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Progress in Lattice QCD

- physics, algorithms, and machines -

How we in Japan met Giorgio

- From late 80's to early '90, we had intensive interactions on lattice QCD
  - Common interest in many topics of lattice QCD
    - High temperature behavior of QCD
    - Hadron mass spectrum
    - Hadron-hadron interactions etc
  - Common interest in machine building
    - APE series since early 80's
    - PACS series since late 70's
- Giorgio's visit to Kyoto and Tsukuba in '91
    - "Finite size effect for hadron masses in lattice QCD"

      M. Fukugita, H. Mino, M. Okawa, G. Parisi, A. Ukawa. Phys.Lett.B294:380-384,1992.

Lattice QCD over the years...

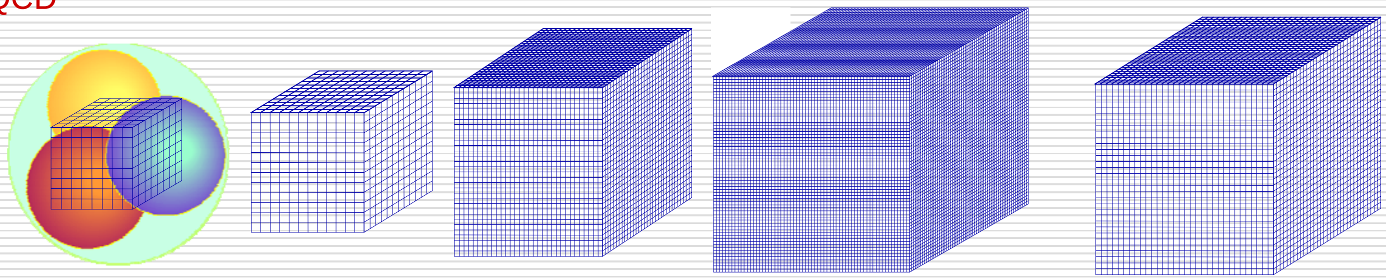
1975 1980 1985 1990 1995 2000 2005 2010

1973 QCD

1974 lattice QCD

Physics

1st spec calculation 1981 Hamber-Parisi Weingarten



Lattice size L	0.8fm	1.6fm	2.4fm	3.0fm	3.0fm
	8^3x16	16^3x32	24^3x48	64^3x118	32^3x64

Algorithms

Nf=0 quenched

Nf=2 u,d

Nf =#sea quarks

Nf=2+1 u,d,s

Machines

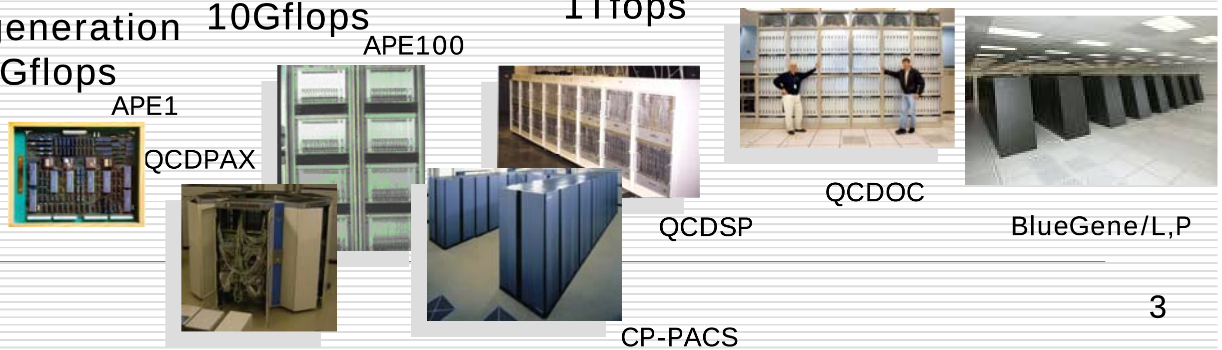
1st generation 1Gflops

2nd generation 10Gflops

3rd generation 1Tfops

4th generation 10Tfops

100Tflops

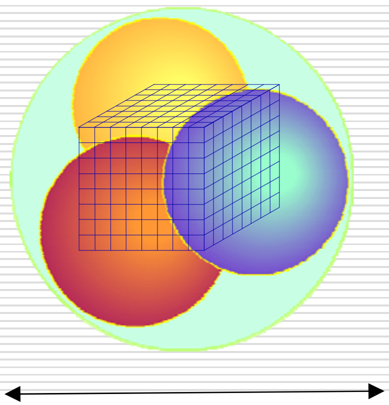


Giorgio and lattice QCD (I)

- First numerical calculation of hadron masss in lattice QCD (1981)
  - H. Hamber and G. Parisi, "Numerical Estimates of Hadronic Masses in a Pure SU(3) Gauge Theory", Nov 1981, Phys. Rev. Lett. 47, 1792, 1981
  - D. Weingarten, "Monte Carlo Evaluation of Hadron Masses in Lattice Gauge Theories with Fermions", Oct 1981. Phys.Lett.B109:57,1982.

- Limited in several respects:
  - Lattice size $4^4 \sim 8^4$
  - Physical size $L \sim 1\text{fm}$
  - Quenched approximation, i.e., no sea quarks

- But a giant step forward



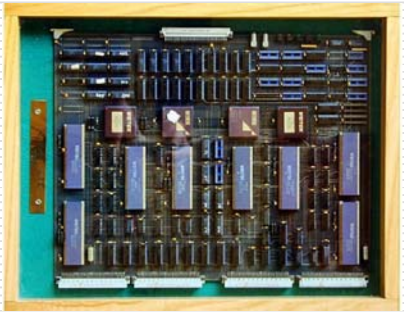
$size \approx 2 \times 10^{-15} m$

VAX

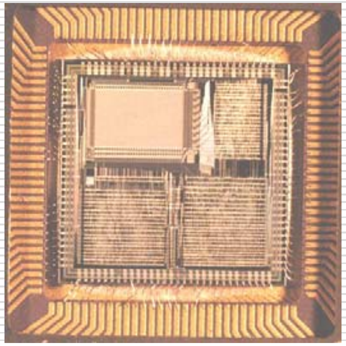
$speed \approx 1\, Mflops$

Giorgio and lattice QCD (II)

- Development of APE series of parallel computers optimized for lattice QCD (1984)
  - Massively parallel architecture suitable for lattice QCD
  - Optimized for complex arithmetic
  - TAO Language and compiler



APE1 (1987)



APE100 (1994)

APEmille (2000)

APENEXT(2005)

- Parallel development in USA and in Japan

What I wish to do today

Review the essential points of progress which took place over the last ten years or so

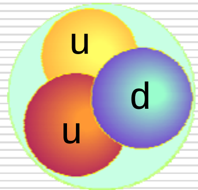
- Lattice QCD as computation and machine trends
- Physical point simulation and hadron spectrum
- Chiral symmetry and $K \to \pi\pi$ decay
- (Hot/Dense QCD)
- Conclusions

A bit of reminder

The Standard Model

- Matter particles
  - 6 quarks $\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} s \\ c \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$
  - 6 leptons $\begin{pmatrix} e \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$

Hadons(proton, neutron,pion etc) are cpmposites of 3 or 2 quarks



proton=uud

- Particles mediating interactions
  - photon γ EM
  - Weak bosons $W, \quad Z$ Weak interactions
  - gluons g Strong interaction

Weinberg-Salam theory (photon, Weak bosons)

Quantum Chromodynamics (QCD) (gluons)

- Gauge field theory based on

$$\underbrace{SU(3)}_{QCD} \otimes \underbrace{SU(2) \otimes U(1)}_{EM+Weak}$$

Quantum Chromodynamics

Gross-Wilczek-Politzer 1973

- Quantum field theory of quarks and gluon fields

$q_f(x)$ Quark field
$A_\mu(x)$ Gluon field
} defined over 4-dim space time

$$L_{QCD} = \frac{1}{8\pi\alpha_s} Tr\left(F_{\mu\nu}F_{\mu\nu}\right) + \sum_f \bar{q}_f\left(\gamma_\mu \cdot \left(\partial_\mu - iA_\mu\right) + m_f\right)q_f$$ QCD lagrangian

$$\left\langle O\left(A, \bar{\psi}, \psi\right)\right\rangle = \frac{1}{Z}\int dA d\bar{q} dq O\left(A, \bar{q}, q\right) e^{-\int d^4 x L_{QCD}}$$ Physical quantities by Feynman path integral

- Knowing

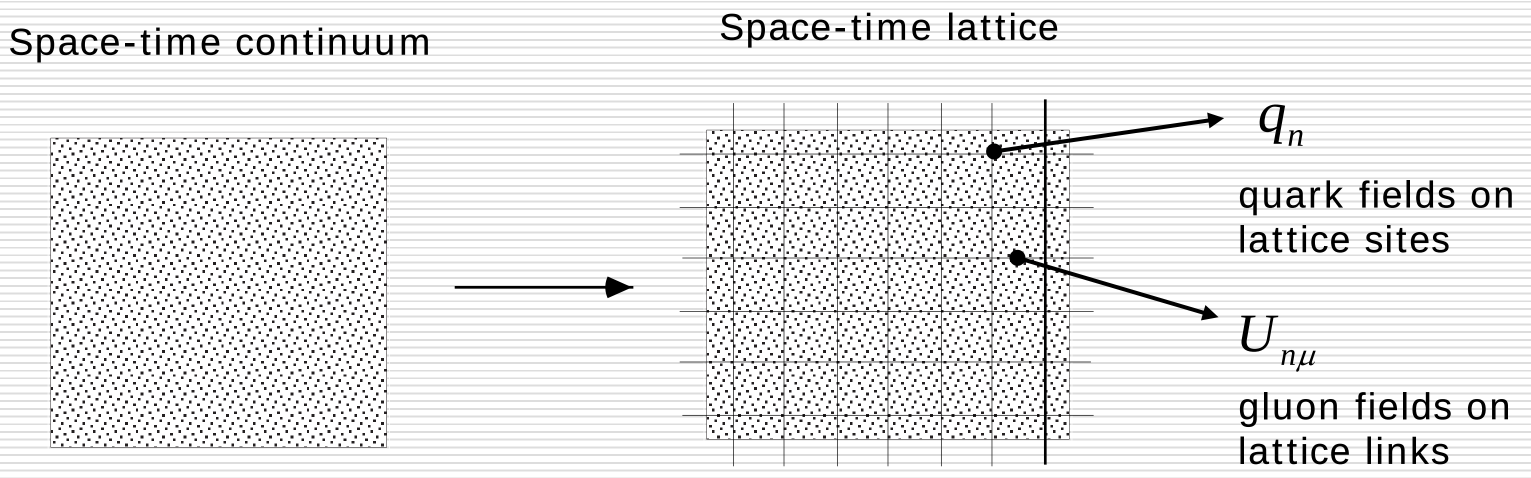
1 coupling constant $\alpha_s = \frac{g_s^2}{4\pi}$
and
6 quark masses $m_u, m_d, m_s, m_c, m_b, m_t$

will allow full understanding of hadrons and their strong interactions

"fulfilling Yukawa's dream of 1934 in a refined way"

QCD on a space-time lattice

K. G. Wilson 1974



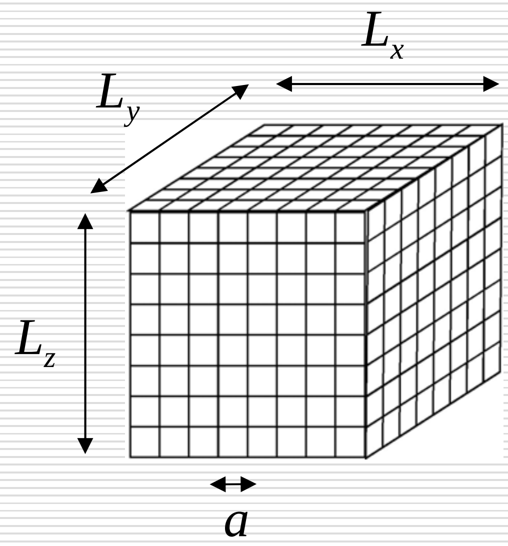
- Feynman path integral
  - Action $S_{QCD} = \frac{1}{g_s^2}\sum_P tr(UUUU) + \sum_f \bar{q}_f(\gamma \cdot U + m_f)q_f$
  - Physical quantities as integral averages → **Monte Carlo calculation of the averages**

$$\langle O(U,\bar{q},q)\rangle = \frac{1}{Z}\int \prod_{n\mu} dU_{n\mu} \prod_n d\bar{q}_n dq_n \; O(U,(U,\bar{q},q))e^{-S_{QCD}}$$

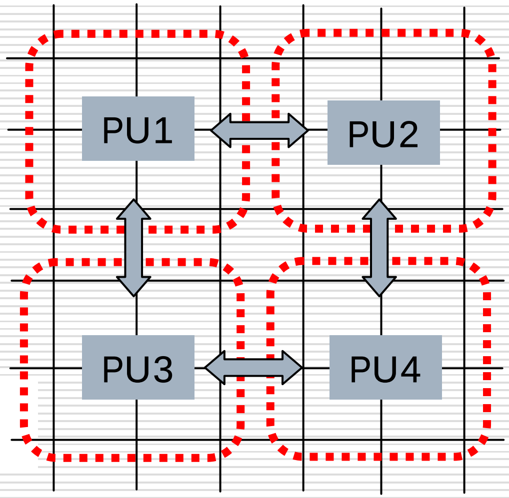
Lattice QCD as computation and machine trends

Lattice QCD as computation(I)

- 4-d simple cubic lattice
  - Lattice volume $V = L_x \times L_y \times L_z \times L_t$
  - Lattice spacing a



- Parallelization
  - Mapping space-time lattice to processor array
  - QCD is a local field theory; only nearest neighbor interactions
    ⇒only nearest neighbor data communication needed



⇒ **Highly parallelizable and scalable**

Lattice QCD as computation(II)

- Quark fields, being anticommuting, needs a special trick

$$\int \prod_n d\bar{q}_n dq_n \, e^{-\sum_{n,m} \bar{q}_n D_{nm}(U) q_m} = \det D(U) = \int \prod_n d\bar{\phi}_n d\phi_n \, e^{-\sum_{n,m} \bar{\phi}_n \left(\frac{1}{D(U)}\right)_{nm} \phi_m}$$

Grassmann rep　　　　　　　　　　Boson rep

- Need to invert the lattice Dirac operator D(U)

$$\sum_m D_{nm}(U) x_m = \phi_n \Rightarrow x_n = \left(\frac{1}{D(U)}\right)_{nm} \phi_m$$

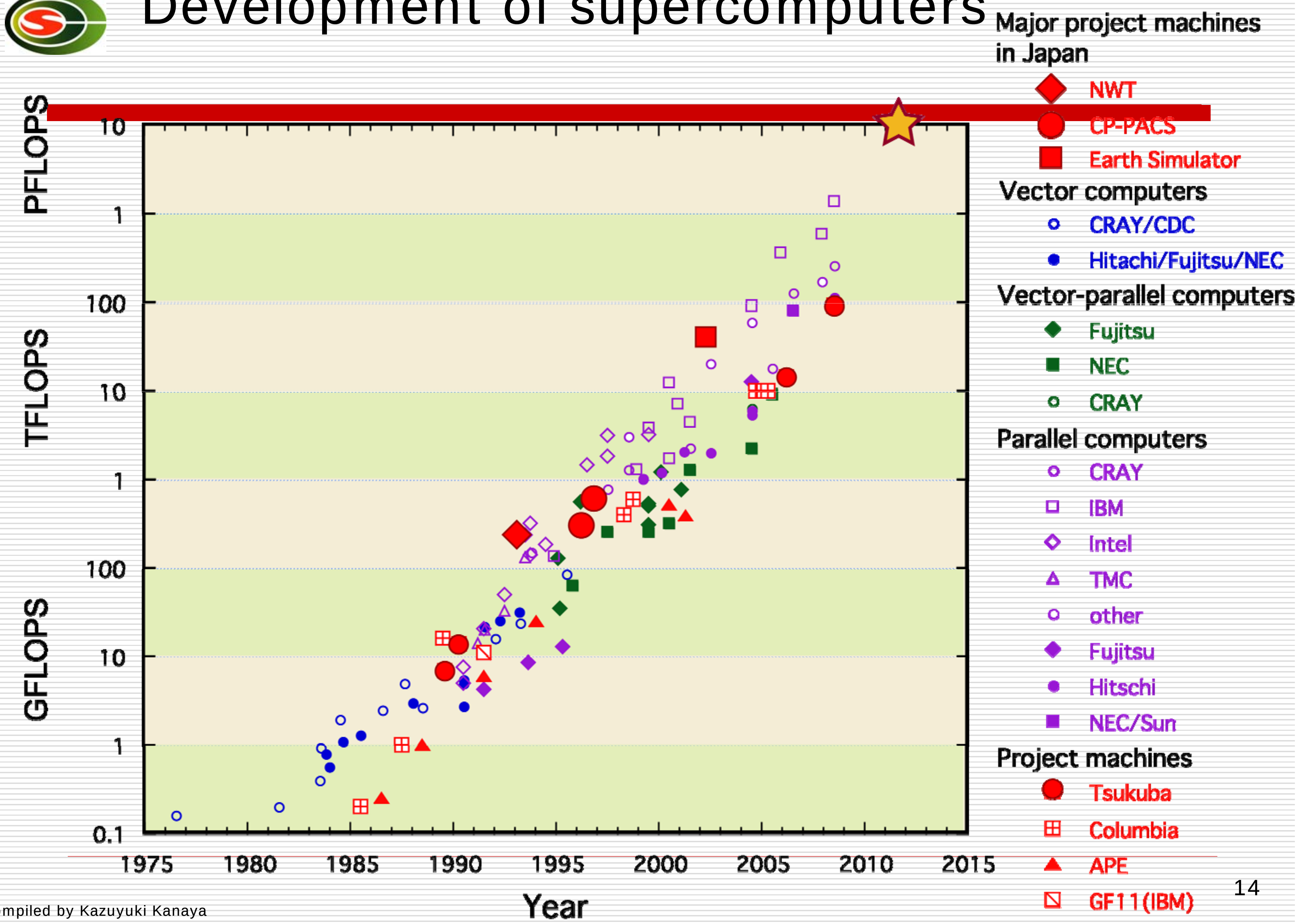
Core calculation of QCD

  - $D(U) \approx i\gamma \cdot (\partial - igA(x)) + m_q$　　complex12Vx12V dim matrix
  - Condition number ～ $1/m_q$

    With iterative solvers, #arithmetic ops rapidly increases when quark mass m_q becomes small (up, down, strange)

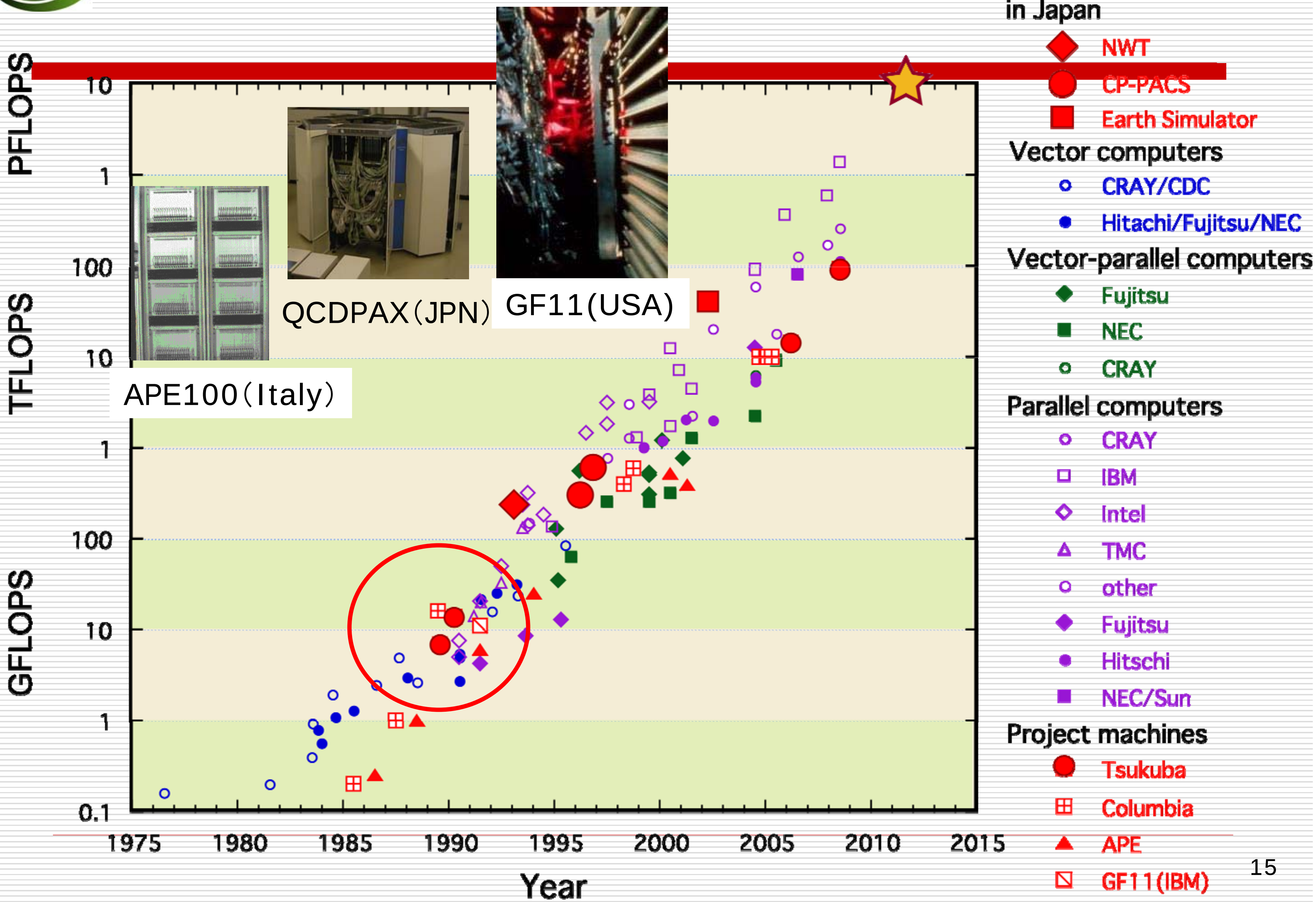
→ Computationally very intensive

Development of supercomputers

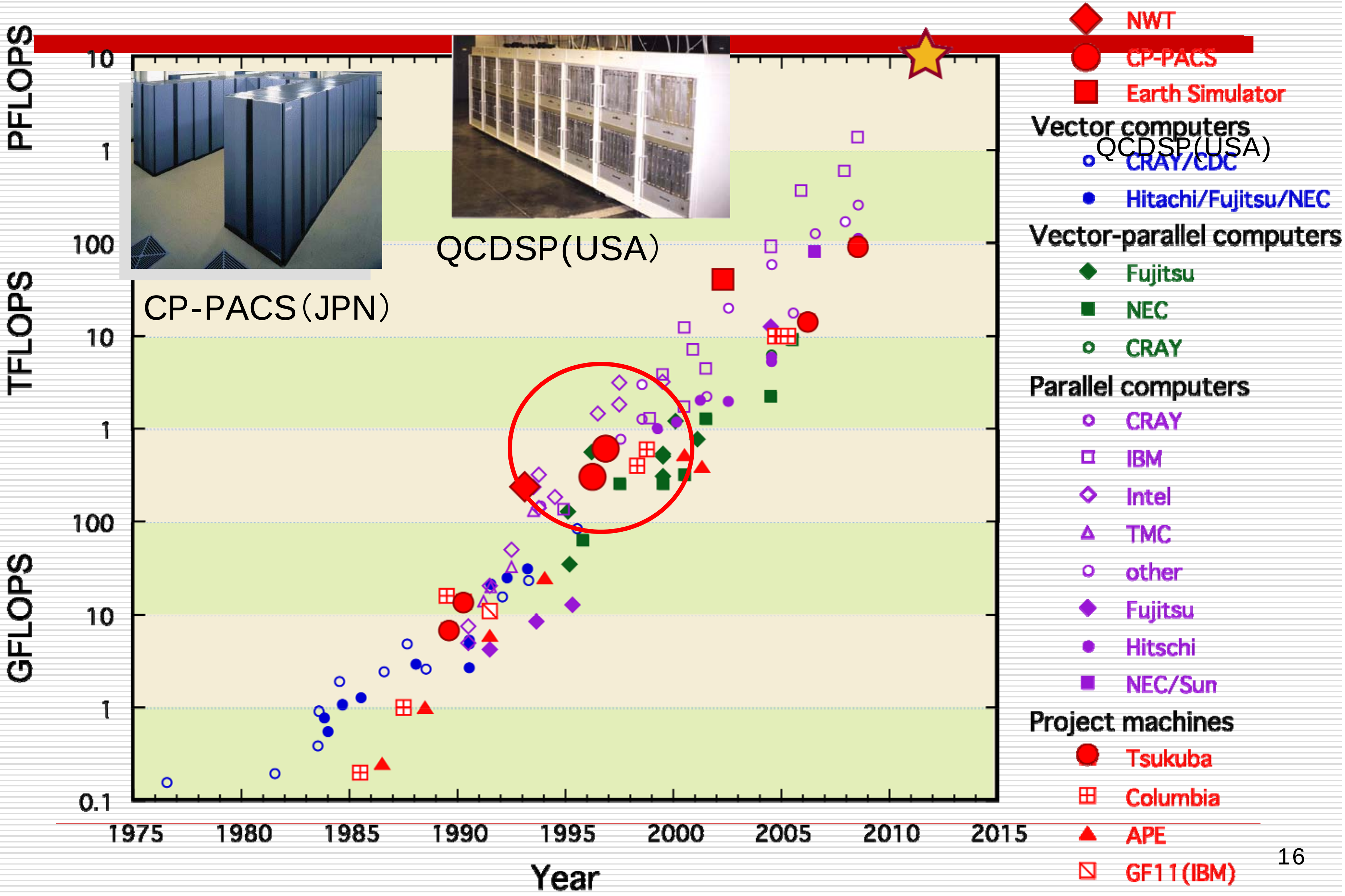


Compiled by Kazuyuki Kanaya

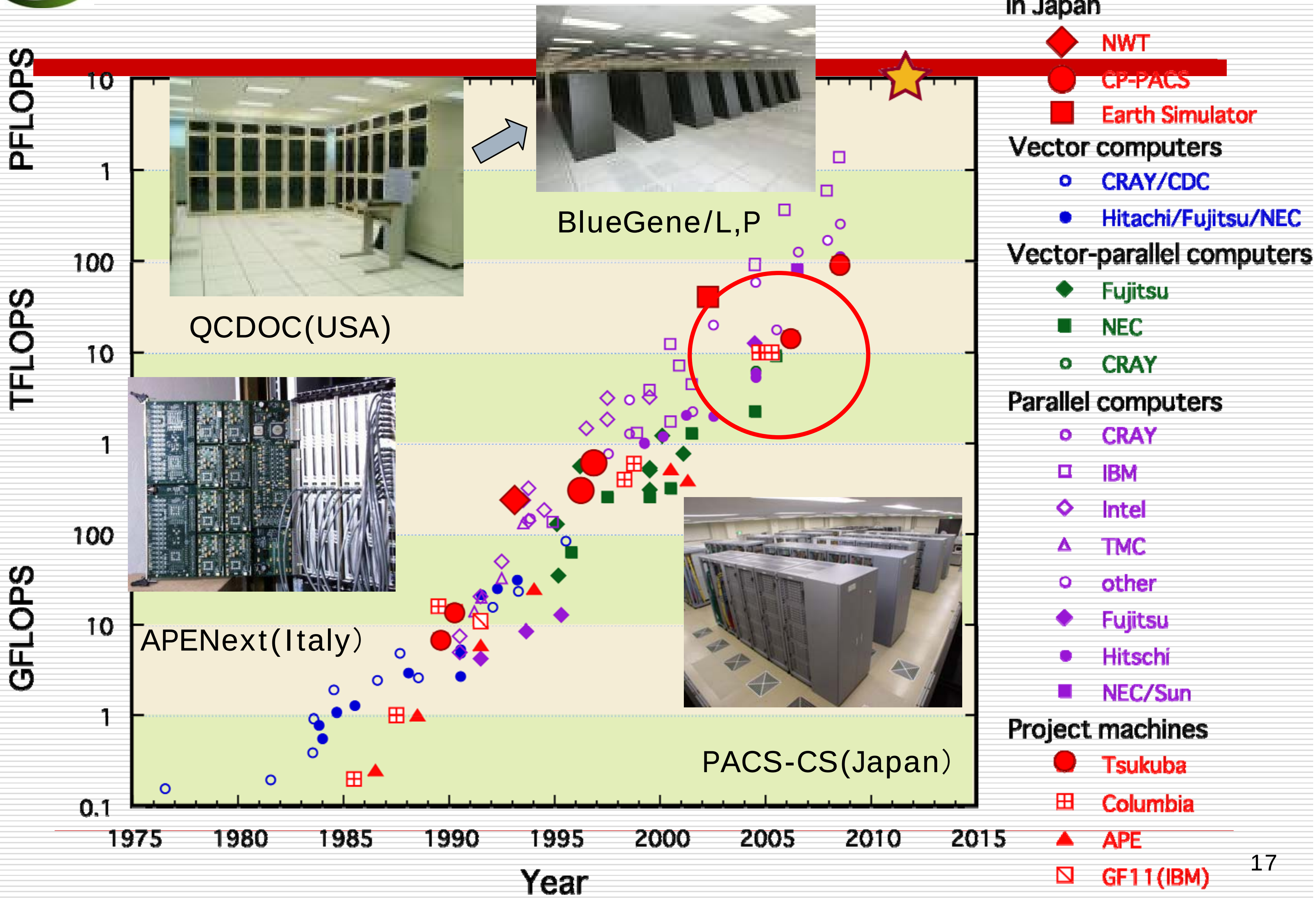
1990's 2^{nd} generation QCD machines
O(10)Gflops



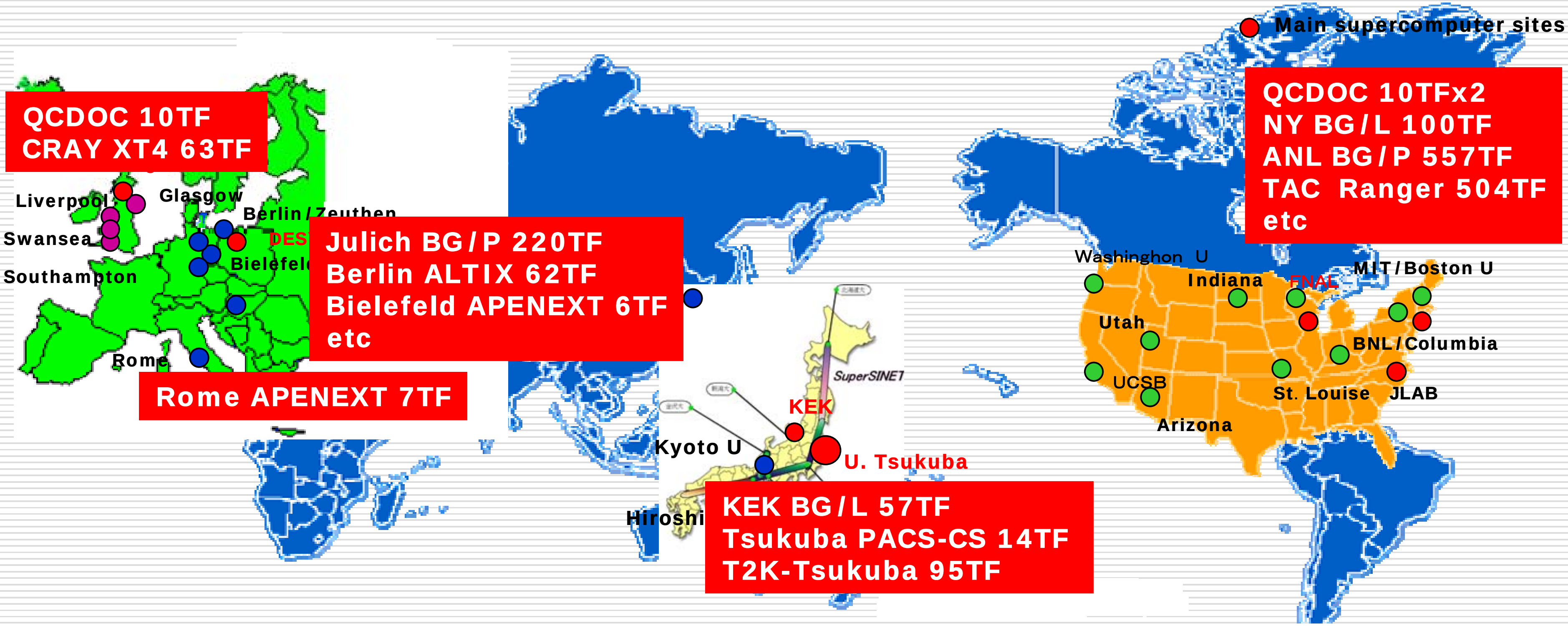
Late 1990's 3rd generation QCD machines O(500)Gflops



2000's 4th generation QCD machines O(10)Tflops



Current computing resources for lattice QCD in the World



- About 10 major cites scattered in USA, EU(UK, Germany, Italy etc), Japan
- In total 500～600Tflops in peak speed (US300Tf、EU150Tf、Japan100Tf)
- Data sharing through *ILDG (International Lattice Data Grid)*

Future:petascale computing

- Peta-scale computing is around the corner(2010),
  - National “Big Gun” projects
    - USA: Road Runner (Cell-based cluster), BlueGene/Q, ...
    - Japanese Project
  - Commercial clusters based on multi-core CPU’s (Intel, AMD)
- New projects for lattice QCD
  - QPACE Project (QCD Parallel Computing on the CELL)
    - CELL-based cluster/200Tflops in 2009
  - Pet-APE Project (Petaflops Processor Array Experiment)
    - Reference platform for 2009-2014
  - GPGPU?
    - Many-core high speed graphic cards/software development

Will lattice QCD be able to take back the leadership role in High Performace Computing?

Physical point simulation and hadron spectrum

From quenched to full QCD simulation

- Quenched QCD=ignore quark contribution,
  i.e., no sea quarks Parisi, Weingerten 1980
- Hybrid Monte Carlo algorithm for full QCD
  Duane-Kennedy-Pendleton (1987)
  - Molecular dynamics with action as pseudo hamiltonian

$$\frac{d}{d\tau}U_{n\mu} = -iU_{n\mu}P_{n\mu} \qquad S_{QCD} = \frac{1}{g_s^2}\sum_P tr(UUUU) + \sum_{n,m}\bar{\phi}_n\left(\frac{1}{D(U)}\right)_{nm}\phi_m$$

$$\frac{d}{d\tau}P_{n\mu} = -\frac{\partial S_{QCD}}{\partial U_{n\mu}}$$

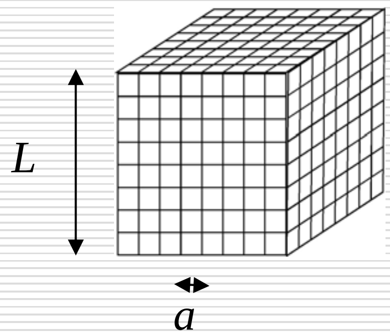
$$= \frac{1}{g^2}(UUUU\ )_{n\mu} + \bar{\phi}\left(\frac{1}{D(U)}\right)\frac{\partial D(U)}{\partial U_{n\mu}}\left(\frac{1}{D(U)}\right)\phi$$

Gluon force quark force

  - #arithmetic ops dominated by the inversion of D(U)

Scaling law for #arithmetic ops for HMC

- parameters:
  - Quark mass m_q or m_π / m_ρ
  - Lattice size $L\,(fm)$
  - Lattice spacing $a\,(fm)$



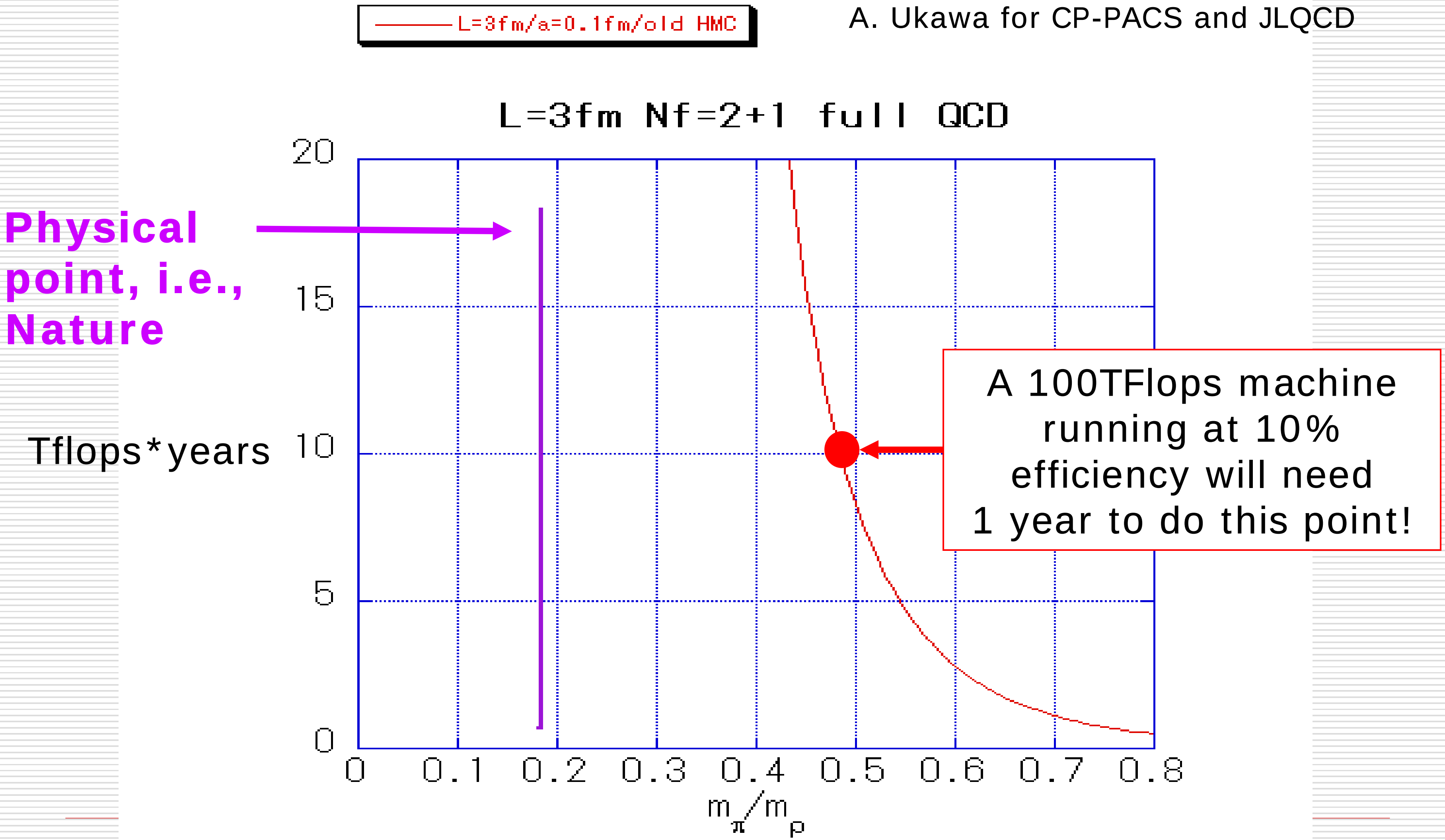
- An empirical formula for #arithmetic ops of HMC algorithm(2001)

$$\#FLOP's \approx 2.8\left[\frac{\#conf}{1000}\right]\cdot\left[\frac{m_\pi/m_\rho}{0.6}\right]^{-6}\cdot\left[\frac{L}{3\,fm}\right]^{5}\cdot\left[\frac{a}{0.1\,fm}\right]^{-7} Tflops\cdot year$$

  - Severe scaling toward small pion mass /large volume/small lattice spacing

"Berlin wall" at Lattice 2001@Berlin

A. Ukawa for CP-PACS and JLQCD



Recent developments

- Molecular dynamics equation of HMC once again

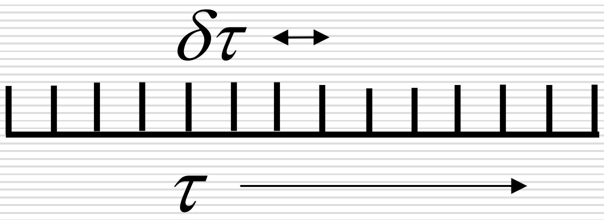
$$\frac{d}{d\tau}U_{n\mu} = -iU_{n\mu}P_{n\mu}$$

quark force

$$\frac{d}{d\tau}P_{n\mu} = F_{n\mu} = \frac{1}{g^2}\left(UUUU\right)_{n\mu} + \bar{\phi}\left(\frac{1}{D(U)}\right)\frac{\partial D(U)}{\partial U_{n\mu}}\left(\frac{1}{D(U)}\right)\phi$$

gluon force

  - Molecular dynamics equation is integrated in discrete steps



  - Stability condition $$\delta\tau \cdot \left|F_{n\mu}\right| < 1$$

UV/IR separation of quark force

$$\det D = \int d\phi_{UV}^{+} d\phi_{UV} \exp\left(-\phi_{UV}^{+}\frac{1}{D_{UV}}\phi_{UV}\right)\cdot\int d\phi_{IR}^{+} d\phi_{IR} \exp\left(-\phi_{IR}^{+}\frac{1}{\underbrace{D/D_{UV}}_{D_{IR}}}\phi_{IR}\right)$$

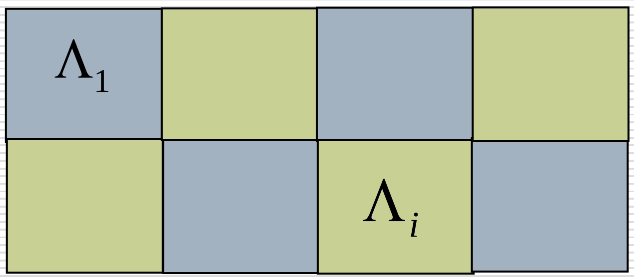
$$\Longrightarrow \quad F_{quark} = F_{UV} + F_{IR}$$

- Hasenbusch preconditioner — M. Hasenbusch(2001)

$$D_{UV} = D\left(m_q \to m'_q > m_q\right)$$

Use heavier quark mass to separate out UV modes

- Luescher domain decomposition — M. Luescher(2005)



$$D_{UV} = \sum_i D_{\Lambda i}$$

Restricted to each domain
→ UV part of D

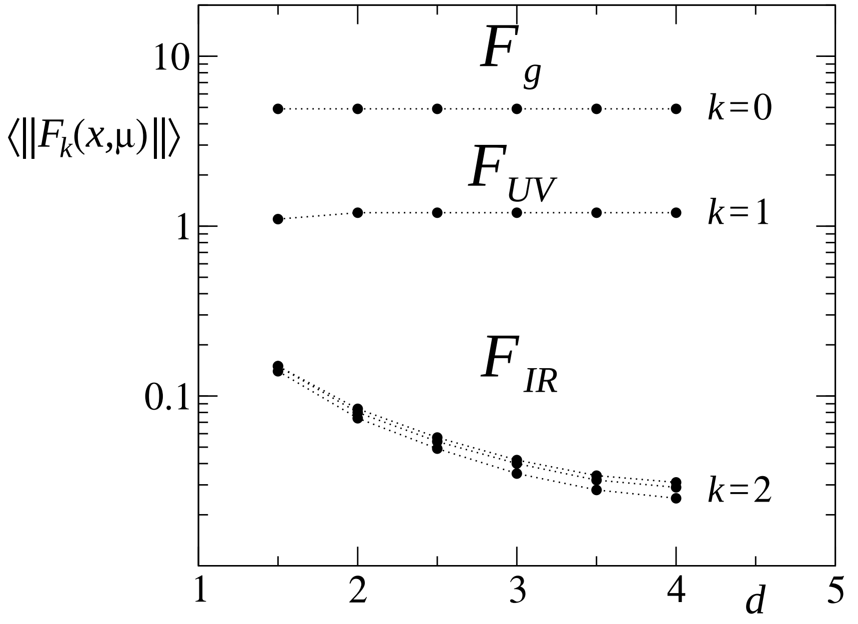
UV/IR separation and multi-time step

M. Luescher、Comput.Phys.Commun. 165 (2005) 199

- Magnitude of force is ordered

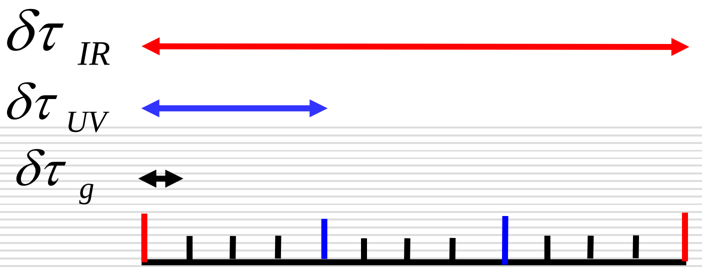
$$F_g : F_{UV} : F_{IR} \approx 5:1:0.2 = 25:5:1$$

$\frac{m_\pi}{m_\rho} \approx 0.7-0.4$ and $a^{-1} \approx 2.4\text{GeV}$



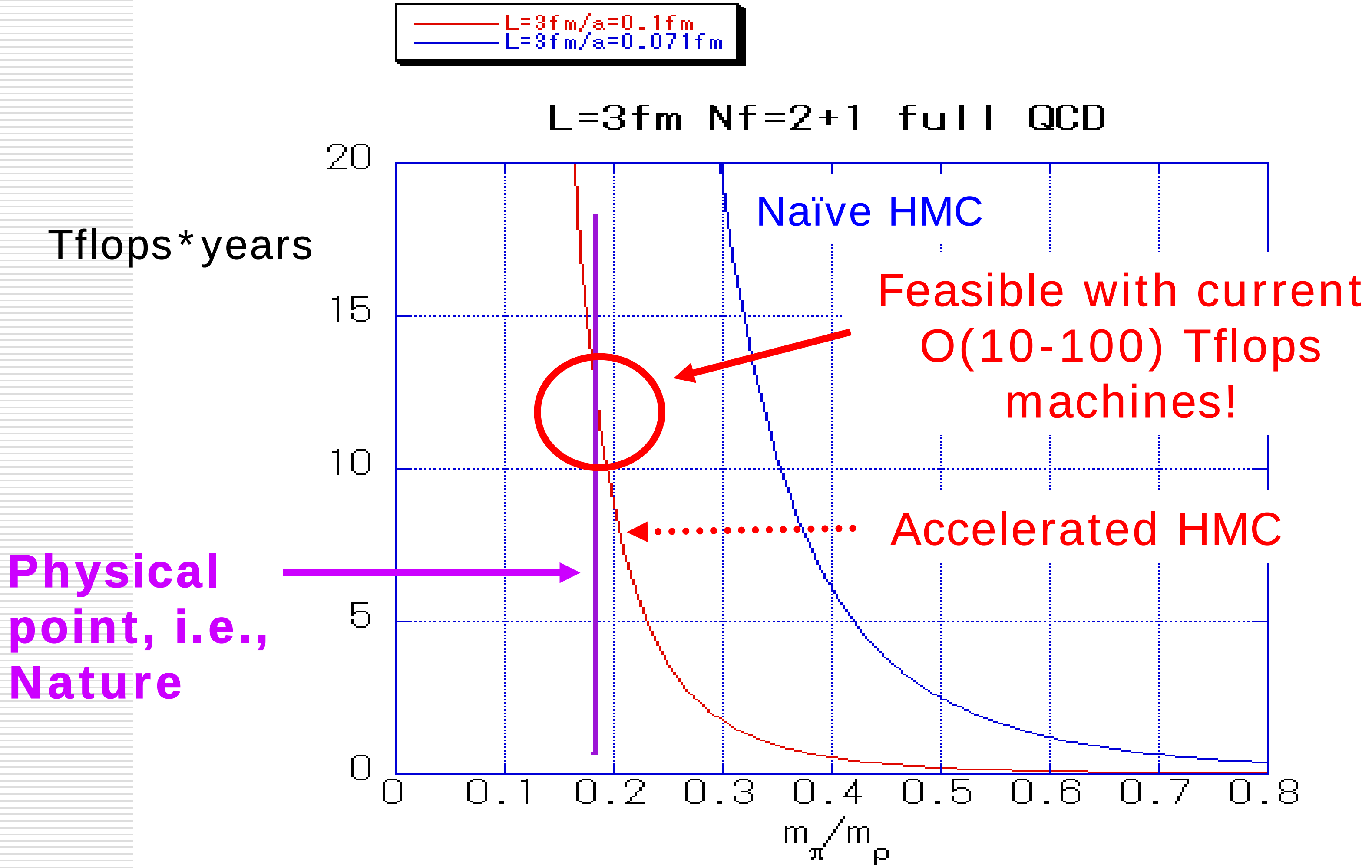
- Time step can be enlarged for smaller force; multi-time step evolution

$$\delta\tau_g : \delta\tau_{UV} : \delta\tau_{IR} \approx 1:5:25$$



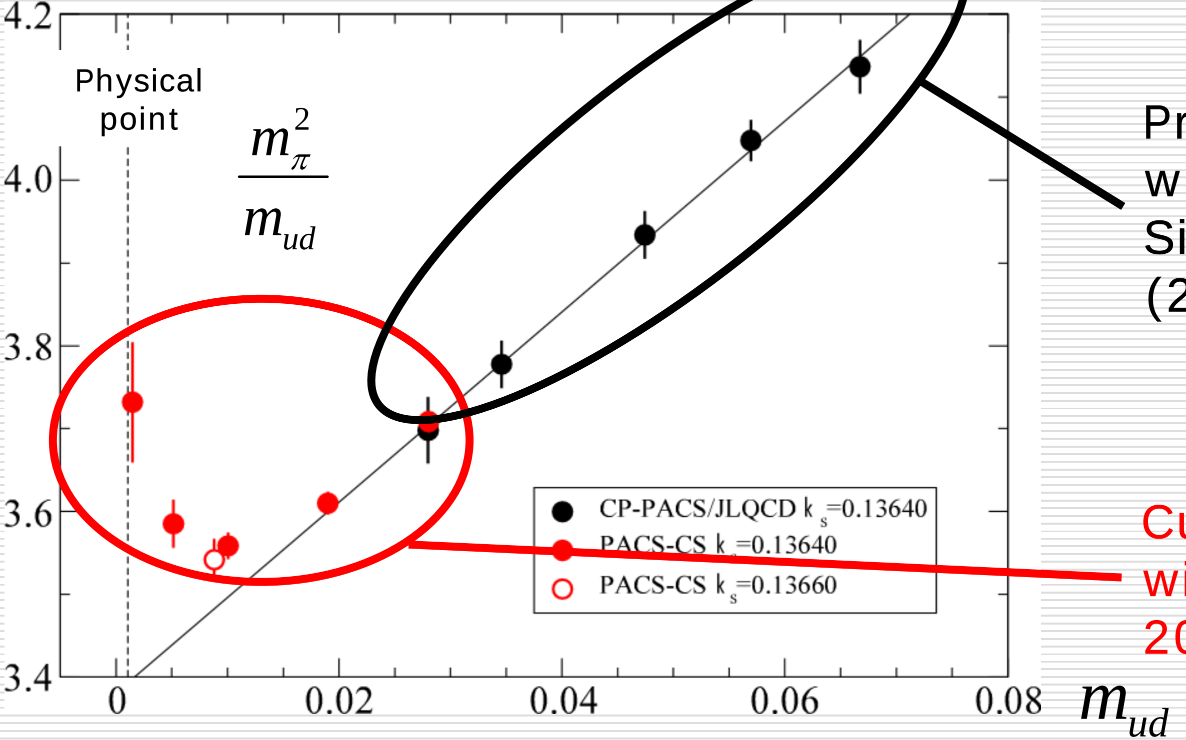
- The most time consuming IR quark part needs calculated less oftenthan the naïve single time step HMC

Acceleration of HMC



Why so important? Example (1)

Pion mass



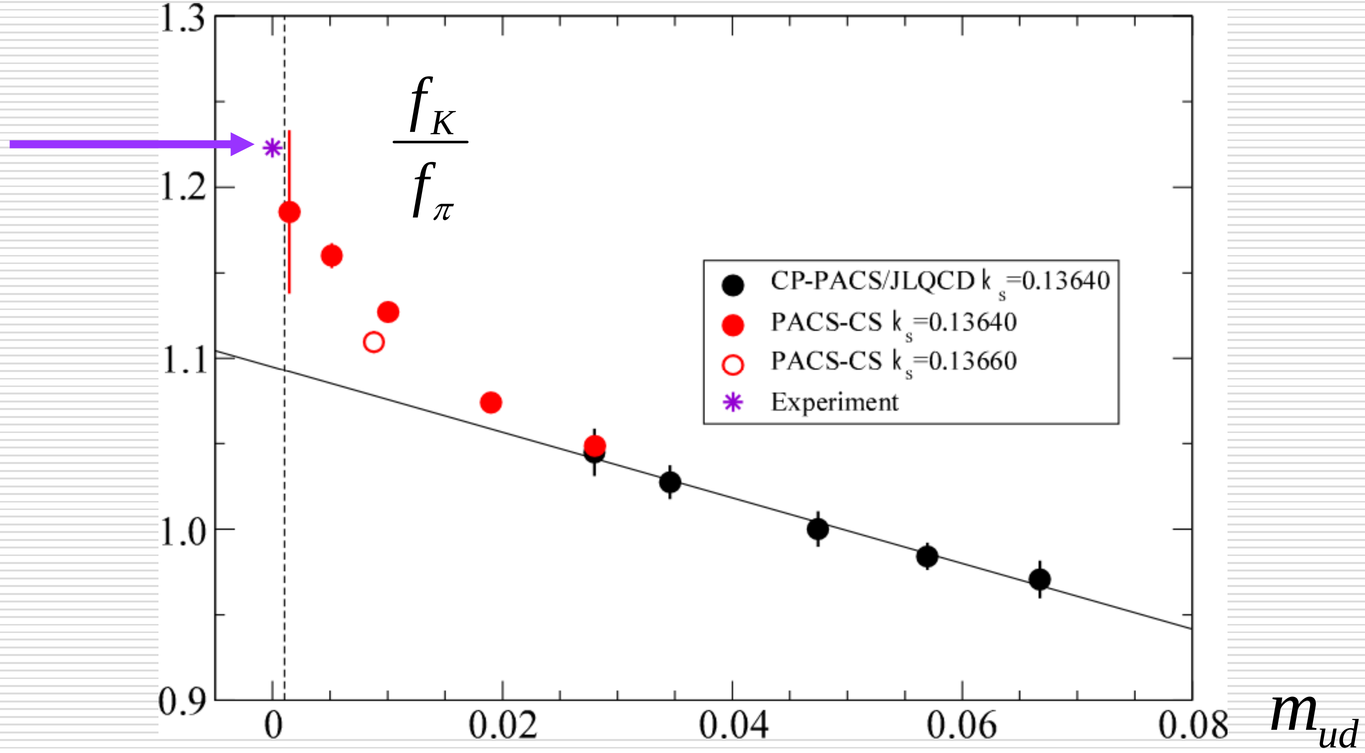
- Chiral logarithm at zero quark mass finally confirmed and controlled

$$\frac{m_\pi^2}{m_{ud}} \propto 1 + \frac{\tilde{m}_\pi^2}{(4\pi f)^2}\log\frac{\tilde{m}_\pi^2}{\mu^2} + \cdots, \qquad \tilde{m}_\pi^2 = 2Bm_{ud}$$

Why so important? Example (2)

Decay constant of K and π

実験値



$$\frac{f_K}{f_\pi} \propto 1 + \frac{5}{4}\frac{\tilde{m}_\pi^2}{(4\pi f)^2}\log\frac{\tilde{m}_\pi^2}{\mu^2} + \cdots$$

Recent large-scale Nf=2+1 calculations

- Features
  - Fully incorporates dynamical effects of up, down, strange sea quarks, hence called "Nf=2+1"
  - Pion mass reaching down to $m_\pi \approx 200-300 MeV$

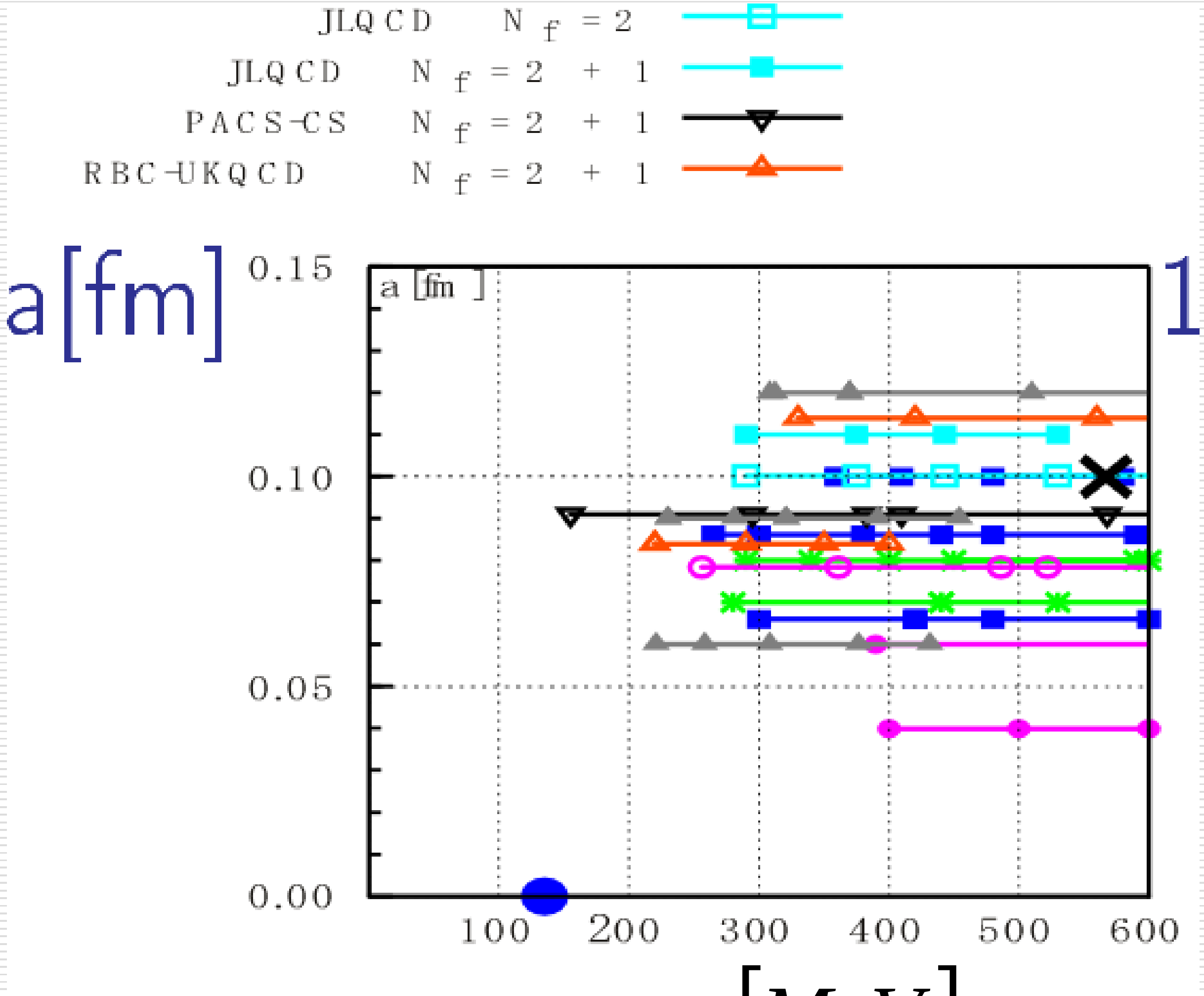
    PACS-CS even attempting the physical point $m_\pi \approx 140 MeV$
  - Lattice size to avoid finite size effects $m_\pi L \approx 3-4$

- Collaborations

Collaborations	action	a (fm)	L (fm)	m_π (MeV)
MILC	Kogut-Susskind	0.06	4	220
PACS-CS	wilson-clover	0.09	3	155
BMW	wilson-clover	0.09	4	190
RBC-UKQCD	domain-wall	0.085	4	220
JLQCD	overlap	0.11	1.8	290

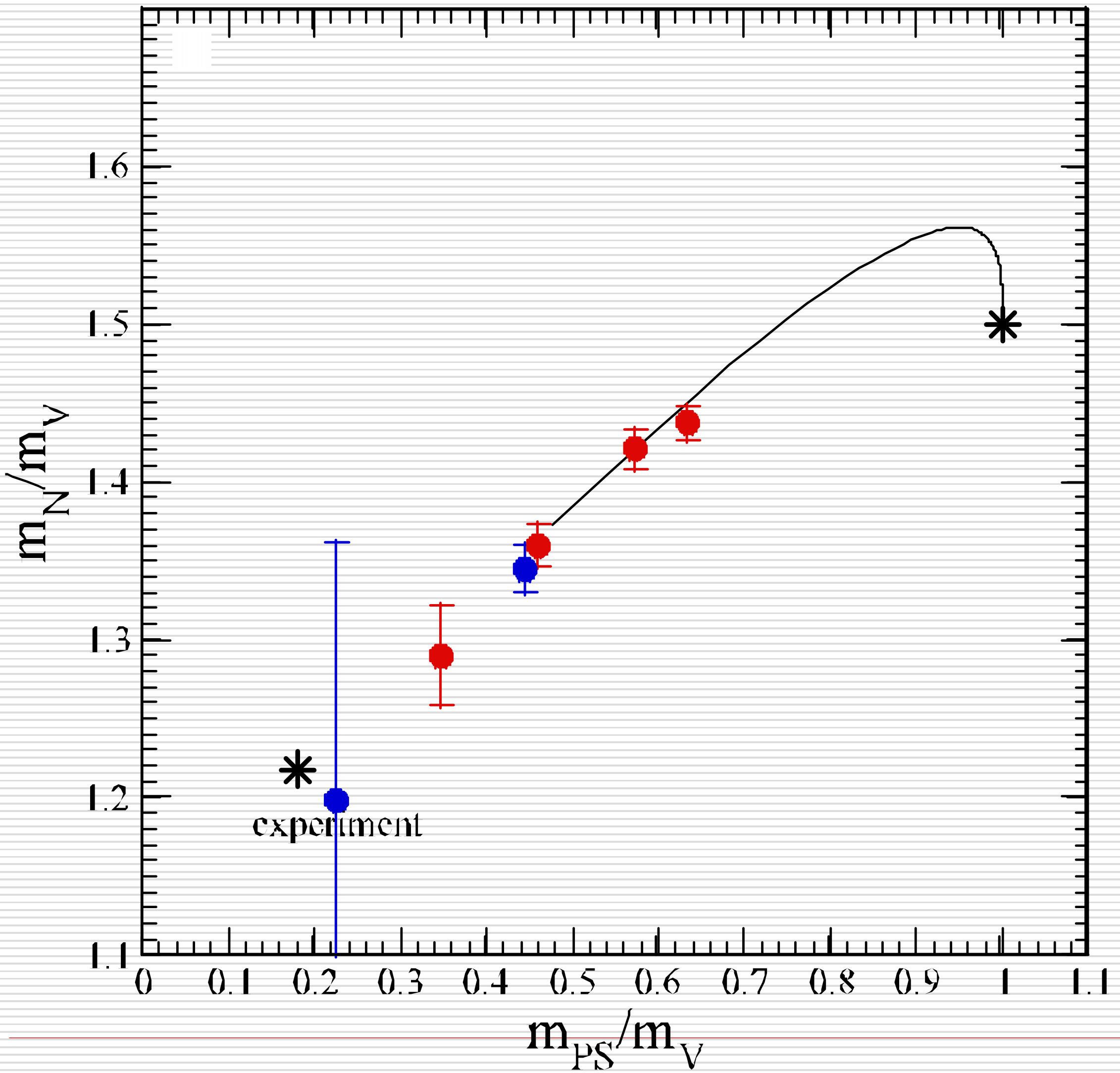
Lattice spacing and pion mass

K. Jansen, plenary talk at Lattice08



$m_\pi [MeV]$

Edinburgh plot today



PACS-CS Collaboration

Wilson-clover action

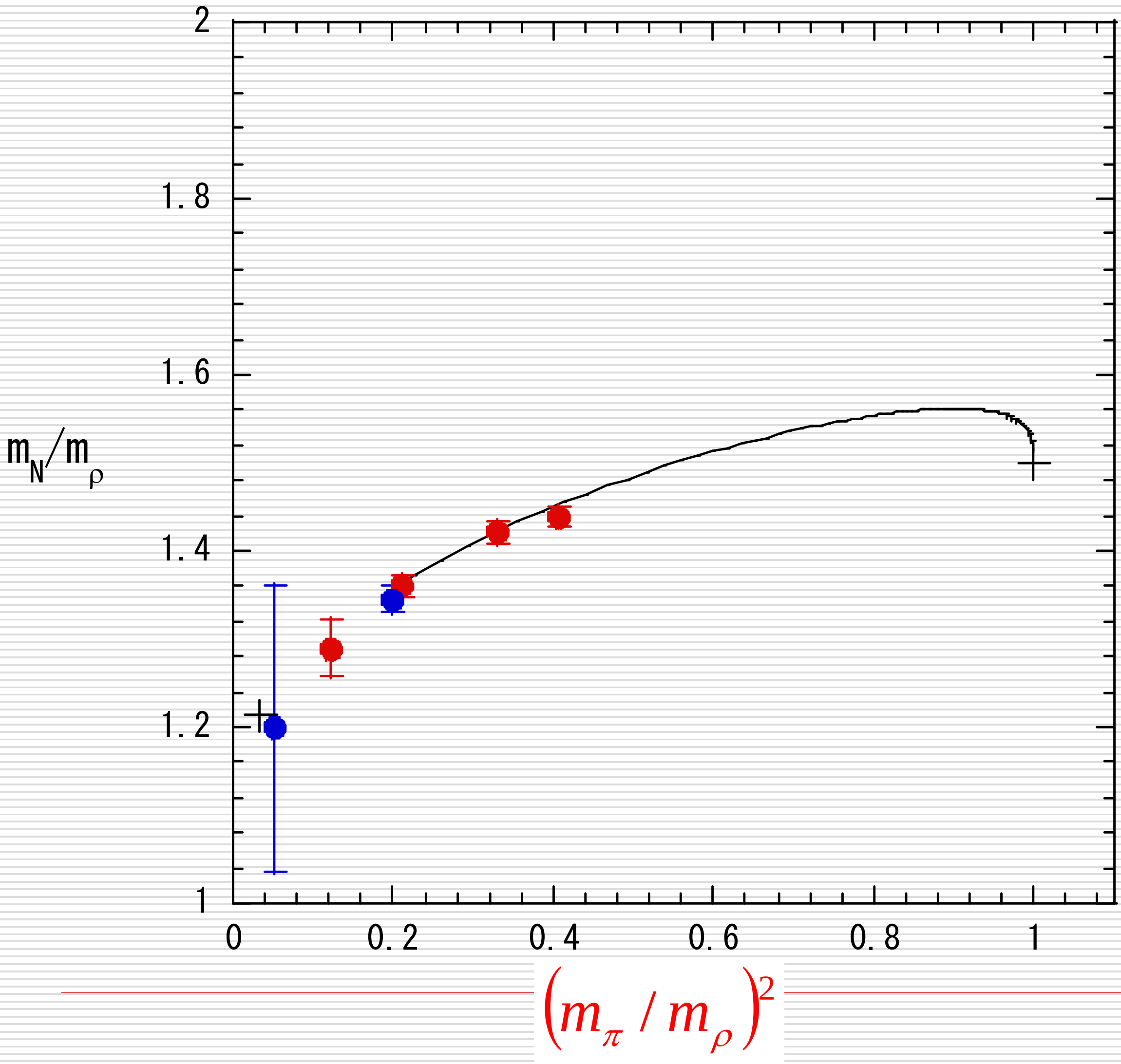
Nf=2+1 full QCD

32^3x64

a=0.09fm

hep-lat arXiv:0807.1661v1

Or if you prefer APE plot …



PACS-CS Collaboration
Wilson-clover action
Nf=2+1 full QCD
32^3x64
a=0.09fm

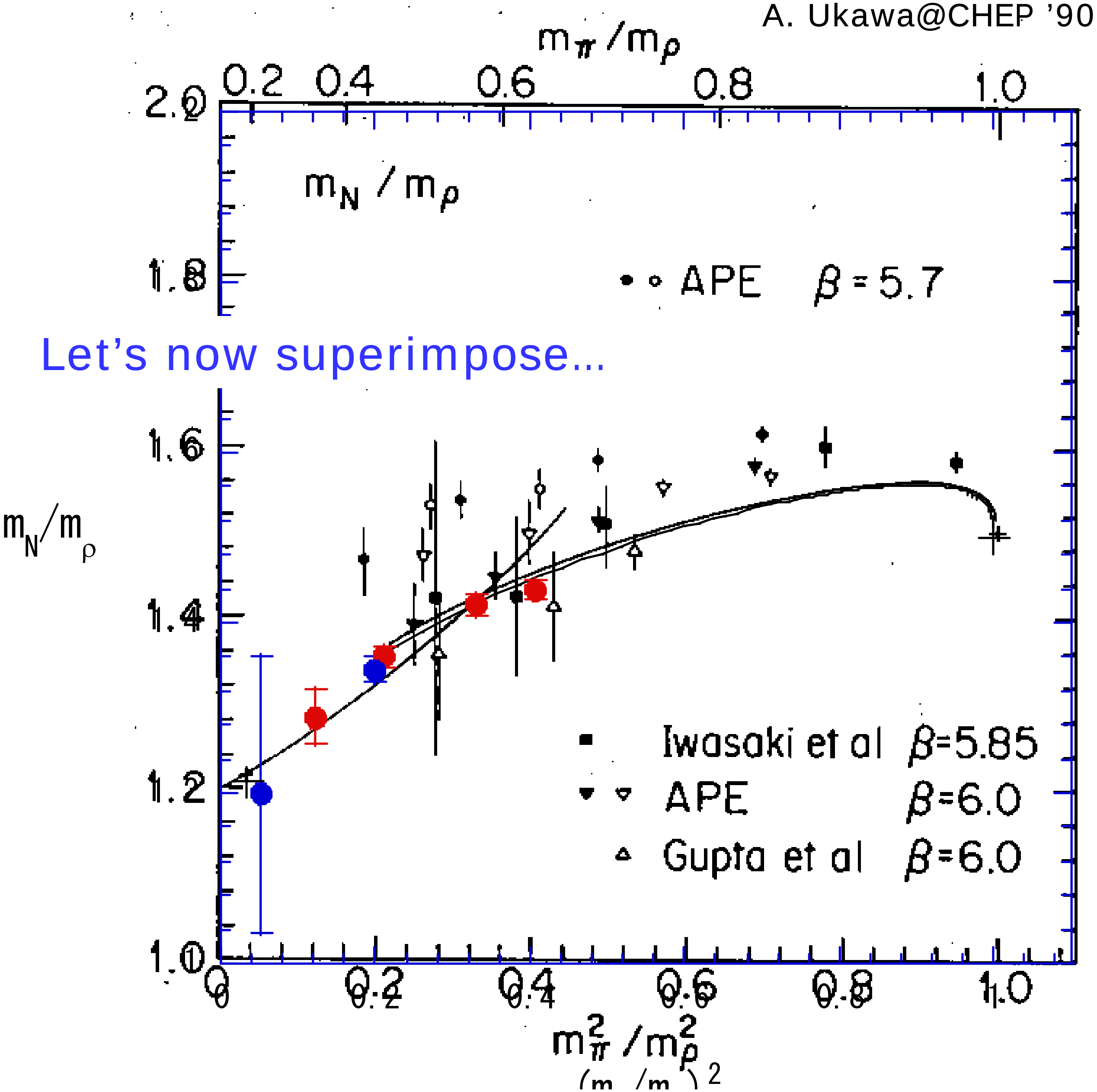
hep-lat arXiv:0807.1661v1

And this is how it was in 1990

all data in quenched QCD

A. Ukawa@CHEP '90

Let's now superimpose...



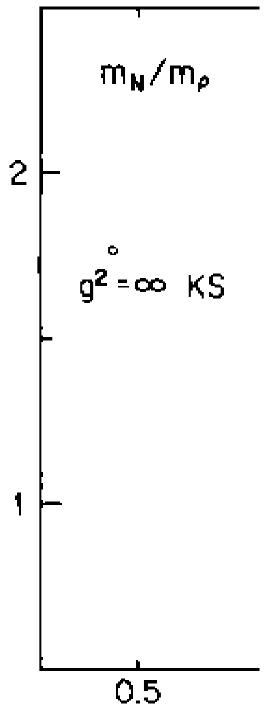
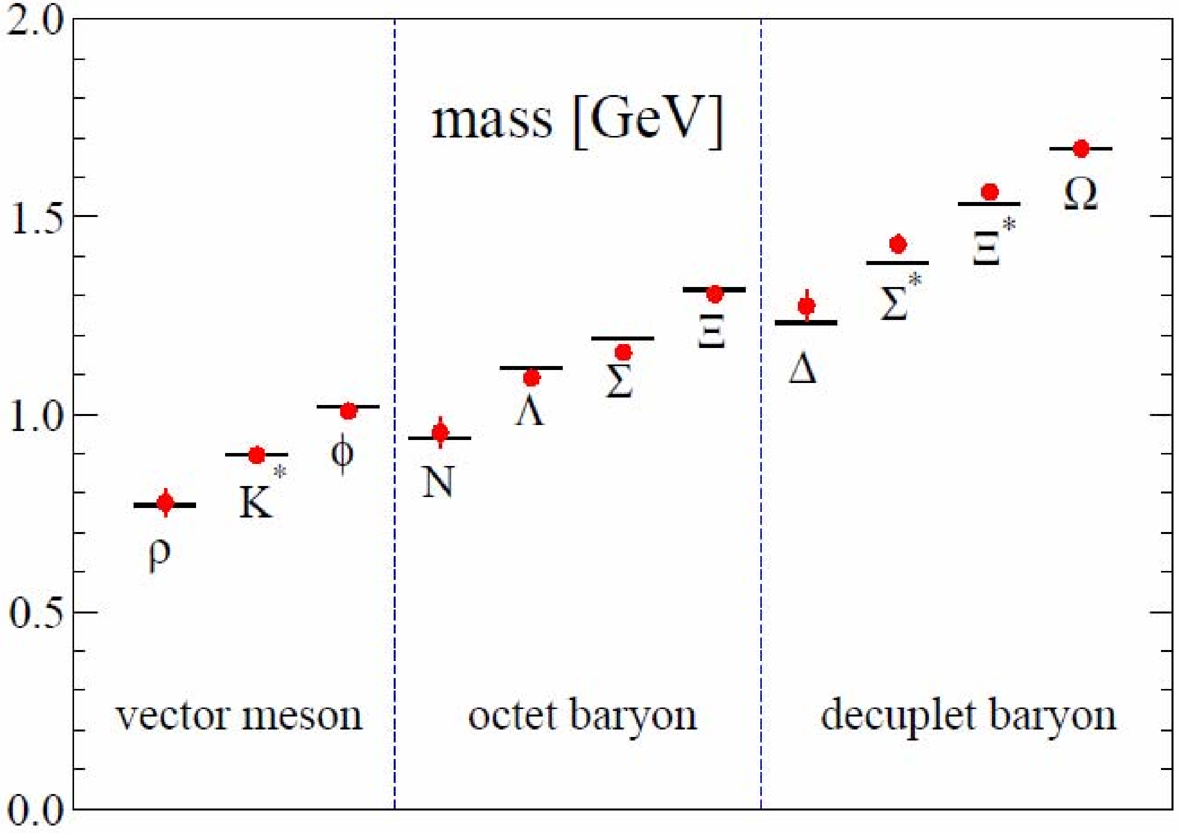


Figure 2: m
a^{-1} in the qu
mass data in t
$m_H a = B \cdot m_\rho a + ($
The data are
at $g^2 = \infty$[35,36

Hadron spectrum(I)

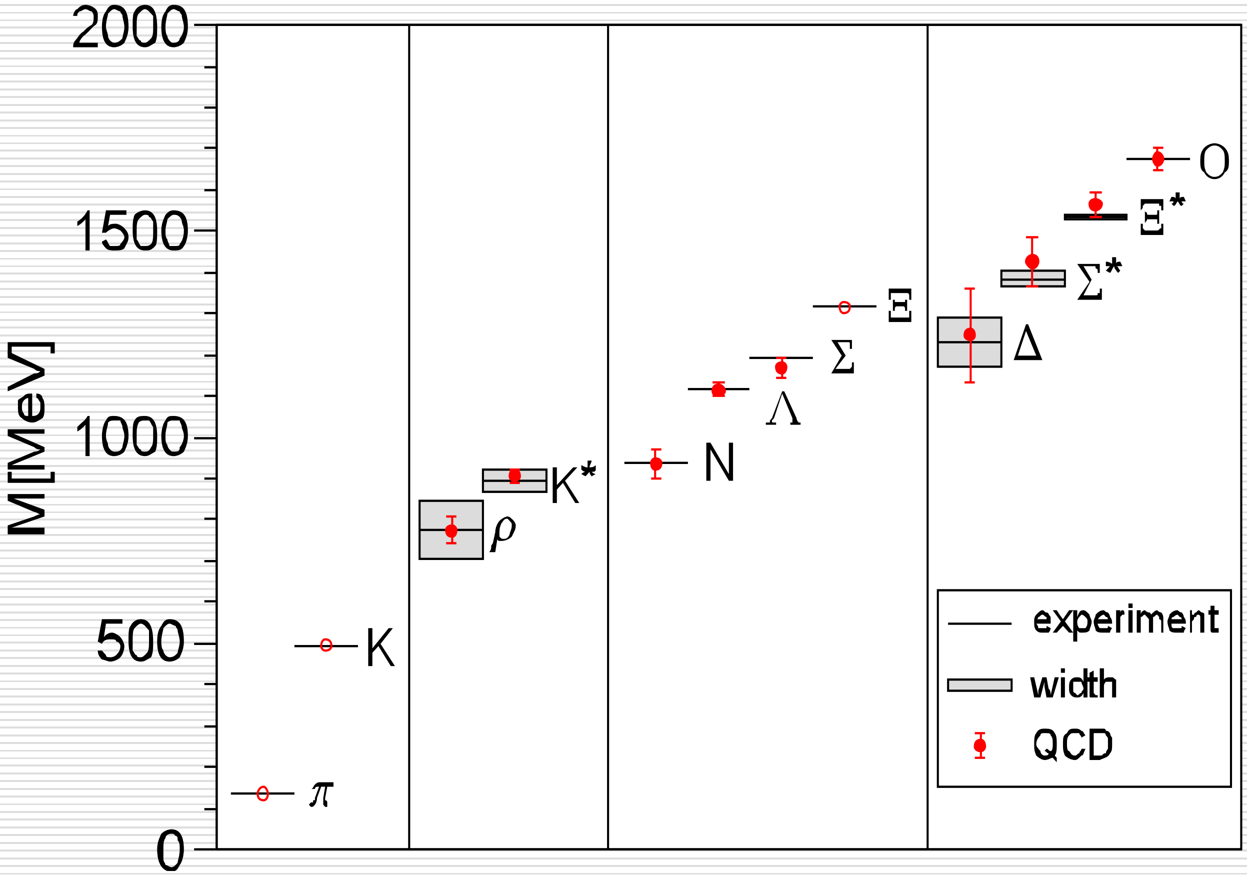


PACS-CS Collaboration
Wilson-clover action
Nf=2+1 full QCD
32^3x64
a=0.09fm

hep-lat arXiv:0807.1661v1

Hadron spectrum (II)

Butapest-Marseille-Wuppertal Collaboration @ Lattice08

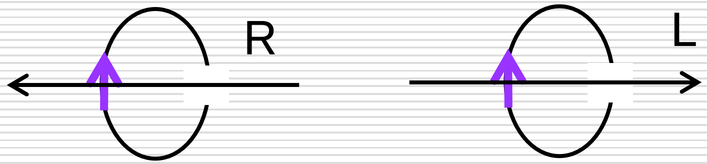


Wilson-clover action
Nf=2+1 full QCD
3 lattice spacing and
continuum
extrapolated

Chiral symmetry and
K→$\pi\pi$ decay

Chiral symmetry and QCD

- Important symmetry for quarks $q \to e^{i\alpha\gamma_5} q$



- Spontaneously broken in Nature $\langle 0|\bar{q}q|0\rangle \neq 0$
  - Nambu-Goldstone boson = pion
  - Small pion mass related to small up, down quark masses

$$m_{up,down} \approx 3MeV,\ m_{strange} \approx 70MeV$$

$$m_{up,down} / m_{proton} \approx 0.003,\ m_{strange} / m_{\Omega} \approx 0.05$$

- Controls much of the low-energy dynamics of the strong interaction
- One of the key features of weak interaction

Nielesen-Ninomiya theorem

- Nielesen-Ninomiya (1981)
  Lattice fermion action satisfying
  - Chiral symmetry
  - Lattice translational invariance
  - Locality

  necessarily has even number of states with the same flavor content,

  i.e., exact chiral symmetry without doubling is not possible on the lattice

- Conventional fermion action
  - Wilson-clover action
    No doubling but explicitly broken by a mass-like term
  - Kogut-Susskind (staggered) action
    Only U(1) chiral symmetry and 4-fold doubling

The way out

- Ginsparg-Wilson relation (1982)

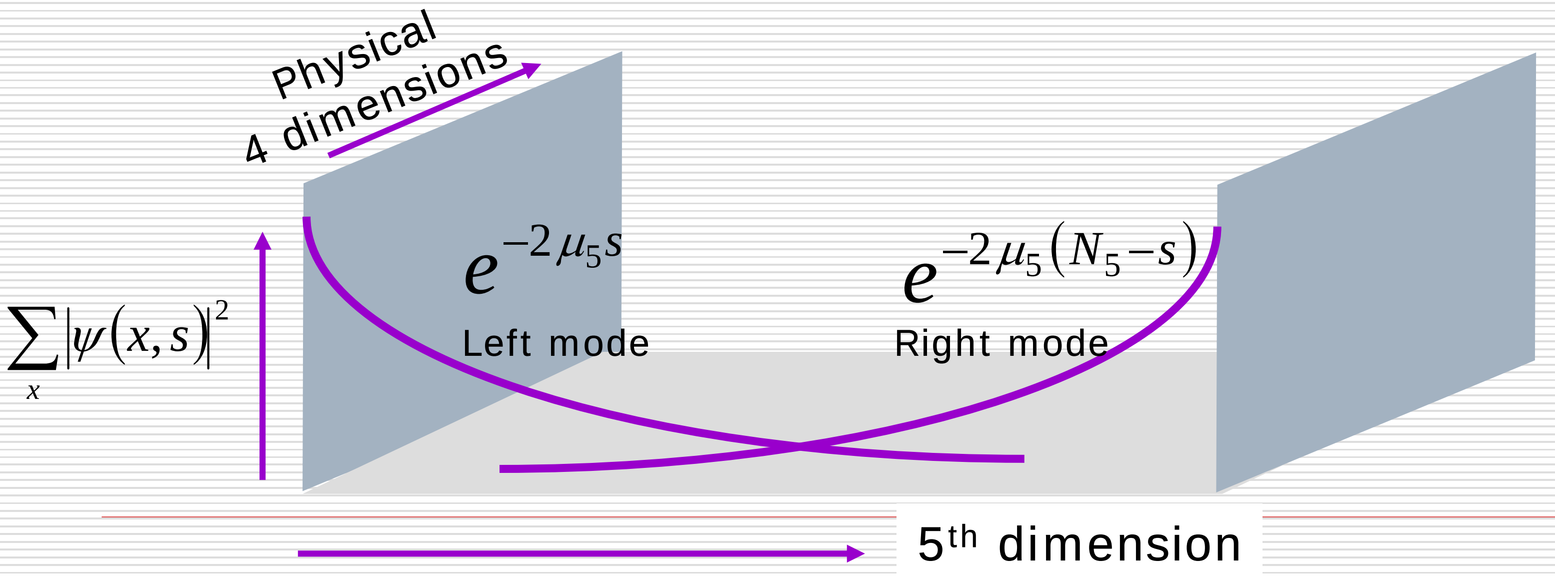
$$D\gamma_5 + \gamma_5 D = 2aD\gamma_5 D \quad \text{or} \quad \gamma_5 \left(D^{-1}\right)_{n,n'} + \left(D^{-1}\right)_{n,n'}\gamma_5 = 2a\gamma_5\delta_{n,n'}$$

  - Avoids the Nielesen-Ninomiya theory via an ultra-local term which breaks chiral symmetry
  - requires infinitely many fields to satisfy the relation (hence needs more computer power to simulate)

- Explicit realizations
  - Domain-wall fermion Kaplan('92)/Furman-Shamir('94)
  - Overlap formalism Neuberger-Narayanan('92,'97)
  - Fixed point action Hasenfratz-Neidermyer('94)

Domain-wall fermion action

- Kaplan(1992)/Furman-Shamir(1994)
  - 5dimensional operator with $D = D^{4d}_{Wilson}\left(-M_W\right) + D^{5d}$
    - 4dim Wilson operator with negative mass
    - 5dim hopping term to control chirality
  - Left and right chiral modes separately bound to the opposite wall in the 5th dimension
  - Exact chiral symmetry for infinite 5th dimension



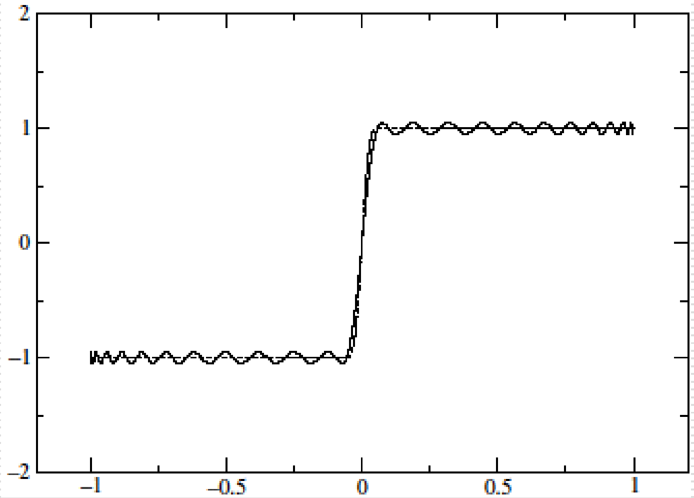
Overlap fermion action

- Neuberger-Narayanan(1998)

$$D = \frac{1}{a}\left[1 + \gamma_5 \,\mathrm{sgn}\left(aH_W\right)\right],\ aH_W = \gamma_5\left(aD_{wilson} - 1\right)$$

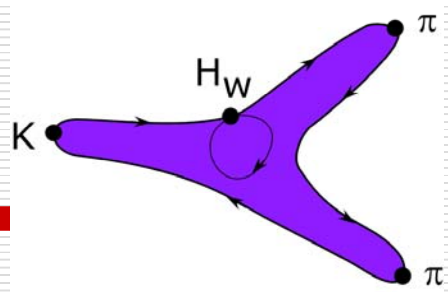
  - Rational approximants such as Zolotarev to approximate the sign function

$$\mathrm{sgn}(x) = x\left(p_0 + \sum_{\ell=1}^{N} \frac{p_\ell}{x^2 + q_\ell}\right)$$



  - Exact chiral symmetry
    by control of near zero modes

K→π π decay



- Weak interaction decays of K mesons
  - ΔI=1/2 rule $$\frac{\mathrm{Re}\,A_0\left(K\to\pi\pi(I=0)\right)}{\mathrm{Re}\,A_2\left(K\to\pi\pi(I=2)\right)}\approx 22$$
  - CP violation $$\frac{\varepsilon'}{\varepsilon}=\begin{cases}(20.7\pm2.8)\times10^{-4} & KTeV\ \mathrm{experiment}\ (FNAL)\\ (15.3\pm2.6)\times10^{-4} & NA48\ \mathrm{experiment}\ (CERN)\end{cases}$$
- Crucial numbers to verify the Standard Model understanding of CP violation
- Chiral symmetry crucial because of the chiral structure of weak interactions
  - In the absence of chiral symmetry, mixing of wrong chirality operators destroys signal in numerical simulations
  - NO success with conventional fermion actions in the I=0 channel

First attempt with domain wall QCD (2003)

- RBC Collaboration, Phys.Rev. D68 (2003) 114506
- CP-PACS Collaboration, Phys.Rev. D68 (2003) 014501

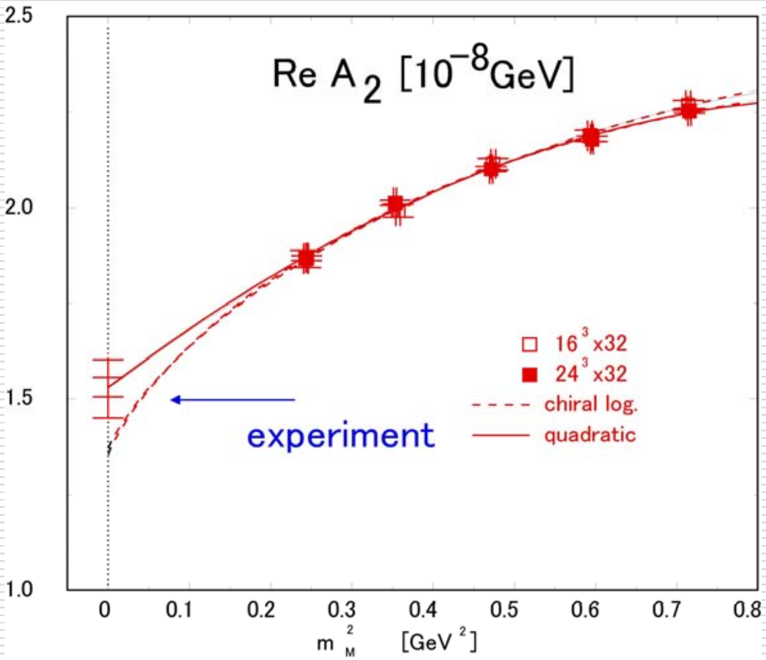
$$H_W = \frac{G_F}{\sqrt{2}} \sum_{i=1}^{10} c_i(\mu / m_W) Q_i(\mu)$$

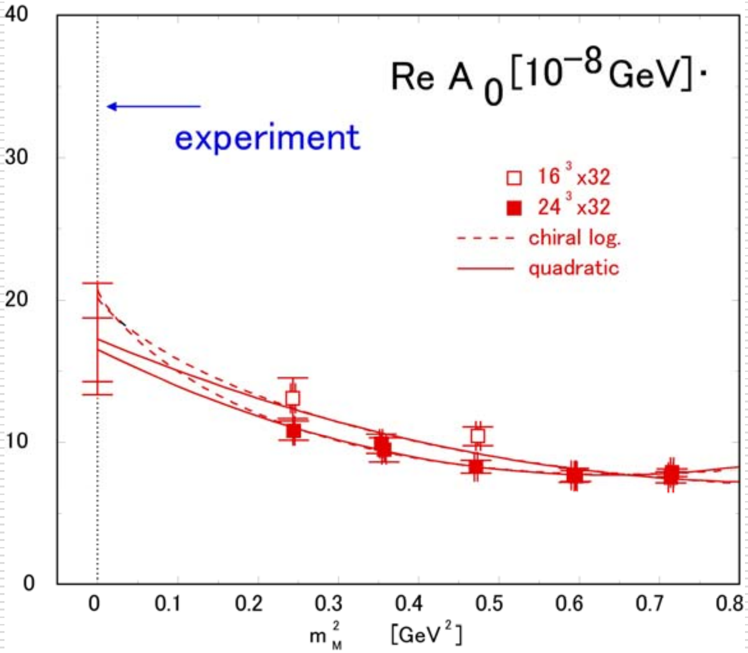
- quenched approximation
- Very complicated involving
  - 2 current-current operators   Q1, Q2
  - 4 QCD penguin operators   Q3, Q4, Q5, Q6
  - 4 EM penguin operators   Q7, Q8, Q9, Q10
- Used K-pi method,
  - i.e., uses chiral symmetry to reduce $K\rightarrow\pi\pi$ decay amplitude to $K\rightarrow\pi$ matrix element, and calculates the latter

$$\langle K | Q_i | \pi\pi \rangle \longleftrightarrow \langle K | Q_i | \pi \rangle, \quad \langle K | Q_i | 0 \rangle$$

Result: ΔI=1/2 rule

- Reasonable agreement with experiment for I=2
- About half of experiment for I=0
- RIKEN-BNL-Columbia obtains a somewhat different result (smaller I=2 and larger I=0)



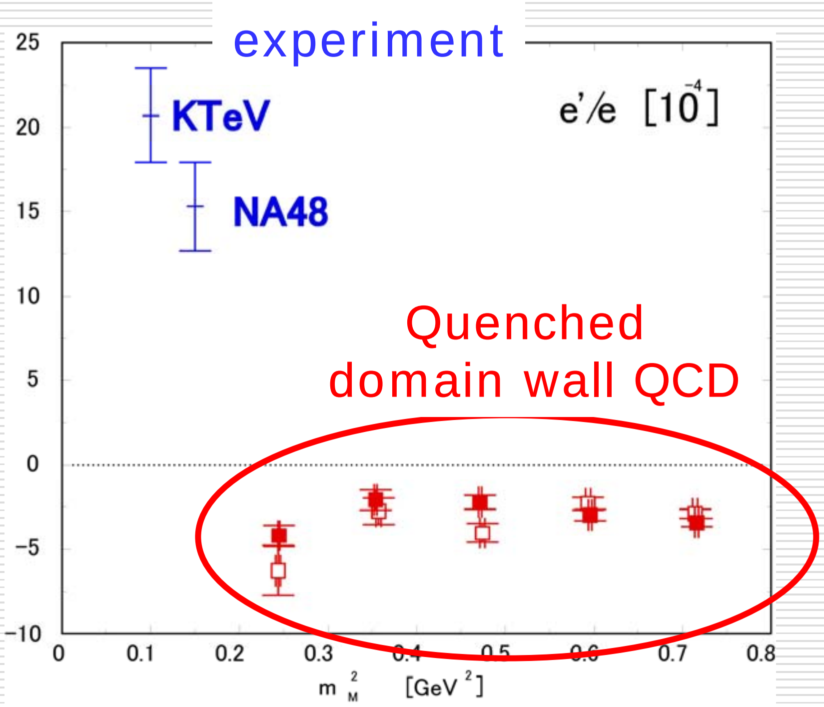


Result: CP violation parameter ε'/ε

- Small and negative in disagreement with experiment

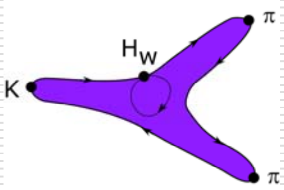
$$\frac{\varepsilon'}{\varepsilon} = \frac{\omega}{\sqrt{2}|\varepsilon|}\left[\frac{\operatorname{Im} A_2}{\operatorname{Re} A_2} - \frac{\operatorname{Im} A_0}{\operatorname{Re} A_0}\right]$$

- Possible reasons
  - connected with insufficient enhancement of ΔI=1/2 rule?
  - Quenched artifact?
  - K-pi method does not work?
- Has remained a major challenge for lattice QCD



Recent developments with direct K→π π amplitude

C. Lellouche and M. Luescher (2001)



- Finite-size formula for direct K→π π amplitude

$$\left|A_{physical}(K\to\pi\pi)\right|^2 = 8\pi\left(\frac{E_{\pi\pi}}{p}\right)^3\left\{p\frac{\partial\delta(p)}{\partial p}+q\frac{\partial\phi(q)}{\partial q}\right\}\left|\langle K|H_W|\pi\pi\rangle_{lattice}\right|^2$$

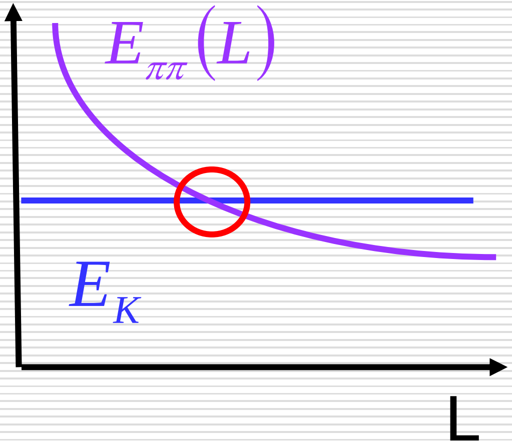
Physical amplitude

Finite volume lattice amplitude

$$p^2 = E_{\pi\pi}^2/4 - m_\pi^2,\quad q^2 = (pL/2\pi)^2$$

$$\tan\phi(q) = -\frac{q\pi^{3/2}}{Z_{00}(1;q^2)},\quad Z_{00}(1;q^2) = \frac{1}{\sqrt{4\pi}}\sum_{\vec{n}\in Z^3}\frac{1}{\vec{n}^2-q^2}$$

$$\delta(p) = n\pi - \phi(q)$$ Phase shift



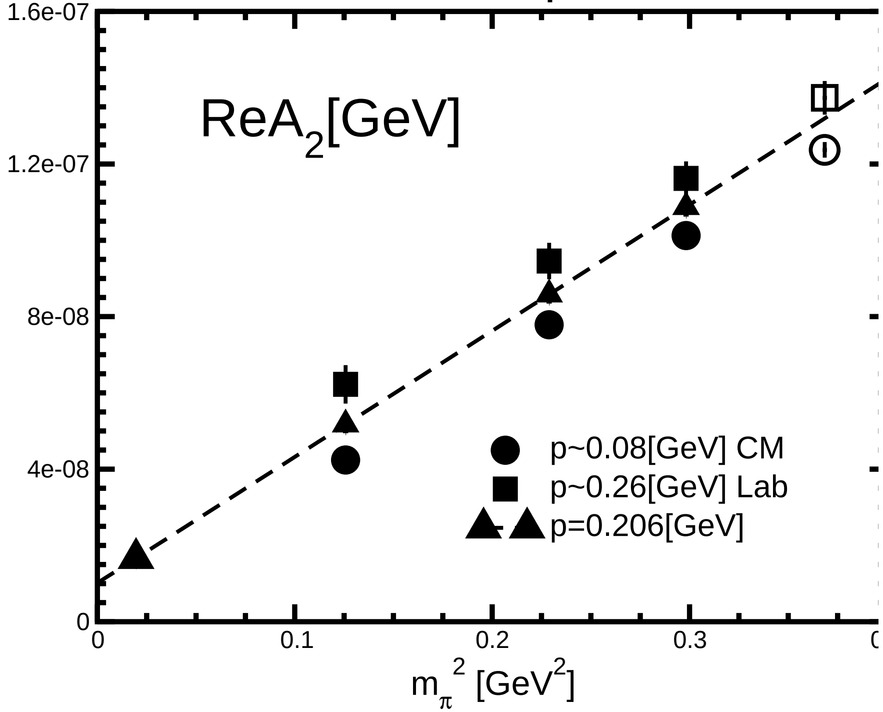
Requires $E_K = E_{\pi\pi}(L)$

Application for I=2 channel with domain wall QCD

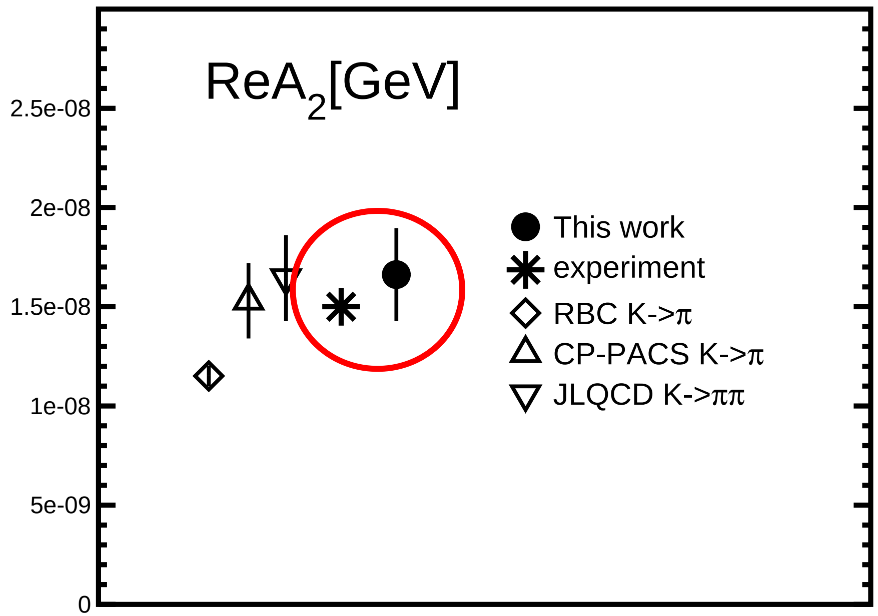
T. Yamazaki, Archive 0807.3130 (2008)

- Only I=2 channel at present, for which previous attempts yielded reasonable results
- But an encouraging start toward a direct K→π π calculation in Nf=2+1 full QCD; expect such a calculation in a few year's time

Pion mass dependence



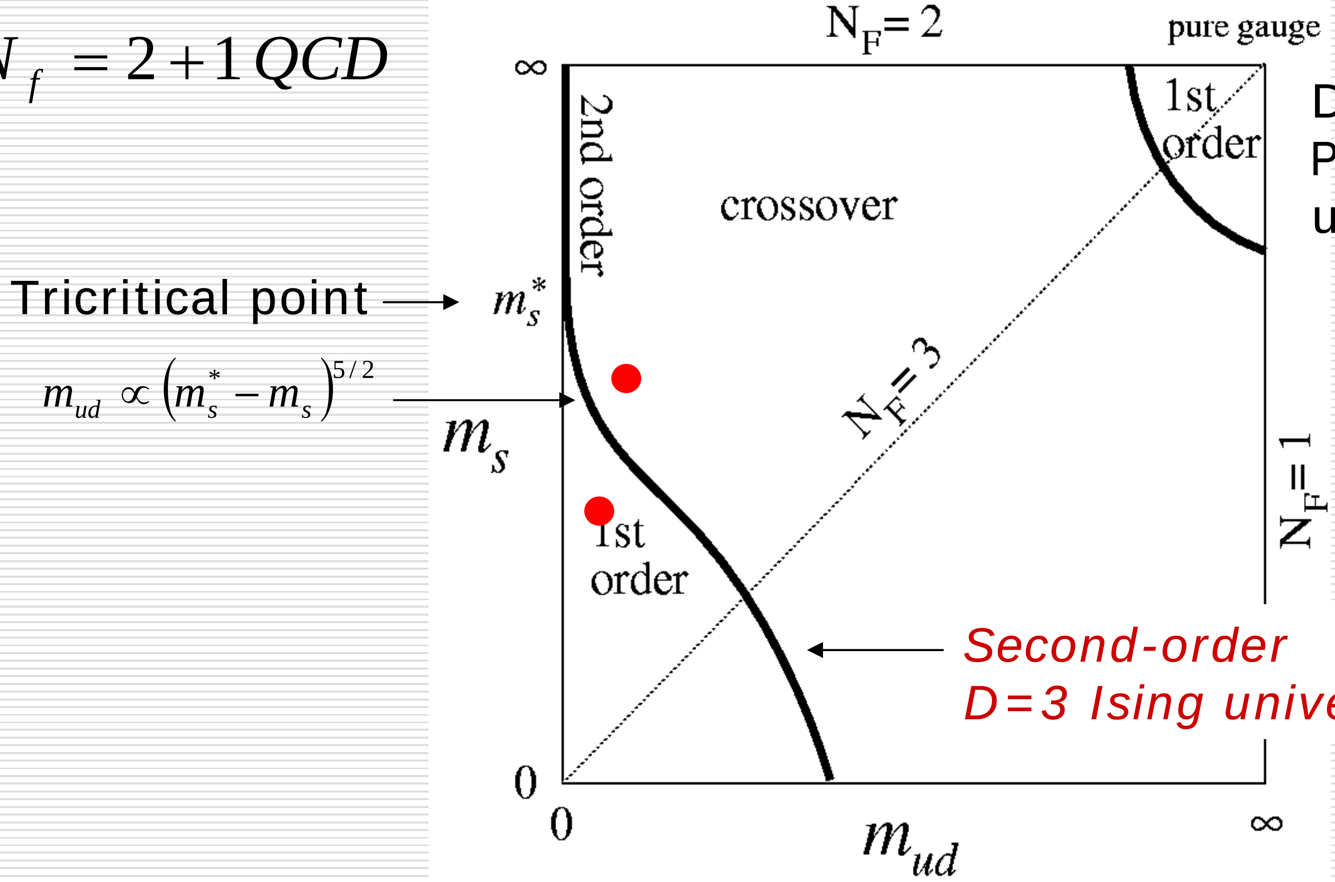
Result for I=2



Hot/Dense QCD

Finite-temperature phases and Tc at $\mu_B=0$

$N_f = 2+1\, QCD$



D=3 Z(3) Potts universality

Second-order D=3 Ising universality

Theory predictions Consistent with most of simulations so far.

Where is the physical point?

Nature of the transition for the physical point

- Long-standing issue addressed by a large number of simulations since mid 1980's
  - Bielefeld group, MILC Collaboration, JLQCD Collaboration etc

- "crossover and no real phase transition" has been a general concensus.

- Recent work employs dynamical staggered up, down, strange quark with realistically small quark masses
  - Wuppertal group, Y. Aoki, G. Endrodi, Z. Fodor, S.D. Katz, K. Szabo,

    Nature 443 (2006) 675-678; PLB643 (2006) 46-54;Lattice08
  - Bielefeld-RBRC-BNL Collaboration, M. Cheng et al,

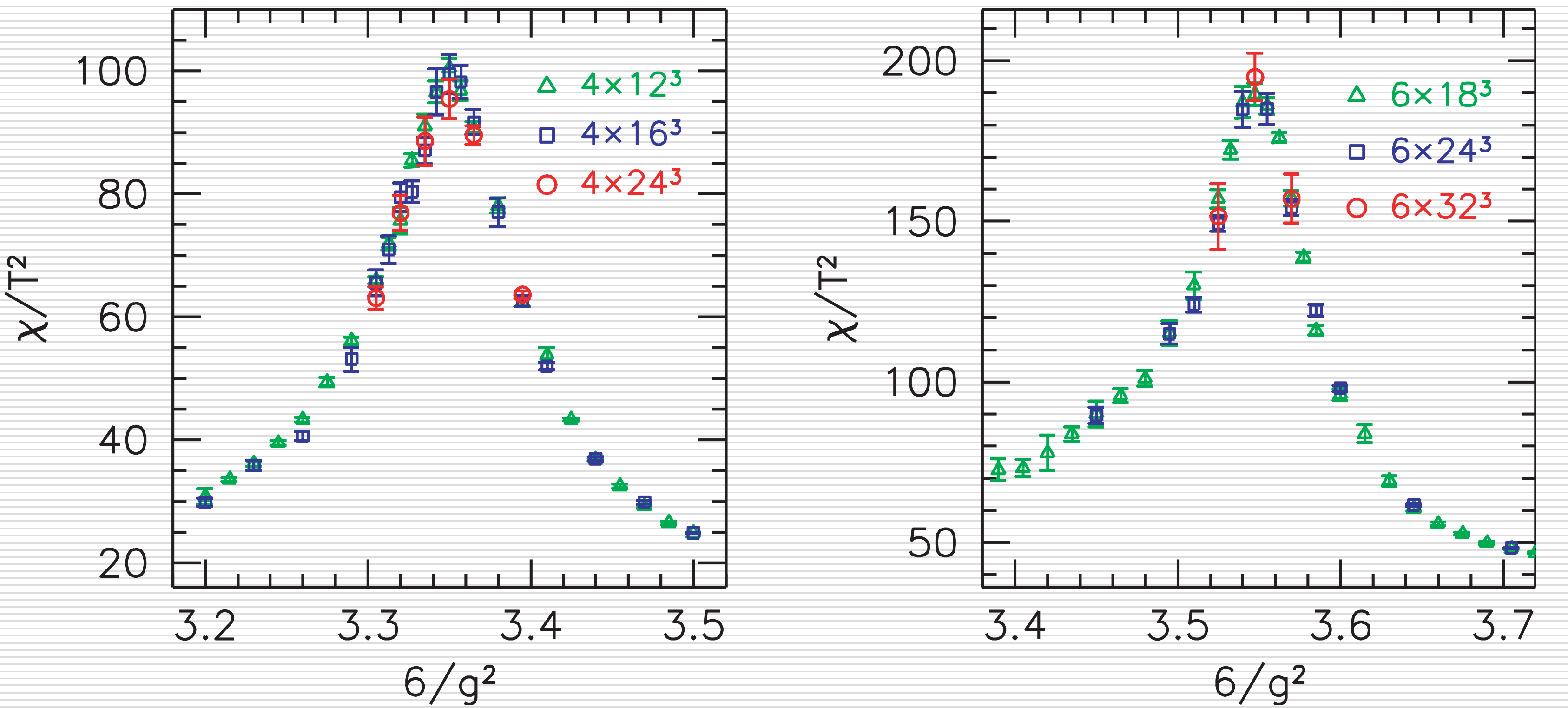
    PR D75 (2007) 034506, D74 (2006) 054507
  - HOTQCD Collaboration

    Lattice07;Lattice08

Finite-size analysis of chiral susceptibility

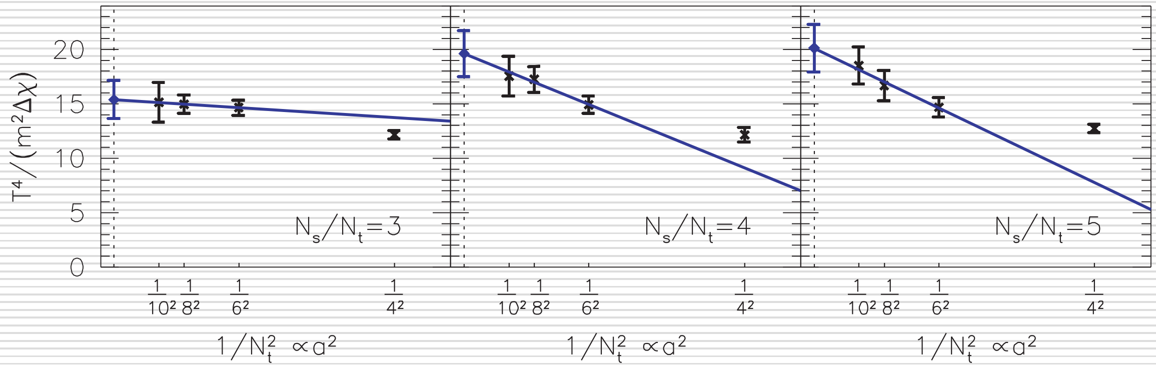
Y. Aoki et al, Nature 443 (2006) 675-678

- Constant behavior for increasing volume shows a crossover at finite lattice spacing



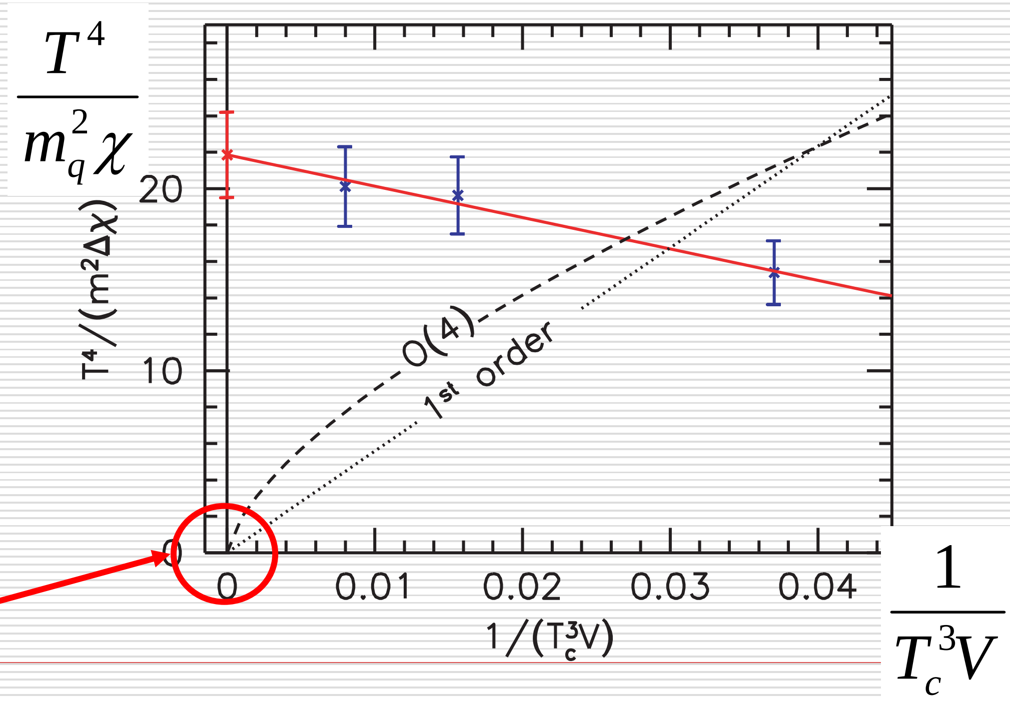
Continuum limit extrapolation

Y. Aoki et al, Nature 443 (2006) 675-678



Extrapolation to the continuum

- Chiral susceptibility is finite for infinite volume in the continuum
- "Crossover", at least for staggered action



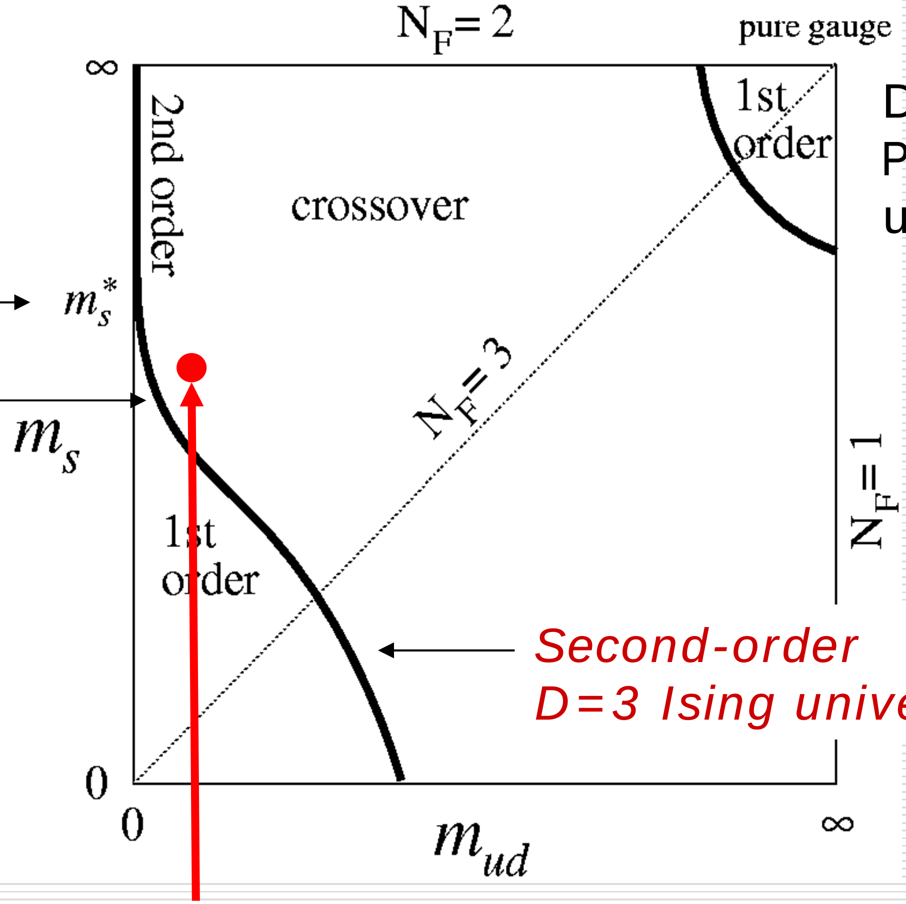
Infinite volume

Finite-temperature phases and Tc at $\mu_B=0$

$N_f = 2+1\, QCD$

Tricritical point →

$m_{ud} \propto \left(m_s^* - m_s\right)^{5/2}$



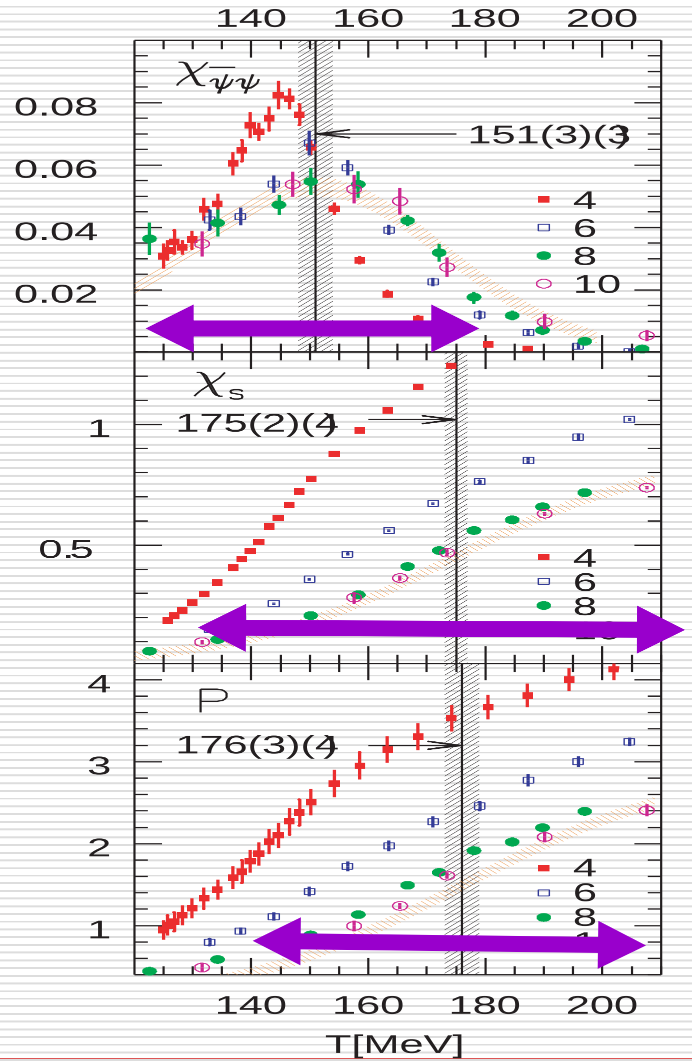
D=3 Z(3) Potts universality

Second-order D=3 Ising universality

physical point according to staggered action

Transition temperature

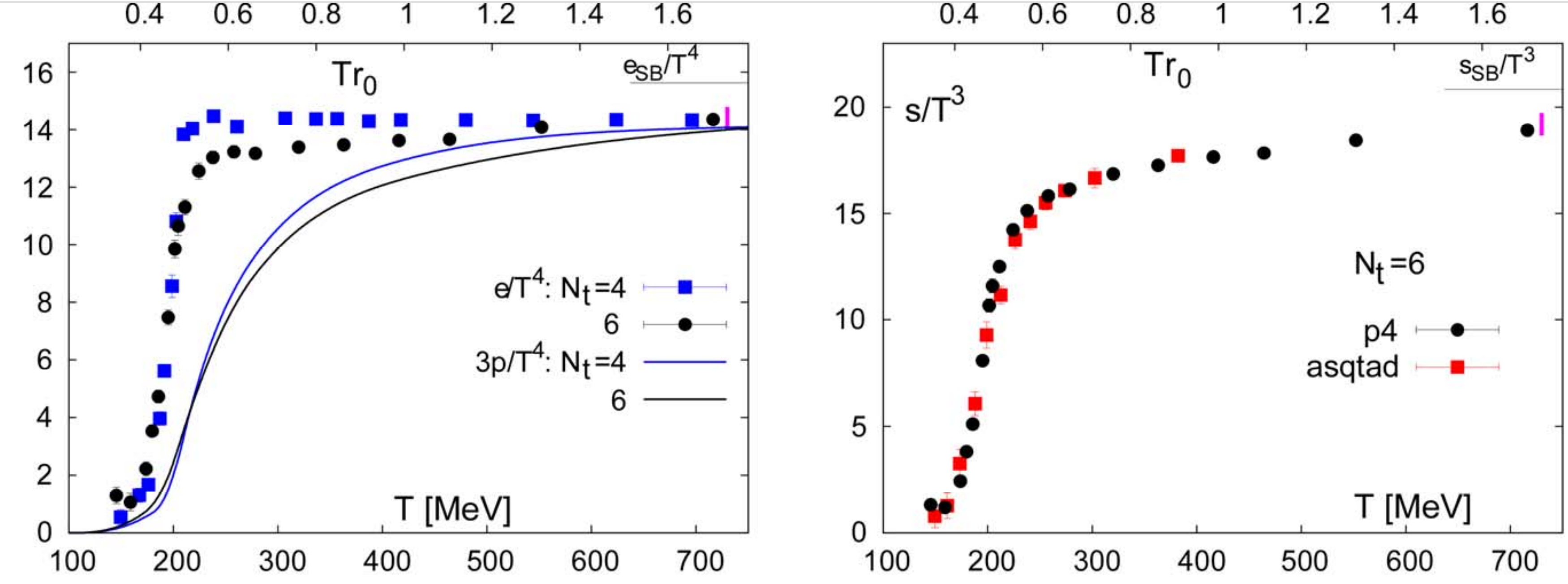
- Point of dispute in recent literature
  - Wuppertal Group
  - RBC-Bielefeld, HOTQCD
- My view
  - No sharp definition since crossover; fairly broad in practice
  - May sizably depend on the quantity
  - Scale setting also introduces uncertainties



Y. Aoki etal PL B643 (2006) 46-54

Equation of state

- Recent results from MILC and Bielefeld-RBRC
- Agreement between the two groups; qualitatively no change since previous results



p4: RBC-Bielefeld, preliminary

asqtad: C. Bernard et al., PRD75, 094505 (2007)

entropy density: p4 vs. asqtad

F. Karsch, Lattice 2007, Regensburg, July 2007 – p. 20/33

How much do we trust staggered results?

- Theoretical uncertainties with the staggered simulations
  - Only U(1)xU(1) chiral symmetry out of $SU(N_f)xSU(N_f)$
  - Fractional power of quark determinant $[detD(U)]^{Nf/4}$ to "adjust" the #flavor
- Does it converge to the correct QCD in the continuum limit?
  - OK perturbatively, but is it at the non-perturbative level?
  - Lots of discussions in the the community, not yet settled:
    - Lattice06
      S. Sharpe, "Rooted staggered fermions: good, bad, or ugly?"
    - Lattice07
      M. Creutz, "Why rooting fails"
      A. Kronfeld, "Lattice QCD with Staggered Quarks: Why, Where, and How"
- Clearly desirable to work with chiral action:
  - Domain-wall
  - Overlap

Much work and many simulations already done at T=0, so hot/dense QCD is the next natural target.

Transport coefficients in QGP

- Very limited work over the years
  - Karsch, Wylde, PRD35 (1987)2518
  - S. Gupta, PLB597(2004)57
  - Nakamura, Sakai, PRL94 (2005)072305
  - Aarts, Allton. Foley, Hands, Kim, PRL99(2007)022002
  - H. B. Meyer, hep-lat/0704.1801
- resurgence of interest due to RHIC experiment

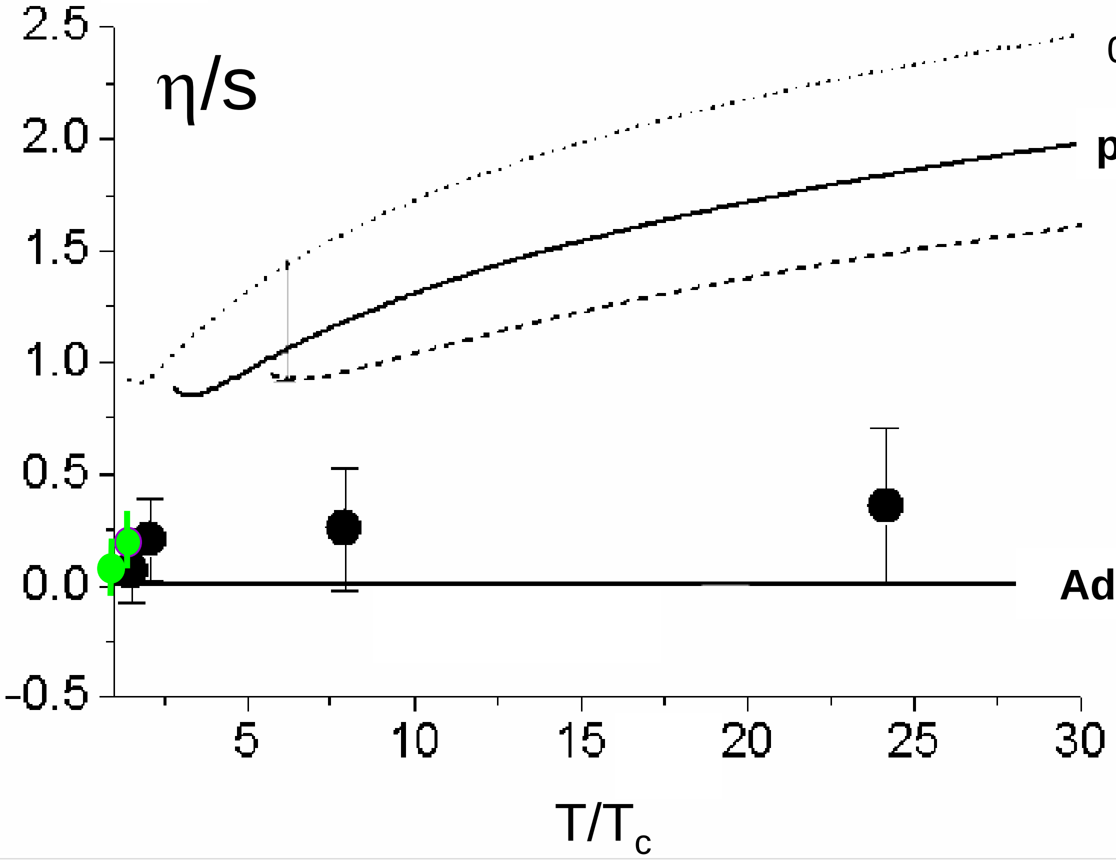
  e.g.,

  $$\ell \approx \frac{\eta}{sT} \quad \text{mean free path} \qquad R \approx \left(\frac{\eta + \varsigma}{s} \frac{1}{T} \frac{1}{\tau} \right)^{-1} \quad \text{Reynolds number}$$

  η :shear viscosity, s: entropy density
- Application of the Kubo formula

$$\eta = \lim_{\omega \to 0} \frac{\rho_{12,12}(\omega, 0)}{2\omega} \qquad \rho_{\mu\nu,\rho\sigma}(\omega, \vec{p}) = \int d^4x \exp(ip \cdot x) \langle T_{\mu\nu}(x) T_{\rho\sigma}(0) \rangle$$

Quenched result for shear viscosity



Baym, Monien, Pethick, Ravenhall, PRL64 (1990)1867.
Arnold, Moore & Yaffe, JHEP 0305(2003)051.

Kovtun, Son & Starinets, PRL94(2005)111601

● 24^3x8 — Nakamura, Sakai, hep-lat/0510100

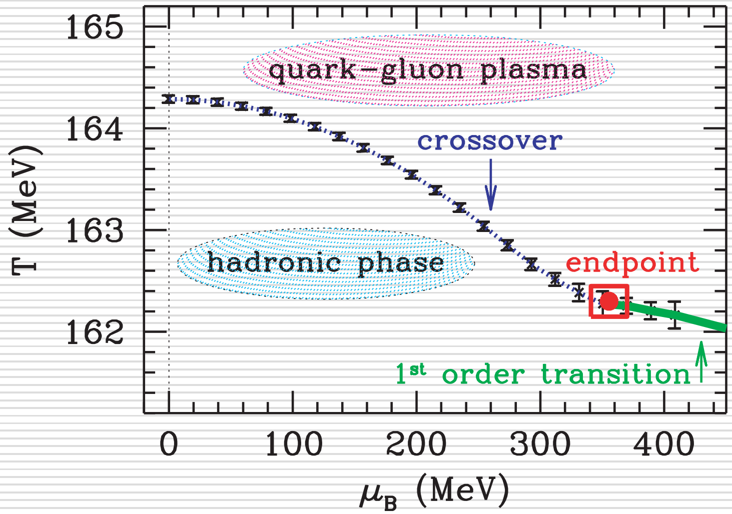
● 20^3x8,28^3x8 — H. Myer, hep-lat/0704.1801

Comments on finite-density QCD

- The *"sign problem"*, i.e., large phase fluctuation of the quark determinant detD for non-zero density

$$Z_{QCD} = \int \prod dU_{n\mu} \det D[U] \exp(-S_{gluon}[U])$$

Z. Fodor, S. Katz, JHEP 0404 (2004) 050



- Slow but steady progress over the years for *not too large baryon density:*
  - Estimate of the end point of the 1st order line on the T-μ plane
- Still no real prospect for large baryon number density

2-parameter reweigting method:
Z. Fodor, S. Katz, JHEP 0404 (2004) 050
Nf=2+1, Nt=4

$$(T_E, \mu_E) = (162 \pm 2, 360 \pm 40) MeV$$

Taylor expansion method:
C. Allton etal, Phys.Rev. D71 (2005) 054508
Nf=2, Lt=4

Try Complex Langevin algorithm?

Parisi-Wu (1981), Klauder, Parisi, ...

Recent work Aarts-Stamatescu archive0807.1597

Conclusions

Where we stand now

- *Realistic calculation directly at the physical point finally in sight*
  - Fruit of continuous effort over 25 years toward:

    Better physics understanding

    Better algorithms

    More powerful machines

- *Change of philosophy from "simulation" to "calculation"*
  - No more approximations/extrapolations
  - Gluon configuration produced is Nature itself

Where do we go

- *Expect that the fundamental issues of lattice QCD as particle theory makes major progress over the next five year range*
  - Single hadron properties
  - Weak interaction aspects such as $K \to \pi\pi$ decays
  - Hot/dense QCD with chiral lattice action on large lattices
- *Vast area of multi-hadron systems/atomic nuclei lies in wait for nuclear physics colleagues to explore*
  - Nuclear force from lattice QCD
  - Exotic nuclei with unusual n/p ratios/strangeness etc

Nuclear force from lattice QCD(2007)

N. Ishii, S. Aoki, T. Hatsuda, PRL 99, 022001 (2007)

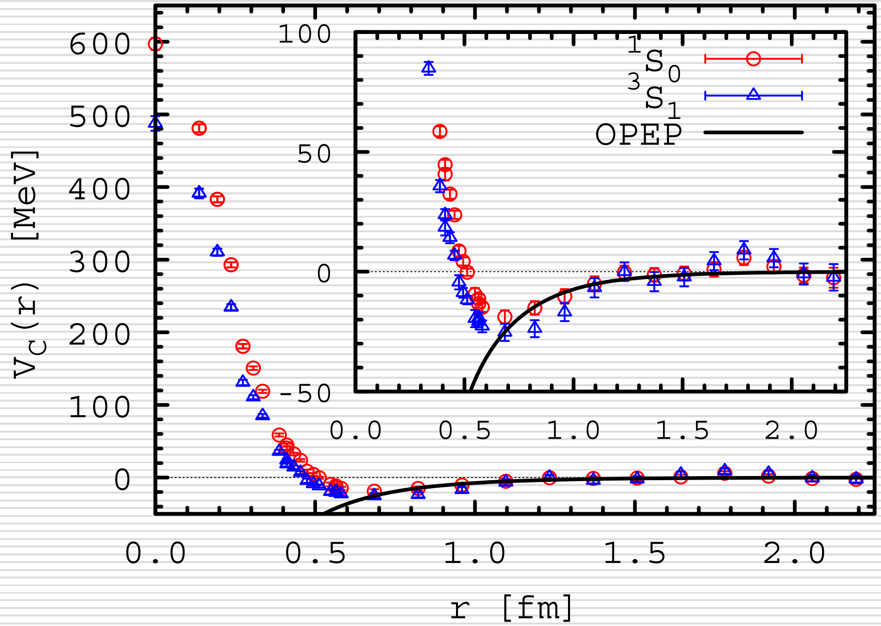
- 2-nucleon BS amplitude from lattice QCD

$$\phi(r) = \frac{1}{L^3} \sum_{\vec{x} \in L^3} \langle 0 | N(\vec{x} + r) N(\vec{x}) | NN \rangle$$

- Extraction of potential from an effective Schrodinger eq.

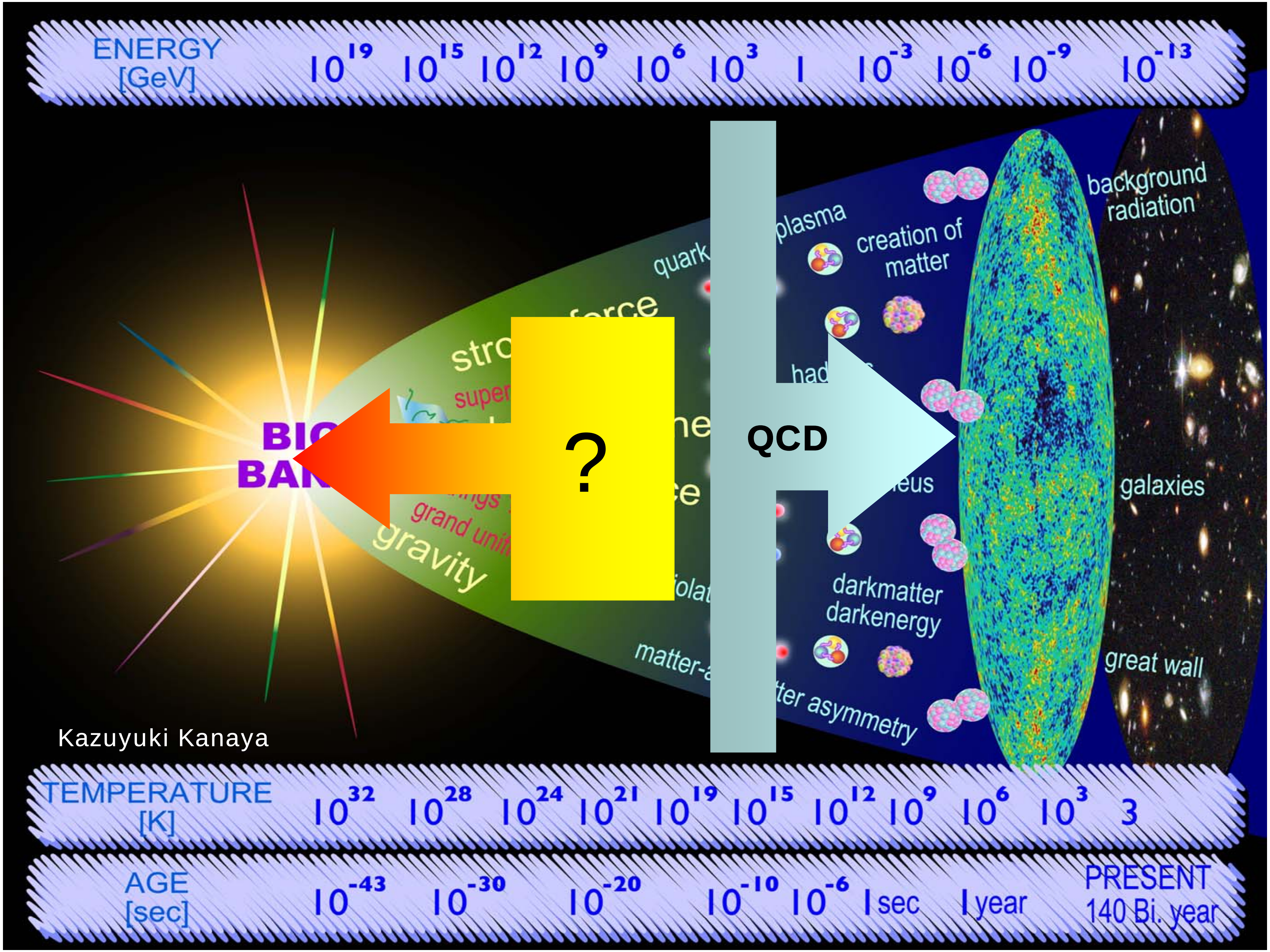
$$V(r) = E + \frac{1}{2\mu} \frac{\nabla^2 \phi(r)}{\phi(r)}$$

- Impact and prospects
  - Derivation of the hard core
  - Extension to hyperon-nucleon potential etc



Quenched QCD 32^4 lattice

$m_\pi / m_\rho = 0.595$


ENERGY [GeV]
10^19 10^15 10^12 10^9 10^6 10^3 1 10^-3 10^-6 10^-9 10^-13
plasma
creation of matter
background radiation
quark
?
QCD
galaxies
gravity
grand unif
darkmatter darkenergy
great wall
matter-a
ter asymmetry
Kazuyuki Kanaya
TEMPERATURE [K]
10^32 10^28 10^24 10^21 10^19 10^15 10^12 10^9 10^6 10^3 3
AGE [sec]
10^-43 10^-30 10^-20 10^-10 10^-6 1sec 1year
PRESENT 140 Bi. year