Precision Calculation of Electromagnetic Observables for Light Nuclei

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Outline

1. Motivation
2. Ab initio calculations
   - Similarity Renormalization Group (SRG)
   - No-Core Shell Model (NCSM)
3. Electromagnetic Observables
4. Results
   - Electromagnetic Observables of Deuteron
   - M1 Observables of ^{6}Li
   - E2 Observables of ^{12}C
   - M1 Observables of Light Nuclei
5. Summary and Outlook

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- precision calculation of electromagnetic **observables**

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  - → accessible in **experiments**
  - → examples: electromagnetic moments, transition strengths
- **ab initio calculations** from first principles
  - → **consistent treatment** of electromagnetic observables
  - → neglected contributions

Ab initio nuclear-structure calculations

Chiral Effective Field Theory (χEFT)

$\rightarrow$ construct **consistent** interaction and electromagnetic operator

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Importance-Truncated No-Core Shell Model (IT-NCSM)

$\rightarrow$ solve many-body Schrödinger equation

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Continuous unitary transformation and flow equation

$$\mathbf{H}_\alpha = \mathbf{U}_\alpha^\dagger \mathbf{H}_0 \mathbf{U}_\alpha \qquad \Rightarrow \qquad \frac{\mathrm{d}}{\mathrm{d}\alpha}\mathbf{H}_\alpha = [\boldsymbol{\eta}_\alpha, \mathbf{H}_\alpha]$$

with initial conditions

$$\mathbf{H}_{\alpha=0} = \mathbf{H}$$

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- antihermitian generator $\boldsymbol{\eta}_\alpha = m_\mathrm{N}^2 \, [\mathbf{T}_\mathrm{rel}, \mathbf{H}_\alpha]$

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- so far: **bare operator** with evolved eigenstates

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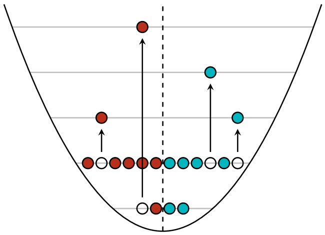

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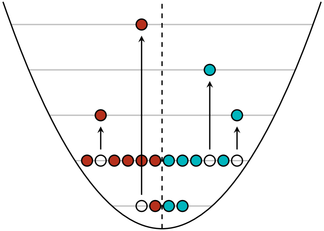

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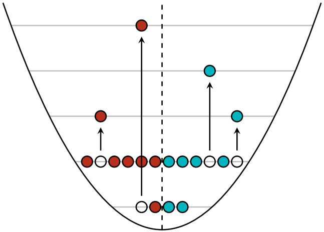

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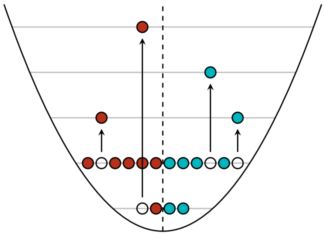
- **limited** by model space size



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- solve EV problem in new model space

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Magnetic dipole operator

$$\mathbf{M}_{10} = \sqrt{\frac{3}{4\pi}}\mu_{\mathrm{N}} \sum_{i=1}^{A} \left[\{g_{\mathrm{p}}\mathbf{s}_{\mathrm{z}}(i) + \mathbf{l}_{\mathrm{z}}(i)\}\mathbf{\Pi}_{\mathrm{p}}(i) + g_{\mathrm{n}}\mathbf{s}_{\mathrm{z}}(i)\mathbf{\Pi}_{\mathrm{n}}(i)\right]$$

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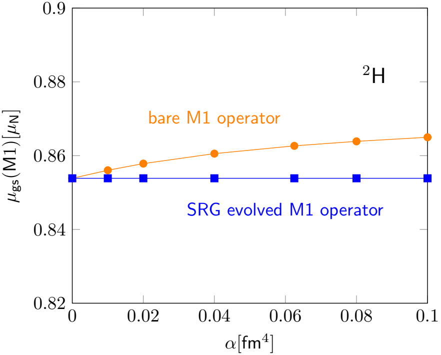
Magnetic dipole moment

$$\mu = \sqrt{\frac{4\pi}{3}}\,\langle J, M_{\mathrm{J}} = J|\mathbf{M}_{10}|J, M_{\mathrm{J}} = J\rangle$$

Electromagnetic Observables of Deuteron

Magnetic dipole moment



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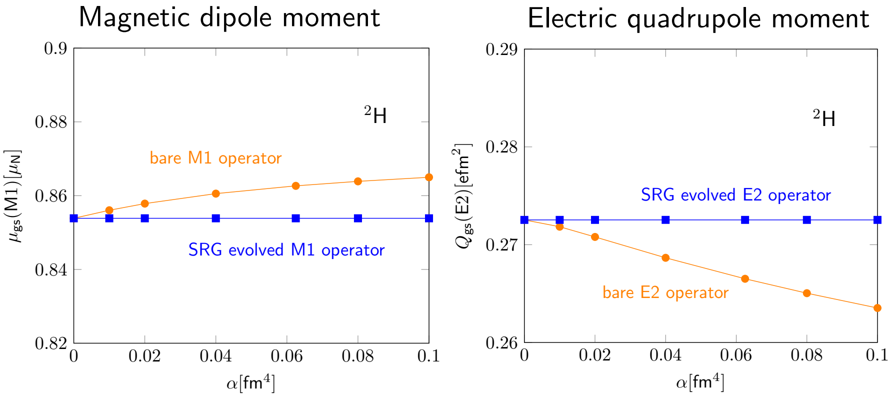

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M1 Observables of ^{6}Li

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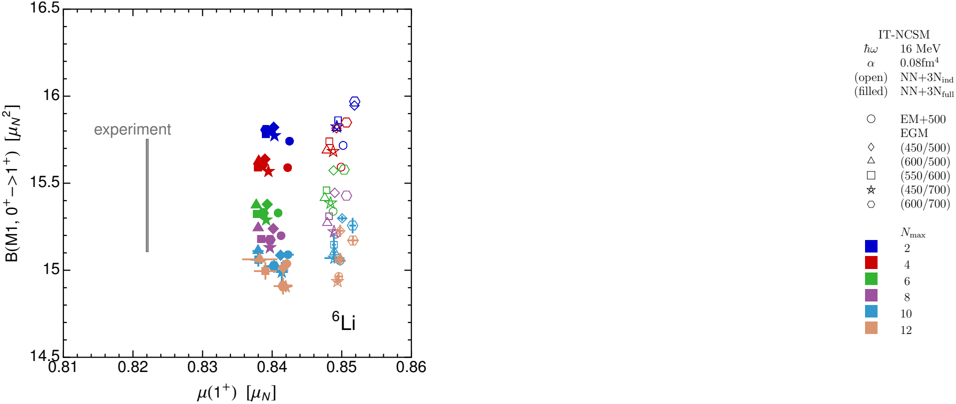
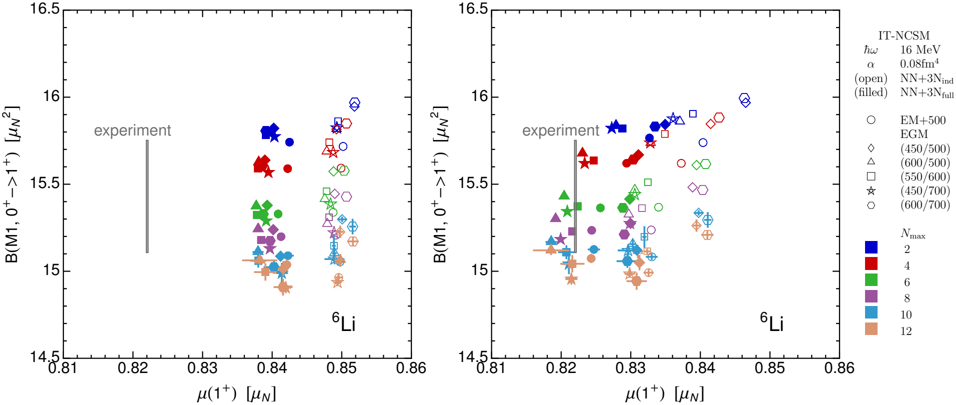


M1 Observables of ^{6}Li

bare M1 operators | SRG evolved M1 operators



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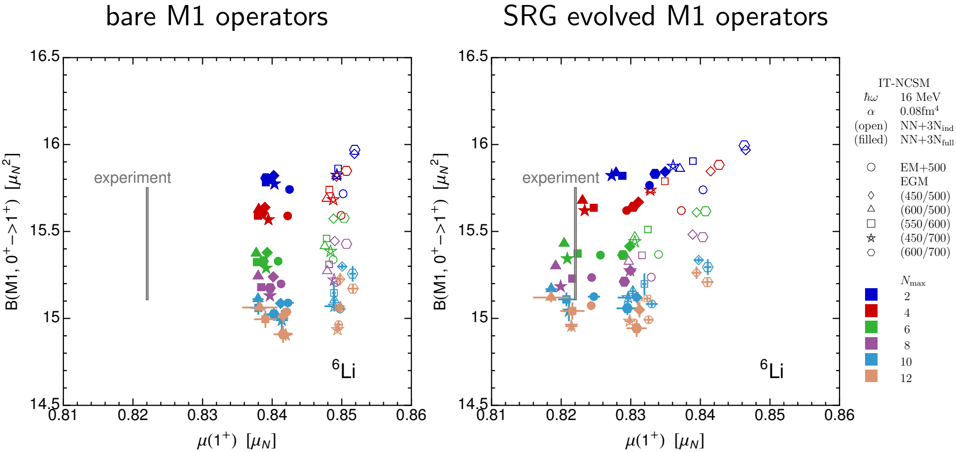

- contributions to μ by using **consistent** SRG transformation
- **small** contributions to **transition strength**

M1 Observables of ^{6}Li

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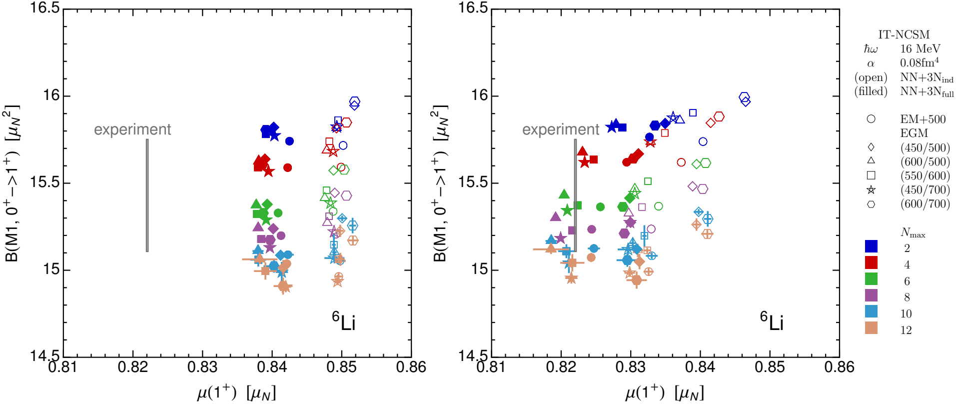
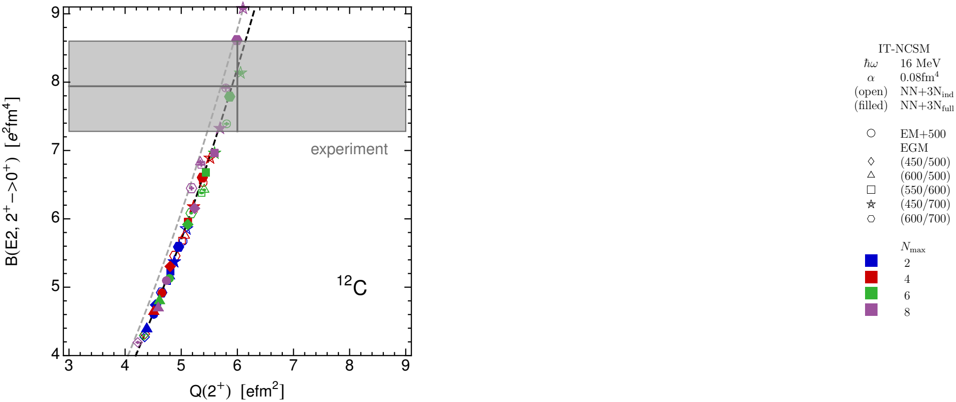


- contribution of **chiral currents**: [S. Pastore et al., Phys.Rev.C**87**, 035503(2013)]
  - → to magnetic-dipole moment: weak
  - → to transition strength: not negligible

E2 Observables of ^{12}C

bare E2 operators



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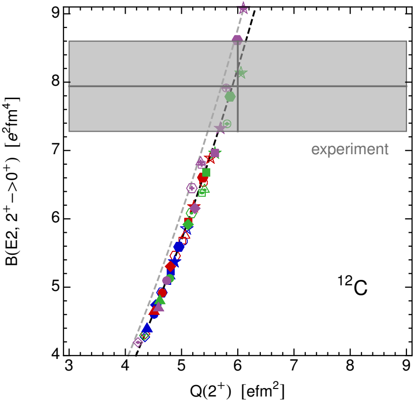

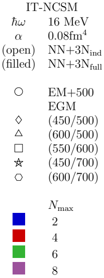

- description of **strong correlation** by rotor model

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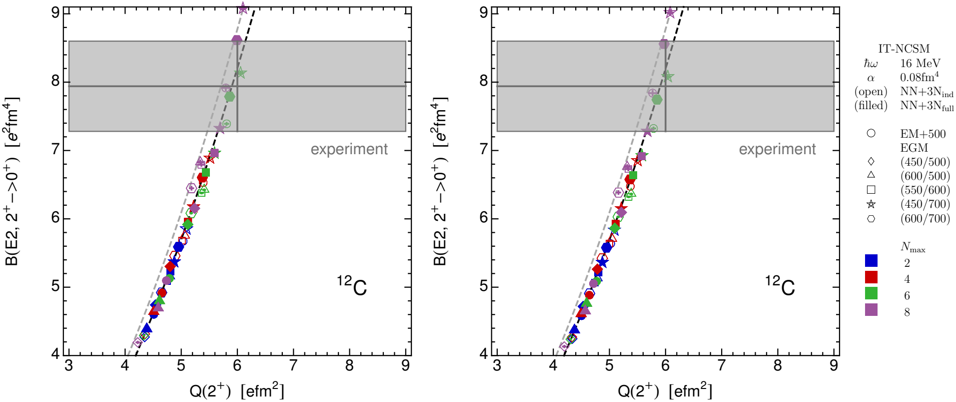

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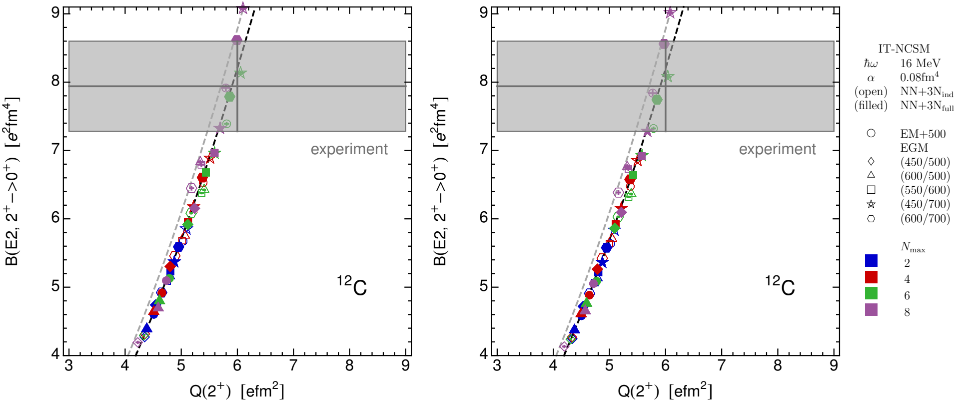
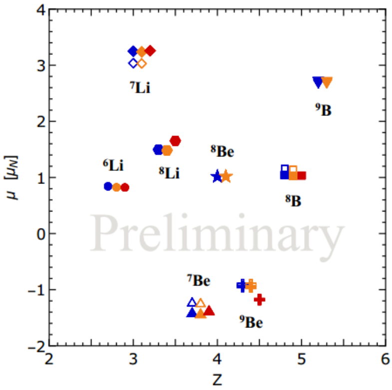


- description of **strong correlation** by rotor model
- contributions from consistent SRG are **small**

Magnetic Dipole Moments of Light Nuclei



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EM+500

$\hbar\omega$	16 MeV
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bare
SRG evolved
experiment

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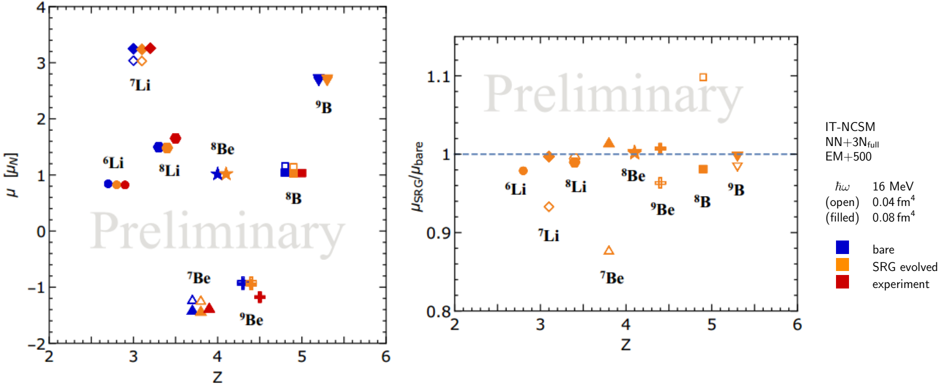

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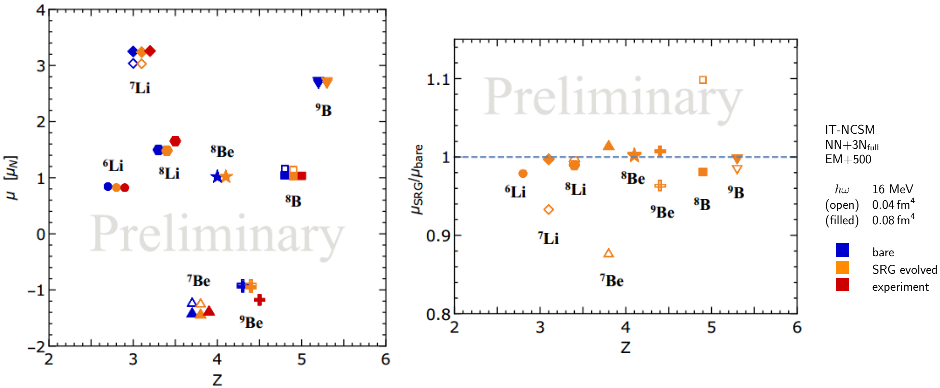

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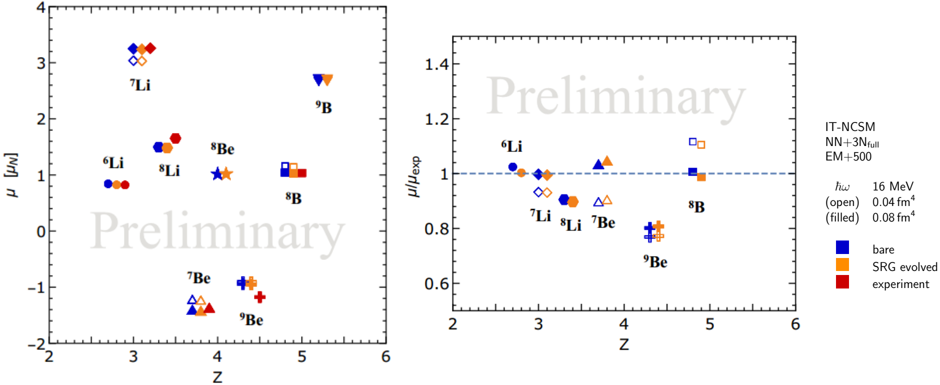

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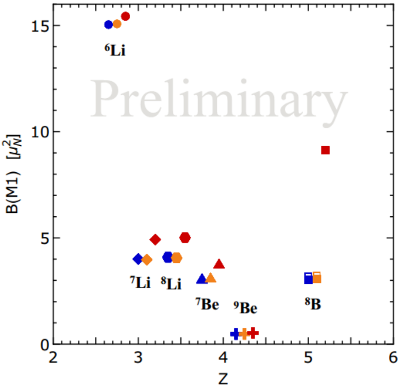
$M1$ Transition Strengths of Light Nuclei



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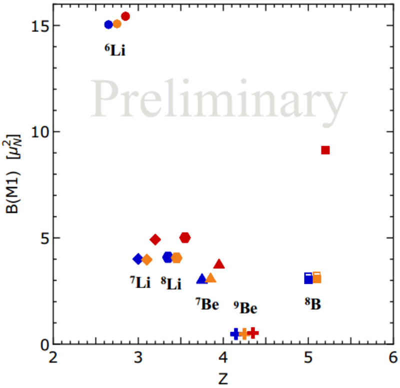

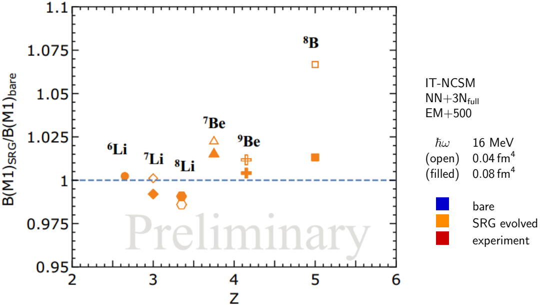

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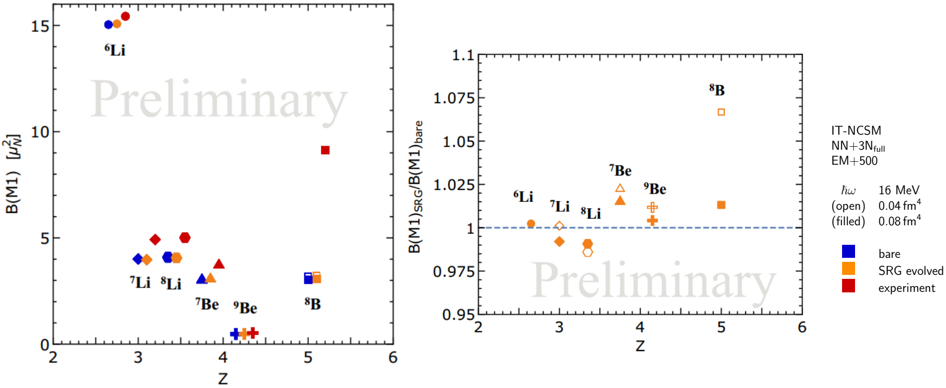

- **small** contributions through **consistent SRG**

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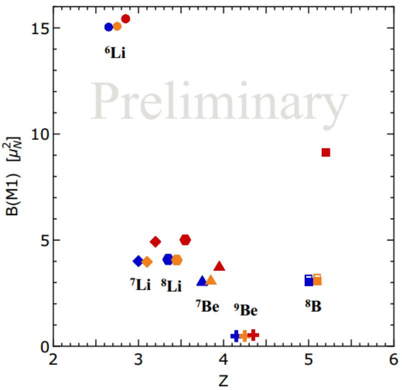

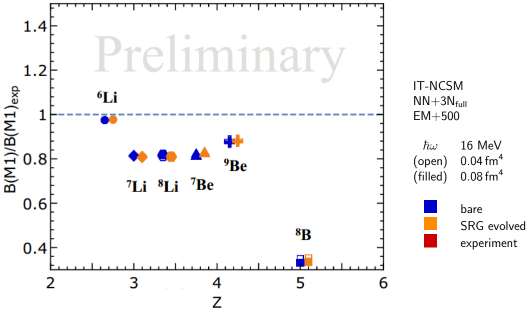

- **underestimation** of $M1$ transition strengths

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Outlook

next step to improve consistency → include two-body currents from chiral EFT

$$M_{lm} \propto \int \left(\vec{r} \times \vec{j}(\vec{r})\right) \vec{\nabla} r^l Y_{lm}(\theta, \phi)\, \mathrm{d}^3 r$$

Epilog

- **Thanks to my group**
  - S. Alexa, E. Gebrerufael, **T. Hüther**, **R. Roth**, S. Schulz, H. Spielvogel, C. Stumpf, A. Tichai, K. Vobig, R. Wirth

    Institut für Kernphysik, TU Darmstadt

- **Thank you for your attention!**

COMPUTING TIME

JURECA | LOEWE-CSC | LICHTENBERG

Deutsche Forschungsgemeinschaft