Interfacing Many-Body methods and EFT interactions

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Doctoral Training Program
ECT*

Outline

1. Reminders on Nuclear interactions
   - Historical review of interactions
   - Generalities about EFT
   - Going to many-body observables

2. Interfacing ab initio many-body methods with EFT
   - Self-Consistent Green's Function for nuclear matter
   - Recent results about neutron matter

Outline

Brief history of nuclear interactions

- 1935 : Yukawa potential

  $\rightarrow$ Birth of Meson Theory

- 40s -50s : Pions Theories

  $\rightarrow$ Troubles with multi-pions, anti-nucleons diagrams, ...

- 60s -00s : One-Boson-Exchange Model $(\rho, \sigma, \omega, \dots)$

  $\rightarrow$ Good data fitting

  $\rightarrow$ But no systematic

  $\rightarrow$ Need separate framework for 3-body interactions

} No renormalization group invariance

- 90s - today : EFT based interactions [S.Weinberg 90 91]

  $\rightarrow$ *Pragmatic* view : Break RG invariance + estimate error

  [Entem, Machleidt 03] [Epelbaum, Glöckle, Meißner 05]

  $\rightarrow$ *Canonical* view : Modify power counting and adapt Many-body methods

  [Kaplan, Savage, Wise 96] [Nogga, Timmermans, van Kolck 05]

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Motivations for interaction based on EFTs

Folk Theorem

The quantum field theory generated by the most general Lagrangian with some assumed symmetries will produce the most general S matrix incorporating quantum mechanics, Lorentz invariance, unitarity, cluster decomposition and the assumed symmetries. [S.Weinberg 79]

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Inputs

- ▷ Degrees of Freedom
  - Nucleons
  - Pions
  - ...
- ▷ Symmetries
  - Space-time
  - Internal
- ▷ LECs $\rightarrow$ Fit to data

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Advantages

- Direct connection to underlying theory
- Hierarchy of terms in potential
- Same framework for all A-body interactions
- Estimations of uncertainty and range of validity

How to make an EFT well-defined

Regularization procedure needed

$\rightarrow$ Regulator = modify high energy physics

$\rightarrow$ Introduce an arbitrary cut-off scale Λ

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Matching procedure

$\rightarrow$ Compute observables $\mathrm{O}\left(c_i(\Lambda), \Lambda\right)$ at a given order N

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$\rightarrow$ High energy physics taken into account in $c_i(\Lambda)$

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Renormalization features

- High energy physics fully included in LECs
- Independence of the observables from the regularization

EFTs for nuclear systems

- Low energy observables in nuclear systems : $Q \ll m_\pi$

Degrees of freedom : non-relativistic nucleons (+ photons, ...)

$$\rightarrow \quad \mathcal{L}_{\not{\pi}EFT} = N^\dagger (i\partial_0 + \frac{\vec{\nabla}^2}{2m_N}) N + C_0 (N^\dagger N)^2 + D_0 (N^\dagger N)^3 + \dots$$

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Truncation scheme

$\rightarrow$ What diagrams to compute for N^{th} order ?

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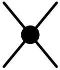
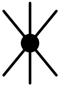
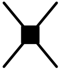
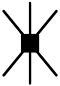
Consistency check

- Convergent observables in negative power of Λ
  - → Observable $\perp\!\!\!\perp$ higher energy scale (except through LECs)
- Convergence of observable to experiment when N^{th} increases
  - → Fail ? Try another guess on LECs
  - → No consistent power counting works ? might be wrong D.o.F

$\not{\pi}$*EFT* for Nuclear systems

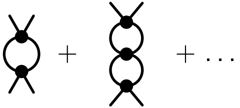
- Guess on LECs size → Proposed power counting for $\not{\pi}$*EFT*

[Van Kolck 97] [Kaplan, Savage, Wise 98] [Bedaque, Hammer, van Kolck 98 99] ...

	2-body	3-body	4-body
LO			X
NLO		X	?
N^2LO	X		?

Un-natural scattering length *a*

⇓



at LO

- Consistency check : LO $A = 4, 6, 16$ & NLO up to $A = 3$

[Contessi, Lovato, Pederiva, Roggero, Kirsher, van Kolck 17] [Vanasse et al. 13][König et al. 16]

Need to extend consistency check to general *A*-body observables !

This is where *ab initio* many-body methods enter the game

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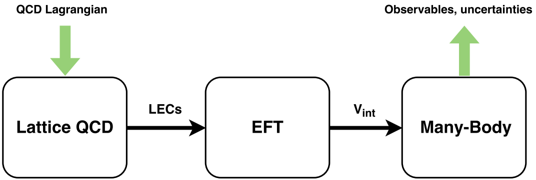
Interface EFT interaction / Many-body methods

- Traditional view : V_{int} as black box



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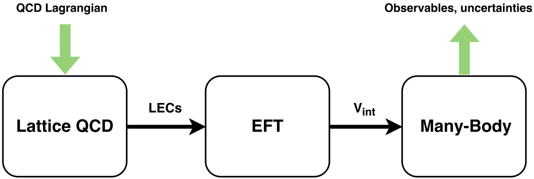
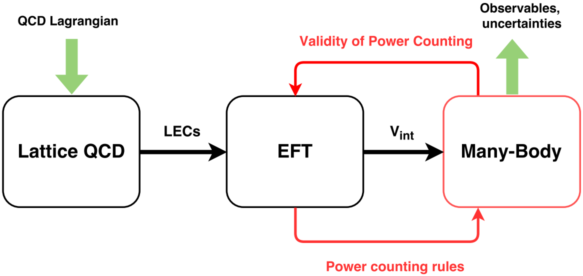


- Adapt many-body scheme to assess proposed power counting



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General perturbation theory

Choose a split of the hamiltonian $H \equiv H_0 + V$ and consider (un)correlated states, related by the Lippmann-Schwinger equation

$$\begin{aligned} H_0|\phi\rangle &\equiv E|\phi\rangle \\ H|\psi\rangle &\equiv E|\psi\rangle \\ V|\psi\rangle &\equiv T|\phi\rangle \end{aligned} \quad \Longrightarrow \quad \begin{aligned} |\psi\rangle &= |\phi\rangle + \frac{1}{E - H_0} V|\psi\rangle \\ T &= V + \frac{1}{E - H_0} T \end{aligned}$$

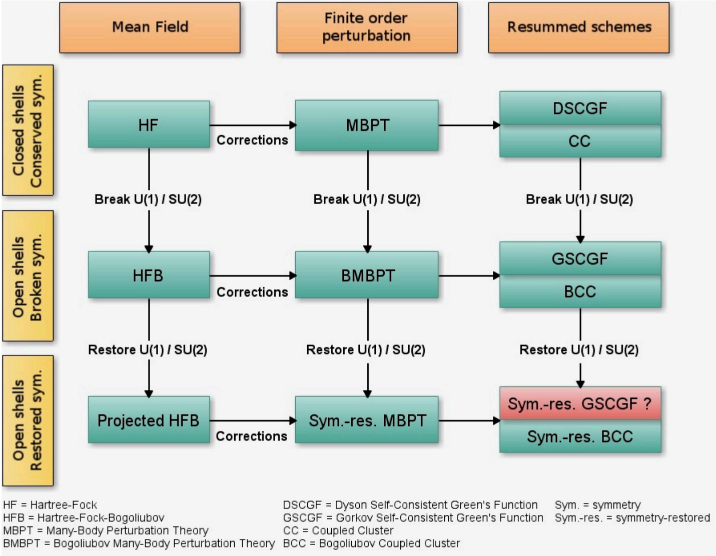
Thus we get the perturbation expansion

$$T = V + V\frac{1}{E - H_0}V + V\frac{1}{E - H_0}V\frac{1}{E - H_0}V + \dots$$

Main features

- $\rightarrow$ Choice of $H_0 \sim$ From what state to start the expansion
- $\rightarrow$ Order of expansion

Brief overview of many-body perturbation methods



Courtesy of P. Arthuis, T. Duguet

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- Work on Green's functions, e.g. one-body Green's function

$$G_{\alpha\beta}(t,t') \equiv -i \frac{\langle \Psi_0^N | \mathcal{T}[a_\alpha(t) a_\beta^\dagger(t')] | \Psi_0^N \rangle}{\langle \Psi_0^N | \Psi_0^N \rangle}$$

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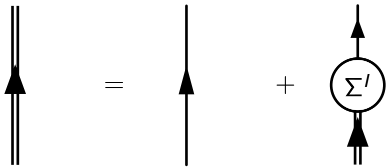
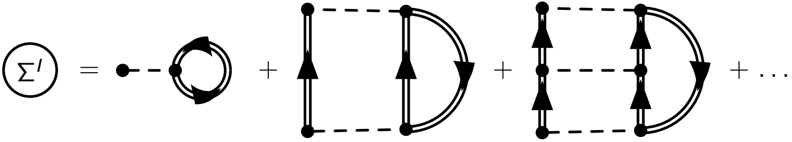
Dyson equation to be solved self consistently

$$G = G^0 + G^0 \Sigma^I G$$



Ladder approximation of the self-energy

Ladder ansatz for the self-energy



SCHF $\quad$ 2^{nd} *Born*

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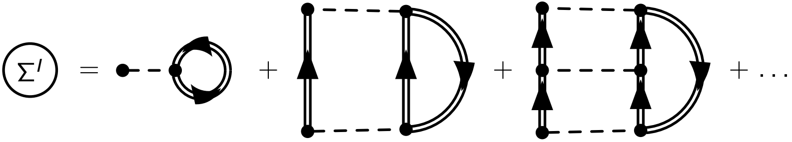

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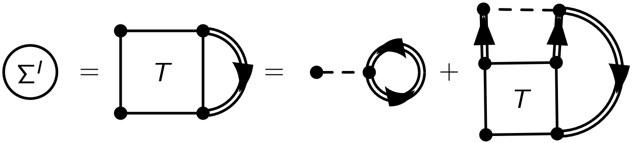
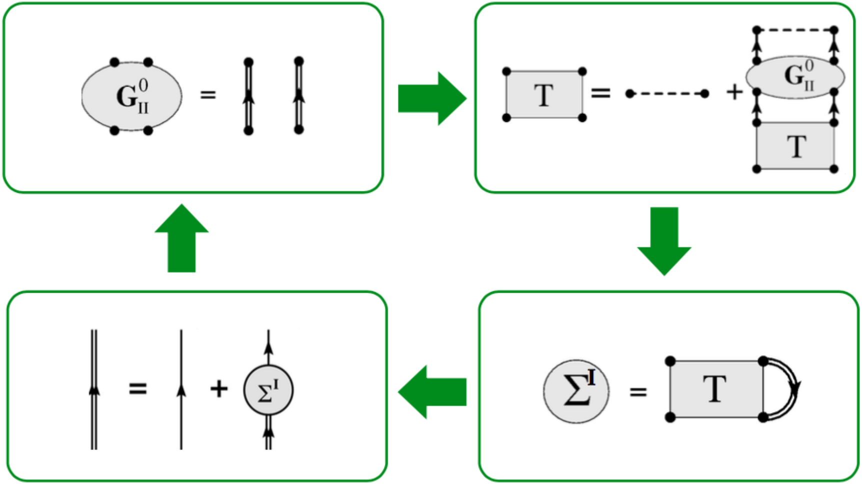
The self-consistent equation (T-matrix formulation)



Implementation of full Ladder calculations



System considered

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  - un-natural scattering length $a = -18.9$ fm $\rightarrow$ multi-loop diagram LO
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δ-shell

$$V(k,k';r_c) = C_0(r_c)\,\frac{\sin(kr_c)}{kr_c}\frac{\sin(k'r_c)}{k'r_c}$$

$$\frac{1}{C_0(r_c)} = \frac{m_N}{4\pi}\left(\frac{1}{a} - \frac{1}{r_c}\right)$$

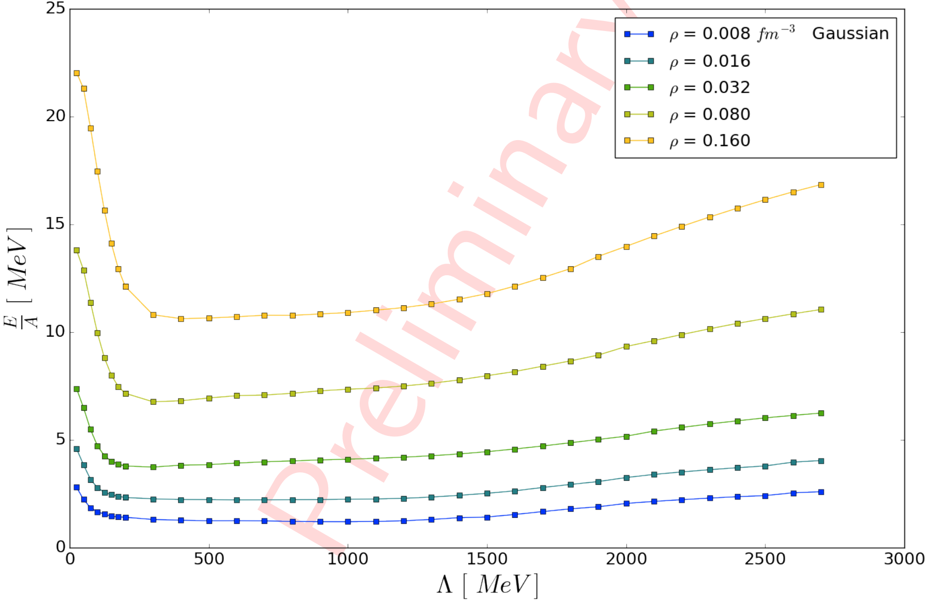
Gaussian

$$V(k,k';\Lambda) = C_0(\Lambda)\ \exp-\frac{k^2}{\Lambda^2}\exp-\frac{k'^2}{\Lambda^2}$$

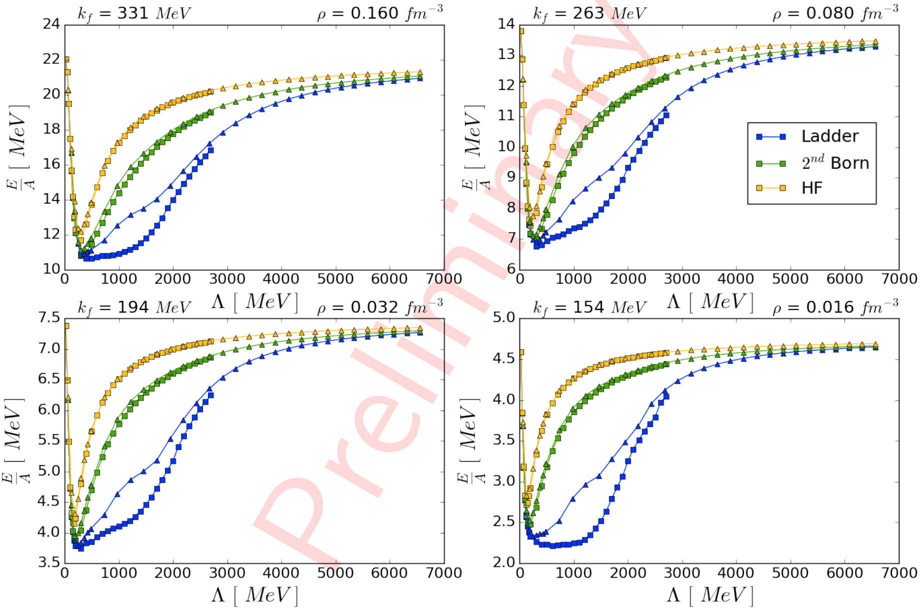
$$\frac{1}{C_0(\Lambda)} = \frac{m_N}{4\pi}\left(\frac{1}{a} - \frac{2}{\pi}\Lambda\int_0^\infty e^{-2x^2}dx\right)$$

Outline

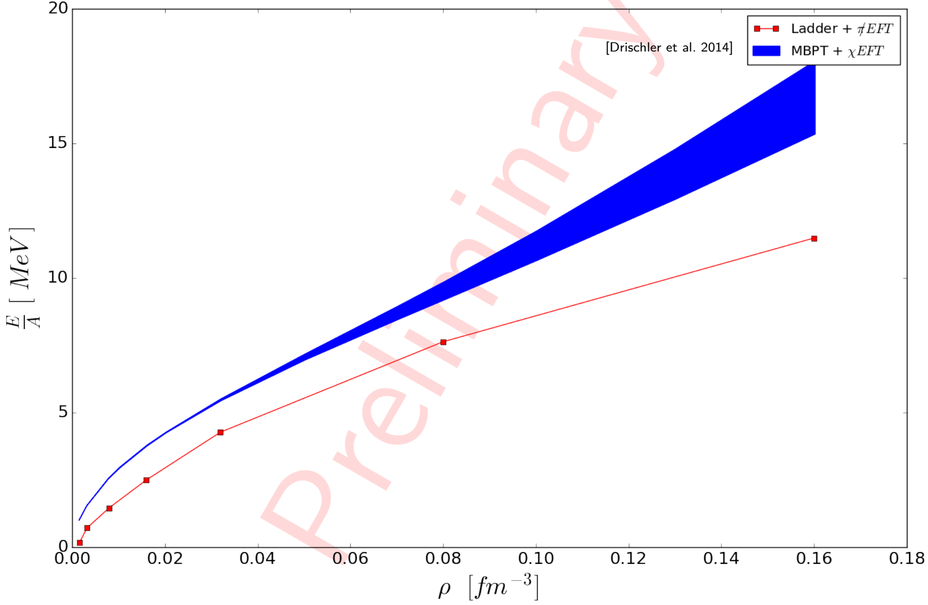
Assessing renormalization with ladder approximation



Comparison of the many-body approaches



Equation of state of neutron matter



Summary and perspectives

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  → Better understand the apparent failure of Ladder approximation
  → Is there a consistent (non-complete) subset of diagrams ?
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  - Analytical developments
    - → Better understand the apparent failure of Ladder approximation
    - → Is there a consistent (non-complete) subset of diagrams ?
  - Exact calculation to benchmark preliminary results
  - Extend study to symmetric matter/NLOs
  - What about error estimations ?
    - → *Still a debated issue*

Thank you !

Advisor : V. Somà

Co-advisor : T. Duguet

B. Bally and P. Arthuis

Collaborators : U. Van Kolck, M. Pavon Valderrama
J. Yang