

# Microscopic description of fission mass yields

**Anna Zdeb**

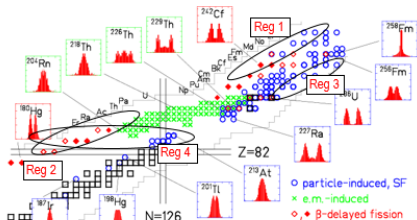
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Lublin, Poland



June 27, 2017  
ECT\* DTP, Trento

# Motivation



K.H. Schmidt *et al.*, *Nucl. Phys.* **A665** 221 (2000)

- ▶ Fission fragment mass yield is one of the basic, measurable observables
- ▶ The shape of observed fragment mass distribution allows to determine the type of fission (symmetric, asymmetric, bimodal)
- ▶ Accuracy of reproduction of the experimental mass yields is a test of the theoretical models
- ▶ The proper theoretical description of fission mass distribution is still pending

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Model HFB

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Dynamics + RNR

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Time of evolution

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Parity dependence

Mixed states

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# Potential energy surface - HFB + Gogny D1S

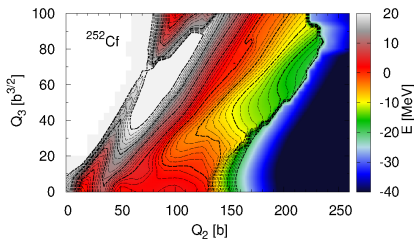
$$\delta \left[ \left\langle \Phi(Q_{20}, Q_{30}) \left| \hat{H} - \lambda_Z \hat{Z} - \lambda_N \hat{N} - \sum_i \lambda_i \hat{Q}_i \right| \Phi(Q_{20}, Q_{30}) \right\rangle \right] = 0$$

The quadrupole moment:

$$\hat{Q}_{20} = \sqrt{\frac{16\pi}{5}} \sum_{i=1}^A r_i^2 Y_{20}$$

The octupole moment:

$$\hat{Q}_{30} = \sqrt{\frac{4\pi}{7}} \sum_{i=1}^A r_i^3 Y_{30}$$



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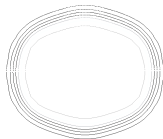
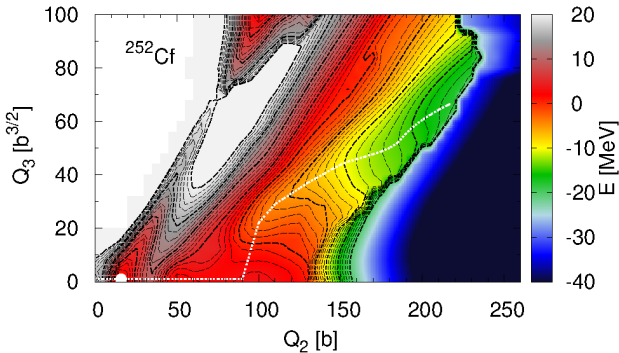
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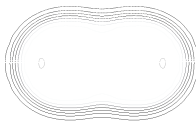
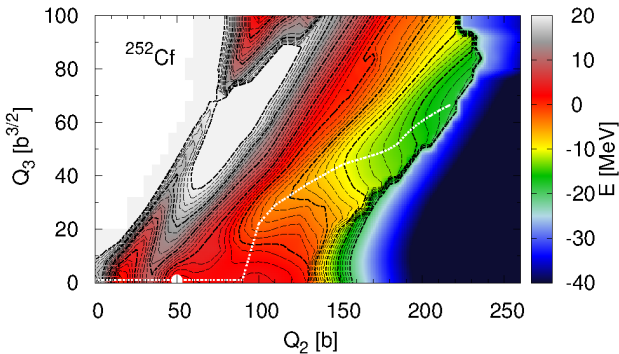
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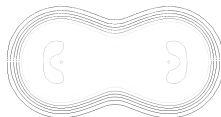
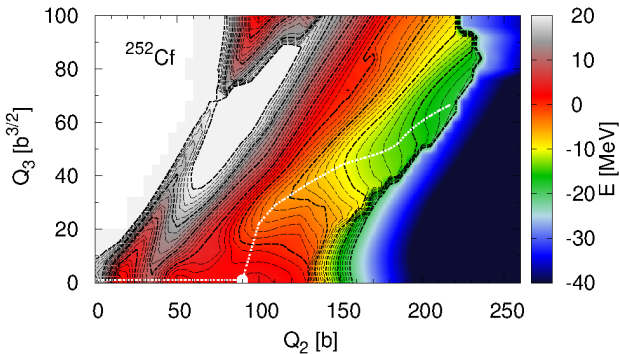
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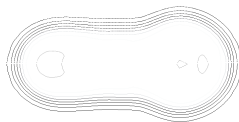
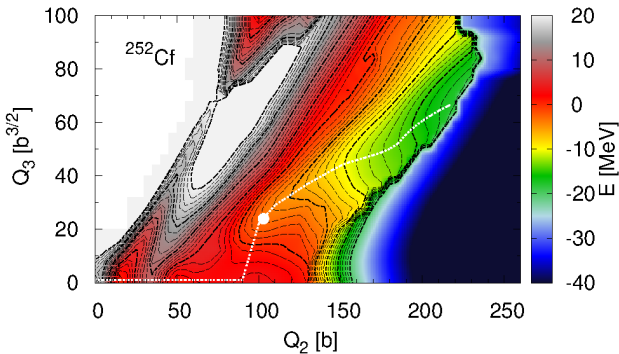
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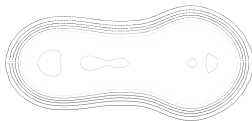
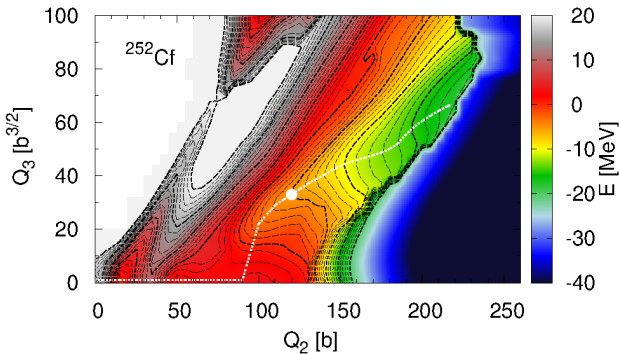
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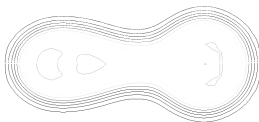
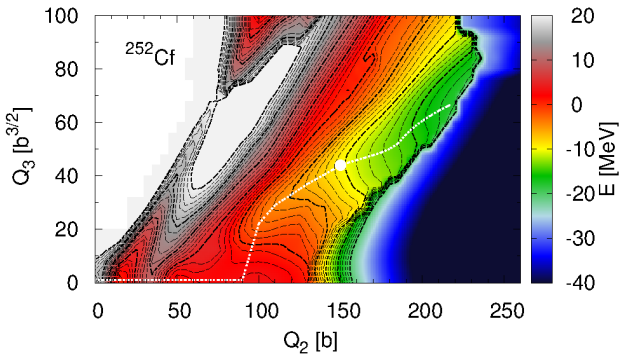
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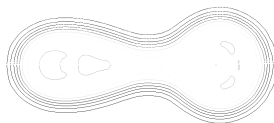
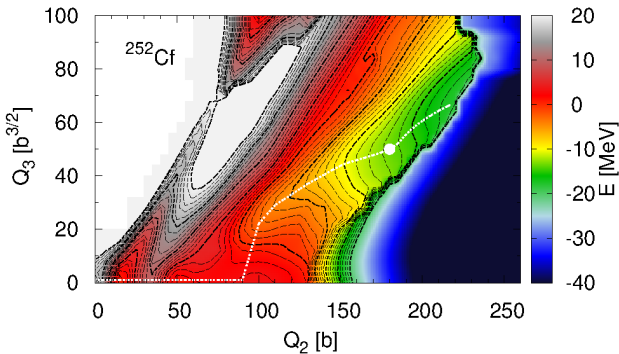
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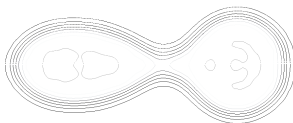
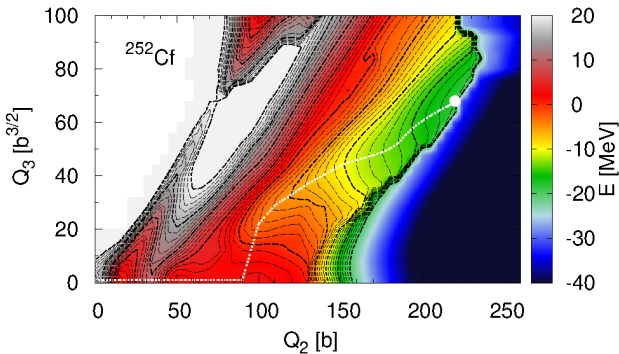
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# Kształty przedrozszczeniowe

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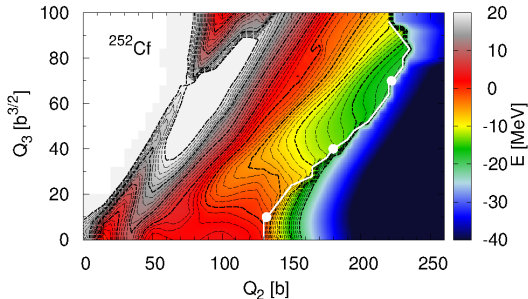
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$Q_2 = 130 \text{ b}, Q_3 = 10 \text{ b}^{3/2}$



$A_H / A_L = 131 / 121$

$Q_2 = 180 \text{ b}, Q_3 = 40 \text{ b}^{3/2}$



$A_H / A_L = 134 / 118$

$Q_2 = 220 \text{ b}, Q_3 = 70 \text{ b}^{3/2}$



$A_H / A_L = 142 / 110$

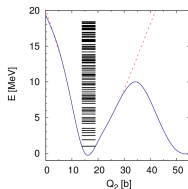
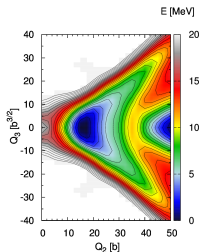


# Fission dynamics

$$\hat{H}_{coll} = -\frac{\hbar^2}{2\sqrt{\gamma}} \sum_{i,j=2}^3 \frac{\partial}{\partial Q_{i0}} \sqrt{\gamma} B_{ij}(Q_{20}, Q_{30}) \frac{\partial}{\partial Q_{j0}} + V(Q_{20}, Q_{30})$$

$$\gamma \equiv \det G(Q_2, Q_3), \quad B_{ij} = \mathcal{M}_{ij}^{-1}$$

$$\hat{H}'_{coll} g_n^\pi(Q_2, Q_3, t=0) = E_n^\pi g_n^\pi(Q_2, Q_3, t=0)$$



$$\hat{H}_{coll} g(Q_2, Q_3, t) = i\hbar \frac{\partial g(Q_2, Q_3, t)}{\partial t}$$



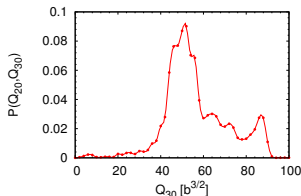
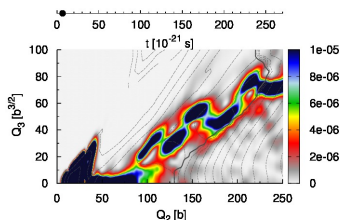
# Probability current density

$$\vec{J}(Q_{20}, Q_{30}, t) = \frac{\hbar}{2i} \sqrt{\gamma} B(Q_{20}, Q_{30}) \times$$

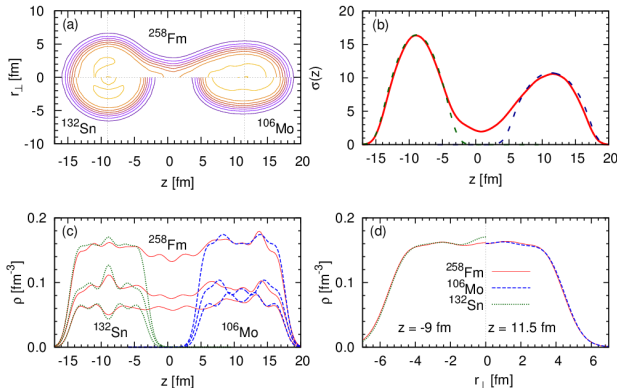
$$[g^*(Q_{20}, Q_{30}, t) \nabla g(Q_{20}, Q_{30}, t) -$$

$$g(Q_{20}, Q_{30}, t) \nabla g^*(Q_{20}, Q_{30}, t)]$$

$$P(Q_{20}^{SC}, Q_{30}^{SC}) = \int_{t=0}^{t=T^{propag}} \vec{J}(Q_{20}^{SC}, Q_{30}^{SC}, t) \cdot \vec{n} dt$$

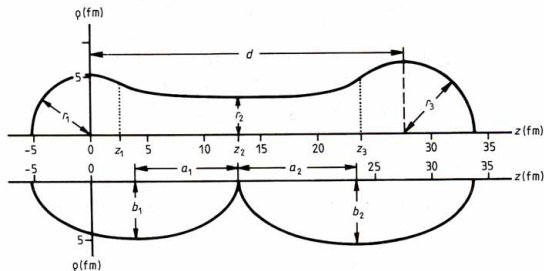


# Pre-scission configuration



$$\sigma(z) = 2\pi \int_0^{\infty} r_{\perp} \rho(z, r_{\perp}) dr_{\perp}$$

# Neck rupture probability



$$P(z) \sim \exp[-\sigma(z)]$$

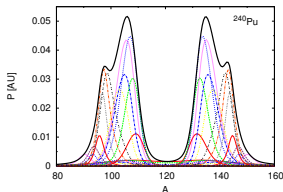
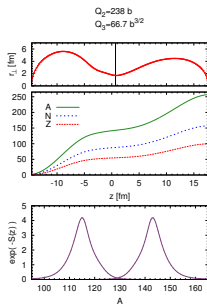


# Random Neck Rupture Mechanism

$$\sigma(z) = 2\pi \int_0^{\infty} r_{\perp} \rho(z, r_{\perp}) dr_{\perp}$$

$$P(A_1/A_2) = \exp\left[\frac{-2\gamma\sigma(z)}{T}\right] \Rightarrow$$

$$P(Q_{20}^{SC}, Q_{30}^{SC}) = \int_{t=0}^{t=T_{propag}} \vec{J}(Q_{20}^{SC}, Q_{30}^{SC}, t) \cdot \vec{n} dt \Rightarrow$$



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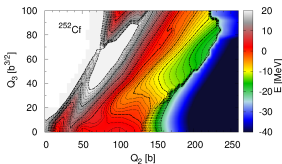
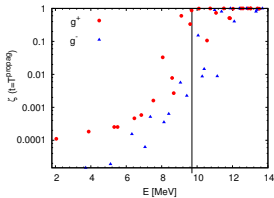
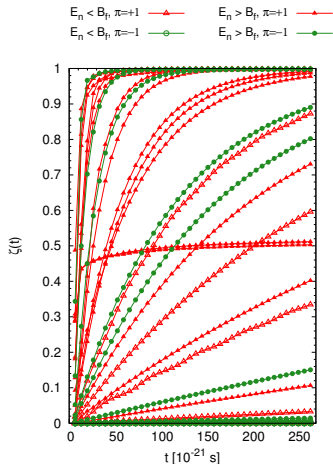
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# The tunneling probability



$$\zeta(t) = 1 - \int |g^\pi(q_{20}, q_{30}, t)|^2 dq_{20} dq_{30}$$



# Time of evolution

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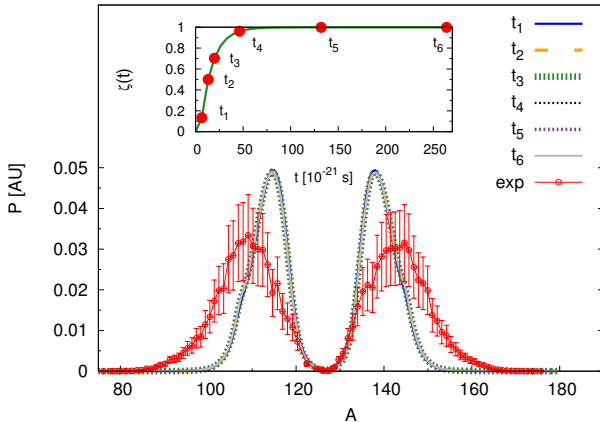
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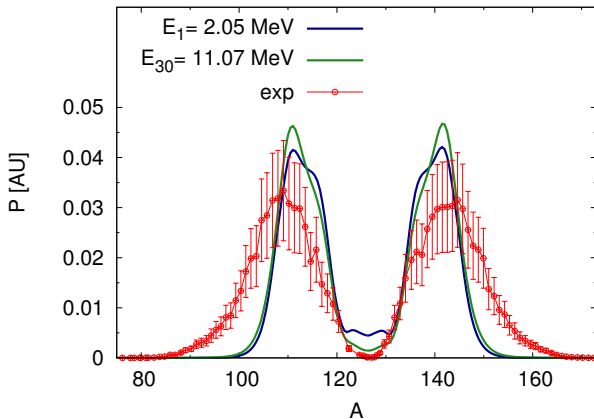
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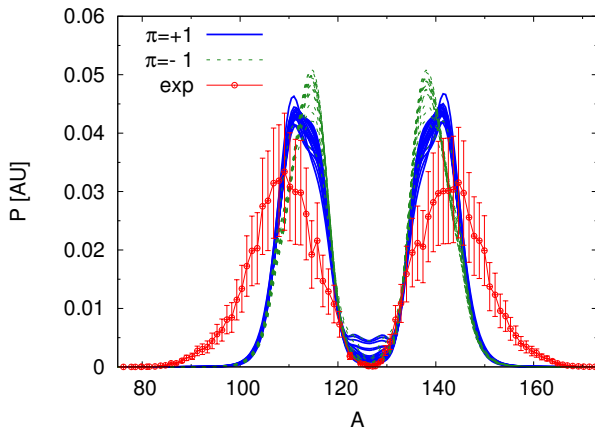
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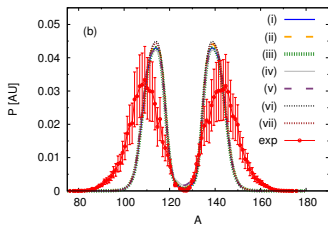
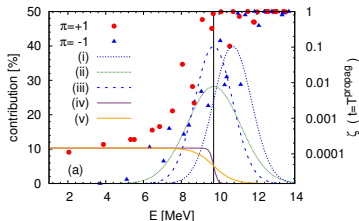
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# Mixed states

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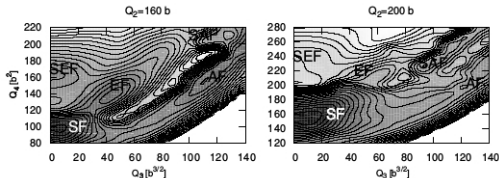
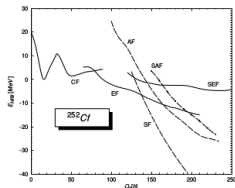
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- ▶ The peaks positions are well reproduced
- ▶ There is no strong correlation between mass distribution and initial conditions
- ▶ Theoretical mass yields are too narrow in comparison to the experimental ones
- ▶ To improve the accuracy of data reproduction model extensions are required



# Third dimension - $Q_{40}$

The hexadecapole moment: 
$$\hat{Q}_{40} = \sqrt{\frac{4\pi}{9}} \sum_{i=1}^A r_i^4 Y_{40}$$



**Super-asymmetric fission path**



**Thank you!**



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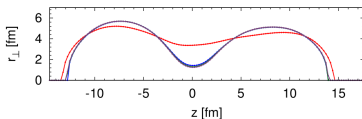
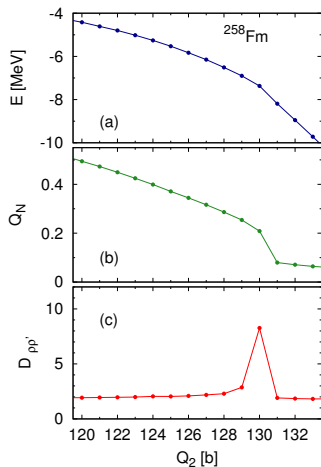
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# Searching for pre-scission points



$$D_{\rho\rho'} = \int_0^{\infty} |\rho(\vec{r}) - \rho'(\vec{r})| d\tau$$

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