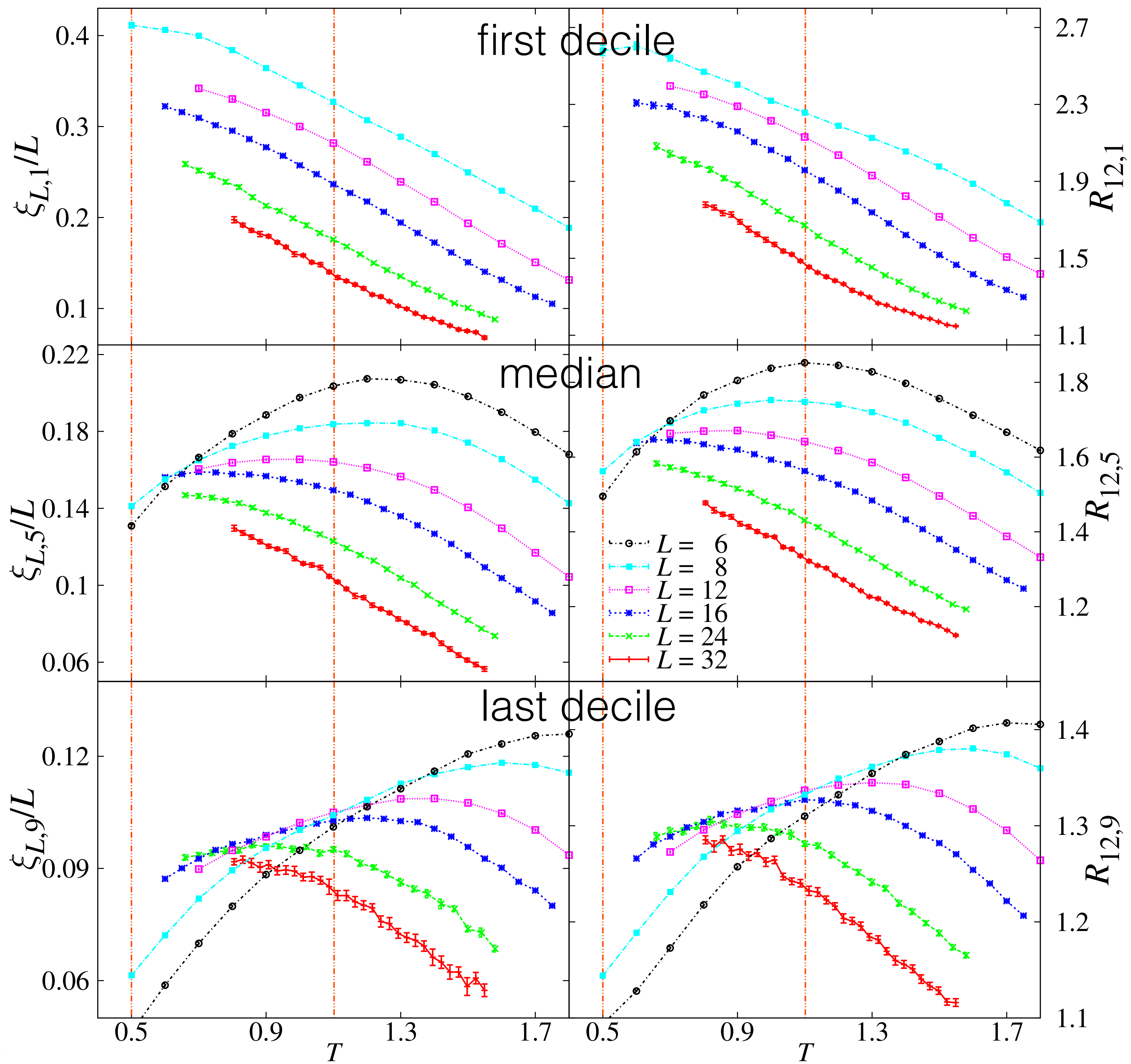
D=3 spin glasses in uniform field

- For T>Tc

$$\lim_{L\to\infty} \tilde{q}_i = q_{\mathrm{EA}}$$

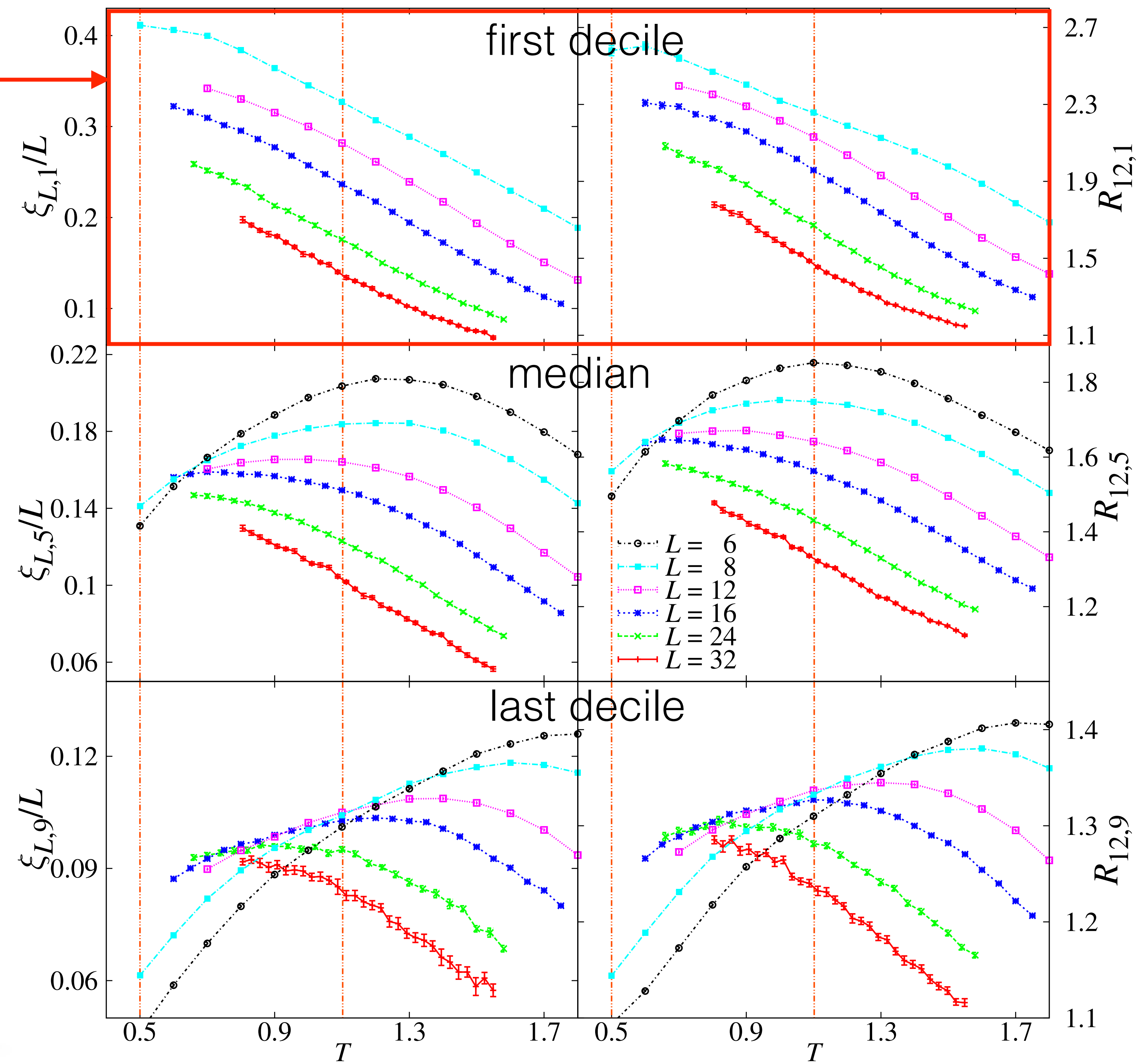
however numerical data sensibly depends on the decile!

Janus collaboration, JSTAT (2014)



D=3 spin glasses in uniform field

this minority dominates the average and hides the phase transition!



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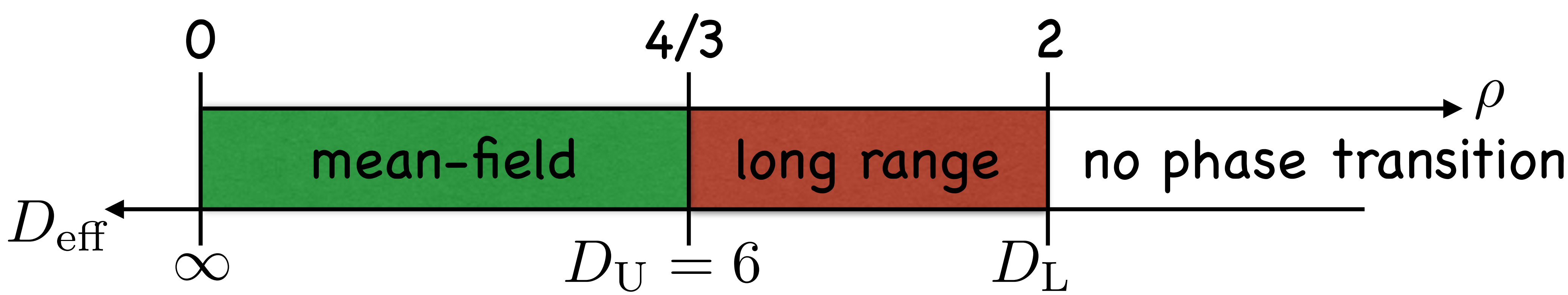
Janus collaboration, JSTAT (2014)

Long range interacting D=1 spin glasses

- Original proposal (Kotliar, 1983): $J_{ij} \sim N(0, |i-j|^{-\rho})$
  - computationally expensive, running times $O(N^2)$
- Our proposal (Leuzzi, Parisi, FRT, Ruiz-Lorenzo, PRL, 2008)
  - $J_{ij} = \pm 1$ and $\mathbb{P}[J_{ij} \neq 0] \propto |i-j|^{-\rho}$
  - computationally efficient, running times O(N)
  - mean-field limit ($\rho \to 0$) recovers SG on random graphs



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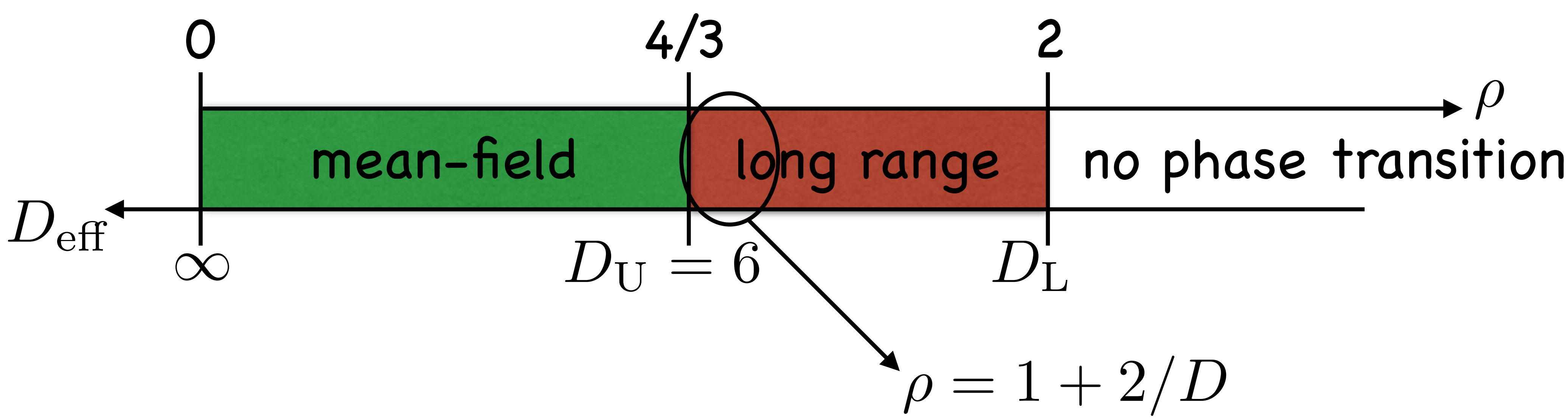


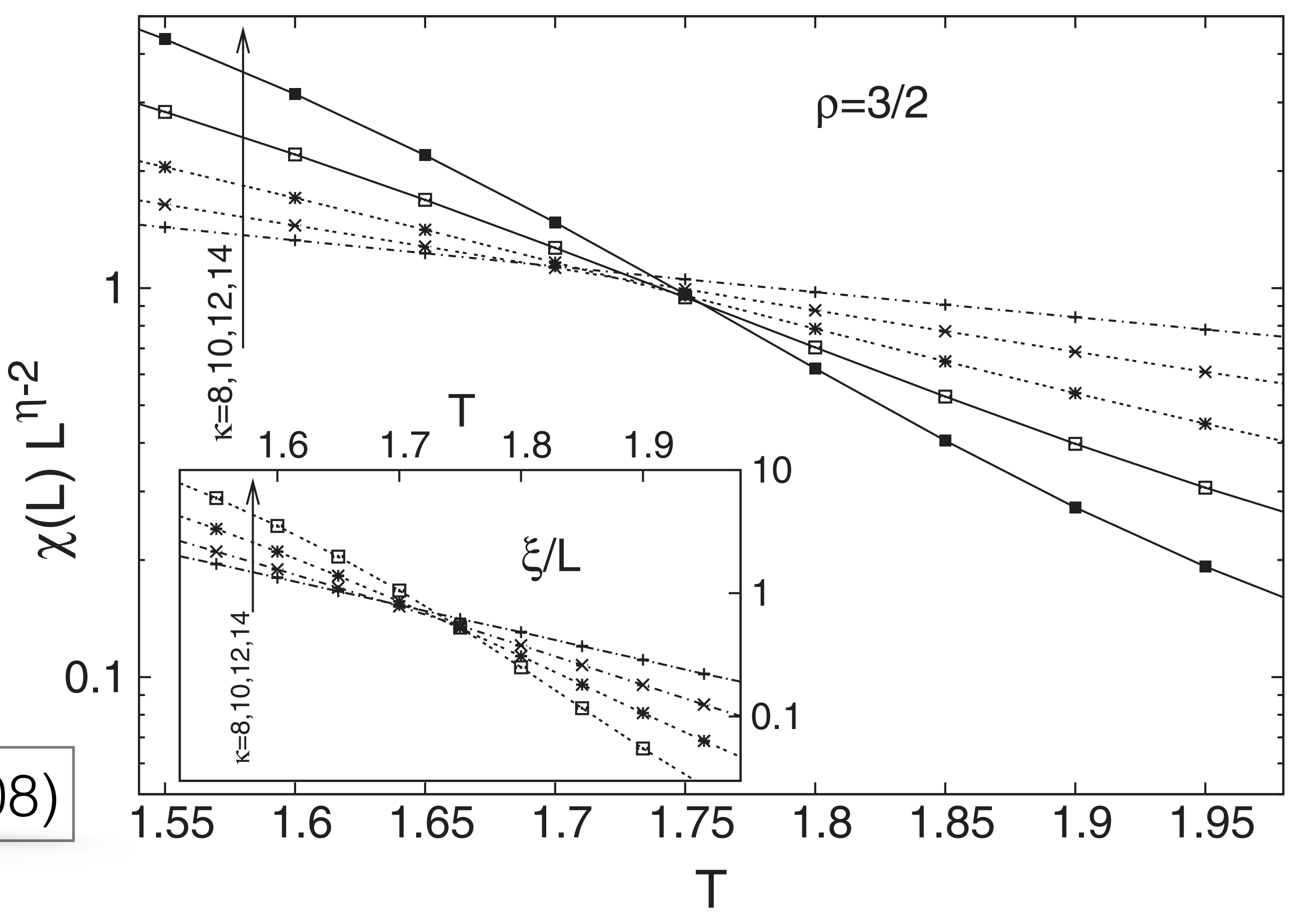
Long range interacting D=1 spin glasses

- Many pros:
  - very large lattice sizes, up to L = O(10^4)
  - we can increase the effective dimension at no cost (we are interested in an eventual phase transition for D<6)
  - $\eta = 3 - \rho$ and does not renormalize outside mean-field
  - two scaling-invariant observables

    $\chi/L^{2-\eta}$

    ξ/L

    that cross at the critical point

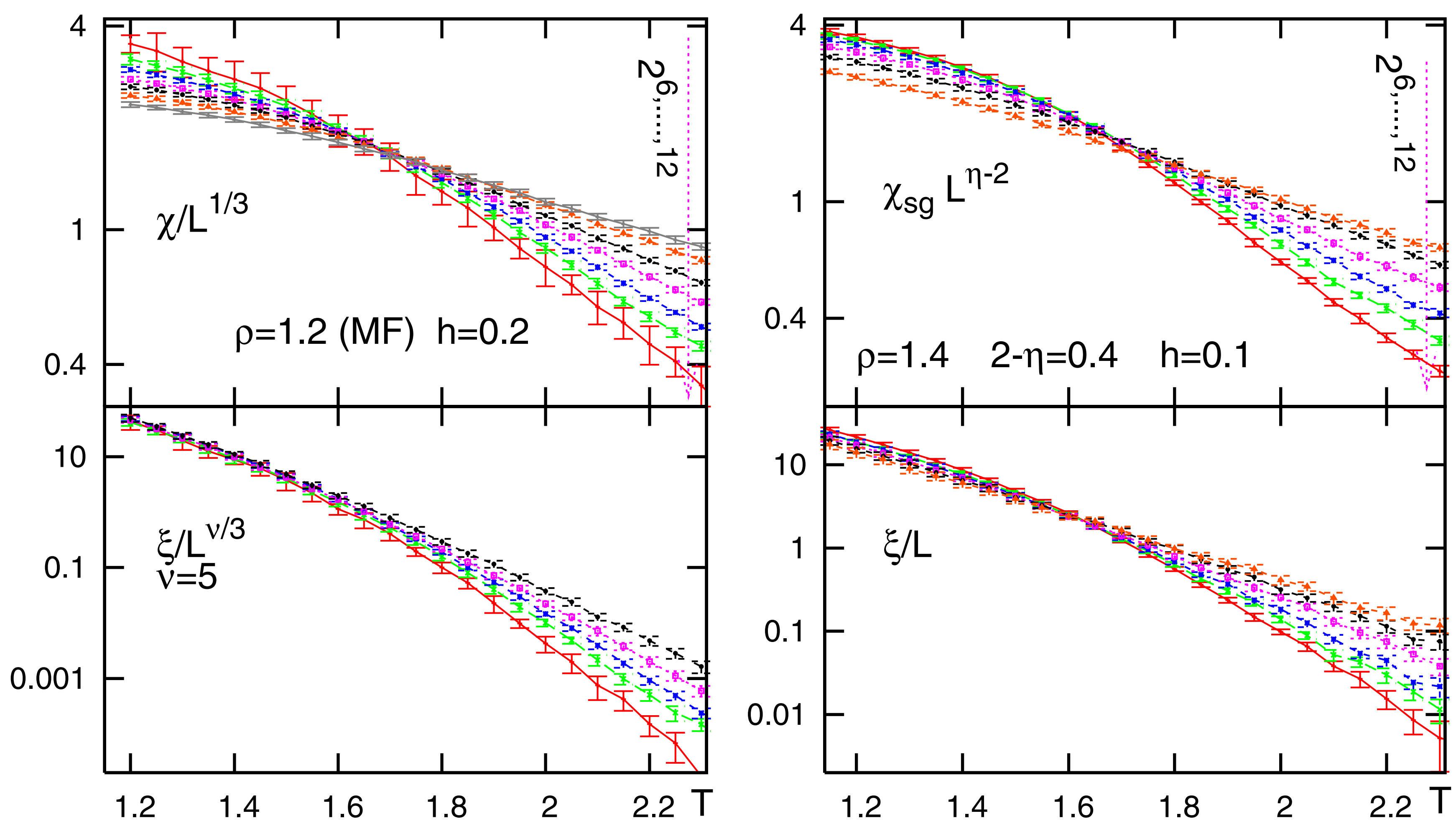


Leuzzi et al. PRL (2008)

Long range D=1 spin glasses with field

Leuzzi, Parisi, FRT, Ruiz-Lorenzo, PRL (2009)

- Standard tools of analysis:
  - clear phase transition in the mean-field region
  - phase transition outside the mean-field region for small fields
  - no phase transition for larger fields (e.g. $\rho = 1.4,\ h = 0.2$)



Long range D=1 spin glasses with field

- A new tool of analysis conditioning on the overlap

$$G(r|q) = \langle q_0 q_r | q \rangle = \mathbb{E}[s_0 t_0 s_r t_r | \underline{s} \cdot \underline{t} = N\, q]$$

$$\widehat{G}(k|q) = FT[G(r|q)] \qquad \widehat{G}(k=0|q) = q^2$$

$$\chi(q) = \widehat{G}(k = \frac{2\pi}{L}|q) \qquad \chi_{\mathrm{SG}} = \int dP(q)\, \chi(q)$$

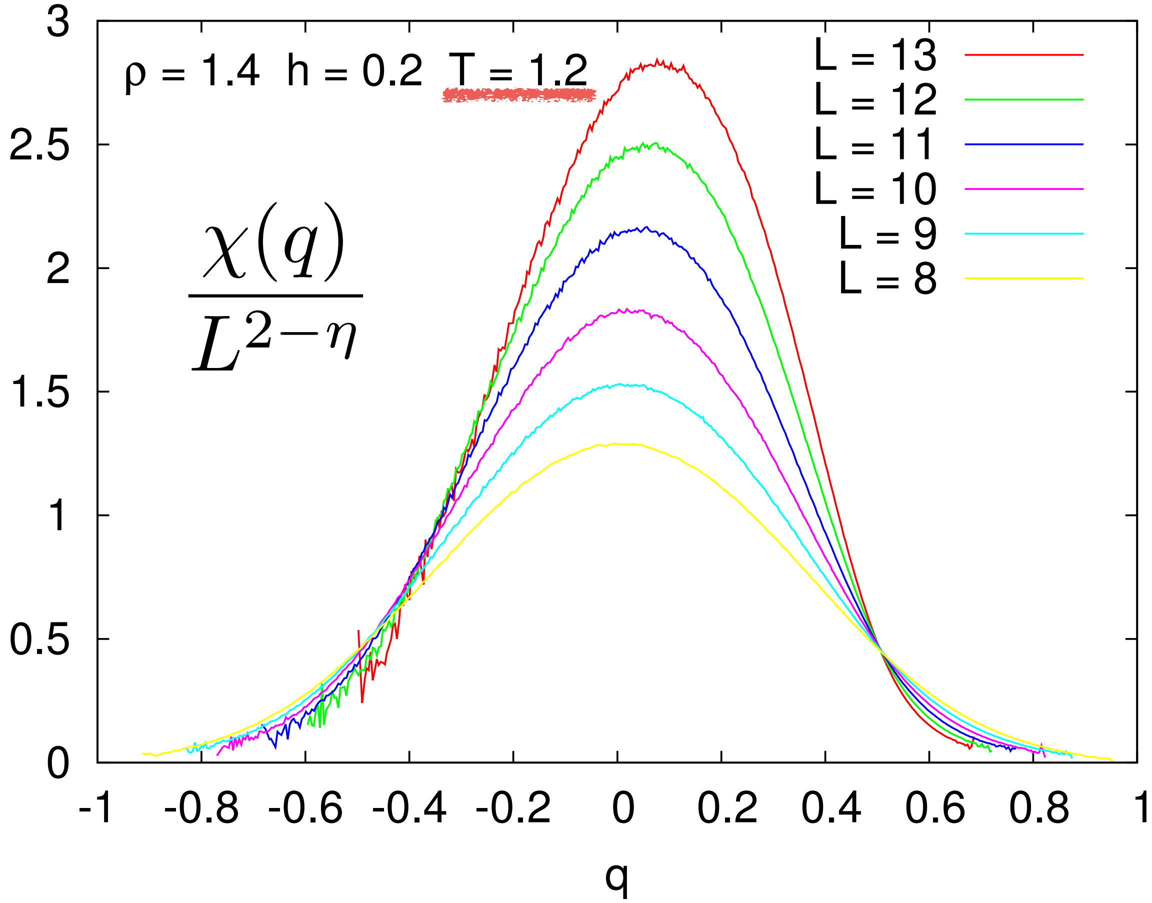
$\chi(q)$ fluctuates much less than χ_{SG}

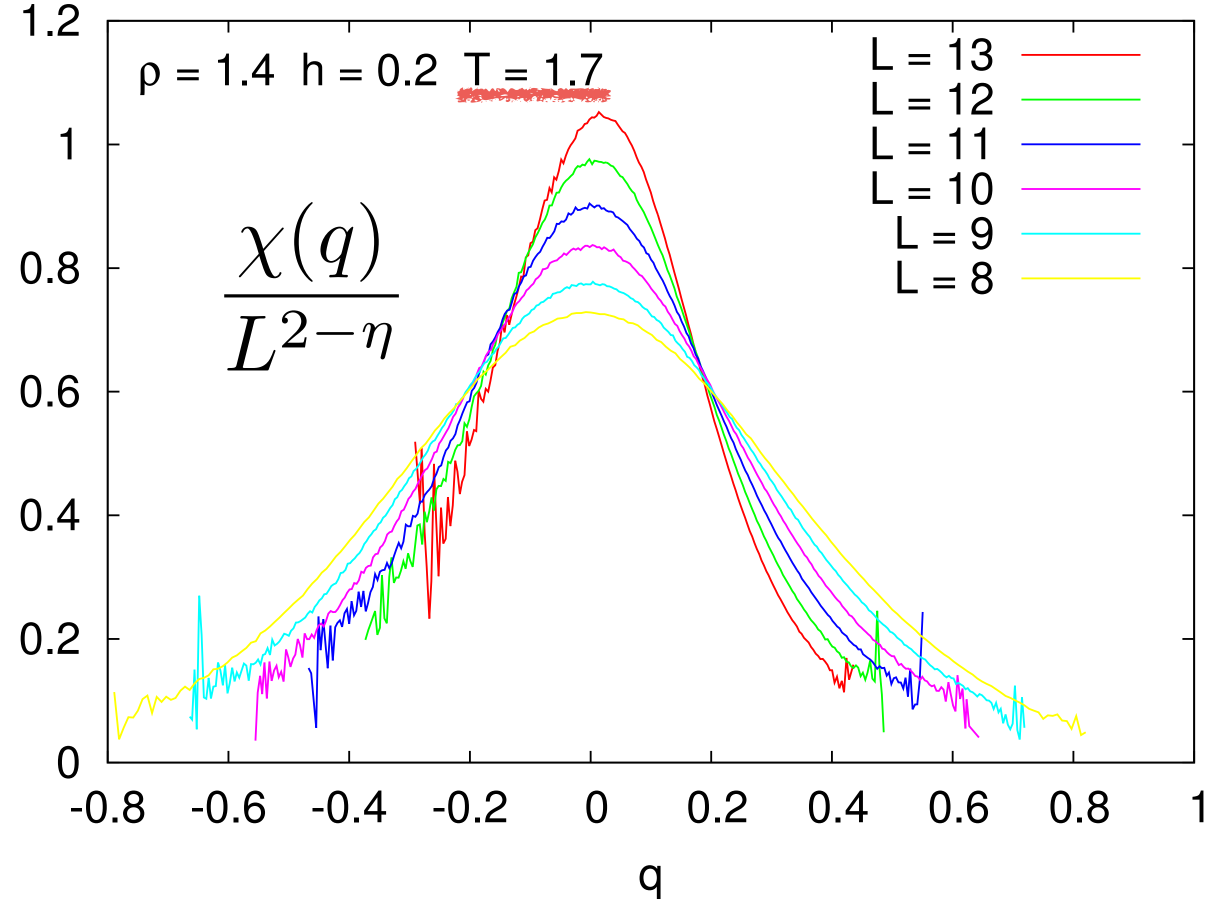
- Claim/conjecture: $G(r|q)$ and $\chi(q)$ are self-averaging

- At criticality: $P(q) = \delta(q - q_{\mathrm{EA}}) \implies \chi_{\mathrm{SG}} = \chi(q_{\mathrm{EA}}) \propto L^{2-\eta}$

Long range D=1 spin glasses with field

- SG phase transition lowering the conditioning overlap!
- Even in the paramagnetic phase!





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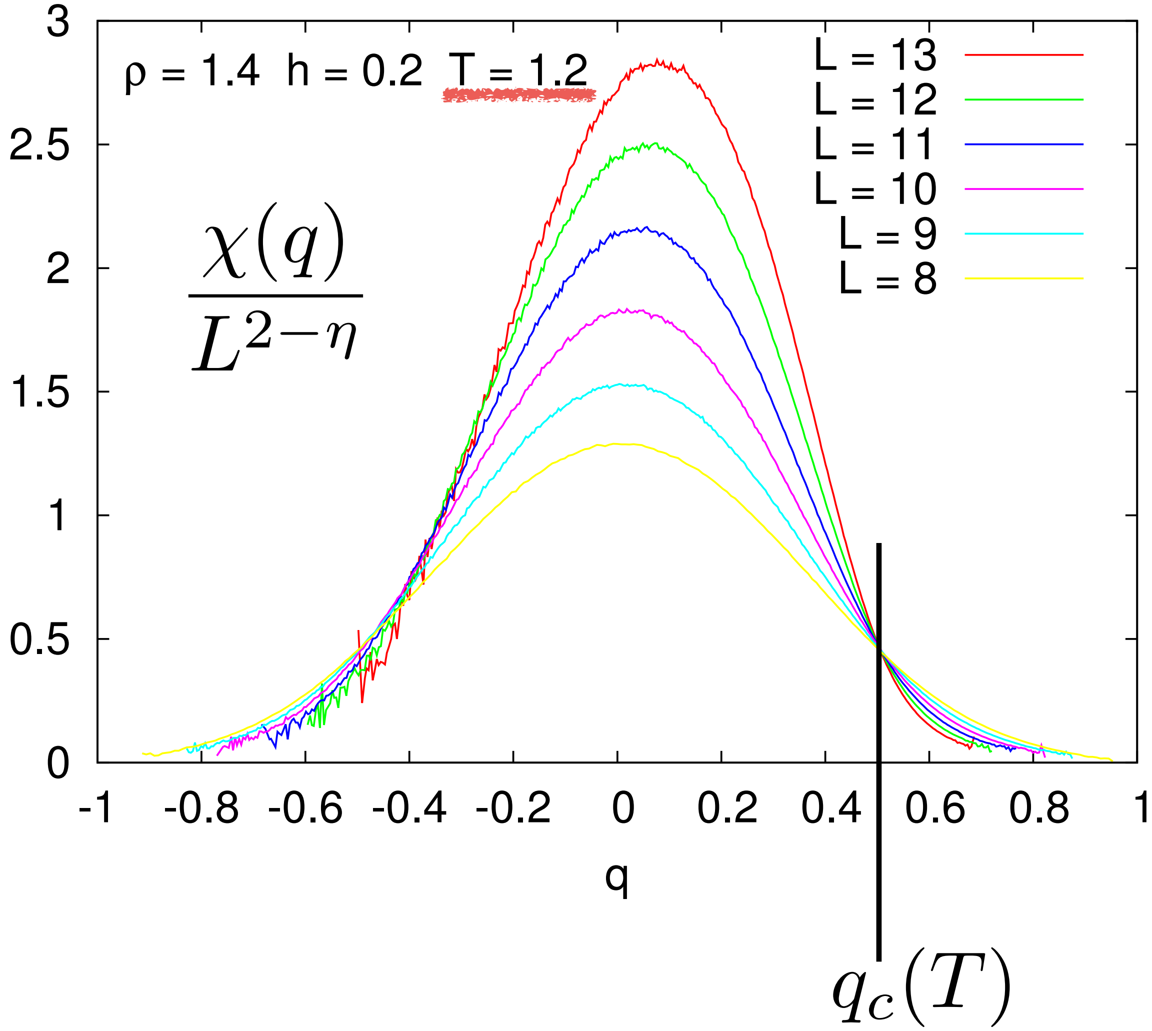
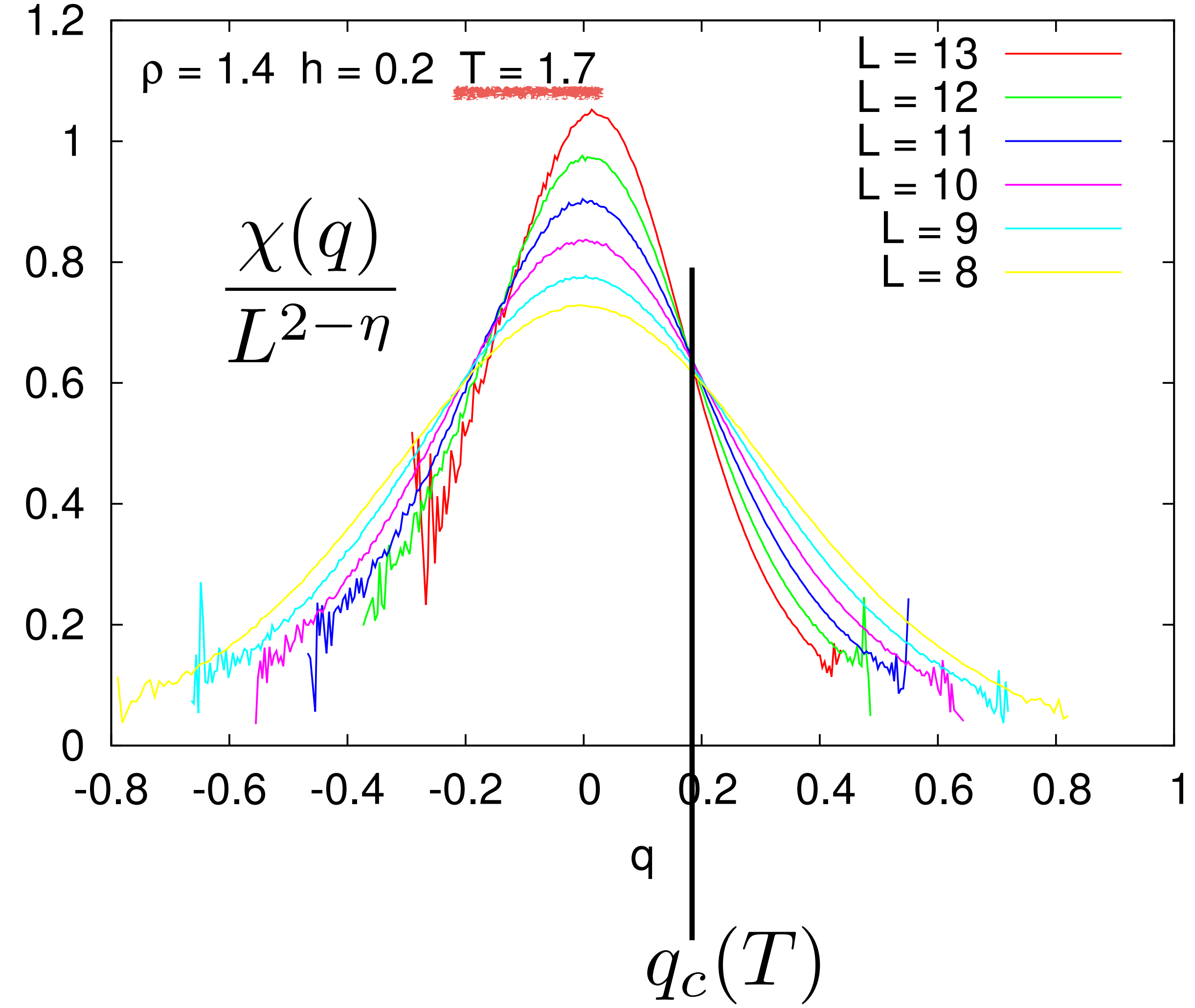
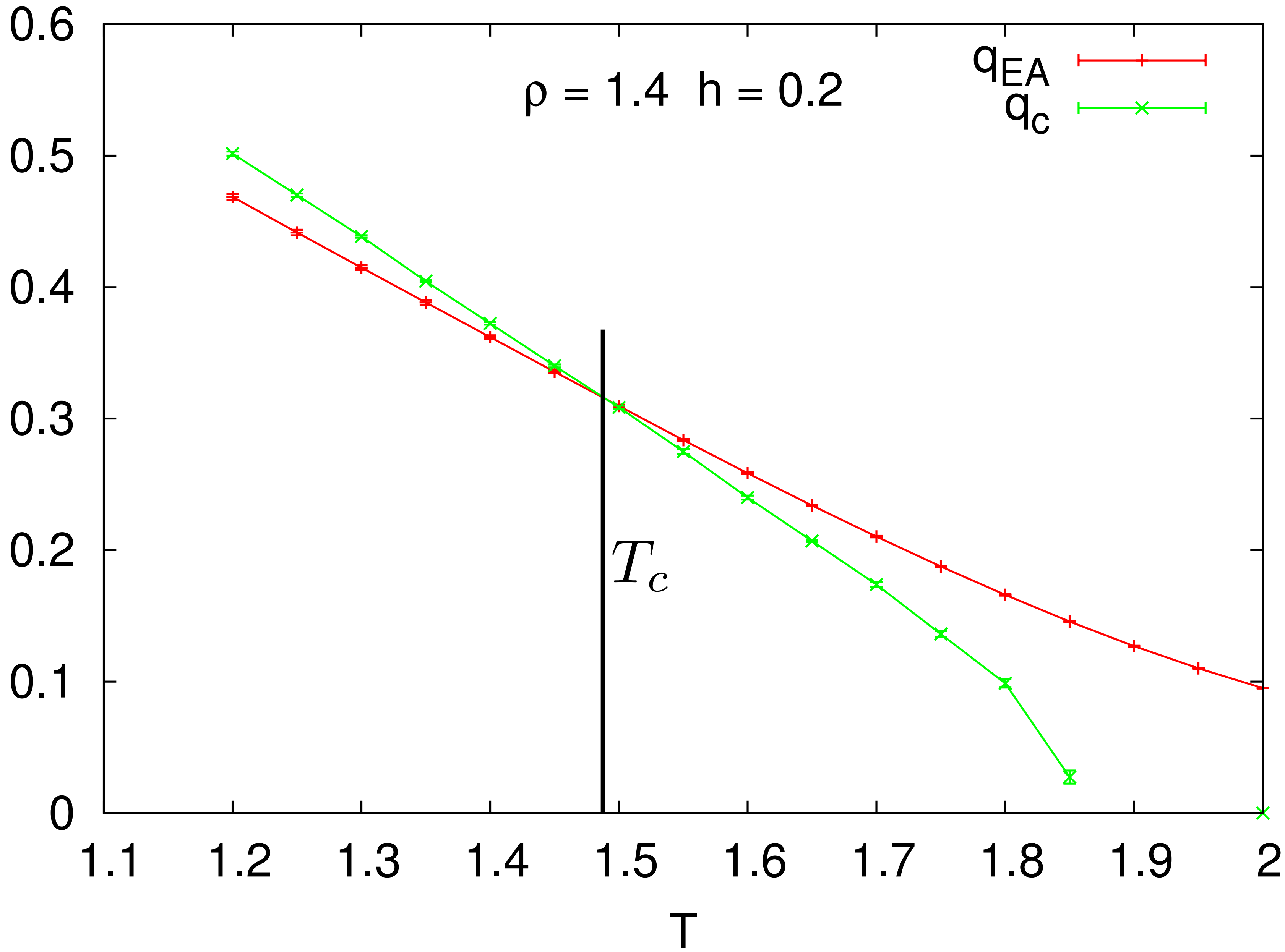




Long range D=1 spin glasses with field

- Very robust estimate of the critical temperature



References to cited works

- Diluted one-dimensional spin glasses with power law decaying interactions
  L. Leuzzi, G. Parisi, F. Ricci-Tersenghi and J.J. Ruiz-Lorenzo
  Phys. Rev. Lett. 101, 107203 (2008)

- Ising Spin-Glass Transition in a Magnetic Field Outside the Limit of Validity of Mean-Field Theory
  L. Leuzzi, G. Parisi, F. Ricci-Tersenghi and J.J. Ruiz-Lorenzo
  Phys. Rev. Lett 103, 267201 (2009)

- Finite size scaling of the de Almeida-Thouless instability in random sparse networks
  H. Takahashi, F. Ricci-Tersenghi and Y. Kabashima
  Phys. Rev. B 81, 174407 (2010)

- A numerical study of the overlap probability distribution and its sample-to-sample fluctuations in a mean-field model
  G. Parisi and F. Ricci-Tersenghi
  Phil. Mag. 92, 341 (2012)

- Thermodynamic glass transition in a spin glass without time-reversal symmetry
  Janus Collaboration
  PNAS 109, 6452 (2012)

- The three-dimensional Ising spin glass in an external magnetic field: the role of the silent majority
  Janus Collaboration
  J. Stat. Mech. P05014 (2014)