Muon capture on deuteron and ^{3}He

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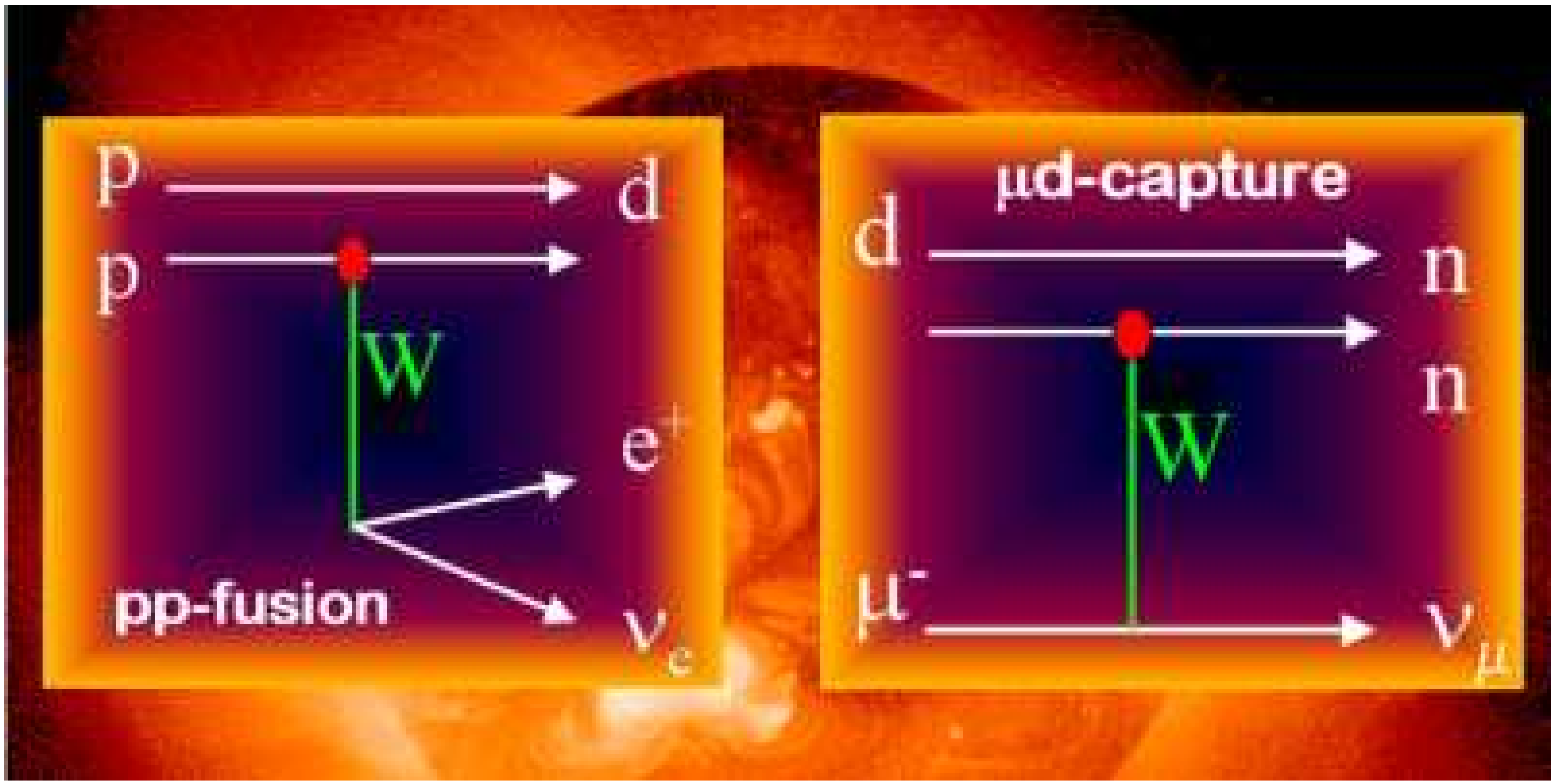
- $\mu^- + d \rightarrow n + n + \nu_\mu$
- $\mu^- + {}^3\mathrm{He} \rightarrow {}^3\mathrm{H} + \nu_\mu$ (70%)
- $\mu^- + {}^3\mathrm{He} \rightarrow n + d + \nu_\mu$ (20%)
- $\mu^- + {}^3\mathrm{He} \rightarrow n + n + p + \nu_\mu$ (10%)

Motivation



http://www.npl.illinois.edu/exp/musun/

Same “ingredients” !

Outline

- Experimental situation
- Present calculation: SNPA and χEFT*
- Results
- Comparison with previous theoretical calculations
- Conclusions and outlook

Experimental situation: $\mu^- + d$

Two hyperfine states: 1/2 and 3/2 $\Rightarrow \Gamma^D$ and Γ^Q

From theory: $\Gamma^D \simeq 400\ \mathrm{s}^{-1}$ $\Gamma^Q \simeq 10\ \mathrm{s}^{-1}$ $\Rightarrow$ only Γ^D

- Wang *et al.*, PR **139**, B1528 (1965): $\boxed{\Gamma^D = 365(96)\ \mathrm{s}^{-1}}$
- Bertini *et al.*, PRD **8**, 3774 (1973): $\boxed{\Gamma^D = 445(60)\ \mathrm{s}^{-1}}$
- Bardin *et al.*, NPA **453**, 591 (1986): $\boxed{\Gamma^D = 470(29)\ \mathrm{s}^{-1}}$
- Cargnelli *et al.*, Workshop on fundamental μ physics, Los Alamos, 1986, LA10714C: $\boxed{\Gamma^D = 409(40)\ \mathrm{s}^{-1}}$
- MuSun Collaboration: result to come!

Experimental situation: $\mu^- + {}^3\text{He} \rightarrow {}^3\text{H} + \nu_\mu$

Total capture rate: Γ_0

- Folomkin *et al.*, PL **3**, 229 (1963): Γ_0=1410(140) s^{-1}
- Auerbach *et al.*, PR **138**, B127 (1967): Γ_0=1505(46) s^{-1}
- Clay *et al.*, PR **140**, B587 (1965): Γ_0=1465(67) s^{-1}
- Ackerbauer *et al.*, PLB **417**, 224 (1998): Γ_0=1496(4) s^{-1}

Theoretical calculations: common "ingredients"

- Accurate nuclear wave functions
  - AV18(+UIX) or N3LO(+N2LO)
  - $A = 3 \to$ HH method
- Nuclear weak transition operators: $[\rho^{(A,V)}, \mathbf{j}^{(A,V)}]$
  - Standard Nuclear Physics Approach - SNPA
  - Hybrid Chiral Effective Field Theory Approach - χEFT*

SNPA and χEFT* used for $p + p \to d + e^+ + \nu_e$ and $p + {}^3\text{He} \to {}^4\text{He} + e^+ + \nu_e$ (*hep*)[1]

[1] Schiavilla *et al.*, PRC **58**, 1263 (1998); Marcucci *et al.*, PRC **63**, 015801 (2000); Park *et al.*, PRC **67**, 055206 (2003)

Nuclear transition operators: SNPA

- One-body operators: NRR of $j_i^\mu \to O(1/m^2)$
- Two-body $\rho^{(V)}$ and $\mathbf{j}^{(V)}$: CVC $\to$ EM operators (MEC+$\underline{\mathbf{j}^{(V)}(\Delta)}$)
- Two-body $\rho^{(A)}$: PCAC + low-energy theorem $\to$ π-exchange and short-range terms from v_c and v_{LS} of NN potential
- Two-body $\mathbf{j}^{(A)}$: π- and ρ-exchange, $\pi\rho$ mechanism, and $\underline{\mathbf{j}^{(A)}(\Delta)}$

Δ-isobar contributions to $\mathbf{j}^{(V)}$

Δ-isobar d.o.f. in the wave function:

- Full $N + \Delta$ coupled-channel calculation
- Perturbation theory (static Δ approximation)
- Transition-correlation-operator (TCO) method

	$\mu(^3\text{H})$	$\mu(^3\text{He})$
1b	2.5745	-1.7634
2b-Δ(PT)	3.0260	-2.2068
2b-Δ(TCO)	2.9337	-2.1079
Exp.	2.9790	-2.1276
1b+2b+3b	**2.9525**	**-2.1299**

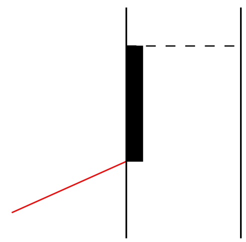
AV18/UIX

$\Rightarrow$ Also $\mathbf{j}_{ijk}^{(V)}$ from UIX [1]

[1] Marcucci *et al.*, PRC **72**, 014001 (2005)

Δ-isobar contributions to $\mathbf{j}^{(A)}$

Largest contribution from



with unknown coupling constant g_A^*

$\Rightarrow$ g_A^* fit to observable: GT_{exp} of tritium β-decay

Potential Model	Method	g_A^*
AV18/UIX	TCO	2.87[1]
	PT	1.17[1]
AV18/UIX	PT	1.21(9)

[1] Marcucci *et al.*, PRC **63**, 015801 (2000)

Nuclear transition operators: χEFT* (I)

χEFT $\rightarrow$ systematic chiral expansion in Q/Λ for Hamiltonian and electroweak current operator

hybrid χEFT $\equiv$ χEFT*

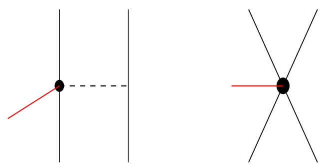
- AV18 w.fs. $\simeq \sum$red.+irred. diagrams up to ALL orders
- Transition operator obtained via systematic chiral expansion up to $O(Q^3) \equiv$ N^{3}LO
- If inconsistency significant $\rightarrow$ strong Λ-dependence: $\Lambda = 500 - 800$ MeV

Nuclear transition operators: χEFT* (II)

- One-body operators ≡ SNPA except for $\mathbf{j}_{1b}^{(V)} \to O(1/m^3)$ RC[1]
- Two-body $\rho^{(A)} \to$ soft π-exchange dominant
- Two-body $\rho^{(V)} = 0$ at N^{3}LO
- Two-body $\mathbf{j}^{(V)} \to$ CVC $\Rightarrow$ EM current: $1\pi \leftarrow pp\,\&\,hep$
  vertex corrections to 1π ($1\pi C$), 2π, CT[1]
  two LECs in CT (g_{4S} & g_{4V}) which cannot be determined from
  NN data $\Rightarrow$ from $\mu(A=3)$

[1] Song *et al.*, PRC **79**, 064002 (2009)

Nuclear transition operators: χEFT* (III)

- Two-body $\mathbf{j}^{(A)}$ 

one LEC (d_R) $\Rightarrow$ from GT_{exp} of tritium β-decay

AV18/UIX [N3LO/N2LO]

	$\Lambda = 500$ MeV	$\Lambda = 600$ MeV	$\Lambda = 800$ MeV
d_R	0.97(7)	1.75(8) [1.00(9)]	3.89(10)
g_{4S}	0.69(1)	0.55(1) [0.11(1)]	0.25(2)
g_{4V}	2.065(6)	0.793(6) [3.124(6)]	-1.07(1)

Results: $\mu^- + d$ (SNPA and χEFT*) (I)

SNPA(AV18)	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$	$^1D_2$	$^3F_2$	Total
g_A=1.2654(42)	246.6(7)	20.1	46.7	71.6	4.5	0.9	390.4(7)
g_A=1.2695(29)	246.8(5)	20.1	46.8	71.8	4.5	0.9	390.9(7)
χEFT*(AV18)	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$	$^1D_2$	$^3F_2$	Total
$\Lambda = 500$ MeV	250.0(8)	19.9	46.2	71.2	4.5	0.9	392.7(8)
$\Lambda = 600$ MeV	250.0(8)	19.8	46.3	71.1	4.5	0.9	392.6(8)
$\Lambda = 800$ MeV	249.7(7)	19.8	46.4	71.1	4.5	0.9	392.4(7)
χEFT*(N3LO)	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$	$^1D_2$	$^3F_2$	Total
$\Lambda = 600$ MeV	250.5(7)	19.9	46.4	71.5	4.4	0.9	393.6(7)

$\Rightarrow$ $\Gamma^D = 392(2)\ \mathrm{s}^{-1}$

Previous theoretical calculations for $\mu^- + d$

$\simeq$ 1990: Kubodera/Japan group and Truhlik/Prague group

however

old potentials (RSC, Bonn B, Paris), MEC not fitted to GT_{exp} $\Rightarrow$
differences respect to *pp* and *hep* calculations

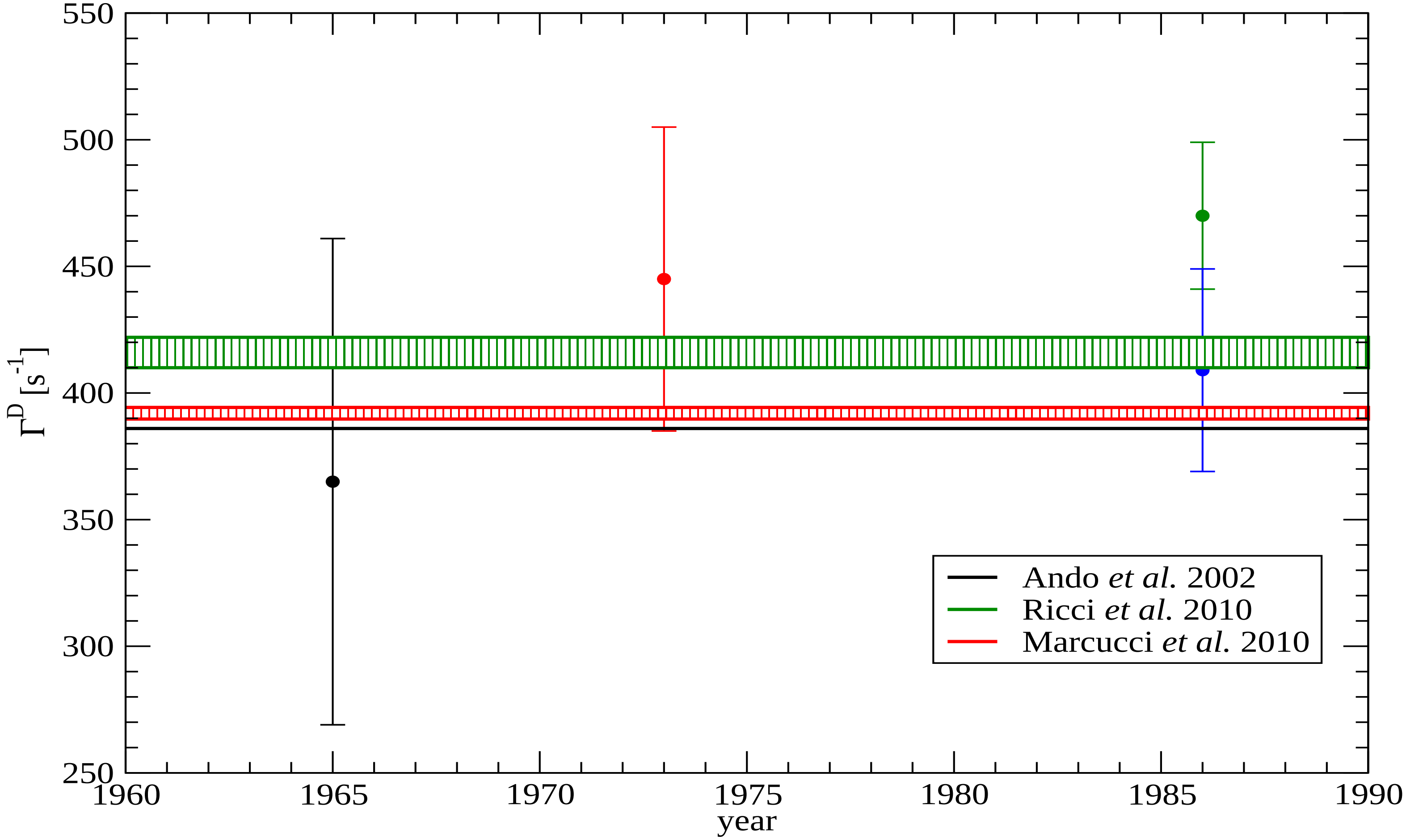
Ando *et al.*, PLB **533**, 25 (2002) [χEFT*, AV18, $\Lambda = 500 - 800$ MeV]

- only $^1S_0$ capture & no "new" $\mathbf{j}^{(V)}$ $\Rightarrow$ $\Gamma^D(^1S_0)= 245(1)$ s^{-1}
- $L \geq 1$ contributions taken from Tatara *et al.*, PRC **42**, 1694 (1990):
  $\Gamma^D(L \geq 1) = 141$ s^{-1} $\Rightarrow$ $\Gamma^D = 386$ s^{-1}

Ricci *et al.*, NPA **837**, 110 (2010) [$\sim$SNPA, NJI and NJ93]

- $\Gamma^D(^1S_0)= 254(3)$ s^{-1}
- $\Gamma^D = 416(6)$ s^{-1} with NJI
- large model dependence ~ 7 s^{-1}

Results: $\mu^- + d$ (SNPA and χEFT*) (II)



Results: $\mu^- +^3$ He (SNPA and χEFT*) (I)

SNPA(AV18/UIX)	Γ_0
g_A=1.2654(42)	1486(8)
g_A=1.2695(29)	1486(5)
χEFT*(AV18/UIX)	Γ_0
Λ = 500 MeV	1487(8)
Λ = 600 MeV	1488(9)
Λ = 800 MeV	1488(8)
χEFT*(N3LO/N2LO; Λ=600 MeV)	1480(9)

$\Rightarrow$ Γ_0= 1484(13) s^{-1}

vs. Γ_0(exp)=1496(4) s^{-1}

Previous theoretical calculations: $\mu^- + {}^3\text{He} \rightarrow {}^3\text{H} + \nu_\mu$

$\Gamma_0 = 1484(13)\ \text{s}^{-1}$

Marcucci *et al.*, PRC **66**, 054003 (2002) [SNPA, AV18/UIX & AV14/TM]

Gazit, PLB **666**, 472 (2008) → [χEFT*, AV18/UIX (EIHH)]

	AV18	AV14	AV18/UIX	AV14/TM
Marcucci *et al.* (2002)	1441(7)	1444(7)	1484(8)	1486(8)
Gazit (2008)			1499(16)	

Note:

1. Marcucci *et al.* (2002) → $\mathbf{j}^{(V)}(\Delta)$ in PT
2. agreement with Gazit (2008), but "new" $\mathbf{j}^{(V)}$?

Conclusions

- Theoretical study of $\mu^- + d \to n + n + \nu_\mu$ and $\mu^- + {}^3\text{He} \to {}^3\text{H} + \nu_\mu$
- Realistic Hamiltonians (AV18 or N3LO + UIX or N2LO)
- SNPA and χEFT* nuclear currents
- $\Gamma^D = 392(2)\ \text{s}^{-1}$ and $\Gamma_0 = 1484(13)\ \text{s}^{-1}$
- Agreement with experiment (but for $\mu^- + d$ needed more accurate results → MuSun!)

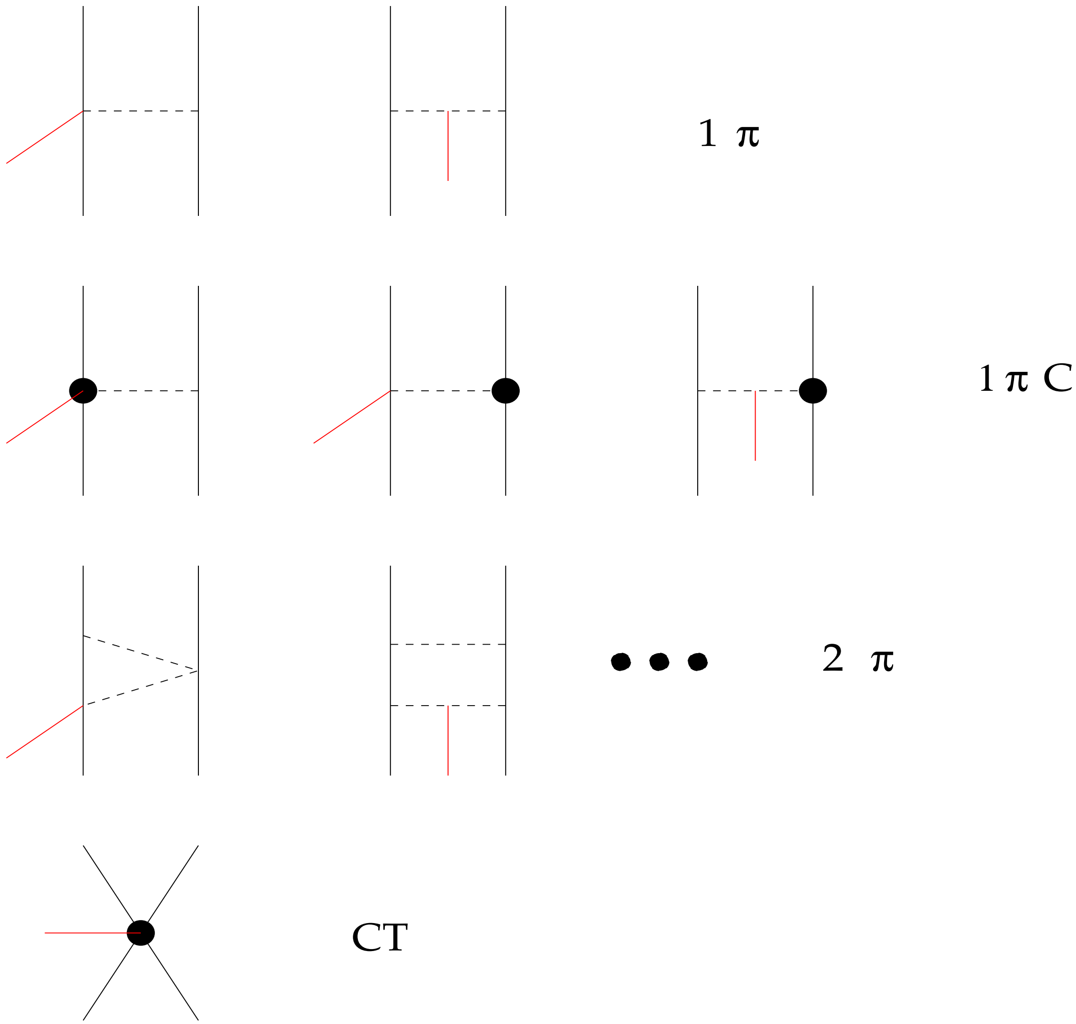
Outlook

- Same calculation for $\mu^- + {}^3\mathrm{He} \rightarrow n + d + \nu_\mu$[1] and $\mu^- + {}^4\mathrm{He} \rightarrow n + {}^3\mathrm{He} + \nu_\mu$
- Repeat the calculation within a non-hybrid χEFT approach: see for instance L. Girlanda's talk

[1] Kuhn *et al.*, PRC **50**, 1771 (1994); Skibiński *et al.*, PRC **59**, 2384 (1999)

EXTRA SLIDES

Nuclear transition operators: χEFT*



Results: $\mu^- + d$ (SNPA and χEFT*) (III)

AV18

SNPA

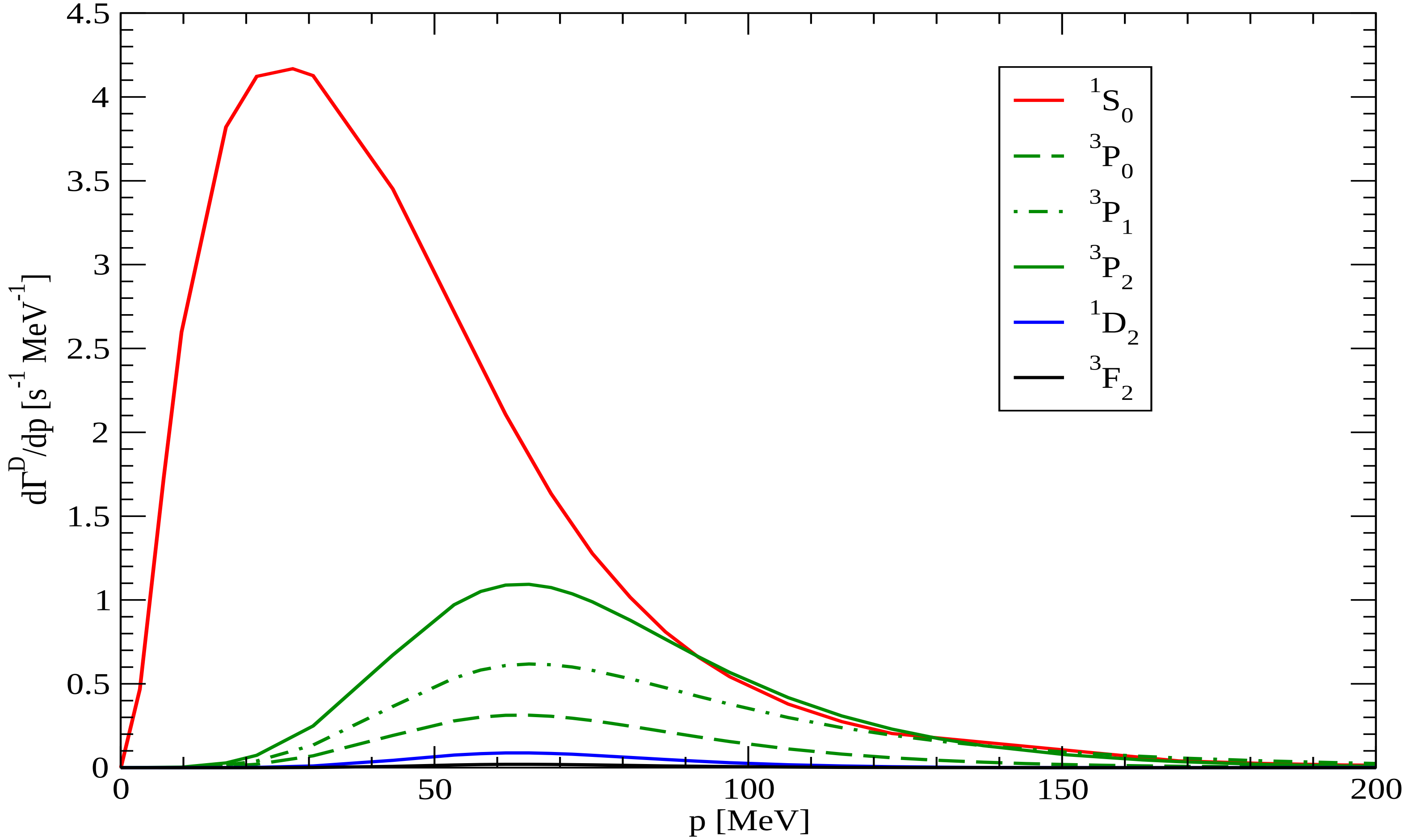
	$^1S_0$	$^3P_2$
IA+RC	293.0	82.6
PS	233.2	71.6
mesonic	237.5	71.4
$\Delta_{PT}(A$-w/o PS$)+\Delta_{PT}(V)$	248.3(8)	71.6
$\Delta_{PT}(A$-w/o PS$)+\Delta_{TCO}(V)$	247.7(8)	71.6
$\Delta_{PT}(A$-w PS$)+\Delta_{TCO}(V)$	246.7(7)	71.6

χEFT*

Λ=600 MeV	$^1S_0$	$^3P_2$
IA+RC	292.5	82.5
PS	232.7	71.6
mesonic	250.0(8)	71.1

Note:

1. IA+RC: SNPA vs. χEFT*
2. Ando *et al.* (2002): $\Gamma^D(^1S_0)$=245(1) s^{-1}
   $\rightarrow$ "new" $j^{(V)}$


dΓ^D/dp [s^-1 MeV^-1]
p [MeV]
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
0
50
100
150
200
1S0
3P0
3P1
3P2
1D2
3F2

$\mu^- + {}^3\text{He} \to {}^3\text{H} + \nu_\mu$ and the PS axial coupling

$$j^{A\mu} = \overline{u}(\mathbf{p}')\gamma_5\left[G_A(q^2)\gamma^\mu + \frac{G_{PS}(q^2)}{m_\mu}q^\mu\right]\tau^\pm u(\mathbf{p})$$

$$\text{NRR} \to \mathbf{j}_i^A = -G_A(q^2)\sigma_i e^{i\mathbf{q}\cdot\mathbf{r}_i}\tau_{i,\pm} + \text{RC}(1/m^2) - \frac{G_{PS}(q^2)}{2mm_\mu}\mathbf{q}\sigma_i\cdot\mathbf{q}e^{i\mathbf{q}\cdot\mathbf{r}_i}\tau_{i,\pm}$$

$$G_A(q^2) = \frac{g_A}{(1+q^2/\Lambda_A^2)^2} \qquad \Lambda_A \simeq 1\,\text{GeV}$$

$$G_{PS}(q^2) = -\frac{2m_\mu m}{m_\pi^2+q^2}G_A(q^2) \qquad \text{PCAC and GT relation}$$

$$R_{PS} \equiv \frac{G_{PS}}{G_{PS}^{PCAC}} \Rightarrow R_{PS}^{\text{SNPA}} = 0.94 \pm 0.06$$

$$R_{PS}^{\text{EFT}^*} = 0.98 \pm 0.08$$

Results: $\mu^- + ^3$He (SNPA and χEFT*) (II)

AV18/UIX

	Γ_0	$L_1(A)$	$M_1(V)$
IA+RC	1530	0.4056	0.1127
PS	1316	0.2589	
mesonic	1385	0.2619	0.1315
$\Delta_{PT}(A$-w/o PS)+$\Delta_{PT}(V)$	1501	0.2811	0.1370
$\Delta_{PT}(A$-w/o PS)+$\Delta_{TCO}(V)$	1493		0.1376
$\Delta_{PT}(A$-w PS)+$\Delta_{TCO}(V)$	1486	0.2742	

Λ=600 MeV	Γ_0	$L_1(A)$	$M_1(V)$
IA+RC	1517	0.4056	0.1082
PS	1303	0.2589	
mesonic	1488	0.2810	0.1317