

Electromagnetic currents from the method of unitary transformation

Stefan Kölling

Forschungszentrum Jülich,
& HISKP (Theorie) Bonn

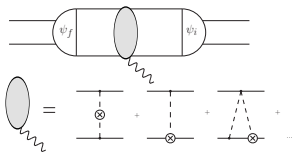
e-mail: s.koelling@fz-juelich.de

in collaboration with: D. Rozpedzik, J. Golak, E. Epelbaum, H. Krebs and U.-G. Meißner

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Motivation

- Following Weinberg^{'90,'91,'92} successful application of ChPT to calculate nuclear forces.
- Calculate only **irreducible** kernel and iterate.
→ Method of **unitary transformation**^{Epelbaum et al. '98}.
- **Consistent derivation** of electromagnetic-current J^μ



- $\vec{\nabla} \cdot \vec{J} = -i [H, \rho]$
- Treat em-interaction as perturbation
- Convolute between wave-functions.

- Define effective current with **unitary transformation**

$$\eta V_{\text{eff}} \eta = \eta U^\dagger \eta U^\dagger H U \eta U' \eta - H_0, \quad U = \begin{pmatrix} \eta (1 + A^\dagger A)^{-\frac{1}{2}} & -A^\dagger (1 + AA^\dagger)^{-\frac{1}{2}} \\ A (1 + A^\dagger A)^{-\frac{1}{2}} & \lambda (1 + AA^\dagger)^{-\frac{1}{2}} \end{pmatrix},$$

$$\eta J_{\text{eff}}^\mu \eta = \eta U^\dagger \eta U^\dagger J^\mu U \eta U' \eta,$$

with projectors η (λ) on the purely nucleonic (rest) subspace.

Lagrangian we use

$$\mathcal{L} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)} + \mathcal{L}_{NN\gamma}^{(2)},$$

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{F_\pi^2}{4} \langle D_\mu U D^\mu U^\dagger + \chi_+ \rangle,$$

$$\begin{aligned} \mathcal{L}_{\pi\pi}^{(4)} = & \frac{l_3}{16} \langle \chi_+ \rangle^2 + \frac{l_4}{16} \left(2 \langle D_\mu U D^\mu U^\dagger \rangle \langle \chi_+ \rangle + 2 \langle \chi^\dagger U \chi^\dagger U + \chi U^\dagger \chi U^\dagger \rangle - 4 \langle \chi^\dagger \chi \rangle \right) \\ & + i \frac{l_6}{2} \langle f_{\mu\nu}^R D^\mu U D^\nu U^\dagger + f_{\mu\nu}^L (D^\mu U)^\dagger D^\nu U \rangle, \end{aligned}$$

$$\mathcal{L}_{\pi N}^{(1)} = \bar{N} [i(v \cdot D) + \tilde{g}_A (S \cdot u)] N,$$

$$\begin{aligned} \mathcal{L}_{\pi N}^{(3)} = & \bar{N} \left[d_{16} S \cdot u \langle \chi_+ \rangle + i d_{18} S^\mu [D_\mu, \chi_-] \right. \\ & + d_6 v^\nu [D^\mu, \tilde{f}_{\mu\nu}^+] + d_7 v^\nu [D^\mu, \langle f_{\mu\nu}^+ \rangle] + d_8 \epsilon^{\mu\nu\alpha\beta} v_\beta \langle \tilde{f}_{\mu\nu}^+ u_\alpha \rangle + d_9 \epsilon^{\mu\nu\alpha\beta} v_\beta \langle f_{\mu\nu}^+ \rangle u_\alpha \\ & \left. + d_{20} i S^\mu v^\nu [\tilde{f}_{\mu\nu}^+, v \cdot u] + d_{21} i S^\mu [\tilde{f}_{\mu\nu}^+, u^\nu] + d_{22} S^\mu [D^\nu, f_{\mu\nu}^-] \right] N, \end{aligned}$$

$$\mathcal{L}_{NN}^{(0)} = -\frac{1}{2} C_S \bar{N} N \bar{N} N + 2 C_T \bar{N} S_\mu N \bar{N} S^\mu N,$$

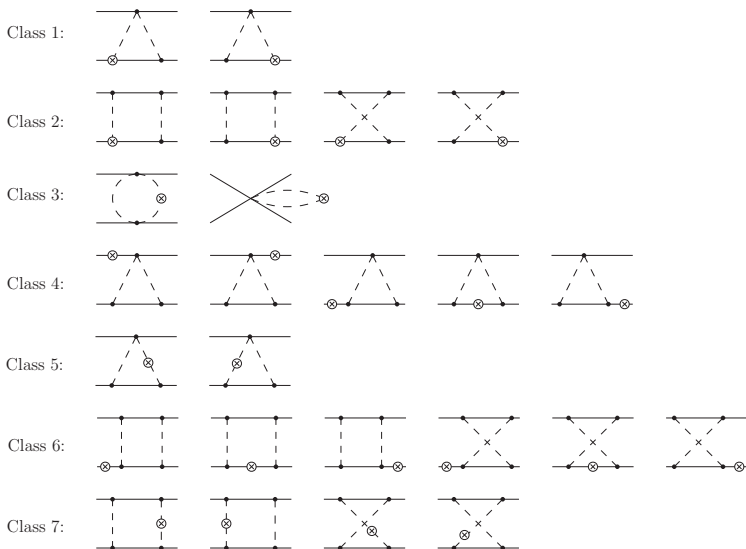
$$\mathcal{L}_{NN\gamma}^{(2)} = e \epsilon_{\mu\nu\rho\sigma} v^\mu f^{\nu\rho} \left[\tilde{C}_1 \bar{N}_\nu S^\sigma N_\nu \bar{N}_\nu N_\nu + \tilde{C}_2 \left(\bar{N}_\nu S^\sigma \tau^3 N_\nu \bar{N}_\nu N_\nu - \bar{N}_\nu S^\sigma N_\nu \bar{N}_\nu \tau^3 N_\nu \right) \right],$$

with the β -functions (Gasser et al. '02)

$$d_i = d_i^f(\mu) + \frac{\beta_i}{F_\pi^2} L, \quad l_i = l_i^f(\mu) + \gamma_i L$$

$$\beta_7 = \beta_8 = \beta_9 = \beta_{18} = \beta_{22} = 0, \quad \beta_6 = -\frac{1}{6} - \frac{5}{6} g_A^2, \quad \beta_{16} = \frac{1}{2} g_A + g_A^3, \quad \beta_{21} = -g_A^3, \quad \gamma_4 = 2, \quad \gamma_6 = -\frac{1}{3}.$$

Two-Pion exchange currents



Two-Pion exchange currents in configuration-space

$$\vec{J}_{c1}(\vec{r}_{10}, \vec{r}_{20}) = e \frac{g_A^2 M_\pi^7}{128\pi^3 F_\pi^4} \left[\vec{\nabla}_{10} [\vec{r}_1 \times \vec{r}_2]^3 + 2 [\vec{\nabla}_{10} \times \vec{\sigma}_2] \tau_1^3 \right] \delta(\vec{x}_{20}) \frac{K_1(2x_{10})}{x_{10}^2} + (1 \leftrightarrow 2),$$

$$\begin{aligned} \vec{J}_{c2}(\vec{r}_{10}, \vec{r}_{20}) &= -e \frac{g_A^4 M_\pi^7}{256\pi^3 F_\pi^4} (3\nabla_{10}^2 - 8) \left[\vec{\nabla}_{10} [\vec{r}_1 \times \vec{r}_2]^3 + 2 [\vec{\nabla}_{10} \times \vec{\sigma}_2] \tau_1^3 \right] \delta(\vec{x}_{20}) \frac{K_0(2x_{10})}{x_{10}} \\ &+ e \frac{g_A^4 M_\pi^7}{32\pi^3 F_\pi^4} \left[\vec{\nabla}_{10} \times \vec{\sigma}_1 \right] \tau_2^3 \delta(\vec{x}_{20}) \frac{K_1(2x_{10})}{x_{10}^2} + (1 \leftrightarrow 2), \end{aligned}$$

$$\vec{J}_{c3}(\vec{r}_{10}, \vec{r}_{20}) = -e \frac{M_\pi^7}{512\pi^4 F_\pi^4} [\vec{r}_1 \times \vec{r}_2]^3 (\vec{\nabla}_{10} - \vec{\nabla}_{20}) \frac{K_2(x_{10} + x_{20} + x_{12})}{(x_{10} x_{20} x_{12})(x_{10} + x_{20} + x_{12})} + (1 \leftrightarrow 2),$$

$$\begin{aligned} \vec{J}_{c5}(\vec{r}_{10}, \vec{r}_{20}) &= -e \frac{g_A^2 M_\pi^7}{256\pi^4 F_\pi^4} (\vec{\nabla}_{10} - \vec{\nabla}_{20}) \left[[\vec{r}_1 \times \vec{r}_2]^3 \vec{\nabla}_{12} \cdot \vec{\nabla}_{20} - 2\tau_1^3 \vec{\sigma}_2 \cdot [\vec{\nabla}_{12} \times \vec{\nabla}_{20}] \right] \\ &\times \frac{K_1(x_{10} + x_{20} + x_{12})}{(x_{10} x_{20} x_{12})} + (1 \leftrightarrow 2), \end{aligned}$$

$$\begin{aligned} \vec{J}_{c7}(\vec{r}_{10}, \vec{r}_{20}) &= e \frac{g_A^4 M_\pi^7}{512\pi^4 F_\pi^4} (\vec{\nabla}_{10} - \vec{\nabla}_{20}) \left[[\vec{r}_1 \times \vec{r}_2]^3 \vec{\nabla}_{12} \cdot \vec{\nabla}_{10} \vec{\nabla}_{12} \cdot \vec{\nabla}_{20} + 4\tau_2^3 \vec{\sigma}_1 \cdot [\vec{\nabla}_{12} \times \vec{\nabla}_{10}] \right. \\ &\left. \times \vec{\nabla}_{12} \cdot \vec{\nabla}_{20} \right] \frac{x_{10} + x_{20} + x_{12}}{x_{10} x_{20} x_{12}} K_0(x_{10} + x_{20} + x_{12}) + (1 \leftrightarrow 2) \end{aligned}$$

$$\vec{J}_{c4}(\vec{r}_{10}, \vec{r}_{20}) = \vec{J}_{c6}(\vec{r}_{10}, \vec{r}_{20}) = 0,$$

with $\vec{r}_{1/2/0}$ the positions of the first/second nucleon/the photon, and $\vec{x}_{10} = M_\pi (\vec{r}_1 - \vec{r}_0)$, $\vec{x}_{20} = M_\pi (\vec{r}_2 - \vec{r}_0)$,

$\vec{x}_{12} = M_\pi (\vec{r}_1 - \vec{r}_2)$ and $\vec{\nabla}_{ij} \equiv \partial / \partial \vec{x}_{ij}$ and $x_{ij} \equiv |\vec{x}_{ij}|$.

All derivatives have to be evaluated as if the **variables were independent**.

Two-Pion exchange currents in configuration-space Ctd.

$$\rho_{c1}(\vec{r}_{10}, \vec{r}_{20}) = \rho_{c2}(\vec{r}_{10}, \vec{r}_{20}) = \rho_{c3}(\vec{r}_{10}, \vec{r}_{20}) = 0,$$

$$\rho_{c4}(\vec{r}_{10}, \vec{r}_{20}) = e \frac{g_A^2 M_\pi^7}{256\pi^2 F_\pi^4} \tau_1^3 \delta(\vec{x}_{20}) \left(\nabla_{10}^2 - 2 \right) \frac{e^{-2x_{10}}}{x_{10}^2} + (1 \leftrightarrow 2),$$

$$\rho_{c5}(\vec{r}_{10}, \vec{r}_{20}) = -e \frac{g_A^2 M_\pi^7}{256\pi^2 F_\pi^4} \tau_2^3 \delta(\vec{x}_{20}) \left(\nabla_{10}^2 - 2 \right) \frac{e^{-2x_{10}}}{x_{10}^2} + (1 \leftrightarrow 2),$$

$$\begin{aligned} \rho_{c6}(\vec{r}_{10}, \vec{r}_{20}) = & -e \frac{g_A^4 M_\pi^7}{256\pi^2 F_\pi^4} \delta(\vec{x}_{20}) \left[\tau_1^3 \left(2\nabla_{10}^2 - 4 \right) + \tau_2^3 \vec{\sigma}_1 \cdot \vec{\nabla}_{10} \vec{\sigma}_2 \cdot \vec{\nabla}_{10} - \tau_2^3 \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right] \frac{e^{-2x_{10}}}{x_{10}^2} \\ & - e \frac{g_A^4 M_\pi^7}{128\pi^2 F_\pi^4} \delta(\vec{x}_{20}) \tau_1^3 \left(3\nabla_{10}^2 - 11 \right) \frac{e^{-2x_{10}}}{x_{10}} + (1 \leftrightarrow 2), \end{aligned}$$

$$\begin{aligned} \rho_{c7}(\vec{r}_{10}, \vec{r}_{20}) = & -e \frac{g_A^4 M_\pi^7}{512\pi^3 F_\pi^4} \left[(\tau_1^3 + \tau_2^3) \left(\vec{\nabla}_{12} \cdot \vec{\nabla}_{10} \vec{\nabla}_{12} \cdot \vec{\nabla}_{20} + \vec{\nabla}_{12} \cdot \left[\vec{\nabla}_{10} \times \vec{\sigma}_1 \right] \vec{\nabla}_{12} \cdot \left[\vec{\nabla}_{20} \times \vec{\sigma}_2 \right] \right) \right. \\ & \left. + \left[\vec{\tau}_1 \times \vec{\tau}_2 \right]^3 \vec{\nabla}_{12} \cdot \vec{\nabla}_{10} \vec{\nabla}_{12} \cdot \left[\vec{\nabla}_{20} \times \vec{\sigma}_2 \right] \right] \frac{e^{-x_{10}}}{x_{10}} \frac{e^{-x_{20}}}{x_{20}} \frac{e^{-x_{12}}}{x_{12}} + (1 \leftrightarrow 2). \end{aligned}$$

- Results also available in momentum-space, expressed in standard loop-function $L(q)$, $A(q)$ and three-point functions [S.K. et al. '09](#).
- Can be easily treated numerically.
- **Continuity-equation is fulfilled** → Current is consistent with potential obtained within the method of unitary transformation

Additional transformations for the em current

- U is only the minimal unitary transformation, can choose **additional transformations** U'_{em}

$$U'_{\text{em}} = e^{S'}, \quad \text{with } S'(\mathcal{A}) \rightarrow 0 \quad \text{for } \mathcal{A} \rightarrow 0,$$

$$U'_{\text{em}} \quad \text{s.t. transformed Hamiltonian is block-diagonal}$$

$$\eta V_{\text{eff}} \eta \rightarrow \eta U'_{\text{em}} \eta \underbrace{U^\dagger H U \eta U'_{\text{em}} \eta - H_0}_{\text{contains } V_{1\pi}} \supset J_{\text{eff}},$$

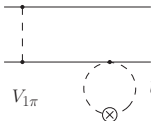


Diagram showing two energy levels. A dashed vertical line connects a lower level to an upper level, labeled $V_{1\pi}$. A dashed circle with a cross inside is on the lower level, labeled $U'_{\text{em}} \sim \beta_1$.

$$S'_1 = \beta_1 \eta \left[J_{02}^{-1} \frac{\lambda^2}{E_\pi^2} H_{22}^2 - H_{22}^2 \frac{\lambda^2}{E_\pi^2} J_{02}^{-1} \right] \eta,$$

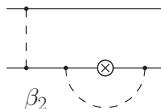


Diagram showing two energy levels. A dashed vertical line connects a lower level to an upper level, labeled $V_{1\pi}$. A dashed semi-circle with a cross inside is on the lower level, labeled β_2 .

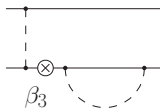


Diagram showing two energy levels. A dashed vertical line connects a lower level to an upper level, labeled $V_{1\pi}$. A dashed semi-circle with a cross inside is on the lower level, labeled β_3 .

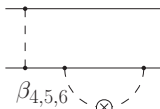
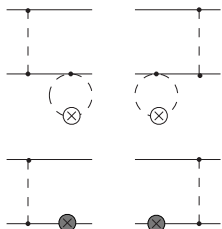


Diagram showing two energy levels. A dashed vertical line connects a lower level to an upper level, labeled $V_{1\pi}$. A dashed semi-circle with a cross inside is on the lower level, labeled $\beta_{4,5,6}$.

Determination of β_s

- Constraints by renormalizability!



- These diagrams receive contributions from S'_1 .

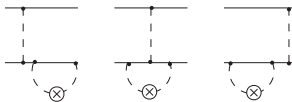
$$\vec{J}_{c5} = e \frac{g_A^2 i}{16F_\pi^4} (\beta_1 - 1) [\vec{\tau}_1 \times \vec{\tau}_2]^3 \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_2 \cdot \vec{q}_2}{q_2^2 + M_\pi^2}$$

$$\times \int \frac{d^3 l}{(2\pi)^3} \vec{l} \frac{\omega_+ - \omega_-}{\omega_+ \omega_- (\omega_+ + \omega_-)^2} + (1 \leftrightarrow 2)$$

$$\omega_\pm^2 = (\vec{l} \pm \vec{k})^2 + 4M_\pi^2, \quad \vec{k} = \text{Photon-momentum.}$$

- The divergent part of the diagrams has to be absorbed into LECs.
- The divergent part (β -functions) of the LECs is already known Gasser et al. '02.
- Contributions from LECs vanish in this case.
- Have to choose $\beta_1 = 1$ to guarantee renormalizability.

Determination of β s Ctd.

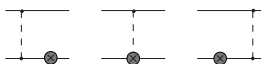


$$\vec{J}_{C7 \text{ div}} =$$

$$-e \frac{i g_A^4}{32 F_\pi^4 \pi^2} [\vec{\tau}_1 \times \vec{\tau}_2]^3 \frac{\vec{\sigma}_2 \cdot \vec{q}_2}{q_2^2 + M_\pi^2} \left[\vec{k} \times [\vec{q}_2 \times \vec{\sigma}_1] \right] \frac{1}{d-4}$$

$$+ e \frac{5i g_A^4}{192 F_\pi^4 \pi^2} [\vec{\tau}_1 \times \vec{\tau}_2]^3 \frac{\vec{\sigma}_2 \cdot \vec{q}_2}{q_2^2 + M_\pi^2} (\beta - 1) \vec{k} \vec{q}_2 \cdot \vec{\sigma}_1 \frac{1}{d-4} + (1 \leftrightarrow 2),$$

Where we choose $\beta_4 = \beta_5 = \beta_6 = \beta$



- Left and right diagram do not contribute.

- The diagram in the middle contributes among other things the following LECs

$$\vec{J} = -2e \frac{i g_A}{F_\pi^2} (d_8 \tau_2^3 + d_9 (\vec{\tau}_1 \cdot \vec{\tau}_2)) \frac{\vec{\sigma}_2 \cdot \vec{q}_2}{q_2^2 + M_\pi^2} \left[\vec{q}_2 \times \vec{k} \right]$$

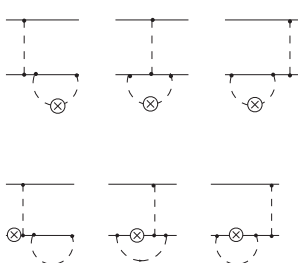
$$- e \frac{i g_A}{4 F_\pi^2} (2d_{21} + d_{22}) [\vec{\tau}_1 \times \vec{\tau}_2]^3 \frac{\vec{\sigma}_2 \cdot \vec{q}_2}{q_2^2 + M_\pi^2} \left[\vec{k} \times [\vec{q}_2 \times \vec{\sigma}_1] \right]$$

$$+ e \frac{i g_A d_{22}}{4 F_\pi^2} [\vec{\tau}_1 \times \vec{\tau}_2]^3 \frac{\vec{\sigma}_2 \cdot \vec{q}_2}{q_2^2 + M_\pi^2} \left(\vec{\sigma}_2 k^2 - \vec{q}_2 \vec{\sigma}_2 \cdot \vec{k} \right) + (1 \leftrightarrow 2)$$

We have to choose $\beta = 1!$

Determination of β s Ctd.

These diagrams have also a (*finite*) contribution to the charge density



$$\rho_{c7} = -e \frac{g_A^4}{8F_\pi^4} \frac{\vec{\sigma}_2 \cdot \vec{q}_2}{q_2^2 + M_\pi^2} \tau_2^3 \int d^3l \frac{1}{\omega_+^2 \omega_-^2} \left[\vec{\sigma}_1 \cdot \vec{l} \vec{l} \cdot \vec{q}_2 - \vec{\sigma}_1 \cdot \vec{k} \vec{k} \cdot \vec{\sigma}_2 \right] + (1 \leftrightarrow 2).$$

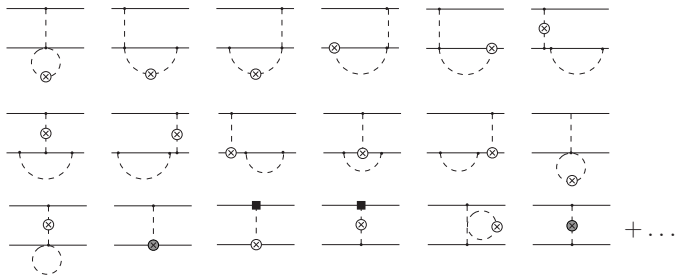
- Omitted most of the diagrams for brevity (in the figure).
- S'_2 and S'_3 could **potentially contribute, but do not.**

$$\rho_{c6} = e \frac{g_A^4}{4F_\pi^4} \tau_2^3 \frac{\vec{\sigma}_2 \cdot \vec{q}_2}{q_2^2 + M_\pi^2} \vec{\sigma}_1 \cdot \vec{q}_2 \frac{1}{3} \int \frac{d^3l}{(2\pi)^3} \frac{l^2}{\omega_l^4} + (1 \leftrightarrow 2).$$

Partial Summary

By choosing $\beta_1 = \beta_4 = \beta_5 = \beta_6 = 1$ we can get rid of divergencies. β_2 and β_3 remain undetermined.

Additional loop contributions Ctd



- Plus the contributions from δZ_π and $\delta(g_A/F_\pi)$.
- LECs d_{16} , l_3 and l_4 disappear, **only remaining term**

The remaining divergence can be absorbed in l_6 .

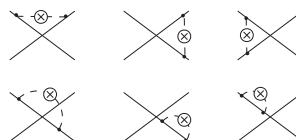
Summary OPE Ctd.

One-Pion Exchange current

- All divergencies can be canceled by additional unitary transformations
- or by LECs with predetermined β -functions
- Contributions from LECs d_8 , d_9 , d_{18} , d_{21} , d_{22} and l_6
- Continuity equation is fulfilled.


One-Pion exchange with LO contact potential

Diagrams with C_S and C_T



$$\begin{aligned} \vec{j} &= e \frac{g_A^2}{16F_\pi^2 \pi^2} C_T (\tau_1^3 - \tau_2^3) [(\vec{\sigma}_1 - \vec{\sigma}_2) \times \vec{k}] (2L(k) - 1), \\ \rho &= -e \frac{g_A^2}{32F_\pi^4 \pi} C_T (\tau_1^3 + \tau_2^3) \left[\vec{\sigma}_1 \cdot \vec{\sigma}_2 (3M_\pi + (3k^2 + 4M_\pi^2)A(k)) \right. \\ &\quad \left. + \vec{\sigma}_1 \cdot \vec{k} \vec{\sigma}_2 \cdot \vec{k} \left(\frac{M_\pi - (4M_\pi^2 + k^2)A(k)}{k^2} + 2A(k) \right) \right]. \end{aligned}$$

- Contributions from additional unitary transformations cancel!
- Divergent part can be absorbed in **contact currents**.



$$\rho = e \frac{g_A^2}{8F_\pi^4 \pi} C_T (\tau_1^3 + \tau_2^3) \vec{\sigma}_1 \cdot \vec{\sigma}_2 M_\pi.$$

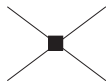
Only dependent on C_T .



→ These diagrams vanish.

Contact currents

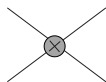
- Seven contact contributions to the potential.



$$V_{\text{contact}} = C_1 q^2 + C_2 k'^2 + (C_3 q^2 + C_4 k'^2) (\vec{\sigma}_1 \cdot \vec{\sigma}_2) + iC_5 \frac{\vec{\sigma}_1 + \vec{\sigma}_2}{2} \cdot [\vec{k}' \times \vec{q}] \\ + C_6 (\vec{q} \cdot \vec{\sigma}_1) (\vec{q} \cdot \vec{\sigma}_2) + C_7 (\vec{k}' \cdot \vec{\sigma}_1) (\vec{k}' \cdot \vec{\sigma}_2),$$

$$\vec{k}' = \frac{\vec{p} + \vec{p}'}{2}, \quad \vec{q} = \vec{p}' - \vec{p}.$$

- Via a gauge transformation and a Fierz-reshuffling, we obtain

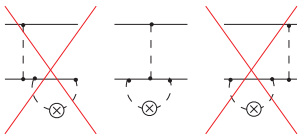


$$\vec{J}_{\text{contact}} = +i(C_2 + 3C_4 + C_7) \frac{e}{16} [\vec{\tau}_1 \times \vec{\tau}_2]^3 (\vec{q}_1 - \vec{q}_2) \\ - i(-C_2 + C_4 + C_7) \frac{e}{16} [\vec{\tau}_1 \times \vec{\tau}_2]^3 (\vec{q}_1 - \vec{q}_2) \vec{\sigma}_1 \cdot \vec{\sigma}_2 \\ - C_5 i \frac{e}{16} (\tau_1^3 - \tau_2^3) [(\vec{\sigma}_1 + \vec{\sigma}_2) \times (\vec{q}_1 - \vec{q}_2)] \\ + iC_7 \frac{e}{16} [\vec{\tau}_1 \times \vec{\tau}_2]^3 \left[\vec{\sigma}_1 \cdot \vec{\sigma}_2 \cdot (\vec{q}_1 - \vec{q}_2) + \vec{\sigma}_2 \cdot \vec{\sigma}_1 \cdot (\vec{q}_1 - \vec{q}_2) \right].$$

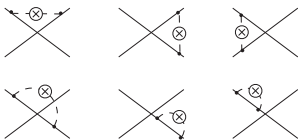
Plus two contact currents that cannot be obtained from gauge transformations

$$\vec{J}_{\text{contact}} = -ei\tilde{C}_1 [(\vec{\sigma}_1 + \vec{\sigma}_2) \times \vec{k}] - ei\tilde{C}_2 (\tau_1^3 - \tau_2^3) [(\vec{\sigma}_1 - \vec{\sigma}_2) \times \vec{k}].$$

Comparison with Pastore et al.



- Pastore et al. (2009) do not take into account these two diagrams.
- In our formalism, however, the contribution from the middle diagram is exactly canceled from the left and right diagrams.
- \Rightarrow Different isospin structure of the loop contributions!
- Different contributions from LECs?



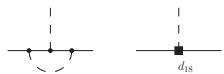
- Loops with leading-order contact interaction depend on C_5 .
- This is similar to the situation with the potential.
- Again a different isospin structure!
- Contact terms agree with ours.

Determining the LECs

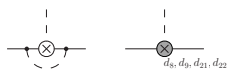
- In the NLO pion exchange current unknown 8 LECs appear:



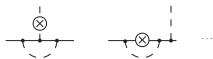
- l_6 is related to pion vector form factor \rightarrow **well known**.



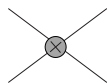
- d_{18} is related to Goldberger-Treiman discrepancy
 $g_{\pi N}/m_N = g_A/F_\pi(1 - 2M_\pi^2 d_{18}/g_A) \rightarrow$ **relatively well known**.



- d_8, d_9, d_{21} and d_{22} are related to pion-photoproduction on one nucleon, **poorly known**.



- Calculate **full pion-photoproduction** amplitude and fix these constants to data!



- \tilde{C}_1 can contribute to elastic ed -scattering, contribution to d -magnetic moment.

- \tilde{C}_2 from d -breakup reaction at threshold.

Conclusion and outlook

Conclusion

- We derived the **full NLO em-current** including two-pion exchange, one-pion exchange and contact terms.
- An explicit **check of renormalizability** of the one- and two-pion exchange contributions was performed.
- Expressions are given in momentum-space in terms of loop-functions $L(q)$, $A(q)$ and three-point functions.
- We analytically carried out the Fourier-transform to arrive at **very compact expressions in configuration-space**.
- The current fulfills the continuity-equation, i.e. is **consistent with the potential**.
- The two-pion exchange current corresponds to the result of Pastore et al.
- The one-loop current is different.

Outlook

- Calculation of pion-photoproduction off nucleons to determine LECs.
- Calculation of ed -scattering observables.
- Inclusion of Δ -degrees of freedom.
- Going to the sub-leading loop-order.