Electromagnetic currents from χEFT

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work done in collaboration with:
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which builds on previous work by
S. Pastore, R. Schiavilla, and J.L. Goity

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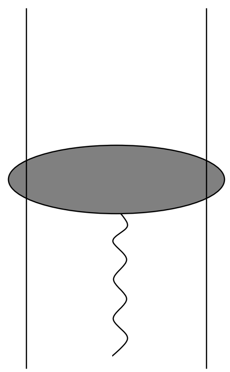
Outline

- Description of the framework
- Classification and renormalization of the counterterms
- Treatment of the recoil corrections
- LECs' determination and predictions in the hybrid approach
- Conclusions and Outlook



compute in time-ordered perturbation theory the irreducible part of the amplitude

$$\langle N'N'|T|NN;\gamma\rangle = \langle N'N'|H_I \sum_n \left(\frac{1}{E - H_0 + i\eta} H_I\right)^n |NN;\gamma\rangle$$

written in terms of the current as $-\frac{\hat{\mathbf{e}}_{\mathbf{q}\lambda}}{\sqrt{2\omega_q}} \cdot \mathbf{j}$

H_I contains the interactions of pions, nucleons and photons coming from $\mathcal{L}_{\rm eff}$, the χPT Lagrangian

The infinite number of chirally symmetric terms can be ordered according to increasing numbers of derivatives and nucleon fields

Recoil corrections to reducible diagrams are also taken into account

Chiral counting

A given diagram counts as $O(Q^\nu)$ with

$$\nu = \sum_i \left(d_i - \frac{b_i}{2} \right) - (V - 1) + 3L$$

using the topological identities

$$2I_N + E_N = \sum_i n_i, \quad 2I_\pi = \sum_i b_i, \quad L = I_\pi + I_N - V + 1$$

we have

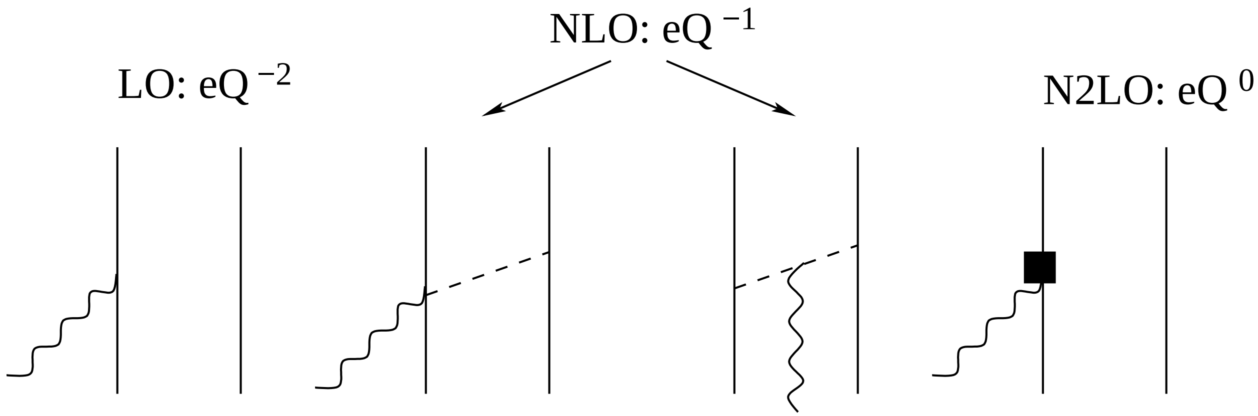
$$\nu = \sum_i \left(d_i + \frac{n_i}{2} - 2 \right) - \frac{E_N}{2} + 2L$$

Chiral symmetry ensures that $d_i + \frac{n_i}{2} - 2 \geq 0$

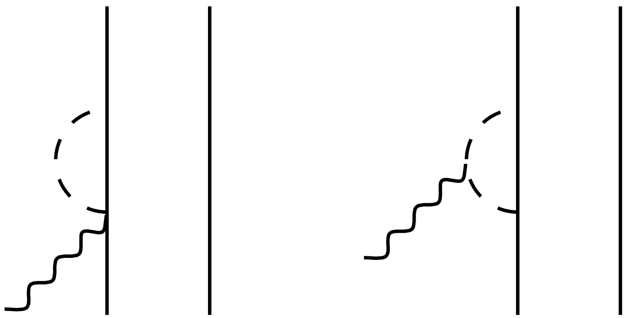
$\Longrightarrow$ at a given order only finitely many diagrams contribute

N2LO Currents

- tree diagrams

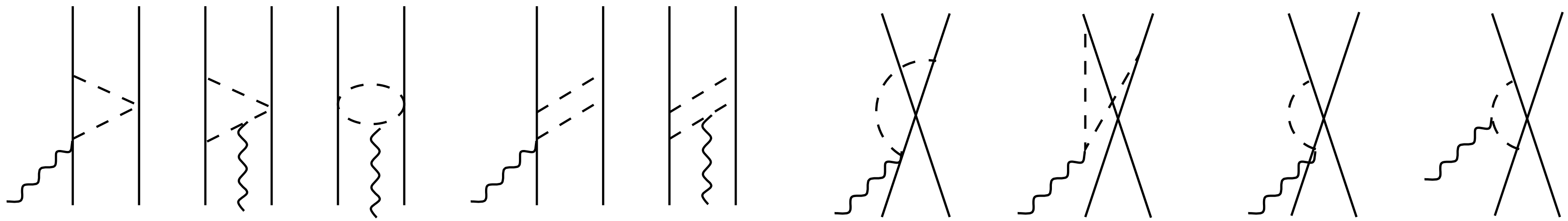


- 1-loop disconnected

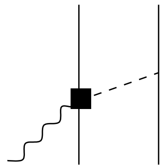


N3LO Currents

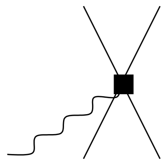
- 1-loop



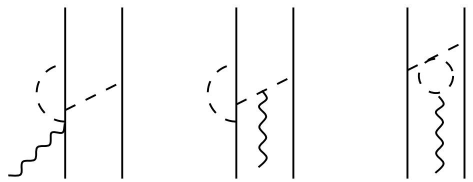
- tree diagrams with one sub^{2}leading vertex (3 LECs)



- contact interaction with 2 derivatives (2 LECs from non-minimal couplings)



- 1-loop renormalization of tree-level currents

 + ...

Renormalization requires to include a complete list of counterterms
Symmetries require the list to be minimal

Consider e.g. the subleading contact interactions: they stem from the gauging of 2-derivative 2-nucleon Lagrangian ($14 \to 12 \to 7$ terms), and from non-minimal coupling terms: parity, time-reversal, rotations $\Longrightarrow$

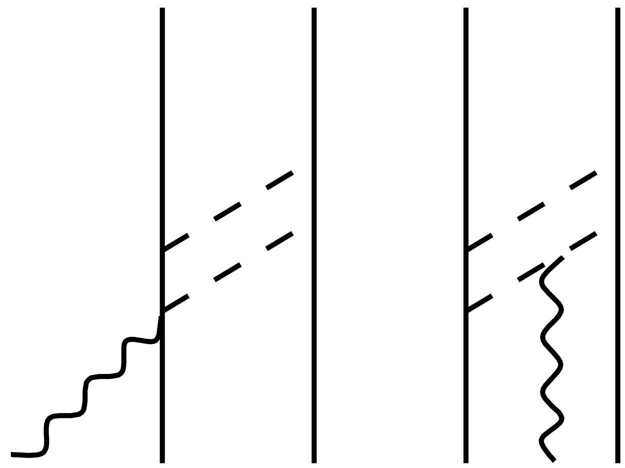
$$\epsilon_{ijk} F_{ij} N^\dagger \sigma_k N N^\dagger N \left\{ \begin{array}{c} \mathbf{1} \otimes \mathbf{1} \\ \tau^a \otimes \tau^a \\ \tau^3 \otimes \mathbf{1} \pm \mathbf{1} \otimes \tau^3 \end{array} \right\}, \quad F_{ij} N^\dagger \sigma_i N N^\dagger \sigma_j N \left\{ \epsilon^{3ab} \tau^a \otimes \tau^b \right\}$$

Using Fierz-type identities encoding fermionic antisymmetry we are left with 2 structures

Renormalization

Divergences in the loop integrals can be absorbed by a renormalization of the counterterms

This is not true for the single diagrams individually: we have to sum both these diagrams in order to find a suitable counterterm renormalization. This provides a non-trivial check of our calculation.



At the end, typically,

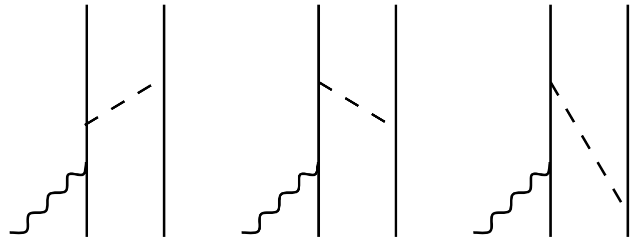
$$C_6 = \bar{C}_6 + \frac{3g_A^4}{8\pi^2 F_\pi^4}\mu^{-\epsilon}\left(-\frac{2}{\epsilon} + \gamma - \log\pi + \log\frac{m_p i^2}{\mu^2} - \frac{4}{3}\right)$$

with $\bar{C}_6$ finite and μ-independent

Recoil corrections to reducible diagrams

Reducible:

$$\left(\frac{1}{E - E_2 - \omega_\pi - E_1'} + \frac{1}{E - E_2' - \omega_\pi - E_1^*}\right)\frac{1}{E - E_2 - E_1^*}$$



Irreducible: $\frac{1}{-\omega_\pi}\frac{1}{-\omega_\pi}$

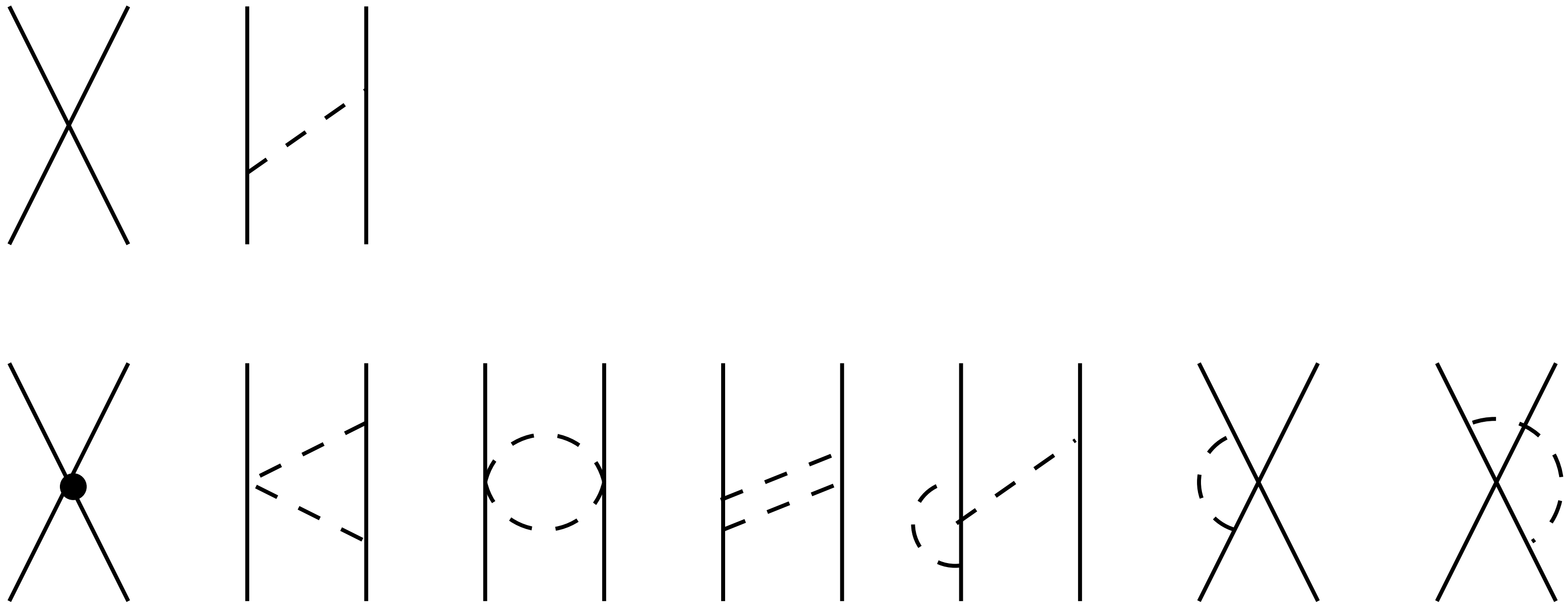
Expand in $E_N/\omega_\pi \Longrightarrow$ Reducible: $-\frac{2}{\omega_\pi}\frac{1}{E-E_2-E_1^*} - \frac{1}{\omega_\pi^2} + O(p^{-1})$
cancelling the irreducible contribution

Such complete or partial cancellations are ubiquous in our calculation.

Recoil corrections to reducible diagrams are a distinctive feature of our framework: they remove the explicit energy dependence in the potential, and allow to recover the results obtained with the method of the unitary transformation (see next talk by S. Koelling).

NN potential at N2LO

The same machinery is applied for the calculation of the NN potential



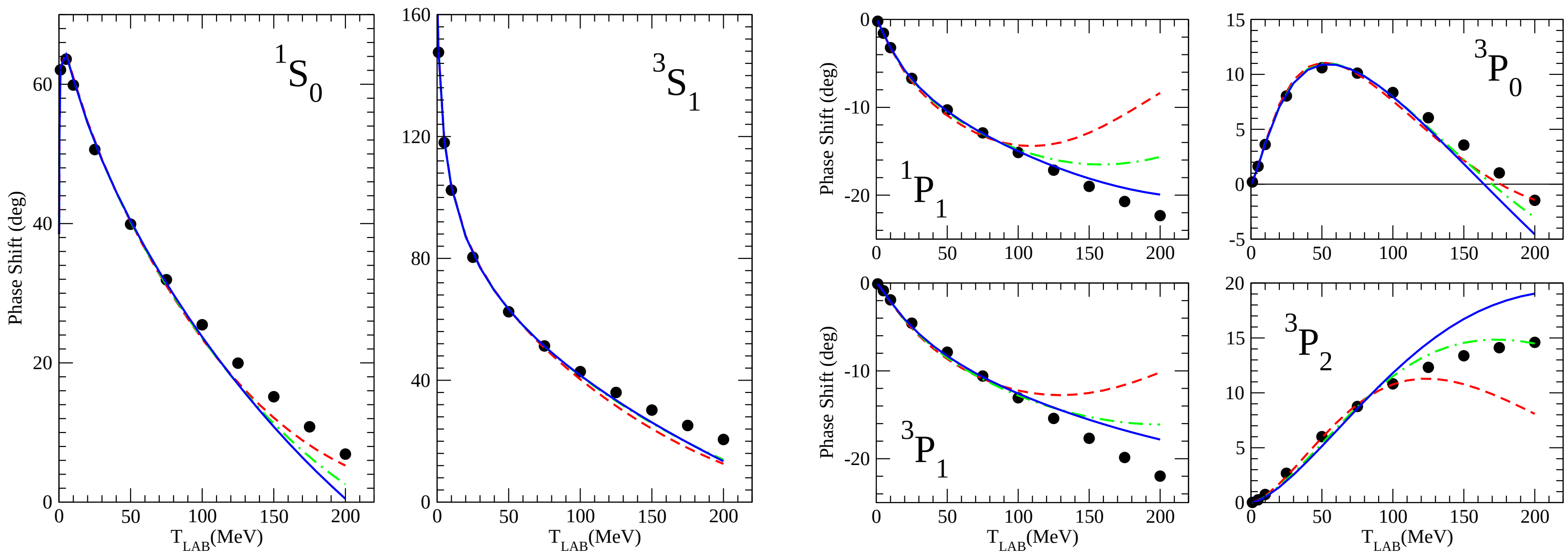
7 LECs parametrize the contact interaction in the c.o.m. frame

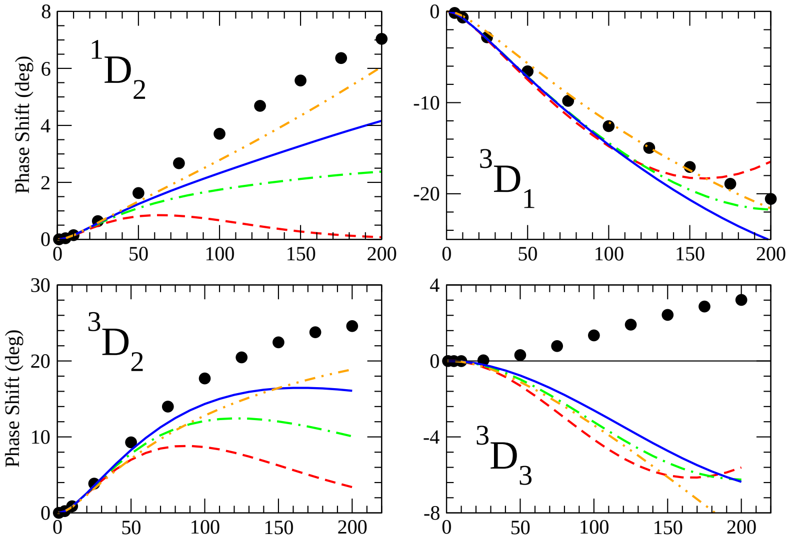
5 LECs are fixed in terms of C_S and C_T through Poincaré symmetry

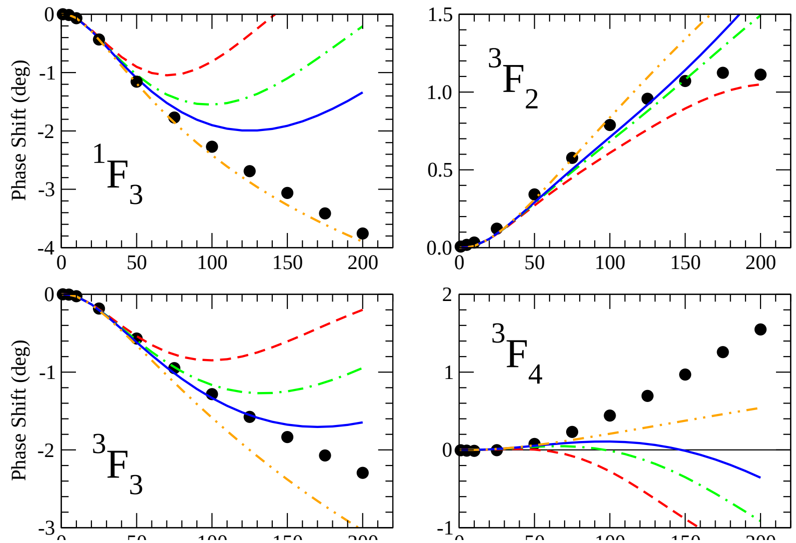
The resulting potential satisfies the continuity equation giving current conservation

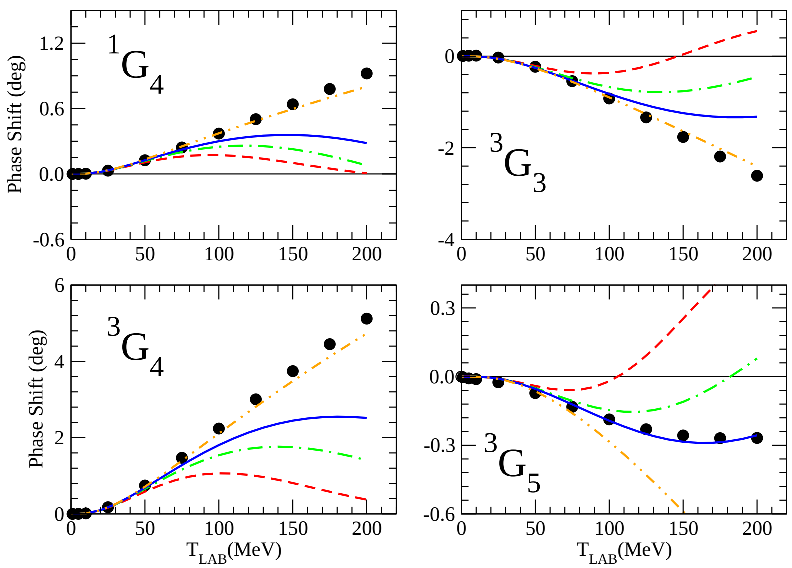
Determining the LECs

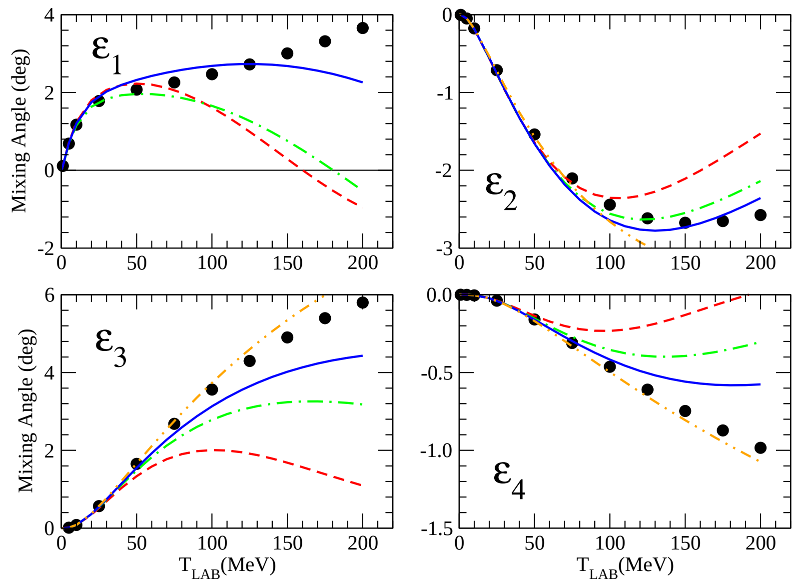
We fit the 9 contact LECs to the NN phaseshifts up to 100 MeV for $\Lambda = 500 - 700$ MeV



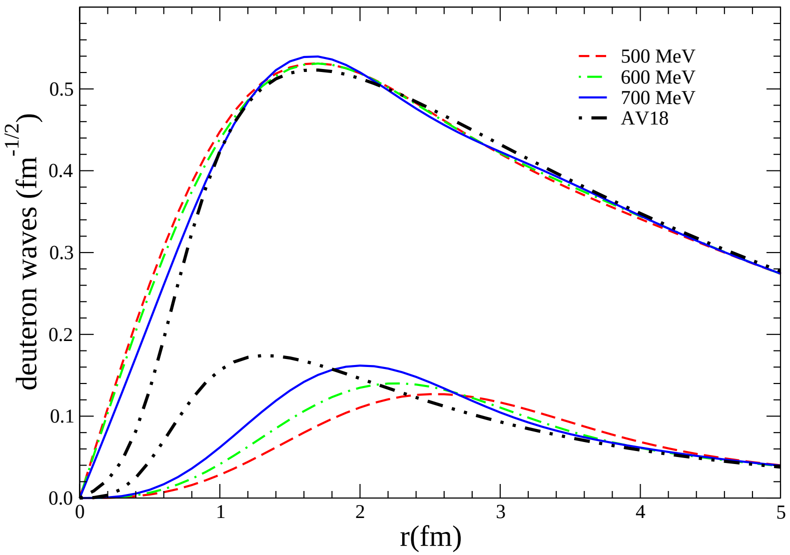

Phase Shift (deg)
$^1D_2$
$^3D_1$
$^3D_2$
$^3D_3$


Phase Shift (deg)
$^1F_3$
$^3F_2$
$^3F_3$
$^3F_4$


Phase Shift (deg)
$^1G_4$
$^3G_3$
$^3G_4$
$^3G_5$
T_{LAB}(MeV)


Mixing Angle (deg)
ε_1
ε_2
ε_3
ε_4
T_{LAB}(MeV)

Deuteron



	Λ (MeV)			
	500	600	700	Expt
B_d (MeV)	2.2244	2.2246	2.2245	2.224575(9)
η_d	0.0267	0.0260	0.0264	0.0256(4)
r_d (fm)	1.943	1.947	1.951	1.9734(44)
μ_d (μ_N)	0.860	0.858	0.853	0.8574382329(92)
Q_d (fm^2)	0.275	0.272	0.279	0.2859(3)
P_D (%)	3.44	3.87	4.77	

We are left with 5 constants to determine

1 combination is fixed by resonance saturation. 4 constants fixed from nuclear em properties in $A = 2, 3$

Preliminary studies in nd and n^3He for AV18/UIX and N3LO/N2LO show reduced Λ dependence and reasonable agreement with data
It seems that the LECs contribution is the dominant one

Conclusions

- We have derived in a unique framework a NN potential and consistent em currents, which satisfy conservation up to N3LO
- Recoil corrections to reducible diagrams are a distinctive feature of our approach, allowing to recover results obtained within the method of unitary transformation
- The obtained result is the complete one at order eQ: in particular $3N$ currents vanish at this order, due to peculiar cancellations
- Dominance of the LECs contributions might indicate that the inclusion of the Δ is mandatory to improve convergence
- Despite the lack of accuracy of the N2LO NN potential, it will be interesting to perform non-hybrid studies in light systems