Nuclear effects on the determination of neutrino oscillation parameters

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Special thanks to Maria Barbaro, Omar Benhar, Juan Antonio Caballero, Enrique Fernandez Martinez and Jose Udias

Main motivation of this work

comparing Fermi gas (FG) and advanced nuclear model predictions for physically interesting neutrino observables

- this is relevant because many MonteCarlo codes, used to study the sensitivity to still *unknown* parameters at future ν facilities are based on FG
- impossible to discuss all recent nuclear models
- focus the attention on two different approaches

ν FLAVOUR CONVERSION has been confirmed in many experiments

$$U = R_{23}(\theta_{23}) R_{13}(\theta_{13}, \delta) R_{12}(\theta_{12})$$

The neutrino oscillation probability (in matter)

$$P_{\alpha\beta} = |A_{\alpha\beta}|^2 = \sum_{i,j} \tilde{U}^*_{\alpha i} \tilde{U}_{\beta i} \tilde{U}_{\alpha j} \tilde{U}^*_{\beta j} \exp\left(i \frac{\tilde{m}_j^2 - \tilde{m}_i^2}{2E} L \right)$$

E is the neutrino energy, L is the baseline length, $\tilde{m}_i$ and $\tilde{U}_{\beta j}$ are the mass of the ith neutrino mass eigenstate and the mixing matrix in matter

- Usual assumption: U is a 3×3 unitary mixing matrix
- three angles θ_{ij} and one CP phase δ

$\Downarrow$

the standard framework implies 7 parameters to describe ν oscillation in matter

Global 3 ν fit to the world neutrino data

At 1σ (3σ) — M. C. Gonzalez-Garcia and M. Maltoni, Phys. Rept. **460**, 1 (2008)

- well known parameters

$$\Delta m^2_{21} = 7.67\,^{+0.22}_{-0.21}\,\left({}^{+0.67}_{-0.61}\right) \times 10^{-5}\ eV^2\,,$$

$$\Delta m^2_{31} = \begin{cases} -2.37 \pm 0.15\,\left({}^{+0.43}_{-0.46}\right) \times 10^{-3}\ eV^2 & \text{(inverted hierarchy)}\,, \\ +2.46 \pm 0.15\,\left({}^{+0.47}_{-0.42}\right) \times 10^{-3}\ eV^2 & \text{(normal hierarchy)}\,, \end{cases}$$

$$\theta_{12} = 34.5 \pm 1.4\,\left({}^{+4.8}_{-4.0}\right)\,,$$

$$\theta_{23} = 42.3\,^{+5.1}_{-3.3}\,\left({}^{+11.3}_{-7.7}\right)\,,$$

- poor and unknown parameters

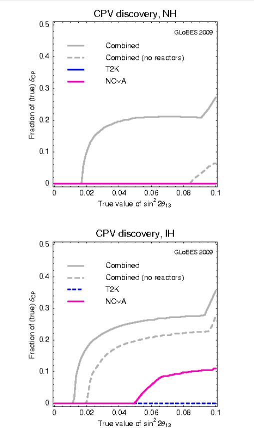
$$\theta_{13} = 0.0\,^{+7.9}_{-0.0}\,\left({}^{+12.9}_{-0.0}\right) \qquad \text{recent claim}: \sin^2\theta_{13} = 0.016 \pm 0.01 \text{at } 1\,\sigma$$

$$\delta_{\mathrm{CP}} \in [0,\ 360]\ (\text{unknown}) \qquad \text{G. L. Fogli et at., arXiv}: 0806.2649$$

$$\mathrm{sign}(\Delta m^2_{31}) \qquad \text{octant of } \theta_{23} \qquad \text{Majorana or Dirac Neutrinos?}$$

Great interest on θ_{13} and δ

some hints at incoming experiments?



modified from **P. Huber et al. JHEP 0911:044,2009**

Many future experiments will look for a precise measurement of θ_{13}.
In the standard parametrization, large θ_{13} means good chance to reveal the CP violation in the leptonic sector

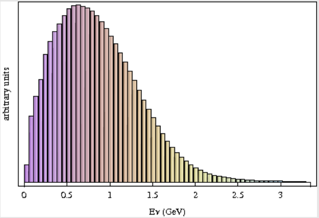
One needs to control:

- flux composition
- detector response
- nuclear cross sections

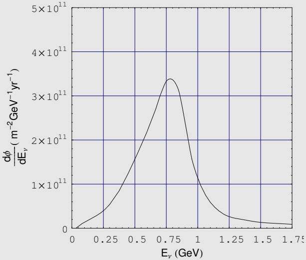
The importance of cross sections in the QE region

many current and planned experiments use a ν flux at energies $\lesssim$ 1 GeV

MiniBoone



T2K-I



and many others (NOνA, high γ β-beams...)

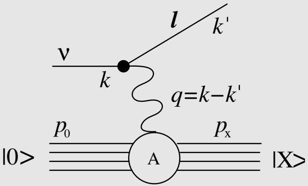
- very few neutrino scattering data

important to estimate precisely the ν-nucleus cross sections in the QE region

The QE region

- at low energies ($E_\nu \leq 0.6 - 0.7$ GeV): the dominant contribution comes from **quasi-elastic** scattering;
- at higher energies: **inelastic production** of charged leptons (via resonance excitation) + inelastic production of π^0 also contribute
- negligible deep inelastic scattering contribution at $\mathcal{O}(1)$ GeV

formalism to describe inclusive $\nu + A \to l + X$ reaction



$$\frac{d^2\sigma}{d\Omega dE_l} = \frac{G_F^2\,|V_{ud}|^2}{16\,\pi^2}\,\frac{|\mathbf{k}'|}{|\mathbf{k}|}\,L_{\mu\nu}W_A^{\mu\nu}$$

$$L^{\mu\nu} = 8\left[k'_\mu\,k_\nu + k'_\nu\,k_\mu - (k\cdot k') - i\,\varepsilon_{\mu\nu\alpha\beta}\,k'^{\beta}\,k^\alpha\right]$$

$$W_A^{\mu\nu} = \sum_X \langle 0|J_A^{\mu\,\dagger}|X\rangle\langle X|J_A^\nu|0\rangle\delta^{(4)}(p_0 + q - p_X)$$

The Impulse Approximation

the problem is the calculation of the hadronic tensor $W_A^{\mu\nu}$

- for $|\mathbf{q}| < 0.5\,\mathrm{GeV}$ NMBT + nonrelativistic wave functions + expansion of the current operator in powers of $|\mathbf{q}|/m_N$ **Carlson&Schiavilla, Rev. Mod. Phys. 70, 743 (1998)**
- for larger $|\mathbf{q}|$ (the energy regime we are interested in) we can no longer describe the final states $|X\rangle$ in terms of nonrelativistic nucleons

$$\Downarrow$$

we need a set of simplifying assumptions to describe relativistic motion of final state particles and the occurrence of inelastic processes

the Impulse Approximation

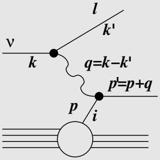
target nucleus seen as a collection of individual nucleons
$$J_\mu \to \sum_i j^i_\mu$$

but see the Ankowsky's talk and Ankowsky et al., 1001.0481

scattered nucleons and recoiling system $\mathcal{R}$ evolve independently of one another
$$|X\rangle \to |i, p^{'}\rangle \otimes |\mathcal{R}, p_{\mathcal{R}}\rangle$$
(no Final State Interactions)

The Spectral Function Approach

Benhar et al., Phys.Rev.D72:053005,2005

$\sigma \sim \sum_i \Big|$  $\Big|^2$

$$\frac{d^2\sigma_{IA}}{d\Omega dE_l} = \int d^3p\, dE\, P(\mathbf{p},E)\, \frac{d^2\sigma_{\rm elem}}{d\Omega dE_l}$$

$$\frac{d^2\sigma_{\rm elem}}{d\Omega dE_l} = \frac{G_F^2\, V_{ud}^2}{32\,\pi^2}\, \frac{|k'|}{|k|}\, \frac{1}{4\, E_{\mathbf{p}}\, E_{|\mathbf{p+q}|}}\, L_{\mu\nu} W_A^{\mu\nu}$$

$$W_A^{\mu\nu} = \frac{1}{2} \int d^3p\, dE\, P(\mathbf{p},E) \frac{1}{4\, E_{\mathbf{p}}\, E_{|\mathbf{p+q}|}}\, W^{\mu\nu}(\tilde{p},\tilde{q})$$

- $P(\mathbf{p},E)$ is the target spectral function: probability distribution of finding a nucleon with momentum $\mathbf{p}$ and removal energy E in the target nucleus

it encodes all the informations about the initial struck particle

The Spectral Function

A. Ramos, A. Polls, W. H. Dickhoff, Nucl., Phys. A503, (1989) 1
O. Benhar, A. Fabrocini, S. Fantoni, Nucl., Phys. A505, (1989) 267
O. Benhar, A. Fabrocini, S. Fantoni and I. Sick, Nucl., Phys. A579, (1994) 493

- the calculation of $P(\mathbf{p}, E)$ for any A is a complicated task
- for nuclei from Carbon to Gold has been modeled using the Local Densitiy Approximation (LDA)

$$P_{LDA}(\mathbf{p}, E) = P_{MF}(\mathbf{p}, E) + P_{corr}(\mathbf{p}, E)$$

measured contribution corresponding to low momentum nucleons, occupying the shell model states

high momentum nucleons calculable using the result of uniform nuclear matter "recomputed" for a finite nucleus of mass number A

The Relativistic Mean Field approximation

already introduced in the M.B. Barbaro's talk
model based on
J. M. Udias et al.,Phys. Rev. C 64 , 024614 (2001);
C. Maieron et al., Phys. Rev. C 68 , 048501 (2003);
M. C. Martinezet al., Phys. Rev. C 73 , 024607 (2006)

Still using the impulse approximation

- The nuclear current is obtained as a sum over individual single-nucleon currents

$$J^\mu_N(\nu,\vec{q}) = \int d\vec{p}\,\bar{\psi}_F(\vec{p}+\vec{q})\hat{J}^\mu_N(\nu,\vec{q})\psi_B(\vec{p})$$

ψ_B = wave function for initial bound nucleons

ψ_F = wave function for final bound nucleons

$\hat{J}^\mu_N(\nu,\vec{q})$ = relativistic nucleon current operator = $F_1(Q^2)\gamma_\mu + i\frac{k}{2m}F_2(Q^2)\sigma_{\mu\nu}q^\nu + \dots$

- Matrix elements can be computed having the wave functions of the initial and the final nucleons (besides form factors)

The Relativistic Mean Field approximation

- both bound and scattered nucleons feel the same 'potentials' which represent the nuclear medium;
- these potential are computed from lagrangians describing interactions among nucleons via boson exchange (σ, ω);
- being a relativistic model, ψ_B and ψ_F are solutions of Dirac-like equations

$$\Downarrow$$

solving Dirac-like equations with scalar-vector (S-V) potentials:

$$\begin{aligned} \tilde{E}\gamma_0 - \vec{p}\cdot\vec{\gamma} - \tilde{M} &= 0 \\ \tilde{E} &= E - V(r) \\ \tilde{M} &= M - S(r) \end{aligned}$$

The Relativistic Fermi Gas Model

- many MonteCarlo codes (GENIE, NuWro, Neut, Nuance) use some version of the Fermi model
  - target nucleons are moving (Fermi motion) subject to a nuclear potential (binding energy)
  - the ejected nucleon does not interact with other nucleons (Plane Wave Impulse Approximation)
  - Pauli blocking reduces the available phase space for scattered particle
- in terms of Spectral Function:

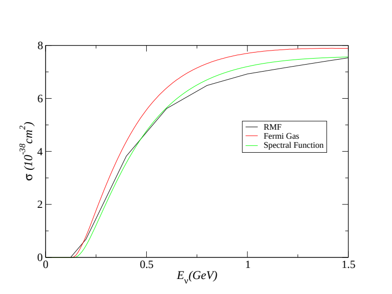
$$P_{RFGM} = \left(\frac{6\pi^2 A}{p_F^3}\right)\theta(p_F - \vec{p})\delta(E_{\vec{p}} - E_B + E)$$

where

$$\begin{aligned} p_F &= \text{Fermi momentum} && (225 \text{ MeV for Oxygen}) \\ E_B &= \text{average binding energy} && (25 \text{ MeV for Oxygen}) \\ E &= \text{removal energy} \end{aligned}$$

The ν-nucleus cross sections ($\nu A \to \mu X$)

- some of the *qualitative* impacts of several nuclear models on the ν observables can already be understood at the "cross section" level
- however the *quantitative* differences should be carefully evaluated



- as expected, FG overstimates the xsection over the whole QE energy regime
- $m_A \sim 1$ GeV in any of the models
- dipole form factors
- same pattern for $\bar{\nu}$

The βBeam concept

- Concept introduced by Zucchelli, **Phys.Lett.B532:166-172,2002**
- it involves producing a beam of β-unstable heavy ions (i.e.,^{6}He and ^{18}Ne), accelerating them to some reference energy, and allowing them to decay in the straight section of a storage ring, resulting in a very intense ν_e neutrino beam

$\Downarrow$

  - *pure* ν fluxes (e.g., only one neutrino species, in contrast to a conventional super-beam where contamination of other neutrino species is inevitable)
  - *systematics free*, since the spectrum can be calculated exactly (again, in contrast with a conventional beam, where knowledge of the spectrum always involves a sizable systematic uncertainty).

in the ion rest-frame:

$$\frac{dN^{\rm rest}}{d\cos\theta dE_\nu} \sim E_\nu^2(E_0 - E_\nu)\sqrt{(E_\nu - E_0)^2 - m_e^2}$$

in the laboratory frame:

$$\left.\frac{d\Phi^{\rm lab}}{dSdy}\right|_{\theta\simeq 0} \simeq \frac{N_\beta}{\pi L^2}\frac{\gamma^2}{g(y_e)}y^2(1-y)\sqrt{(1-y)^2 - y_e^2}$$

The βBeam concept

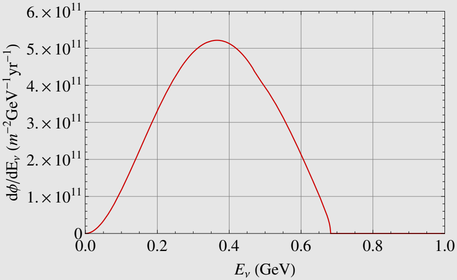
- the value of the Lorentz boost factor γ and the source-detector distance L determine the neutrino spectra
- interested in $\nu_e \longrightarrow \nu_\mu$ oscillation
- leading terms in $P_{\nu_e \nu_\mu}$ depend on θ_{13}^2 and $\theta_{13} \cdot \sin\delta$

- here we focus on $(\gamma, L) = (100, 732\,Km)$

-(ν-$\bar{\nu}$) spectra very similar

- QE events

- very low backgrounds



- warning: working in the region where IA starts to be inadequate
  use this βBeam as a prototype!

The CP discovery potential

Definition

for any θ_{13} is the ensemble of true values of δ_{CP} for which the 3σ CL do not touch $\delta_{CP} = 0, \pi, \pm\pi$

the precision measurement should be enough to establish $\delta_{CP} \neq 0, \pi, \pm\pi$

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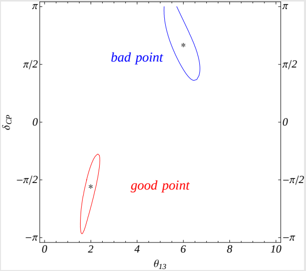
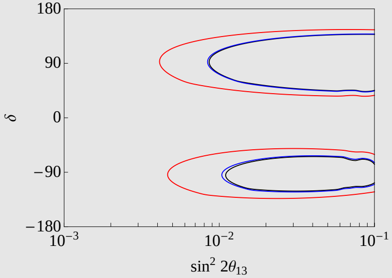


The CP discovery potential

- We simulate at the same time $\nu_e \to \nu_\mu$ and the CP-conjugate channel and compute event rates (μ in the final state) after interaction with Oxygen
- “Points“ inside the curves represent values of δ_{CP} for which leptonic CP violation can be established at 3σ CL



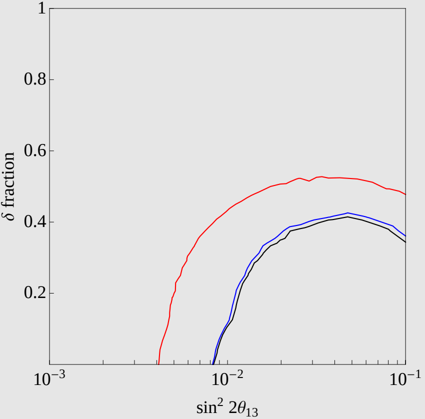
- RED: Fermi Gas
- BLACK: Spectral Function
- BLUE: RMF

1- the FG performs *too* well compared with the other two models

2- at $\delta \sim \pm\pi/2$ the largest discrepancy: 25-30% better!

The CP discovery potential

More evident if we compute the fraction of *good* δ's over the total



- RED: Fermi Gas
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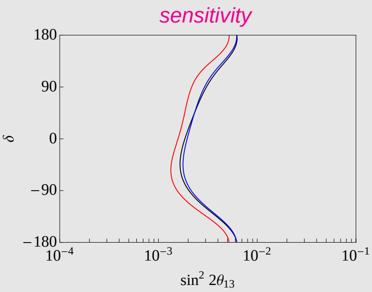
- BLUE: RMF

The sensitivity to θ_{13}

- same analysis for θ_{13}

Definition

for any δ_{CP} is the ensemble of true values of θ_{13} for which the 3σ CL do not touch $\theta_{13} = 0$



- RED: Fermi Gas
- BLACK: Spectral Function
- BLUE: RMF

- a bit less evident than before: something of $\mathcal{O}(10)\%$

A combined analysis

What about a simultaneous fit to θ_{13} and δ_{CP} ?

To see the impact of various models:

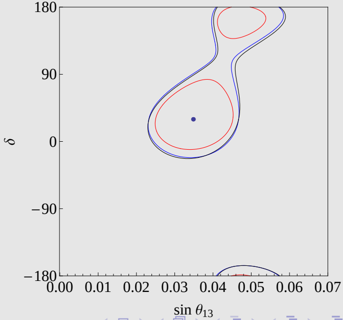
- we first fix some *true* value $(\theta_{13}, \delta_{CP}) = (3^o, 30^o)$
- then we study the capability of the facility to *measure* them

RED: Fermi Gas

BLACK: Spectral Function

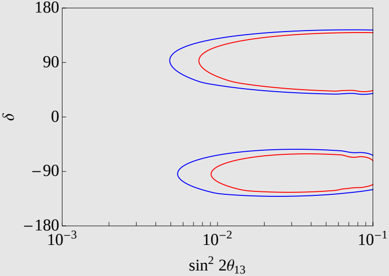
BLUE: RMF

- much better precision at 3σCL for FG

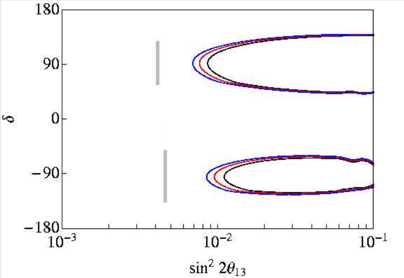


Generalizing the previous results

- same effects with ^{56}Fe target
  blue: FG, red: SF



- mild dependence on the axial mass
  SF with blue: m_A=1.2 GeV, red: m_A=1.1 GeV
  black: m_A=1.0 GeV



Summary and conclusions

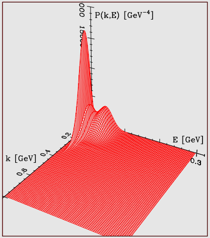
- *Summary*
  - We studied the impact of nuclear effects on the determination of various neutrino parameters
  - In particular, we compare the FG results (widely adopted in MonteCarlo codes) with the SF and RMF approaches
  - The different behaviour of the cross sections translates into overestimated sensitivity to θ_{13} and δ_{CP}
  - Although we focused on Oxygen, the same pattern is observed for other nuclear targets
- *Conclusions*
  - It could be necessary to implement more realistic nuclear effects in MC codes
  - It is also necessary to study the DIS region, where the future Neutrino Factories will work

Benhar et al.,Nucl. Phys. A 579 (1994) 493

Phys. Rev D72 (2005) 053005

- overwhelming evidence from electron scattering that the energy-momentum distribution of nucleons in the nucleus is quite different from that predicted by Fermi gas
- the most important feature is the presence of strong nucleon-nucleon (NN) correlations (virtual scattering processes leading to the excitation of the participating nucleons to states of energy larger than the Fermi energy)

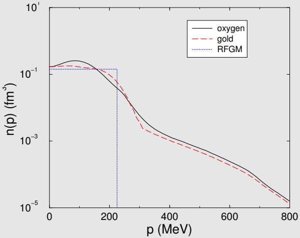
spectral function extends to $|\mathbf{p}| \gg p_F$ *and* $E \gg \varepsilon$



momentum distribution

$$n(\mathbf{p}) = \int dE \; P(\mathbf{p}, E)$$

$\Longrightarrow$



$$\frac{d^2\sigma_{\rm elem}}{d\Omega dE_l} = \frac{G_F^2\, V_{ud}^2}{32\,\pi^2}\,\frac{|k^{'}|}{|k|}\,\frac{1}{4\,E_{\mathbf{p}}\,E_{|\mathbf{p}+\mathbf{q}|}}\,L_{\mu\nu}W^{\mu\nu}$$

- The hadronic tensor is decomposed in structure functions as usual

$$\begin{aligned} W^{\mu\nu} \quad &= \quad -g^{\mu\nu}\,W_1 + \tilde{p}^{\mu}\,\tilde{p}^{\nu}\,\frac{W_2}{m_N^2} + i\,\varepsilon_{\mu\nu\alpha\beta}\,\tilde{q}^{\alpha}\,\tilde{p}^{\beta}\,\frac{W_3}{m_N^2} + \tilde{q}^{\mu}\,\tilde{q}^{\nu}\,\frac{W_4}{m_N^2} + \\ &\quad (\tilde{p}^{\mu}\,\tilde{q}^{\nu} + \tilde{p}^{\nu}\,\tilde{q}^{\mu})\,\frac{W_5}{m_N^2} \end{aligned}$$

- the formalism can be applied to **both** elastic and anelastic processes specifying the form of the structure functions W_i