QMC Methods in dense Neutron Matter

Doctoral Training Program

ECT*: 12/06 – 30/06 2017

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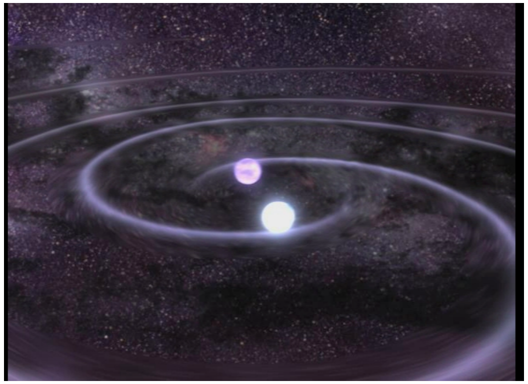
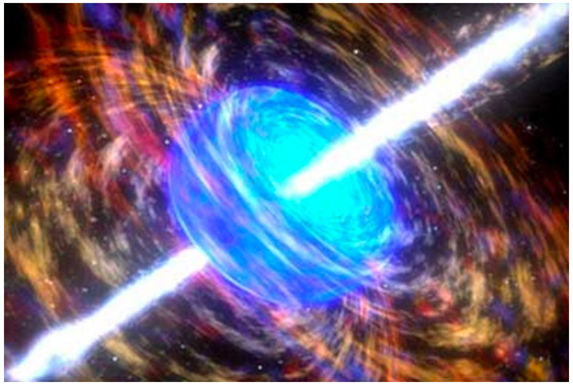
Some Motivations

- Study Equation of State (EoS) of Pure Neutron Matter and effects of Spin.
- Derive ground state (GS) and dynamical properties of homogeneous baryonic matter, which is related to neutrino-nucleon scattering rate and to Neutrino Mean Free Path (NMFP) in compact stars.
- Spin polarization may play a role (e.g. magnetars).
- For GS properties ab initio methods, while for excited states we need Mean Field Approximation.

Some Motivations/2

- Implication on high energy astrophysical phenomena.



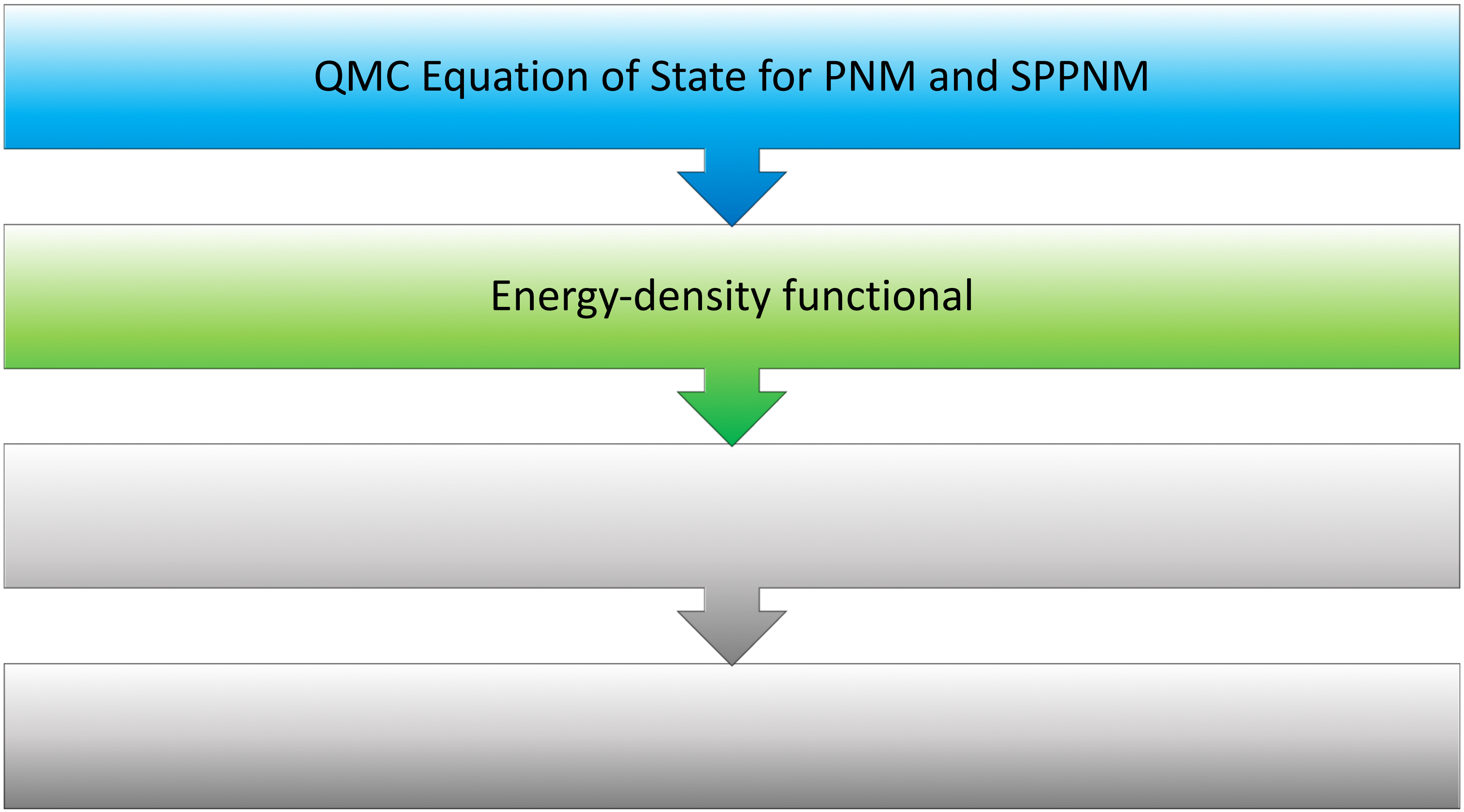


[image credit: NASA]

Overview

QMC Equation of State for PNM and SPPNM

Overview


QMC Equation of State for PNM and SPPNM
Energy-density functional

Overview

QMC Equation of State for PNM and SPPNM

Energy-density functional

Time Dependent Local Density Approximation response in spin channel

Overview

QMC Equation of State for PNM and SPPNM

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Energy-density functional

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Evaluation of Neutrino Mean Free Path in neutron matter

Overview

QMC Equation of State for PNM and SPPNM

Neutron matter

- Neutron matter can be modeled as a periodic system of *N* neutrons interacting by an Hamiltonian of the form:

$$H = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \cdots$$

We used two different kind potentials:

- phenomenlogical AV8’+UIX
- chiral EFT N2LO local

Argonne AV8' + Urbana UIX

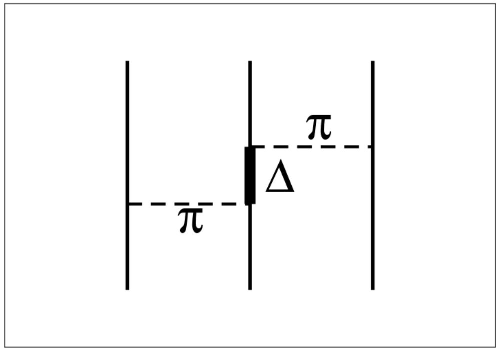
- AV8' is a reprojection of the full AV18. It can be written as a sum of operators in coordinate space:

$$O_{i,j}^{p=1,8} = \left(1, \sigma_i \cdot \sigma_j, S_{ij}, L_{ij} \cdot S_{ij}\right) \times \left(1, \tau_i \cdot \tau_j\right)$$

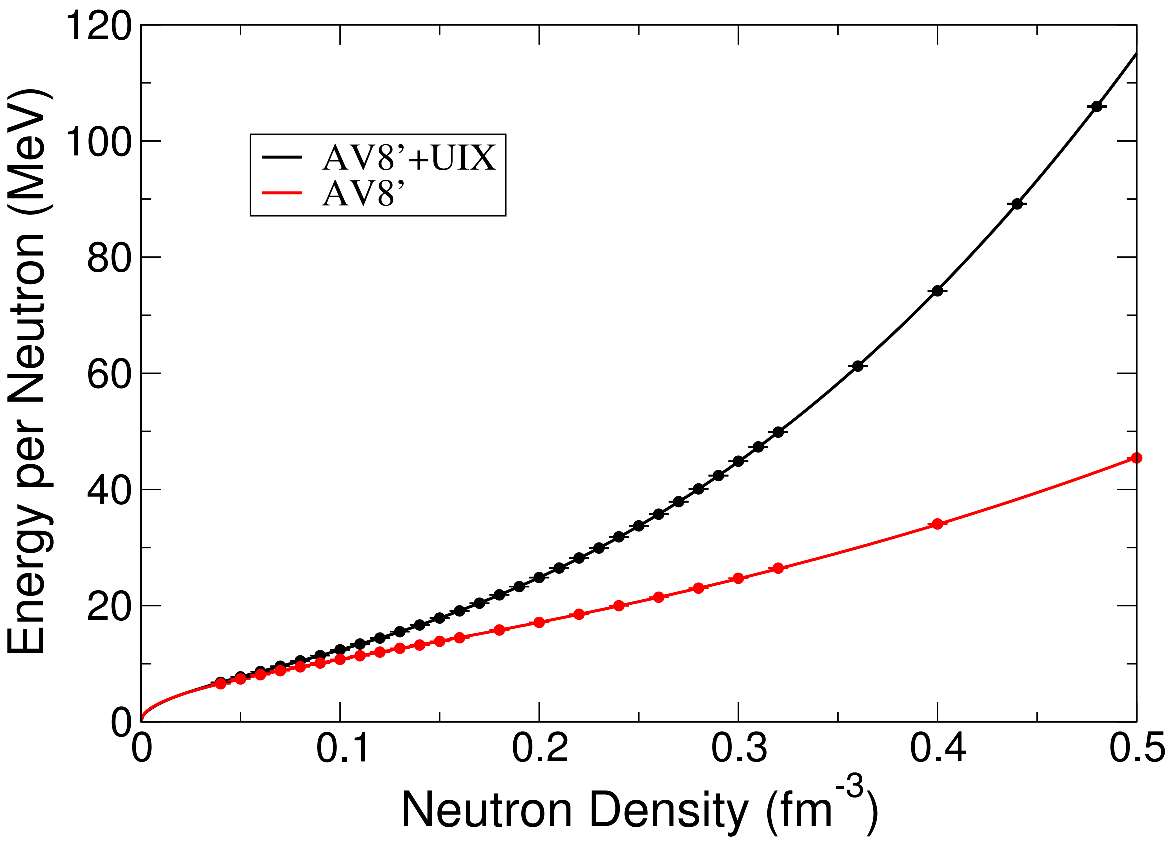
  multiplied by radial function $V_p(r_{ij})$.

- UIX introduced to correct limitations of 2- body.

$$V_{ijk} = V_{ijk}^{2\pi,P} + V_{ijk}^{R}$$

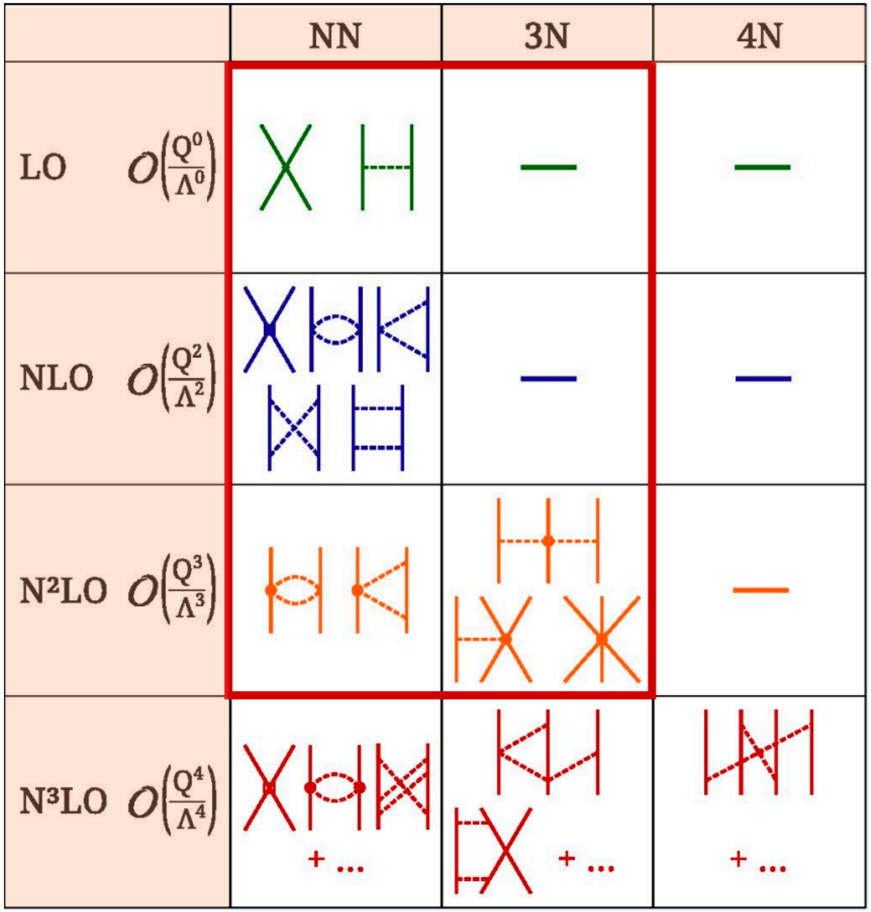


Effects of 3-body forces PNM



[Gandolfi et al. Eur. Phys. J. A, 50(2):10, (2014)]

Chiral EFT – N2LO (Local)

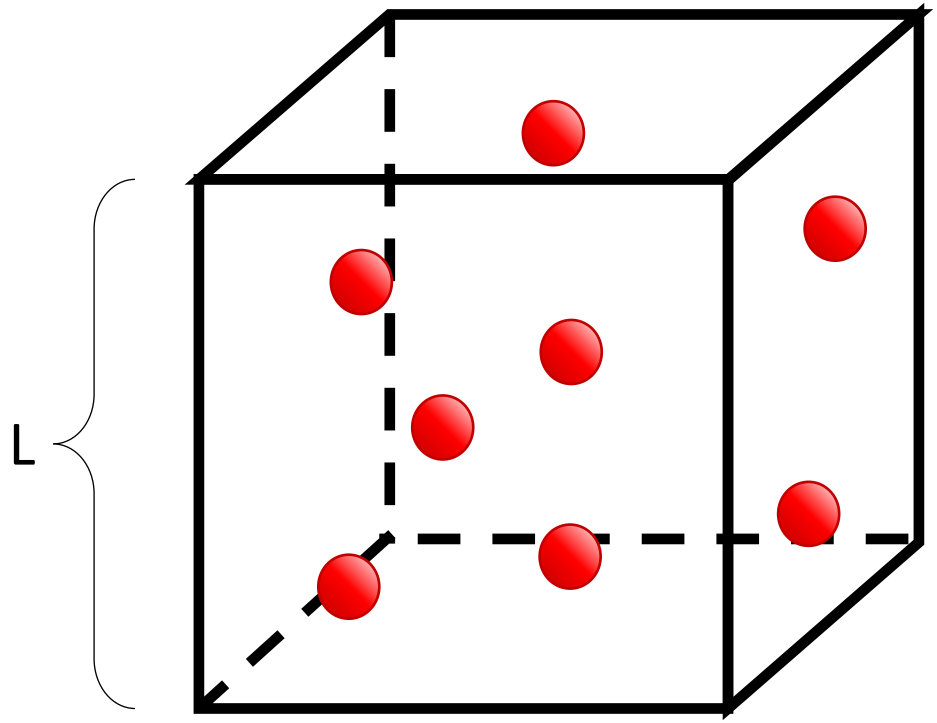


- Local in order to use QMC.
- It is possible to remove all nonlocal operators up to N2LO.
- LECs fitted to phase shifts.
- Choice D2,E1 with $R_0=R_{3N}=1.0$ fm.

[Lynn et al. PRL 116, 062501 (2016), Gezerlis et al. PRL 111, 032501 (2013)]

Neutron matter

- Infinite neutron matter = particle in a box:
  - 33 – SPPNM
  - 66 – PNM
- Periodic Boundary Condition.



Quantum Monte Carlo

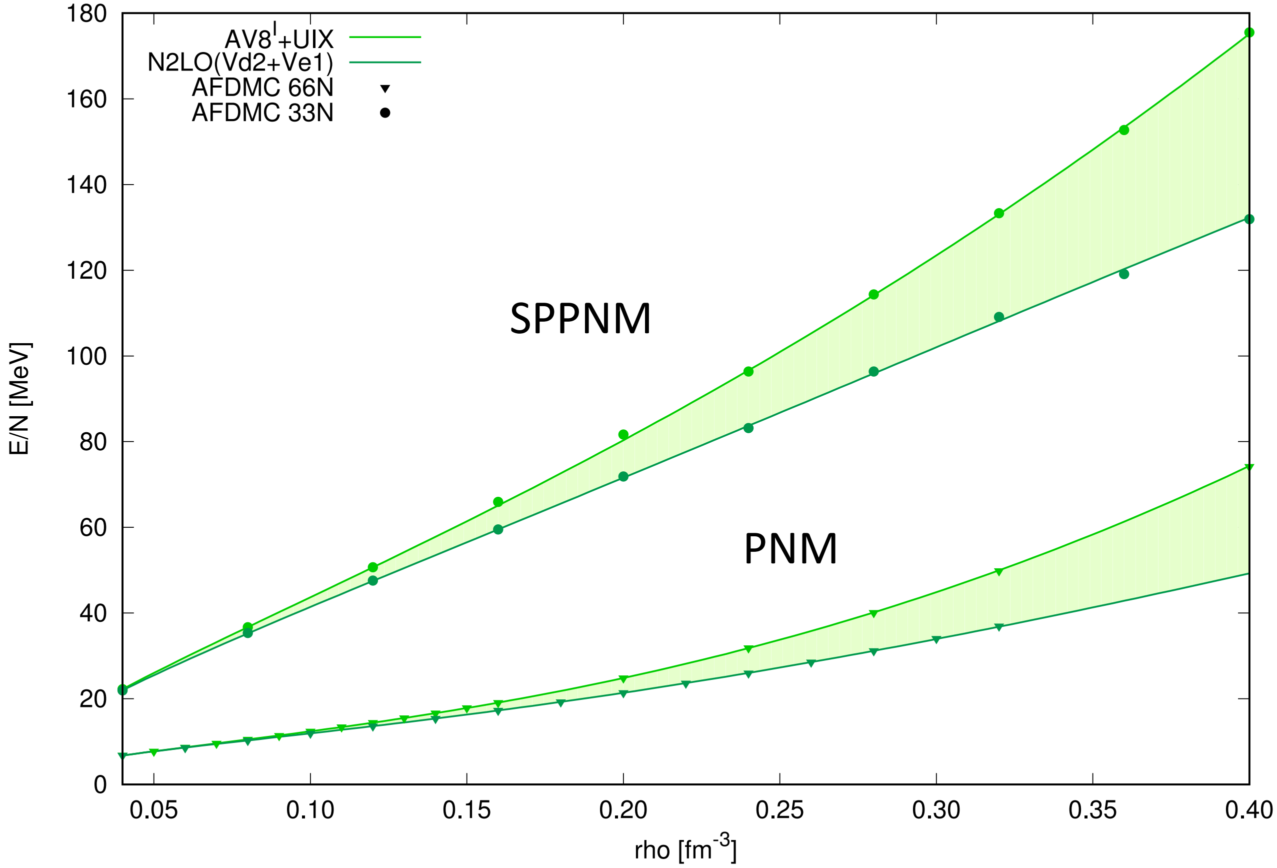
- Trial wave-function:

$$\psi_T(\vec{R}, S) = \phi_S(\vec{R})\phi_A(\vec{R}, S)$$

where the first term is a Jastrow operatorial correlations function and the second term is a Slater determinant of plane waves.

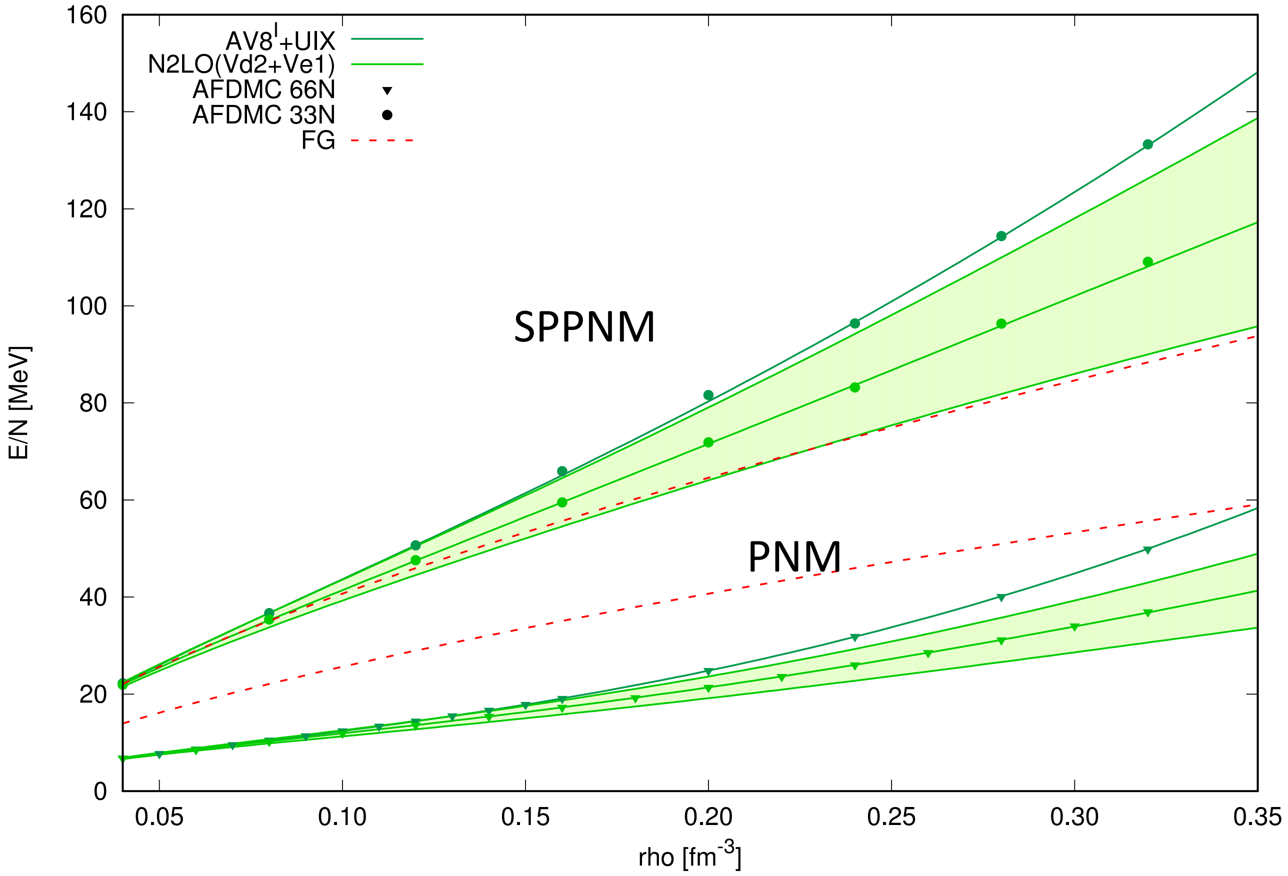
- Computational procedure:
  1. VMC – optimization of parameters;
  2. VMC – sampling configurations for AFDMC;
  3. AFDMC – project on ground state.

EoS for neutron matter



PNM results from S.Gandolfi

EoS + uncertainties



Errors computed according to [Epelbaum et al. Eur. Phys. J. A, 51, 53 (2015)]

Overview

QMC Equation of State for PNM and SPPNM

Energy-density functional

Energy Density Functional (EDT)

- We define the energy density functional as:

$$E(\rho,\xi) = T_0(\rho,\xi) + \int \epsilon_V(\rho,\xi)\rho \, d\vec{r}$$

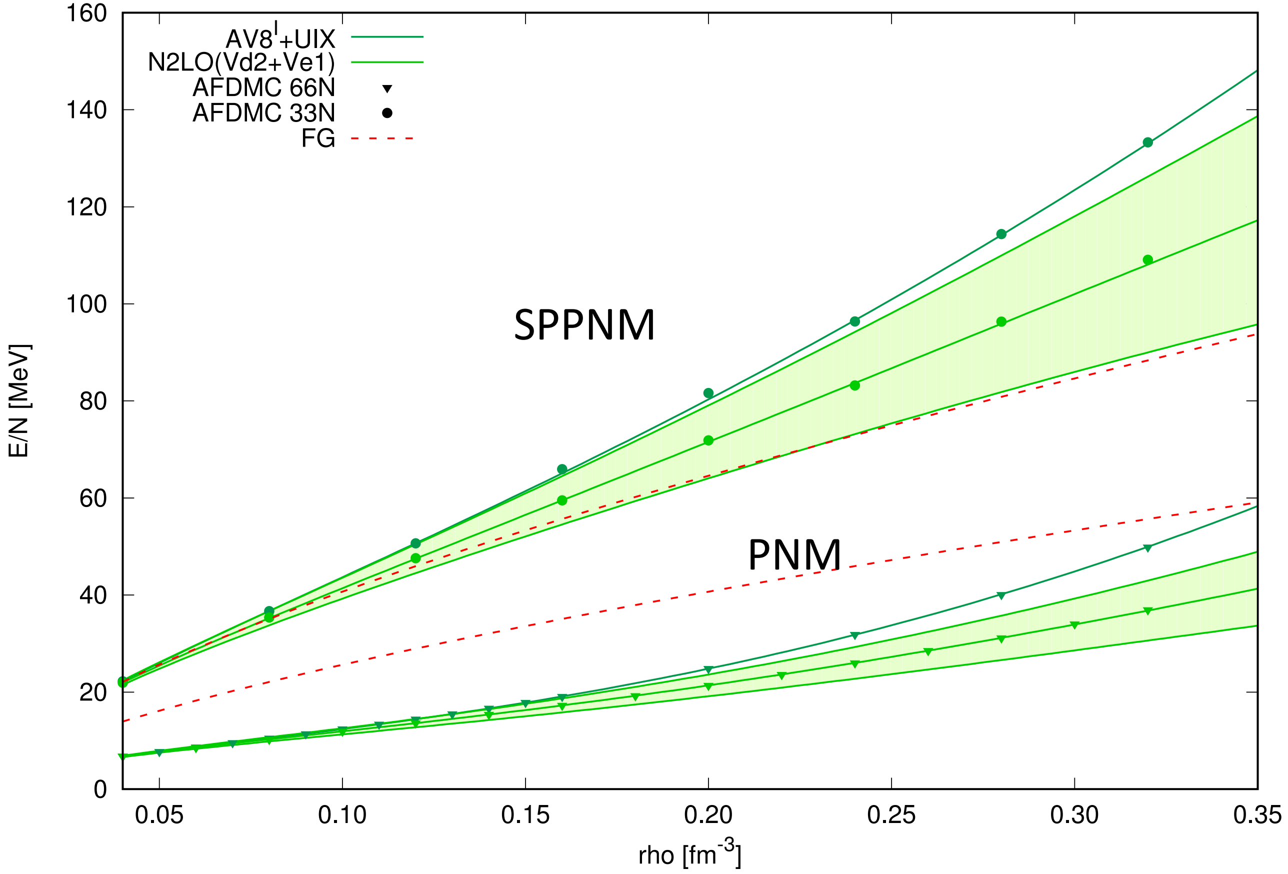
, with the interaction part assumed to be of the form:

$$\epsilon_V(\rho,\xi) = \epsilon_0(\rho) + \xi^2[\epsilon_1(\rho) - \epsilon_0(\rho)]$$

where:

$$\epsilon_q(\rho) = \epsilon_q^0 + a_q(\rho-\rho_0) + b_q(\rho-\rho_0)^2 + c_q(\rho-\rho_0)^3$$

EoS + EDF



AV8'+UIX
N2LO(Vd2+Ve1)
AFDMC 66N
AFDMC 33N
FG
SPPNM
PNM
E/N [MeV]
rho [fm-3]
0
20
40
60
80
100
120
140
160
0.05
0.10
0.15
0.20
0.25
0.30
0.35

Response function in NM

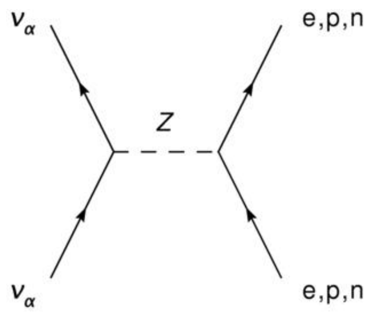
- We are interested in studying the density response of the system. For nucleons the response can be splitted in different operators: e.g. “Fermi”, ”Gamow-Teller”, “Neutral-vector” and “Neutral-axial-vector”.

- What is the relation between weak scattering processes and nuclear density response?
  - Weimber-Salam Lagrangian

Weimberg-Salam Model

- We have a coupling between nucleons and neutrinos through weak currents.
- E.g. for a lepton weak neutral current the coupling Lagrangian density is:

$$L_W = \frac{G_W}{\sqrt{2}} \bar{\psi}_\nu(x)\gamma_\mu(1-\gamma_5)\psi_\nu(x)\frac{1}{2}\bar{\psi}_n(x)\gamma^\mu(1-C_A\gamma_5)\psi_n(x)$$



Currents in NR-limit

- The WS Lagrangian couples neutrinos to *density* and *spin-density* fluctuations of neutrons.
- In fact, in the **non-relativistic limit** the baryonic current may be approximated by:

$$\bar{\psi}_n(x)\gamma^\mu(1 - C_A\gamma_5)\psi_n(x) \sim \psi_n^\dagger(x)\psi_n(x)\delta_0^\mu - C_A\psi_n^\dagger(x)\sigma_i\psi_n(x)\delta_i^\mu$$

- We have two contribution: density fluctuations and spin-density fluctuations.

ν scattering rate

- Applying Fermi golden rule, we get the neutrino scattering rate from a system of neutrons.

$$\sigma = \frac{G^2}{2}\frac{1}{E}\int dq \int d\omega (E-\omega) q \left(1 + \frac{E^2 + (E-\omega)^2 - q^2}{2E(E-\omega)}\right) S(q,\omega)\,,$$

where $S(q,\omega)$ is the dynamical structure factor (DSF) for the excitation operators describing the process (i.e. density and spin-density excitations).

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QMC Equation of State for PNM and SPPNM

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Energy-density functional

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Time Dependent Local Density Approximation response in spin channel

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Evaluation of Neutrino Mean Free Path in neutron matter

Time Dependent Local Density Approximation

- We use Time Dependent Local Spin Density Approximation (TDLSA) to compute the response function and the DSF.
- We have worked out the response functions in the longitudinal and transverse spin channels.
- Following *Kohn-Sham* method we introduce LSDA for homogeneous neutron matter using the energy density functional introduced before and we apply the Hohenmber-Kohn theorem which provides a variational principle on energy-density functional.

Response functions

- In the logitudinal channel, one can write expressions for $\chi^{n_\downarrow}$ and $\chi^{n_\uparrow}$ in function of the response function of the free Fermi gas χ_0 and combining them we get the density-density χ_s and vector-density/vector-density χ_v response functions for arbitrary spin polarization.
- A similar derivation can be done for transverse channel (there is an extra effective vector potential accounting for the equilibrium spin polarization).

Excitation strengths and sum rules

- The dynamic structure factor (DSF) is:

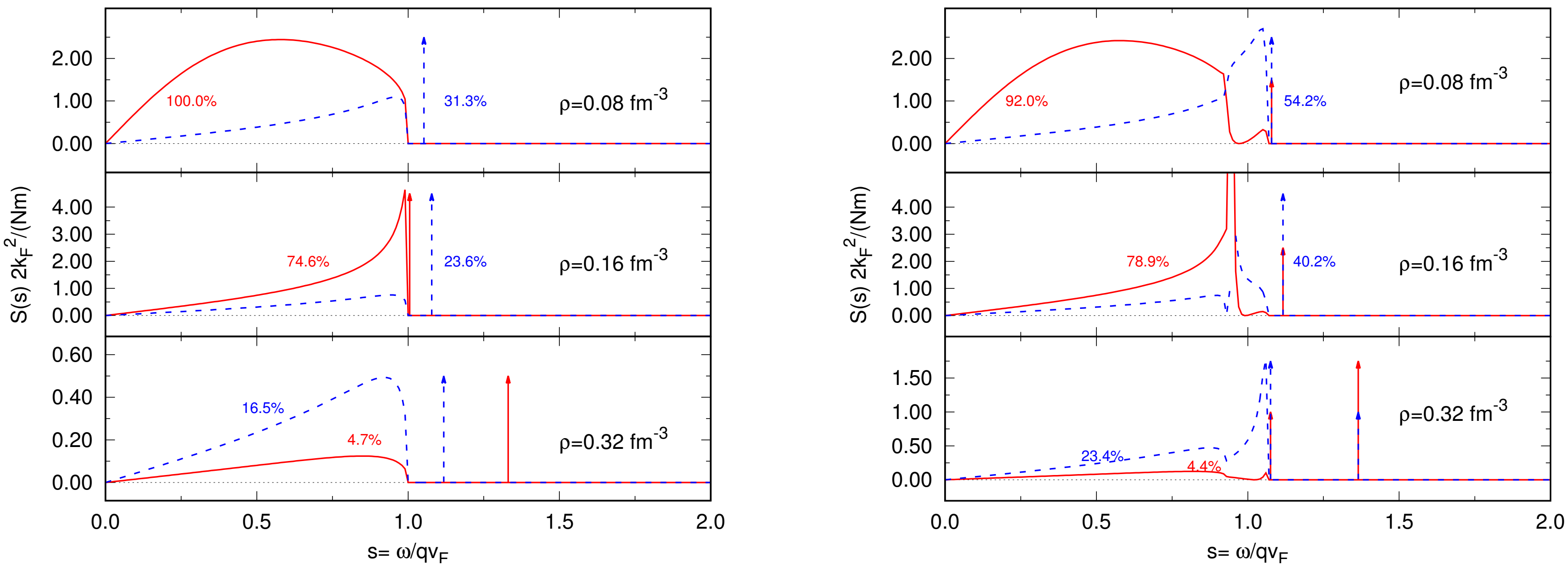
$$S^{s,v}(q,\omega) = -\frac{1}{\pi} Im[\chi^{s,v}]$$

- From the DSF one can also compute the energy-weighted sum rules:

$$m_k^{s,v} = \int_0^\infty d\omega\, \omega^k\, S^{s,v}(q,\omega) = \sum_n \omega_{n0}^k\ |< 0|F^{s,v}|n > |^2$$

- The poles of $\chi(q,\omega)$ give the spectrum and the dispersion $\omega(q)$ of the collective excitations.

Longitudinal Response (AV8'+UIX)



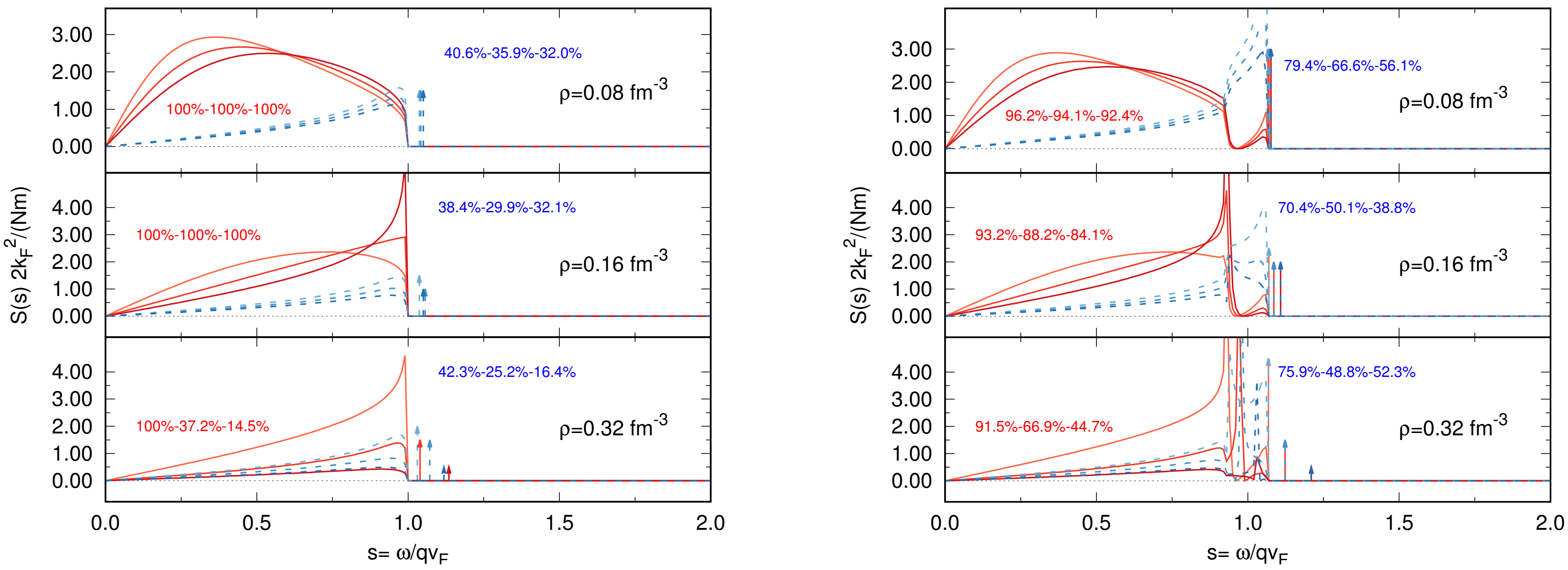
Left Panel: PNM ($\xi = 0$)

Right Panel: ($\xi = 0.2$)

density

spin-density

Longitudinal Response (χ-EFT)



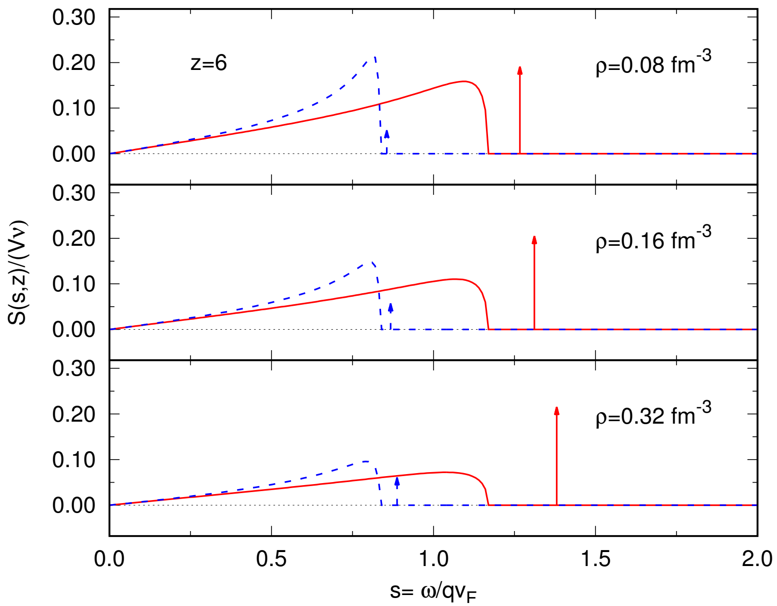
Left Panel: PNM ($\xi = 0$)

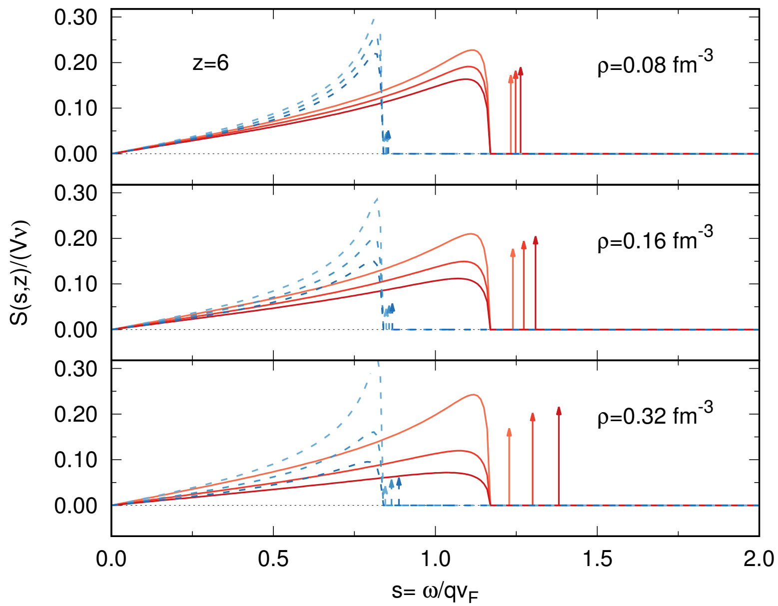
Right Panel: ($\xi = 0.2$)

density

spin-density

Transverse Response





Left Panel: AV8'+UIX

Right Panel: χ-EFT

$\Delta S_z = -1\ (s > 0)$

$\Delta S_z = +1\ (s < 0)$

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Energy-density functional

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Time Dependent Local Density Approximation response in spin channel

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Evaluation of Neutrino Mean Free Path in neutron matter

Neutrino Mean Free Path

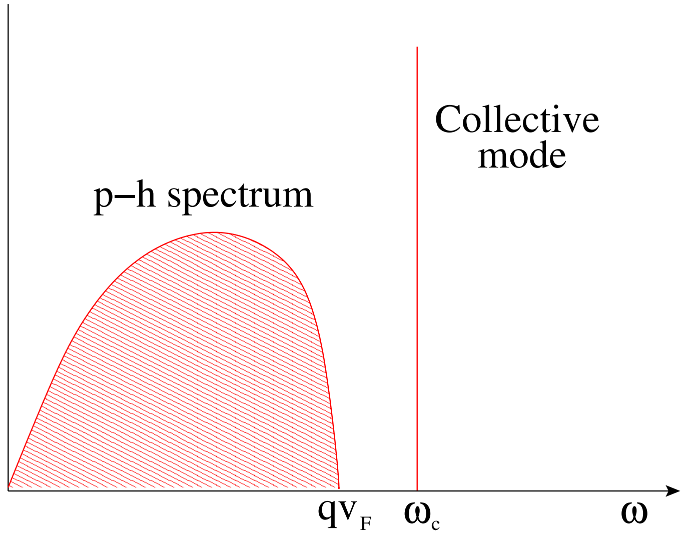
- The scattering rate of neutrinos we obtained:

$$\sigma = \frac{G^2}{2}\frac{1}{E}\int dq \int d\omega (E-\omega) q \left(1 + \frac{E^2 + (E-\omega)^2 - q^2}{2E(E-\omega)}\right) S(q,\omega)\,,$$

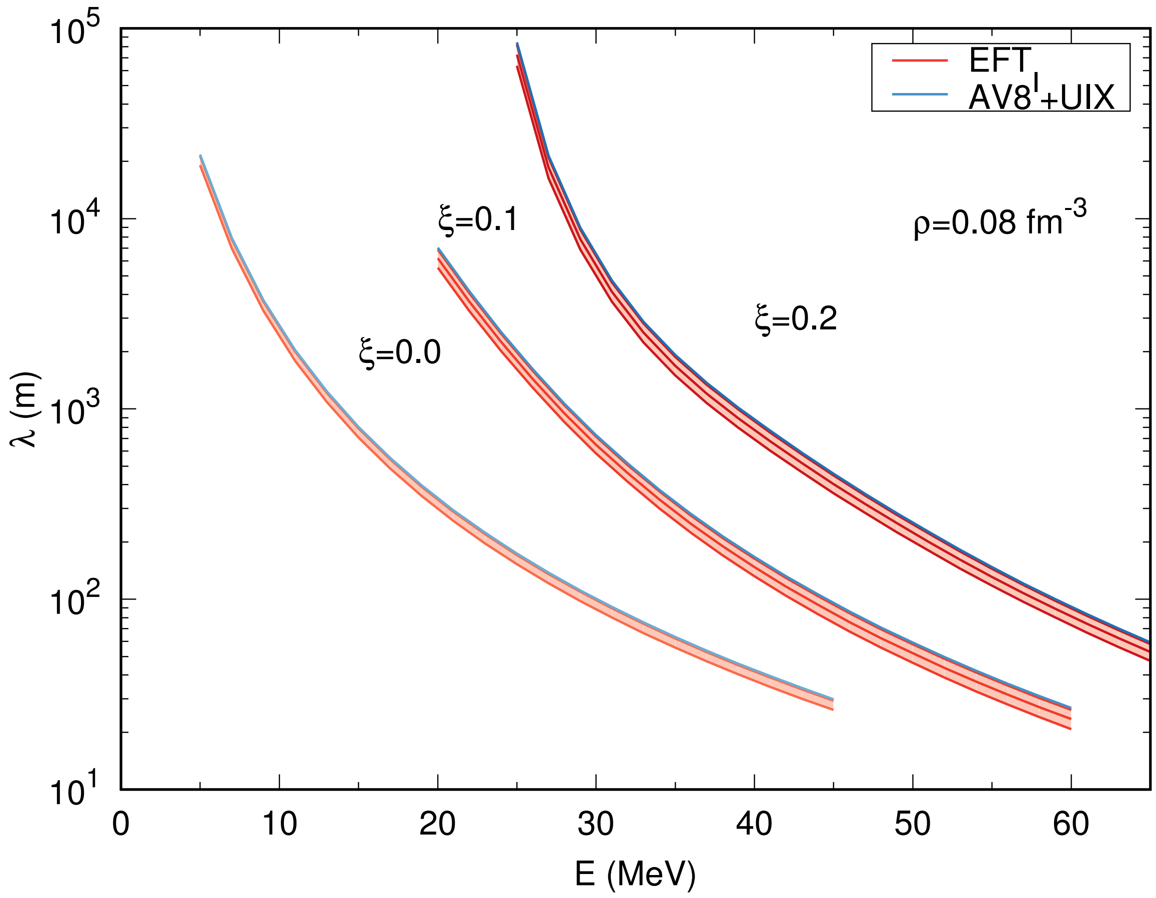
- The neutrino mean free path λ is related to σ by:

$$\lambda = \frac{1}{\sigma\rho}$$

- NB: The integration has to performed on kinematically accessible regions to neutrinos.



ν Mean Free Path (transverse)



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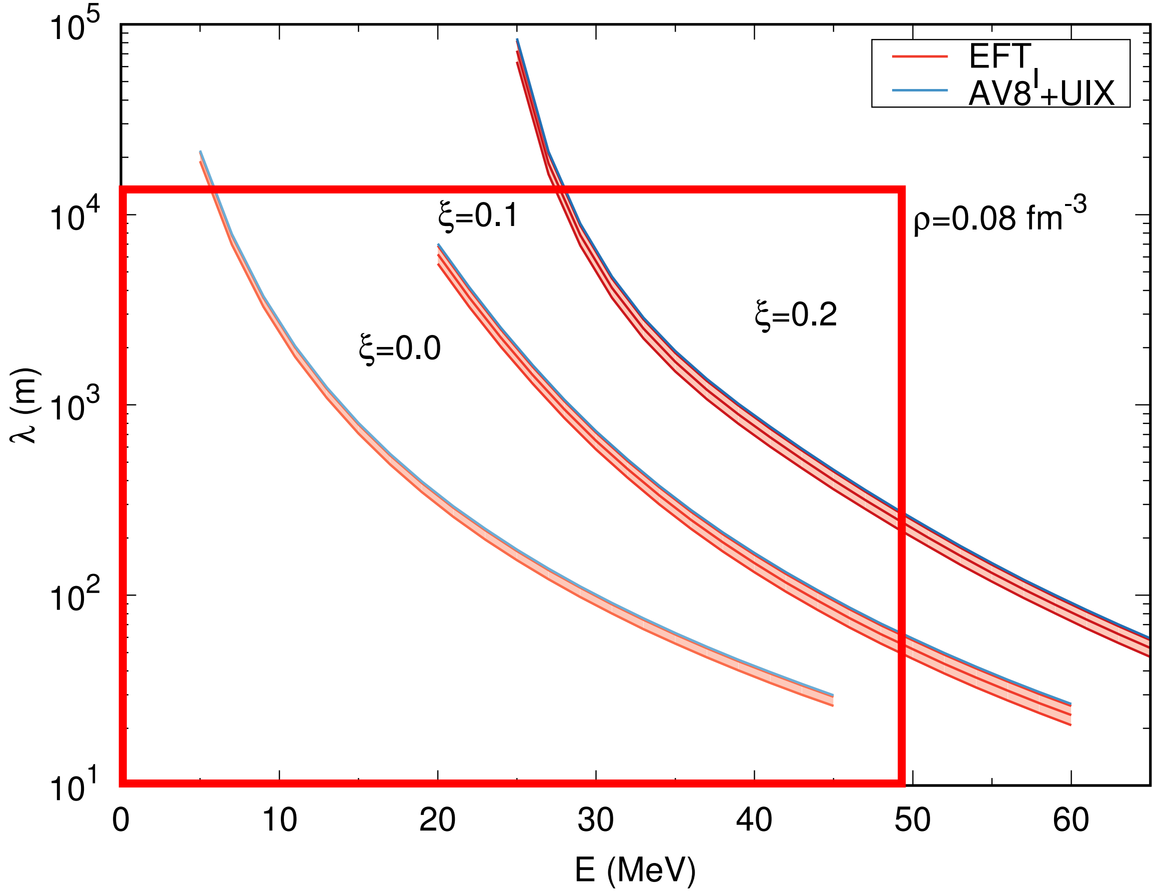

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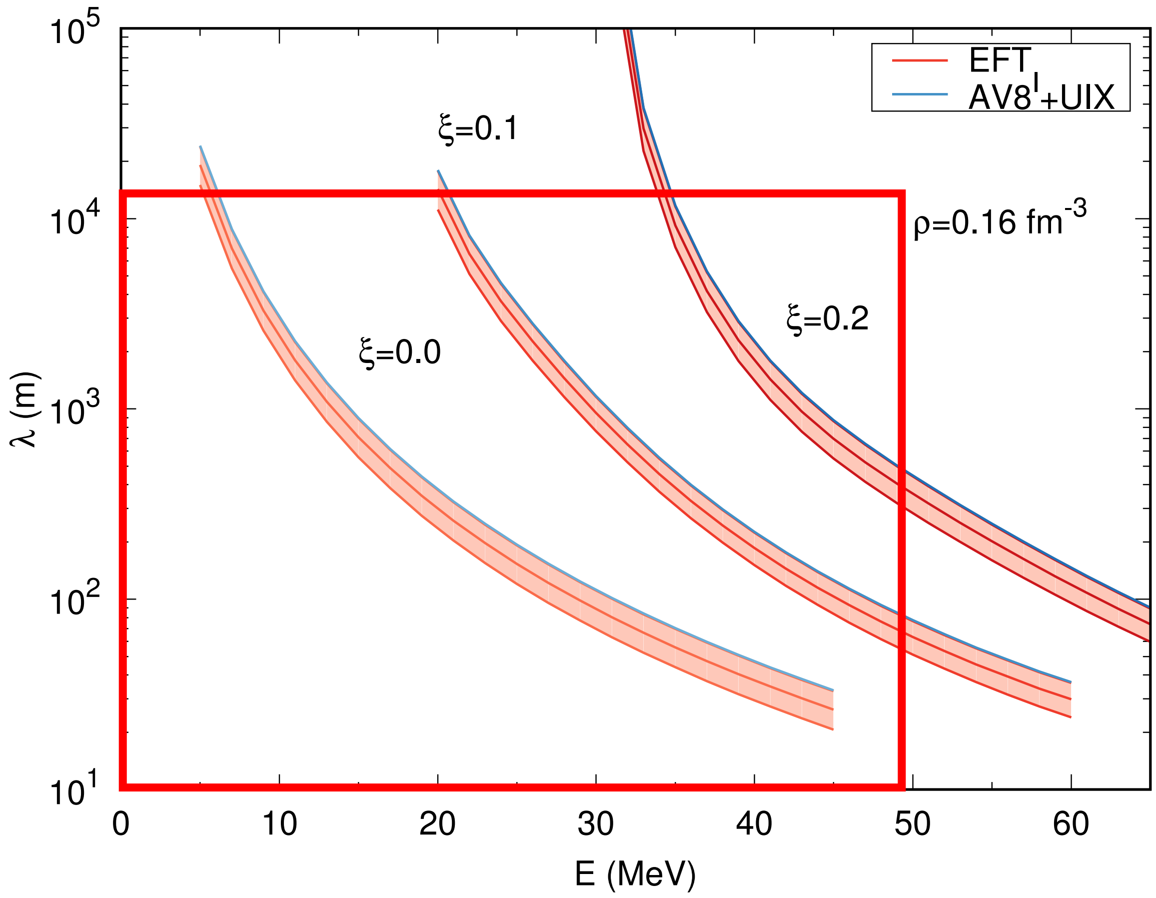

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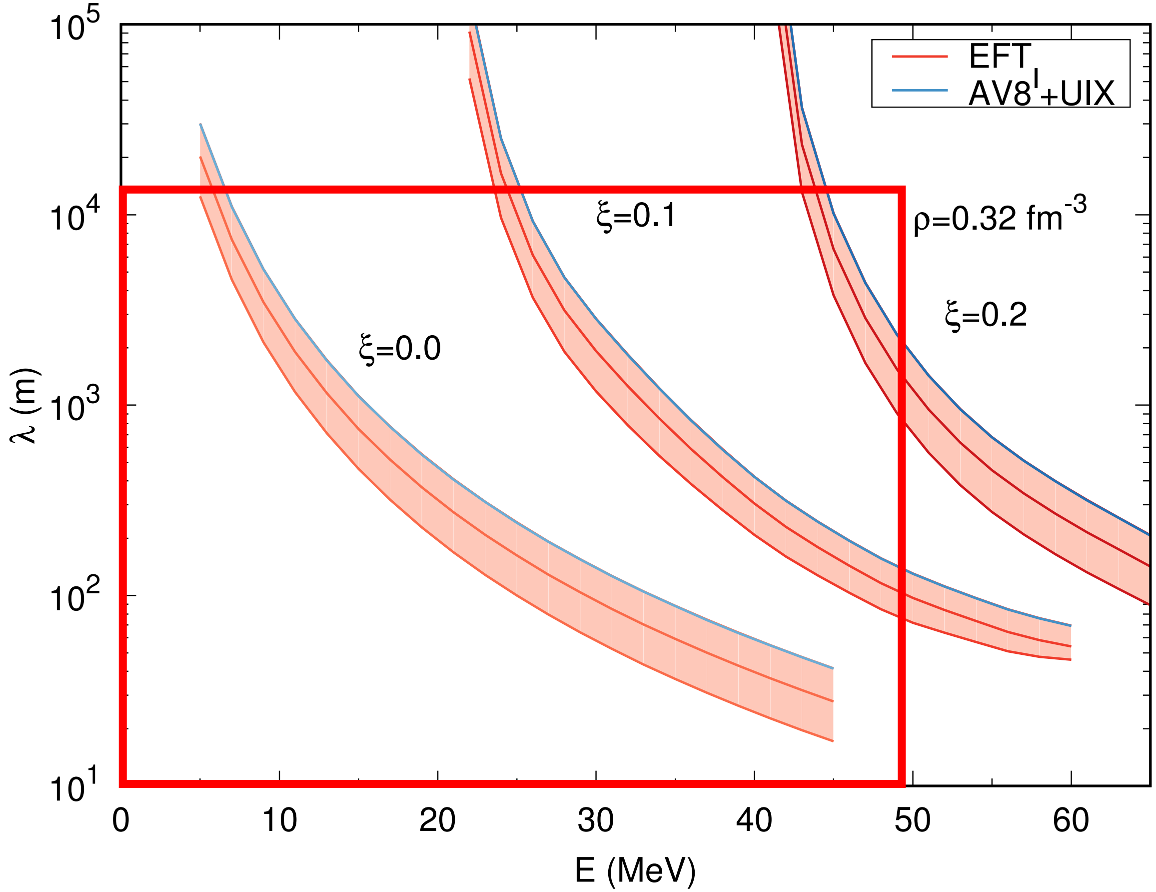

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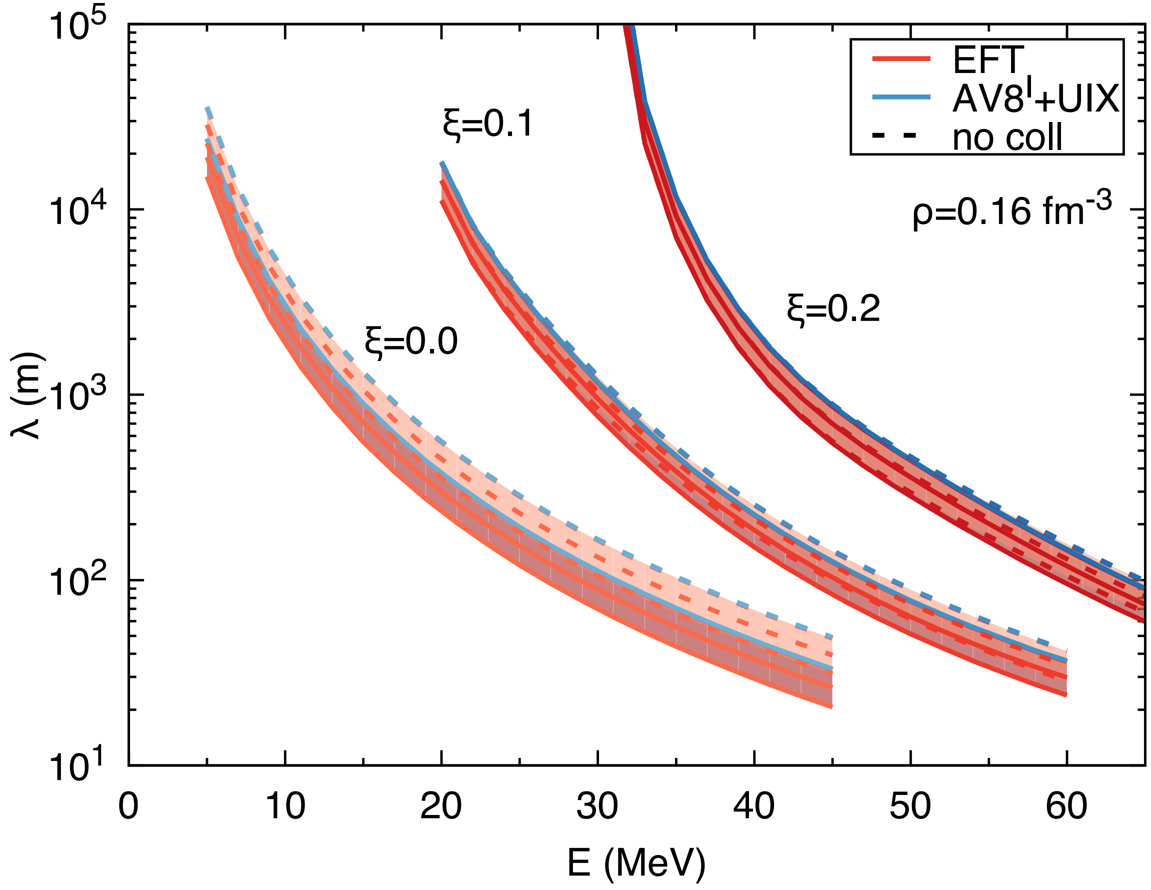
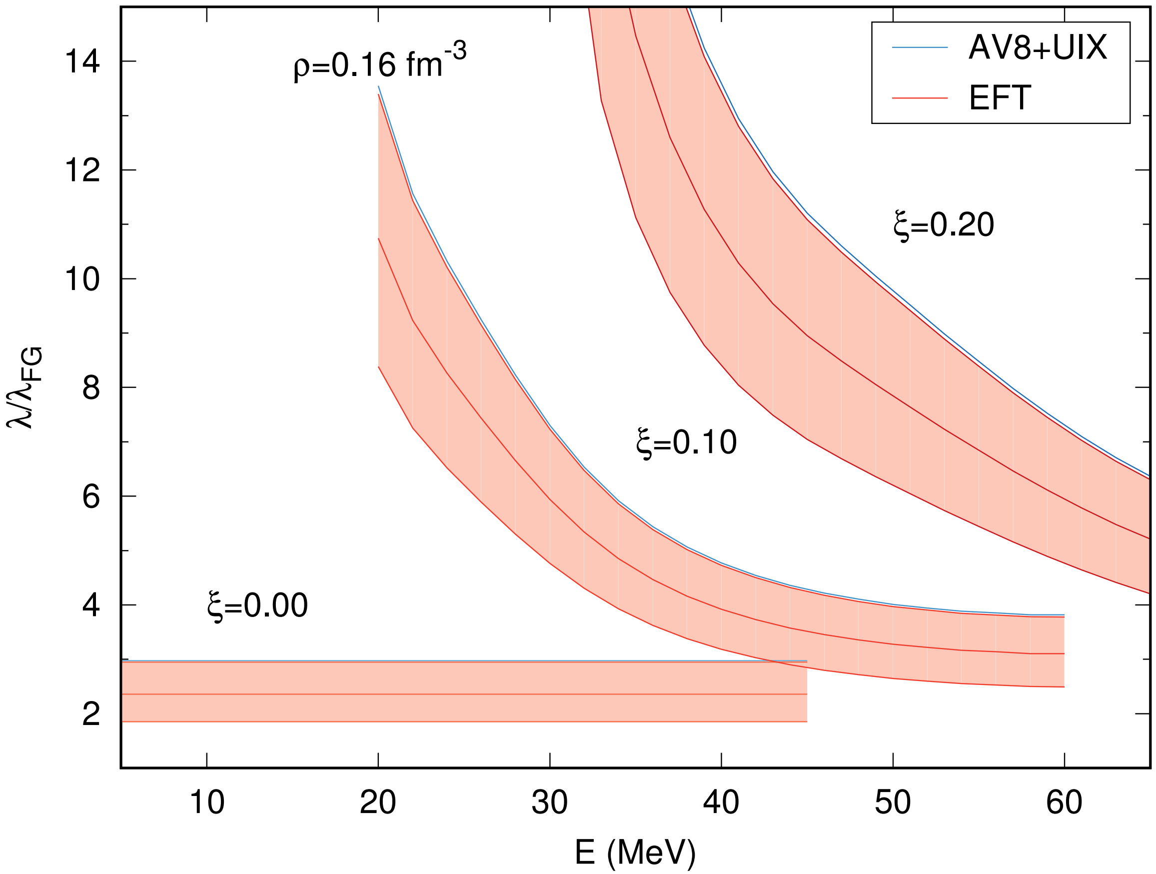


ν Mean Free Path (transverse)



Conclusions

- Response function in longitudinal and transverse channels have been computed for neutrinos in pure neutron matter starting from QMC calculations.
- TDLDA was applied succesfully to estimate the response function of arbitrary spin polarized neutron matter.
- We computed NMFP in the transverse channel (longitudinal soon™).
- Matter essentially transparent at NS core condition, while relevant effects could be seen in the crust.