Time Reversal Violation in two Nucleons Systems

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Introduction

The Time Reversal Violation in Nuclei

- In the Standard Model (SM) it is possible to introduce *P*-violating and *C*-conserving terms and therefore time reversal violating (TRV):
    - phase of the CKM matrix
    - phase of the neutrino's mixing matrix
    - θ-term
- The effects of the CKM and neutrino's mixing matrix phases give small contributions in observables which do not involve flavour changes
- Light nuclei can be good candidates to investigate TRV effects from θ:
    - Permanent Electric Dipole Moment (EDM)
    - Spin rotation (this work)
- Related issues:
    - "Strong CP" problem ($\bar{\theta} < 10^{-10}$ from neutron EDM)
    - possible effects beyond the SM [J. Bsaisou, *et al.*, 2015; E. Mereghetti and U. van Kolck, 2015]

The TRV Lagrangian in the χEFT [J. Bsaisou, *et al.*, 2015]

$$\overline{\mathcal{L}}_{\rm QCD} = \overline{q} i \gamma^\mu D_\mu q - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}^{\mu\nu}_a \overbrace{- \overline{q}\mathcal{M}q - \theta \frac{g^2}{64\,\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathcal{G}^a_{\mu\nu}\mathcal{G}^a_{\rho\sigma}}^{\mathcal{L}^{\mathcal{M}}_{\rm QCD}}$$

$$\mathcal{M} = e^{i\rho}\begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$$

$$\Downarrow \quad U(1)_A$$

$$\mathcal{L}^{\mathcal{M}}_{\rm QCD} = -\overline{q}(\overline{m}1 + \epsilon\,\overline{m}\,\tau_3 - i\frac{\overline{\theta}\,\overline{m}}{2}(1-\epsilon^2)\gamma^5 1)q$$

$$\overline{m} = (m_u + m_d)/2 \qquad \overline{\theta} = 2\rho - \theta$$

$$\Downarrow \quad \chi\text{EFT } (\bar{\theta} \text{ as external source})$$

At nuclear level $N = (n, p)$

$$\mathcal{L}^{(\rm TRV)} = \overline{N}(\overline{g_0}^\theta\,\vec{\tau}\cdot\vec{\pi} + \overline{g_1}^\theta\pi_3)N + M\overline{\Delta}^\theta\pi_3\pi^2 + \frac{\overline{C_1}^\theta}{2\Lambda^2_\chi f_\pi}\overline{N}N\partial_\mu(\overline{N}\gamma^\mu\gamma^5 N) + \frac{\overline{C_2}^\theta}{2\Lambda^2_\chi f_\pi}\overline{N}\vec{\tau}N\partial_\mu(\overline{N}\vec{\tau}\gamma^\mu\gamma^5 N)$$

Construction of TRV potential

T matrix potentials

Let us consider the scattering $NN \to N'N'$

- Transition amplitude $S = 1 - 2\pi i \delta(E_i - E_f) T$
- Using the time-ordered perturbation theory from $\mathcal{L}(\pi, \mathrm{N})$ we get

$$T_{\mathrm{EFT}} = \sum_n T^{(n)}, \qquad T^{(n)} \sim Q^n$$

- Condsidering a NN system which interacts with a potential V we solve the Schröedinger Equation

$$T = V + V \frac{1}{E_0 - H_0 + i\epsilon} T$$

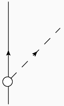
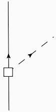
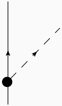
$$T = V + V \frac{1}{E_0 - H_0 + i\epsilon} V + V \frac{1}{E_0 - H_0 + i\epsilon} V \frac{1}{E_0 - H_0 + i\epsilon} V + \cdots$$

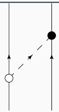
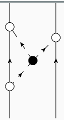
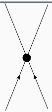
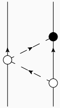
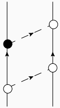
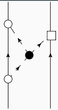
- Assuming

$$V = \sum_n V^{(n)}, \qquad V^{(n)} \sim Q^n$$

We define V such that $T = T_{\mathrm{EFT}}$ order by order

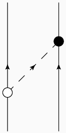
Chiral counting of the “Time-ordered” diagrams

PC vertex		TRV vertex
 $\sim Q^{\frac{1}{2}}$	 $\sim Q^{\frac{1}{2}}$	 $\sim Q^{-\frac{1}{2}}$

Order	Chiral Power	TRV diagrams
LO	Q^{-1}	
NLO	Q^{0}	
N2LO	Q^{1}	   

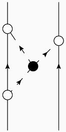
The TRV potential

Q^{-1}
(LO)



$$V_{\rm TRV}^{(-1)} = -\frac{g_A \bar{g}_0^\theta}{2 f_\pi}(\vec{\tau}_1\cdot\vec{\tau}_2)\frac{i(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\boldsymbol{k}}{\omega_k^2} - \frac{g_A\bar{g}_1^\theta}{4f_\pi}\Big[(\tau_{1z}+\tau_{2z}) \times \frac{i(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\boldsymbol{k}}{\omega_k^2} + (\tau_{1z}-\tau_{2z})\frac{i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{k}}{\omega_k^2}\Big]$$

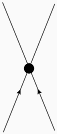
Q^0
(NLO)



$$V_{\rm TRV}^{(0)} = \frac{5 g_A^3 M \bar{\Delta}^\theta}{4 f_\pi}\frac{\pi}{\Lambda_\chi^2}\Big[(\tau_{1z}+\tau_{2z})\frac{i(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\boldsymbol{k}}{\omega_k^2} + (\tau_{1z}-\tau_{2z}) \times \frac{i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{k}}{\omega_k^2}\Big]\left(1-\frac{2m_\pi^2}{s^2}\right)s^2 A(k)$$

$$A(k) = \frac{1}{2k}\arctan\left(\frac{k}{2m_\pi}\right) \qquad s = \sqrt{4\,m_\pi^2 + k^2}$$

Q
(N2LO)



$$V_{\rm TRV}^{(1)} = -\frac{\bar{C}_1^\theta}{2\Lambda_\chi^2 f_\pi} i\boldsymbol{k}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2) - \frac{\bar{C}_2^\theta}{2\Lambda_\chi^2 f_\pi} i\boldsymbol{k}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)(\vec{\tau}_1\cdot\vec{\tau}_2)$$

5 LECs

NLO complete, N2LO two pions exchange in progress

The potentials in configuration space

- The loop divergences are corrected through dimensional regularization
- To solve the Schröedinger equation we need the potential in configuration space

The potential is valid only for $Q \ll \Lambda_\chi$
$\Rightarrow$ we introduce a cut-off $C_{\Lambda_F}(k) = \exp(-(k/\Lambda_F)^4)$

- The Fourier transform results

$$V(r) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} V(k) C_{\Lambda_F}(k)$$

- The observables should not depend on Λ_F

The $\vec{n} - \vec{p}$ spin rotation

Calculation of the observables

We solve the Schröedinger equation

$$\Big(-\frac{\nabla^2}{2\mu}+V(\mathbf{r})\Big)\psi(\mathbf{r})=E\psi(\mathbf{r}) \qquad V=V_{\mathrm{PC}}+V_{\mathrm{TRV}}$$

where $V_{\mathrm{PC}} \to V(\mathrm{N3LO})$ [Entem & Machleidt, 2011]

- We use the Köhn variational principle to find the scattering states
- From the asymptotic behaviour of $\psi(\boldsymbol{r}) \to S$-matrix
- Scattering amplitude $f(\theta) \propto S$

Spin rotation

Ultracold neutron beam ($E \simeq 0.0001$ MeV) which pass through an hydrogen gas layer of width d

$\Rightarrow$ refraction index n [P. K. Kabir, 1982]

$$\psi_{in} = e^{ip_n z}|\chi\rangle \;\Rightarrow\; \psi_{out} = e^{ip_n(z-d)} e^{ip_n d n}|\chi\rangle$$

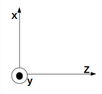
$|\chi\rangle =$ intial spin state

$$n-1 = \frac{2\pi N}{p_n^2} f(0) = \frac{2\pi N}{p_n^2}\Big(f_0 + \underbrace{f_M(\boldsymbol{\sigma}\cdot\boldsymbol{S})}_{\text{spin interaction}} + \overbrace{f_P(\boldsymbol{\sigma}\cdot\boldsymbol{p}_n)}^{\text{PV}} + \underbrace{f_T\boldsymbol{\sigma}\cdot(\boldsymbol{p}_n\times\boldsymbol{S})}_{\text{TRV}}\Big)$$

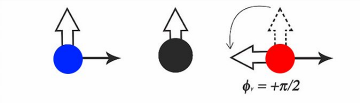
- $f(0)$ forward scattering amplitude
- p_n neutron momentum
- σ spin operator of the incoming neutron
- S spin operator of the proton
- $N = 0.4 \cdot 10^{23}$ cm^{-1} gas density

TRV spin rotation

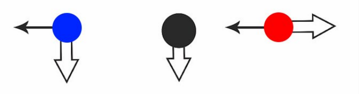
- initial state:
  $\Uparrow \vec{p}, \Uparrow \vec{n} \parallel x-$axis
- final state: $\Uparrow \vec{p} \parallel x-$axis



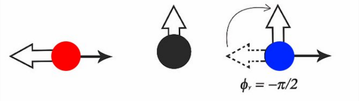
Original process
(we suppose that the spin rotates counterclokwise around the y-axis)



Time-reversal



Rotation of 180° around the y-axis



$$f(0) = f_0 + f_M\sigma_x + f_P\sigma_z + f_T\sigma_y$$

$\Rightarrow$ spin rotation term around the y-axis

$$\psi_{out} = e^{ip_n(z-d)} e^{i\frac{2\pi Nd}{p_n} f_t\sigma_y} |\chi\rangle$$

Results

The rotation around the y-axis is linearly dependent on TRV LECs

$$\frac{\mathrm{d}\phi_y}{\mathrm{d}z} = \overline{g}_0^\theta d_0 + \overline{g}_1^\theta d_1 + \overline{\Delta}^\theta d_2 + \overline{C}_1^\theta d_3 + \overline{C}_2^\theta d_4$$

Λ_F (MeV)	d_0	d_1	d_2	d_3	d_4
450	4.59	0	0	0.13	0.09
500	4.69	0	0	0.12	0.09
600	4.62	0	0	0.12	0.08

The coefficients d_i are in units of Rad m^{-1}

- PC potential [Entem & Machleidt, 2011]
- No contribution from the LECs $\overline{g}_1^\theta$ and $\overline{\Delta}^\theta$
- The ratio $d_3/d_0 \sim 0.03$ and $d_4/d_0 \sim 0.02$

Results

Using the estimates of the LECs in term of $\bar{\theta}$ [J. Bsaisou *et al.*, 2015]

$$
\begin{aligned}
\overline{\Delta}^{\theta} &= (0.37 \pm 0.09) \cdot 10^{-3}\bar{\theta} \\
\overline{g_0}^{\theta} &= (0.0155 \pm 0.0019)\bar{\theta} \\
\overline{g_1}^{\theta} &= (0.0034 \pm 0.0011)\bar{\theta} \\
\overline{C}^{\theta}_{1,2} &\simeq (3 \cdot 10^{-2})\bar{\theta}
\end{aligned}
$$

Λ_F(MeV)	$\mathrm{d}\phi_y/\mathrm{d}z$(Rad m^{-1})
450	$(7.12 \pm 0.87) \cdot 10^{-2}\,\bar{\theta}$
500	$(7.27 \pm 0.89) \cdot 10^{-2}\,\bar{\theta}$
600	$(7.16 \pm 0.88) \cdot 10^{-2}\,\bar{\theta}$

- The estimated value of $\bar{\theta} \lesssim 10^{-10}$ so we expect $\mathrm{d}\phi_y/\mathrm{d}z \lesssim 10^{-11}$
- Any signal that $\mathrm{d}\phi_y/\mathrm{d}z \gtrsim 10^{-11} \Rightarrow$ BSM effects

Conclusions

Conclusions

- Independent derivation of the TRV *NN* potential at NLO + contact terms
  - N2LO in progress
- Explorative study of $\vec{n} - \vec{p}$ spin rotation
  - This effect could be enhanced in $\vec{n} - \vec{A}$ [V. Gudkov, 1992]
- Calculation of light nuclei EDMs are planned (d,^{3}H,^{3}He)
  - Proposal for new storage-rings dedicated to the measurement of the d, ^{3}H, ^{3}He, ^{6}Li EDMs (estimated precision $\sim 10^{-16} e$ fm) [Y. K. Semertzidis, 2011]